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Possible biases in scaling-based estimates of mountain-glacier contribution to the sea level

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Abstract. Predicting mountain-glacier contribution to sea-level rise involves computing global-scale glacier loss under a given climate-change scenario. Such calculations are usually done with low-complexity and computationally-efficient approximate models of glacier dynamics. A statistical power-law relation between glacier volume and area (and/or length) is the basis of several such models. We simulate transient response of an ensemble of 551 glaciers from Ganga basin, the Himalaya, using a scaling-based method and a two-dimensional ice-dynamical model based on shallow-ice approximation (SIA). A comparison of the model outputs suggests that the scaling-based method systematically underestimates long-term ice loss due to a violation of the assumed time-invariant scaling. We derive expressions for the response time and climate sensitivity of glaciers simulated using a time-invariant scaling assumption, and validate them with results from the scaling-based simulation of the ensemble of glacier. These expressions are modified empirically to obtain similar parameterisations of the response properties of glaciers simulated with SIA. These new parameterisation yields a linear-response model which significantly reduces the above biases, while retaining the advantage of numerical efficiency.

1 Introduction

Shrinking mountain glaciers contributed significantly to the global eustatic sea-level rise in the recent past and this trend is expected to continue for the next hundred years or so (Meier, 1984; van de Wal and Wild, 2001; Raper and Braithwaite, 2006; Cogley, 2009; Hirabayashi et al., 2010; Radić and Hock, 2011; Slangen and van de Wal, 2011; Jacob et al., 2012; Marzeion et al., 2012; Huss and Hock, 2015; Hock et al, 2019). The reliability of the predicted sea-level change is, thus, intimately tied to the accuracy of the predicted ice-loss from mountain glaciers globally.

Instantaneous (annual) glacier surface mass balance can be calculated readily using data from climate model simulations. However, an accurate prediction of the long-term evolution of a glacier would require simulating the decadal-scale changes in glacier area and hypsometry (Raper and Braithwaite, 2006; Radić et al., 2008; Huss et al., 2010; Giesen and Oerlemans, 2013; Radić et al., 2014; Huss and Hock, 2015) - ideally by solving the dynamical ice-flow equations (Oerlemans, 2001; Cuffey and Patterson, 2010). However, the numerical cost of such a computation on a global scale is prohibitive, even if a simplified approximate description of the full ice-flow equations like shallow-ice approximation (SIA) were to be used. Further, an uncertain glacier bedrock limits the benefit of using physically-based ice-flow models (Farinotti et al., 2016). The existing global-scale estimates of the mountain-glacier contribution to sea-level rise mostly rely on low-dimensional approximate parameterisations

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of the glacier dynamics (van de Wal and Wild, 2001; Raper and Braithwaite, 2006; Hirabayashi et al., 2010; Radić and Hock, 2011; Slangen and van de Wal, 2011; Marzeion et al., 2012; Giesen and Oerlemans, 2013; Huss and Hock, 2015; Hock et al, 2019). Several of these parameterisations are based on an statistical area-volume (or area-volume-length) scaling relation for any set of mountain glaciers (Chen and Ohmura, 1990; Bahr et al., 2015). Empirical prescriptions for distributing the annual ice-loss over glacier surface, or equivalently, adjusting the hypsometry of the transient glaciers are also used (Raper and Braithwaite, 2006; Radić et al., 2008; Hirabayashi et al., 2010; Huss et al., 2010; Giesen and Oerlemans, 2013; Huss and Hock, 2015).

1.1 Area-volume scaling for mountain glaciers

The statistical power-law relationship between glacier area and volume was established empirically (e.g., Chen and Ohmura, 1990),

$$V = cA^{\gamma}, \tag{1}$$

where, V and A are glacier area (km^2) and volume (km^3), respectively. γ is a dimensionless scaling exponent expected to be in the range $1.17 < \gamma < 1.5$ (Bahr et al., 2015). The scaling exponent γ can be estimated using dimensional analysis (Bahr et al., 2015), if an empirical sub-linear scaling of glacier width with glacier area (Bahr, 1997) is assumed. c (km$^{3-2\gamma}$) is a dimensionful fitting parameter (Bahr et al., 2015). The above relation is statistical in nature, and allows estimation of the total volume of a large set of glaciers fairly accurately. However, it would have considerable uncertainty when volume of any individual glacier in the set is considered (Bahr et al., 2015). The scaling relation is used to predict glacier area change given the volume change, with c and γ assumed to be time-invariant constants (e.g., Radić et al., 2007).

According to the theoretical arguments by Bahr et al. (2015), γ indeed is a time-independent constant with $\gamma = 1 + \frac{m+1}{m+n+3}$, where n is the power-law exponent of Glenn's rheology of ice and m is the scaling exponent of ablation rate with glacier length. However, the dimensionful scale factor c may vary with time (Bahr et al., 2015) for a set of non-steady glaciers. For example, dimensional arguments do not rule out the possibility that $c = c(t/\tau)$, where t/τ is the dimensionless ratio of time t to the response time τ of the glacier. An unaccounted-for time-dependence of the scale factor c would lead to a systematic bias in the predicted glacier change as discussed later in this paper.

1.2 Motivation for the present study

The performance of the scaling relation (eq. 1) in describing transient glacier response have previously been investigated by comparing results of scaling-based model with those from simulations based on dynamical ice-flow models. Dynamical models with various levels of complexity, e.g., SIA, higher order approximations or full Stokes' evolution in one to three dimensions (Radić et al., 2007, 2008; Adhikari and Marshall, 2012; Farinotti and Huss, 2013) were used for this purpose. However, the above studies assumed that scaling exponent γ may vary with time, which violates the theoretical arguments of Bahr et al. (2015). In addition, most of the above investigations were confined to synthetic glaciers having idealised geometries. Possible uncertainties introduced by a time-invariant scaling-based parameterisation of the evolution of glaciers with realistic geometries

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were considered only by Farinotti and Huss (2013). The authors provided thorough assessment of the uncertainty arising out of the inaccuracies of the parameters c and γ. The spirit of the present study is quite similar to that of Farinotti and Huss (2013) mentioned above. However, the authors did not consider the possibility of a systematic bias in the scaling based models given the known violation of the time-invariant scaling assumption (Radić et al., 2007, 2008; Adhikari and Marshall, 2012) as discussed above. A time-independent c can lead to systematic bias even in the case when c and γ are known accurately for a given set of glaciers. One of our two main objectives of the present study is to investigate this possible bias of scaling-based models.

Apart from the scaling-based methods, there are other empirical methods for mimicking glacier evolution that use specific algorithms for distributing the net mass loss over the glacier surface (Raper and Braithwaite, 2006; Huss et al., 2010). Yet another viable alternative is obtained by assuming small deviations from a steady-state glacier (Oerlemans, 2001; Harrison et al., 2001; Lüthi, 2009). This so-called linear-response theory for glacier area or volume-change, however, requires the knowledge of the response time and climate sensitivity of glacier volume and area. A lack of accurate and numerically convenient parameterisations of these dynamical properties limits the application of such linear-response models for sea-level rise predictions (Harrison et al., 2001; Lüthi, 2009). The other objective of the present study is to come up with an accurate parameterisation of the relevant linear-response properties, and thus improve the performance of the linear-response model.

We emphasise that the points of departures of this study from that of Farinotti and Huss (2013) are that, 1) Following the theoretical guidelines of Bahr et al. (2015) c is considered to be the sole fit-parameter, 2) Given the known violation of the assumed time-invariance of area-volume scaling, possible systematic biases in scaling-derived glacier-loss predictions are investigated, and 3) possible new parameterisations of glacier-response properties are explored in order to improve the accuracy of the linear-response model.

1.3 Outline of the study

Here, we simulate the response of an ensemble of 551 clean-ice glaciers in the Himalaya to a step change in equilibrium-line altitude (ELA) using 2-dimensional SIA model. The accuracy of a scaling-based model (Radić et al., 2007) in reproducing the long-term loss in total glacier area and volume as obtained with SIA, is assessed. First we discuss the basics of the area-volume scaling method for glacier evolution, deriving theoretical predictions for dynamical response properties of glaciers evolving under a time-invariant scaling. These theoretical predictions are verified with a scaling-based simulation of the response of the same 551 glaciers to a step-change in ELA. The scaling-based theoretical expressions for glacier response properties are then empirically extended in order to obtain accurate parameterisations the linear-response properties of the SIA-simulated glaciers. The linear-response model the long-term total shrinkage of glaciers as predicted by the scaling-based method (Radić et al., 2007), and the linear-response model are compared with the corresponding SIA results.

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2 Dynamical consequences of a time-independent area-volume scaling

Let us consider a set of glaciers that are responding to a warming climate while conforming to a time-independent area-volume scaling (Eq. 1). For small fractional changes in area and volume of the glaciers, the two are related as follows.

$$\Delta V \approx c\gamma A^{\gamma-1}\Delta A = \gamma\frac{V}{A}\Delta A = \gamma h \Delta A, \tag{2}$$

where, ΔV and ΔA are the changes in area and volume, and the mean ice thickness is $h = V/A$. The scaling factor c is assumed to be a time-independent constant here.

The above equation is the basis of the scaling-based glacier-evolution schemes, allowing computation of area change during a time interval from the given volume change during that interval. The particular model of Radić et al. (2007) that is considered here, assumes that the area loss takes place only in the lowest elevation band/s near the glacier terminus and updates glacier hypsometry accordingly (Radić et al., 2007).

2.1 The rates of area and volume change

A consequence of the above scaling relation (Eq. 2) is that the instantaneous rates of area and volume changes are related as,

$$\dot{V} = \gamma c A^{\gamma-1}\dot{A} = \gamma h \dot{A}, \tag{3}$$

where, $\dot{V}$ and $\dot{A}$ denote the rate of changes of volume and area. If the mean specific balance rate is δb, then the annual rate of mass loss is $\dot{V} = \delta b A$. This, together with Eq. 3, implies,

$$\dot{A} = \frac{\delta b}{\gamma h}A \tag{4}$$

$$= \frac{\delta b}{\gamma c}A^{2-\gamma}. \tag{5}$$

Thus, a consequence of the time-invariant scaling hypothesis is that the rate of area change must scale with glacier area with an exponent $(2-\gamma)$. This is consistent with empirical observations (Banerjee and Kumari, 2019). As the scale factor of this power-law relation is proportional to the mean mass balance, Eq. 4 may be a convenient way of obtaining mean regional thinning rates from relatively straightforward remote-sensing measurements of the rate of area change. However, this relation is accurate only to the extent the assumption of the time-independence of the scale-factor c is valid.

2.2 The area response time

As response time is defined with respect to a steady-state glacier, let us consider a steady glacier and apply a constant perturbation for time $t > 0$ (i.e. a step change in ELA). Let's take the corresponding instantaneous net negative balance of the glacier at $t = 0$ to be $\delta b_0 A$. The perturbation would asymptotically $(t \to \infty)$ lead to a shrinkage of glacier area by $\Delta A_\infty = A(0) - A(t \to \infty)$ and ice volume by ΔV_∞ such that (Harrison et al., 2001),

$$\Delta A_\infty b_t + \beta \Delta V_\infty \approx -\delta b_0 A. \tag{6}$$

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Here, b_t is the ablation rate near the terminus and β is the balance gradient. The area response time of the glacier can be estimated as $\tau_A \approx \Delta A_\infty / \dot{A}$. Therefore, using the expressions for $\dot{A}$ (Eq. 4) and ΔA_∞ (Eq. 6), respectively, we obtain,

$$\tau_A = -(\frac{b_t}{\gamma h} + \beta)^{-1} = \tau^*. \tag{7}$$

Here, the symbol τ^* is a convenient shorthand notation for the time scale $-(\frac{b_t}{\gamma h} + \beta)^{-1}$. In the above derivation, ΔV_∞ that appears in eq. 6 is eliminated with the help of eq. 2. The resultant expression for response time (Eq. 7 is comparable with that derived by Harrison et al. (2001) or Lüthi (2009).

2.3 Linear response model for area and volume change

Within a linear-response model the instantaneous change in volume (ΔV) as after a steady glacier is perturbed by a small step change in ELA is, for example, given by Lüthi (2009) as,

$$\Delta V(t) = \Delta V_\infty (1 - e^{-t/\tau_v}), \tag{8}$$

where, τ_v is the volume response time and V_∞ is the volume sensitivity.

Now, for small fractional changes in area it is straightforward to show that Eq. 8 implies,

$$\Delta A(t) = \Delta A_\infty (1 - e^{-t/\tau_v}), \tag{9}$$

by expressing $V(t), V(0)$ and $V(t \to \infty)$ in terms of $A(t), A(0)$, and $A(t \to \infty)$, respectively, using the scaling relation (Eq. 1), That is, within the scaling assumption, the area response time, τ_A, is the same as the volume response time, τ_V,

$$\tau_A = \tau_V = \tau^*. \tag{10}$$

This also implies that the ratio $\Delta V(t)/\Delta A(t)$ is a time independent constant. In fact, Eq. 3 requires that this constant ratio is given by γh. While h is not strictly time independent for a set of shrinking glaciers, for a small fractional change in area the corresponding fractional changes in mean thickness is going to be significantly smaller. This is because, from eq. 1, $h = V/A = cA^{\gamma-1}$, or $\frac{\Delta h}{h} = (\gamma - 1)\frac{\Delta A}{A}$, and $(\gamma - 1)$ is a small quantity for typical values of γ.

2.4 Climate sensitivity of area and volume

The climate sensitivity of the area is given by ΔA_∞. An expression for the asymptotic fractional change in area is obtained by eliminating ΔV_∞ from Eqs. 6, and using the expression for τ_A (Eq. 7),

$$\frac{\Delta A_\infty}{A} = \frac{\tau^* \beta \delta E}{\gamma h} = \alpha^*. \tag{11}$$

Here, we have used $\delta b_0 \approx \beta \delta E$ for a step change in ELA by δE, and the RHS is denoted by α^* for convenience.

The corresponding expression for $\frac{\Delta V_\infty}{V}$ is then obtained using Eq. 2,

$$\frac{\Delta V_\infty}{V} = \gamma \alpha^*. \tag{12}$$

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Again, Eq. 12 is comparable to the expression of volume sensitivity as derived by (Harrison et al., 2001). The minor differences are due to the time-invariant scaling assumption made here.

Please note that strictly speaking, the climate sensitivity of area and volume with respect to a change in ELA should be defined as $\frac{\Delta A_\infty}{\delta E}$ and $\frac{\Delta V_\infty}{\delta E}$, respectively. However, in this paper we use ΔA_∞ and ΔV_∞ as the corresponding sensitivities to simplify notation.

3 Numerical methods

3.1 2-dimensional SIA model

We consider all the 814 glaciers larger than 2 km^2 in the Ganga basin, the central Himalaya from RGI 6.0 inventory (RGI, 2017), and simulated them one by one using an automated procedure. We defined the simulation domain for each of the glaciers with a one-pixel-wide (pixel size of 100 m $\times$ 100 m) buffer around the corresponding RGI 6.0 outlines. The corresponding ice-free bedrock is obtained using available ice-thickness estimates for each of the individual glaciers in the RGI inventory (Kraaijenbrink et al., 2017) and ASTER GDEM (ASTER GDEM, V003) which are down-sampled to 100 m resolution.

The ice-flow dynamics was implemented within two dimensional shallow-ice approximation (e.g., Le Meur et al., 2004) as an easy-to-implement numerically efficient non-linear diffusion problem (Oerlemans, 2001). While SIA may not be the best method for simulating valley glaciers due to its limitation in describing ice-flow that is influenced by longitudinal stresses and/or steep bedrock slopes (Le Meur et al., 2004), there is enough evidence in the literature that SIA does a reasonable job of describing both steady and transient dynamics of valley glaciers (e.g., Vieli and Gudmundsson, 2004; Le Meur et al., 2004).

The equations of motions were integrated using a linearised implicit finite-difference scheme (Hindmarsh and Payne, 1996) with no-slip boundary condition at ice-bedrock interface. An iterative conjugate-gradient method was employed within the implicit scheme, with a spatial grid-size of 100m$\times$100m and time steps of 0.01 years. During the simulations, ice-mass conservation was monitored and was within one parts per 10^8 at each time step.

The value of Glenn's flow-law exponent is assumed to be 3. Since, we are interested in the generic behaviour of a set of glaciers with realistic geometries, we do not attempt any tuning of the flow parameters to obtain realistic values of ice thickness or surface velocity of the chosen glaciers. Rather, the rate constant appearing Glenn's law was picked randomly from the set $\{0.5, 0.6, ...1.4, 1.5\} \times 10^{-24}$ Pa^{-3}s^{-1} for each of the glaciers.

The following elevation-dependent idealised linear mass-balance profile was used for all of the glaciers,

$$b(z) = Max\{\beta(z-E), b_0\}. \qquad (13)$$

Here, β is the balance gradient, which was again picked randomly for any given glacier, with the allowed set of values {0.005, 0.006, ... 0.009, 0.010} yr^{-1} for the glaciers. z and E are the surface elevation and the equilibrium-line altitude (ELA) measured in meters. b_0 is a cutoff on maximum accumulation taken to be 1.0 m/yr.

The value of E for each of the glaciers was fixed as follows. E is at first set equal to the glacier median elevation. Then SIA simulation was run starting with an empty bedrock, and was continued till a steady state was reached. E was then moved up

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or down and the simulation was repeated to obtain the corresponding steady states. The iterations were continued till a steady extent that was similar to the present glacier extent (RGI, 2017) was reached. Once the desired steady state was found, the glaciers were perturbed by a 50 m step rise in ELA, and then for the subsequent 500 years, annual values of area and volume were recorded.

Out of the total 814 glaciers, there were 120 glaciers where either our algorithm for finding a steady-state similar to present extent did not converge or the final steady state glacier geometry was not realistic. As a result, among 814 Ganga basin glaciers that are larger than 2 km^2, we could simulate only 694 glaciers. Out of these, the fractional changes in glacier area at $t = 500$ were more than 50% for 91 glaciers and they were excluded. An additional 52 glaciers had response time larger than 300 years and were also not considered. Finally, we were left with an ensemble of 551 Himalayan glaciers, which is 68% of the initial set. The area of these modeled glaciers were in the range $2.5 - 89.5$ km^2 with a median value 5.7 km^2. The total area and volume of these glaciers were 5143.7 km^2 and 602.7 km^3, respectively. As an ensemble consisting 552 glaciers can be considered large enough to test the statistical area-volume scaling, we did not do a detailed glacier-by-glacier analysis of the reasons behind the failure of the algorithm for some of the glaciers.

In the 2-d SIA-based dynamic glacier model described above, we have neglected the effects of debris cover on ablation, avalanche contribution to accumulation and ice flow due to sliding. While these processes are rather important in the Himalaya (Banerjee and Shankar, 2013; Laha et al., 2017), this was done to keep the model as simple as possible. Therefore, the simulated glaciers are not faithful copies of the actual Himalayan glaciers. On glaciers that are extensively debris-covered and/or receives strong accumulation contribution from avalanches in reality, our modelled ELA is likely to have a negative bias. Moreover, these ignored processes may either lead to a different scaling exponent and/or scale factor, or add additional scatter to the scaling plot for the real Himalayan glaciers. However, These simplifications do not weaken our study, as we restrict our aim to simulating the response of a large number of synthetic clean glaciers with realistic geometries and simplified mass balance profiles to a hypothetical step-change in ELA using different methods and comparing the results.

3.2 Scaling and hypsometric-adjustment-based models

The response of the set of 551 steady-state glaciers to a 50 m instantaneous rise in ELA were recomputed with a scaling-based approach (Radić et al., 2007). The SIA-derived initial steady-state volume, area, and hypsometry (with bin size of 25 m) for each of the glaciers were used as the starting point. At any time t, the elevation-dependent mass-balance function was summed over the instantaneous glacier hypsometry to obtain the volume loss in a given balance year. The computed annual volume loss was converted to a corresponding area loss using Eq. 2 for the scaling-based method (Radić et al., 2007). The reduction in area was assumed to have taken place in the lowest elevation band of a glacier (Radić et al., 2007). The scaling exponent was fixed at $\gamma = 1.286$ because of the linear mass balance profiles of the simulated glacier ($m = 1$). The annual resolution time series of computed area and volume were recorded for 500 years for each of the glaciers after the perturbation (a 50 m step-change in ELA) was applied.

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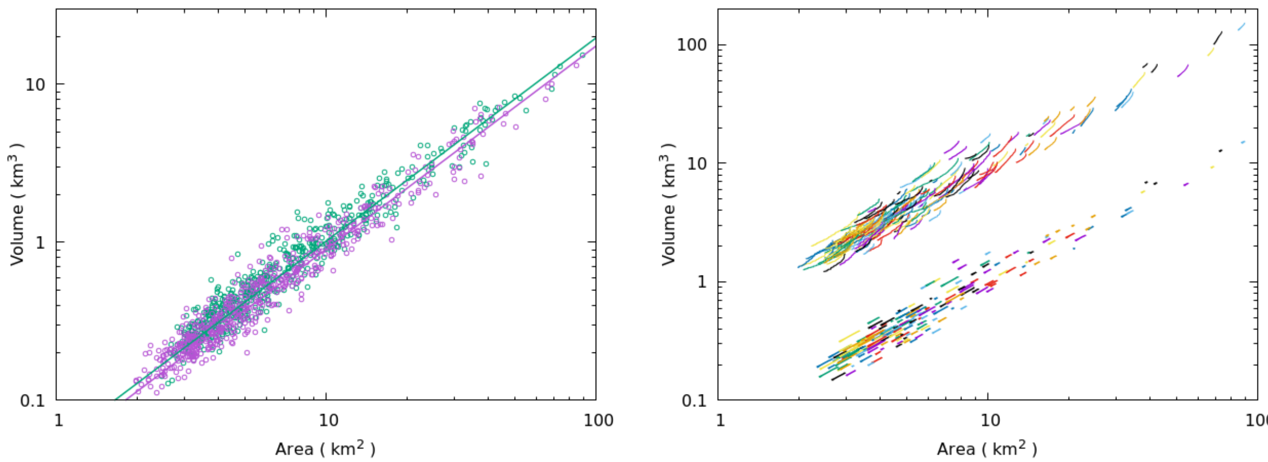

Figure 1. A) The area-volume scaling for 551 glaciers simulated with SIA for the steady state at $t = 0$ yr (purple circles) and the transient states at $t = 500$ yr (green circles). γ is taken to be 1.286 and best-fit c values are 0.053 ± 0.001 and 0.047 ± 0.001 km$^{3-2\gamma}$, respectively, at $t = 0$ and $t = 500$ years. For both the fits $R^2 = 0.90$. B) The simulated evolution of 200 randomly chosen glaciers in the $V - A$ plane with SIA (thin lines) and scaling (thick line) models. See text for further details. Note that the SIA-derived volumes are scaled by a factor of 10 for better visualisation.

3.3 Linear-response model

For each of the 551 glaciers, the time series of volume and area, as obtained using both SIA and scaling-based methods, were separately fitted to linear-response forms analogous to eq. 8 to obtain the corresponding best-fit values of the four linear-response parameters: the climate sensitivities $(\Delta V_\infty, \Delta A_\infty)$ and response times (τ_V, τ_A).

The best-fit linear-response properties obtained from the scaling evolution were used to verify the theoretical expressions obtained from scaling theory as given in the previous section. Subsequently, the best-fit response time and climate sensitivities obtained from the SIA simulations were used to check if the same relations describe the SIA results or not, in order to obtain empirical parameterisations for the response properties. All these fits were performed in log-log scale and R^2 of the fits were noted.

These best-fit empirical parameterisations of climate sensitivity and response time obtain by fitting the SIA results were used to compute the linear-response model predictions for the time series of total area and volume of the 551 glaciers perturbed by a 50 m step-change in ELA at $t = 0$. To assess the uncertainty of the linear-response model results, a random Gaussian noise were added to the best-fit empirical parameters to generate an ensemble of 100 independent copies of the linear-response model. The standard deviation of this Gaussian noise for any given fit parameter was set equal to its standard error.

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4 Results and discussions

4.1 Area-volume scaling of glaciers simulated with SIA

As discussed in the introduction, the area and volume of the 551 glaciers are expected to follow a power-law relation (eq. 1) with scaling exponent $\gamma = 1 + \frac{m+1}{m+n+3} = 1.286$. Indeed, the ensemble of glaciers modelled with SIA conform to a power-law relation $V = cA^{1.286}$ at any time t. For example, fig. 1a shows the power-law behaviours at $t = 0$ and $t = 500$ years. The corresponding linear fits in log-log scale have R^2 value of 0.90.

Another interesting trend that is evident from fig. 1a is that the best-fit c is time-dependent and decreases systematically with time as glaciers shrink. For example, the best-fit c-values are 0.053 ± 0.001 and 0.047 ± 0.001 km$^{3-2\gamma}$, respectively, at $t = 0$ and $t = 500$ years (fig. 1a). This implies a $\sim$13% reduction in c for the ensemble over 500 years subsequent to the step-change in ELA. The time dependent c is consistent with arguments of Bahr et al. (2015) and those in the introduction of this paper.

The systematic decline in c for an ensemble of shrinking glaciers leads to a systematic bias in any scaling-based glacier evolution model where the time-invariant c is a basic requirement. A decreasing c would imply, $\frac{\Delta V}{V} = \gamma\frac{\Delta A}{A} + \frac{\Delta c}{c}$. Since the changes in all the three quantities are negative, for any given $|\Delta A|$, the corresponding $|\Delta V|$ is underestimated whenever the second term on the RHS is assumed to be zero, e.g. in eq. 2. As eq. 2 is the basis of all scaling-based glacier models, this implies that all such models would have the above bias and predict thicker glaciers in the future as climate warms up. Due to a corresponding surface-elevation feedback related to thicker glaciers, the bias is likely to get amplified, leading to significant underestimation for future glacier mass loss under warming climate. Our comparison of the results from SIA and scaling-based simulations as shown in Fig. 1b are consistent with above arguments.

4.2 The climate sensitivity of glacier area and volume

For the scaling-based simulation of glacier response, the fitted asymptotic fractional changes in glacier area and volume are related to each other (fig. 2a) as, $\frac{\Delta V_\infty}{V} = (1.276 \pm 0.002)\frac{\Delta A_\infty}{A}$. This is in line with the prediction of eq. 2.

As discussed in the previous subsection, a decreasing c is expected to lead to a larger asymptotic fractional volume change for glaciers simulated with SIA. This is exactly what is seen in fig. 2b, which shows $\frac{\Delta V_\infty}{V} = (1.87 \pm 0.02)\frac{\Delta A_\infty}{A}$.

Note that apart from the contribution of fractional change in c to the tune of $\sim$13%, a further amplification of the volume loss due to thickness feedback discussed before also plays a role. Both these effects are not captured by the time-independent scaling assumption, leading to a underestimation of glacier volume change in scaling-based methods. It is interesting that despite a changing c in SIA simulations, the two quantities $\frac{\Delta V_\infty}{V}$ and $\frac{\Delta A_\infty}{A}$, are still proportional to each other (fig. 2b) . We do not have a clear explanation of this effect as yet.

In fig. 2b, about 30 data points with close to the largest value of $\frac{\Delta A_\infty}{A}$ (shown with black circles in the figure) were not included in the fit. The cut-off $\frac{\Delta V_\infty}{V} < 0.5$, prevents sampling of the full vertical scatter in this range, which may create bias in the linear fit and therefore, these points were not considered for the fit.

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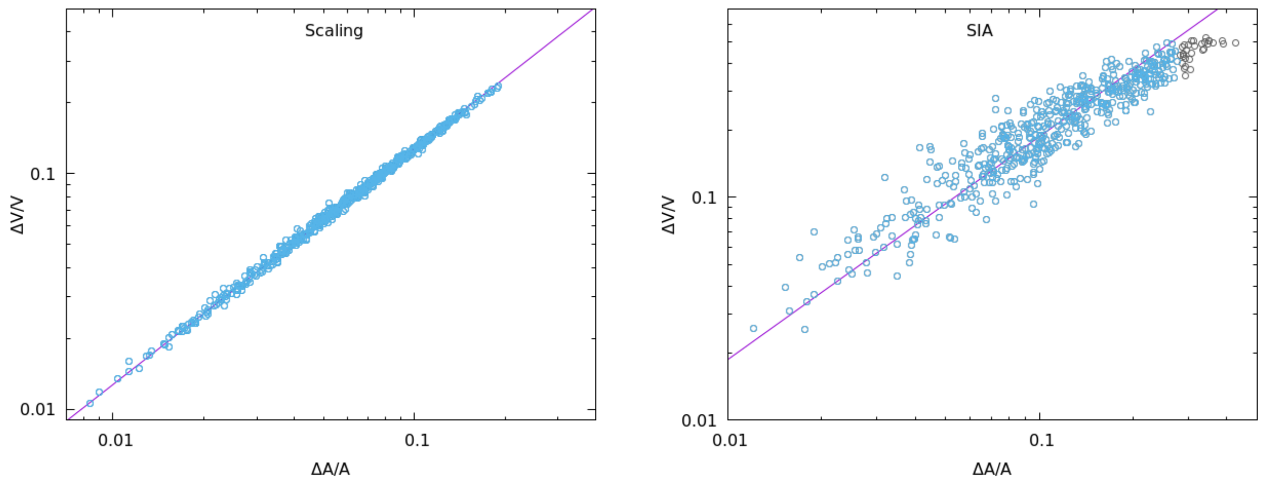

Figure 2. The relationship between the asymptotic fractional changes in glacier area ($\frac{\Delta V_\infty}{V}$) and volume ($\frac{\Delta A_\infty}{A}$) as obtained from, A) scaling and B) SIA-based evolution. The solid lines are best-fit straight lines, A) $\frac{\Delta V_\infty}{V} = (1.276 \pm 0.002)\frac{\Delta A_\infty}{A}$, and B) $\frac{\Delta V_\infty}{V} = (1.87 \pm 0.02)\frac{\Delta A_\infty}{A}$. R^2 values of the fits are 0.99 and 0.85, respectively. In sub-figure B, the points denoted by black circles were not used for fitting. See text for details.

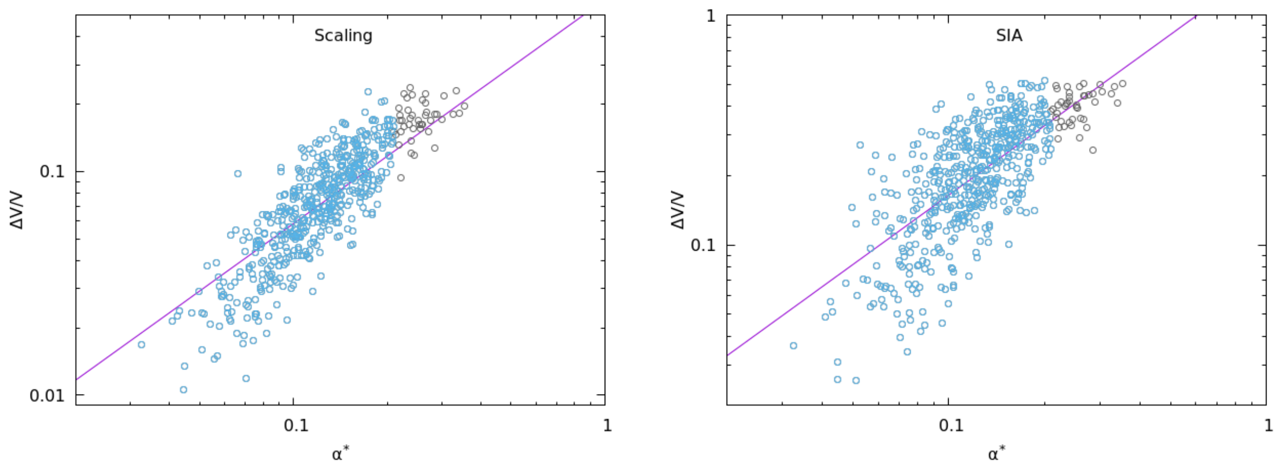

Figure 3. The fractional climate sensitivity of glacier area as obtained from, A) scaling and B) SIA simulations are compared with the theoretically predicted value of $\alpha^* = \frac{\beta\delta E\tau^*}{\gamma h}$ (see text for details) . The best-fit straight lines are, A) $\frac{\Delta V_\infty}{V} = (0.585 \pm 0.009)\alpha^*$, and B) $\frac{\Delta V_\infty}{V} = (1.65 \pm 0.03)\alpha^*$. The corresponding R^2 are 0.65 and 0.48, respectively.

Fig. 3a shows that the prediction of eq. 12 for climate sensitivity of glaciers works for the scaling-based model, though the prefactor is not 1, i.e., $\frac{\Delta V_\infty}{V} = (0.585 \pm 0.009)\alpha^*$. The R^2 of the fit is 0.65. A similar behaviour is also seen for the SIA-derived estimates of $\frac{\Delta V_\infty}{V}$ (fig. 3b), though there is more noise and consequently a smaller R^2 of 0.48.

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The above figure (fig. 3b) may also suggest that a more general power-law function of α^* with a power-law exponent larger than 1, may provide better estimates of $\frac{\Delta V_\infty}{V}$, particularly so for SIA-based simulations. Again, we do not have a theoretical argument for such a power-law behaviour and did not explore this further here. α^* being dimensionless, dimensional arguments does note rule out possible a power-law behaviour.

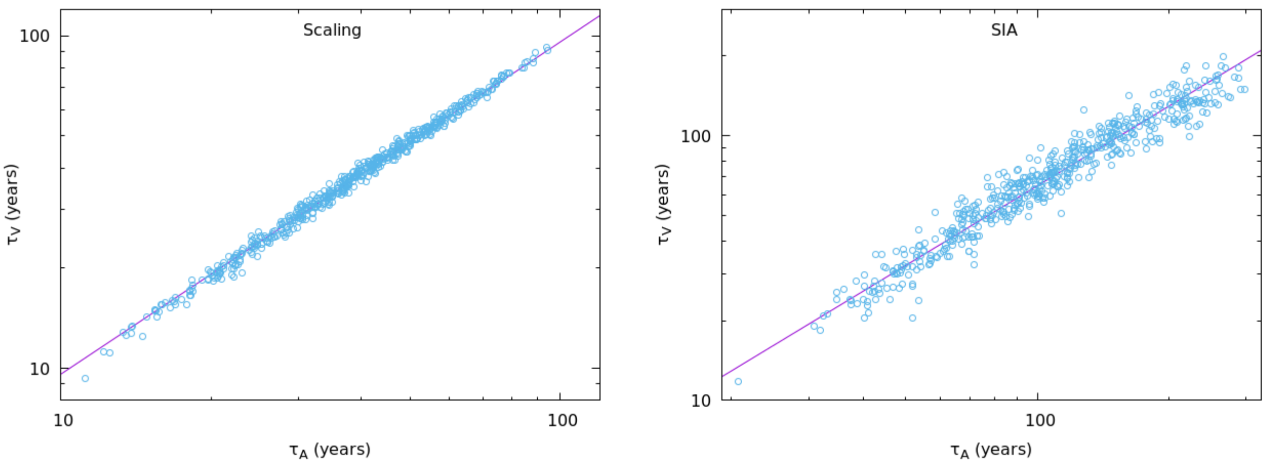

Figure 4. The relationship between the area and volume response time as obtained from, A) scaling and B) SIA-based evolution. The solid lines are best-fit straight lines passing through origin with slope of 0.959 ± 0.001 and 0.647 ± 0.003, respectively. R^2 values for the fits are 0.99 and 0.94, respectively.

4.3 Area and volume-response time

The theoretical prediction of eq. 10 works reasonably well for the scaling-based evolution (fig. 4a), with best-fit functions $\tau_V = (0.959 \pm 0.001\tau_A$ and (fig. 5b) with $\tau_V = (0.945 \pm 0.006)\tau^*$. The above fits have R^2 values of 0.99 an 0.88, respectively.

For SIA evolution, τ_V and τ_A are not equal (fig. 4b). However, τ_V is still proportional to τ_A with best-fit relation being $\tau_V = (0.647 \pm 0.003)\tau_A$, with an R^2 of 0.94. τ_V and τ_A are both proportional to τ^* to a good approximation (fig. 5b) for SIA evolution as well, although the proportionality constant is not 1 contrary to eq. 10.

Apart from the fact that scaling-based evolution leads to an underestimation of glacier response time as described above, an important point of departure of SIA results from that of scaling is that area-response time is larger than the volume-response time in SIA. The difference between area and volume response for mountain glaciers is a known fact (e.g., Banerjee and Shankar, 2013).

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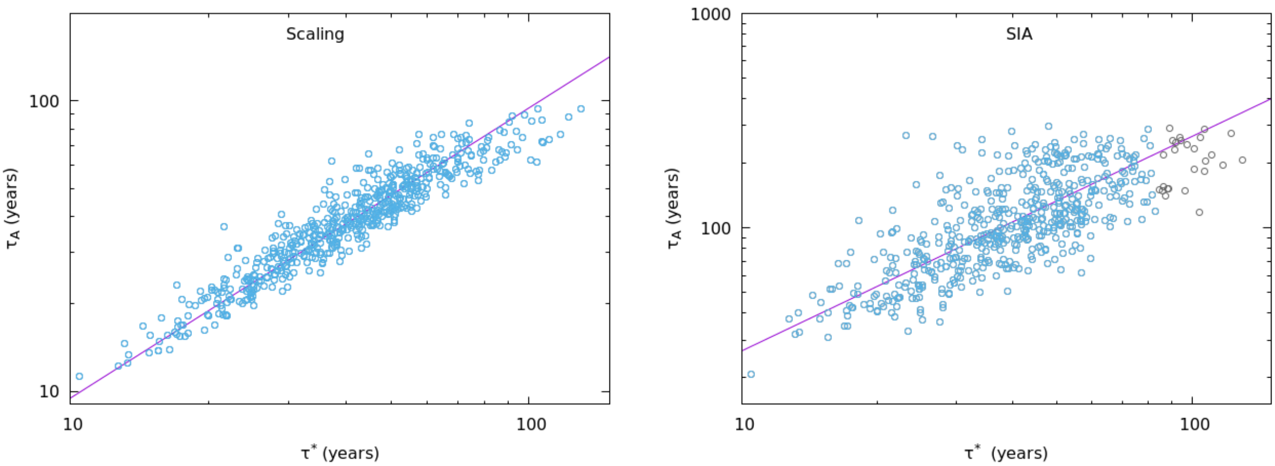

Figure 5. The area and volume response time as obtained from, A) scaling and B) SIA-based evolution are compared with theoretical prediction $\tau^* = \frac{-1}{(\beta + b_t/\gamma h)}$ (see text for details). The best-fit straight lines in log-log scale yield, A) $\tau_A = (0.945 \pm 0.006)\tau^*$ and B) $\tau_A = (2.67 \pm 0.04)\tau^*$, respectively, for scaling and SIA evolution. R^2 of the fits are 0.88 and 0.52, respectively.

4.4 Estimating total glacier loss using scaling and linear response theory

Starting with an initial volume (area) of 603 km^3 (5144 km^2) the 551 glaciers simulated by SIA loses 123 km^3 (521 km^2) of volume (area) in 500 years after the step-change in ELA by 50 m, with most of the changes taking place during the first couple of centuries (fig. 6).

As shown in fig 6, both scaling-based and linear-response model underestimates the long-term change in total area for the same 50 m rise in ELA, with respective values of the predicted area change being 264 and 478 km^2. The scaling model prediction is off by a factor of $\sim \frac{1}{2}$, while the linear response model is within 10% of the SIA values. Very similar trends are seen for the magnitude of volume change as well.In fact, here the scaling-model estimates for long-term change is smaller by a factor of $\sim \frac{1}{3}$. In comparison, the linear-response model predictions are within about 15% of the SIA values.

This relatively strong underestimation in the scaling model results is consistent with the systematic bias introduced by a time-invariant scaling assumption as discussed above. This also suggests that there might be significant negative bias of mountain glacier contribution to sea-level rise as computed by scaling-based methods. It is also clear that the linear-response model performs quite well in reproducing SIA results. We have verified the linear-response model obtained by fitting the SIA simulation results for the ensemble of 551 central Himalayan glaciers, similarly outperforms the scaling-based method for another set of 143 glaciers from the western Himalaya (figure A2).

We note that the aforementioned bias of the scaling-based method is present even when multi-decadal predictions of glacier volume loss is concerned (fig. 6). However, up to $t \lesssim 100$ years the predicted area loss curves are similar for both scaling-based and linear response models. A problem that is apparent with scaling-model predictions is the generally quicker response of the

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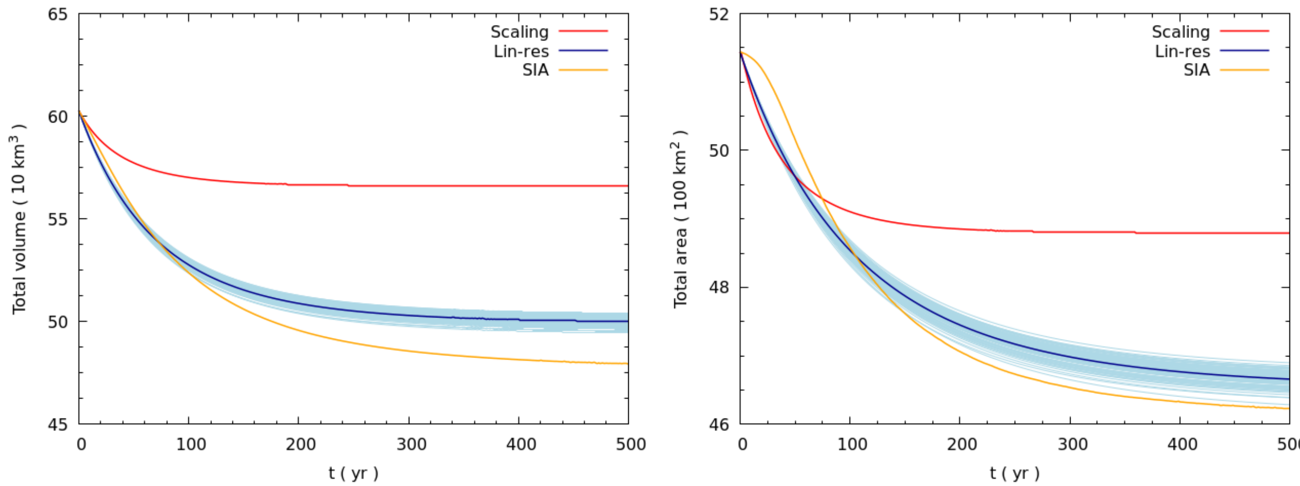

Figure 6. The evolution of the total glacier volume (A), and (B) glacierised area for the ensemble of 551 glaciers simulated with three different methods, namely, SIA, scaling and linear-response model, are shown with orange, red, and blue solid lines, respectively. See text for details.

glaciers evolving under time-independent scaling evolution (fig. 6)). This is consistent with the discussions above that time-invariant scaling-based models underestimates glacier response time. The difference in response time of total area and volume, as seen in SIA simulations, is not captured by the scaling-based model.

4.5 Limitation of the present study

Some of the limitations of the present study, namely, 1) use of SIA that considers only the horizontal shear stresses to describe ice deformation, 2) ignoring the sliding contribution to glacier flow, and 3) the effect of debris-cover and avalanche activity on the mass-balance profile, have already been discussed. Another important simplification is the use of a simple linear mass-balance profile with a cut-off and considering step-changes in ELA as the only mass-balance forcing. In reality, the mass-balance profile and its temporal variability are going to be more complicated. In fact, the mass-balance forcing are likely to vary between regions as well, driving a variable climate response of glaciers. More detailed studies that relaxes some of the above mentioned assumptions are needed to check the validity of and to refine the linear-response model described here.

It is possible or even likely that the inclusion of some the above processes may influence the scaling behaviour by changing the the value of the scaling parameters or by intruding more scatter in the fits. However, based on the argument outlined in the text before, and the SIA model result that the scaling factor c is time-dependent may be quite general. If that is so, then the biases would be present in predictions from time-invariant scaling assumption, even if the magnitude of the bias is different from what is discussed in this paper.

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Since the results of this paper are based on simulation of an ensemble consisting of 551 synthetic glaciers with geometries similar to glaciers in the central Himalaya (and tested on 143 glaciers in the western Himalaya), it remains to be investigated if the results described here depend on the regional characteristics of glaciers to some extent.

5 Summary and Conclusions

In this paper, we perform 2d SIA-based transient simulations of 551 synthetic glaciers with geometries similar to glaciers in the Ganga basin, central Himalaya. The long-term response of these glaciers to 50 m ELA change reveal that the initial ($t = 0$ year) and final states ($t = 500$ years) obey area-volume scaling with the predicted exponent of 1.286. However, the scale factor c reduces by 13% over this period. This changing c introduces a bias in the area and volume change whenever scaling-based models, that assumes c to be time-independent, are used. The scaling method employed here underestimates the long-term changes in glacier area and volume over several centuries quite strongly. Significant differences are also present over the multi-decadal scale. These results points to the possibility of significant biases in predictions of future sea-level change or future extent of glaciers in various glacierised regions, whenever a area-volume scaling-based method is used.

In addition to identifying the bias in scaling-based models, we derived here expressions for glacier-response properties under a time-invariant scaling assumption and verified them with results from a scaling-based simulation of the ensemble of glaciers mentioned above. These expressions were also utilised to obtain best-fit parameterisation of the linear-response properties of glaciers simulated by SIA. A comparison between the response properties obtained for the above two methods confirms the systematic underestimation of climate sensitivity and response time under time-invariant scaling assumption. The response properties obtained from the analysis of SIA-based simulation results lead to a linear-response model that performs quite well in reproducing the time-series of total area and volume. This linear-response model could potentially be useful in predicting large-scale glacier change or global sea-level rise accurately as it reduces the biases that are inherent in scaling-based methods and, at the same time, retains the advantage of computational efficiency.

Code availability. The codes for the low-dimensional models used in this paper are available upon request.

Author contributions. AB designed the study, did the theoretical analysis, and wrote the paper. AJ and DP developed the codes. AJ ran the simulations. All three authors contributed to the analysis of the simulated data and discussions.

Competing interests. We declare that there are no competing interest.

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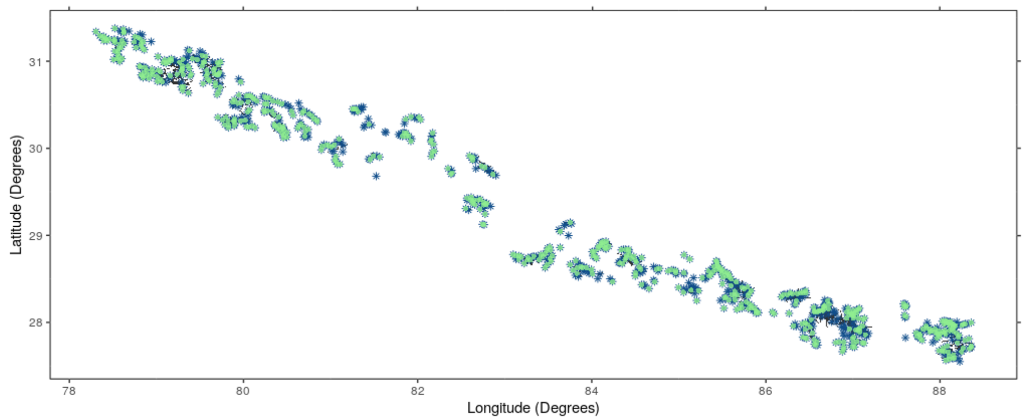

Figure A1. The location 551 simulated glaciers from Ganga basin, the Central Himalaya are shown with filled green circles. The dark blue stars denote all the glaciers in the region with area more than 2 km^2 as per RGI 6.0 inventory.

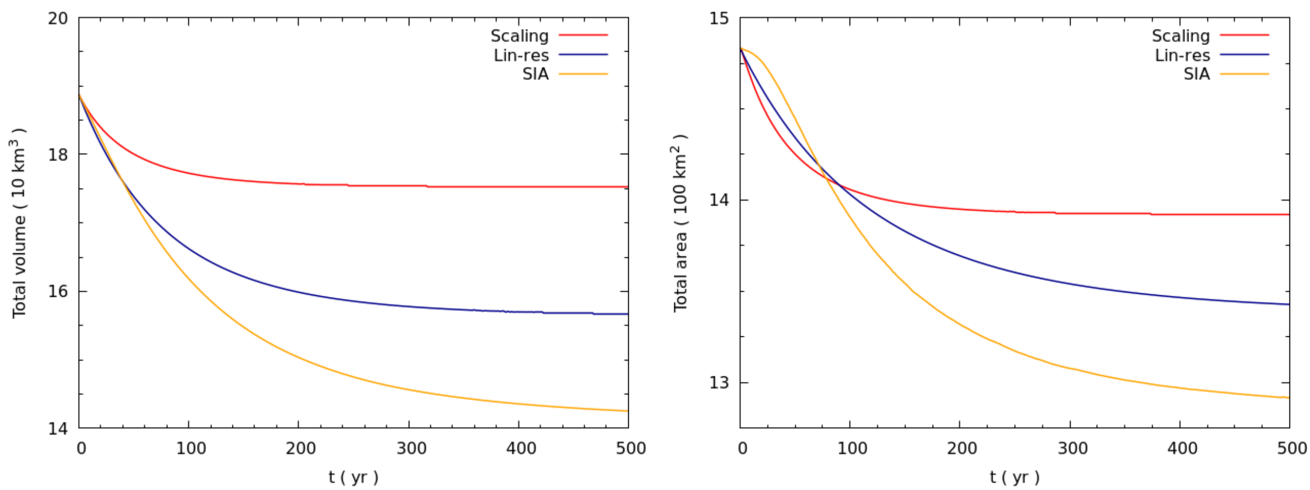

Figure A2. The evolution of the total glacier volume (A), and (B) glacierised area for an ensemble of 143 glaciers from the western Himalaya. These glaciers simulated with three different methods, namely, SIA, scaling and linear-response model. The corresponding results are shown with orange, red, and blue solid lines, respectively. These glaciers are located within 31°N-33°N and 77°E-79°E. The modelled area ranges between 2.7 to 134.4 km^2. The selection criteria, detail of the models and parameters used, and the climate forcing are exactly the same as that used for the ensemble of 551 central Himalayan glaciers described in the main text.

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