Reply to the Editor

December 15, 2020

Dear Editor,

Our manuscript with the title 'Sensitivity of ice sheet surface velocity and elevation to variations in basal friction and topography in the Full Stokes and Shallow Shelf Approximation frameworks using adjoint equations' submitted to The Cryosphere has been revised following the all suggestions and comments by you.

In particular, we state from the beginning that we are interested in the perturbations in the grounded ice due to the perturbations at its base. The forward equations for FS have been expanded and the criteria satisfied by the grounding line in FS and SSA are now given explicitly.

We hope that after these revisions the paper can be accepted for publication. We wish to thank you and the referees once more for your work which helped us improve the presentation.

Best regards,
Cheng Gong, Nina Kirchner, Per Lötstedt

Comments by the editor

After a careful read of the last version of your manuscript, based on the two last reviews, I think there is still room for improvements. I have one main remark (which was already a point in my first review) as well as few minors points that should be accounted for before the final decision for publication is taken.

The main point concern the treatment of the grounding line dynamics for the transient cases (both FS and SSA). Indeed, solving for the GL dynamics in FS would require to solve the evolution equation for the bottom boundary z_b, in a similar form of (4a) for h. But this equation is missing in (4)? And one should also take care to enforce that $h > z_b >= b$. For the SSA equations (8), equation (8a), using h_t, implicitly assume that all changes in H impacts the upper surface elevation h, which is certainly true for grounded ice (where $z_b = b$), but doesn't hold anymore for floating shelves (where $z_b > b$), and where flotation criteria gives h and z_b as a function of H. So I think that h_t should write H_t in (8a) and that the flotation criteria should be given to determine h from z_b in the

grounded and floating cases. On the same lines, how is determined x_{GL} in the analytical solution (12)? The proposed solution depends on the value of x_{GL} but nothing is said on how it is obtained.
Response: Here is our response to the major remarks by the editor:

1. We mention in the revised abstract and Introduction that we are interested in the perturbations in the grounded ice due to perturbations at the base. Changes in C and b on the ground will have some effect on the floating ice but it will be less than on the grounded part. The boundary conditions at the wetted boundary of the floating ice are found in the new eq (5) and the complementarity condition defining the grounding line in FS is introduced. A new paragraph is written about the SSA equations and their grounding line in Sect. 2.2. We write in the beginning of Sect. 3 that we are interested in perturbations in the grounded part of the ice where $\delta C \neq 0$ and δb directly affect the ice velocity and thickness. Thus, we can integrate over the ice from b to h in the z-direction.

2. We remark after former (12) (and present (13)) that x_{GL} and $H(x_{GL})$ are assumed to be known. An alternative would be to use $H(0)$ at $x = 0$ as in Nye (1959) for the scaling (or boundary data) of the ice. The formula using $H(x_{GL})$ is more accurate compared to numerical solutions of FS and SSA. In the examples in Sect. 3.2.2, the solutions are perturbed around the computed steady state with computed x_{GL} and $H(x_{GL})$.

3. There is a new paragraph in the end of Sect. 2.1 mentioning inequalities that the FS solution should fulfill, e.g. $H \geq 0$ and a bounded η. This should be guaranteed by the numerical method.

4. h_t is replaced by H_t in former (8) with a remark after the equations that we are interested in the grounded part where $h_t = H_t$.

Minors remarks

- page 4, line 12: not sure that ISCAL needs to be mentioned here as it is specific to the Elmer/Ice context

  Response: The sentence about ISCAL is expanded to explain the general applicability and where it is implemented.

- page 5, lie 17: In a two dimensional vertical ice $->$ For a two dimensional flow line geometry?

  Response: Changed.

- Eq. (2): I don't think that ν is used further in the manuscript so may be not necessary to define it?

  Response: It was also used in eq (6). But, no more than these two places. We decide to remove it.

- page 6, line 1: the equation for the friction law should be given

  Response: The expression of the friction law is added, the definition of the operator **T** is moved, accordingly.

- page 9, line 7: MISMIP is already used above. The acronym should be define at its first use.

  Response: Thanks. We have moved the definition to the first place it appears.

- Eq. (12): how do you explain that your analytical solution (starting from the same set of equations) is different than the one of Greve and Blatter (2019). Especially the uniform ice shelf thickness looks like a strange solution?

  Response: This is our approximated solution and the comparison to the solution in Greve and Blatter (2009) is mentioned above Fig. 2 and after eq (D4). We are only interested in the friction and the grounded ice, so this solution on the floating ice is just an approximation of the solution by Greve and Blatter. They assume that the thickness varies linearly, $H_x = const.$ while we let $H_x = 0$. We obtain $h_x = 0$ from the equations implying a flat upper surface. For a better approximation of the thickness on the floating ice, more work is needed.

- Eq. (14): should it be b or z_b for the integral born?

  Response: Our focus is on the grounded ice where $C > 0$ and $z_b = b$. See also the response to the editor's major concerns. The floating ice equation for z_b could have been included with a multiplier in (14) but the perturbations δb and δC are not meaningful under the floating ice. We could also have added an equation for the calving front in the functional. Taking into account the moving boundaries of the floating ice at the base and the front would probably need a separate paper.

- page 17, line 21: same questioning as above. You are mentioning a change in the bottom topography by delta b, whereas it should be delta z_b?

  Response: There is a discussion of this issue in the answer above. We consider b and C as external, given data which are perturbed.

- page 25, line 12: how the GL will move should be given in the initial set of equations.

  Response: How the grounding line is determined is found in new paragraphs in Sects. 2.1 and 2.2. We tell in a few words how this is done numerically in Sects. 3.1.2 and 3.2.4.