# Interactive comment on "Possible biases in scaling-based estimates of mountain-glacier contribution to the sea level" by Argha Banerjee et al.

Argha Banerjee, Ajinkya Jadhav, and Disha Patil

We thank referee Eviatar Bach for his critical comments (RC1) on the manuscript. Below we provide point-by-point reply to his comments. The referee comments are highlighted with *red italicised fonts*.

*The paper is interesting, well-written, and has the potential to be useful for understanding biases in scaling-based projections of glacier volume evolution. However, I have a few major concerns which prevent me from recommending publication.*

**Major issues:**

*1. For the linear-response model based projections, the authors write that they fit the four parameters (area and volume sensitivities and response times) for each glacier based on the SIA data. They then validate the projections obtained using these parameters on the same SIA data. This is using the same data for fitting and validation,so it is not surprising that it replicates the data fairly well. Testing this method requires validation data that is not part of the fitting. A possible way to do this would be to only use a portion of the time-series of each glacier to fit, and validate on the rest (for example, fit on the first 50 years, project into the future, and validate on the 450 remaining years).*

*There is this this sentence which I was not clear on: "We have verified the linear-response model obtained by fitting the SIA simulation results for the ensemble of 551central Himalayan glaciers, similarly outperforms the scaling-based method for another set of 143 glaciers from the western Himalaya (figure A2)." Were the parameters obtained for the central Himalayan glaciers somehow extrapolated to the western Himalayan ones? Or were the parameters fit for every western Himalayan glacier as well? It is not clear from the description. If the authors use an extrapolation method, it would be important to describe it.*

Let us clarify that our workflow is as follows:

- The response of 551 Central Himalayan glaciers to a 50 m step-change in

ELA is modeled with SIA to generate time series of area and volume.

- The area and volume evolution curves are fitted to obtain $\Delta V_\infty, \Delta A_\infty, \tau_A$, and $\tau_V$ for each of the glaciers.
- The response coefficients of the set of 551 glaciers obtained above are used to arrive at the following best-fit parameterisations. $\frac{\Delta V_\infty}{V} = (1.65 \pm 0.03)\alpha^*$
  $\frac{\Delta A_\infty}{A} = (1.87 \pm 0.02)^{-1}\frac{\Delta V_\infty}{V}$
  $\tau_A = (2.67 \pm 0.04)\tau^*$
  $\tau_V = (0.647 \pm 0.003)^{-1}\tau_A$
  where, $\tau^* = -(\frac{b_t}{\gamma h} + \beta)^{-1}$ and $\alpha^* = \tau^*\beta\delta E/\gamma h$.
- These four expressions are then used to run the linear-response model first for the same 551 glaciers (main fig 6), and subsequently, for 143 western Himalayan glaciers (main fig A2). (We emphasise that for modeling the second set of 143 glaciers, the four equations stated above, the ones that were obtained from results for the set of 551 glaciers, were used without any further calibration).

We agree with the referee that a favourable comparison between linear-response model and SIA results for the set of 551 central Himalayan glaciers is not an independent validation of the linear response model. However, the reasonable performance of the exact same parameterisations for 143 western Himalayan glaciers can be considered an independent validation of the linear-response model presented here. We propose to move the figure A2 to the main text to highlight this.

To avoid the possibility of any confusion, we shall tabulate these expressions and summarise the rationale in the discussion section of the revised manuscript. We shall also discuss how this model can be applied to any other set of glaciers, and details like the data requirements etc. as suggested by the referee in his comments below.

*2. The linear-response method is being proposed as an alternative to scaling-based methods for projecting glacier volume evolution. However, I am not clear on how this would be implemented in practice. The climate sensitivities $\Delta V_\infty$ and $\Delta A_\infty$ characterize the response of an initially steady-state glacier to a perturbation in the ELA. How can this be used to project evolution of a glacier that is already transient, and in a situation where it is not a single perturbation in the ELA, but that the ELA is continually rising?*

Linear response model is excepted to be a good approximation even if the initial state is a transient one, as long as the glacier being modelled is close to a steady state. It is not essential that the initial state is strictly a steady state. However, an additional initial condition is needed to apply linear-response model to a transient state. A continuous ELA change can be implemented as the sum total of a series of discrete steps every year (say). The net response is given by a superposition of suitably delayed response profiles due to each of the steps. The above statements follow from the general solution of the linear-response equation,

$$\Delta V(t) = \Delta V(0) e^{-t/\tau} + \frac{\Delta V_\infty}{\delta E} \int_0^t \Delta E(t') e^{-(t-t')/\tau} dt'.$$

Here, $\frac{\Delta V_\infty}{\delta E}$ is the climate sensitivity of glacier volume. $\Delta V(0)$ is the initial departure from a steady state that can be obtained from the observed rate of volume change as $\Delta V(0) = -\tau \frac{dV(0)}{dt}$. A similar expression can be written down for area evolution as well. We propose to include these details in the discussion section of the revised manuscript.

*3. Furthermore, it seems that the linear-response method would require a relatively long time-series of the area and volume evolution of each glacier in order to fit the parameters, which is often not available. I would like to see a discussion of the data requirements and feasibility for use in sea-level projections.*

In principle, no further tuning is required as the paramterisation of the linear response properties of mountain glaciers have been obtained with the help of the ensemble of SIA glaciers. One only needs a method to compute balance gradient ($\beta$), melt rate near terminus ($b_t$), and mean thickness ($h$) for each of the glacier to obtain the response properties. However, since the parameterisation is obtained for an ensemble of Himalayan glaciers, there is a need to validate the parameterisations for various other regions in the world where the mass-balance profiles or the typical glacier geometry could be different. That can be achieved with similar SIA-based tw-dimensional simulation of a large enough ensemble of glaciers.

To apply the model on a global scale, data on glacier area and volume, and their rate of changes are required at the initial epoch to start the model run. Additionally, a mass-balance model is required to compute $\beta$ and $b_t$ for each of the glacier. Another requirement is a prescription to determine the change in terminus elevation as volume and area changes. Most of these inputs are quite similar to that needed for any analogous scaling-based model (eg, Radic et al., 2007). Again, we shall include these details in the discussion section of the revised manuscript.

*4. Although it is true that there are a priori arguments for what the scaling exponent gamma should be, in practice it can be quite different, even for simulated glaciers (e.g., Radic et al., 2007). So for use of the scaling-based method as a statistical projection method it would be more fair, in order to compare to the linear-response method, to estimate the scaling exponent from the SIA runs. Although Radic et al. (2007, 2008)showed that the volume evolution over 100 years is not very sensitive to the exponent,this may be different for the 500-year simulations. Also, how were the constants of proportionality c determined for each glacier? In fact, we could expect that fitting the exponent and constant of* *proportionality individually for each glacier, as would be possible if a sufficiently long time-series data were available for every glacier, would considerably reduce the bias of the scaling-based method.*

In the manuscript we followed the prescription of Bahr et al. (2015) and fitted for only $c$, setting $\gamma$ equal to its theoretical value. We agree that in practice $\gamma$ and $c$ both are to be fitted for. For our synthetic glaciers simulated with SIA such a fit led to best-fit $\gamma$ that was essentially the same as the theoretically predicted value - likely due to the idealisation involved in the models.

We have used a single fit-parameter $c$ for all glaciers (Bahr et al., 2015). However, we emphasise that scaling evolution only requires the value of $\gamma$, and does not require that of $c$ due to the assumed time-invariance (eq. 2 in the manuscript).

We disagree with the referee that fitting for $c$ and $\gamma$ for individual glaciers using available long-term observations can cure the bias in the scaling-based methods. It was clearly demonstrated in this manuscript that $c$ is time-dependent. Therefore a constant $c$ assumption would always lead to some bias in the long run. However, over shorter time scales, the suggested prescription may be useful - particularly if the drift in $c$ is negligible.

### Other issues:

*1. The authors remove some glaciers from consideration in several parts of the paper, such as those that had fractional changes of more than 50% over 500 years, and those with response times higher than 300 years. Also, in another part of the paper, glaciers with large values of $\Delta A_\infty/A$ are removed, and another cut-off on $\Delta V_\infty/V$ is imposed. I don't see an adequate justification of why these were removed,and doing so biases the results.*

We agree with the reviewer that rejecting several modeled glaciers at various stages is confusing. The criticism is well taken and our arguments and proposed changes as detailed below.

First of all, the cut-off on fractional changes in area/volume (assumed to be 50% in the manuscript) are necessary as linear-response theory only works for relatively small deviations. We checked that our results is more or less independent of the actual value of the cut-off. For example, the model performance for the set of glaciers with fractional changes of less than 20% are shown in Figure 1 below.

We propose to remove the cut-offs used while fitting for glacier response properties (the excluded points were denoted by black circles in main fig 2, 3 and 5). This lead to small insignificant changes in the best-fit coefficients appearing in the expressions for linear-response properties.

A cut-off of $\tau_A < 300$ years (Please note that as $\tau_A > \tau_V$, it is enough to put a cut-off on only $\tau_A$) was applied as our simulation period is 500 years, and response times much larger than that may not be estimated accurately. The actual value of the cut-off is somewhat arbitrary though. For example, we had

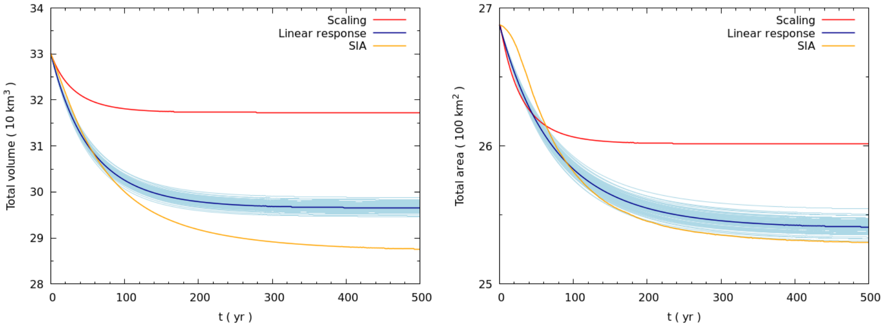

Figure 1: A comparison of model performance for the set of 246 glaciers with fractional changes of less 20%.

verified that setting the cut-off at, say 500 years, does not affect the linear-response parameterisations within a few percent of so. The nature of model outputs is insensitive to the precise cut-off value (see Figure 2 below). However, to be on the safe side we have used a response-time cutoff of 300 years. It can

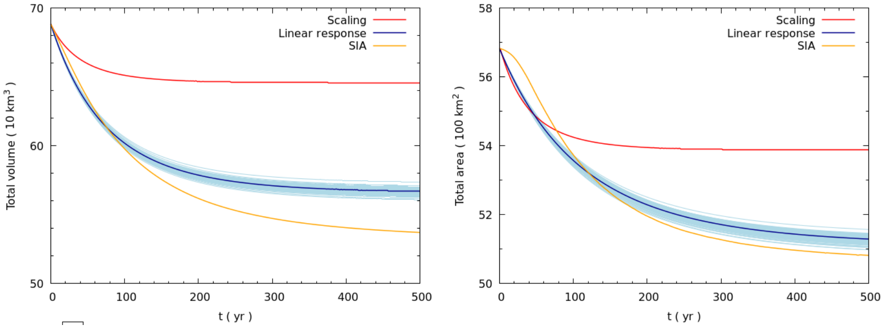

Figure 2: A comparison of model performance for the set of 582 glaciers with response time less than 500 years.

be seen Figure 3A below that without the best-fit $\tau_A$ for individual glaciers are likele overestimated when they are larger than 500 years or so. As a result the linear relationship between $\tau_A$ and $\tau_V$ breaks downhere. This is also confirmed by Figure 3B below, where we have shown the best-fit exponential decay lead to larger estimates for $\tau_A$ when a small part of the intial profile is fitted for. Based of these arguments, we believe an appropriate cutoff on response time - one that is less than 500 years or so - is necessary to obtain accurate parameterisation of the same.

We propose to include all the details mentioned here related to cutoffs applied as supplementary material with the revised manuscript.

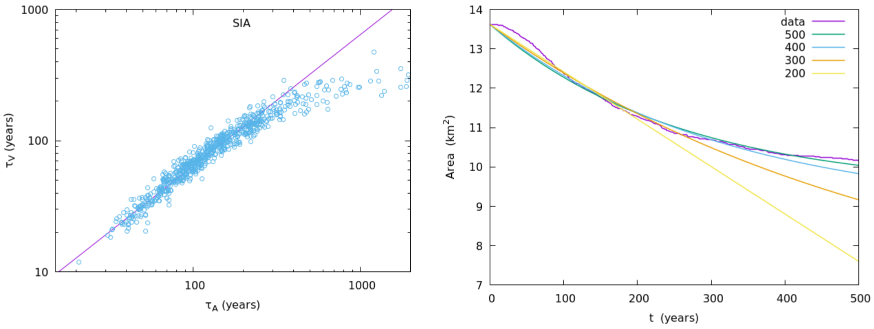

Figure 3: (A) For glaciers with $\tau_A$ comparable or larger than simulation period of 500 years, best-fit $\tau_A$ values are overestimated. This leads to a breakdown of the linear relationship between $\tau_A$ and $\tau_V$. (B) As an illustration, we show fits to the timeseries of area for one of the 551 glacier over the first 200, 300, 400, and 500 years (solid lines) which obtain response time of $7\times10^6$, 557, 312, and 257 years, resepctively.

*2. "The minor differences are due to the time-invariant scaling assumption made here." Please clarify in more detail what is the difference between your derivations and those of Harrison (2001).*

The differences arise out of our use of scaling to relate $\Delta V$ and $\Delta A$, namely, $\Delta V = \gamma h \Delta A$, that is used to obtain the expression of $\tau_v = -(\frac{b_t}{\gamma h} + \beta)^{-1}$.

In contrast, Harrison et al. (2001) used $\Delta V = H\Delta A$ where $H$ is a "thickness scale" to get, $\tau_v = -(\frac{b_t}{H} + \beta)^{-1}$.

Since sensitivities are proportional to response time the same factor of $\gamma$ also appears in our expression of climate sensitivities. That is not the case for corresponding expressions of Harrison et al. (2001).

*3. In Fig. 1B, scaling the SIA results by 10 for visual comparison is confusing. It's also hard to distinguish which are the thick and thin lines.*

We proposed to replace the figure with the following one (see Figure 4 below) where we plotted results for 10 glaciers and did not scale the SIA results.

**Minor issues: ...**

We thank the referee for pointing out the typographical errors which we shall correct in the revised version of this manuscript.

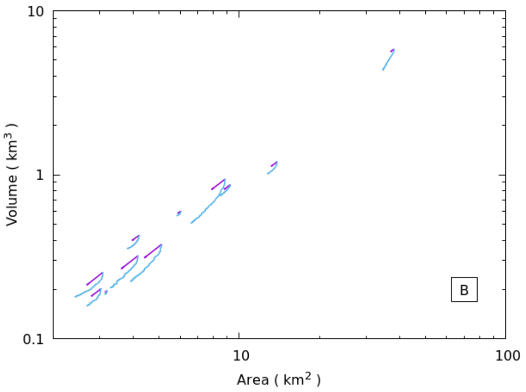

Figure 4: Evolution of 10 randomly chosen glacier in the $V - A$ plane for SIA and scaling based models.