Dear Editor,

We gratefully acknowledge critical inputs from both the reviewers.   Based on their suggestions, we have made the following important changes in this revised version.

1. Spelt out our objectives at the beginning clearly in order to clarify the experimental design.

2. Repeated experiments to simulate 810 out of 814 Himalayan glaciers. We have now used 703 glaciers for our analysis.

3. Added finer details about the experimental methods and results in the main text. Also, added 10 supplementary figures to clarify various things.

4. Expanded the discussions about the theoretical/numerical results, including how gradual steepening of the retreating glaciers can contribute to a decline in *c*.

5. Acknowledged and emphasised the idealisation involved the set of simulated “synthetic Himalayan glaciers”, and possible dependence of the results on the chosen ensemble/model.

7. Explained how the long-term biases in scaling models based on the feedback of shrinking ablation zone on net mass-balance. The scaling models assume an equality of the area and volume response times.In contrasting, the SIA model predicts a larger area response time.

7. Discussed additional results from 1-d flowline model simulation of highly idealised glaciers to support the general applicability of our results.

8. Discussed how results from the GlacierMIP support our claim of possible biases in scaling models.

9. Provided methodological details about the application of the linear-response model to real transient glaciers that are forced by an arbitrary time-dependent ELA.

10. Added a careful itemised summary of the results obtained in the end.

With these changes we hope to have addressed the issues raised by the reviewers. We look forward to further comments on the experiments, results, and conclusions as presented in the revised manuscript.

Our point-by-point replies to the reviewer’s comments (in red) are appended below.

Regards,
Argha

Replies to comments by reviewer #1 (Eviatar Bach):

Major issues:

1. For the linear-response model based projections, the authors write that they fit the four parameters (area and volume sensitivities and response times) for each glacier based on the SIA data. They then validate the projections obtained using these parameters on the same SIA data. This is using the same data for fitting and validation,so it is not surprising that it replicates the data fairly well. Testing this method requires validation data that is not part of the fitting. A possible way to do this would be to only use a portion of the time-series of each glacier to fit, and validate on the rest (for example, fit on the first 50 years, project into the future, and validate on the 450 remaining years).

There is this this sentence which I was not clear on: "We have verified the linear-response model obtained by fitting the SIA simulation results for the ensemble of 551central Himalayan glaciers, similarly outperforms the scaling-based method for another set of 143 glaciers from the western Himalaya (figure A2)." Were the parameters obtained for the central Himalayan glaciers somehow extrapolated to the western Himalayan ones? Or were the parameters fit for every western Himalayan glacier as well? It is not clear from the description. If the authors use an extrapolation method, it would be important to describe it.

We have calibrated the model using data from 703 Central Himalayan glaciers, and validated it (without any further calibration) on an different set of 204 glaciers in the western Himalaya. We have clarified this in L83-85 and L193-197 of the revised manuscript to avoid any confusion.

2. The linear-response method is being proposed as an alternative to scaling-based methods for projecting glacier volume evolution. However, I am not clear on how this would be implemented in practice. The climate sensitivities ΔV_∞ and ΔA_∞ characterize the response of an initially steady-state glacier to a perturbation in the ELA. How can this be used to project evolution of a glacier that is already transient, and in a situation where it is not a single perturbation in the ELA, but that the ELA is continually rising?

We have added a section (3.4) titled "Applying the linear-response model to real glaciers", where we explain how the model can be applied to transient glaciers that are forced by any arbitrary time-varying ELA perturbation.

3. Furthermore, it seems that the linear-response method would require a relatively long time-series of the area and volume evolution of each glacier in order to fit the parameters, which is often not available. I would like to see a discussion of the data requirements and feasibility for use in sea-level projections.

Following the standard paradigm of linear-response theory, the response parameters can obtained from an SIA simulation of the step-change response of steady glaciers without any time-series data – exactly the way we have done here for the 703 glaciers here. Of course, the SIA model can be tuned using surface velocity and ice-thickness data wherever available (which we have not done here) to obtain more accurate parameterisation of the linear-response properties for any given set of glaciers. We have discussed this issue in L407-412.

Other issues:

1. The authors remove some glaciers from consideration in several parts of the paper, such as those that had fractional changes of more than 50% over 500 years, and those with response times higher than 300 years. Also, in another part of the paper, glaciers with large values of $\Delta A_\infty/A$ are removed, and another cut-off on $\Delta V_\infty/V$ is imposed. I don't see an adequate justification of why these were removed,and doing so biases the results.

We have now explained (L143-146) that is it is necessary to remove the glaciers with 'large' changes as linear response theory do not apply to them. Additionally, supplementary fig. S6 is included, where a cut off 0f 20% is used, to demonstrate that our results do not depend on the cutoff chosen.

The transient runs are extended to 1000 years, extending the cut-off on response time to 500 years so that only 9 glaciers are removed. Supplementary fig. S7 presents justification why this cutoff is needed.

In the revised version, we now consider 703 glaciers. We have compared the area and slope distributions of these 703 and 810 glaciers (supplementary fig. S8) .

2. "The minor differences are due to the time-invariant scaling assumption made here." Please clarify in more detail what is the difference between your derivations and those of Harrison (2001).

The difference between the expressions is mentioned now (L 256).

3. In Fig. 1B, scaling the SIA results by 10 for visual comparison is confusing. It's also hard to distinguish which are the thick and thin lines.

We have updated fig 1B based on the above criticism.

We have also corrected the typographical/grammatical errors pointed out by the reviewer.

Replies to the comments of reviewer #2 (Anonymous):

[1] State-of-the art.

The authors compare the outcome of V-A scaling with results from a SIA model. V-A has indeed been used in some important regional-to global studies in the past (e.g. Marzeion et al., 2012; Radić et al., 2014) due its computational efficiency. Moreover, with spatial estimates of ice thickness lacking for individual glaciers at the time, V-A methods offered a good alternative to estimate the volume of a glacier (and its changes through time). However, increasing computational performance and new glacier-specific inventories on e.g. ice thickness (Huss & Farinotti, 2012; Farinotti et al., 2019)and mass balance (e.g. Brun et al., 2017; Braun et al., 2019; Dussaillant et al., 2019; Zemp et al., 2019), now allow for far more sophisticated methods to simulate the dynamic evolution of glaciers. This includes methods based on imposing observed geometry changes in which the glacier geometry is explicitly accounted for(e.g. Huss & Hock, 2015; Rounce et al., 2020a, 2020b)and more recently also flowline models in which glacier dynamics (i.e. mass transfer within a glacier) are included when projecting glacier changes at regional to global scales (Maussion et al., 2019; Zekollari et al., 2019). When reading this manuscript, it seems like V-A scaling is a state-of-the art approach, and that you compare it to something more sophisticated (2-D SIA model). This comparison would have been very relevant a few years ago, when V-A scaling was state-of-the art (I am for instance thinking about the excellent work realized by Surendra Adhikari during his PhD; see e.g. Adhikari & Marshall, 2012), but has, in my opinion, lost some of its interest by now. With the new glacier-specific ice thickness estimates and other information derived from remote sensing becoming widely available (outlines, surface topography, ice thickness derived from this), the importance of V-A scaling methods is now strongly reducing and is likely to continue doing so so in the near future (see e.g. discussion by Haeberli, 2016). I do therefore have some reservations whether the ‘The Cryosphere’ is the ideal medium to share these (somewhat outdated?) findings. This concern is furthermore strengthened by my doubts about the experimental setup and the validity of your main conclusions as elaborated in the following points.

We do not agree with the reviewer that scaling models have lost significant, and need not be studied. We are not aware of any global estimate of sea-level contribution of glaciers based on flow models studies. Most of the available recent estimates (e.g., a recent intercomparison study: Hock et al., 2019; a global-scale vulnerability study: Kulp and Strauss, 2019), strongly rely on scaling models. However, we do acknowledge the potential of the 1-d and 2-d ice-flow models and revised the introduction accordingly (L26).

[2] The experimental setup:

a. Comparing different methods and models is always quite complicated. This is especially the case when considering 'real' cases (glaciers with real geometries in your case). A study such as the one presented here would have greatly benefited from an idealized setup, which would have made comparisons more straightforward and allowed to disentangle differences between simulations obtained fromV-A scaling and those relying on2-D SIA modelling:see e.g. Leysinger Vieli and Gudmundsson (2004)and Adhikari and Marshall (2012). Here a 'selection' of glaciers is considered, due to some 'problems' occurring when considering all glaciers in the region (see point 2b), which makes it even questionable how representative these are for this given region. With idealized glacier geometries, you could have explored the effect of glacier size, surface slope,...on the discrepancies between V-A based results and SIA modelled results more carefully.

As we have explained in the revised text our main objectives are (L64-70),
1. To obtain analytical predictions for climate sensitivity and response time of glaciers in a scaling model.
2. To compare the climate sensitivity and response time of a large number of synthetic glaciers with realistic geometries, as obtained from a scaling model and a 2-d SIA model.
3. To investigate the possibility of long-term biases in scaling model estimates of changes in glacier area and volume with respect to corresponding SIA results.
4. To find convenient parameterisation of glacier response properties obtained from the SIA simulations, and develop an accurate linear-response model.

Idealised models are inadequate for the objectives 4 above (and also for 2 and 3).

However, motivated by the reviewer's suggestion, we included idealised flowline model simulations that supports our conclusions. (sect. 3.3 and supplementary fig. S10).

b. Several arbitrary steps and decisions are made in the manuscript. A few examples of decisions that are hard to understand / seem not well funded:

ol. 181-182: you exclude glaciers with a large change in area over the 500-year time period? Why? This seems arbitrary, but you must have a reason for this. Moreover, how this this influence your results? This makes the sample less representative …

We have now explained (L143-146) that it is necessary to remove the glaciers with 'large' changes as a linear-response theory cannot be applied to them. Additional data is presented in the supplementary fig. S6 to demonstrate that our results do not depend on the cutoff chosen.

The set of selected 703 glaciers include 86% of all 814 Ganga basin glaciers (larger than 2 sq km) and cover 89% of their area. These two sets have similar distribution of slope and area as well (supplementary fig. S8) so that the selected ensemble can be considered representative.

o l. 182-183: why do you exclude glaciers with long response times? Again, this makes your sample less representative (you probably exclude a certain type of glaciers, likely those that are gently sloping: see e.g. Haeberli & Hoelzle, 1995). Is this because these glaciers are not in steady state after 500 years? If so, you should simply run your experiments for longer and not exclude these glaciers.

We have extended the runtime to 1000 years and the cut-off to 500 years so that only 9 glaciers out of the total 814 (~1% number-wise and ~2% area-wise) are left out now. The rationale behind this is explained with the help of supplementary fig S7 (also, L146-148 in the main text).

The selected 703 glaciers have similar slope distribution as that of all the 810 glaciers as shown in Supplementary fig. S8.

o Figure 1: you show '200 randomly chosen glaciers': why? Should show them all!

o l.249-250: 'In fig. 2b, about 30 data points,...were not included in the fit': why? You mention something about possibly creating a bias in the linear fit in the next sentence, but I do not see where this would result from / what the problem could be.

In the revised version, data for all the 703 glaciers are shown in fig 1, and no data points are excluded from the fits.

c. The Setup of your SIA model is not fully clear.

oYou mention that for > 100 cases 'our algorithm for finding a steady-state similar to present extent did not converge or the final steady state glacier geometry was not realistic': how is this possible? How can a simple SIA solution not 'converge' to steady state(in fact, even analytical solutions may exist that do not even require running the SIA model to find the steady state: see e.g. Jouvet & Bueler, 2012)? And what do you consider 'not being realistic'?

We have now updated our algorithm so that 810 out of 814 glaciers are included in the experiment. We have only excluded only 3 glaciers where bedrock noise/steepness led to violation of mass conservation – a known issue with SIA model (Jarosch et al., 2013), and another one due to a mapping error. This is discussed in the text (L134-136) and in the supplementary (fig. S2).

Which boundary conditions did you use to ensure mass conservation (e.g. to ensure specific ice-free regions do not become ice-covered)? You mention that mass conservation was monitored (l. 162-163): but how do you do this (this is not so straightforward to do...)? Did you check that the integrated SMB over your glacier is zero for the steady states (which it should be)?

Would also be good if you could consider some benchmark experiments (e.g. Jarosch et al., 2013)to make sure your model is mass conserving.

Boundary conditions are now described in the text (L127-128: noslip BC at the bedrock, and noflux BC at the domain boundary).

The algorithm used for checking conservation is given in L130-135, along with corresponding plots in supplementary fig S5. Ice conservation was explicitly verified to be satisfied up to 1 part in 10^9. The steady-state mass-balance was zero.

o Why do you randomly pick the values for the rate factor in Glen's flow law (not 'Glenn' + add a reference to the original studies, e.g. Glen, 1955)? The value of the rate factor will have a large influence on the local ice thickness and on thus the glacier volume. By picking this randomly: could be 'off' quite a lot from the 'reference/observed' volume of the glacier. Why do you not match this to the reference volume from every glacier that you have from Kraaijenbrink et al. (2017)?

Is this also not problematic when working with single values for c and g later in your analyses for all glaciers (e.g. for the best fits): you make some glaciers too thin and some too thick.

We did not calibrate the rate factor (or the balance gradient) to match the available volume and/or velocity estimate to avoid the associated computational cost. Tuning the rate factor to fit the thickness may not be a good idea, as it may lead to unrealistically small glacier velocities, and thus, unrealistic response properties.

We have acknowledged that the resultant ensemble is not a faithful copy of the Himalayan glaciers (L108-110, and Sect. 3.5). However, this ensemble serves the present purpose, as we are interested in a set of synthetic glaciers with realistic geometries. Plots comparing the area and ice thickness of SIA simulated steady glaciers, and the corresponding estimates from Kraaijenbrink et al. (2017) are added (fig S3, S8).

We have also added references to emphasise that the range of values of rate constant and balance gradient is realistic (L122-125).

The problem of large uncertainties in estimated volume of individual glaciers using a scaling relation with a single *c* is wellknown (e.g. Bahr et al., 2015). Since we are considering the scaling models that use this formulation, we stick to the above statistical interpretation of the scaling relation. This is clearly stated at the outset (L46-56).

d. Lack of in-depth analyses. Often you seem to be perplexed by some findings yourself and leave important questions unanswered, which is unsatisfying for the reader. This questions the thoroughness of your approach, e.g.:

We have tried to improve our discussions.

ol. 186-187: '...we did not do a detailed glacier-by-glacier analysis of the reason behind the failure of the algorithm'... Well, you should do this! May be something intrinsically wrong with your setup (e.g. in terms of mass conservation, boundary conditions; see 2c). If this is the case, this is likely to have direct consequences for your results and for some of your conclusions…

As described in our replies above,

* We have now include 810 glaciers out of the total of 814 in our analysis.
* The procedure to check mass-conservation is explained in the text, and corresponding plots are added to the supplementary.
* Boundary condition is stated in the methods section.
* Details of the three excluded glaciers are provided.

ol. 247-248: 'We do not have a clear explanation of this effect as yet': ...

ol. 256: 'Again, we do not have a theoretical argument for such a power-law behavior and did not explore this further here': …

ol. 304-305: '..., it remains to be investigated if the results described here depend on the regional characteristics of glaciers to some extent':...

The above comments have been deleted/modified in the present version.

[3] The **main conclusion** drawn your manuscript, and which also appears in the title, is that using V-A scaling methods (with 'time-invariant scaling') are likely to underestimate the future sea level contribution from glaciers.

a. I am not sure that the material you presented is convincing enough to support this statement and that the experimental setup is adequate (see previous point).

We hope to have answered this criticism in the replies above.

b. Another major concern that I have is: if this would be the case: why do we not see this when comparing outcomes of V-A scaling estimates compared to more sophisticated methods relying on retreat parameterizations (Huss & Hock, 2015)or flowline models (Maussion et al., 2019)? The first phase of the GlacierMIP project (Hock et al., 2019), in which future large-scale glacier simulations from the literature were compared, did not reveal a tendency for V-A scaling methods to underestimate the

contribution to sea-level rise (SLR). Also in the second phase of the GlacierMIP experiments, in which several ice dynamic (vs. V-A) were included and in which coordinated experiments were performed, no clear tendency can be seen when considering V-A scaling vs. methods in which the glacier geometry (and in some cases also ice dynamics) are explicitly considered. From the material at hand, I would rather tend to believe the outcomes from GlacierMIP than the main conclusions put forward here when it comes to the implications of using V-A scaling for future sea level projections.

Please refer to Table 3 of Hock et al. (2019). It is clear that scaling-model estimates for changes in both area and volume are always lower than that obtained in GloGEM. Please see the discussion in L335-344.

c. You draw your main conclusion (that the loss from V-A scaling with time-invariant scaling is underestimated vs. SIA) from two steady states: an initial one and a final one. You present your results like transient results (e.g. in plots, when describing response times,in section 4.1. describing that cis time-dependent and decreases with time,in section 4.4.,....etc.), but in the end, **it boils down to the fact that the volume of the final steady state with time-invariant V-A scaling is 'too large'**(compared to the SIA). Due to this, the transient volume loss when evolving to this steady state is underestimated (always with respect to SIA results). The main question that you thus need to address is: **why is the V-A scaled final steady state too big?** I am not an expert in V-A scaling, but I would find it surprising that this issue has not been addressed in other V-A scaling studies and that no solutions to this problem have been formulated. In the end, from my understanding, what happens is that many glaciers that reduce in size lose their lowest part, which are often the most gently sloping parts of the glacier and where the highest ice thickness is thus found (in most ice thickness reconstructions this clearly appears, where in the end, a large part of the reconstruction results from the negative correlation between the surface slope and the local ice thickness; see Farinotti et al., 2017). It is thus to be expected that the V-A scaling that you use to create the initial steady state does not hold for the final one. This is something that would need to be explored in more detail, and for which studies in which the volume scaling also uses information from other glacier characteristics (e.g. the glacier slope) could be useful (Grinsted, 2013; Zekollari & Huybrechts, 2015; see e.g. Fig. 9a in the latter, which summarizes the main point made here).

1. It is inadequate to say the only problem with scaling model is that they underestimate climate sensitivity of volume as the reviewer suggested here. We have demonstrated that, the scaling model,

* underestimate both area and volume sensitivities,
* underestimate both volume and area response times, and
* assume area and volume response time to be equal to each other.

We believe there are no existing prescription that allows correcting all these biases within a scaling model framework.

2. We have acknowledged that a gradual steepening of the shrinking glaciers is a major factor behind the decline in c (L276-282).
However, as we have shown using 1-d flowline models, even for transient glaciers with the same linear bedrock slope similar deviations/biases are seen (supplementary fig. S10). Slope-dependent corrections, therefore, cannot cure all the biases of scaling models.

3. We have argued that a significantly faster area response in scaling models compared to that in the SIA model, lead to a subdued volume response through the feedback of a shrinking ablation zone on the net negative balance (L242-247, and L294-304).

4. As stated clearly in the objective, instead of investigating the extended scaling model, we chose to focussed on a simple linear-response model and establish it as possible alternative which reduces the above biases.

[4] Unclarities in the manuscript. I found the text difficult to follow and quite often had to re-read sentences several times before being able to grasp their meaning. A few examples include:

a. l. 8-9: '..and validate them with results from scaling-based simulation of the ensemble of glacier'

b. l.84-85: '...are then empirically extended in order to obtain accurate parameterisations the linear-response properties of the SIA-simulated glaciers'

c.l.86-87: 'The linear-response model the long-term total shrinkage of glaciers as predicted by the scaling-based method (Radićet al., 2007), and the linear-response model are compared with the corresponding response'd.....etc.
See also comments on specific sections below.This makes it tedious to go through the manuscript. Furthermore, there a substantial number of grammatical errors, some of which (but not all) have already been pointed out by the first reviewer.

Also, many figures cannot be interpreted/read independently, without having to refer to the caption. It would be good if all essential information (e.g. meaning of colors used, R^2 values, equations,...etc.) could be directly included in the figure. Some other comments for specific sections(non-exhaustive list and not focusing on grammatical errors)

We have tried to improve the language, clarity, and organisation of the revised manuscript.
All the figures have been modified based on the above suggestions.

o1. Introduction:
'methods solving the dynamical ice-flow equations' à'numerical cost of such a computation on a global scale is prohibitive': well is not really the case anymore. In general: would be good to acknowledge regional-to global studies in which ice flow is explicitly accounted for (Clarke et al., 2015; Maussion et al., 2019; Zekollari et al., 2019).

o2. Quite abstract and thus very difficult to go through for someone who is not an expert in V-A scaling. Could make it less technical by for instance adding some additional information that links the various parts.
We have referred to these approaches in the revised version.

o1.2. Motivation for the present study: difficult to follow the first paragraph: be more specific when you refer to c and gamma not continuously mix with other terminology 'time invariant scaling-based parameterisation', '...given the known violation of the time-invariant scaling assumption'.

We have defined the scaling models in the beginning as the ones that assume a constant c and γ to describe an ensemble of glaciers interpreting the scaling relation statistically. We clarify that the present paper only considers this statistical interpretation of the scaling relation. We have tried to improve the Introduction section in general.

o3.1.: 2-dimensional SIA model:

•l.152-154: where did you get the ice thickness from? From Kraaijenbrink et al. (2017)directly? As the ice thickness is quite crucial in your story (it determines the volume...), why did you not consider the consensus estimate of Farinotti et al. (2019), which is freely available?

This is because Kraaijenbrink et al. (2017) was available when the study was initiated. It also had debris-cover information that we we intended to incorporate. Moreover, given the other idealisations in our ensemble of synthetic glaciers and our stated objectives in the study, any reasonable bed rock is fine. So we stick our original choice of bedrock.

•SIA: refer to the original work by Hutter (1983)also.

•You neglect basal sliding (l. 161). Justification? Could refer to other studies where this is done, like e.g. Gudmundsson(1999) and Clarke et al. (2015).

The available flowline model studies of Himalayan glaciers typically includes sliding. We have not performed a sensitivity analysis. So at the moment, we simply state that we dropped the sliding term for simplicity.

•l.168-178: this is related to the SMB, which you apply in all cases(i.e. also for the linear-response model and the V-A scaling, right?). Not sure this section is correctly placed here in the '2-dimensional SIA model' section.

The mass-balance function is moved out of the section describing SIA model.

•l.183: through several exclusion you keep 68% of the initial glaciers... How much does this represent in terms of glacier volume and glacier area when compared to the total glacier sample?

We have now included 86% glaciers in our study. The distribution of slope, and area of the two sets with 703 and 810 glaciers are shown in the supplementary S8 .

•l.188-196: you explain some simplifications related to debris cover, avalanche and sliding have been made and that this may influence your results. Well, you have made much larger simplifications than this: e.g. linear SMB profiles with strongly imposed max. SMB, steady state assumptions for glaciers,... à not even worth mentioning these more detailed simplifications in my opinion. With all these simplifications, would have been better to opt for idealized setup likely (see main comment 2a).

We have already explained why we prefer the present setup over idealised glacier before.

At least for Himalayan glaciers, factors like avalanches and debris cover can be very important.

We have used the standard paradigm of defining system response around steady-states. In fact, most of the knowldge about glacier response is based on response properties of steady-state glaciers. A detailed discussion of this issue is added (L171-180).

We have included 1-d idealised flowline model simulation now to show that our results are not specific to the ensemble of synthetic glaciers used (Supplementary fig. S10).

•l. 194: 'These simplifications do not weaken our study': not sure you can judge on this yourself...

oSection 3.2.:

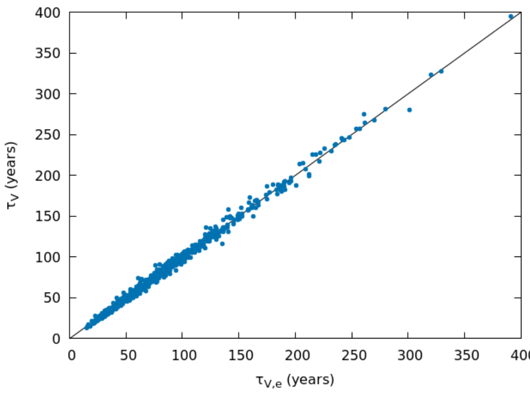
•l.204: 'was fixed at... because...': don't understand the causality (i.e. link between cause and consequence).

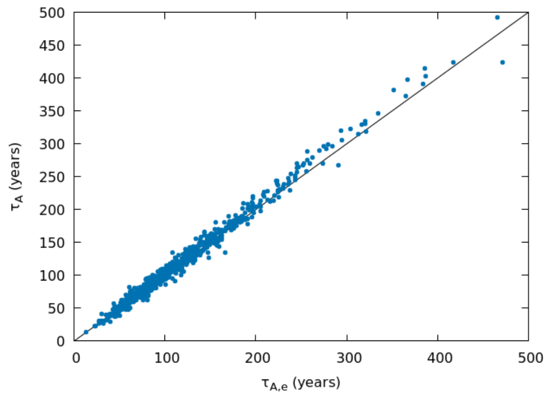
•Figure 1: SIA-derived volumes are scaled by a factor 10: why? Does not really make sense and unclear when just looking at the figure without reading the caption... Axes should be correct in the figure and not only for a part of the data you show..Also illustrates the unclarity in the figures mentioned in main comment 4 (problem that figures cannot be interpreted without referring to their caption).

We have made appropriate changes to address these concerns.

OSection 3.3.:
•l.208-210: complicated way to say that you consider e-folding time scales. Would reformulate this and add references for thistoe.g. Leysinger Vieli & Gudmundsson(2004).

We do not use e-folding time, but fit the linear response for directly. These two methods would give the same result for a purely exponential response. Our approach may be more suited for calibrating the linear-response model, as it minimises the RMSE between the linear-response and SIA model outputs. (L171-180)





As shown above, the differences between our best-fit response time (vertical axis) and the corresponding e-folding time (horizontal axis) is less than a couple of per cent on the average. Note that in the above plots the 1:1 lines (black).

oSection 4.1.:

•l.225: V=cA$^{1.286}$: not sure I understand. Does this statement apply for the initial and/or final steady state volumes? And can all the volumes be described with this single relationship? Is the fact that quite different rate factors are used not a problem for this (see main comment, point 2c)?

As explained in L263-268, the set of 703 glaciers at any instant (intial, final, steady or nonsteady) follow this relation statistically with different best-fit *c*. The same is confirmed in fig 1a. This is consistent with the theory of Bahr et al. (2015).

•l.227 + l.230 + l.232: here you mention that c is time-dependent. Not sure you can say that it is time dependent: simply results from the fact that final steady state volume for V-A scaling is 'overestimated' (vs. SIA). As a result the evolution to this steady state is different. See main comment 3c for this.

It is clear from figure 1 that *c* is time-dependent. For example, the best-fit *c* has two different values at t=0 and t=500 years. We have already replied the other comment before.

•l.235-237: relates to main comment 3c again. If you do not modify the V-A scaling, then problems will arise when considering the same glacier that is much smallerin a warmer climate (when rising the ELA in your case): you typically lose the lower parts where most volume is and volume will thus be 'overestimated'. Is this not accounted for in some way in future glacier evolutions based on V-A scaling? As a part of this discussion, studies in which V-A scaling is extended with other glacier characteristics (such as the surface slope; Grinsted, 2013; Zekollari & Huybrechts, 2015)would be good to include. Such relationshipswhich could prove to remain valid over time, even without changing scaling and exponents.

We agree with the reviewer that scaling model description can be improved by incorporating the slope-dependence of *c*. We now acknowledged that a major part of the time dependence of c is due to steepening of glacier slope with time (L276-283). However, a slope-dependent correction is unlikely to get rid of all the scaling model biases pointed out in this study. For example, as shown in our idealised flowline model results (supplementary fig. S8), similar biases can be present even for a set of glaciers with the same slope.

We have now emphasised that scaling models implicitly assume,

$\tau_A=\tau_V$.

However, the area-response time is longer in reality (SIA). This limitation would introduce complex time-dependence in the relationship between A and V for real glaciers (Fig 1b). We have now argued that a faster area loss in the early stages of the response as simulated by scaling models, provides a feedback to the net negative balance, and leads to a subdued long-term volume loss (L242-247 and L294-305)

OSection 4.2.:

•l. 243: 'This is exactly what is seen in Fig. 2b, which shows...': I cannot directly see this...

•l.244: 'change in c to the tune of ~13%': what does this mean?

•l.255: 'The above figure': will depend where your figure comes in final manuscript...

oSection 4.3:

•Iwas wondering what the pointis thatyou want to make with this section? It is known from literature that volume responds faster than area (e.g. Oerlemans, 2001; Leysinger Vieli & Gudmundsson, 2004).

•l.260-264: relationship between volume and area response times. How does this compare to the relationship others have found in the literature?

We have updated the text based on these comments, and added necessary discussions and references (L294-298).

OSection 4.4.:

•l.271-272: 'with most of the changes taking place during the first couple of centuries': this is not a result/finding.. This directly results from the e-folding time-scale when forcing a steady state glacier with an instantaneous forcing in SMB.

•l.273: 'underestimates the long-term change': not about reaction/response. This is direct consequence of fact that final steady state volume is too large (see main comment 3c)

•l.279-280: '...suggests that there might be significant negative biases of mountain glacier contribution to sea-level rise as computed by scaling-based methods'(+ section 4.5, l.300-302): well, do not see this in GlacierMIP phase 1+2... Is a very strong statement to make and should be sure that it is well-founded.

We reworded the above statements, and added discussions referring to the trends seen in Hock et al., (2019) in support of the above claim. (L334-349)

oSection 4.5:

•l.296-297: 'More detailed studies that relaxes some of the above mentioned assumptions are needed...': not sure what you mean by this. Would also make sense that you dig into this: e.g. by focusing on real transient response vs. comparing two steady states (what you do now and then translate into an analysis of the transient response resulting from this: see main comment 3c).

We have added a section on how the steady-state response properties can be used to obtain transient response (section 3.4) to clarify the issue.

•l.299: ‘intruding more scatter in the fits’: what does this mean?
The sentence is removed.

oSummary and Conclusions:

•l.309-310: scale factor reduces over time. Well, not sure the time dimension is adequate here. Boils down to having a final steady state that would require a smaller value for c: see main comment 3c.

If initial and final states have different best-fit *c*, then that implies *c* is time dependent.

We agree that having a smaller c and having a thinner final state are equivalent – it is not possible to assign one as the cause and the other as the effect. We have now based our explanation on the theoretical and numerical results showing that the scaling models assume the area and volume response to be equal. In contrast, within SIA, area response time is ~1.5 times larger than the corresponding volume response time. This lead to a faster initial area loss in scaling which reduces net volume loss due to a feedback on net negative balance.

•l.324: computational efficiency. OK, still important, but is not really a limitation anymore, due to which V-A scaling becomes less important (and also driven bythe release ofnew datasets with regional-to global spatial coverage at individual glacier level: see main comment 1).

Given that most, if not all, available model estimates for glacier contribution to sea-level rise are based on low-dimensional models, computational efficiency may be an important factor.

•Code availability: for which models is the code available? Seems to suggest that the SIA code is not available. Not sure if this fully agrees with the policies of The Cryosphere: see www.the-cryosphere.net/about/data_policy.html

We shall make all codes available upon possible publication of the manuscript.

Possible biases in scaling-based estimates of mountain-glacier contribution to ~~the sea level~~sea-level rise

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Abstract. ~~Predicting mountain-glacier contribution~~ Low-complexity glacier models are used to compute the contribution of mountain glaciers to sea-level rise ~~involves computing global-scale glacier loss under a given~~ given a climate-change scenario. ~~Such calculations are usually done with low-complexity and computationally-efficient approximate models of glacier dynamics. A statistical power-law relation~~ A majority of these models are based on statistical scaling relations between glacier volume~~and area(~~, area, and/or length~~) is the basis of several such models. We simulate transient response of an ensemble of 551 glaciers from Ganga basin, the Himalaya, using a scaling-based method and a two-dimensional ice-dynamical model based on~~ . In this paper, the response properties of glaciers are theoretically analysed within a time-independent volume-area scaling assumption. The theoretical results are validated with a scaling model simulation of the response of 703 synthetic Himalayan glaciers from the Ganga basin to a step-change in climate. The same numerical experiment repeated with a 2-d shallow-ice approximation (SIA) ~~. A comparison of the model outputs suggests that the scaling-based method systematically underestimates~~ model, obtains about three times larger climate sensitivity and response time than that predicted by the scaling model. There is a corresponding low bias in the scaling model estimates of the long-term ~~ice loss due to a violation of the assumed time-invariant scaling. We derive expressions for the response time and climate sensitivity of glaciers simulated using a~~ loss of the total glacier area and volume. Also, the scaling model predicts the area and volume response times to equal to each other, while the SIA model obtains area response time that is about 1.5 times larger than the corresponding volume response time. Consequently, the transient glaciers simulated with SIA exhibit a systematic violation of time-invariant scaling~~assumption, and validate them with results from the scaling-based simulation of the ensemble of glacier. These expressions are modified empirically to obtain similar parameterisations of the response properties of glacierssimulated with SIA. These new parameterisation yields~~ . The SIA results are used to obtain parameterisations of climate sensitivity and response time of glaciers, leading to a linear-response model which ~~significantly reduces the above biases, while retaining the advantage of numerical efficiency~~outperforms the scaling model in reproducing the SIA results. This is confirmed by an experiment on an independent set of 204 glaciers from the Western Himalaya. This linear-response model may be useful for predicting the sea-level contribution from shrinking mountain glaciers.

1 Introduction

Shrinking mountain glaciers have contributed significantly to ~~the~~ global eustatic sea-level rise in the recent past, and this trend is expected to continue for the next hundred years or so ~~(Meier, 1984; van de Wal and Wild, 2001; Raper and Braithwaite, 2006; Cogley, 2009~~ The reliability of the predicted global sea-level change is, thus, intimately tied to the accuracy of the predicted total ice-loss from mountain glaciers~~globally~~.

Instantaneous (annual) glacier surface mass balance can be calculated readily using ~~data from climate model simulations. However, an accurate~~ climate model outputs. In contrast, any prediction of the long-term evolution of a glacier ~~would require simulating the decadal-scale~~ requires simulating the slow (decadal) changes in glacier area and ~~hypsometry (Raper and Braithwaite, 2006; R~~ ~~ideally~~ geometry. Ideally, this is to be done by solving the dynamical ice-flow equations ~~(Oerlemans, 2001; Cuffey and Patterson, 2010)~~(e.g. However, the numerical cost of such a computation on a global scale ~~is prohibitive~~creates a bottleneck, even if ~~a simplified approximate description of the full~~ simplified approximate descriptions of the ice-flow equations~~like~~, like, shallow-ice approximation (SIA) (Hutter, 1983) or its higher order variants were to be used ~~. Further~~(Egholm et al., 2011; Clarke et al., 2015). One-dimensional SIA-based modelling tools are promising developments in this regard (Maussion et al., 2019; Zekollari et al., 2019; Roun The uncertainties associated with various input parameters, e.g., an uncertain glacier bedrock~~limits~~, limit the benefit of using the physically-based ice-flow models ~~(Farinotti et al., 2016). The~~ as well (Farinotti et al., 2016).

Due to the above difficulties, the existing global-scale estimates of the ~~mountain-glacier contribution~~ contributions of shrinking mountain-glaciers to sea-level rise mostly rely on low-dimensional approximate parameterisations of ~~the glacier dynamics (van de Wal and Wild, 2001; Raper and Braithwaite, 2006; Hirabayashi et al., 2010; Radić and Hock, 2011; Slangen and van de~~ ~~Several~~ glacier dynamics (Radić et al., 2014). The results from these simplified models provide critical inputs for assessing regional to global vulnerability to sea-level rise (e.g., Kulp and Strauss, 2019). While some of these parameterisations are ~~based on an statistical area-volume (or area-volume-length) scaling relation for any set of mountain glaciers (Chen and Ohmura, 1990; Bahr et al.,~~ ~~Empirical prescriptions for distributing the annual ice-loss over glacier surface, or equivalently,~~ empirical prescriptions for adjusting the hypsometry of the transient glaciers ~~are also used (Raper and Braithwaite, 2006; Radić et al., 2008; Hirabayashi et al., 2010; Hu~~

1.1 ~~Area-volume scaling for mountain glaciers~~

~~The statistical power-law relationship between glacier area and volume was established empirically (e.g., Chen and Ohmura, 1990),~~

$$\cancel{V = cA^{\gamma},}$$

~~where,~~ (Raper and Braithwaite, 2006; Huss et al., 2010; Huss and Hock, 2015), a majority of them are primarily based on a statistical volume-area (or volume-area-length) scaling relation. This volume-area scaling equation relates glacier volume V ~~and~~ to glacier area A ~~are glacier area (km^2) and volume (km^3), respectively.~~ as,

$$V = cA^{\gamma}, \tag{1}$$

where, γ is a dimensionless scaling exponent~~expected to be in the range $1.17 < \gamma < 1.5$ (Bahr et al., 2015). The scaling exponent γ can be estimated~~ , and c is a scale factor (Bahr et al., 2015). This relation was established empirically (e.g., Chen and Ohmura, 19 and subsequently proved using dimensional analysis ~~(Bahr et al., 2015), if an~~ (Bahr et al., 1997, 2015). The derivation utilised the empirical sub-linear scaling of glacier width ~~with glacier area (Bahr, 1997)is assumed. c (km$^{3-2\gamma}$) is a dimensionful fitting parameter (Bahr et al., 2015). The above relation is statistical in nature, and allows estimation of the total volume of a large set of glaciers fairly accurately. However, it would have considerable uncertainty when volume of any individual glacier in the set is considered (Bahr et al., 2015). The scaling relation is used to predict glacier area change given the volume change, with c and γ assumed to be time-invariant constants (e.g., Radić et al., 2007).~~ and ablation rate with the glacier length (Bahr, 1997).

~~According to the theoretical arguments by Bahr et al. (2015), γ indeed is a time-independent constant with~~ Theoretically, the scaling exponent γ is time-independent, and can be expressed as $\gamma = 1 + \frac{m+1}{m+n+3}$ ~~, where~~ (Bahr et al., 2015). Here, n is the power-law exponent of ~~Glenn~~Glen's rheology of ice (Glen, 1955), and m is the scaling exponent of ablation rate with glacier length ~~. However, the dimensionful scale factor~~ (Bahr, 1997). For an individual glacier, the scale-factor c ~~may vary with time (Bahr et al., 2015)for a set of non-steady glaciers. For example, dimensional arguments do not rule out the possibility that $c = c(t/\tau)$, where t/τ is the dimensionless ratio of time t to the response time τ of~~ captures the control of all the glacier-specific factors (except area) on its volume (Bahr et al., 2015). There is no available theoretical prescription for obtaining the value of c for an arbitrary glacier. c may be calibrated for a particular glacier based on available independent measurements of area and volume during an epoch, but its time dependence can be accessed only with a detailed model simulation (Bahr et al., 2015).

For a large enough ensemble, glacier area typically spans a few orders of magnitude. However, the ~~glacier. An unaccounted-for time-dependence of~~ corresponding c values vary over a relatively restricted range (Bahr et al., 2015). This allows an approximate statistical description of any set of glaciers using eq. 1, where a single best-fit c and a fixed γ is used (Bahr et al., 2015). Such a best-fit scaling relation provides a fairly accurate estimate of the ~~scale factor~~ total ice volume of a large set of glaciers, but the corresponding predictions for the individual glaciers have relatively large uncertainties (Bahr et al., 2015). Note that there is no theoretical constraint for c ~~would lead to a systematic bias in the predicted glacier change as discussed later in this paper~~to be time-independent for a given set of non-steady glaciers (Bahr et al., 2015).

It is the above statistical interpretation of the scaling relation, where a best-fit time-invariant c and a constant γ is used to describe an ensemble of glaciers, that is exploited in the scaling-based approximate models of glacier dynamics (e.g., Radić et al., 2007). Hereinafter, we refer to the class of models that are based on such an approach (e.g., Radić et al., 2007), as "scaling models". As the present study investigates the possibility of biases in scaling model predictions of the sea-level rise contribution of mountain glaciers, we restrict ourselves to the above statistical interpretation of the scaling relation.

1.1 ~~Motivation for the present study~~

The performance of ~~the scaling relation (eq. 1) in describing~~ scaling models in simulating the transient glacier response have previously been ~~investigated by comparing results of scaling-based model with those from simulations based on~~ tested against various dynamical ice-flow models ~~. Dynamical models with various levels of complexity,~~ (e.g., SIA, higher order approximations~~or full Stokes' evolution~~ , or Stokes' model) in one to three dimensions ~~(Radić et al., 2007, 2008; Adhikari and Marshall, 2012; Farinot~~

~~used for this purpose. However, the above studies assumed that scaling exponent γ may vary with time, which violates the theoretical arguments of Bahr et al. (2015). In addition, most of the above investigations were confined to synthetic glaciers having idealised geometries. Possible~~ using both idealised (Radić et al., 2007; Adhikari and Marshall, 2012) and realistic geometries (Radić et al., 2008; Farinotti and Huss, 2013). The uncertainties introduced by a ~~time-invariant scaling-based~~ scaling-model parameterisation of the evolution of glaciers with realistic geometries were considered ~~only~~ by Farinotti and Huss (2013). The ~~authors provided thorough assessment of the uncertainty arising out of the inaccuracies of the parameters c and γ. The~~ spirit of the present study is quite similar to that of Farinotti and Huss (2013)~~mentioned above. However, the authors did not consider the possibility of a systematic bias in the scaling based models given the known violation of the time-invariant scaling assumption (Radić et al., 2007, 2008; Adhikari and Marshall, 2012) as discussed above. A time-independent c can lead to systematic bias even in the case when~~ , except that we are investigating the possible intrinsic biases of scaling models in a situation where the parameters (c and γ) are known accurately~~for a given set of glaciers. One of our two main~~ . The specific objectives of the present study ~~is to investigate this possible bias of scaling-based models.~~

~~Apart from the scaling-based methods, there are other empirical methods for mimicking glacier evolution that use specific algorithms for distributing the net mass loss over the glacier surface (Raper and Braithwaite, 2006; Huss et al., 2010). Yet another viable alternative is obtained by assuming small deviations from a steady-state glacier (Oerlemans, 2001; Harrison et al., 2001; Lüth~~ ~~This so-called~~ are,

1. To obtain analytical predictions for climate sensitivity and response time of glaciers in a scaling model.

2. To compare the climate sensitivity and response time of a large number of synthetic glaciers with realistic geometries, as obtained from a scaling model and a 2-d SIA model.

3. To investigate the possibility of long-term biases in scaling model estimates of changes in glacier area and volume with respect to corresponding SIA results.

4. To find convenient parameterisations of glacier response properties obtained from the SIA simulations, and develop an accurate linear-response ~~theory for glacier area or volume-change, however, requires the knowledge of~~ model.

Note that the last objective involves a linear-response model which is a low-complexity model obtained in the limit of a relatively small deviation around a steady state (e.g., Oerlemans, 2001). To apply this model on a large number of glaciers, the response time and climate sensitivity ~~of glacier volume and area~~need to be specified for each of them. A lack of accurate and ~~numerically convenient~~ numerically-convenient parameterisations of these dynamical properties ~~limits the application of such linear-response models for sea-level rise predictions (Harrison et al., 2001; Lüthi, 2009). The other objective of the present study is to come up with an accurate parameterisation of the relevant linear-response properties , and thus improve the performance of the linear-response model.~~ may have limited their application (Harrison et al., 2001; Lüthi, 2009; Bach et al., 2019). Here, we aim to obtain parameterisations of the glacier response properties using results from 2-d SIA simulations of a large ensemble of synthetic glaciers with realistic geometries.

~~We emphasise that the points of departures of this study from that of Farinotti and Huss (2013) are that, 1) Following the theoretical guidelines of Bahr et al. (2015) c is considered to be the sole fit-parameter, 2) Given the known violation of the assumed time-invariance of area-volume scaling , possible systematic biases in scaling-derived glacier-loss predictions are investigated, and 3)possible new parameterisations of~~ The paper is organised as follows. First, we theoretically derive the glacier-response ~~properties are explored in order to improve the accuracy of the linear-response model.~~

1.1 ~~Outline of the study~~

~~Here, we simulate~~ properties within a time-invariant scaling assumption (sect. 2.1 and 3.1). Then, we compare the performance of a representative scaling model (Radić et al., 2007) with that of a 2-dimensional SIA model, in simulating the response of ~~an ensemble of 551 clean-ice~~ 703 idealised Himalayan glaciers in the ~~Himalaya to a step change in equilibrium-line~~ Ganga basin to a hypothetical step rise in equilibrium line altitude (ELA) ~~using 2-dimensional SIA model~~ (sect. 2.2 and 3.2). We use the response properties obtained from the scaling model to test the above analytical expressions for glaciers-response properties. The SIA results are used to obtain parameterisations for the linear-response properties of glaciers. The accuracy of ~~a scaling-based model (Radić et al., 2007)~~ the scaling model and a linear-response in reproducing the SIA-derived long-term loss ~~in~~ of total glacier area and volume ~~as obtained with SIA, is assessed . First we discuss the basics of the area-volume scaling method for glacier evolution, deriving theoretical predictions for dynamical response properties of glaciersevolving under a time-invariant scaling. These theoretical predictions are verified with a scaling-based simulation of the response of the same 551 glaciers to a step-change in ELA. The scaling-based theoretical expressions for glacier response properties are then empirically extended in order to obtain accurate parameterisations the~~ is assessed for the above 703 glaciers. The performance of the linear-response ~~properties of the SIA-simulated glaciers. The~~ model is also tested for an independent set of 204 glaciers in the western Himalaya without any further calibraton. We also discuss the applicability of the linear-response model ~~the long-term total shrinkage of glaciers as predicted by the scaling-based method (Radić et al., 2007), and the linear-response model are compared with the corresponding SIA results~~for actual computation of future glacier loss for a set of transient glaciers forced by any arbitrary time-variation ELA (sect. 3.3).

2 ~~Dynamical consequences of a time-independent area-volume scaling~~

~~Let us~~

2 Methods

2.1 Theoretical methods

For a theoretical analysis of the glacier-response properties implied by a scaling model, we consider a set of hypothetical glaciers that are responding to a warming climate ~~while conforming to a time-independent area-volume scaling (Eq~~such that the volume-area scaling relation (eq. 1) ~~. For small~~ is valid, and c is a given time-invariant constant. Then, the fractional changes in area and volume of ~~the glaciers, the two~~ these glaciers, in the limit of small changes, are related as follows.

$$\Delta V \approx c\gamma A^{\gamma-1}\Delta A = \gamma \frac{V}{A}\Delta A = \gamma h \Delta A, \tag{2}$$

where, ΔV and ΔA are the changes in area and volume, and the mean ice thickness is $h = V/A$. The ~~scaling factor c is assumed to be a time-independent constant here.~~

~~The~~ above equation is the basis of the ~~scaling-based glacier-evolution schemes, allowing computation of area change during a time interval from the given volume change during that interval. The particular model of Radić et al. (2007) that is considered here, assumes that the area loss takes place only in the lowest elevation band/s near the glacier terminus and updates glacier hypsometry accordingly (Radić et al., 2007).~~

2.2 ~~The rates of area and volume change~~

~~A consequence of the above scaling relation (Eq. 2) is that the instantaneous rates of area and volume changes are related as,~~

$$\dot{V} = \gamma c A^{\gamma-1}\dot{A} = \gamma h \dot{A},$$

~~where, $\dot{V}$ and $\dot{A}$ denote the rate of changes of volume and area. If the mean specific balance rate is δb, then the annual rate of mass loss is $\dot{V} = \delta b A$. This, together with Eq. 4, implies,~~

$$\begin{aligned} \dot{A} &= \frac{\delta b}{\gamma h} A \\ &= \frac{\delta b}{\gamma c} A^{2-\gamma}. \end{aligned}$$

~~Thus, a consequence of the time-invariant scaling hypothesis is that the rate of area change must scale with glacier area with an exponent $(2-\gamma)$. This is consistent with empirical observations (Banerjee and Kumari, 2019). As the scale factor of this power-law relation is proportional to the mean mass balance, Eq. 5 may be a convenient way of obtaining mean regional thinning rates from relatively straightforward remote-sensing measurements of the rate of area change. However, this relation is accurate only to the extent the assumption of the time-independence of the scale-factor c is valid.~~ scaling models of glacier evolution (e.g., Radić et al., 2007). We have derived analytical expressions for glacier response time and climate sensitivity starting from this equation, essentially following the line of arguments by Harrison et al. (2001).

2.2 ~~The area response time~~

~~As response time is defined with respect to a steady-state glacier, let us consider a steady glacier and apply a constant perturbation for time $t > 0$ (i.e. a step change in ELA). Let's take the corresponding instantaneous net negative balance of the glacier at $t = 0$ to be $\delta b_0 A$. The perturbation would asymptotically $(t \to \infty)$ lead to a shrinkage of glacier area by $\Delta A_\infty = A(0) - A(t \to \infty)$ and ice volume by ΔV_∞ such that (Harrison et al., 2001),~~

$$\Delta A_\infty b_t + \beta \Delta V_\infty \approx -\delta b_0 A.$$

~~Here, b_t is the ablation rate near the terminus and β is the balance gradient. The area response time of the glacier can be estimated as $\tau_A \approx \Delta A_\infty / \dot{A}$. Therefore, using the expressions for $\dot{A}$ (Eq. 5)~~

2.2 Numerical methods

We simulated the response of an ensemble of synthetic clean glaciers with realistic geometries to a hypothetical step-change in ELA using three different methods (scaling, SIA, and ~~ΔA_∞ (Eq. 7), respectively, we obtain,~~

$$\gamma h + \beta)^{-1} = \tau^*.$$

(

~~Here, the symbol τ^* is a convenient shorthand notation for the time scale $-(^{b_t}\gamma h + \beta)^{-1}$. In the above derivation, ΔV_∞ that appears in eq. 7 is eliminated with the help of eq. 2. The resultant expression for response time (Eq. 8 is comparable with that derived by Harrison et al. (2001) or Lüthi (2009).~~

2.3 ~~Linear response model for area and volume change~~

~~Within a~~ linear-response ~~model the instantaneous change in volume (ΔV) as after a steady glacier is perturbed by a small step change in ELA is, for example, given by Lüthi (2009) as,~~

$$\Delta V(t) = \Delta V_\infty (1 - e^{-t/\tau_v}),$$

~~where, τ_v is the volume response time and V_∞ is the volume sensitivity.~~

~~Now, for small fractional changes in area it is straightforward to show that Eq. 9 implies,~~

$$\Delta A(t) = \Delta A_\infty (1 - e^{-t/\tau_v}),$$

~~by expressing $V(t)$, $V(0)$ and $V(t \rightarrow \infty)$ in terms of $A(t)$, $A(0)$, and $A(t \rightarrow \infty)$, respectively, using the scaling relation (Eq. 1), That is , within the scaling assumption, the area response time, τ_A, is the same as the volume response time, τ_V,~~

$$\tau_A = \tau_V = \tau^*.$$

~~This also implies that the ratio $\Delta V(t)/\Delta A(t)$ is a time independent constant. In fact, Eq. 4 requires that this constant ratio is given by γh. While h is not strictly time independent for a set of shrinking glaciers, for a small fractional change in area the corresponding fractional changes in mean thickness is going to be significantly smaller. This is because, from eq. 1, $h = V/A = cA^{\gamma-1}$, or $\frac{\Delta h}{h} = (\gamma - 1)\frac{\Delta A}{A}$, and $(\gamma - 1)$ is a small quantity for typical values of γ.~~

2.3 ~~Climate sensitivity of area and volume~~

~~The climate sensitivity of the area is given by ΔA_∞. An expression for the asymptotic fractional change in area is obtained by eliminating ΔV_∞ from Eqs. 7, and using the expression for τ_A (Eq. 8),~~

$$\tau^* \beta \delta E \gamma h = \alpha^*.$$

(

~~Here, we have used $\delta b_0 \approx \beta \delta E$ for a step change in ELA by δE, and the RHS is denoted by α^* for convenience.~~

~~The corresponding expression for $\frac{\Delta V_\infty}{V}$ is then obtained using Eq. 2,~~

$$\frac{\Delta V_\infty}{V} = \gamma \alpha^*.$$

~~Again, Eq. 13 is comparable to the expression of volume sensitivity as derived by (Harrison et al., 2001). The minor differences are due to the time-invariant scaling assumption made here.~~

~~Please note that strictly speaking, the climate sensitivity of area and volume with respect to a change in ELA should be defined as $\frac{\Delta A_\infty}{\delta E}$ and $\frac{\Delta V_\infty}{\delta E}$, respectively. However, in this paper we use ΔA_∞ and ΔV_∞ as the corresponding sensitivities to simplify notation.~~

3 ~~Numerical methods~~

2.1 ~~2-dimensional SIA model~~

~~We consider all the 814 glacierslarger than~~ models). For this exercise, we considered all the 814 glaciers larger than 2 km^2 ~~in the Ganga basin, the central Himalaya from RGI 6.0 inventory (RGI, 2017), and simulated them one by one using an automated procedure. We defined the simulation domain for each of the glaciers with a one-pixel-wide (pixel size of 100 m × 100 m) buffer around the corresponding RGI 6.0 outlines. The corresponding~~ in the Ganga basin, the central Himalaya (Supplementary fig. S1). The ice-free ~~bedrock is obtained using available~~ bedrock for each of the glacier was obtained using available ice-thickness ~~estimates for each of the individual glaciers in the RGI inventory (Kraaijenbrink et al., 2017) and ASTER GDEM (ASTER GDEM, V003) which are down-sampled to 100 m resolution~~estimates (Kraaijenbrink et al., 2017) and surface elevation (ASTER GDEM, V003). The following idealised elevation-dependent linear mass-balance profile was used,

$$b(z) = Max\{\beta(z - E), b_0\}. \tag{3}$$

Here, β is the balance gradient, z is the surface elevation, and E is the equilibrium-line altitude (ELA). b_0 is a cutoff on maximum accumulation taken to be 1.0 m/yr. The choice of β is described later. In our mass-balance model, we neglected complicating factors like supraglacial debris cover and its effects on ablation, and the avalanche contribution to accumulation. Overall, the simulated glaciers can not be considered faithful copies of the actual Himalayan glaciers. Rather, they constituted an ensemble of synthetic glaciers with realistic geometries (e.g., Farinotti and Huss, 2013) to be used here for a comparative study of the performance of the three models.

2.0.1 A 2-d SIA model

The ice-flow dynamics was implemented within ~~two dimensional shallow-ice approximation (e.g., Le Meur et al., 2004) as an easy-to-implement~~ a two dimensional SIA (Hutter, 1983) as a numerically efficient non-linear diffusion problem (Oerlemans, 2001). While SIA may not be the best method for simulating valley glaciers due to its limitation in describing ice-flow ~~that is~~ influenced by longitudinal stresses and/or steep bedrock slopes (Le Meur et al., 2004), there is enough evidence in the literature that SIA does a reasonable job of describing both the steady and transient dynamics of valley glaciers ~~(e.g., Vieli and Gudmundsson, 2004; Le Meur et al., 2004).~~

~~The equations of motions were integrated using a linearised implicit finite-difference scheme (Hindmarsh and Payne, 1996) with no-slip boundary condition at ice-bedrock interface. An iterative conjugate-gradient method was employed within the implicit scheme, with a spatial grid-size of 100m×100m and time steps of 0.01 years. During the simulations, ice-mass conservation was monitored and was within one parts per 10^8 at each time step~~(e.g., Vieli and Gudmundsson, 2004; Le Meur et al., 2004; Radić et al., 20
The contribution of sliding to the flow was neglected here for simplicity.

The value of ~~Glenn~~Glen's flow-law exponent ~~is~~ was assumed to be ~~3. Since, we are interested in the generic behaviour of a set of glaciers with realistic geometries, we do not attempt any tuning of the flow parameters to obtain realistic values of ice thickness or surface velocity~~ 3 (e.g., Oerlemans, 2001). For the sake of simplicity, we did not tune any of ~~the chosen glaciers. Rather,~~ the ~~rate constant appearing Glenn~~model parameters to match the observed ice-thickness and/or flow velocity on any of these glaciers. The only exception was ELA which was tuned to obtain the initial steady state as described below. In order to avoid possible dependence of the results on any specific choice of parameters, we picked the parameters related to mass balance and flow from random distributions. The rate constant of Glen's law was picked randomly from the set ~~$\{0.5, 0.6, ...1.4, 1.5\} \times 10^{-24}$~~ $\{0.5, 0.6, ..., 1.4, 1.5\} \times 10^{-24}$ $\mathrm{Pa}^{-3}\mathrm{s}^{-1}$ for each of the glaciers.

~~The following elevation-dependent idealised linear mass-balance profile was used for all of the glaciers,~~

~~$b(z) = Max\{\beta(z-E), b_0\}.$~~

~~Here, β is the balance gradient , which was again picked randomly for any given glacier, with the allowed~~ This range of values is comparable to those used to model mountain glaciers previously (Radić et al., 2008). The balance gradient β was also picked randomly from the set of values $\{0.005, 0.006, ..., 0.009, 0.010\}$ yr^{-1} for ~~the glaciers. z and E are the surface elevation and the equilibrium-line altitude (ELA) measured in meters. b_0 is a cutoff on maximum accumulation taken to be 1.0 m/yr.~~ each glacier. This range of β-values is comparable to the observed mass-balance gradients in the Himalaya (e.g., Wagnon et al., 2013).

~~The value of E for each of the glaciers was fixed as follows. E is at first set equal to the glacier median elevation. Then~~ The model was integrated using a linearised implicit finite-difference scheme (Hindmarsh and Payne, 1996), with a no-slip boundary condition at the ice-bedrock interface and a no-flux boundary condition at the domain boundary. An iterative conjugate-gradient method was employed within the implicit scheme, with a spatial grid-size of 100 m×100 m and time steps of 0.01 years. To avoid the known problem of a possible violation of mass conservation in SIA on steep terrains (Jarosch et al., 2013), we smoothed the bedrock with a centrally-weighted 3×3 moving-window averaging. In addition, conservation of ice was explicitly monitored by tracking the total accumulation and ablation on the glacier surface, and the ice flux out of the glacier boundary in the ablation zone. The cumulative net gain of ice matched the total ice in the domain to within one part per 10^9 at any time t. Only on three glaciers (out of the total of 814), a violation of conservation due to steep bedrock was observed, and these three were not considered in our analysis (supplementary figure S2). One more glacier had to be removed where an erroneously mapped truncated tributary lead to an unrealistic piling up of ice (Supplementary fig. S2).

The SIA simulation was run starting with an empty bedrock, ~~and was continued till~~ with the initial E being the median elevation. The simulation was continued until a steady state was reached. Subsequently, E was ~~then~~ moved up or down, and the simulation was repeated~~to obtain the corresponding steady states. The iterations were continued till a steady extent that~~, until the extent of the steady state was similar to the present glacier extent (RGI, 2017) ~~was reached~~(Supplementary fig. S2). Once the desired steady state was found (See supplementary fig. S3 for a few examples), the glaciers were perturbed by a 50 m step rise in ELA~~, and then for the subsequent 500 years,~~. Subsequently, the annual values of area and volume were recorded for the next 1000 years (Supplementary fig. S4). The mean and standard deviation of the modelled ELA for these 810 glaciers were 5480 and 445 m, respectively.

Out of the total ~~814 glaciers, there were 120 glaciers where either our algorithm for finding a steady-state similar to present extent did not converge or the final steady state glacier geometry was not realistic. As a result, among 814 Ganga basinglaciers that are larger than 2 km^2, we could simulate only 694 glaciers . Out of these, the fractional changes~~ 810 simulated glaciers from the Ganga basin, on 98 glaciers the fractional change in glacier area at ~~$t = 500$ were~~ $t = 1000$ was more than 50%~~for 91 glaciers and they were excluded~~, and these were excluded from the analysis. This was necessary as a linear-response model can only be applied to glaciers with small relative changes (Oerlemans, 2001). We confirmed that the nature of our results does not depend on the precise value of this cutoff (Supplementary fig. S6). An additional ~~52~~ 9 glaciers had response time larger than ~~300 years and were also not considered.~~ 500 years and they were removed. This was done to avoid a possible overestimation of the response time whenever its magnitude was comparable to or larger than the total simulation period of 1000 years (supplementary fig. S7). The removal of these 9 glaciers led to a reduction in the number (total area) of simulated glaciers by only $\sim 1\%(\sim 2\%)$.

Finally, we were left with an ensemble of ~~551 Himalayan glaciers , which is 68% of the initial set. The area of these modeled glaciers were~~ 703 synthetic Himalayan glaciers (Supplementary fig. S1), with area in the range ~~$2.5 - 89.5$~~ of $2.2 - 156.0$ km^2 ~~with~~ (a median value ~~5.7~~ 5.5 km^2). The steady glaciers modelled with SIA had, on the average, 1.25 times larger area and 1.66 times larger ice-thickness (supplementary figs. S3, S8) compared to the corresponding estimates of Kraaijenbrink et al. (2017). The higher thickness of the modelled glaciers can be ascribed to a larger modelled area, a steady mass balance, and an uncalibrated SIA model. The total area and volume of these ~~glaciers were 5143.7~~ 703 synthetic glaciers were 6865 km^2 and ~~602.7~~ 847 km^3, respectively. ~~As an ensemble consisting 552 glaciers can be considered large enough to test the statistical~~

~~area-volume scaling, we did not do a detailed glacier-by-glacier analysis of the reasons behind the failure of the algorithm for some of the glaciers.~~

~~In the 2-d SIA-based dynamic glacier model described above, we have neglected the effects of debris cover on ablation, avalanche contribution to accumulation and ice flow due to sliding. While these processes are rather important in the Himalaya (Banerjee and Shankar, 2013; Laha et al., 2017), this was done to keep the model as simple as possible. Therefore, the simulated glaciers are not faithful copies of the actual Himalayan glaciers. On glaciers that are extensively debris-covered~~ This set covered 86% of the total 810 glaciers number-wise, and ~~/or receives strong accumulation contribution from avalanches in reality, our modelled ELA is likely to have a negative bias. Moreover, these ignored processes may either lead to a different scaling exponent and /or scale factor, or add additional scatter to the scaling plot for the real Himalayan glaciers. However, These simplifications do not weaken our study, as we restrict our aim to simulating the response of a large number of synthetic clean glaciers with realistic geometries and simplified mass balance profiles to a hypothetical step-change in ELA using different methods and comparing the results~~89% area-wise. The distributions of glacier area and mean slope for the two sets of 810 and 703 synthetic glaciers are shown in supplementary fig. S8.

2.1 ~~Scaling and hypsometric-adjustment-based models~~

2.0.1 Scaling model

The response of the ~~set of 551~~ above set of 703 steady-state glaciers to a 50 m instantaneous rise in ELA ~~were recomputed with a scaling-based approach (Radić et al., 2007)~~was also computed with a scaling model (Radić et al., 2007). The SIA-derived initial steady-state volume, area, and hypsometry (with the bin size of 25 m) for each of the glaciers were used as the starting point. For any of the modelled glaciers, the scaling and SIA models used the same mass-balance parameters. At any time t ~~, the elevation-dependent~~ during the evolution, the mass-balance function (eq.3) was summed over the instantaneous glacier hypsometry to obtain the ~~volume loss in a given balance year. The computed annual volume loss was converted to a~~ net volume loss for that time step. The corresponding area loss was then obtained using Eq. 2~~for the scaling-based method (Radić et al., 2007)~~. The reduction in the area was assumed to have taken place in the lowest elevation band~~of a~~ /s of each glacier (Radić et al., 2007). The scaling exponent was fixed at $\gamma = 1.286$ because of the ~~linear mass balance~~ assumed linear mass-balance profiles of the simulated ~~glacier (~~glaciers (i.e., $m = 1$). The ~~annual resolution~~ annual-resolution time series of ~~computed~~ area and volume were recorded for ~~500~~ 1000 years for each of the glaciers~~after the perturbation (a 50 m step-change in ELA) was applied. A) The area-volume scaling for 551 glaciers simulated with SIA for the steady state at $t = 0$ yr (purple circles) and the transient states at $t = 500$ yr (green circles). γ is taken to be 1.286 and best-fit c values are 0.053 ± 0.001 and 0.047 ± 0.001 km$^{3-2\gamma}$, respectively, at $t = 0$ and $t = 500$ years. For both the fits $R^2 = 0.90$. B) The simulated evolution of 200 randomly chosen glaciers in the $V - A$ plane with SIA (thin lines) and scaling (thick line) models. See text for further details. Note that the SIA-derived volumes are scaled by a factor of 10 for better visualisation.~~ .

2.1 ~~Linear-response model~~

2.0.1 Glacier response properties

For each of the ~~551~~ 703 glaciers, the time series of volume and area ~~,~~ as obtained using ~~both SIA and scaling-based methods~~the SIA and scaling models, were separately fitted to linear-response forms ~~analogous to~~ (e.g., eq. 9 below) to obtain the corresponding best-fit values of the four linear-response parameters ~~:~~ (the climate sensitivities ~~$(\Delta V_\infty, \Delta A_\infty)$and response times (τ_V, τ_A)~~and the response times of area and volume) for each of them (supplementary fig. S4).

Please note that applying a step change in ELA to a steady-state glacier to obtain the step-response function is a standard prescription for obtaining glacier response properties (Oerlemans, 2001; Vieli and Gudmundsson, 2004; Harrison et al., 2001; Bach et al., 2 Within a linear-response assumption, the step-responses of volume and area have an exponential form (e.g., eq. 9 below). The asymptotic exponential decay time is the response time of the glacier, and the asymptotic magnitude of the decay is the climate sensitivity. Because of the deviations of the simulated response from a pure exponential decay (supplementary fig. S4), the best-fit response time may be slightly different from the e-folding time, which has been used in some of the previous studies (e.g., Vieli and Gudmundsson, 2004; Bach et al., 2019). However, we take the best-fit asymptotic decay time to be the response time. By definition, it minimises the deviation between the predictions of the SIA and linear-response models, and thus, improves the performance of the latter in reproducing SIA results to some extent. We confirm that the difference between the above two definitions of the response time is small.

The best-fit linear-response properties obtained from the scaling ~~evolution~~ model results for the 703 glaciers were used to verify the corresponding theoretical expressions obtained from scaling theory ~~as given in the previous section.Subsequently~~(eqs. 8, 11, 12, 13 below). On the other hand, the best-fit response ~~time~~ times and climate sensitivities obtained from the SIA simulations of the 703 glaciers were used to ~~check if the same relations describe the SIA results or not, in order to obtain empirical parameterisations for the response properties. All these~~ fit for empirical relations that are motivated by the corresponding expressions derived from the scaling theory. All the above fits were performed in log-log scale, and R^2 of the fits were noted.

~~These~~

2.0.2 A linear-response model

The best-fit empirical parameterisations ~~of~~ for climate sensitivity and response time ~~obtain~~ obtained by fitting the SIA results, were used to ~~compute the~~ obtain a linear-response model~~predictions for the time series of total area and volume of the 551 glaciers perturbed by~~ . This model was applied to simulate the response of the above 703 synthetic Himalayan glaciers to a 50 m step-change in ELA at $t = 0$. To assess the uncertainty of the linear-response model ~~results~~output, a random Gaussian noise were added to the best-fit empirical parameters to generate an ensemble of 100 independent ~~copies of the~~ linear-response model outputs. The standard deviation of this added Gaussian noise for ~~any~~ a given fit parameter was set equal to ~~its standard error~~ standard error of that parameter.

3 ~~Results and discussions~~

2.1 ~~Area-volume scaling of glaciers simulated with SIA~~

~~As discussed in the introduction, the~~ To test the applicability of the above linear-response model that was calibrated using SIA results for the 703 central Himalayan glaciers, the model was applied to a different set of 204 glaciers from the western Himalaya without any further calibration. For these western Himalayan glaciers, SIA and scaling model simulations were also performed following the same procedures as detailed above. The time series of total area and volume of ~~the 551 glaciers~~ these 204 western Himalayan glaciers obtained using the three different models were then compared.

3 Results and Discussions

3.1 Theoretical results

Below, we derive some relevant consequences of the time-invariant scaling assumption, including expressions for the climate sensitivity and response time of area and volume. These results are expected to ~~follow a power-law relation (~~be generally valid for all scaling models that are based on eq. 2.

3.1.1 The rates of area and volume change

Eq. 2, which was derived from eq. 1 ~~) with scaling exponent $\gamma = 1 + \frac{m+1}{m+n+3} = 1.286$.Indeed, the ensemble of glaciers modelled with SIA conform to a power-law relation $V = cA^{1.286}$ at any time t. For example, fig~~assuming a time-independent c, implies,

$$\dot{V} = \gamma c A^{\gamma-1}\dot{A} = \gamma h \dot{A}. \tag{4}$$

Here, $\dot{V}$ and $\dot{A}$ denote the corresponding rates of change of glacier volume and area, respectively. If the net specific balance is δb (in m/yr), then the annual rate of volume loss $\dot{V} = \delta b A$. This, together with eq. 4, implies,

$$\dot{A} = \frac{\delta b}{\gamma h} A \tag{5}$$

$$= \frac{\delta b}{\gamma c} A^{2-\gamma}. \tag{6}$$

Thus, in the scaling models the rate of change of area scales with glacier area with an exponent $(2-\gamma)$. This is consistent with empirical observations for real glaciers as well (Banerjee and Kumari, 2019). As the scale factor $\frac{\delta b}{\gamma c}$ in the right-hand side (RHS) of eq. 5 is proportional to the net specific mass balance, this may be a convenient way of obtaining mean regional thinning rates from relatively straightforward remote-sensing measurements of the rate of area change. However, the accuracy of this relation is contingent on the validity of the assumption of a time-independent c.

3.1.2 Area response time

To compute the area response time, let us consider a constant perturbation, i.e., a step change in ELA applied to a steady glacier for time $t \geq 0$ (e.g., Oerlemans, 2001). Let's denote the corresponding instantaneous net negative balance at $t = 0$ by $\delta b_0 A$, the asymptotic ($t \to \infty$) shrinkage of glacier area by $\Delta A_\infty \equiv A(0) - A(t \to \infty)$, and that of ice volume by ΔV_∞. Then, we have (Harrison et al., 2001),

$$\Delta A_\infty b_t + \beta \Delta V_\infty \approx -\delta b_0 A. \tag{7}$$

Here, b_t is the ablation rate near the terminus. The area response time of the glacier can be expressed as $\tau_A \approx \Delta A_\infty / \dot{A}$. Therefore, using the above expressions for $\dot{A}$ (Eq. ~~1a shows the power-law behaviours~~ 5) and ΔA_∞ (Eq. 7), we obtain,

$$\gamma h + \beta)^{-1} \equiv \tau^*. \tag{8}$$

Here, the symbol τ^* is a convenient shorthand notation for the time scale $-(\frac{b_t}{}\gamma h + \beta)^{-1}$. In the above derivation, ΔV_∞ that appears in eq. 7 is eliminated with the help of eq. 2. Eq. 8 is comparable with the expression of area response time as given by Harrison et al. (2001), or Lüthi (2009).

3.1.3 Volume response time

The instantaneous change in volume ($\Delta V(t)$) for a steady glacier perturbed by a small step change in ELA at $t = 0$ ~~and $t = 500$ years. The corresponding linear fits in log-log scale have R^2 value of 0.90~~is given by,

$$\Delta V(t) = \Delta V_\infty (1 - e^{-t/\tau_v}), \tag{9}$$

where, τ_v is the volume response time and ΔV_∞ is the volume sensitivity (e.g., Lüthi, 2009). Now, $V(t)$, $V(0)$, and $V(t \to \infty)$ appearing in eq. 9 can be expressed in terms of $A(t)$, $A(0)$, and $A(t \to \infty)$, respectively, with the help of corresponding scaling relations (eq. 1). This, in the limit of a small fractional changes in area, yields,

$$\Delta A(t) = \Delta A_\infty (1 - e^{-t/\tau_v}). \tag{10}$$

Comparing the above two equations, and using eq. 8 one obtains,

$$\tau_A = \tau_V = \tau^*. \tag{11}$$

This implies that all scaling models implicitly assume the area and volume response times of a glacier to be equal to each other. However, it is known that for mountain glaciers area response time is larger than the volume response time within a SIA model (Oerlemans, 2001; Vieli and Gudmundsson, 2004). Therefore, the assumed equality of the two response times in scaling models (eq. 11) contradicts the existing SIA results. This is an intrinsic bias that is present in any scaling model.

~~Another interesting trend that is evident from fig~~After a step change in ELA, as the ablation zone shrinks, the initial net negative balance of a glacier gradually decays to zero over a period determined by the corresponding response time. A longer area response time in SIA implies that this reduction in the ablation zone is slower here than that in a scaling model. A corresponding feedback of a larger ablation zone on the net mass balance should then lead to a higher long-term volume loss in a SIA model than that in a scaling model. This indicates the possibility of a low bias in scaling model estimates of the climate sensitivity of volume, or equivalently, that in the long-term changes in glacier volume due to any rise in ELA.

3.1.4 Climate sensitivity of area and volume

An expression for the climate sensitivity of glacier area (ΔA_∞), which is the asymptotic change in area due a change in ELA by δE, is obtained by eliminating ΔV_∞ from eq. ~~1a is that the~~ 7 using eq. 2,

$$\tau^* \beta \delta E \gamma h \equiv \alpha^*. \qquad (12)$$

Here, we have used the definition of τ^* (Eq. 8), and that $\delta b_0 \approx \beta \delta E$ for a step change in ELA by δE. The RHS of the above equation is denoted by α^* for convenience.

The corresponding expression for $\frac{\Delta V_\infty}{V}$ is then obtained using Eq. 2,

$$\frac{\Delta V_\infty}{V} = \gamma \alpha^*. \qquad (13)$$

Again, Eq. 13 is comparable to the expression of volume sensitivity as derived by (Harrison et al., 2001), where the authors used an arbitrary thickness scale H, instead of the denominator of γh appearing in the definition of α^* above.

Please note that strictly speaking, the climate sensitivity of area and volume with respect to a change in ELA should be defined as $\frac{\Delta A_\infty}{\delta E}$ and $\frac{\Delta V_\infty}{\delta E}$, respectively. However, in this paper, we use ΔA_∞ and ΔV_∞ as the corresponding sensitivities to simplify the notation.

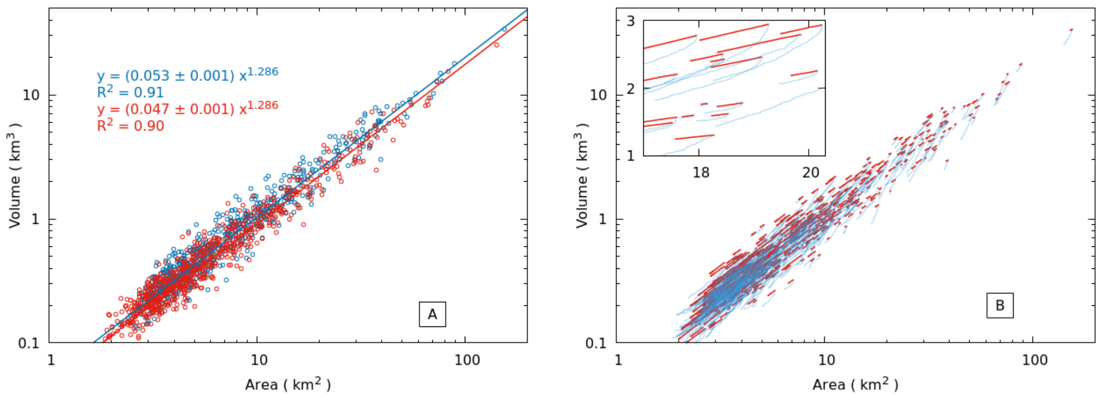

Figure 1. A) Glacier volume as a function of area for the 703 Himalayan glaciers simulated with SIA at $t = 0$ yr (blue circles), and at $t = 500$ yr (red circles) are plotted along with the corresponding best-fit scaling relations (blue and red solid lines). The corresponding fitted functions, and R^2 values are shown in blue and red texts, respectively. B) The trajectories of the 703 glaciers in the $V - A$ plane as simulated with SIA (thick red lines) and scaling (thin blue lines) models. The inset is a zoomed-in version of the same plot, but with a linear scale.

3.2 Numerical results

3.2.1 Volume-area scaling and a time-dependent scale factor in the SIA model

Following eq. 1, a power-law relation between the area and volume of the 703 glaciers with an exponent $\gamma = 1 + \frac{m+1}{m+n+3} = 1.286$, is expected (as $m = 1$ and $n = 3$). The ensemble of glaciers modelled with SIA did conform to above power-law relation $V = cA^{1.286}$ at any time t with a single best-fit c~~is time-dependent and decreases systematically with time as glaciers shrink~~. The scale factor slowly decreased with time. For example, fig. 1a shows the power-law fits at $t = 0$ and $t = 500$ years ($R^2 = 0.9$), where the best-fit c-values ~~are~~ were 0.053 ± 0.001 and ~~0.047 ± 0.001~~ 0.47 ± 0.001 km$^{3-2\gamma}$, respectively~~, at $t = 0$ and $t = 500$ years (fig. 1a)~~. This implies a $\sim$~~13~~11% reduction in c for the ensemble over the period of 500 years ~~subsequent to~~ after the step-change in ELA ~~. The time dependent~~ was applied. A time-dependent c is consistent with the theoretical arguments of Bahr et al. (2015)~~and those in the introduction of this paper~~.

The slow and systematic decline in c for ~~an~~ the ensemble of shrinking glaciers ~~leads to a systematic bias in any scaling-based glacier evolution model where the~~ simulated with SIA model contradicts the basic assumption of scaling models of a time-invariant c~~is a basic requirement~~. A decreasing c would ~~imply,~~ mean eq. 2 is violated, with $\frac{\Delta V}{V} = \gamma \frac{\Delta A}{A} + \frac{\Delta c}{c}$. ~~Since the changes in~~ Note that all the three ~~quantities are negative~~fractional changes involved in this relation are negative. Therefore, for any given $|\Delta A|$, the corresponding $|\Delta V|$ is ~~underestimated whenever the second term on the RHS~~ going to be larger in SIA model than that in a scaling model where $\frac{\Delta c}{c}$ is assumed to be zero ~~, e.g. in~~ (eq. 2~~. As eq. 2 is the basis of all scaling-based glacier models, this implies that all such models would have the above bias and predict thicker glaciers in the future as climate warms up. Due to a~~). Even though the decline in c is only about 11%, it may be associated with a stronger low bias in the long-term change predicted by scaling models. This is because a larger volume change in SIA would lead to a thinner glacier, and a corresponding surface-elevation feedback ~~related to thicker glaciers, the bias~~ to mass balance is likely to ~~get amplified, leading to significant underestimation for future glacier mass loss under warming climate. Our comparison of the results from SIA and scaling-based simulations as shown in Fig. 1b are consistent with above arguments. The relationship between the asymptotic fractional changes in glacier area ($\frac{\Delta V_\infty}{V}$) and volume ($\frac{\Delta A_\infty}{A}$) as obtained from, A) scaling and B) SIA-based evolution. The solid lines are best-fit straight lines, A) $\frac{\Delta V_\infty}{V} = (1.276 \pm 0.002)\frac{\Delta A_\infty}{A}$, and B) $\frac{\Delta V_\infty}{V} = (1.87 \pm 0.02)\frac{\Delta A_\infty}{A}$. R^2 values of the fits are 0.99 and 0.85, respectively. In sub-figure B, the points denoted by black circles were not used for fitting. See text for details.~~

3.3 ~~The climate sensitivity of glacier area and volume~~

~~For the scaling-based simulation of glacier response, the fitted asymptotic fractional changes in glacier area and volume are related to each other (fig. **??**a) as, $\frac{\Delta V_\infty}{V} = (1.276 \pm 0.002)\frac{\Delta A_\infty}{A}$. This is in line with the prediction of eq. 2.~~ amplify the corresponding long-term mass loss over time.

~~As discussed in the previous subsection, a decreasing c~~ The dependence of the glacier-specific scale factor on the mean slope is known (Bahr et al., 2015) and has been incorporated in modified scaling relations where volume is a power-law function both area and slope (e.g., Grinsted, 2013; Zekollari and Huybrechts , 2015). For the simulated 703 glaciers, the mean slope increases with time as area is lost preferentially from the gently-sloping lower ablation zone. For example, the median slope of the 703 simulated glaciers reduced from 0.41 at $t = 0$ to 0.37 at $t = 500$ years. This $\sim 10\%$ reduction in slope is expected to lead to a ~~larger asymptotic fractional volume change for glaciers simulated with SIA. This is exactly what is seen in fig. **??**b, which shows $\frac{\Delta V_\infty}{V} = (1.87 \pm 0.02)\frac{\Delta A_\infty}{A}$.~~

~~Note that apart from the contribution of fractional change in~~ $\sim 5\%$ decline in c ~~to the tune of ~13%, a further amplification of the volume loss due to thickness feedback discussed before also plays a role. Both these effects are not captured by the time-independent scaling assumption, leading to a underestimation of glacier volume change in scaling-based methods. It is interesting that despite a changing~~ (Bahr et al., 2015) . So, at least part of the time dependence of c ~~in~~ for transient glaciers in SIA simulation is explained by the slope-dependence of c. However, there may be other factors contributing to the decline in c for the transient glaciers as discussed below.

3.2.1 Area and volume response times

The theoretical prediction for glacier area and volume response time (eq. 11) worked rather well for the scaling model results (figs. 2C, and 2D), with best-fit relations of $\tau_V = (0.996 \pm 0.001)\tau_A$ with $R^2 = 0.995$, and $\tau_V = (0.942 \pm 0.006)\tau^*$ with $R^2 = 0.89$.

For SIA simulations, the ~~two quantities $\frac{\Delta V_\infty}{V}$ and $\frac{\Delta A_\infty}{A}$, are~~ data showed that $\tau_A > \tau_V$, and that the two response times were still proportional to each other (fig. ~~**??**b). We do not have a clear explanation of this effect as yet.~~

~~In~~ 3C: $\tau_V = (0.687 \pm 0.004)\tau_A$, with $R^2 = 0.94$). Also, τ_V was proportional to τ^* to a good approximation (fig. ~~**??**b, about 30 data points with close to the largest value of $\frac{\Delta A_\infty}{A}$ (shown with black circles in the figure) were not included in the fit.~~

~~The cut-off $\frac{\Delta V_\infty}{V} < 0.5$, prevents sampling of the full vertical scatter in this range, which may create bias in the linear fit and therefore, these points were not considered for the fit. The fractional climate sensitivity of glacier area as obtained from, A) scaling and B) SIA simulations are compared with the theoretically predicted value of $\alpha^* = \frac{\beta\delta E\tau^*}{\gamma h}$ (see text for details) . The best-fit straight lines are, A) $\frac{\Delta V_\infty}{V} = (0.585 \pm 0.009)\alpha^*$, and B) $\frac{\Delta V_\infty}{V} = (1.65 \pm 0.03)\alpha^*$. The corresponding R^2 are 0.65 and 0.48, respectively.~~ 3D: $\tau_V = (2.56 \pm 0.04)\tau^*$, with $R^2 = 0.94$). Interestingly, the value of the proportionality constant in the latter relation as obtained from SIA was about 2.7 times larger than the corresponding value obtained in the scaling model. This underlines the relatively large underestimation of volume response time by the scaling model. Similarly, the area response time was about 3.9 times larger in the SIA simulation than the corresponding scaling model value. This implies that for a given ELA perturbation, the glacier response is much faster in the scaling model compared to that in the SIA model for the ensemble of 703 synthetic glaciers.

~~Fig.~~ Apart from the overall underestimation of area and volume response times by the scaling model, another serious limitation of scaling models that emerges from the above analysis is that here the area and volume response times are equal to each other (eq. **??**~~a shows that the prediction of eq.~~11, and fig. ~~13 for climate sensitivity of glaciers works for the scaling-based model, though the prefactor is not 1, i.e. , $\frac{\Delta V_\infty}{V} = (0.585 \pm 0.009)\alpha^*$. The R^2 of the fit is 0.65. A similar behaviour is also seen for the SIA-derived estimates of $\frac{\Delta V_\infty}{V}$~~ 3C). In contrast, the SIA model predicted $\tau_A \approx 1.5\tau_V$. The ratio of the two response times obtained from the 2-d SIA model here is generally consistent with earlier results based on 1-d flowline models (Oerlemans, 2001; Vieli and Gudmundsson, 2004). The equality of the two response times in the scaling model led to a linear trajectory in $V - A$ plane for the transient glaciers (fig. **??**b)~~, though there is more noise and consequently a smaller R^2 of 0.48.~~

~~The above figure (fig. **??**b) may also suggest that a more general power-law function of α^* with a power-law exponent larger than 1, may provide better estimates of $\frac{\Delta V_\infty}{V}$, particularly so for SIA-based simulations. Again, we do not have a theoretical argument for such a power-law behaviour and did not explore this further here. α^* being dimensionless, dimensional arguments does note rule out possible a power-law behaviour.~~

~~The relationship between the area and volume response time as obtained from, A) scaling and B) SIA-based evolution. The solid lines are best-fit straight lines passing through origin with slope of 0.959 ± 0.001 and 0.647 ± 0.003, respectively. R^2 values for the fits are 0.99 and 0.94, respectively.~~

3.3 ~~Area and volume-response time~~

~~The area and volume response time as obtained from, A) scaling and B) SIA-based evolution are compared with theoretical prediction $\tau^* = \frac{-1}{(\beta + b_t/\gamma h)}$ (see text for details). The best-fit straight lines in log-log scale yield, A) $\tau_A = (0.945 \pm 0.006)\tau^*$ and B) $\tau_A = (2.67 \pm 0.04)\tau^*$, respectively, for scaling and SIA evolution. R^2 of the fits are 0.88 and 0.52, respectively.~~

~~The theoretical prediction of eq. 11 works reasonably well for the scaling-based evolution (fig~~1B). While in SIA model, a relatively larger area response time, together a slow initial changes in area (supplementary figs. S4, S10), led to curved $V - A$ plane trajectories for individual transient glaciers. In particular, a slowly changing area means the $V - A$ trajectories bend downward causing c to reduce for the transient ensemble (fig **??**a)~~, with best-fit functions $\tau_V = (0.959 \pm 0.001\tau_A$ and~~ 1).

Moreover, At the early stages of response, glaciers simulated by a scaling model lose area much quicker than those simulated by an SIA model (fig. ~~??b)with $\tau_V = (0.945 \pm 0.006)\tau^*$. The above fits have R^2 values of 0.99 an 0.88, respectively~~1B).

~~For SIA evolution, τ_V and τ_A are not equal (fig. ??b) . However, τ_V is still proportional to τ_A with best-fit relation being $\tau_V = (0.647 \pm 0.003)\tau_A$, with an R^2 of 0.94~~The associated net mass-balance feedbacks then lead to a subdued long-term volume response in scaling model, and a comparatively stronger volume response in the SIA model, just as predicted in sect. 3.1.3. ~~τ_V and τ_A are both proportional to τ^* to a good approximation~~

3.2.1 The climate sensitivity of glacier area and volume

For the 703 glaciers simulated by the scaling model, the fitted asymptotic fractional changes in area and volume, or equivalently, the corresponding (fractional) climate sensitivities, were proportional to each other (fig. ~~??b)for SIA evolution as well, although the proportionality constant is not 1 contrary to~~ 2A: $\frac{\Delta V_\infty}{V} = (1.232 \pm 0.002)\frac{\Delta A_\infty}{A}$, with R^2=0.997). Here, the best-fit constant of proportionality was close to $\gamma = 1.286$, as predicted by eq. ~~11~~2.

~~Apart from the fact that scaling-based evolution leads to an underestimation of glacier response time as described above, an important point of departure of SIA results from that of scaling is that area-response time is larger than the volume-response time in SIA.The difference between area and volume response for mountain glaciers is a known fact (e.g., Banerjee and Shankar, 2013)~~In contrast, the SIA simulations obtained $\frac{\Delta V_\infty}{V} = (1.93 \pm 0.02)\frac{\Delta A_\infty}{A}$, with R^2=0.85 (fig. 3A). In this case, the constant of proportionality was $\sim 1.5\gamma$, compared to the corresponding value of $\sim \gamma$ in the scaling model. This larger value of the ratio of the two climate sensitivities in SIA model is consistent with the observed decline in c for the transient glaciers simulated with this model (fig. 1). Please note that no theoretical prediction is available for the ratio of asymptotic fractional changes in volume and area in a SIA model.

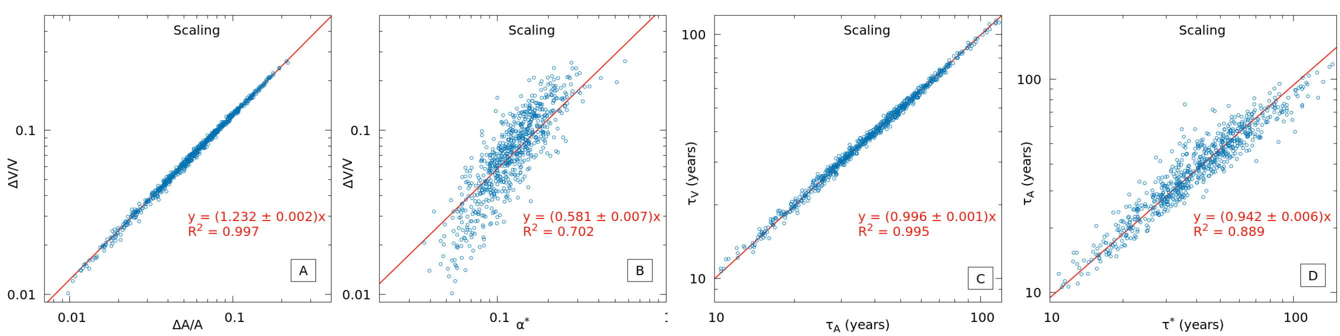

Figure 2. Scaling model simulations of the 703 synthetic Himalayan glacier show that, (A) the best-fit (fractional) climate sensitivities of area and volume are proportional to each other, (B) The climate sensitivity of volume is proportional to $\alpha^* \equiv \frac{\beta\delta E\tau^*}{\gamma h}$, (C) The response times associated with glaciers area and volume are approximately equal, and (D) the volume response time is approximately equal to $\tau^* \equiv -(\frac{b_t}{h}\gamma h + \beta)^{-1}$. In all the above plots, the corresponding best-fit curves are shown with red lines. The fit parameters and R^2 of the fits are also given. These numerical trends are consistent with theoretical results derived in sect. 3.1.

Fig. 2B shows that in the scaling model, climate sensitivity of glacier volume is proportional to α^* ($\frac{\Delta V_\infty}{V} = (0.581 \pm 0.007)\alpha^*$, with $R^2 = 0.7$). This is in line with eq. 13, except that the constant of proportionality is significantly less than γ. A similar proportionality between the SIA-derived best-fit $\frac{\Delta V_\infty}{V}$ and α^* is shown in fig. 3B, with $\frac{\Delta V_\infty}{V} = (1.71 \pm 0.03)\alpha^*$. However, in this case the fit is relatively noisy with $R^2 = 0.48$.

The above relations suggest that the climate sensitivity of volume in the SIA simulation was about 2.9 times larger than that in the scaling model. Similarly, the climate sensitivity of glacier area obtained from the SIA model was also about 3.2 times larger than that obtained from the scaling model. This trend of a relatively large (by about a factor of about 3) underestimation of climate sensitivity of glacier volume and area by the scaling model is consistent with the effects of a relatively faster shrinkage of the ablation zone in the early stages of the response as discussed in 3.1.3 and 3.2.2.

3.3 ~~Estimating total glacier loss using scaling and linear response theory~~

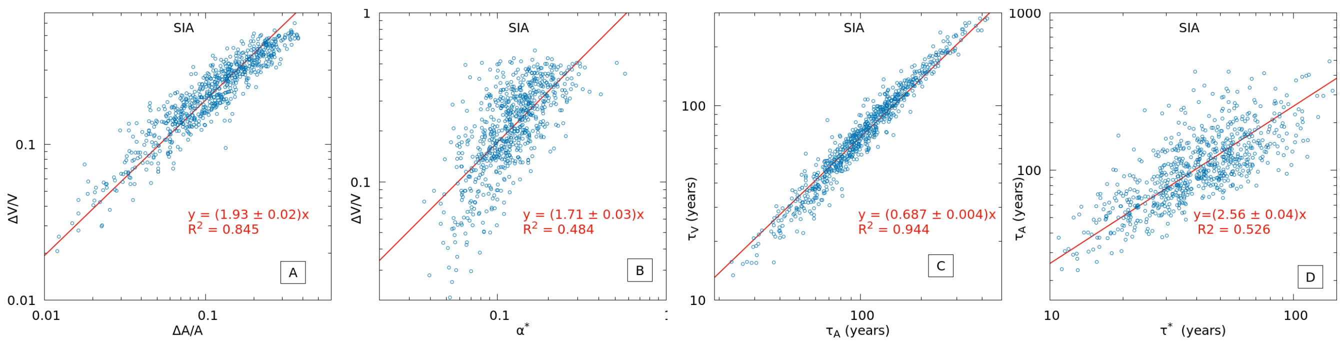

Figure 3. Results from the SIA simulations of the 703 synthetic Himalayan glacier show that, (A) The climate sensitivities of area and volume are proportional to each other, (B) The climate sensitivity of glacier volume is proportional to $\alpha^* = \frac{\beta\delta E\tau^*}{\gamma h}$, (C) The response times associated with glaciers area and volume are proportional to each other, and (D) The volume response time is proportional to $\tau^* = -(\frac{b_t}{h}\gamma h + \beta)^{-1}$. The fitted functions are shown with red lines. The corresponding fit parameters and R^2 of the fits are also given. See text for detailed discussions.

3.2.1 The total glacier loss estimated using the three models

Starting with an initial volume (area) of ~~603~~ 847 km^3 (~~5144~~ 6865 km^2)~~the 551~~, the 703 glaciers simulated by SIA ~~loses 123~~ lost a total of 194 km^3 (~~521~~ 726 km^2) of volume (area) in 500 years ~~after the step-change~~ due to the step-rise in ELA by 50 m~~, with most of the changes taking place during the first couple of centuries (fig. 4).~~

. As shown in fig 4, both ~~scaling-based and~~ the scaling and the linear-response ~~model underestimates~~ models underestimated the long-term change in total area ~~for the same 50 m rise in ELA, with respective values of the predicted area change being 264 and 478~~ in this experiment, with estimated area changes of 352 and 621 km^2~~. The scaling model prediction is off by a factor of $\sim \frac{1}{2}$~~, respectively. The scaling-model prediction for area change was only 48% of the corresponding SIA estimate, while the ~~linear response model is within 10% of the SIAvalues. Very similar trends are~~ linear-response model estimate was

86% of that of SIA. Similar trends were seen for the ~~magnitude of~~ magnitudes of estimated volume change as well.~~In fact, here the scaling-model estimates for long-term change is smaller by a factor of $\sim\frac{1}{3}$. In comparison~~, with the respective scaling and linear-response model estimates being $\sim$ 27% and $\sim$ 75% of the corresponding SIA prediction (fig 4). We confirmed that the nature of the above results does not depend on the chosen cut-off of 50% change that was used to select the 703 glaciers (Supplementary fig. S6). In fact, with a smaller cut-off, the linear-response model ~~predictions are within about 15% of the SIA values.~~

estimates were even closer to the corresponding SIA estimates (Supplementary fig. S6). This is expected as linear-response models are derived in the limit of small fractional changes (Oerlemans, 2001).

~~The evolution of the total glacier volume (A), and (B) glacierised area for the ensemble of 551 glaciers simulated with three different methods, namely, SIA, scaling and linear-response model, are shown with orange, red, and blue solid lines, respectively. See text for details.~~

~~This relatively strong underestimation in the scaling model results~~ The low-bias in the long-term changes of glacier area and volume computed with the scaling model is consistent with the ~~systematic bias introduced by a time-invariant scaling assumption as discussed above. This also suggests that there might be significant negative bias~~ underestimation of corresponding climate sensitivities by this model (sect. 3.2.3). This indicates the possibility of a negative bias in scaling model estimates of mountain glacier contribution to sea-level rise as ~~computed by scaling-based methods~~well. As an example, let us consider a recent comparison (Hock et al, 2019) of projected end-of-the-century sea-level rise contribution of glaciers from 6 different models that include a hypsometric-adjustment-based model (Huss and Hock, 2015) and 5 other models which are all based on some form of scaling. It is ~~also clear~~ seen that the former model consistently predicted the largest change under various climate scenarios (e.g., Table 3 of Hock et al (2019)). This may be an indication that biases qualitatively similar to that discussed here, are generally present in scaling models. Based on our results, the potential biases in the scaling models may be clearer in long-term simulations over multiple centuries. On shorter time scales of multiple decades, an underestimation of response times by about a factor of 3 (sect. 3.2.2) to some extents compensates for a corresponding underestimation of the climate sensitivities (sect. 3.2.3), and the deviation between the SIA and scaling models are not that prominent (fig. 4).

Please note that depending on the details of the scaling and SIA models that are being compared or the set of glaciers that are being simulated, the actual magnitude of the biases in scaling-model derived climate sensitivity, response time, and long-term glacier change could be different from that obtained here. However, based on the theoretical arguments and numerical evidence presented, similar qualitative trends are expected if the above exercise were to be repeated with a more detailed model and/or for a more realistic set of glaciers.

Above results also show that the linear-response model ~~performs quite well in reproducing SIA results . We have verified the~~ outperformed the scaling model, producing a closer match with the SIA results for the 703 synthetic glaciers from the Gangetic Himalaya. However, this linear-response model ~~obtained by fitting the SIA simulation~~ was calibrated using the SIA results for the ~~ensemble of 551 central Himalayan glaciers, similarly outperforms the scaling-based method for another set of 143 glaciers from~~ same set of glaciers. Therefore, this match is not enough to establish the effectiveness of the linear-response model. To confirm the improved performance of the linear-response model compared to that of the scaling model, we applied these two models without any further calibration, to simulate a different set of 204 glaciers in the western Himalaya (~~figure A2).~~ supplementary fig. S1). In this independent experiment, the linear-response model again outperformed the scaling model in reproducing the corresponding SIA results (supplementary fig. S9). This confirms that the linear-response model can be used for computing long-term glacier changes accurately.

~~We note that the aforementioned bias of the scaling-based method is present even when multi-decadal predictions of glacier volume loss is concerned (fig. 4). However, up to $t \lesssim 100$ years the predicted area loss curves are similar for both scaling-based and linear response models. A problem that is apparent with scaling-model predictions is the generally quicker response of the glaciers evolving under time-independent scaling evolution (fig. 4)). This is consistent with the discussions above that time-invariant scaling-based models underestimates glacier response time. The difference in response time~~

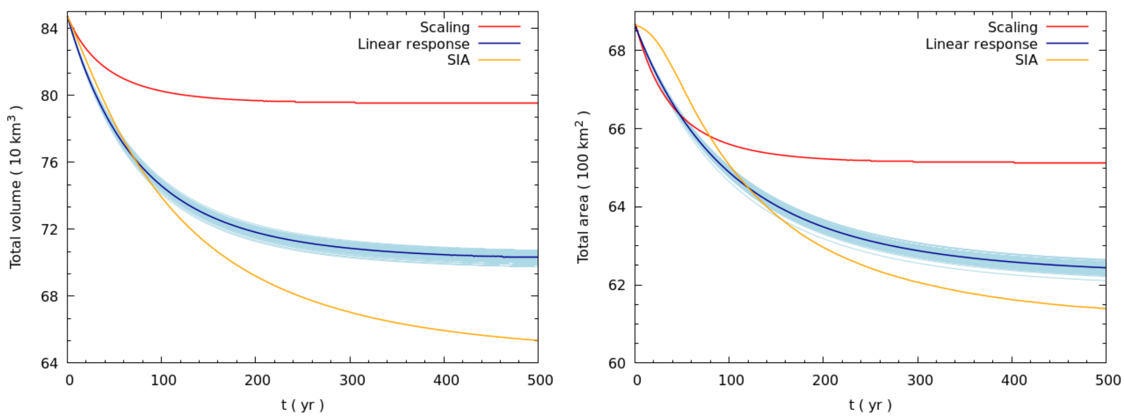

Figure 4. The evolution of the total (A) volume, and (B) area of the ensemble of 703 Himalayan glaciers simulated with three different methods: SIA, scaling, and linear-response models. The uncertainty bands for the linear response model results as also shown. See text for details.

3.3 The effects of glacier geometry

Can the biases in the scaling model described above, be artefacts arising out of some peculiarities of the geometry of the specific set of glaciers being simulated, and are not relevant in general for scaling model computations of global-scale mass loss of mountain glaciers? To explore that possibility, we simulated the response of a set of highly idealised synthetic glaciers using both a flowline model (Banerjaee, 2017) and the above scaling model (Radić et al., 2007). Note that this flowline model included sliding as well. All of these synthetic glaciers have the same constant-width, the same linear bedrock with constant slope, and the same linear mass-balance profile. Only the ELA was varied between glaciers. Even for this highly idealised set of glaciers, the scaling model estimates for the evolution of total area and volume ~~, as seen in SIA simulations, is not captured by the scaling-based model~~ showed biases compared to that obtained from the flowline model (supplementary fig. S9), and these biases were qualitatively very similar to those observed in figs. 1 and 4. Again, the scaling model predicted relatively smaller climate sensitivities, a relatively faster area response, and a low-bias in the long-term changes, compared to corresponding flowline model estimates (supplementary fig. S9).

3.4 ~~Limitation of the present study~~

~~Some of the limitations of~~ The above flowline model experiment provides an additional piece of evidence that the scaling-model biases discussed in this paper are in general expected to be present in scaling model simulations of any set of glaciers. We emphasise that even though biases are expected to be qualitatively similar to that presented here, the ~~present study, namely, 1) use of SIA that considers only the horizontal shear stresses to describe ice deformation, 2) ignoring the sliding contribution to glacier~~ magnitude of the biases are likely to depend on the detailed characteristics (related to geometry, flow, and ~~3) the effect of debris-cover and avalanche activity on the mass-balance profile, have already been discussed. Another important simplification is the use of a simple linear mass-balance profile with a cut-off and considering step-changes in ELAas the only mass-balance forcing.In reality, the~~ mass-balance ~~profile and its temporal variability are going to be more complicated. In fact, the mass-balance forcing are likely to vary between regions as well, driving a variable climate response~~ processes) of ~~glaciers. More detailed studies that relaxes some of the above mentioned assumptions are needed to check the validity of and to refine~~ the glaciers studied and the models used.

3.4 The linear-response model, and its application to real glaciers

As described above, we have used results from the 2-d SIA model simulations of the response of 703 synthetic Himalayan glaciers to a 50 m step change in ELA, to obtain the following best-fit paramterisations of the glacier response properties (i.e., $\frac{\Delta V_\infty}{V}$, $\frac{\Delta A_\infty}{A}$, τ_A and τ_V).

$$\frac{\Delta V_\infty}{V} \approx (1.71 \pm 0.03)\alpha^*, \tag{14}$$

$$\frac{\Delta V_\infty}{V} \approx (1.93 \pm 0.02)\frac{\Delta A_\infty}{A}, \tag{15}$$

$$\tau_V \approx (2.56 \pm 0.04)\tau^*, \tag{16}$$

$$\tau_V \approx (0.687 \pm 0.004)\tau_A. \tag{17}$$

Here, as defined before, $\tau^* \equiv -(\frac{b_t}{h}\gamma h + \beta)^{-1}$, $\alpha^* \equiv \frac{\beta \delta E \tau^*}{\gamma h}$, and $\delta E = 50$ m. With the help of these paramterisations, it is possible to compute the evolution glacier volume and area accurately given a glacier and any arbitrary ELA forcing function. For this the following general solution of the linear-response ~~model described here.~~ equation is used.

$$\Delta V(t) = \Delta V(0) e^{-t/\tau_V} + \frac{\Delta V_\infty}{\delta E} \int_0^t \Delta E(t') e^{-(t-t')/\tau_V} dt' \tag{18}$$

Here, $\Delta E(t)$ is the given (arbitrary) ELA forcing function. This equation simply states that, any continuous ELA change can be interpreted as the sum total of a series of discrete steps, and the corresponding net response is given by a superposition of suitably delayed responses due to each of the steps. An analogous expression can be written down for the area evolution as well by replacing all the V's in the above equation with A's.

~~It is possible or even likely that the inclusion of some the above processes may influence the scaling behaviour by changing the~~ Please note that the ~~value of~~ above formulation does not require the initial state to be steady. As long as the glacier is close to a steady state, a linear-response theory will be a good approximation (Oerlemans, 2001). However, an additional initial condition, i.e., the value of $\Delta V(0)$, is needed to apply the ~~scaling parameters or by intruding more scatter~~ linear-response model to transient glaciers. $\Delta V(0)$ is the initial departure from a steady state, and can be obtained from the observed rate of volume loss ($\dot{V}$) simply as, $\Delta V(0) = -\tau_V \dot{V}$. Thus, the linear-response model can be used to evolve the area and volume of a real set of glaciers for any arbitrary time-dependent ELA forcing given the initial rates of change of volume and area, initial thickness, mass-balance gradient, and melt rate near glacier terminus.

Due to the noise present in the fits (fig. 3), the linear-response model predictions for an individual glacier would have significant uncertainties. However, ~~based on the argument outlined in the text before,~~ for a large set of glaciers, the linear-response model provides accurate estimates of the total area and volume evolution (fig. 4, supplementary figs. S6 and S9).

3.5 Limitation of the present study

Because of the idealised descriptions of ice flow and the mass-balance profile (as discussed in sect. 2.2), and the absence of model calibration to match the available observed data of surface velocity, ice thickness, recent mass balance etc., the ~~SIA model result that the scaling factor c is time-dependent may be quite general.If that is so, then the biases would be present in predictions from time-invariant scaling assumption, even if~~ glaciers simulated here are not faithful copies of the Himalayan ones. For a set of more realistic glaciers, the magnitude of the ~~bias is different from what is discussed in this paper.~~ corresponding biases in scaling-model derived climate sensitivity and response time could be different from that obtained here. However, based on the theoretical arguments and numerical evidence presented, similar qualitative trends are expected if the above exercise were to be repeated for a more realistic model that includes higher order mechanics, flow due to sliding, a more realistic mass-balance model, and so on. Similarly, The parameterisations for the linear-response properties given here are obtained from 2-d simulations of 703 synthetic Himalayan glaciers with some idealisations (sect. 2.2) and without any tuning of model parameters. The fit-parameters in eqs. 14-17 may be different for a different set of glaciers. The paramterisations may also change if a more detailed and calibrated model of the same glaciers is used. However, the protocol used here to obtain the parameterisation for linear response-properties can be directly applied without any change for any set of glaciers and for any ice-flow/mass-balance model.

~~Since the results of this paper are based onsimulation of an ensemble consisting of 551 synthetic glaciers with geometries similar to glaciers in the central Himalaya (and tested on 143 glaciers in the western Himalaya), it remains to be investigated if~~

4 Summary and Conclusions

We performed a theoretical analysis of the response of mountain glaciers within a time-independent scaling assumption. In addition, the ~~results described here depend on the regional characteristics of glaciers to some extent~~step-response of 703 steady-state synthetic Himalayan glaciers with realistic geometries and idealised mass-balance profiles were simulated with three different models: a scaling model, a 2-d SIA model, and a linear-response model. The results obtained are as follows.

– Analytical expressions for climate sensitivity and response time of glacier area and volume are derived within a time-independent scaling assumption. These expressions are validated using results from the scaling model simulation of the ensemble of 703 glaciers.

5 ~~Summary and Conclusions~~

~~In this paper, we perform 2d SIA-based transient simulations of 551 synthetic glaciers with geometries similar to glaciers in the Ganga basin, central Himalaya. The long-term response of these glaciersto 50 m ELA change reveal~~

– The response of the glaciers simulated with the 2-d SIA model reveals that the initial ~~($t = 0$ year) and final states ($t = 500$ years) obey area-volume scaling with the predicted exponent of 1.286. However, the scale factor c reduces by 13% over this period. This changing c introduces a~~ steady states and the transient states follow the volume-area scaling relation, with the best-fit scale factor reducing slowly with time.

– For the ensemble of glaciers studied, the scaling model obtains relatively smaller climate sensitivities of glacier area and volume by a factor of about 3, compared to that from the SIA model. This results in a low bias in the ~~area~~ long-term changes predicted by the scaling model.

– For the ensemble of glaciers studied, the scaling model underestimates volume (area) response time by a factor $\sim$2.7 (3.9) compared to the corresponding SIA estimates.

– For the scaling model, $\tau_A \approx \tau_V$, and $\frac{\Delta V_\infty}{V} \approx \gamma \frac{\Delta A_\infty}{A}$. In contrast, for the SIA simulations, $\tau_A \approx 1.5\tau_V$ and ~~volume change whenever scaling-based models, that assumes~~ $\frac{\Delta V_\infty}{V} \approx 1.5\gamma \frac{\Delta A_\infty}{A}$.

– The relatively larger ratio of the two response times in the SIA simulations, along with an initial slow change in area, leads to curved $V - A$ trajectories, a decreasing c~~to be time-independent, are used. The scaling method employed here underestimates the~~ , and a relatively larger long-term volume loss for the transient glaciers due to a corresponding mass-balance feedback.

– A linear-response model based on the parameterisations of SIA-derived response properties helps reduce the biases in the predicted long-term ~~changes in glacier areaand volume over several centuries quite strongly. Significant differences are also present over the multi-decadal scale. These results points to~~ glacier changes that are present in the scaling model results. The improved performance of this model is validated on an independent set of 204 glaciers in the western Himalaya.

Based on the theoretical arguments and numerical evidence presented here, it is possible that qualitatively similar biases may generally be present in the ~~possibility of significant biases in predictions of future~~ long-term glacier changes computed with scaling models. However, the actual magnitude of such biases in scaling models may be different from that obtained here for a set of synthetic Himalayan glaciers with idealised mass balance. Possible biases in scaling models may, in turn, lead to a low bias in the corresponding estimates of the long-term sea-level ~~change or future extent of glaciers in various glacierised regions, whenever a area-volume scaling-based method is used~~rise contribution from shrinking mountain glaciers.

~~In addition to identifying the bias in scaling-based models, we derived here expressions for glacier-response properties under a time-invariant scaling assumption and verified them with results from a scaling-based simulation of the ensemble of glaciers mentioned above. These expressions were also utilised to obtain best-fit parameterisation of the linear-response properties of glaciers simulated by SIA. A comparison between the response properties obtained for the above two methods confirms the systematic underestimation of climate sensitivity and response time under time-invariant scaling assumption. The response properties obtained from the analysis of SIA-based simulation results lead to a linear-response model that performs quite well in reproducing the time-series of total~~ On a multidecadal scale, a faster response due to shorter response times in the scaling model can compensate for the effects of smaller climate sensitivities to some extent. However, the low biases in scaling model derived changes in glacier area and volume ~~. This~~ are likely to become apparent over longer time scales of multiple centuries. The linear-response model presented above could potentially be useful in predicting ~~large-scale glacier change or global~~ the long-term global glacier change and/or sea-level rise ~~accurately as it reduces the biases that are inherent in scaling-based methods and , at the same time, retains the advantage of computational~~ due to its accuracy and numerical efficiency.

Code availability. The codes for the various models used in this paper shall available upon publication.

~~The location 551 simulated glaciers from Ganga basin, the Central Himalaya are shown with filled green circles. The dark blue stars denote all the glaciers in the region with area more than 2~~ ~~km^2~~ ~~as per RGI 6.0 inventory.~~

~~The evolution of the total glacier volume (A), and (B) glacierised area for an ensemble of 143 glaciers from the western Himalaya. These glaciers simulated with three different methods, namely, SIA, scaling and linear-response model. The corresponding results are shown with orange, red, and blue solid lines, respectively. These glaciers are located within 31°N-33°N and 77°E-79°E. The modelled area ranges between 2.7 to 134.4~~ ~~km^2~~~~. The selection criteria, detail of the models and parameters used, and the climate forcing are exactly the same as that used for the ensemble of 551 central Himalayan glaciers described in the main text.~~

Author contributions. AB designed the study, did the theoretical analysis, and wrote the paper. AJ and DP wrote the codes. AJ, DP, and AB ran the simulations. All the three authors contributed to the analysis of the simulated data and discussions.

Competing interests. We declare that there are no competing interests.

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