Dear Dr Radic,

Thank you for providing us with an opportunity to submit a revised version of the manuscipt. We have modified our manuscript based on the referees' comments, and the changes are as follows.

1. We have provided clear description of the methodology adopted for the linear-response model simluation of the western Himalayan glaciers.

2. The discussion of the intercomparison studies have been expanded which now include reference to Marzeion et al. (2020) as well. We have acknowldeged the future potential of flow-based models while discussing the motivation behind the present work.

In the present version, we have corrected a couple of errors in eq. 16 and 18, and fixed a bug that was present in the scaling model code. We have updated the figures and text describing linear-response and scaling model results accordingly. Please note that updated results did not lead to any changes in the basic results, disccussions, and conclusions.

Also, while we had simulated 204 glaciers from the western Himalaya, results from only 164 were used for the comparison shown in fig S9. The remaining glaciers were not considered due to more 50% long-term changes taking place on these glaciers. We have corrected the text and captions accordingly.

Please note that apart from the experiments with step-changes in ELA to study the glacier response properties, we have also done a comparison of the three models for continuous linear changes in ELA as disccussed later in this document. This was in response to one of the comments by the anonymous referee. The biases of the scaling model and the improved performance of the linear response model can be seen in these experiments as well.

We look forward to your response on the revised manuscript.

Best regards,
Argha Banerjee

Reply to the comments by editor Valentina Radic

Your revised manuscript received two reviews. While the referees acknowledge the efforts you have put into the revisions they still remain critical about the study. Please see their responses below. I am willing to provide you a chance to submit your responses to these comments and revise the manuscript accordingly. In particular, pay attention to the referees' comments regarding the (1) usefulness of V-A scaling in the light of existing more advanced approaches, and (2) the clarity on how the parameters in the V-A scaling are calibrated over the Himalayan glaciers. It is likely that your revised manuscript, once received, will be sent for another round of reviews.

We thank the editor for providing us with an opportunity to submit a revised manuscipt. We have modified our manuscript based on the referees' comments to the best of our ability.

Replying to the comment 1) by the editor, we would like to argue that the present study, which deals with glacier response properties within scaling theory and possible biases of corresponding long-term glacier change estimates, is relevant due to the folllowing reasons.

There are numerous examples of coexistence of 'simple' and 'adavnaced' models in most branches of physical sciences, including in glaciology. For example, all the ice-flow models in the Marzeion et al (2020) uses simple temperature-index models for mass-balance computation, and not more advanced energy-balance models that have been there for decades. We are confident that the low-complexity glacier-evolution models would remain in use in the future, and in that context our results are relevant and valuable.

The reliance on multi-model ensemble averages imply that a variety of glacier model would continue to be used even as flowline models capable of simulating glaciers on a global scale are available. The scaling models are lilkely to contribute significantly to such multi model averages in the future. As we have pointed out in the text, about 2/3 and 1/2 of all the models used in latest sea-level rise estimates by Hock et al (2019) and Marzeion et al (2020), respectively, were scaling models. They are also being used for hydrological simulations. This makes it important to develope and critical investigate of low-complexity models, rather than focusing exclusively on SIA models.

There are two other contributions presented in the paper that may be important. First, the linear-response model presented here is an accurate low-complexity alternative to model glacier evolution. As such, this may be an useful addition to the ensemble of low-complexity models used for computing sea-leve rise. Second, our systematic derivation of the response properties of glaciers under a scaling assumption, and parameterisations for SIA derived response properties, we believe, are useful additions to the exisitng literature on glacier response properties.

In response to the comment 2) by the editor, we have now included additional discussions about the calibration of the linear-response model using SIA results for the central Himalayan glaciers, the application of the model to simulate western Himalayan glaciers, and possible application of the linear-response model to other glacierised regions. Hope this has led to improved clarity of our presentation of the linear-response model. We thank referee Eviator Bach for pointing this out.

Our detailed (in **black**) to the comments by the referees (in **red**), along with details of the corresponding changes in the manuscript are given below.

Reply to the comments by referee Eviator Bach

However, I am still unclear on two issues, which I believe have not been adequately addressed from the last revision:

1. How is the linear response model applied in projecting the volume loss? In the paper, the authors write that "For each of the 703 glaciers, the time series of volume and area as obtained using the SIA and scaling models, were separately fitted to linear-response forms (e.g., eq. 9 below) to obtain the corresponding best-fit values of the four linear-response parameters (the climate sensitivities and the response times of area and volume) for each of them". If the linear-response model is fit individually to each glacier, how was it then applied to the "204 glaciers from the western Himalaya without any further calibration"? Was some sort of average chosen for the linear-response parameters?

We apologise for the confusion. We have now tried to clarify the procedure as follows.

L194
"The best-fit empirical parameterisations for climate sensitivity and response time obtained by fitting the SIA results as described above (given in eqs. 14−17 later), were used to run a linear-response model simulation for any given glacier. This model was applied to simulate the response of the above 703 synthetic Himalayan glaciers to a 50 m step-change in ELA at t = 0. We emphasise that for the linear-response model, we do not use the best-fit the response properties of the individual glacier derived from the SIA simulations. Rather, the parameterisations of the same obtained by fitting the SIA-derived response properties (given in eqs. 14−17 later) were utilised. These parameterisations thus allow the model to be applied to any other set of Himalayan glaciers without the need for simulating them with SIA first."

L 204
"To test the applicability of the above linear-response model that was calibrated using SIA results for the 703 central Himalayan glaciers, the same model was applied to a different set of 204 glaciers from the western Himalaya. The parameterisations developed for the central Himalayan glaciers as discussed above (given in eqs. 14−17 later) were used to estimate the response properties of each of these western Himalayan glaciers using input values of corresponding mass-balance gradient, mean thickness and ablation rate near the terminus. For these western Himalayan glaciers, SIA and scaling model simulations were also performed following the procedures as detailed above."

L374
"To confirm the improved performance of the linear-response model compared to that of the scaling mode we applied both the models to simulate a different set of 164 glaciers in the western Himalaya (supplementary fig. S1). The best-fit linear-response properties obtained from SIA simulation of the 703 central Himlayan glaciers were first fitted to obtain four equations (eqs. 14−17) that relates the response properties toβ, γ, h and b_t as described before. The same equations were used to estimate response properties of each of the 164 western Himalayan glaciers as required for the linear-response model simulations."

2. How you are proposing to apply the linear-response model for a real glacier? Are you suggesting that, for each glacier, an SIA model should be run to generate a time-series, fit the linear-response parameters, and then use the linear-response model to project into the future? If so, what is the advantage of this approach over just running the SIA model into the future for each glacier? This

should be made explicit.

The parameterisation provided in eqs. 14-17 can be used to estimate the response properties of any glacier as long as estimates of mean thickness, balance gradient, gamma, and ablation rate near the terminus are available - Just as we have demonstrated for the western Himalayan glaciers. However, since the above paramterisations were derived for conditions typical of Himalayan glaciers, it may be necessary to test the parameterisations by SIA simulation of a few tens of glaciers spanning a wide range of area and slope values, before applying it to any other region. We have now discussed this in the manuscript (L422):

“Since the above parameterisation of linear-response perperties (eqs. 14−17) are derived from SIA simulations of an ensemble of Himalayan glaciers, when applying them to any other glacierised region in world, it may be necessary to simulate a few tens of glaciers (having a representative range of area and slope) from that region using SIA first, and confirm the accuracy of the above parameterisations.”

Typographical issues:
– Bach et al. is cited as 2019 in the text, but it is from 2018
– paramterisations -> parameterisations

We have corrected these errors.

Reply to comments by anonymous reviewer

Banerjee and colleagues have put a substantial effort in updating their manuscript in order to answer the issues raised by both reviewers. Through this, they have addressed some issues raised (e.g. more clarity on mass conservation, some unclear statements were removed and clarified). However, some important foundations of the story are still problematic.

We thank the reviewer for the kind comments. Our replies to his/her specific criticisms are given below.

Scaling methods are losing significance now that methods arise in which the glacier geometry is explicitly accounted for. The authors state that they are not aware of any studies in which sea-level contribution is calculated based on flow models. OGGM was applied globally (Maussion et al., 2019), and was used to project the future contribution to sea level from all glaciers (Marzeion et al., 2020).
The argument that an idealized setup is not useful to compare various methods does not hold in my opinion.

Despite the advent flow-based models, the scaling models continue contribute significantly to the multi-model mean. The recent intercomparison studies by Hock et al (2019) and Marzeion et al (2020), report multimodel ensembles where 45 - 66% members were scaling-based models. Thus, it is important to probe the scaling model assumptions, and possible prediction biases in such models. We have refered to the flowline model studies appropriately now. Please refer to our replies to the comment (1) by the editor above for our detailed reponse on this issue.

We never claimed that idealised setup is not useful for model comparisons. They certainly are, and that is why we have used them in this study to confirm the general nature of our results based on the earlier suggestions by the reviewer (please refer to the highly ideliased 1-d setup presented in supplementary fig. S10). However, an idealised setup is inadequate to obtain parameterisation of response properties (e.g., eqs. 14-17) that can be applied to real glaciers. Our objective of establishing a linear-response model necessitates the realistic geometries as explained before.

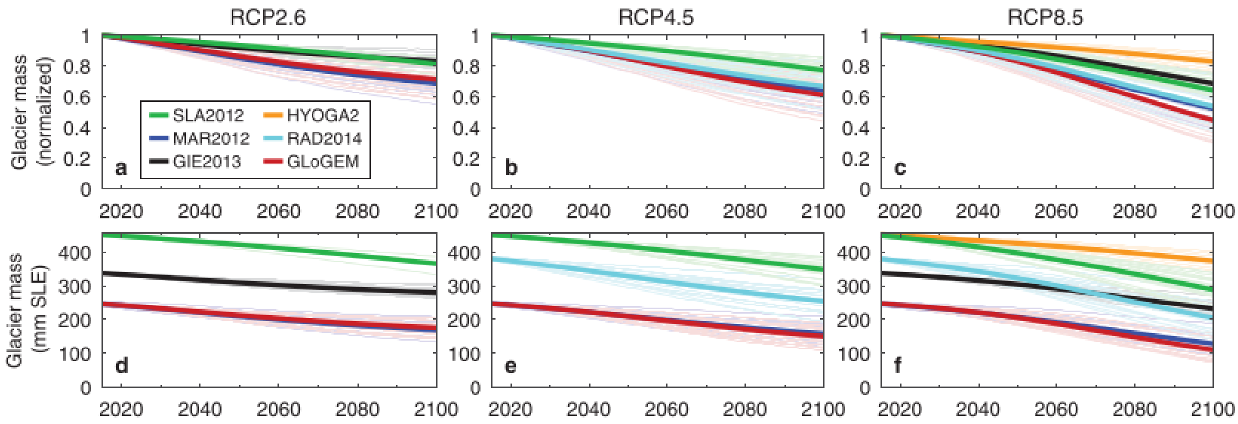
To make claims about the suitability of scaling-based methods for sea level contributions based on Himalayan glaciers (or any mountain glaciers) does not really make sense. When it comes to sea-level contribution studies, one should focus on ice caps and big Arctic glaciers (i.e. not mountain glaciers), which contain the almost entirety of the worldwide glacier volume (see e.g. Table 1 in Farinotti et al., 2019). i.e. you investigate the effect of scaling-based methods on the future evolution of mountain glaciers: sea level is out of the context here.

We beg to disagree with the above point of view. First, the realtive contribution of any set of glaciers to sea-level rise over a given span of time depends on both their climate sensitivity and response time, apart from their volume. Typically glaciers across all size classes are considered in the exisiting sea-level rise computations. Second, all of our discussions are relevant and useful to model any glacier (excluding ice-sheets and ice caps) independent of its size and location, including the ones refered to by the referee.

In their answers Banerjee and colleagues suggest that there is a clear evidence of underestimation of glaciers relying on V-A scaling arguments in GlacierMIP (Hock et al., 2019). This is not very clear, and moreover, the problem with GlacierMIP is that the setup was very different for the various models (i.e. it is difficult to compare a V-A scaling and another type of model if the forcing is totally different). The good news is that this has partly been solved in the second phase of GlacierMIP (Marzeion et al., 2020). Another advantage of this new study is that there are more models to compare and that the comparisons can also be made at the regional level. All the data is freely available. The authors would have to look into this, but I m afraid that also here there is no clear sign of V-A scaling based methods vs. Others.

We agree with the referee that it is hard to establish systematic biases among different models unless the models are all applied to the same set of glaciers, with the same initial volume, and forced by the same climate forcing. Though we have done exactly that this in our study of the sythetic glaciers, the strategies in the above intercomparison studies are different. So it is not expected that the above studies would help in concluding about the scaling model biases that we present in out manuscript. We have acknowledged that in the present manuscript (L356-L363).

However, as we have argued in the text, the intercomparison studies referred to above do not rule out the possibility of a systematic bias in scaling models. For example, even as some of the scaling models start with an initial volume that is ~200% larger than that used in GLoGEM model (see the relavant portion of fig3 of Hock et al. (2019) reproduced below), more often than not GloGEM yielded higher estimates of fractional change on a global scale.



Similarly, the percentage changes under three clmiate scenarios presented in Fig 5 and 11 of Hock et al. (2019) showed larger estimated global change wer obtaine with GloGEM model. The trend is particularly clear in figure 11 which compared runs forced with the same GCM outputs.

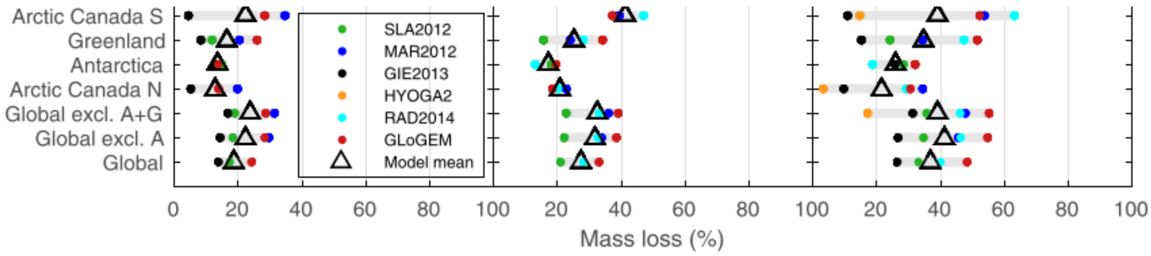


Fig. 5. Projected mass losses by 2100 in percent of the glacier mass in year 2015 for 19 RGI regions from six glacier models using three RCP emission scenarios. Dots mark the multi-GCM means for each glacier model connected by gray bars, and triangles show their arithmetic mean. Regional results are sorted by the glacier models' mean mass loss according to the RCP8.5 scenario. Results are also shown for all regions combined (global), and all regions excluding the Antarctica periphery (A), and excluding the Antarctica and Greenland periphery (A + G). Note that not all glacier models compute all regions or use all three emission scenarios. The data are available in the Supplementary Material.

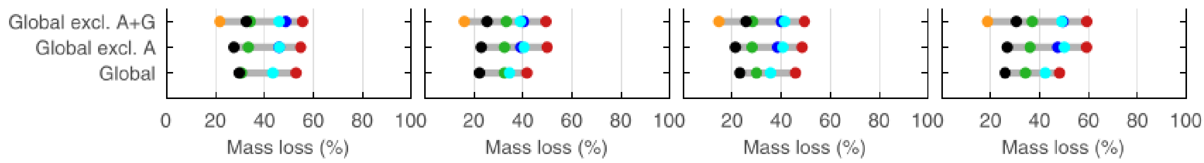
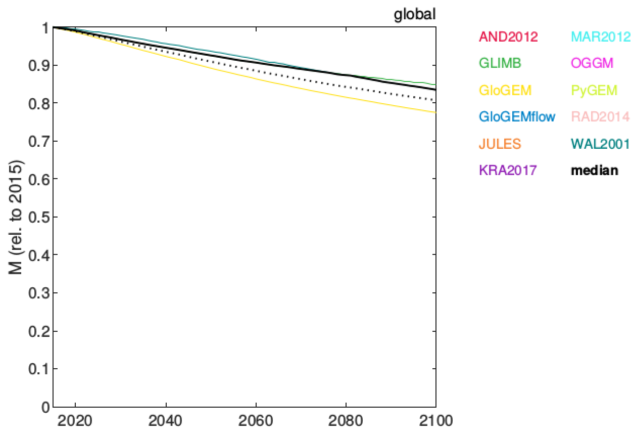
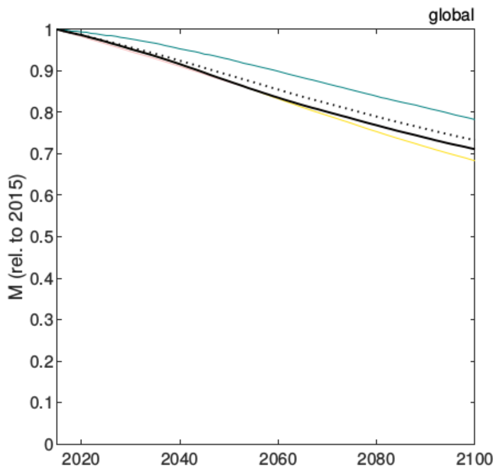
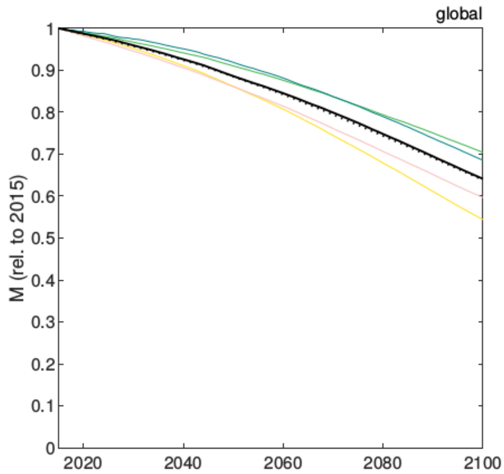


Fig. 11. Projected mass losses by 2100 in percent of the glacier mass in year 2015 for 19 RGI regions and globally excluding Antarctic and Greenland periphery (A + G) based on the four GCMs that were used by all six glaciers models. Results are based on RCP8.5. Dots mark the results for each glacier model connected by gray bars. Regional results are sorted as in Figure 5.

Very similar trends are seen in estimated global-scale fractional glacier loss by 2100 under different climate scenario as given in Marzeion et al. (2020). The relevant subfigures of fig. S17, S18 and S20 reproduced below showed higher changes in GloGEM compared to the scaling-based GLIMB, WAL2001 and RAD2014 models. However, we agree with the referee that at regional scales, the above trend is not always seen.







Overall, the above intercomparison studies at least does not rule out the possiblity of a systematic bias in the scaling model. And as mentioned above, to reach a definite conclusion, the different models should be applied to the same set of glaciers that are initialised with the same volume and hypsometry, and are forced by the same climate data.

Some parts remain vague. It is for instance unclear how your algorithm failed for many glaciers, and now that you have performed an update of your algorithm this is solved...

In the earlier version of the paper the steady-state criterion was that the aboslute fractional changes in mean glacier thickness has to be less than 10^{-6}. In the present run, we have changed it to a criterion on specific mass balance: any state with an abolute mass balance less than 10^{-4} m/yr is assumed to be steady. This reduced the computation time required to find the steady state - particularly for intermediate states with mean ice thickness less than 100 m. This helped in finding the right steady state for most of the glaciers within a reasonable computation time. Please note that tuning the ELA to obtain a steady glacier having extent similar to the present one is the most expensive piece in our computation. For each trial ELA value, the model needs to be run for several hundreds to a few thousand year until the corresponding steady state is reached.

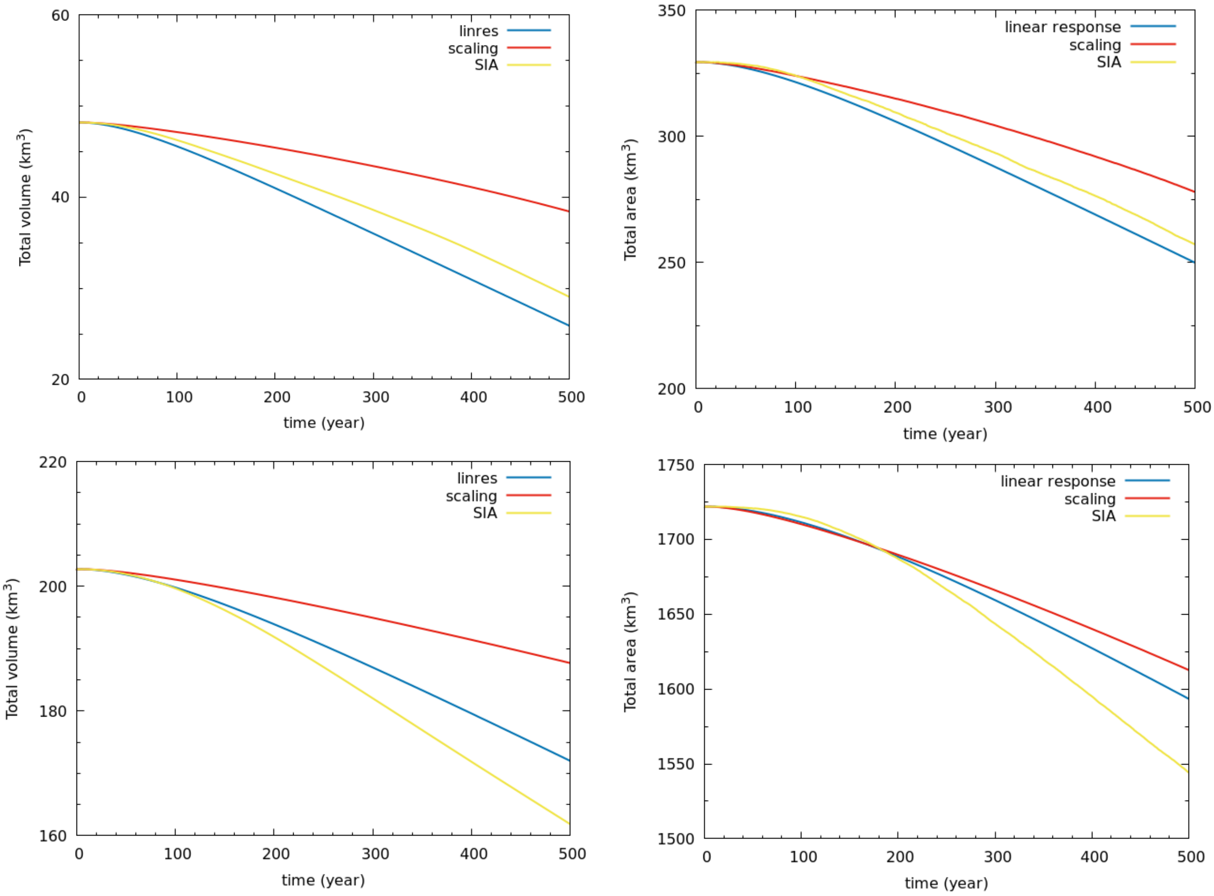
It remains problematic to see that the model was not calibrated for individual glaciers. The argument to avoid the associated computational cost is not a very solid one... Such models are computationally cheap to run and given the relatively limited sample of glaciers considered (within the framework of regional- to global studies), this should not be a problem. By having a realistic geometry, the velocities will automatically also be relatively close to the observations (and the argument Tuning the rate factor to fit the thickness may not be a good idea, as it may lead to unrealistically small glacier velocities, and thus, unrealistic response properties does therefore not hold).

As we have argued in our previous reply, while a calibration of the SIA model is necessary to obtain accurate glacier-change predictions for the Himalaya under climate change, it is not needed to investigate relative biases between models. This, in fact, is consistent with the reviewer's assertion above about the effectiveness of idealised setup. What we need here is a set of synthetic glaciers with realistic geometries and other physical properties. Details of the modeled glaciers provided in the supplementary demonstrate that the ensemble studied here serves that purpose adequately.

While we agree that realisitc geometry gets the shape of the velocity profile right, it would not get the corresponding scale right without an appropriately chosen glacier-specific rate factor. And varying the rate factor would also impact the ice thickness. Therefore, we believe, it is important to calibrate for both the velocity and thickness data.

The conclusions are in the end still based on a comparison of the evolution between steady states (which boils down to comparing steady states). The lack of real transient analyses makes it difficult to support any claims related to validity of transient models (which are used for sea level rise studies).

To our knowledge, all discussions of response properties of mountain glaciers in the literature are based on studying transitions from one steady state to another. The trasient response of a system is assumed to be determined by the response properties that are defined with respect to the initial and final steady states, which is then calculated using eq. 18 or its analogs. We have simply followed that standard paradigm here.
However, based on the reviewer's suggestion we have performed some additional transient analysis. The figures below show the computed transient response of the glaciers in the western Himalaya to a linear rise in ELA at the rate of 50m per century (top row), and 10 m per century (bottom row), resepectively. The ELA change were applied continuously over the simulation period of 500 years. (As in the experiments discussed in the main text and the supplementary material, for each of these experiments below we leave out the glaciers with more than 50% change at 500 years mark).



As expected, the outcomes of these transient experiments are consistent with the general results obtained in the manuscript based on an analysis of the step response of steady-state glaciers as simulated by the three different models.

In conclusion, I am still not convinced by the statement that V-A scaling methods are likely to underestimate the future sea level contribution from glaciers. And even with a good modelling setup and clear presentation of your results, I do not think that any conclusions on sea level contribution validity of different models can be obtained from a study on Himalayan glaciers (or any other mountain glaciers, given the limited total volume of ice stored in these ice bodies).'

We have provided the following convincing evidence to support the claim of possible biases in scaling model.

1) The theoretical results on response properties of scaling-model glaciers,

2) The numerical comparison scaling model resutls with that from a 2-d SIA model simulation for a set of 703 and 164 synthetic Himalayan glaciers responding to step (and linear) change in ELA,

3) numerical evidence from a comparison of 1-d (SIA) flowline model and a scaling model wihtin a highly idealised setup,

They all support our conclusions. As we have discussed in the manuscript, and in our replies above, the intercomparison studies may also be consistent with our result. However, it is also true that these intercomparison studies are not ideally suited to make any strong conclusion on this debate due to the differences in initial conditions, climate forcing and so on, between the models.

The past contribution of mountain glaciers to sea-level rise has been estimated to be 30% since 1900, and their contribution to potential sea-level rise is expected to remain significant for this century at least. And of course, our results are not restricted to the central Himalayan glaciers studied, and of relevance to any glaciers (other than Ice-sheets and Ice caps) that follow the volume-area scaling that is discussed in the text.

References

Hock, R., Bliss, A., Marzeion, B., Giesen, R. H., Hirabayashi, Y., Huss, M., et al. (2019). GlacierMIP – A model intercomparison of global-scale glacier mass-balance models and projections. Journal of Glaciology . https://doi.org/10.1017/jog.2019.22

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Possible biases in scaling-based estimates of mountain-glacier contribution to sea-level rise

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Abstract. ~~Low-complexity~~ Approximate glacier models are used to compute the contribution of mountain glaciers to sea-level rise given a climate-change scenario. A majority of these models are based on statistical scaling relations between glacier volume, area, and/or length. In this paper, the response properties of glaciers ~~are theoretically analysed within~~ resulting from a time-independent volume-area scaling assumption are theoretically analysed. The theoretical results are validated with a scaling model simulation of the response of 703 synthetic Himalayan glaciers from the Ganga basin to a step-change in climate. The same ~~numerical~~ experiment repeated with a 2-d shallow-ice approximation (SIA) model, obtains about ~~three~~ two times larger climate sensitivity and response time than that predicted by the scaling model. ~~There is a corresponding~~ This indicates a possible low bias in the scaling model estimates of the long-term loss of ~~the total~~ glacier area and volume. ~~Also, the scaling model predicts~~ Scaling models predict the area and volume response times to be equal to each other, while the SIA model obtains area response time that is about 1.5 times larger than the corresponding volume response time. Consequently, the transient glaciers simulated with SIA exhibit a systematic violation of time-invariant scaling. The SIA results are used to obtain parameterisations of climate sensitivity and response time of ~~glaciers, leading to a~~ the glaciers in terms of corresponding ablation rate near the terminus, mass-balance gradient, and mean thickness. A linear-response model ~~which~~ based on these paramterisations outperforms the scaling model in reproducing ~~the SIAresults~~glacier response as simulated with SIA. This is confirmed ~~by an experiment on an independent set of 204~~ in an independent experiment with a set of 164 glaciers from the Western Himalaya. This linear-response model may be useful for predicting the sea-level contribution from shrinking mountain glaciers.

1 Introduction

Shrinking mountain glaciers have contributed significantly to global eustatic sea-level rise in the recent past, and this trend is expected to continue for the next hundred years or so ~~(Meier, 1984; van de Wal and Wild, 2001; Raper and Braithwaite, 2006; Cogley, 2009;~~ The reliability of the predicted global sea-level change is, thus, intimately tied to the accuracy of the predicted total ice-loss from mountain glaciers for any given climate scenario.

Instantaneous (annual) glacier surface mass balance can be calculated readily using climate model outputs. In contrast, any prediction of the long-term evolution of a glacier requires simulating the slow (decadal) changes in glacier area and geometry. Ideally, this is to be done by solving the dynamical ice-flow equations (e.g., Oerlemans, 2001). However, the numerical cost of such a computation on a global scale ~~creates a bottleneck~~is high, even if simplified approximate descriptions of the ice-flow equations, like, shallow-ice approximation (SIA) (Hutter, 1983) or its higher order variants were to be used (Egholm et al., 2011; Clarke et al., 2015). One-dimensional SIA-based modelling tools are promising developments in this regard (Maussion et al., 2019; Zekollari et al., 2019; Rounce et al., 2020) and have recently been used for global sea-level rise computation (Marzeion et al., 2020). The uncertainties associated with various input parameters, e.g., an uncertain glacier bedrock, limit the benefit of using the physically-based ice-flow models ~~as well (Farinotti et al., 2016).~~

~~Due to the above difficulties, the existing~~ (Farinotti et al., 2016). Consequently, a majority of the recent global-scale estimates of the contributions of shrinking mountain-glaciers to sea-level rise ~~mostly rely~~ relies on low-dimensional approximate parameterisations of glacier dynamics ~~(Radić et al., 2014)~~(Radić et al., 2014; Hock et al, 2019; Marzeion et al., 2020). The results from these simplified models ~~provide~~ have provided critical inputs for assessing regional to global vulnerability to sea-level rise (e.g., Kulp and Strauss, 2019)~~.~~ , and contributed strongly to the multimodel ensemble-averaged predictions of future sea-level rise (Hock et al, 2019; Marzeion et al., 2020).

While some of ~~these parameterisations~~ the above parameterisations of glacier dynamics are empirical prescriptions for adjusting the hypsometry of the transient glaciers (Raper and Braithwaite, 2006; Huss et al., 2010; Huss and Hock, 2015), a majority of them are primarily based on a statistical volume-area (or volume-area-length) scaling relation. This volume-area scaling equation relates glacier volume V to glacier area A as,

$$V = cA^{\gamma}, \qquad (1)$$

where, γ is a dimensionless scaling exponent, and c is a scale factor (Bahr et al., 2015). This relation was established empirically (e.g., Chen and Ohmura, 1990), and subsequently proved using dimensional analysis (Bahr et al., 1997, 2015). The derivation utilised the empirical sub-linear scaling of glacier width and ablation rate with the glacier length (Bahr, 1997).

Theoretically, the scaling exponent γ is time-independent, and can be expressed as $\gamma = 1 + \frac{m+1}{m+n+3}$ (Bahr et al., 2015). Here, n is the power-law exponent of Glen's rheology of ice (Glen, 1955), and m is the scaling exponent of ablation rate with glacier length (Bahr, 1997). For an individual glacier, the scale-factor c captures the control of all the glacier-specific factors (except area) on its volume (Bahr et al., 2015). There is no available theoretical prescription for obtaining the value of c for an arbitrary glacier. c may be calibrated for a particular glacier based on available independent measurements of area and volume ~~during~~ over an epoch, but its time dependence can be accessed only with a detailed model simulation (Bahr et al., 2015).

For a large enough ensemble, glacier area typically spans a few orders of magnitude. However, the corresponding c values vary over a relatively restricted range (Bahr et al., 2015). This allows an approximate statistical description of any set of glaciers using eq. 1, where a single best-fit c and a fixed γ is used (Bahr et al., 2015). Such a best-fit scaling relation provides a fairly accurate estimate of the total ice volume of a large set of glaciers, but the corresponding predictions for the individual glaciers have relatively large uncertainties (Bahr et al., 2015). Note that there is no theoretical constraint for c to be time-independent for a given set of non-steady glaciers (Bahr et al., 2015).

It is the above statistical interpretation of the scaling relation, where a best-fit time-invariant c and a constant γ is used to describe an ensemble of glaciers, that is exploited in the scaling-based approximate models of glacier dynamics (e.g., Radić et al., 2007). Hereinafter, we refer to the ~~class of~~ models that are based on such an approach (e.g., Radić et al., 2007), as "scaling models". As the present study investigates the possibility of biases in scaling model predictions of the sea-level rise contribution of mountain glaciers, we restrict ourselves to the above statistical interpretation of the scaling relation.

The performance of scaling models in simulating the transient glacier response have previously been tested against various dynamical ice-flow models (e.g., SIA, higher order approximations, or Stokes' model) in one to three dimensions using both idealised (Radić et al., 2007; Adhikari and Marshall, 2012) and realistic geometries (Radić et al., 2008; Farinotti and Huss, 2013). The uncertainties introduced by a scaling-model parameterisation of the evolution of glaciers with realistic geometries were considered by Farinotti and Huss (2013). The spirit of the present study is quite similar to that of Farinotti and Huss (2013), except that we are investigating the possible intrinsic biases of scaling models in a situation where the parameters (c and γ) are known accurately. The specific objectives of the present study are,

1. To obtain analytical predictions for climate sensitivity and response time of glaciers in a scaling model.

2. To compare the climate sensitivity and response time of a large number of synthetic glaciers with realistic geometries, as obtained from a scaling model and a 2-d SIA model.

3. To investigate the possibility of long-term biases in scaling model estimates of changes in glacier area and volume with respect to corresponding SIA results.

4. To find convenient parameterisations of glacier response properties obtained from the SIA simulations, and develop an accurate linear-response model.

Note that ~~the last objective involves~~ a linear-response model ~~which~~ introduced in the last objective is a low-complexity model obtained in the limit of a relatively small deviation around a steady state (e.g., Oerlemans, 2001). To apply this model ~~on~~ to a large number of glaciers, the response time and climate sensitivity need to be specified for each of them. A lack of accurate and numerically-convenient parameterisations of these dynamical properties may have limited their application (Harrison et al., 2001; Lüthi, 2009; Bach et al., 2018). Here, we aim to obtain parameterisations of the glacier response properties as functions of a few easily accessible properties of the glaciers, using results from 2-d SIA simulations of a large ensemble of synthetic glaciers with realistic geometries.

The paper is organised as follows. First, we theoretically derive the glacier-response properties within a time-invariant scaling assumption (sect. 2.1 and 3.1). Then, we compare the performance of a representative scaling model (Radić et al., 2007) with that of a 2-dimensional SIA model, in simulating the response of 703 idealised Himalayan glaciers in the Ganga basin to a hypothetical step rise in equilibrium line altitude (ELA) (sect. 2.2 and 3.2). We use the response properties obtained from the scaling model to test the above analytical expressions for glaciers-response properties. The corresponding SIA results are used to obtain parameterisations for the linear-response properties of realistic glaciers. The accuracy of the scaling model and a linear-response model in reproducing the SIA-derived long-term loss of total glacier area and volume is assessed for the above 703 glaciers. The performance of the linear-response model is also tested for an independent set of ~~204~~ 164 glaciers in the western Himalaya without any further ~~calibraton~~calibration. We also discuss the applicability of the linear-response model for actual computation of future glacier loss for a set of transient glaciers forced by any arbitrary time-variation ELA (sect. 3.3).

2 Methods

2.1 Theoretical methods

For a theoretical analysis of the glacier-response properties implied by a scaling model, we consider a set of hypothetical glaciers that are responding to a warming climate such that the volume-area scaling relation (eq. 1) is valid, and c is a given time-invariant constant. Then, the fractional changes in area and volume of these glaciers, in the limit of small changes, are related as follows.

$$\Delta V \approx c\gamma A^{\gamma-1}\Delta A = \gamma\frac{V}{A}\Delta A = \gamma h\Delta A, \tag{2}$$

where, ΔV and ΔA are the changes in area and volume, and the mean ice thickness is $h = V/A$. The above equation is the basis of the scaling models of glacier evolution (e.g., Radić et al., 2007). We have derived analytical expressions for glacier response time and climate sensitivity starting from this equation, essentially following the line of arguments by Harrison et al. (2001).

2.2 Numerical methods

We simulated the response of an ensemble of synthetic clean glaciers with realistic geometries to a hypothetical step-change in ELA using three different methods (scaling, SIA, and linear-response models). For this exercise, we considered all the 814 glaciers larger than 2 km^2 in the Ganga basin, the central Himalaya (Supplementary fig. S1). The ice-free bedrock for each of the glacier was obtained using available ice-thickness estimates (Kraaijenbrink et al., 2017) and ~~surface elevation~~ surface-elevation data (ASTER GDEM, V003). The following idealised elevation-dependent linear mass-balance profile was used,

$$b(z) = Max\{\beta(z-E), b_0\}. \tag{3}$$

Here, β is the balance gradient, z is the surface elevation, and E is the equilibrium-line altitude (ELA). b_0 is a cutoff on maximum accumulation taken to be 1.0 m/yr. The choice of β is described later. In our mass-balance model, we neglected complicating factors like supraglacial debris cover and its effects on ablation, and the avalanche contribution to accumulation. Overall, the simulated glaciers ~~can not~~ cannot be considered faithful copies of the actual Himalayan glaciers. Rather, they constituted an ensemble of synthetic glaciers with realistic geometries (e.g., Farinotti and Huss, 2013) to be used here for a comparative study of the performance of the three models.

2.2.1 A 2-d SIA model

The ice-flow dynamics was implemented within a two dimensional SIA (Hutter, 1983) as a numerically efficient non-linear diffusion problem (Oerlemans, 2001). While SIA may not be the best method for simulating valley glaciers due to its limitation in describing ice-flow influenced by longitudinal stresses and/or steep bedrock slopes (Le Meur et al., 2004), there is enough evidence in the literature that SIA does a reasonable job of describing both the steady and transient dynamics of valley glaciers (e.g., Vieli and Gudmundsson, 2004; Le Meur et al., 2004; Radić et al., 2008). The contribution of sliding to the flow was neglected here for simplicity.

The value of Glen's flow-law exponent was assumed to be 3 (e.g., Oerlemans, 2001). For the sake of simplicity, we did not tune any of the model parameters to match the observed ice-thickness and/or flow velocity on any of these glaciers. The only exception was ELA which was tuned to obtain the initial steady state as described below. In order to avoid possible dependence of the results on any specific choice of parameters, we picked the parameters related to mass balance and flow from random distributions. The rate constant of Glen's law was picked randomly from the set $\{0.5, 0.6, ..., 1.4, 1.5\} \times 10^{-24}$ Pa^{-3}s^{-1} for each of the glaciers. This range of values is comparable to those used to model mountain glaciers previously (Radić et al., 2008). The balance gradient β was also picked randomly from the set of values {0.005, 0.006, ..., 0.009, 0.010} yr^{-1} for each glacier. This range of β-values is comparable to the observed mass-balance gradients in the Himalaya (e.g., Wagnon et al., 2013).

The model was integrated using a linearised implicit finite-difference scheme (Hindmarsh and Payne, 1996), with a no-slip boundary condition at the ice-bedrock interface and a no-flux boundary condition at the domain boundary. An iterative conjugate-gradient method was employed within the implicit scheme, with a spatial grid-size of 100 m$\times$100 m and time steps of 0.01 years. To avoid the known problem of a possible violation of mass conservation in SIA on steep terrains (Jarosch et al., 2013), we smoothed the bedrock with a centrally-weighted 3×3 moving-window averaging. In addition, the conservation of ice was explicitly monitored by tracking the total accumulation and ablation on the glacier surface, and the ice flux out of the glacier boundary in the ablation zone. The cumulative net gain of ice matched the total ice in the domain to within one part per 10^9 at any time t. Only on three glaciers (out of the total of 814), a violation of conservation due to steep bedrock was observed, and these three were not considered in our analysis (supplementary figure S2). One more glacier had to be removed where an erroneously mapped truncated tributary ~~lead~~ led to an unrealistic piling up of ice (Supplementary fig. S2).

The SIA simulation was run starting with an empty bedrock, with the initial E being the median elevation. The simulation was continued until ~~a~~ an approximate steady state was reached such that the absolute value of the net specific balance was less than 10^{-4} m/year. Subsequently, E was moved up or down, and the simulation was repeated ~~,~~ until the extent of the steady state was similar to the present glacier extent (RGI, 2017) (Supplementary fig. S2). Once the desired steady state was found (See supplementary fig. S3 for a few examples), the glaciers were perturbed by a 50 m step rise in ELA. Subsequently, the annual values of area and volume were recorded for the next 1000 years (Supplementary fig. S4). The mean and standard deviation of the modelled ELA for these 810 glaciers were 5480 and 445 m, respectively.

Out of the total 810 simulated glaciers from the Ganga basin, on 98 glaciers the fractional change in glacier area at $t = 1000$ was more than 50%, and these were excluded from the analysis. This was necessary as a linear-response model can only be applied to glaciers with small relative changes (Oerlemans, 2001). We confirmed that the nature of our results does not depend on the precise value of this cutoff (Supplementary fig. S6). An additional 9 glaciers had response time larger than 500 years and they were removed. This was done to avoid a possible overestimation of the response time whenever its magnitude was comparable to or larger than the total simulation period of 1000 years (supplementary fig. S7). The removal of these 9 glaciers led to a reduction in the number (total area) of simulated glaciers by only $\sim 1\% (\sim 2\%)$.

Finally, we were left with an ensemble of 703 synthetic Himalayan glaciers (Supplementary fig. S1), with area in the range of $2.2 - 156.0$ km^2 (a median value 5.5 km^2). The steady glaciers modelled with SIA had, on the average, 1.25 times larger area and 1.66 times larger ice-thickness (supplementary figs. S3, S8) compared to the corresponding estimates of Kraaijenbrink et al. (2017). The higher thickness of the modelled glaciers can be ascribed to a larger modelled area, a steady mass balance, and an uncalibrated SIA model. The total area and volume of these 703 synthetic glaciers were 6865 km^2 and 847 km^3, respectively. This set covered 86% of the total 810 glaciers number-wise, and 89% area-wise. The distributions of glacier area and mean slope for the two sets of 810 and 703 synthetic glaciers are shown in supplementary fig. S8.

2.2.2 Scaling model

The response of the above set of 703 steady-state glaciers to a 50 m instantaneous rise in ELA was also computed with a scaling model (Radić et al., 2007). The SIA-derived initial steady-state volume, area, and hypsometry (with the bin size of 25 m) for each of the glaciers were used as the starting point. For any of the modelled glaciers, the scaling and SIA models used the same mass-balance parameters. At any time t during the evolution, the mass-balance function (eq.3) was summed over the instantaneous glacier hypsometry to obtain the net volume loss for that time step. The corresponding area loss was then obtained using Eq. 2. The reduction in the area was assumed to have taken place in the lowest elevation band/s of each glacier (Radić et al., 2007). The scaling exponent was fixed at $\gamma = 1.286$ because of the assumed linear mass-balance profiles of the simulated glaciers (i.e., $m = 1$). The annual-resolution time series of area and volume were recorded for 1000 years for each of the glaciers.

2.2.3 Glacier response properties

For each of the 703 glaciers, the time series of volume and area as obtained using the SIA and scaling models, were separately fitted to linear-response forms (e.g., eq. 9 below) to obtain the corresponding best-fit values of the four linear-response parameters (the climate sensitivities and the response times of area and volume) for each of them (supplementary fig. S4).

Please note that applying a step change in ELA to a steady-state glacier to obtain the step-response function is a standard prescription for obtaining glacier response properties (Oerlemans, 2001; Vieli and Gudmundsson, 2004; Harrison et al., 2001; Bach et al., 2018). Within a linear-response assumption, the step-responses of volume and area have an exponential form (e.g., eq. 9 below). The asymptotic exponential decay time is the response time of the glacier, and the asymptotic magnitude of the decay is the climate sensitivity. Because of the deviations of the simulated response from a pure exponential decay (supplementary fig. S4), the best-fit response time may be slightly different from the e-folding time, which has been used in some of the previous studies (e.g., Vieli and Gudmundsson, 2004; Bach et al., 2018). However, we take the best-fit asymptotic decay time to be the response time. By definition, it minimises the deviation between the predictions of the SIA and linear-response models, and thus, improves the performance of the latter in reproducing SIA results to some extent. We confirm that the difference between the above two definitions of the response time is small.

The best-fit linear-response properties obtained from the scaling model results for the 703 glaciers were used to verify the corresponding theoretical expressions obtained from scaling theory (eqs. 8, 11, 12, 13 below). On the other hand, the The best-fit response times and climate sensitivities obtained from the SIA simulations of the 703 glaciers were used to fit for empirical relations that are motivated by the corresponding expressions derived from the scaling theory. These fitted forms would allow estimation of the response properties of any given glaciers as functions of properties like mean thickness, mass balance gradient and so on. All the above fits were performed in log-log scale, and R^2 of the fits were noted.

2.2.4 A linear-response model

The best-fit empirical parameterisations for climate sensitivity and response time obtained by fitting the SIA results as described above (given in eqs. 14−17 later), were used to obtain run a linear-response model simulation for any given glacier. This model was applied to simulate the response of the above 703 synthetic Himalayan glaciers to a 50 m step-change in ELA at $t = 0$. We emphasise that for the linear-response model, we do not use the best-fit the response properties of the individual glacier derived from the SIA simulations. Rather, the parameterisations of the same obtained by fitting the SIA-derived response properties (given in eqs. 14−17 later) were utilised. These parameterisations thus allow the model to be applied to any other set of Himalayan glaciers without the need for simulating them with SIA first.

To assess the uncertainty of the linear-response model output, a random Gaussian noise were added to the best-fit empirical parameters to generate an ensemble of 100 independent linear-response model outputs. The standard deviation of this added Gaussian noise for a given fit parameter the uncertainty of each of the fit parameters was set equal to standard error of that parameterthe corresponding standard error, and the 95% uncertainty band for the linear-response model outputs were generated using a Monte Carlo method.

To test the applicability of the above linear-response model that was calibrated using SIA results for the 703 central Himalayan glaciers, the same model was applied to a different set of 204 glaciers from the western Himalayawithout any further calibration. The parameterisations developed for the central Himalayan glaciers as discussed above (given in eqs. 14−17 later) were used to estimate the response properties of each of these western Himalayan glaciers using input values of corresponding mass-balance gradient, mean thickness and ablation rate near the terminus. For these western Himalayan glaciers, SIA and scaling model simulations were also performed following the same procedures as detailed above. The glaciers that showed more than 50% change at the 500 year mark in the corresponding SIA simulations were left out as before, and the time series of total area and volume of these 204 total volume of 164 western Himalayan glaciers obtained using the three different models were then compared.

3 Results and Discussions

3.1 Theoretical results

Below, we derive some relevant consequences of the time-invariant scaling assumption, including expressions for the climate sensitivity and response time of area and volume. These results are expected to be generally valid for all scaling models that are based on eq. 2.

3.1.1 The rates of area and volume change

Eq. 2, which was derived from eq. 1 assuming a time-independent c, implies,

$$\dot{V} = \gamma c A^{\gamma-1} \dot{A} = \gamma h \dot{A}. \tag{4}$$

Here, $\dot{V}$ and $\dot{A}$ denote the corresponding rates of change of glacier volume and area, respectively. If the net specific balance is δb (in m/yr), then the annual rate of volume loss $\dot{V} = \delta b A$. This, together with eq. 4, implies,

$$\dot{A} = \frac{\delta b}{\gamma h} A \tag{5}$$

$$= \frac{\delta b}{\gamma c} A^{2-\gamma}. \tag{6}$$

Thus, in the scaling models the rate of change of area scales with glacier area with an exponent $(2-\gamma)$. This is consistent with empirical observations for real glaciers as well (Banerjee and Kumari, 2019). As the scale factor $\frac{\delta b}{\gamma c}$ in the right-hand side (RHS) of eq. 5 is proportional to the net specific mass balance, this may be a convenient way of obtaining mean regional thinning rates from relatively straightforward remote-sensing measurements of the rate of area change. However, the accuracy of this relation is contingent on the validity of the assumption of a time-independent c.

3.1.2 Area response time

To compute the area response time, let us consider a constant perturbation, i.e., a step change in ELA applied to a steady glacier for time $t \geq 0$ (e.g., Oerlemans, 2001). Let's denote the corresponding instantaneous net negative balance at $t = 0$ by $\delta b_0 A$, the asymptotic ($t \to \infty$) shrinkage of glacier area by $\Delta A_\infty \equiv A(0) - A(t \to \infty)$, and that of ice volume by ΔV_∞. Then, we have (Harrison et al., 2001),

$$\Delta A_\infty b_t + \beta \Delta V_\infty \approx -\delta b_0 A. \tag{7}$$

Here, b_t is the ablation rate near the terminus. The area response time of the glacier can be expressed as $\tau_A \approx \Delta A_\infty / \dot{A}$. Therefore, using the above expressions for $\dot{A}$ (Eq. 5) and ΔA_∞ (Eq. 7), we obtain,

$$\tau_A = -(\frac{b_t}{\gamma h} + \beta)^{-1} \equiv \tau^*. \tag{8}$$

Here, the symbol τ^* is a convenient shorthand notation for the time scale $-(\frac{b_t}{\gamma h}+\beta)^{-1}$. In the above derivation, ΔV_∞ that appears in eq. 7 is eliminated with the help of eq. 2. Eq. 8 is comparable with the expression of area response time as given by Harrison et al. (2001), or Lüthi (2009).

3.1.3 Volume response time

The instantaneous change in volume ($\Delta V(t)$) for a steady glacier perturbed by a small step change in ELA at $t = 0$ is given by,

$$\Delta V(t) = \Delta V_\infty (1 - e^{-t/\tau_v}), \tag{9}$$

where, τ_v is the volume response time and ΔV_∞ is the volume sensitivity (e.g., Lüthi, 2009). Now, $V(t), V(0)$, and $V(t \to \infty)$ appearing in eq. 9 can be expressed in terms of $A(t), A(0)$, and $A(t \to \infty)$, respectively, with the help of corresponding scaling relations (eq. 1). This, in the limit of a small fractional changes in area, yields,

$$\Delta A(t) = \Delta A_\infty (1 - e^{-t/\tau_v}). \tag{10}$$

Comparing the above two equations, and using eq. 8 one obtains,

$$\tau_A = \tau_V = \tau^*. \tag{11}$$

This implies that all scaling models implicitly assume the area and volume response times of a glacier to be equal to each other. However, it is known that for mountain glaciers area response time is larger than the volume response time within a SIA model (Oerlemans, 2001; Vieli and Gudmundsson, 2004). Therefore, the assumed equality of the two response times in scaling models (eq. 11) contradicts the existing SIA results. This is an intrinsic bias that is present in any scaling model.

After a step change in ELA, as the ablation zone shrinks, the initial net negative balance of a glacier gradually decays to zero over a period determined by the corresponding response time. A longer area response time in SIA implies that this reduction in the ablation zone is slower here than that in a scaling model. A corresponding feedback of a larger ablation zone on the net mass balance should then lead to a higher long-term volume loss in a SIA model than that in a scaling model. This indicates the possibility of a low bias in scaling model estimates of the climate sensitivity of volume, or equivalently, that in the long-term changes in glacier volume due to any rise in ELA.

3.1.4 Climate sensitivity of area and volume

An expression for the climate sensitivity of glacier area (ΔA_∞), which is the asymptotic change in area due a change in ELA by δE, is obtained by eliminating ΔV_∞ from eq. 7 using eq. 2,

$$\frac{\Delta A_\infty}{A} = \frac{\tau^* \beta \delta E}{\gamma h} \equiv \alpha^*. \tag{12}$$

Here, we have used the definition of τ^* (Eq. 8), and that $\delta b_0 \approx \beta \delta E$ for a step change in ELA by δE. The RHS of the above equation is denoted by α^* for convenience.

The corresponding expression for $\frac{\Delta V_\infty}{V}$ is then obtained using Eq. 2,

$$\frac{\Delta V_\infty}{V} = \gamma \alpha^*. \tag{13}$$

Again, Eq. 13 is comparable to the expression of volume sensitivity as derived by (Harrison et al., 2001), where the authors used an arbitrary thickness scale H, instead of the denominator of γh appearing in the definition of α^* above.

Please note that strictly speaking, the climate sensitivity of area and volume with respect to a change in ELA should be defined as $\frac{\Delta A_\infty}{\delta E}$ and $\frac{\Delta V_\infty}{\delta E}$, respectively. However, in this paper, we use ΔA_∞ and ΔV_∞ as the corresponding sensitivities to simplify the notation.

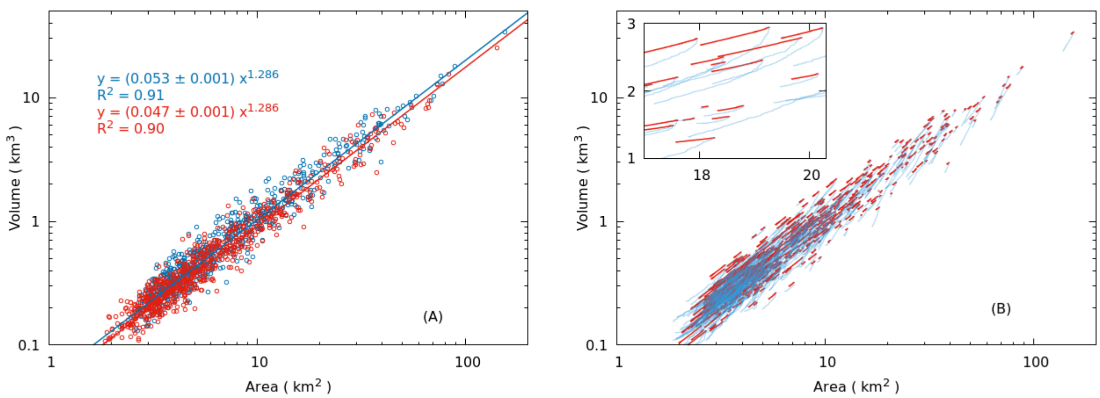

Figure 1. A) Glacier volume as a function of area for the 703 Himalayan glaciers simulated with SIA at $t = 0$ yr (blue circles), and at $t = 500$ yr (red circles) are plotted along with the corresponding best-fit scaling relations (blue and red solid lines). The corresponding fitted functions, and R^2 values are shown in blue and red texts, respectively. B) The trajectories of the 703 glaciers in the $V - A$ plane as simulated with SIA (thick red lines) and scaling (thin blue lines) models. The inset is a zoomed-in version of the same plot, but with a linear scale.

3.2 Numerical results

3.2.1 Volume-area scaling and a time-dependent scale factor in the SIA model

Following eq. 1, a power-law relation between the area and volume of the 703 glaciers with an exponent $\gamma = 1 + \frac{m+1}{m+n+3} = 1.286$ ~~, is expected (~~is expected as $m = 1$ and $n = 3$~~)~~. The ensemble of glaciers modelled with SIA did conform to above power-law relation $V = cA^{1.286}$ at any time t with a single best-fit c. The scale factor slowly decreased with time. For example, fig. 1a shows the power-law fits at $t = 0$ and $t = 500$ years ($R^2 = 0.9$), where the best-fit c-values were 0.053 ± 0.001 and 0.47 ± 0.001 km$^{3-2\gamma}$, respectively. This implies a $\sim$11% reduction in c for the ensemble over the period of 500 years after the step-change in ELA was applied. A time-dependent c is consistent with the theoretical arguments of Bahr et al. (2015).

The slow and systematic decline in c for the ensemble of shrinking glaciers simulated with SIA model contradicts the basic assumption of scaling models ~~of a time-invariant~~ that c is time-invariant. A decreasing c would mean eq. 2 is violated, with

$\frac{\Delta V}{V} = \gamma \frac{\Delta A}{A} + \frac{\Delta c}{c}$. Note that all the three fractional changes involved in this relation are negative. Therefore, for any given $|\Delta A|$, the corresponding $|\Delta V|$ is going to be larger in SIA model than that in a scaling model where $\frac{\Delta c}{c}$ is assumed to be zero (eq. 2). Even though the decline in c is only about 11%, it may be associated with a stronger low bias in the long-term change predicted by scaling models. This is because a larger volume change in SIA would lead to a thinner glacier, and a corresponding surface-elevation feedback to mass balance is likely to amplify the corresponding long-term mass loss over time.

The dependence of the glacier-specific scale factor on the mean slope is known (Bahr et al., 2015) and has been incorporated in modified scaling relations where volume is a power-law function both area and slope (e.g., Grinsted, 2013; Zekollari and Huybrechts , 2015). For the simulated 703 glaciers, the mean slope increases with time as area is lost preferentially from the gently-sloping lower ablation zone. For example, the median slope of the 703 simulated glaciers reduced from 0.41 at $t = 0$ to 0.37 at $t = 500$ years. This $\sim 10\%$ reduction in slope is expected to lead to a $\sim 5\%$ decline in c (Bahr et al., 2015) . So, at least part of the time dependence of c for transient glaciers in SIA simulation is explained by the slope-dependence of c. However, there may be other factors contributing to the decline in c for the transient glaciers as discussed below.

3.2.2 Area and volume response times

The theoretical prediction for glacier area and volume response time (eq. 11) worked rather well for the scaling model results (figs. 2C, and 2D), with best-fit relations of ~~$\tau_V = (0.996 \pm 0.001)\tau_A$ with $R^2 = 0.995$, and $\tau_V = (0.942 \pm 0.006)\tau^*$ with $R^2 = 0.89$~~ $\tau_V = (0.914 \pm 0.002)\tau_A$ with $R^2 = 0.99$, and $\tau_A = (1.066 \pm 0.008)\tau^*$ with $R^2 = 0.80$.

For SIA simulations, the data showed that $\tau_A > \tau_V$, and that the two response times were still proportional to each other (fig. 3C: $\tau_V = (0.687 \pm 0.004)\tau_A$, with $R^2 = 0.94$). Also, ~~τ_V~~ τ_A was proportional to τ^* to a good approximation (fig. 3D: ~~$\tau_V = (2.56 \pm 0.04)\tau^*$, with $R^2 = 0.94$~~ $\tau_A = (2.56 \pm 0.04)\tau^*$, with $R^2 = 0.53$). Interestingly, the value of the proportionality constant in the latter relation as obtained from SIA was about ~~2.7~~ 2.4 times larger than the corresponding value obtained in the scaling model. This underlines the relatively large underestimation of ~~volume~~ area response time by the scaling model. Similarly, the ~~area~~ volume response time was about ~~3.9~~ 1.8 times larger in the SIA simulation than the corresponding scaling model value. This implies that for a given ELA perturbation, the glacier response is much faster in the scaling model compared to that in the SIA model for the ensemble of 703 synthetic glaciers.

Apart from the overall underestimation of area and volume response times by the scaling model, another serious limitation of scaling models that emerges from the above analysis is that here the area and volume response times are equal to each other (eq. 11, and fig. 3C). In contrast, the SIA model predicted $\tau_A \approx 1.5\tau_V$. The ratio of the two response times obtained from the 2-d SIA model here is generally consistent with earlier results based on 1-d flowline models (Oerlemans, 2001; Vieli and Gudmundsson, 2004). The equality of the two response times in the scaling model led to a linear trajectory in $V - A$ plane for the transient glaciers (fig. 1B). While in SIA model, a relatively larger area response time, together a slow initial changes in area (supplementary figs. S4, S10), led to curved $V - A$ plane trajectories for individual transient glaciers. In particular, a slowly changing area means the $V - A$ trajectories bend downward causing c to reduce for the transient ensemble (fig 1). Moreover, At the early stages of response, glaciers simulated by a scaling model lose area much quicker than those simulated by an SIA model (fig. 1B). The associated net mass-balance feedbacks then lead to a subdued long-term volume response in scaling model, and a comparatively stronger volume response in the SIA model, just as predicted in sect. 3.1.3.

3.2.3 The climate sensitivity of glacier area and volume

For the 703 glaciers simulated by the scaling model, the fitted asymptotic fractional changes in area and volume, or equivalently, the corresponding (fractional) climate sensitivities, were proportional to each other (fig. 2A: $\frac{\Delta V_\infty}{V} = (1.232 \pm 0.002)\frac{\Delta A_\infty}{A}$ $\frac{\Delta V_\infty}{V} = (1.383 \pm$ with R^2=0.9970.99). Here, the best-fit constant of proportionality was close to $\gamma = 1.286$, is close to, but about 8% larger than $\gamma = 1.286$ as predicted by eq. 2.

In contrast, the SIA simulations obtained $\frac{\Delta V_\infty}{V} = (1.93 \pm 0.02)\frac{\Delta A_\infty}{A}$, with R^2=0.85 (fig. 3A). In this case, the constant of proportionality was $\sim 1.5\gamma$, compared to the corresponding value of $\sim \gamma$ in the scaling model. This larger value of the ratio of the two climate sensitivities in SIA model is consistent with the observed decline in c for the transient glaciers simulated with this model (fig. 1). Please note that no theoretical prediction is available for the ratio of asymptotic fractional changes in volume and area in a SIA model.

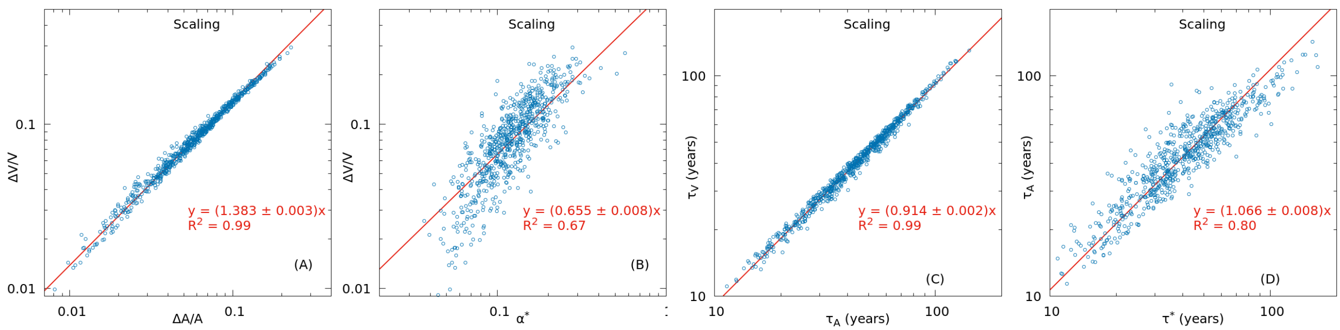

Figure 2. Scaling model simulations of the 703 synthetic Himalayan glacier show that, (A) the best-fit (fractional) climate sensitivities of area and volume are proportional to each other, (B) The climate sensitivity of volume is proportional to $\alpha^* \equiv \frac{\beta\delta E\tau^*}{\gamma h}$, (C) The response times associated with glaciers area and volume are approximately equal, and (D) the volume response time is approximately equal to $\tau^* \equiv -(\frac{b_t}{\gamma h} + \beta)^{-1}$. In all the above plots, the corresponding best-fit curves are shown with red lines. The fit parameters and R^2 of the fits are also given. These numerical trends are consistent with theoretical results derived in sect. 3.1.

Fig. 2B shows that in the scaling model, climate sensitivity of glacier volume is proportional to α^* ($\frac{\Delta V_\infty}{V} = (0.581 \pm 0.007)\alpha^*$, with $R^2 = 0.7$ $\frac{\Delta V_\infty}{V} = (0.655 \pm 0.008)\alpha^*$, with $R^2 = 0.67$). This is in line with eq. 13, except that the constant of proportionality is significantly less than γ. A similar proportionality between the SIA-derived best-fit $\frac{\Delta V_\infty}{V}$ and α^* is shown in fig. 3B, with $\frac{\Delta V_\infty}{V} = (1.71 \pm 0.03)\alpha^*$. However, in this case the fit is relatively noisy with $R^2 = 0.48$.

The above relations suggest that the climate sensitivity of volume in the SIA simulation was about 2.9 2.6 times larger than that in the scaling model. Similarly, the climate sensitivity of glacier area obtained from the SIA model was also about 3.2 1.9 times larger than that obtained from the scaling model. This trend of a relatively large (by about a factor of about

~~3~~2) underestimation of climate sensitivity of glacier volume and area by the scaling model is consistent with the effects of a relatively faster shrinkage of the ablation zone in the early stages of the response as discussed in 3.1.3 and 3.2.2.

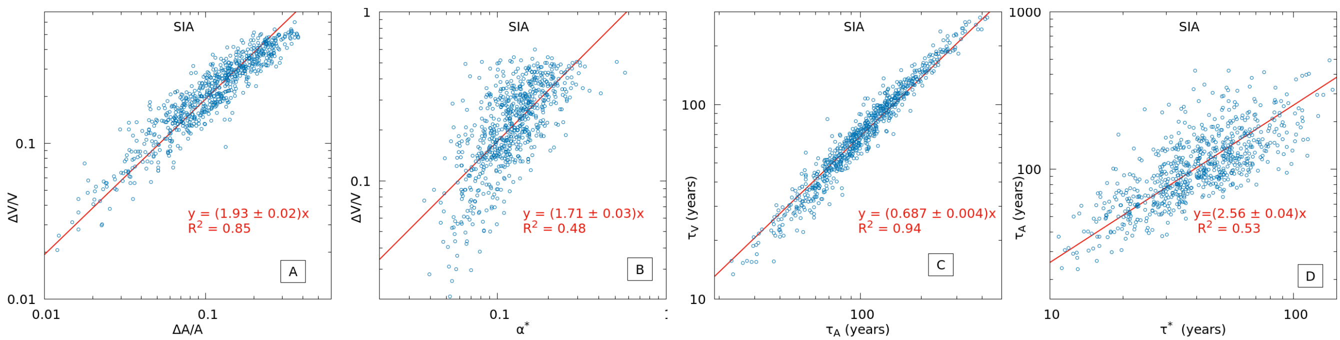

Figure 3. Results from the SIA simulations of the 703 synthetic Himalayan glacier show that, (A) The climate sensitivities of area and volume are proportional to each other, (B) The climate sensitivity of glacier volume is proportional to $\alpha^* = \frac{\beta \delta E \tau^*}{\gamma h}$, (C) The response times associated with glaciers area and volume are proportional to each other, and (D) The volume response time is proportional to $\tau^* = -(\frac{b_t}{\gamma h} + \beta)^{-1}$. The fitted functions are shown with red lines. The corresponding fit parameters and R^2 of the fits are also given. See text for detailed discussions.

3.2.4 The total glacier loss estimated using the three models

Starting with an initial volume (area) of 847 km^3 (6865 km^2), the 703 glaciers simulated by SIA lost a total of 194 km^3 (726 km^2) of volume (area) in 500 years due to the step-rise in ELA by 50 m. As shown in fig 4, both the scaling and the linear-response models underestimated the long-term change in total area in this experiment, with estimated area changes of ~~352 and 621~~ 334 and 623 km^2, respectively. The scaling-model prediction for area change was only ~~48~~46% of the corresponding SIA estimate, while the linear-response model estimate was 86% of that of SIA. Similar trends were seen for the magnitudes of estimated volume change as well, with the respective scaling and linear-response model estimates being ~~~ 27~~~ 31% and ~ 75% of the corresponding SIA prediction (fig 4). We confirmed that the nature of the above results does not depend on the chosen cut-off of 50% change that was used to select the 703 glaciers (Supplementary fig. S6). In fact, with a smaller cut-off, the linear-response model estimates were even closer to the corresponding SIA estimates (Supplementary fig. S6). This is expected as linear-response models are derived in the limit of small fractional changes (Oerlemans, 2001).

The low-bias in the long-term changes of glacier area and volume computed with the scaling model is consistent with the underestimation of corresponding climate sensitivities by this model (sect. 3.2.3). This indicates the possibility of a negative bias in scaling model estimates of mountain glacier contribution to sea-level rise as well. As an example, let us consider a recent comparison (Hock et al, 2019) of projected end-of-the-century sea-level rise contribution of glaciers from 6 different models that include a hypsometric-adjustment-based model (Huss and Hock, 2015) and 5 other models which are all based on some form of scaling. It is seen that the former model consistently predicted the largest ~~change~~ fractional change of global glacier volume and area under various climate scenarios (~~e.g.,~~ Table 3 of Hock et al (2019)). ~~This may be an indication that biases qualitatively similar to that discussed here, are generally present~~ The same trend was confirmed in a susbset of all the runs where the models were forced by the same global climate model outputs (Figure 11 of Hock et al (2019)). In another recent comparison, similar trends are seen as far as global-scale fractional volume loss by 2100 are concerned (Figure S17−S20 of Marzeion et al. (2020)), though on a regional scale there are differences. However, it is difficult to draw a definite conclusion about any potential bias in scaling models ~~. Based on our results, the~~ from the above-mentioned studies as there are wide differences among the models in terms of the initial conditions, climate forcing, and mass-balance parameterisations used. For example many of the scaling models in Hock et al (2019) had a much larger initial global glacier volume, by up to a factor of about 2, than that in the non-scaling one. An intercomparison of the models where the same set of glaciers, with the same initial geometry and volume were simulated under the same mass-balance forcing - similar to the strategy used in the present study - is neccessary to identify possible biases that may be present in the exisiting scaling models. The potential biases in the scaling models may be clearer in long-term simulations over multiple centuries. On shorter time scales of multiple decades, an underestimation of response times by about a factor of ~~3~~ 2 (sect. 3.2.2) to some ~~extents~~ extent compensates for a corresponding underestimation of the climate sensitivities (sect. 3.2.3), and the ~~deviation~~ deviations between the SIA and scaling models are not that prominent (fig. 4).

Please note that depending on the details of the scaling and SIA models that are being compared, or the set of glaciers that are being simulated, the actual magnitude of the biases in scaling-model derived climate sensitivity, response time, and long-term glacier change could be different from that obtained here. However, based on the theoretical arguments and numerical evidence presented, similar qualitative trends are expected if the above exercise were to be repeated with a more detailed model and/or for a more realistic set of glaciers.

Above results ~~also~~ show that the linear-response model outperformed the scaling model, producing a closer match with the SIA results for the 703 synthetic glaciers from the Gangetic Himalaya. However, this linear-response model was calibrated using the SIA results for the same set of glaciers. Therefore, this match is not enough to establish the effectiveness of the linear-response model. To confirm the improved performance of the linear-response model compared to that of the scaling model, we applied ~~these two models without any further calibration,~~ both the models to simulate a different set of ~~204~~ 164 glaciers in the western Himalaya (supplementary fig. S1). The best-fit linear-response properties obtained from SIA simulation of the 703 central Himlayan glaciers were first fitted to obtain four equations (eqs. 14−17) that relates the response properties to β, γ, h and b_t as described before. The same equations were used to estimate response properties of each of the 164 western Himalayan glaciers as required for the linear-response model simulations. In this independent experiment, the linear-response model again outperformed the scaling model in reproducing the corresponding SIA results (supplementary fig. S9). This confirms that the linear-response model, along with eqs. 14−17, can be used for computing long-term glacier changes accurately.

3.3 The effects of glacier geometry

Can the biases in the scaling model described above, be artefacts arising out of some peculiarities of the geometry of the specific set of glaciers being simulated, and are not relevant in general for scaling model computations of global-scale mass loss of mountain glaciers? To ~~explore~~ rule out that possibility, we simulated the response of a set of highly idealised synthetic glaciers using both a flowline model (Banerjee, 2017) and the above scaling model (Radić et al., 2007). Note that this flowline model

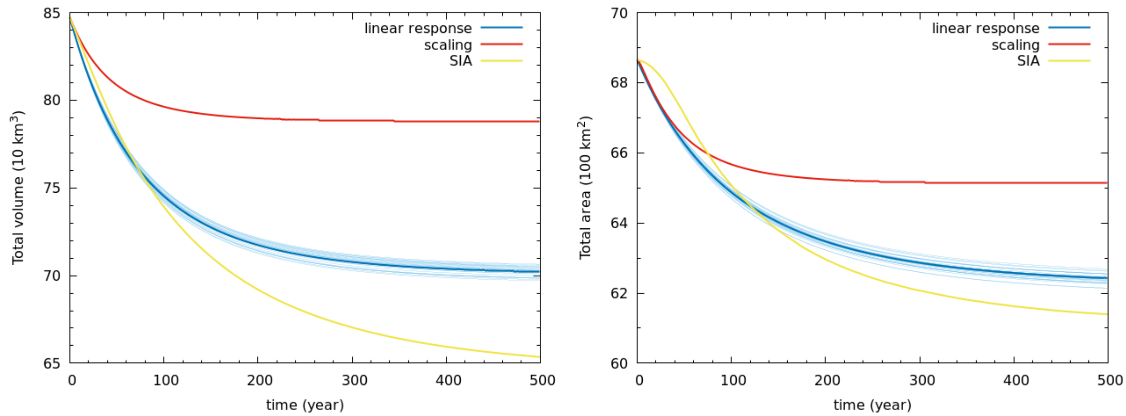

Figure 4. The evolution of the total (A) volume, and (B) area of the ensemble of 703 Himalayan glaciers simulated with three different methods: SIA, scaling, and linear-response models. The uncertainty bands for the linear response model results as also shown. See text for details.

included sliding as well. All of these synthetic glaciers have the same constant-width, the same linear bedrock with constant slope, and the same linear mass-balance profile. Only the ELA was varied between glaciers. Even for this highly idealised set of glaciers, the scaling model estimates for the evolution of total area and volume showed biases compared to that obtained from the flowline model (supplementary fig. S9), and these biases were qualitatively very similar to those ~~observed~~ depicted in figs. 1 and 4. Again, the scaling model predicted relatively smaller climate sensitivities, a relatively faster area response, and a low-bias in the long-term changes, compared to corresponding ~~flowline model~~ flowline-model estimates (supplementary fig. S9).

The above ~~flowline model~~ flowline-model experiment provides an additional piece of evidence that the scaling-model biases discussed in this paper are in general expected to be present in scaling model simulations of any set of glaciers. We ~~emphasise~~ re-emphasise that even though the biases are expected to be qualitatively similar to that presented here, the magnitude of the biases are likely to depend on the detailed characteristics (related to geometry, flow, and mass-balance processes) of the glaciers studied and the models used.

3.4 The linear-response model, and its application to real glaciers

As described above, we have used results from the 2-d SIA model simulations of the response of 703 synthetic Himalayan glaciers to a 50 m step change in ELA, to obtain the following best-fit ~~paramterisations~~ parameterisations of the glacier response properties (i.e., $\frac{\Delta V_\infty}{V}$, $\frac{\Delta A_\infty}{A}$, τ_A and τ_V).

$$\frac{\Delta V_\infty}{V} = (1.71 \pm 0.03)\alpha^*, \tag{14}$$

$$\frac{\Delta V_\infty}{V} = (1.93 \pm 0.02)\frac{\Delta A_\infty}{A}, \tag{15}$$

$$\tau_{\sout{V}A} = (2.56 \pm 0.04)\tau^*, \tag{16}$$

$$\tau_V = (0.687 \pm 0.004)\tau_A. \tag{17}$$

Here, as defined before, $\tau^* \equiv -(\frac{b_t}{\gamma h} + \beta)^{-1}$, $\alpha^* \equiv \frac{\beta \delta E \tau^*}{\gamma h}$, and $\delta E = 50$ m. ~~With the help of these paramterisations~~

With the estimated glacier-specific response properties obtained from eqs. 14−17, it is possible to compute the evolution glacier volume and area accurately ~~given a glacier and~~ for any glacier and for any arbitrary ELA forcing function. For this the following general solution of the linear-response equation is used.

$$\Delta V(t) = \Delta V(0)e^{-t/\tau_V} + \sout{\frac{\Delta V_\infty}{\delta E}}\,\frac{\Delta V_\infty}{\tau_V \delta E}\int_0^t \Delta E(t')e^{-(t-t')/\tau_V}\,dt' \tag{18}$$

Here, $\Delta E(t)$ is the given (arbitrary) ELA forcing function. This equation simply states that, any continuous ELA change can be interpreted as the sum total of a series of discrete ~~steps~~impluses, and the corresponding net response is given by a superposition of suitably delayed responses due to each of the ~~steps~~impulses. An analogous expression can be ~~written down~~ obtained for the area evolution ~~as well~~ just by replacing all the V's in the above equation with A's.

Please note that the above formulation does not require the initial state to be steady. As long as the glacier is close to a steady state, a linear-response theory will be a good approximation (Oerlemans, 2001). However, an additional initial condition, i.e., the value of $\Delta V(0)$, is needed to apply the linear-response model to transient glaciers. $\Delta V(0)$ is the initial departure from a steady state, and can be obtained from the observed rate of volume loss ($\dot{V}$) simply as, $\Delta V(0) = -\tau_V \dot{V}$. Thus, the linear-response model can be used to evolve the area and volume of a real set of glaciers for any arbitrary time-dependent ELA forcing given the initial rates of change of volume and area, initial thickness, mass-balance gradient, and melt rate near glacier terminus.

Since the above parameterisation of linear-response perperties (eqs. 14−17) are derived from SIA simulations of an ensemble of Himalayan glaciers, when applying them to any other glacierised region in world, it may be necessary to simulate a few tens of glaciers (having a representative range of area and slope) from that region using SIA first, and confirm the accuracy of the above parameterisations.

Due to the noise present in the fits (fig. 3), the linear-response model predictions for an individual glacier would have significant uncertainties. However, for a large set of glaciers, the linear-response model provides accurate estimates of the total area and volume evolution (fig. 4, supplementary figs. S6 and S9).

3.5 ~~Limitation~~ Limitations of the present study

Because of the idealised descriptions of ice flow and the mass-balance profile (as discussed in sect. 2.2), and the absence of model calibration to match the available observed data of surface velocity, ice thickness, recent mass balance etc., the glaciers simulated here are not faithful copies of the Himalayan ones. For a set of more realistic glaciers, the magnitude of the corresponding biases in scaling-model derived climate sensitivity and response time could be different from that obtained here. However, based on the theoretical arguments and numerical evidence presented, similar qualitative trends are expected if the above exercise were to be repeated for a more realistic model that includes higher order mechanics, ~~flow due to sliding,~~ a more realistic mass-balance model, and so on. Similarly, The parameterisations for the linear-response properties given here are obtained from 2-d simulations of 703 synthetic Himalayan glaciers with some idealisations (sect. 2.2) and without any tuning of model parameters. The fit-parameters in eqs. 14-17 may be different for a different set of glaciers. The ~~paramterisations~~ parameterisations may also change if a more detailed and calibrated model of the same glaciers is used. However, the protocol used here to obtain the parameterisation for linear response-properties can be directly applied without any change for any set of glaciers and for any ice-flow/mass-balance model. While applying the linear response model to any other region, it may be useful to obtain response properties of a few tens of representative glaciers using flow-model simulations and check if any recalibration of the parameterisation as given in eqs. 14-17 is necessary.

4 Summary and Conclusions

We performed a theoretical analysis of the response of mountain glaciers within a time-independent scaling assumption. In addition, the step-response of 703 steady-state synthetic Himalayan glaciers with realistic geometries and idealised mass-balance profiles were simulated with three different models: a scaling model, a 2-d SIA model, and a linear-response model. The results obtained are as follows.

- Analytical expressions for climate sensitivity and response time of glacier area and volume are derived within a time-independent scaling assumption. These expressions are validated using results from the scaling model simulation of the ensemble of 703 glaciers.
- The response of the glaciers simulated with the 2-d SIA model reveals that the initial steady states and the transient states follow the volume-area scaling relation, with the best-fit scale factor reducing slowly with time.
- For the ensemble of glaciers studied, the scaling model obtains relatively smaller climate sensitivities of glacier area and volume by a factor of about ~~3~~1.9 and 2.6, respectively, compared to ~~that~~ those obtained from the SIA model. This results in a low bias in the long-term changes predicted by the scaling model.
- For the ensemble of glaciers studied, the scaling model underestimates volume (area) response time by a factor ~~~2.7 (3.9~~1.8 (2.4) compared to the corresponding SIA estimates.
- For the scaling model, $\tau_A \approx \tau_V$, and $\frac{\Delta V_\infty}{V} \approx \gamma \frac{\Delta A_\infty}{A}$. In contrast, for the SIA simulations, $\tau_A \approx 1.5\tau_V$ and $\frac{\Delta V_\infty}{V} \approx 1.5\gamma \frac{\Delta A_\infty}{A}$.

– The relatively larger ratio of the two response times in the SIA simulations, along with an initial slow change in area, leads to curved $V - A$ trajectories, a decreasing c, and a relatively larger long-term volume loss for the transient glaciers due to a corresponding mass-balance feedback.

– A linear-response model based on the parameterisations of SIA-derived response properties helps reduce the biases in the predicted long-term glacier changes that are present in the scaling model results for the simulated central Himalayan glaciers. The improved performance of this model is validated on an independent set of ~~204~~ 164 glaciers in the western Himalaya.

Based on the theoretical arguments and numerical evidence presented here, it is possible that qualitatively similar biases may generally be present in the long-term glacier changes computed with scaling models. However, the actual magnitude of such biases in scaling models may be different from that obtained here for a set of synthetic Himalayan glaciers with idealised mass balance. Possible biases in scaling models may, in turn, lead to a low bias in the corresponding estimates of the long-term sea-level rise contribution from shrinking mountain glaciers. On a multidecadal scale, a faster response due to shorter response times in the scaling model can compensate for the effects of smaller climate sensitivities to some extent. However, the low biases in scaling model derived changes in glacier area and volume are likely to become apparent over longer time scales of multiple centuries. The linear-response model presented above could potentially be useful in predicting the long-term global glacier change and/or sea-level rise due to its accuracy and numerical efficiency.

Code availability. The codes for the various models used in this paper shall be made available upon publication.

Author contributions. AB designed the study, did the theoretical analysis, and wrote the paper. AJ and DP wrote the codes. AJ, DP, and AB ran the simulations. All the three authors contributed to the analysis of the simulated data and discussions.

Competing interests. We declare that there are no competing interests.

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