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Discussions

Interactive comment on "Modeling snow slab avalanches caused by weak layer failure – Part II: Coupled mixed-mode criterion for skier-triggered anticracks" *by* Philipp L. Rosendahl and Philipp Weißgraeber

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Part II: Michael Zaiser

Prof. Michael Zaiser has raised questions about the concept of FFM and its implications at hand of two thought experiments.

We used the two thought experiments to further explain the concept of FFM – with special focus on size effects.

We thank the referee for the detailed review of our manuscript. We changed manuscript to provide a clearer introduction of FFM and the size effect example.

Reviewer comments

The paper deals with the formulation of a failure criterion for collapse-like failure of snow and application to skier triggering of avalanches. It combines a fracture mechanical and a strength-of-materials approach to formulate a criterion for weak layer failure that does not rely on assuming a pre-existing flaw. Fracture mechanical and strength-of-materials ('snow stability index') approaches to snow failure have both been used in the snow literature, and sometimes in a manner that confounds their respective domains of application. Thus a unification of sorts is an inherently desirable undertaking.

The authors first discuss contradictions that may arise from an uncritical application of either strength-of-materials or fracture mechanical criteria to situations where those criteria do not apply. A nice example is given by Eqs. (3) and (4) which are particularly instructive as I have encountered very similarly flawed reasoning in a recent manuscript under review for The Cryosphere: Basically, one cannot simply apply an energy argument to an uncracked specimen and deduce a failure stress from it, nor can one invert the argument and convert a failure stress into a fracture toughness by evaluating the overall energy stored within the sample. By contrast, I find the argument surrounding Figure 1 less convincing. Reference is made to Bazant 1984 but, if we adopt Bazant's reasoning I see no reason why the strength of an uncracked specimen as shown in Figure 1 should be size dependent (in fact, Bazant's relation predicts strength to be size independent at small sample sizes whatever the crack length).

It is important to note that Bazant's [1] arguments are based on the consideration of the microstructure of the material. Hence, Fig. 1 (see Figure 1) in his work does not indicate absolute size on the horizontal axis but a nondimensional size parameter ($\mathtt{SIZE} = \lambda = d/d_a$), which is a characteristic structural dimension d normalized by a characteristic dimension of the microstructure d_a. This is a very important difference to Fig. 1 in our manuscript.

Bazant's Fig. 1 implies that when the size of the structure is of the order of the size of its particles $d/d_a \approx 1$ ($\log d/d_a \approx 0$), failure is governed by stress. When the structure is much larger than its microstructure $d/d_a \gg 1$, the problem is dominated by energy. The structures considered in Fig. 1 in our manuscript fall into the transition zone between the two extremes. Hence, Bazant's work supports our argument that fracture processes are always governed by strength and toughness simultaneously, even if one often hides the other.

In fact, he draws the similar conclusions as FFM. To quote his 1984 paper [1]:

> *"2. Dimensional analysis based on the foregoing basic hypothesis shows that, for structures that are geometrically similar (i.e., have the same shape), the nominal stress at failure varies with the structure size as $(1+\lambda/\lambda_0)^{-1/2}$ where λ_0 is a constant and λ is the ratio of the size of the structure to the maximum size of the aggregate."*

That is, as the size of the structure d decreases, λ decreases and, hence, according to the above conclusion, the nominal structural strength increases.

A recent study by Leguillon et al. [2] draws very similar conclusions from FFM. Because the apparent strength of very brittle materials such as ceramics is governed by intrinsic defects such as pores or surface flaws, the authors raised the question of the intrinsic strength of such materials. To answer this, they revisited experimental failure data of surface-flawed metallic ceramics and modeled surface defects of the tested specimens using FFM. The experiments and the FFM model (Figure 2) paint a very similar picture as Bazant's size effect law (Figure 1). That is, when the surface flaws are smaller than the microstructure (obtained, e.g., by surface etching), the problem is governed by (the intrinsic) stress. When the flaws are deep, they act crack-like. The transition resembles

Bazant's size effect law and is well captured by the physical arguments of FFM.

Another example for how FFM describes the transition from stress to energy is given by Weißgraeber et al. [3]. The authors consider ellipses of varying aspect ratios a/b under uniaxial tension (Figure 3). When a narrow ellipsis is oriented perpendicular to the loading direction ($a/b \gg 1$), it behaves crack-like and can be described using linear elastic fracture mechanics (LEFM, dashed line) with the corresponding $1/\sqrt{a}$-size effect. Narrow ellipsis oriented in loading direction ($a/b \ll 1$) represent only very weak stress concentrations and the problem is governed by stress (Strength, dotted line) without size effect. Again the transition ($0.1 \leq a/b \leq 100$) is described by finite fracture mechanics (PM FFM, solid line). The FFM model also captures the size effect of circular holes. This is evident comparing the PM FFM curves at the aspect ratio of $a/b = 1$. Here, the stress at failure is a function of the hole radius $a = b$.

The FFM criterion attempts to resolve the well-known conundrum that exists in theories of fracture, namely that the transition from stress-induced damage accumulation (no crack) to the propagation of a critical crack is not well understood. It does so by combining a strength of materials criterion to obtain an upper estimate of the crack length (a crack can at maximum form over a length where the stress is above sigma_c) with a lower estimate (the ensuing crack must be able to propagate). The crack forms if the lower bound falls below the upper one. Taking the example provided by the authors makes me wonder whether I understand the criterion correctly. Consider the tensile beam of Eqs. (3),(4) of the manuscript. Load this beam at the stress sigma_c such that the stress based criterion for mode I failure is fulfilled over the entire cross section. Finite fracture mechanics seems to indicate that all energy stored in the beam is released if a cross-section spanning crack is formed. If that is true, however, then Eq. (3) entails a critical beam length $l \sim$ (2EG_c/sigma_c^2) below which the beam cannot fail because that energy is insufficient. Now I insert typical values of steel, say KIc = 40 MPa m^1/2, sigma_c = 500 MPa, to obtain with EG_c $\sim$ KIc^2 a critical length in the range of 1 cm. Ehem. Do the authors want to imply that my 1cm tensile sample is too small to break??????? Clearly I must be misunderstanding something.

The suggested thought experiment is an example of a size effect. As shown in many works of Bazant (see, e.g., his review [4]), they exist in virtually any structure (with or without pre-cracks).

Of course also the small tensile steel sample will break. However, it will do so at a slightly higher critical load. A sufficiently long sample will fail at $\sigma = \sigma_c$. One that is shorter than the critical length (at which the energy condition is not satisfied anymore), requires a slightly higher load $\sigma > \sigma_c$ to fulfill the energy balance for crack nucleation. Hence, as the sample size is further decreased below the critical length, the effective sample strength will increase owing to the energy condition. This argument holds provided the sample is still much larger than its microstructure (so that we find ourselves in the transition zone of Bazant's [1] Fig. 1 size effect law).

I have a second objection. Consider again a thin tensile beam with thickness a and length l ≪ a loaded at a stress slightly above sigma_c. Let us now assume the energy criterion is fulfilled: sigma_c^2 l > 2 E G_c. So the beam breaks. Now consider the same beam but embedded as surface fiber into a bending beam as in Figure 1. Let the bending moment be such that a region of thickness a from the surface is above the critical stress. In that case, the energy release will be less than for the free standing beam, and it may well be that the crack cannot form since sigma_c^2 a < 2 E G_c. However, the stress state in the considered volume is identical in both cases. The only thing which the volume elements (and the microstructure, grains, dislocations, atoms......) in the beam know about the outside world is the local stress acting on them. How do they understand that, in the first case, they should form a crack instantaneously, and in the second case, not?

The fundamental principle of linear elastic fracture mechanics is (global) conservation of energy applied to crack growth. This is reflected in Griffith's crack propagation criterion $\mathcal{G}_c = \mathcal{G} = -\mathrm{d}\Pi/\mathrm{d}A$, where Π is the total potential energy of the system. That

is, Griffith's criterion and, hence, fracture mechanics in general, always considers the global energy balance.

Let us apply your thought experiment to a crack tip: If, as you suggest, volume elements only know local stress acting on them, any crack should grow at arbitrarily small loads because crack tip stresses are singular. However, they do not because the global energy balance (reflected by Griffith's criterion) must be satisfied. Finite fracture mechanics uses the Griffith condition in its original sense where it was defined not only for infinitesimal crack growth and applies it to finite crack extension.

Considering strain energy density locally represents another form of a local criterion (the square of a simple stress criterion). Hence, strain energy density concepts cannot be used to address fracture mechanics problems unless a length scale (a critical distance or an area) is assumed.

I kindly request the authors to clarify the above two points.

Once we accept the basic approach, the development of stress and energy based failure criteria looks sensible. Concerning the skier loading, I have a question since the loading model is not very clearly described. I presume the authors assume plane strain conditions in which case F would need to be a load per unit length. On line 11, page 11 there is a mysterious b which seems not defined.

Yes, we consider the weight load of a skier (mg) distributed over an assumed effective length of skis (l_o) which provides a force per unit length that corresponds to the load in a unit out-of-plane width plane strain model. If we use an out-of-plane width $b < l_o$ that is not unity, the total force loading of this strip is given by $F = mgb/l_o$. This is explained in part I but was missing in part II. We have addressed this in our revision of part II.

As to material parameter choices, I commented on that point in relation to the companion paper.

In this second part, we only investigate qualitative effects of model parameters. Of course, the absolute value of model outputs is affected by, e.g., the choice of Young's modulus. However, observations do not change qualitatively. An analysis the model's sensitivity to material parameters, shows, for instance, a rather weak effect of the weak layer's Young's modulus. Hence, it does not seem important to choose a certain set of parameters. We now indicate references for our parameter choices in Table 1.

A minor point: It is noted as an inherent flaw of models that assume subcritical damage accumulation that 'such models would predict avalanche release if only enough skiers ski the same slope in close temporal succession'. While such may not be generic behavior, I cannot see why this should be impossible to happen. I know of several passages in the avalanche literature reporting that several skiers may ski a slope before number X triggers an avalanche, and I have seen it myself happening once.

We agree that this question cannot be resolved conclusively at this point. However, if the general understanding of dry-slab avalanche release comprises damage accumulation by subcritical load, then the consequential release after repeated loading must also be a generally observed feature. The sudden crack formation described by FFM is a though model. It does not necessarily mean that the crack actually appears spontaneously. Instead, by considering only the intact and the fractured state, we can also interpret the crack jump as an accumulation of damage which is just not resolved in time.

[1] Z. P. Bažant. Size Effect in Blunt Fracture: Concrete, Rock, Metal. Journal of Engineering Mechanics, 110(4):518–535, 1984.
[2] D. Leguillon, E. Martin, O. ŠevecÌŇek, and R. Bermejo. What is the tensile strength of a ceramic to be used in numerical models for predicting crack initiation? International Journal of Fracture, 212(1):89–103, 2018.
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[4] Z. P. Bažant. Size effect on structural strength: a review. Archive of Applied Me-

chanics, 69(9-10):703–725, 1999.

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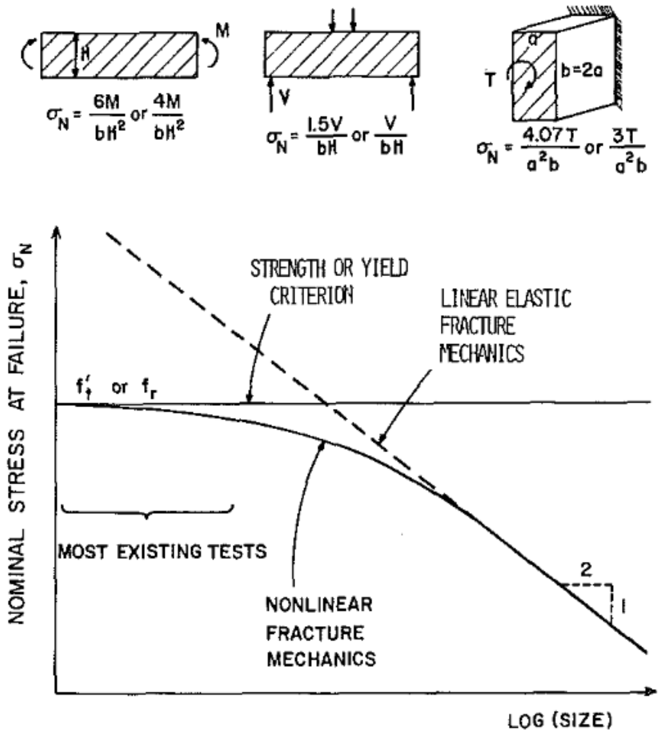

Fig. 1. Fig. 1 by Bazant [1]. The x-axis shows a nondimensional size parameter (SIZE = d/d_a), where d is a structural dimension and d_a th e size of the microstructure.

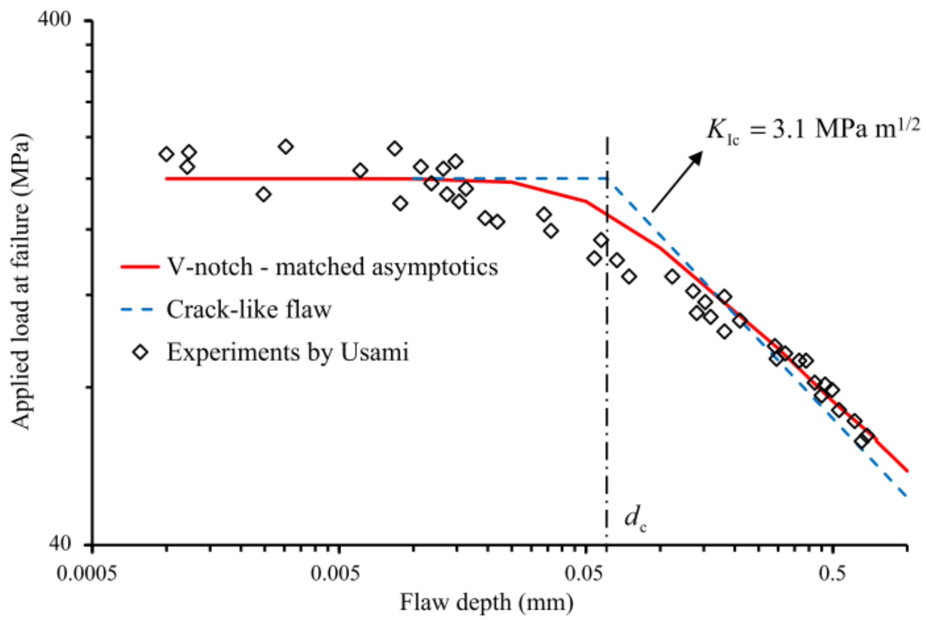

Fig. 2. Using FFM, Leguillon et al. [2] show that failure (vertical axies) is governed by stress (horizontal asymptote) when initial flaws (horizontal axis) become smaller than the microstructure.

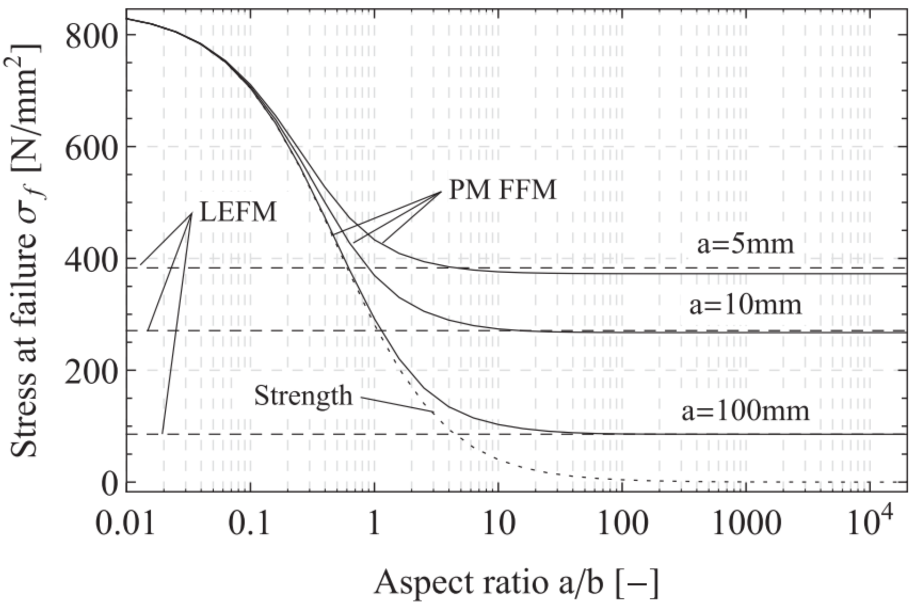

Fig. 3. Stress at failure over aspect ratio a/b of an ellipsis. Transition from stress problem (Strength, dotted) to linear elastic fracture mechanics (LEFM, dashed) captured by FFM (solid)