Sensitivity of the Greenland surface mass and energy balance to uncertainties in key model parameters

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\textbf{Abstract.} We investigate the sensitivity of a distributed glacier surface mass and energy balance model using a variance based analysis, for two distinct periods of the last glacial cycle: present day climate (PD) and the Last Glacial Maximum (LGM). The results can be summarized in three major findings: The sensitivity towards individual model parameters and parameterizations is neither as invariant in space nor as in time. The model is most sensitive to uncertainty related to down-welling long-wave radiation. Turbulent latent heat flux has a sizable contribution to the surface mass balance uncertainty in central Greenland today and over the entire ice sheet during the cold climate of the LGM, in spite of its low impact on the overall surface mass balance of the Greenland ice sheet in modern climate. We conclude that quantifying the model sensitivity is very helpful for tuning model parameters, because it clarifies the relative importance of individual parameters and highlights interactions between them that need to be considered.

\textbf{Copyright statement.} TEXT

\section{Introduction}

Of the many challenges to accurately simulate past variations in the volume of the Greenland ice sheet (GrIS) and to project its future contribution to sea level rise, recent studies agree that the uncertainty associated with surface mass balance (SMB) is among the most important (\cite{Plach2019, Aschwanden2019, Plach2019}). Models to calculate SMB spread a whole range of complexities from empirical index models that only account for air temperature (\cite{Ohmura2001, Zemp2019}), or temperature and solar radiation (\cite{Robinson2011, Bintanja2002, VanDenBerg2011}), to coupled atmosphere-snow models that simulate the snowpack in multiple layers and a full representation of the atmospheric circulation, based on physical first principles (\cite{Lehning2002, Fettweis2007, Noël2018}). On this spectrum, the empirical models perform well for the observational period and when the temperature sensitivity of the SMB is well known (\cite{Fettweis2020}), but are hard to constrain for temporal and spatial climate variations and become unreliable for conditions outside their relatively narrow tuning interval (\cite{vandeBerg2011, Plach2019}). This is unfortunate because while The lack of constraint in empirical models is unfortunate even though their low computational
requirements make these models attractive for the long integration times that are needed to simulate continental ice sheets. Their shortcomings severely limit the usefulness of these results. On the other hand, detailed snow models and especially those coupled with regional atmosphere models are computationally too expensive to run for long periods of time. This situation motivated the development of models that balance the defensible representation of the relevant physical processes with computational efficiency (Krapp et al., 2017; Krebs-Kanzow et al., 2018; Born et al., 2019).

In this study, we use the BErgen Snow SImulator (BESSI), a model that is designed to include all relevant physical mechanisms with reasonable detail but specifically prepared for long integration times by reducing its computational requirements and by strictly conserving mass and energy (Born et al., 2019). Adding to the original model version, we now include three different parameterizations for snow aging based on Oerlemans and Knapp (1998), Aoki et al. (2003), and Bougamont et al. (2005). The turbulent latent heat flux while limiting is now simulated also. For this study the model domain to only was reduced to Greenland at a resolution of 10 km. Three different snow aging parametrizations are used based on Oerlemans and Knapp (1998), Aoki et al. (2003), and Bougamont et al. (2005), but all model changes are applicable to the original setup also.

Multiple studies have investigated the impact of the turbulent latent heat flux on the surface mass balance (Box and Steffen, 2001; Box et al., 2004; Van Den Broeke et al., 2008; Cullen et al., 2014; Noël et al., 2018). They find a relatively small impact of the vapor fluxes on the Greenland ice sheet total mass balance (≈ 5 Gt a⁻¹, Cullen et al. (2014)), but its local impact can be up to 20% of the annual accumulation (Box et al., 2004). The importance of the vapor flux is difficult to assess for different climatic settings, as Box and Steffen (2001) have shown that the choice of the calculation method impacts the results greatly in regions of low flux like the dry interior zone of Greenland. This is exacerbated by the fact that turbulent latent heat fluxes are mostly negative in winter under the present climate conditions, and positive in summer so that the sign of the net flux may change with changing a different climate. During the colder climate of the glacial a much larger impact of the turbulent latent heat flux can be assumed, similarly to the much greater importance it currently has in Antarctica (e.g., Gallet et al., 2014; Van Wessem et al., 2018). A parameterization based on the bulk-method by Rolstad and Oerlemans (2005) has been added to BESSI to simulate the turbulent latent heat flux.

To assess the sensitivity of the new parameterizations and that of BESSI overall, we employ a variance based approach (Saltelli et al., 2000, 2006, 2010; Sauter and Obleitner, 2015), that has previously been used to quantify the sensitivity of a glacier model (Zolles et al., 2019) glacier and ice models (Aschwanden et al., 2019; Bulthuis et al., 2019; Zolles et al., 2019). We extend the sensitivity analysis used by Zolles et al. (2019) to provide spatial patterns of sensitivity indices. Following our model’s design goal to be used over time scales of glacial cycles and accounting for potentially different sensitivities under different climate boundary conditions, we analyze two large ensembles with a total of 16,500 simulations, for present day (PD) climate and that of the Last Glacial Maximum (LGM). The result is rich information on what parameters and parameterizations have the largest impact on the model’s performance, how this sensitivity varies in different regions of Greenland and over time. This knowledge enables a balanced calibration of the model as it prevents parameters to be specifically adjusted to the study location and parameters as the knowledge prevents or reduces over-fitting to a particular study location or time (Beven, 1989).
The revised model is described in section 2. The sensitivity-distributed sensitivity analysis of multiple model output variables to model parameters, including those for the new parameterizations for turbulent latent heat flux and snow aging, is presented in section 3. After that, we discuss our findings in section 4 and conclude in section 5.

2 Model description and study setup

The study uses the efficient mass and energy balance model BESSI, which is designed to simulate the mass balance over long time scales (Born et al., 2019). The energy exchange of the snow with the atmosphere is altered in the model version used here, while the subsurface and internal processes are unchanged from the previously published version. The following model was enhanced to include the turbulent latent heat flux and multiple more complex snow albedo schemes were added. The model description given in the following will focus on the snow-surface interaction with interaction between the snow surface and the atmosphere. For the numerical description and other subsurface processes like firnification and heat conduction see Born et al. (2019).

The model setup used here has a 10 km horizontal grid over the domain of Greenland. The vertical dimension is discretized using a mass-following grid with up to 15 layers in the snow pack (Born et al., 2019). The mass of each layer ranges is 100 - 500 kg m$^{-2}$. Each cell has a default maximum of 300 kg m$^{-2}$, but due to melt and refreezing the mass may decrease or increase, respectively. Cells above 500 kg m$^{-2}$ or below 100 kg m$^{-2}$ are split or merged respectively, to restore the default maximum value. Simulations require daily input of air temperature, total precipitation, solar radiation and its reference height. Humidity is an optional input which is required if the turbulent latent heat flux is computed. All variables are interpolated to the 10x10 km model grid using bi-linear interpolation. The air temperature is vertically downscaled to the actual model topography using a temperature lapse rate of 6.5 K/km for PD and 8.55 K/km for the LGM. The output written by the model may be adjusted by the user ranging from daily over monthly to annual values. Output variables include: surface mass balance, melt of snow, melt of ice, runoff, refreezing, albedo, turbulent latent heat flux, a mask containing snow, land, ice and water as well as the 3-dimensional grid values for snow mass, snow density, snow temperature and liquid water mass.

2.1 Surface energy fluxes

The energy exchange between the surface and the atmosphere comprises five different processes, of which two, of which the precipitation and the turbulent latent heat flux (vapor flux) also imply a change in mass: The short-wave radiation ($Q_{SW}$), the long-wave/thermal radiation ($Q_{LW}$), the turbulent sensible heat flux ($Q_{SH}$), the turbulent latent heat flux ($Q_{L}$) and the heat flux associated with precipitation ($Q_{P}$).

The total surface flux can be expressed as:

$$c_i m_{s,1} \left. \frac{\partial T}{\partial t} \right|_{surface} + Q_{M} = Q_{i} + Q_{M} = Q_{SW} + Q_{LW} + Q_{SH} + Q_{L} + Q_{P}$$  (1)
where the left hand side denotes the resulting temperature change of the snow/ice \((Q_i)\) and the available energy for melting \((Q_M)\) if the melting point is reached. Due to the implicit scheme the model uses, no melt is calculated at first, but only energy fluxes and temperatures (even above 273 K). The actual melt is then calculated explicitly each time step as the excess heat above the melting point. The mass flux of the water vapor is also calculated explicitly.

2.1.1 Short-wave radiation and albedo parameterization

The energy input to the surface from solar radiation is calculated by using a broadband albedo value:

\[
Q_{SW} = (1 - \alpha)SW_{in},
\]

where \(\alpha\) denotes the surface albedo, either \(\alpha_s\) for snow or \(\alpha_i\) for ice and \(SW_{in}\) the incoming short wave radiation at surface height. The albedo value is assumed constant with respect to the solar incidence angle, but undergoes temporal and spatial variations depending on surface properties. We implement four albedo parameterizations of different complexity to simulate the snow albedo. They all have a common maximum albedo value for fresh snow \((\alpha_{fs})\), minimum value for aged snow (firn \(\alpha_{fi}\)), and ice \(\alpha_i\), but vary in how they calculate the aging.

1. **Constant:** This simple parameterization only uses constant values for dry snow \((\alpha_s = \alpha_{fs}, T_s < 273K)\), wet snow \((\alpha_s = \alpha_{fi}, T_s = 273K)\), and ice \(\alpha_i\). This parameterization has been used before in BESSI (Born et al., 2019).

2. **Oerlemans and Knapp (1998):** This parameterization assumes an exponential decay with time of the fresh snow albedo to a final value of old snow albedo (Oerlemans and Knapp, 1998):

\[
\alpha_s = \alpha_{fi} + (\alpha_{fs} - \alpha_{fi})\exp\left(\frac{t_{fs} - t}{t^*}\right), \quad t^* = \begin{cases} 30d, & T_s < 273.15 K \\ 5d, & T_s = 273.15 K \end{cases}
\]

where \(t_{fs}\) denotes the last day of snowfall, \(t\) the current day (time step) with \(t^*\) as the characteristic time in days. This or similar parameterizations are usually optimized for the decay rate \(t^*\) using observations of albedo or mass balance (e.g., Oerlemans and Knapp, 1998; Klok and Oerlemans, 2004; Bougamont et al., 2005). The very fast decay at the melting point was chosen to account for our very large upper grid box which would likely already be wet if the real surface was resolved (0.28 - 1.4 m depending on the mass and density of the box), as the heat capacity of the entire large box may delay the melting on the top of the surface layer. Equation 3 does not consider shallow snow packs, where underlying ice or dirty firn albedo may reduce the albedo.

3. **Bougamont et al. (2005)** modified the parameterization by Oerlemans and Knapp (1998) specifically for the Greenland ice sheet by introducing a snow temperature-dependent decay rate:

\[
t^* = \begin{cases} 100d, & T_s < 263 K \\ 30d + 7d \cdot (273.15K - T_s), & 263 K \geq T_s < 273.15 K \end{cases}
\]
This Equation 4 results in the same \( t^* \) of 30 d as Oerlemans and Knapp (1998) for 273 (eq. 3) up to the melting point (273.15 K). This parameterization furthermore introduces an additional wetness-dependent albedo decay in the case of wet snow, which assumes a thin layer of water at the surface according to

\[
\alpha_s = \alpha_{fi} - (\alpha_{fi} - \alpha_s) \cdot \exp\left(\frac{-w_{surf}}{w^*}\right).
\] (5)

Here \( w_{surf} \) denotes the thickness of the water layer and \( w^* \) is a characteristic water layer thickness. Since BESSI does not explicitly simulate water at the surface, this was adapted for this model by we adapted the liquid water depending part using a simple linear parameterization. The decay rate increases depending on the liquid water content \( \zeta \) (see 2.2 for details about the liquid water content):

\[
t^* = 15 - 14 \cdot \frac{\zeta}{\zeta_{max}} \text{ d}, \quad T_S = 273.273.15 \text{ K}
\] (6)

4. Aoki et al. (2003): The final albedo parameterization available in the model uses both a fixed and a temperature dependent decay rate, rather than keeping it constant or relating it to temperature and time dependent decay rates. There is a linear dependency on temperature in each time step and this parameterization is therefore not exponentially depending on the time since the last snowfall.

\[
\alpha_s(t) = \min\{\alpha_s(t-1) - ((T_S - 273 - 273.15 \text{ K}) \cdot k + c), \alpha_{fi}, \alpha_s(t-1)\}
\] (7)

where \( t \) and \( t-1 \) are the current and the last previous time step, \( \alpha_s \) the snow albedo and \( k = 1.35 \cdot 10^{-3} \text{K}^{-1} \) and \( c = 0.0278 \) two empirically based constants. The values are based on averaged values from Aoki et al. (2003) for different spectral bands. To account for a faster decay in a wet snow packit, the albedo is linearly decreased based on the liquid water content of the topmost layer:

\[
\alpha_s(t) = \alpha_s(t) - (\alpha_s(t) - \alpha_{fi}) \cdot \frac{\zeta}{\zeta_{max}}
\] (8)

If the layer is fully saturated with water the snow albedo instantly drops to its minimum value. In this parametrization the albedo increases if This is done in addition to equation 7 at each time step. The albedo increases when new snowfall occurs, but instead of resetting it to the fresh snow value, it this albedo is incrementally increased depending on the amount of fresh snow to account for thin layers of snow, and the penetration of short-wave radiation into the older subsurface:

\[
\alpha_t(t) = \alpha_{t-1}(t-1) - (\alpha_{fs} - \alpha_{fi}) \cdot (1 - \exp\left(-\frac{-d}{d^*}\right)),
\] (9)

with where \( d \) as the snow depth. The is the amount of new snow, and the characteristic snow depth \( d^* \) is at 3 cm (Oerlemans and Knapp, 1998).
2.1.2 Long-wave radiation

The long-wave radiation is a simple parametrization based on the Stefan-Boltzmann law:

\[ Q_{LW} = \sigma (\epsilon_{atm} T_{atm}^4 - \epsilon_s T_s^4) \]  

(10)

where \( \sigma \) is the Stefan-Boltzmann constant, \( T_{air} \) and \( T_s \) are the 2 m air and snow surface temperature, respectively. The emissivity of snow/ice \( \epsilon_s \) is constant at 0.98.

The incoming long-wave radiation is only depending on the actual air temperature and the atmospheric emissivity \( \epsilon_{atm} \), as the only free model parameter. The lack of confidence in cloud cover of climate models in particular during the last glacial cycle, lead to this decision. Though more complex empirical relations exist (e.g., Liston and Elder, 2006), their applicability for other time scales is questionable. We therefore refrain from using these empirical relationships despite the importance of cloud cover, moisture and aerosols. \( \epsilon_{atm} \) varied over a broad range of 0.6-0.9 following the previous configurations of this BESSI or similar models (Greuell and Konzelmann, 1994; Busetto et al., 2013; Born et al., 2019). Values spanning this entire range may in reality occur simultaneously in different regions of the Greenland ice sheet, but in the current configuration there is only a single atmospheric emissivity value over the entire ice sheet.

2.1.3 Turbulent sensible heat flux

The calculation of the turbulent latent heat flux is based on a bulk method (Braithwaite, 2009) which was applied previously on Greenland as residual method (Rolstad and Oerlemans, 2005). This method assumes a constant turbulent exchange coefficient \( (C_h) \) for sensible heat over time and space. The only dependency in the previously published parametrization is on the local wind speed \( u \) and air temperature \( T_{air} - T_{atm} \):

\[ Q_{SH} = \rho_{air} c_p C_h u (T_{air} - T_s) = D_{SH} (T_{air} - T_s) \]  

(11)

where \( \rho_{air} \) is the density of air and \( c_p \) the heat capacity of air. Since BESSI does not use wind speed as an input field, we simplify the equation with a single free model parameter, the turbulent heat exchange coefficient \( D_{SH} \). This is a free parameter which is subject of the sensitivity analysis. The values given in Table 1 assume an average wind speed of 5 ms\(^{-1}\) if compared to the reported values by Braithwaite (2009). The variation in parameter \( D_{SH} \) therefore accounts for both the variability in average wind speed and the efficiency of the exchange \( C_h \).

2.1.4 Turbulent latent heat flux

The previous version of BESSI did not include turbulent latent heat flux. Here it is an optional. The new model version includes an optimal turbulent latent heat flux subroutine as part of the setup. The implementation is analog to the turbulent sensible heat flux (eq. 11):

\[ Q_L = 0.622 \rho_{air} L_v C_h u (e_{air} - e_s) p^{-1} = D_{LH} (e_{air} - e_s) \]  

(12)
where $D_{ LH}$ is the turbulent latent heat exchange coefficient, and $e$ is the water vapor pressure. The parameterization is based on the bulk formulation of Rolstad and Oerlemans (2005) with the latent heat of vaporization $L_v$ and the air pressure $p$. The latter is calculated from the standard pressure at sea level for each grid point. While Rolstad and Oerlemans (2005) assume the same exchange coefficient $C_h$ for vapor and sensible heat, it has been shown (Greuell and Smeets, 2001) previously that the roughness lengths and the exchange coefficient for momentum and vapor are not necessarily equal (Greuell and Smeets, 2001). Nevertheless, the parameters $D_{ LH}$ and $D_{ SH}$ are inherently connected by the surface structure (snow/ice) and the wind speed. To account for these points the setup uses two free model parameters determining $D_{ LH}$, the turbulent exchange coefficient for sensible heat $D_{ sh}$, and $r_{ lh/sh}$ which are defined by

$$D_{ LH} = r_{ lh/sh} \cdot 0.622 L_v D_{ SH} / c_p,$$

(13)

In the setup of this study there are three parameters determining the turbulent latent heat flux. The ratio, $(r_{ lh/sh})$, accounts for different exchange rates for water vapor and sensible heat. $D_{ SH}$ the absolute exchange strength (roughness, wind, stability). The additional parameter, $(\chi_{ QL})$, switches the subroutine, switches the simulation of the turbulent latent heat flux on and off.

2.1.5 Precipitation heat flux

The heat supplied by precipitation depends on the atmospheric temperature, which we assume to be in balance with the precipitation. An atmospheric temperature of 273.273.15 K is the limit of solid precipitation. In the case of snow fall, the solid mass of the topmost grid cell of the snow pack increases by the amount of snow and the heat added to this box reads:

$$Q_{ P,\text{snow}} = P \rho_w c_i (T_{\text{air atm}} - T_s)$$

(14)

while rain is added as liquid water mass to the same cell

$$Q_{ P,\text{rain}} = P \rho_w c_w (T_{\text{air}} - 273_{\text{atm}} - 273.15 K),$$

(15)

where $P$ is the amount of precipitation, and $\rho_w$ and $c_w$ the density and heat capacity of water. If the rain freezes or percolates further down, the corresponding exchange of latent heat is calculated during the balance calculation, and as outlined in the next section.

2.2 Subsurface percolation and refreezing

This is the only part of the subsurface routine which is explained here. Only a brief overview of the subsurface routine is given in this paper. Full details are available in Born et al. (2019). The water holding capacity of each layer determines the percolation. The maximum liquid
water content $\zeta_{max}$ is the parameter subject to the sensitivity analysis:

$$\zeta = \frac{m_w}{m_s} \frac{1}{\rho_w (\frac{1}{\rho_s} - \frac{1}{\rho_i})}$$

(16)

where $m_w$, $m_s$ and $\rho_w$, $\rho_s$, $\rho_i$ are the mass and density masses and densities of water, snow and ice, respectively. If the liquid water content $\zeta$ exceeds $\zeta_{max}$ the liquid water mass percolates to the next cell below or is treated as runoff when it leaves the bottom cell.

2.3 Global sensitivity analysis

2.3.1 Theoretical background Setup and setup theoretical background

![figure](image_url)

**Figure 1.** The climate model topography for PD (a) is from the ERAinterim data set, and the LGM (b) is from the CCSM4 simulation. The used model topography for PD (c) is ETOPO and ICE-6G (d) for the LGM. For the plot all topography below 5 m was considered sea level for the climate models.

We assess the model sensitivity and uncertainty of BESSI for two different time periods, to verify its applicability over the whole glacial cycle. The PD period using the ERA-interim reanalysis data (Uppala et al., 2011) ranging over 38 years from 1979 to 2017. Our last glacial maximum (LGM, 21,000 years before present) simulations are forced with 30 climate years of the Community Climate System Model version 4 (CCSM4) (Brady et al., 2013). The simulated LGM has an annual mean temperature over the entire model domain of 249 K/-24°C, while the current climate is relatively warm with an annual mean of 262 K/-11°C. The entire model domain was chosen as the ice sheet has different shapes in the two climate states. The precipitation averages are around 400 kg m$^{-2}$ for the LGM and 570 kg m$^{-2}$ for PD. The annual mean solar radiation is about 10% higher for the LGM, though the seasonal cycle deviates drastically from PD. The surface mass balance model uses a static topography, which is based on ETOPO (Amante and Eakins, 2009) for PD and on ICE-6G (Peltier et al., 2015) for the LGM (fig. 1). There is an interpolation artifact in the PD simulation in the far north-west, but as no ice is present in this region of Greenland it does not influence the analysis. BESSI was run for 500 years with the same forcing data looping the forcing data back and forth (1979-2017-1979-2017...) to account for the long response time of the firn cover. After 400 years the firn cover was dynamically (density) and thermodynamically (temperature) stable, even in the regions of very low accumulation. The analyses shown here are entirely based on the last 100 years of every simulation.
The global sensitivity analysis is a variance based method that allows for a full assessment of the model sensitivity over the entire parameter space. In comparison to other methods, all parameters are changing at the same time. In contrast to other sensitivity methods, it assesses the full parameter space simultaneously. The method has been applied previously to snow pack (Sauter and Obleitner, 2015) and recently to alpine glaciers (Zolles et al., 2019). The method is based on algorithms developed by Saltelli et al. (2000, 2006, 2010), utilizing the setup of the ensemble hypercube using (Sobol et al., 2007). (Sobol et al., 2007). To compute both sensitivity indices, the estimator from Sobol et al. (2007) was used. The probabilistic framework provides an estimate of the sensitivity of the model output to the individual input variables, including parameters and data. The GSA is independent of model calibration and tuning. The model output \( Y \) is a function of the input parameters \( X_i: Y = f(X_1, X_2, \ldots, X_n) \). There are two normalized values that quantify the model sensitivity for each input parameter \( X_i \). The first order index denotes the sensitivity of the model towards the parameter \( X_i \) only, while the latter includes all the interactions of \( X_i \):

\[
S_{X_i} = \frac{V_{X_i}(E_{X_i}(Y|X_i))}{V_Y}
\]

\[
S_{T_i} = \frac{E_{X_i}(V_{X_i}(Y|X_i))}{V_Y},
\]

where \( E \) is the expectation value of a given observable such as the SMB. \( V_Y \) is the total variance of the given variable and \( V_{X_i} \) the variance that only depends on the input parameter \( X_i \). \( X_{-i} \) denotes the whole parameter space excluding any variation in \( X_i \). The first order index calculates the mean model output \( (E_{X_i}(Y|X_i)) \) for each representation of \( X_i \) and then assesses the sensitivity by calculating the variance for all values of \( X_i \).

The total index can be compared to the local sensitivity index that is often determined around the optimal model setting, but the GSA presented here does not rely on a predetermined optimal parameter setting. \( V_{X_i}(Y) \) varies the parameter \( X_i \) along its dimension, but is computed for all possible points of the parameter space instead of the optimal one. For the detailed algorithm refer to Saltelli et al. (2010). As under-sampling is assumed by the relatively low numbers of simulations, bootstrapping is applied to the ensemble and multiple sensitivity values are reported. Both indices are normalized with the variance of the whole ensemble \( V_Y \).

To study the applicability of BESSI for the use over the whole glacial cycle the model sensitivity and uncertainty is assessed for two different time periods. To consider a wide range of climate conditions in Greenland two extreme cases of the whole glacial cycle were chosen: the PD period using the ERA-interim reanalysis data (Uppala et al., 2011) ranging over 38 years from 1979 to 2017 and 30 simulation years of the climate of the last glacial maximum (LGM, 21,000 years before present) of the Community Climate System Model version 4 (CCSM4) (Brady et al., 2013). The simulation of the LGM shows a cold state with an annual mean temperature over the model domain of 249, while the current climate is relatively warm with an annual mean of 262. As the ice sheet has different shapes, these climatological comparisons are only done for the entire model.
Table 1. The parameter ranges for the free model parameters are rather broad and based on previously published values (Born et al., 2019). The firn albedo may not exceed the fresh snow albedo, and its value was limited to 0.65 during the LGM. All parameters are distributed following a pseudo random Sobol sequence.

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Abbreviation</th>
<th>range</th>
<th>unit</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>fresh snow albedo</td>
<td>$\alpha_{fs}$</td>
<td>0.65 - 0.9</td>
<td></td>
<td>Cuffey and Paterson (2010)</td>
</tr>
<tr>
<td>2</td>
<td>firn albedo</td>
<td>$\alpha_{fi}$</td>
<td>0.45 - 0.7 (0.65)$^1$</td>
<td></td>
<td>Cuffey and Paterson (2010)</td>
</tr>
<tr>
<td>3</td>
<td>ice albedo</td>
<td>$\alpha_i$</td>
<td>0.3 - 0.4</td>
<td></td>
<td>Cuffey and Paterson (2010)</td>
</tr>
<tr>
<td>4</td>
<td>turbulent heat exchange coefficient</td>
<td>$D_{SH}$</td>
<td>5 - 25</td>
<td>Wm$^{-2}$K$^{-1}$</td>
<td>Braithwaite (2009)</td>
</tr>
<tr>
<td>5</td>
<td>ratio of sensible and latent heat flux</td>
<td>$r_{th/sh}$</td>
<td>0.5 - 1.2</td>
<td></td>
<td>Greuell (1992)</td>
</tr>
<tr>
<td>6</td>
<td>emissivity of the air</td>
<td>$\epsilon_{atm}$</td>
<td>0.6 - 0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>switch for turbulent latent heat flux</td>
<td>$\chi_{QL}$</td>
<td>on / off</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>albedo module</td>
<td>$\chi_{\alpha}$</td>
<td>cnst,Oer,Bou,Aok</td>
<td></td>
<td>Aoki et al. (2003); Born et al. (2019)</td>
</tr>
<tr>
<td>9</td>
<td>maximum liquid water content</td>
<td>$\zeta_{i}$</td>
<td>5 - 15</td>
<td>% pore volume</td>
<td>Greuell (1992; Born et al., 2019)</td>
</tr>
</tbody>
</table>

domain instead of the ice sheet domain. The precipitation averages are around 400 for the LGM and 570 for PD. The annual mean solar radiation is about 10% higher for the LGM, though the seasonal cycle deviates drastically from PD. The surface mass balance model uses a static topography, which is based on ETOPO (Amante and Eakins, 2009) for PD and on ICE 6G (Peltier et al., 2015) for the LGM (fig. 1). There is an interpolation artifact in the PD simulation in the far north, but as no ice is present in this region of Greenland it does not influence the analysis. BESSI was run for 500 years with the same forcing data back and forth to account for the long response time of the firm cover. 400 years was shown to be sufficient to develop a dynamically and thermodynamically stable firm cover, even in the regions of very low accumulation. The analysis shown throughout this study is entirely based on an assessment of the last 100 years of every simulation. We are limiting the detailed discussion to the total index $S_{Ti}$, as BESSI is a highly correlated model.

The model has nine free parameters which are listed together with their plausible range in Table 1. We are using nine free model parameters (tab. 1). The initial ensemble was generated using a Sobol sequence which consisted of 2000 × 9 members for PD and 1000 × 9 for the LGM. This sequence spans a 9-dimensional unit hypercube. It is split For computing both sensitivity indices the estimator from Sobol et al. (2007) was used. It splits the initial sequence into two subsets $A$ and $B$ each consisting of one half of the initial sequence (1000 × 9). For computing both sensitivity indices the estimator from Sobol et al. (2007) was used. It creates $1000/500 \times 9$. Then an additional set of matrices $B_{ij}^A$, which are based on the matrix $B_{ij}$, where the values for parameter for parameter $X_i$ are replaced with those from subset $A$, are created. The matrices $A$, $B$ and $B_{ij}$ are then used to estimate the model sensitivity. A detailed description of the algorithm can be found in (Sobol et al., 2007) and (Saltelli et al., 2010). The whole ensemble consists of $N \cdot (2 + k)$ members, with $N$ being
the base sample (1000 in the case of PD) and \( k \) the amount of parameters. A detailed description of the algorithm can be found in (Sobol et al., 2007) and (Saltelli et al., 2010) (9, tab. 1).

The full ensemble for present day climate has 11000 members, that for LGM climate 5500. The normalized parameter dimensions are initial hypercube with a length of [0,1] in each dimension is linearly transformed to the intervals given in table 1, with the exception of the latent heat flux switch and the albedo module. These two parameters have two, respectively four discrete values, and the parameter space is split equally between them. The model simulations are carried out with the generated parameter matrix. We are using bootstrapping to estimate the sensitivity indices. For each bootstrap the equations 17 and 18 are evaluated. Finally, we report the mean sensitivity indices and their standard deviation. The results were checked for consistency \((\sum S_{Xi} \leq 1, S_{Xi} \leq S_{Ti})\). The ensemble size used during the LGM is on the absolute lower limit of applicability for GSA, as the standard deviation of the sensitivity indices is large (not shown). It works fine for the output quantities analyzed (fig. A1).

The GSA works well for the SMB, latent heat flux and melt, some with increased uncertainty associated to the absolute values, but fails for the 10 m firm temperature for example. Due to the larger ensemble the confidence in PD ensemble is higher, but not by a large enough margin to justify the additional computation time relative to the 5500 members of the LGM ensemble. BESSI is a complex model with all parameters interacting and \( S_{Xi} \) provides very little information. Therefore, the results mainly focus on the total sensitivity index \( S_{Ti} \).

The GSA was computed for five different outputs: albedo, vapor flux, snow melt, surface mass balance and the average surface temperature. These surface temperature, which are based on average yearly sums or averages for SMB, melt and vapor flux or temporal averages for albedo and temperature, over 100 years. Surface temperature results are rather uncertain and only tendencies can be extracted as the surface temperature is largely influenced by the annual cycle and fresh snow reseeding fall on an ice surface.

### 3 Results

#### 3.1 Global sensitivity analysis - GSA

**GSA at PD over elevation bands:** The main focus of the results will be on the surface mass balance and discussion is limited to the total sensitivity index \( S_{Ti} \), due to the low information that can be extracted from \( S_{Xi} \) in complex models. The sensitivity of the SMB changes for different elevation classes for the present day ice sheet is shown in Figure 2. In the region of lowest elevation from 0 - 1000 m the largest sensitivity is associated with the parameter uncertainty of the atmospheric emissivity \( \epsilon_{atm} \) with a normalized total sensitivity index of about 0.8. The second most influential parameter is the turbulent heat exchange coefficient \( D_{SH} \), followed by a slight influence of the snow albedo (fig. 2 a). The general features are similar up to 2000 m (fig. 2 b), with a slight decrease in the sensitivity to \( D_{SH} \). Above in regions above 2000 m (fig. 2 c,d,e) the SMB is sensitive to a much wider range of parameters: the atmospheric emissivity, the fresh snow \( \alpha_{fs} \) and firm albedo \( \alpha_{fi} \), the choice of albedo module \( \chi_{a} \), the turbulent heat flux coefficient, and the turbulent latent heat flux switch \( \chi_{QL} \). While the importance of \( \chi_{QL} \) is increasing with elevation, the importance of \( \alpha_{fi} \) decreases. The sensitivity of the fresh snow albedo \( \alpha_{fs} \) increases up to 0.4 at 2500, but not further. With the increasing sensitivity of multiple parameters the relative sensitivity
Figure 2. The global sensitivity analysis of the SMB was conducted on the PD-SMB, providing the main order effect (circle) and the total effect (triangle). The two indices are displayed for all nine parameters over the different elevation bands ranging from 0-1000, 1000-2000, 2000-2500, 2500-3000 and above 3000 m and the entire ice sheet. The symbol represents the mean value of the sensitivity index with the bars as ± 1σ. The elevation bands are displayed of the present day topography over Greenland are displayed on the right, the analysis is only done for cells where ice is present. A similar figure for the LGM is found in the appendix (A. A1).

of \( \varepsilon_{atm} \) decreases, leading to \( \chi_{QL} \) almost being equally important in the region with the highest average elevation. The local SMB in the regions above 2000 m is impacted by multiple parameters, while the integrated mass balance is dominated by the atmospheric emissivity with the snow albedo and turbulent fluxes having a minor influence.

Spatial pattern of GSA at PD: The global sensitivity maps for all parameters are displayed in Figure 3. The GSA was also calculated for each grid cell individually. The general trends are similar to the elevation averages, but the structure is much more complex. The global sensitivity maps for all parameters are displayed in Figure 3. The model spatial pattern shows important additional details (fig. 2). The SMB is sensitive to \( \varepsilon_{atm} \) over the entire ice sheet. Its impact is only reduced at the west coast glaciers where \( D_{SH} \) is more sensitive and the north west where the choice of albedo module is important. The model is The SMB is also sensitive to \( \chi_{QL} \) in the interior of Greenland, and \( \chi_{QL} \) can be considered a sensitive parameter for most of the ice sheet apart from the regions of very high melt at low elevation. In the interior of very high elevation, the most sensitive of the three snow albedo related parameters is the one for fresh snow \( \alpha_{f_s} \), while at lower elevation elevations below 2000 m the firm snow albedo \( \alpha_{f_i} \) becomes more important and also the chosen albedo parameterizations and the chosen type of albedo parameterization, \( \chi_{a} \), becomes more important, the latter in particular in the north-east, where fresh snow fall is infrequent. The SMB is sensitive to \( D_{SH} \) at very low elevations, the ice caps in the west and on the ice sheet above 1500 m, only
Figure 3. Global sensitivity at PD. The total sensitivity index of the mass balance SMB of every parameter for PD is displayed for every ice covered grid cell. The ice free land is in brown, the ocean in light blue, 1000 m contours are in white. The sensitivity is largest for the atmospheric emissivity $\epsilon_{atm}$, followed by the fresh snow and firn albedo module $\alpha_{fs}/\alpha_i$, the turbulent heat exchange coefficient $D_{SH}$, and the latent heat flux switch $\chi_{QL}$. Absolute values are not to be taken too seriously due sensitive to the setup uncertainty, but the uncertainties of the total sensitivity indices are in the same order of magnitude as shown in Figure 2.
The large impact of the fresh snow albedo, as snowfall is not frequent and decay rates at atmospheric temperature are of the order of a few weeks (Sec. 2.1.1). An exception is found in the north east, which is characterized as coastal), the latent heat flux switch (up to 0.8 coastal), the latent heat flux switch (S_{L subdivisions} mostly > 0.4) and the snow albedo (S_{L subdivisions} ≈ 0.3) during the LGM. The liquid water content ζ_{max}, the ice albedo α_{i}, and the ratio of latent and sensible
Figure 4. Global Sensitivity at the LGM. The total sensitivity index of the SMB of every parameter for PD is displayed for every ice covered grid cell. The ice free land is in brown, the ocean in light blue, 1000 m contours are in white. The total sensitivity index of the surface mass balance for the LGM is more variable than during PD. The model shows the greatest sensitivity towards $QL_{on/off}$ and the atmospheric emissivity $\epsilon_{atm}$ followed by the fresh snow albedo $\alpha_{fs}$. Firn albedo $\alpha_{fi}$ and the albedo module $\chi_{\alpha}$ as well as the turbulent exchange coefficient $D_{SH}$ are important around the margin. The SMB during the LGM shows almost no sensitivity towards the other parameters.
heat exchange coefficient $r_{lh/sh}$ do not only marginally impact the SMB in either of two climate states (fig. 3 and 4). On the local scale $\chi_{QL}$ followed by $\epsilon_{atm}$ and $\alpha_{fs}$ are the main sensitivity components at the LGM (fig. 4). The sensible heat flux exchange coefficient $D_{SH}$ is important along the margin with the largest impact in the south-east. The firn albedo and the albedo module are sensitive parameters along the margin, with a bit higher impact of the module along the western side apart from the precipitation heavy south and southeast, where frequent snowfall repeats the albedo.

The increased importance of $\chi_{QL}$ is a result of a much colder and dryer climate. The SMB is positive over most of Greenland during the LGM, in the absence of melt, even for extreme parameter combinations, so that even for parameter combinations leading to a high energy input. In the absence of melt, the only change in mass balance to the SMB is due to sublimation and hoar formation. The dry climate also favors increased sublimation. While during the PD the sign of the vapor flux also undergoes an intra-annual variability, it higher sublimation than at PD. The vapor flux (sublimation) is mainly a net heat loss for the surface during at the LGM. The other sensitive parameters still determine the strength of the sublimation via the surface temperature, because the exponential impact of the surface temperature via Clausius–Clapeyron relation impacts the surface temperature via the Clausius–Clapeyron relation has an exponential impact on the turbulent latent heat flux more resulting in a greater model sensitivity towards the atmospheric emissivity than the actual exchange coefficient $D_{SH}$ (eq. 13). The incoming long-wave radiation is also the largest energy source for the surface. The large impact of $D_{SH}$ in the south-east is due to the large temperature difference between the surface and the air temperature. This region air, the south-east is dominated by intense precipitation and rather warm air masses even during the LGM. There is a precipitation gradient from the western coast to central Greenland, the fewest precipitation amounts are least precipitation is found in the west of Greenland and the south of Elsmere Island due to the presence of the Laurentine ice sheet over North America at the LGM (not shown). Therefore, the albedo module is more important on the western than the eastern margin. The reduced model sensitivity As frequent precipitation, which is mainly snowfall during the LGM, increases the albedo more frequently. The lower model sensitivity at the LGM towards $\epsilon_{atm}$ is mainly a result of the reduced atmospheric lower air temperature with annual averages being around 10 K lower, resulting in less incoming long-wave radiation and a lower absolute impact of the emissivity.

The sensitivity of Sensitivity of other output variables in addition to SMB: We also studied the sensitivity of other model variables, namely the surface albedo, turbulent latent heat flux, snow melt, and surface temperature will only be briefly discussed and without additional figures. Due to their large size, the corresponding figures (similar to fig. 3 & 4) are included in the supplement, but we include the main findings here. The global sensitivity for the annual average albedo during PD period is mainly influenced by the fresh snow albedo parameter, with only minor importance of the final–firm albedo, ice albedo, the choice of the albedo module, and the atmospheric emissivity at the ice sheet margins. However, this is an interesting result for the model tuning as margin. As a result, the snow albedo should not be tuned with BESSI without incorporating the atmospheric emissivity should be treated similarly to firm and ice albedo for tuning of the albedo routine in addition to the direct albedo related parameters (fig. S. GSA_albedo_ERAi/LGM).

Besides the switch ($\chi_{QL}$) which disables the turbulent latent heat flux $Q_L$ completely, the turbulent latent heat flux is most sensitive to the atmospheric emissivity $\epsilon_{atm}$ followed by the turbulent heat flux exchange coefficient $D_{SH}$, mainly around the margins. In these areas (fig. A A3). Around the margin the ratio of turbulent sensible and latent exchange coefficients $r_{lh/sh}$ is
a parameter of low sensitivity, as are plays a minor role ($S_{i,i} \approx 0.1$). The turbulent latent heat flux is also sensitive to the snow albedo related parameters ($\alpha_f, D_{SH}, \chi_Q$) in the north. The turbulent latent heat flux $Q_L$ is not sensitive neither sensitive the maximum liquid water content $\zeta_{\text{max}}$ globally nor to the ice albedo and $r_{th/sh}$ in the interior of the ice sheet or the maximum liquid water content $\zeta_{\text{max}}$ globally. This shows that the effect of $r_{th/sh}$ on the turbulent latent heat flux as well as the SMB is low, using similar exchange coefficients for moment, temperature and water vapor are justified within the framework. During the LGM this framework. At the LGM, $Q_L$ shows an increased sensitivity to $\alpha_f$, while the $D_{SH}$ is less important around the margin, due to lower atmospheric temperatures and slower albedo decay (not shown). The albedo module and the firm albedo play almost no role in either case.

The average snow temperature is mainly influenced by $\epsilon_{\text{atm}}, \alpha_f, D_{SH}, \chi_Q$, though uncertainties of the sensitivity are rather large, due to temperature resetting in the event of snowfall on ice or shallow snowpacks.

Snow melt is more less closely linked to the SMB, and shows now significant deviation from the sensitivities similar sensitivities as reported for the SMB. Just as expected, the impact of the latent heat flux switch is much lower as it is mainly important in regions with the absence of melt (fig. 3).

### 3.2 Ensemble statistics

**PD regional parameter dependencies:** The GSA clearly highlights that the mass balance is the surface mass balance as most sensitive to the atmospheric emissivity and the latent heat flux switch irrespective of background climate. A drawback of this method is that it provides The GSA method has the drawback that it gives no information about the sign and absolute magnitude of the SMB changes. Parameters Physical processes and associated parameters, which result in either surface heating or cooling, like the turbulent heat fluxes, require a different analysis. Albedo increase and atmospheric emissivity decrease will always result in a more positive will be analysed based on ensemble statistics explained in the following. Conversely, an increase in albedo and a decrease in atmospheric emissivity always lead to an increase in surface mass balance. For an in depth analysis of the individual parameter impacts, Greenland was split The ensemble statistics also gives additional information for parameter sensitivities which were well analyzed with GSA. We split Greenland into 11 different regions for PD and 13 for the LGM (2 more around Elsmere Island) based on elevation and similarity on elevation, ice divides, geography and climatological similarity for this analysis. In particular most of the west coast shows a similar behavior and is therefore only a single region (fig. 5). This is a regional spread of the elevation bands used in Figure 2. For each region the parameter range is split into 20 equally spaced intervals for which the 5, 25, 33, 50, 66, 75, and 95% quantiles are calculated. The binning is necessary as the ensemble was not created with parameters at regularly spaced intervals. It shows the behavior of the entire or a sub-ensemble with regard to one parameter. This is most interesting The most interesting features are seen for the parameters where the effect changes sign depending on the atmospheric conditions, like the turbulent heat exchange coefficient. We discuss the selected region 5 and the turbulent latent heat flux in depth here, while the complete set of figures is included in the supplement for reference.

In Figure 6 the impact of the various parameters on the PD SMB is shown for region 5. This is the western region of Greenland ranging from 1000 - 2000 m. The dominant parameter is the atmospheric emissivity $\epsilon_{\text{atm}}$. Over the range of
plausible values in \( c_{atm} \) reduces the median of the SMB by almost 800 kgm\(^{-2}\) in an accelerating manner. The atmospheric emissivity and the SMB are inversely correlated and the relationship is non-linear with greater effect at larger values. The spread of the ensemble, i.e., the variation variance of the SMB as a result of other parameters, increases too. The increase of the SMB with the snow albedo related parameters \( \alpha_{fi} \) and \( \alpha_{fS} \) is smaller and the width of the distribution decreases (panel a,b). Even as very low albedo parameter values do not necessarily lead to a negative mass balance in this SMB in the western region. An increase in \( D_{SH} \) slightly reduces the median SMB, but the spread is decreasing. This will be discussed in detail later. Ice albedo \( \alpha_i \), liquid water content \( \zeta_{max} \) and \( r_{lh/sh} \) have almost no impact. The SMB increases in the presence of turbulent latent heat flux due to the heat loss of sublimation in this region—region 5. With all albedo modules the ensemble has a wide spread, but the variation is smallest for the time dependent decay (Oerlemans and Knapp, 1998) which also has the highest median mass balance. The parametrization, as it neither has an instant albedo drop upon reaching the melting point (constant) nor accounts for the liquid water mass in the snow pack. The parameterization based on temperature, wetness and time (Aoki) has the lowest median SMB.
Figure 6. The ensemble statistics for the surface mass balance for region 5 (west 1000-2000 m) shows the at PD. The 5/95, 25/75, 33/66 and 50 quantiles are displayed in progressively darker shading. Black points represent the ensemble mean and the grey points correspond to the rest of the ensemble, apart from outliers (max 5 per bin allowed), which are removed to improve readability. Each plot represents the range of one parameter with $\alpha_{fs}$, $\alpha_f$, and $\alpha_i$ in the top row, $D_{SH}$, $r_{th/sh}$ and $\epsilon_{atm}$ in the middle and $Q_{Lon/off}$, $\chi_\alpha$ and $\zeta_{max}$ at the bottom. As $Q_{Lon/off}$ and $\chi_\alpha$ have two and four discrete values, the parameter range is not split in 20 intervals.
The strong impact of the emissivity on the surface energy balance and therefore also SMB is due to the larger annual average of the incoming long-wave radiation relative to the other fluxes. At very low atmospheric emissivity $\epsilon_{atm}$, short-wave, precipitation, and turbulent fluxes. In the PD climate, the largest energy source during the PD climate is reduced, and hardly any melt occurs in this region—for the snow is incoming long wave radiation. As atmospheric emissivity $\epsilon_{atm}$ decreases less energy is available for melt in region 5. In the absence of melt, surface mass balance response is only due to the sublimation and since this process—the vapor flux has a modest absolute impact on the SMB, the spread of the ensemble is low. Vice versa, at large $\epsilon_{atm}$ values a lot of grid cells will experience a heat up and potential melt of the snow pack early—warming, and potentially early melting of the snow pack, occurs in many grid cells leading to a positive feedback effect with lower albedo. In agreement with the GSA (fig. 3) the firn albedo $\alpha_f$ is almost as important as fresh snow albedo $\alpha_f$. The ELA is located in this region region 5 and snow temperatures are quite rather warm, resulting in a large impact of snow albedo decay and its parametrization parameterization. The constant albedo parametrization has quite a wide spread, as it parameterization spreads more than 1000 kg m$^{-2}$ as the albedo is very sensitive around the ELA, changing instantly from $\alpha_f$ to $\alpha_f$ when the snow-pack reaches the melting point. While the albedo parameterizations based on Oerlemans and Knapp (1998) and Bougamont et al. (2005) have a temporal decay relative to the instant one in the constant case, with Bougamont having

Bougamont et al. (2005) has a snow temperature dependent increased decay rate and even more in the wet case. This leads to the low SMB tail of it relative to the other, leading to less skewed ensemble than of the other albedo parameterizations. The last albedo parameterization parameterization decays faster than the other two at warmer temperatures, leading to already lower albedo than all other parameterizations albedo parameterizations before the melting point is reached. Additionally, there is a difference between the models in case of snowfall. Oerl. const and Bou reset to $\alpha_f$ and decay thereafter, while Aoki increases the albedo after snowfall incremental with the amount of snowfall.

**Differences of the LGM regional parameter dependencies:** During the LGM the western region between 4000–2000 m (not corresponding to the identical geographical area as PD due to topographic changes differences) shows positive, but low SMB with a much lower spread of the ensemble (fig. 7, A A2). There are three distinct changes differences to PD: The impact of $\alpha_f$ and $\chi_\alpha$ is drastically reduced, enabling the latent heat flux results in a decrease of SMB, as sublimation prevails at the LGM and reduces the SMB, and the importance of $\alpha_f$ increases relative to $\epsilon_{atm}$ (fig. 7). The climate in the west of Greenland in the LGM is characterized by a much lower air temperature, slightly more annual mean radiation and lower precipitation. The lower air temperature reduces the impact of $\epsilon_{atm}$ and produces a more positive SEB, which in turn results in lower snow temperatures leading to a slower snow albedo decay (for the albedo subroutines which parameterize the decay), no melting and almost no impact of associated parameters. The incremental increase of snow albedo with snowfall with the Aoki parametrization, gives slightly lower albedo in the dry climate. During the LGM sublimation prevails over the entire year, but. Though in the absence of melt it results in the increase in sublimation results on a mass loss rather than a mass gain as in PD (via cooling and associated reduced melt—), rather than the reduced melt via sublimation during the PD conditions. Furthermore, the non-linearity in the SMB at the LGM almost vanishes for $\epsilon_{atm}$ and $\alpha_f$ as there is hardly any melt and associated snow-ice albedo feedback.
Figure 7. The ensemble statistics for the surface mass balance of the $Q_{Lon}$ sub-ensemble for region 5 (west 1000 – 2000 m) at LGM. The 5/95, 25/75, 33/66 and 50 quantiles are shown in progressively darker shading. Black points represent the ensemble mean and the grey points correspond to the rest of the ensemble, apart from outliers (max five per bin allowed), which are removed to improve readability. Each plot represents the range of one parameter with $\alpha_{fA}$, $\alpha_{fB}$ and $\alpha_3$ in the top row, $D_{sh}$, $r_{vh/sh}$ and $r_{atm}$ in the middle and $Q_{lon/off}$, $\chi_0$ and $\zeta_{max}$ at the bottom. As $Q_{lon/off}$ and $\chi_0$ have two and four discrete values, the parameter range is not split in 20 intervals. Figure A.A2 shows a similar plot for the entire LGM ensemble.
Figure 8. The dependency of the SMB on the turbulent heat exchange coefficient $D_{SH}$ is displayed in a similar matter as Figure 8 for PD. The shading represents the different quantiles: 5/95 (light grey), 25/75 (grey) 33/66 (dark) and the median (solid black line). The black dots are the ensemble mean based on 20 intervals. The panels are sorted by elevation and regions, with the lowest elevation (0-1000 m) in the top row, and the highest elevation at the bottom. Each column is related to a region in Greenland: W-N-E-SE/S (fig. 5).
Impact of the turbulent latent heat flux exchange coefficient on the SMB as an example for PD: The turbulent sensible heat flux may either be a heat loss or heat gain for the surface, making it particularly interesting. The turbulent heat depends on the difference between atmospheric and surface temperature. The strength of the turbulent sensible heat flux is mainly influenced by the turbulent latent heat flux exchange coefficient $D_{SH}$ is the parameter determining its strength. It is very different depending on the region. The turbulent sensible heat flux varies over the different regions (fig. 8), which is why we present a deeper look on this particular parameter. The figure is limited to the Only the sub-ensemble with an active turbulent latent heat flux ($\chi_{QL\text{on sub-ensemble}}$) is shown in Figure 8 because in regions without melt 50% of the simulations ($\chi_{QL\text{ off sub-ensemble}}$) will show a similar mass balance. The overall width of the SMB distribution decreases with altitude. In the lowermost regions in the west and southeast (1,4) there is a trend to more negative mass balances with larger exchange coefficients. Regions 3, 5, 6, 10 and to a lesser extent 2 and 7 show a distinct decrease in surface mass balance spread of the ensemble with increasing $D_{SH}$ and slight decrease of the mean (excluding region 6). The smallest spread of the ensemble is found in the high altitude regions 9, 10, 11. The general slightly negative trend for the SMB with $D_{SH}$ is a result of higher air-temperatures than snow surface temperatures. This shows a very strong influence and a net heating of the turbulent latent heat flux. The negative trend is most pronounced at the lower regions of the west and south, where warm air advection is frequent, resulting in increased melt due to the heat supplied by the sensible heat flux. The regions are also quite moist, in particular the south east (region 4), therefore the leading to a positive turbulent latent heat flux during summer is also positive, which in turn heats the surface even more.

The north-east (region 2, 3, 6, 7) of Greenland is colder and drier resulting in lower temperature and moisture differences between surface and atmosphere. This reduces the effect that decreased turbulent fluxes and therefore the effect of $D_{SH}$ has on the SMB and SEB (surface energy balance) is less. The ensemble spread is narrower at higher altitudes (7-11) due to a generally positive mass balance, so an increased energy input due to any parameter will mainly impact the snow temperature, but in the absence of melt the mass balance does not change significantly. Still the change in temperature temperature change alters sublimation accounting for the remaining variability. This is pronounced in region 10, at low exchange coefficients melt is still possible, if other parameters result in a strong positive energy input, leading to a skewness towards negative SMB. At a stronger exchange the tight coupling with the heat reservoir of the earth-atmosphere limits melt and the skewness towards negative SMBs vanishes, while the median stays almost constant. The parameter $D_{SH}$ is not the main driver for the SMB in this region, rather it region 10, rather the parameter acts as a buffer of the SMB. At strong turbulent sensible heat exchange the surface temperature will be buffered by the air-temperature heat reservoir. This reservoir, The buffering effect is visible for most of the regions, with where air-temperatures most of the year below the melting point (2,3, 5, 7-10).

The earlier discussion mainly focused on the impact of $D_{SH}$ on the surface mass balance via the turbulent sensible heat flux $Q_{SH}$, but it $D_{SH}$ also impacts the turbulent latent heat flux ($Q_{L}$, eq. 12). In a similar manner to Figure 8 the turbulent latent heat flux alone was analyzed, but is not shown here, additional figure in the Supplement). $Q_{L}$ gets more negative with increasing exchange coefficient. There are two effects present, the increased surface water vapor pressure due to higher surface temperature as a result of $Q_{SH}$ and the actual water vapor exchange rate. The annual average of the ensemble over Greenland is a negative turbulent latent heat flux, meaning sublimation occurs more often than condensation. $r_{th/sh}$ does increase $Q_{L}$ in
its absolute value and therefore leads to more sublimation, but the effect on the mean is also lower than on the whole ensemble. At larger exchange rates more mass can be moved and therefore the variation over the ensemble increases as larger surface temperatures are prevailing.

The average SMB increases at low elevation if $\chi_{QL}$ — on, and decreases above 2000 m and is almost constant above 3000 m, where the annual average of the latent heat flux is almost zero. The first is a result of reduced melt due to a negative latent heat flux (sublimation), and it is the SMB increase is therefore most pronounced in the north-east. The latter is a region where melt only occurs for a couple of extreme ensemble members, so the sublimation is the only mass loss and therefore the SMB decreases. The other parameters show analogous behavior to with increased sublimation via increasing $D_{SH}$.

**Ensemble statistics of other parameters for PD:** The other parameter results are consistent with the GSA: An increase in $\epsilon_{atm}$ decreases the SMB. The effect is not purely linear, rather increasingly more melt in case of a high emissivity. This but melting increases a proportionally with higher emissivity. The high impact of $\epsilon_{atm}$ is mainly related to the all year around impact of this parameter altering ice exposure and albedo decay. The snow albedos increase the SMB, with the fresh snow albedo being more important. The SMB ensemble plots do not show any dependency on ice albedo, liquid water content and the mean is also unaffected by $r_{th/sh}$. Additional figures are available in the supplement.

During the LGM the general features are similar to PD. Larger values of $D_{SH}$ reduce the variability/impact of the other parameters, and resulting in slightly lower SMB. Similar to the GSA a shift to lower elevations is occurring. The impact of $D_{SH}$ on the SMB is negligible above 2500 instead of above 3000 m for PD and melt tails are limited to below 2000 m. $\epsilon_{atm}$ has less influence during the colder period, as well as the snow albedo related parameters.

Additionally **Sensitivity of the 10 m firn temperature:** In addition to the SMB, the sensitivity of the 10 m firn temperature was assessed. This was limited to locations where actual firn cores have been taken (Born et al., 2019) not possible for the GSA due the limited sample size. We limited the analysis of the firn temperature to 13 locations in the interior of Greenland. The firn temperature at these mostly central locations is sensitive to three parameters: $\epsilon_{atm}$, $\alpha_{fs}$ and $D_{SH}$. The first two have a rather linear impact with increasing temperature with increasing emissivity and decreasing albedo. The turbulent heat exchange coefficient has a similar effect on the 10 m firn temperature as on the SMB in the shown for region 5. At small $D_{SH}$ the 10 m temperature has a larger variability depending on the other two sensitive parameters, while at large values the air-temperature buffers the snow temperature, even down to 10 m. The sensitive parameters are the main drives of the SEB, which determines the 10 m temperature, in those areas as discussed previously for the SMB. The only difference to regions at higher elevation of the SMB is the insignificance of $\chi_{QL}$. The turbulent latent heat flux has only a minor importance for the SEB, but in the absence of melt it is a requirement for SMB changes. These findings highlight, that by tuning the vapor flux is the only SMB change. Tuning the model for the 10 m firn temperature only provides information about the sensitive parameters could be extracted, which are $\epsilon_{atm}$, $\alpha_{fs}$, and $D_{SH}$.

4 Discussion
The model sensitivity in this study is evaluated based on parameter uncertainties. Cloud cover and the associated atmospheric radiation are big uncertainties in the climate state. This large uncertainties both, in present day and glacial climates. The uncertainty in cloud cover is represented by the large parameter uncertainty of the atmospheric emissivity $\varepsilon_{\text{atm}}$. The SMB is most sensitive to this parameter $\varepsilon_{\text{atm}}$ under PD conditions. The sensitivity of the SMB is not drastically different during the LGM, but lower atmospheric temperatures reduce the impact of $Q_{\text{atm}}$ and $\varepsilon_{\text{atm}}$, the atmospheric emissivity, while also leading to fewer areas where melt and runoff occurs. The relative contribution of the mass flux associated with the turbulent latent heat flux to the SMB increases drastically during the LGM (4 to 15 % of the total mass flux), making it the turbulent latent heat flux switch the model’s most sensitive parameter in large parts of the ice sheet. This ice sheet. The increased importance of $Q_L$ is due to the absence of melt, similar to the highest elevations during PD climate. Additionally, SMB values have a smaller magnitude, as precipitation is less during the LGM and therefore the relative contribution of the vapor fluxes to the SMB is larger. For an accurate modeling of the SMB over the glacial cycle an inclusion of the turbulent latent heat flux is necessary, which may not be as important for a warmer climate.

The sensitivity reported is the result of two components; metric we applied is a relative measurement that depends on two components, the absolute strength of a particular flux and the chosen parameter uncertainty range of the parameter. The latter depends on the subjective choice. We based the parameter values on published common ranges (tab. 1). The incoming long-wave radiation is the largest single energy source for the surface energy balance during PD, ranging from twice the incoming solar radiation around the margin to about 1/3 in the interior. The impact of the atmospheric emissivity is decreasing from the coast inwards for multiple reasons: Firstly, temperatures are higher around the margin leading to increased incoming thermal radiation. Secondly, cloud cover is larger and therefore solar radiation is reduced. Thirdly, at negative SMB the rate of melting depends greatly on the total energy flux and albedo decays faster at a warmer snowpack/ice for a warmer snow pack if more long-wave radiation reaches the surface during winter.

It is important to differentiate between the sensitivity of the Greenland wide integrated mass balance and the local SMB, as well as the impact of the individual fluxes on the absolute SMB.

The Greenland-wide surface mass balance is most sensitive to the atmospheric emissivity during PD (fig. 2), while during the LGM the SMB shows increased sensitivity to the fresh snow albedo, the choice of albedo parameterization, and the turbulent latent heat flux. This is a result of lower atmospheric temperatures and an ice sheet with less melt increasing overall increases the importance of the vapor fluxes similar at to high elevations at PD. While it would be acceptable for the integrated SMB to not be a necessary for models to include $Q_L$ in a warm climate using this model, it is not, though desirable. Conversely, the turbulent latent heat flux $Q_L$ cannot be ignored during a colder and dryer climate state. Additionally, this means that although the cloud cover uncertainty may be similar during the colder period of the LGM the sensitivity of this model towards this uncertainty is lower. Our model towards the emissivity uncertainty is less. Simple surface mass balance models like enhanced temperature index models are likely to create a bias as solar insolation changes are accounted for while the impact of the cloud cover on other components of the energy balance are neglected or
implicitly included in the PDD factor, this is even more true as even under PD conditions the impact of clouds on the SMB is highly variable (Van Tricht et al., 2016; Hofer et al., 2017).

At the local scale such as in the interior of Greenland even in PD climate the vapor flux \( Q_L \) is up to 1/3 of the total SMB, despite its small impact on the Greenland wide integrated scale. Additionally, in addition, in the absence of melt the sublimation and condensation are the only changes to the SMB with a fixed precipitation forcing. Neglecting these processes sublimation and condensation will result in fundamental biases over long-term simulations of the Greenland ice sheet, in particular in its interior. This needs to be considered when tuning surface mass balance models for long time scales. A tuning for the Greenland wide SMB will mainly constrain the most sensitive parameters, which constrain certain key regions of high mass turnover, but not for the bulk surface. Furthermore, the temporal change differences of the sensitivities on the local scale indicates that models of reduced complexity may fail drastically for other time periods (absence of \( Q_L \) for example).

The sensitivity in this study is not defined relative to the absolute flux, it does not tell us that it is most sensitive to the long-wave radiation, but rather analysis shows that the uncertainty of the long-wave radiation has a larger impact on the SMB uncertainty than the uncertainty in the incoming solar radiation on the SMB uncertainty, but as it is not defined relative to the absolute flux, it does not necessarily tell us that the SMB is most sensitive to the long-wave radiation energy component. The impact each energy flux has on the absolute SMB has to be analysed separately. The long-wave radiation dominates, followed by the solar radiation. The increased larger sensitivity of the \( \chi_{Q_L} \) switch during at the LGM is mainly due to its increase on the absolute SMB. In the absence of melt, and with reduced precipitation sublimation accounts for a larger portion of the absolute SMB.

It is beyond the scope of this model BESSI to resolve all the physical processes. We use a simple parameterization for the incoming long-wave radiation which does not accurately represent reality. The atmospheric emissivity is neither constant in space nor time. Area distributed values may work during the observational period, but changes differences in the atmospheric circulation may change alter these patterns over the glacial cycle. We conclude that the overall model uncertainty can effectively be improved by changing the simplified representation of the long-wave radiation flux as a function of atmospheric temperature and emissivity for either a more sophisticated parameterization or to use long-wave radiation as climate model input. If the model is to be tuned for the Greenland-wide SMB it will be biased towards the melt regions around the margin and therefore the atmospheric emissivity. Where possible parameters should be calibrated via quantities which they are sensitive for. Neither of our parameters are to be assumed constant in space, as albedo for example depends very much strongly depends on impurities and snow temperature, but unless the uncertainty of the incoming long-wave radiation is reduced, it is justifiable to work with optimized values from PD. The current model version does not use wind fields, although they impact the SEB via the turbulent fluxes. The strength of the turbulent exchange does not have a large impact on the SMB and the approach to neglect wind speed variability is therefore justified in the context of more uncertain parameters. The found model sensitivity towards the parameters is to be set into context with the assumed forcing (climate) uncertainty.
Summary and conclusions

The surface mass and energy balance model BESSI has been improved by accounting for turbulent latent heat flux and snow aging. The sensitivity of the model towards these changes and other uncertain model parameters was assessed with a variance based sensitivity method based on two ensembles with a total of 16500 simulations. Warm present day climate and the cold last glacial maximum were used to study the change differences of the model response under different PD and LGM boundary conditions. The sensitivity analysis reveals that the inclusion of the turbulent latent heat flux is a necessity to simulate the local SMB and the integrated SMB over the entire Greenland ice sheet. The relative importance of sublimation and condensation is larger in the dry and cold climate of the LGM, as air temperature and precipitation are lower.

The uncertainty associated with cloud cover and atmospheric emissivity creates a dominance of the SMB model uncertainty. With the change in different circulation during the last glacial a changing energy input from the atmosphere to the surface will result in a SMB response. Simple surface mass balance models like enhanced temperature index models are likely to create a bias as solar insolation changes are accounted for while the impact of the cloud cover on other components of the energy balance are neglected or implicitly included in the PDD factor, this is even more true as even under PD conditions the impact of clouds on the SMB is highly variable (Van Triert et al., 2016; Hofer et al., 2017). The sensitivity study further reveals that the uncertainty of the SMB as a result of the atmospheric radiation decreases in colder climate.

The current model version does not use wind fields, although this impacts the SEB via the turbulent fluxes. The strength of the turbulent exchange does not have a big impact on the SMB and the approach to neglect wind speed variability is therefore justified in the context of more uncertain parameters.

We find that uncertainties in the ice albedo, liquid water content and differences of the turbulent fluxes are of minor importance for our and likely also similar models. In order to reduce model uncertainty most effectively, first the larger energy sources of short wave and long wave radiation need to be constrained via the snow albedo and the atmospheric emissivity.

Code availability. The BESSI model code is available on git-hub (https://github.com/TobiasZo/BESSI/tree/TobiasZo---GSA-model-version). Additionally, the github branch also contains the analysis and plotting scripts. It is also available together with the supplement at 10.5281/zenodo.4310369.

Appendix A: Additional plots

Author contributions. TZ implemented the model changes, conducted the ensemble simulations, the sensitivity studies and the data analysis and wrote the main part of the manuscript. AB contributed to all aspects of this study.
Figure A1. The global sensitivity analysis of the LGM-SMB provides the main order effect (circle) and the total effect (triangle). The two indices are displayed for all nine parameters over the different elevation bands ranging from 0-1000, 1000-2000, 2000-2500, 2500-3000, above 3000 m and the entire ice sheet. The symbol represents the mean value of the sensitivity index with the bars as ± 1σ. The elevation bands of the LGM topography over Greenland are displayed on the right, the analysis is only done for cells where ice is present. The uncertainty of the sensitivity indices is larger than for PD due to the smaller ensemble size.

Competing interests. The authors declare that they have no conflict of interest.
Figure A2. The ensemble statistics for the surface mass balance for the entire ensemble at region 5 (west 1000-2000 m) at LGM shows the 5/95, 25/75, 33/66 and 50 quantiles in progressively darker shading. Black points represent the ensemble mean and the grey points correspond to the rest of the ensemble, apart from outliers (max 5 per bin allowed), which are removed to improve readability. Each plot represents the range of one parameter with αfs, αfl and αi in the top row, DSH, τbh/sh and εatm in the middle and QLon/off, Xα and εmax at the bottom. As QLon/off and Xα have two and four discrete values, the parameter range is not split in 20 intervals.
Figure A3. Global sensitivity of the turbulent latent heat flux at PD. The total sensitivity index of $Q_L$ of every parameter for PD is displayed for every ice-covered grid cell. The ice-free land is in brown, the ocean in blue.
References


