Responses to Referee comments by Anonymous Referee #1 on “Brief communication: Time step dependence (and fixes) in Stokes simulations of calving ice shelves” by Brandon Berg and Jeremy Bassis

We thank the anonymous reviewer for their feedback on this manuscript. Our responses to comments are given below, with original comments in black and responses in red. When referenced, line numbers refer to the revised manuscript.

General Comments:

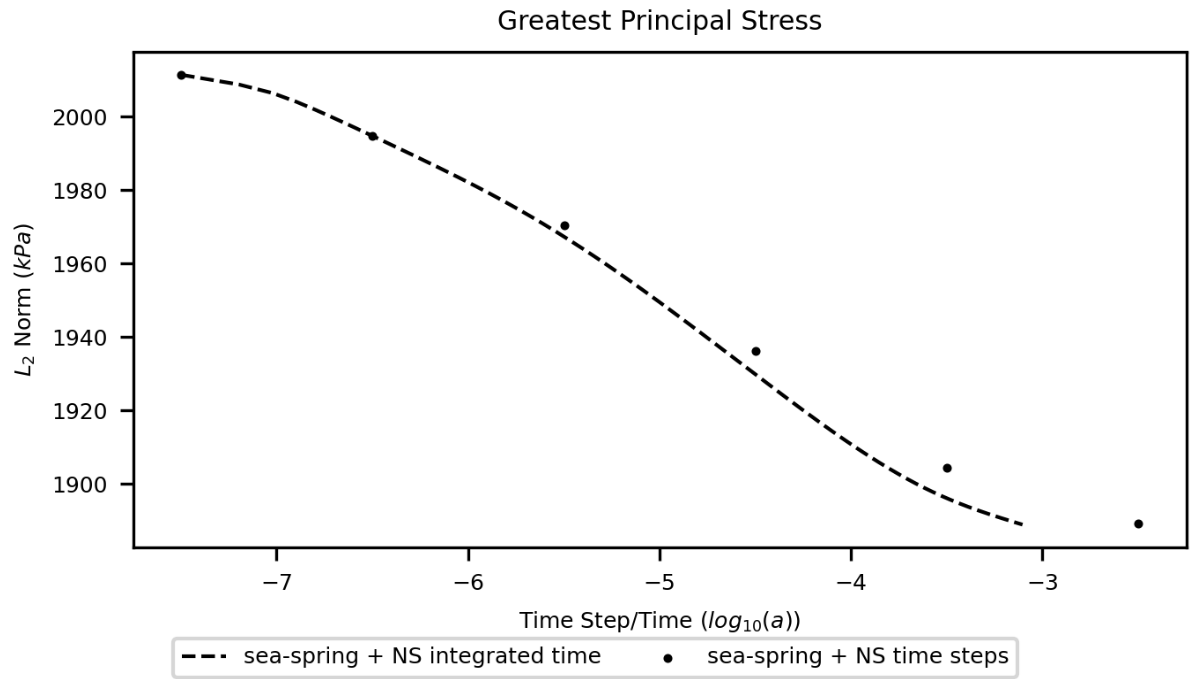
This paper presents a simple method to overcome time step dependence of the solution arising when solving for an ice-shelf which departs significantly for hydrostatic equilibrium. This could be the case for instantaneous non-vertical icebergs calving or supraglacial lake drainage. This is quite a technical paper but as the problem might be encountered by other groups using different Stokes solvers, this brief communication certainly deserves to be published. The overall writing of the paper is quite good even if I think that there is some room for improvement.

We thank the reviewer for their positive comments. We respond in more detail to each comment below.

My main concern is the fact that the time step dependence of the solution is sometimes seen as negative (e.g. title) or positive (e.g. caption Fig. 2). And indeed it is not completely clear from Figs. 2 or 3 to see which of the two solutions is the one that works better.

This is a good point and we agree that the original figures were confusing. We have added text to the Figure 2 caption clarifying that the time step variability for the sea-spring with Navier Stokes solution is connected to the time evolution of the *system*. In the classic Stokes system, the velocity depends solely on the geometry of the ice shelf/glacier and internal properties (e.g., temperature). Hence, the dependence of the velocity field on time step is unphysical. However, when solving the full Navier-Stokes system, the velocity becomes time dependent and, like all numerical ODE integrations, we must take sufficiently small time steps to ensure numerical convergence when integrating with respect to time to find the numerical approximation to the solution. For the sea-spring + NS solution, the velocity tends towards zero as time step size decreases. As a consequence there is no deformation and, for very small times, the ice shelf behaves as a nearly rigid body as it approaches hydrostatic equilibrium. In the sea-spring + NS method, taking small time steps allows us to resolve the quasi-rigid body uplift of the ice shelf as it “bobs” in the water. We have added text in Section 4 of the manuscript clarifying the rigid body behavior at short time steps (lines 132-135).

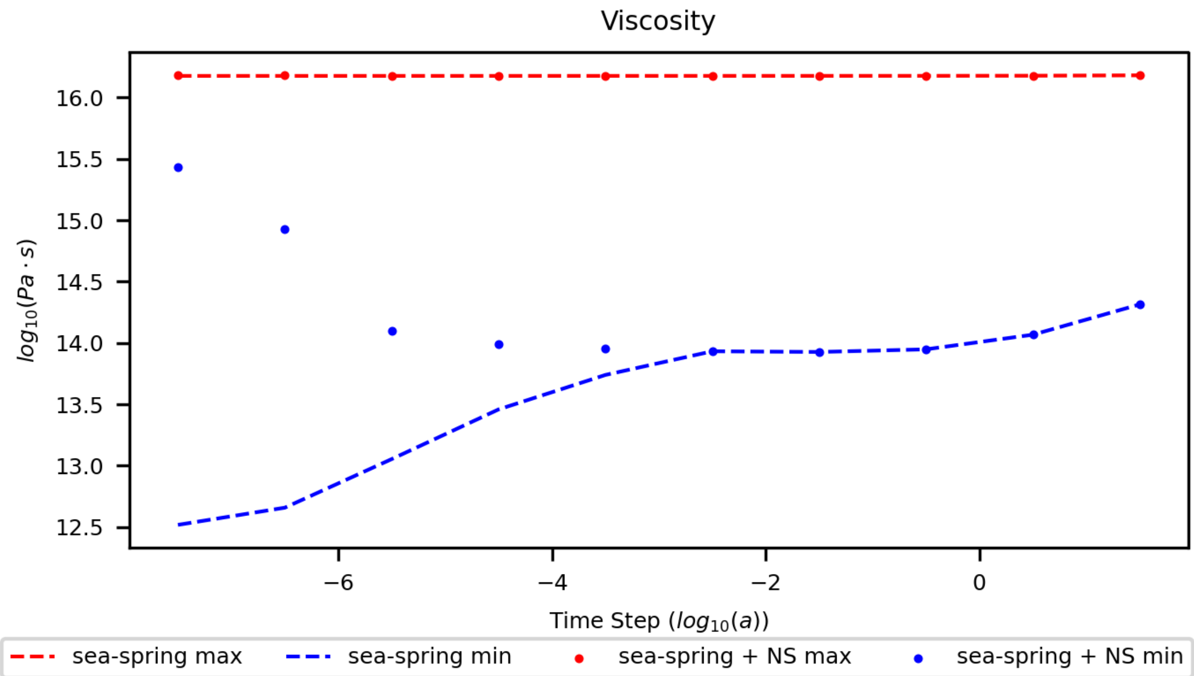
This is illustrated below, where we show a plot of the L2 norm of the greatest principal stress for the sea-spring + NS method. The points represent different time step sizes (as in the manuscript). The dashed line is created by choosing the smallest time step and numerically integrating the system in time. For small times, the solution obtained from taking a single step with different time step size and the solution obtained from numerical integration (with a very small time step size) are similar, but begin to differ as time step size increases because taking large time steps results in less accurate solutions. Thus, the variation from the sea-spring + NS solution at small time steps is consistent with the actual time evolution of the system. This is connected to our discussion in section 4 of the manuscript.



The viscosity has no timestep dependence for the sea-spring solution and it is the sea-spring+NS solution that has no time step dependence for effective strain-rate.

We have chosen to omit plotting the viscosity form the final manuscript, instead replacing it with a plot of vertical velocity because effective strain rate and viscosity display somewhat redundant information and to ease the exposition of this short manuscript. Showing the vertical velocity directly provides a more direct illustration of the problem with the vertical velocity becoming unphysically large as time step size decreases.

However, we include a plot of viscosity below that shows the minimum and maximum value of the viscosity at different time steps for the two methods. Plotting the maximum and minimum values better highlights the time step dependence of the viscosity at small time steps. Note that for small time step sizes (and times) with the sea-spring + NS solution, the motion is quasi-rigid (i.e., deformation is small), strain rates are small and the viscosity increases. The viscosity is ultimately limited by the regularization we use for the rheology at small strain rates. By contrast, for the sea-spring solution, as time step size decreases the increasingly large vertical velocity causes a large strain rate increase. Because the viscosity and strain rate are inversely related, this results in decreasing viscosities.



This is even less clear for stress where both solutions are diverging but presents both a timestep dependence. I would expect more comments on the text on this and how from the figure one can decide which is the working solution.

We have clarified our language and removed most uses of the word “divergent” from the manuscript. The vertical velocity, which we now show in Figure 2 panel (a), becomes unphysically large for small time step sizes for the sea-spring method. Effective strain rate becomes approximately zero for small time step sizes for the sea-spring + NS method because the motion at small times is nearly rigid body. Effective shear stress tends to zero for the sea-spring + NS method for the same reason as the effective strain rate. We also point out in section 4 that while Figure 3 shows higher maximum stresses for the sea-spring method, the L2 norm shows higher stresses for the sea-spring + NS method because of high negative compressive stresses (lines 136-137).

Smaller Points:

page 2, line 41: ", where u1 is a constant"

Comma has been added.

Figures 2 and 3: the quality of Figs. 2 and 3 are very low.

Figures have been changed from .png to .eps to improve viewing quality.

It is not clear from the text and the captions if what is plotted on these figures is the solution after the timestep following the calving event.

Thanks for pointing this out. We now state in the figure captions that the plotted solution is immediately after the (emulated) calving event.

What are the differences of setup between Fig. 2 c and d and Fig. 3?

The difference between Fig. 2 c and d and Fig. 3 is that we plot the L2 norm of the solution in Figure 2 and the L1 norm (maximum) of the solution in Figure 3. This is emphasized in the text and on figure y-axes. We plot the L1 norm (maximum) because this is the criterion that is often used in stress based calving laws, like the Nye zero stress. We show the L2 norm because it is (often) a more robust diagnostic of the behavior of the numerical solution.

I would suggest to modify Pa to MPa or kPa. For the x axis, the caption should tell that time step are varying from xx seconds to xx years?

We have modified the axes for stress to be kPa rather than Pa. Figure caption now states that the time step ranges from 1 second to 30 years.

page 5, line 95: not sure the second sentence of part 3.2 should start with "Furthermore"?

Thanks. “Furthermore” has been removed.

Eq. (9): specify that u_{i-1} is the velocity at previous timestep?

A sentence has been added after the equation defining the variable.

page 6, line 113: "where the damping coefficient is"

Correction has been made.