Review 2 (reviewer comments in italic)

In this study, Zhang et al. present a detailed and thorough analysis of the relation between local ice-shelf buttressing numbers, how they are affected by local perturbations and how they relate to the flux response at the grounding line induced by such a local perturbation. They find that correlations between the flux response (see Reese et al., 2018) and the local buttressing numbers (see Fürst et al., 2016) can be found in very specific cases, but break down for more complicated geometries and when considering regions close to the grounding line. In a second step, they show how the adjoint method could be used to assess the sensitivity of grounding line flux to local perturbations which is shown to be consistent with the computationally more intense perturbation approach (except at the grounding line).

This study presents very interesting results that will help to advance the understanding of ice-shelf buttressing significantly. However, I think that some points should be addressed before it could be published.

We thank the reviewer for all of the helpful comments below.

<u>*Major comments*</u>

Your manuscript would be much easier to read if the central questions and related main findings of your paper were formulated more clearly. This shows for example in your abstract, where you state that you search for causal connections between sub-shelf melting, buttressing and grounding line flux. However, there is no clear answer to that, you rather switch to presenting an alterative method to calculate the grounding line flux sensitivity in the second part of the abstract. This is also reflected, for example, in the formulation of the research questions on page 2, line 47-52.

We have modified the abstract to remove the focus on understanding the causal connections between melt perturbations, buttressing, and grounding line flux. The research questions in the introduction have also been slightly modified (also in response to reviewer no. 1). As discussed further below, we have also significantly updated and improved the content and writing relative to our initial submission.

Section: 4.5: You state that the differences between the adjoint approach and the perturbation approach near the grounding line arise from 'nonlinearities' - more clarification is required at this point. In particular, Figures 13 and 14 show that the adjoint-sensitivites are negative along the grounding line, while Figure 15 indicates in general positive sensitivities in the perturbation approach. I think that the treatment of the grounding line in the sensitivity assessment could be key in explaining these differences.

We interpret the large nonlinearities close to the grounding line as a consequence of 1) relatively large thickness gradients and the rapid transition from vertical-shear-dominated to membrane-stress-dominated flow close to the grounding line, and 2) the changes of grounding line position and local geometry due to the change of thickness in the cells adjacent to the grounding line. This is now more clearly stated in the manuscript.

In terms of the difference between Figs. 13, 14 versus Fig. 15 (11, 12, and 13 in the revised version), the analysis in current Fig. 13 focuses on a small fraction of the total number of perturbation points -- those near the grounding line -- whereas current Figs. 11, 12 include those same points plus many more points (all in the case of MISMIP+) on the ice shelf proper. Thus, there are relatively more negative values obvious in current Fig. 13.

So please explain (1) how you specify the grounding line position in your experiments / model and how grounding line flux is calculated (can the grounding line move in your perturbation experiments?),

We have added a new section, 2.1.1, in which we detail the computation of the GL and the GLF. We also note that a perturbation to the ice thickness at a (triangular) grid cell intersecting the GL will affect both the GL position and length (because the thickness affects the flotation condition, which in turn can affect the position of the GL).

(2) if these differences arise only for cells directly adjacent to the grounding line, and

We observe negative sensitivities only for cells intersecting the grounding line. This can also happen for the perturbation-based sensitivity approach, as shown in Figure 2a (for the Larsen C case, the perturbation points are randomly chosen and we are not reporting results for all of the points at the grounding line). Differences between the adjoint-based and the perturbation-based sensitivities are more pronounced near the grounding line and become smaller far from the grounding line, as shown in revised Figure 13 of the paper.

(3) how this is reflected in the adjoint method. In addition, issues might arise due to the discretization. Perturbations in the ice shelf should theoretically not be able to change the ice thickness at the grounding line or the surface slope upstream, but they do so in numerical models, so it could be argued that including these regions is anyway problematic

The reviewer is correct that perturbations on the ice shelf proper do not change the thickness or slope of the ice at the grounding line while they do in the case where a grid cell being perturbed also intersects with the grounding line (as now explained more carefully in section 2.1.1). For this reason we argue that one should refrain from including cells that intersect the grounding line when performing similar sensitivity analysis (when using either the adjoint- or perturbation-based approach). Further, one should also refine the mesh near the GL to get more accurate sensitivities near the GL. This is now stressed more clearly in section 4.5.

The adjoint method also accounts for possible changes in the GL position/length due to (infinitesimal) changes in the ice thickness at triangular grid cells intersecting the GL. This is now explained in more detail in (new) section 2.1.2 and Appendix C after equation C1.

We further note that, in the numerical model, the thickness on the ice shelf and along the grounding line does not change *prognostically.*

Title: In the manuscript you are not so much analysing the sensitivity of grounding line flux to perturbations itself, but you are rather (1) trying to relate the concept of buttressing numbers to the concept of locally-induced grounding line flux changes and (2) showing that the adjoint method is consistent with the perturbation approach. So I would suggest to reformulate your title to reflect the content better, maybe something in the direction of 'On the complicated relation between local ice-shelf buttressing and induced grounding line flux changes', 'Are there causal connections between local ice-shelf buttressing and locally-induced grounding line flux changes?' or 'Adjoint-based sensitivity of grounding line flux to sub-shelf melting...'

We appreciate the reviewer's suggestion. However, we have decided to keep the original title as we feel that it clearly encompasses and describes our efforts and the main focus of the paper.

page 1, line 7: I'm not sure if this is the correct argument to debunk a correlation between grounding line flux changes and local buttressing numbers. If the ice shelf is locally perturbed and buttressing at the grounding line is reduced, this speeds up ice flow at the grounding line up to the perturbation location. However, the perturbation will reduce the spreading rate and hence tends to reduce velocities at and downstream of the perturbation location. This shows in your figure 7 where velocities increase up to the perturbation location and decrease downstream of it, which is then reflected in a local reduction in longitudinal stresses. This is then interpreted as an increase in the local buttressing number based on, e.g., the flow direction. From this point of view, a reduction in buttressing at the grounding line can consistently be related to an increase in the local buttressing number. Your point here is supported by the fact that you cannot find correlations once you include regions close to the grounding line or you analyse Larsen C. Don't get me wrong, I think that it is a very important point to make that local perturbations increase locally measured buttressing numbers, as I do not think that many people are aware of that (I was not).

In general, the reviewer seems to be largely agreeing with the interpretation and conclusions we present in our paper. That is, that a local *increase* in buttressing number following perturbations is paradoxical with respect to the overall *decrease* in buttressing experienced by the broader ice shelf (leading to an *increase* in ice flux across the grounding line). Further, as we discuss in more detail in our revised manuscript (specifically, in sections 4.3.1 and 4.3.2), the local (at the perturbation location) change in buttressing is not always consistent and is often in opposition to the broader response immediately neighboring it. The reviewer may be proposing that the increase in buttressing number in the local area around a perturbation is a consistent diagnostic that could be used as a proxy for overall increases in grounding line flux. While this may be the case for specific domains and perturbations, we are not comfortable making such a general statement here based on the analysis conducted.

page 1, line 20 and other: please check your references, e.g., Schoof (2012) does not use idealized modelling and Asay-Davis et al. (2016) do not include experiments showing MISI, also Royston and Gudmundsson (2016) analyse diagnostic responses to ice-shelf collapse.

We have changed the Schoof (2012) reference to the more appropriate Schoof (2007) reference. The Asay-Davis et al. (2016) reference was meant to be a placeholder for the MISMIP+ paper (which at the time of this submission, had not yet been submitted). We have updated it to the correct reference for the MISMIP+ experiments (Cornford et al. 2020).

page 2, line 43: 'diagnostic, forward experiments'

Changed.

page 4, line 86: you could add a subsection 'Initial configuration' here to improve readability.

Changed as suggested. A new "Model configuration" subsection (2.2) has been added.

page 5, line 120: You need to multiply Nrp with a time interval (e.g., one year if your flux is given in units per year) to get a dimensionless number.

In Reese et al. (2018), it is implicit that the time period of interest is one year (according to personal communications). Therefore, P should have units of m^3, which are the same units as R. We have explicitly stated that the units of R and P are both in m^3 (in which case their ratio is non-dimensional).

page 5, line 120: Do you exclude changes in grounding lines of ice rises in the Larsen C domain?

For the Larsen C domain, the grounding lines around ice rises are treated in the same way as the "primary" grounding line. Therefore, the cells close to ice rises will also be removed when we pick cells for analyzing based on the distance to the grounding line. We have added an explicit statement about the treatment of ice rises to section 2.1.1.

page 5, line 121: you could add a subsection 'Local buttressing number' here to improve readability.

Since this section is already fairly short, and to avoid breaking up the paper into too many short sections, we've opted to keep the discussion of the local buttressing number in with the discussion of the flux response number (and the perturbation experiments in general).

Figure 2: labels for the colorbars are missing and it would be easier to understand your message if you added the normal directions in the panels (also in Figure 3 and others).

We have added colorbar titles and also labels for the choice of buttressing-number normal direction to Figs. 2, 3 and 4.

page 7, line 156: Why 12km? Does the relation already improve if you remove only cells that are directly linked to the grounding line?

We have substantially revised this portion of the paper because we discovered a new metric that can be used for “removing” these areas from consideration (based largely on whether or not the region is experiencing significant shear and/or close to the grounding line). This is discussed in the update section 4.1 (and new Figure S2).

Figure 4: Please add p-values for your correlation statistics.

The p values, relative to the null hypothesis that N_{rp} is independent of N_b, are 1.23e-59 for $\mathbf{n}_{p1}$, 1.57e-09 for $\mathbf{n}_{p2}$ and 5.20e-31 for $\mathbf{n}_{flow}$. However, we think that it would be misleading to report these exceedingly small p-values in the paper. Here, we are trying to assess whether N_{rp} and N_b are linearly related and for this reason we are using linear regression to compute the fitting line and the correlation coefficient, to quantify the discrepancy from that line. We are not interested in ascribing a statistical interpretation to this line fit (which would then need to be explained / defended). For these reasons, we argue against including p-values in the paper.

page 8, line 174, isn’t this a contradiction to your statement in p7., line 58?

We have changed Line 158 (now Line 198; end of section 4.2) to “... we find that buttressing in this direction is not useful for predicting changes in GLF; compared to $N_b(\mathbf{n}_{p2})$, $N_b(\mathbf{n}_{p1})$ and $N_b(\mathbf{n}_f)$ both show a better correlation with changes in GLF via local, sub-ice shelf melt perturbations.”, for consistency with old line 174.

page 11, line 195, maybe better state that the thickness gradients magnitude increases / decreases, since this is the relevant quantity to drive ice flow.

Because the change in thickness gradient *magnitude* doesn’t allow for any information regarding how the change in ice thickness impacts the *direction* of the ice flow, we prefer to keep the wording here as is. This is important because with no information regarding the sign of the gradient change, it’s not immediately obvious if the change in thickness gradient should lead to an increase or a decrease in the local ice velocity.

page 11, line 204: I do not understand your sentence in brackets, please clarify.

This section has been completely re-written.

Figure 8: why do you analyse buttressing changes in neighboring cells and not in the cell itself? This should not make a difference, given Fig. 7 etc, or do I miss some argument here?

In 4.3.1 and 4.3.2 we now discuss in detail the buttressing changes both at the perturbation location and in the immediately surrounding neighborhood.

page 12, line 218: you could add that this negative correlation is in line with the general understanding of how buttressing reduction affects ice flow.

We have revised this sentence to state this more explicitly (around line 266 in the revised manuscript).

Section 4.3.2.: when calculating buttressing at the grounding line, you have an additional direction that emerges naturally, which is the grounding line normal as used in Gudmundsson (2013). In fact, since the boundary condition at the calving front is formulated in terms of the calving front normal, this is the only direction that guarantees that you get a value of 1 if and only if you do not have any buttressing at the corresponding grounding line location. It is worth checking, how using that normal affects your findings.

We have updated this figure and the discussion around it, including (as suggested) the addition of a subplot showing the value of Eq. 14 when changes in buttressing are calculated in the direction normal to the grounding line (of Fig. 8a).

page 13, line 235: I do not understand your statement here as there is a difference between the first principal component along the grounding line and within the ice shelf?

This statement has been removed.

page 14, line 245: you state that you test experiments with and without perturbing elements that are crossed by the grounding line, but you never refer to these experiments again.

This section has been entirely revised. Updated versions of the figures (and the related discussion) from the original version are now included in the SI. With respect to this comment, we do not include cells that cross the grounding line but rather grid cells that are *close to the grounding line* but still on the ice shelf. These experiments are discussed in more detail in the following two paragraphs. Note that we have also edited this sentence to try to clarify its meaning.

page 15, line 266: It might be worth checking the flow and normal directions as well (similar to Figure S8 for the p1-direction).

The relevant figures are now all contained by Fig. S6 and the related discussion is also in the SI. Note that in those figures, we show correlations for all directions (i.e., 180 degrees around the p1 direction).

page 17, line 300: I suppose that you repeat the perturbations for the different thicknesses?

Correct. The experiments are repeated with different sized thickness perturbations. For clarity, we've added "the only change being the magnitude of the applied perturbation" in Line 329.

page 17, line 314: you refer here to the other methods discussed in the previous sections, i.e., the local buttressing numbers etc?

Correct. We've added a clarifying statement "Two previous approaches for assessing GLF sensitivity to changes in ice shelf buttressing – the flux response number (N_{rp})and the buttressing number (N_b)..." to this effect (line 357).