

1 **Inter-comparison and improvement of two-stream shortwave radiative transfer**  
2 **models for a unified treatment of cryospheric surfaces in ESMs**

3  
4 Cheng Dang<sup>1</sup>, Charles S. Zender<sup>1</sup>, Mark G. Flanner<sup>2</sup>

5  
6 <sup>1</sup> Department of Earth System Science, University of California, Irvine, CA, USA

7 <sup>2</sup> Department of Climate and Space Sciences and Engineering, University of Michigan,  
8 Ann Arbor, MI, USA

9 *Correspondence to:* Cheng Dang (cdang5@uci.edu)

10  
11 **Abstract.** Snow is an important climate regulator because it greatly increases the surface  
12 albedo of large parts of the Earth. Earth System Models (ESMs) often adopt two-stream  
13 approximations with different radiative transfer techniques, the same snow therefore has  
14 different solar radiative properties depending on whether it is on land or on sea ice. Here  
15 we inter-compare three two-stream algorithms widely used in snow models, improve  
16 their predictions at large zenith angles, and introduce a hybrid model suitable for all  
17 cryospheric surfaces in ESMs. The algorithms are those employed by the SNow ICE and  
18 Aerosol Radiative (SNICAR) module used in land models, and by dEdd-AD used in  
19 Icepack, the column physics used in the Los Alamos sea ice model CICE and MPAS-  
20 seaice, and a two-stream discrete ordinate (2SD) model. Compared with a 16-stream  
21 benchmark model, the errors in snow visible albedo for a direct-incident beam from all  
22 three two-stream models are small ( $< \pm 0.005$ ) and increase as snow shallows, especially  
23 for aged snow. The errors in direct near-infrared (near-IR) albedo are small ( $< \pm 0.005$ ) for  
24 solar zenith angles  $\theta < 75^\circ$ , and increase as  $\theta$  increases. For diffuse incidence under  
25 cloudy skies, dEdd-AD produces the most accurate snow albedo for both visible and  
26 near-IR ( $< \pm 0.0002$ ) with the lowest underestimate (-0.01) for melting thin snow.  
27 SNICAR performs similarly to dEdd-AD for visible albedos, with a slightly larger  
28 underestimate (-0.02), while it overestimates the near-IR albedo by an order of magnitude  
29 more (up to 0.04). 2SD overestimates both visible and near-IR albedo by up to 0.03. We  
30 develop a new parameterization that adjusts the underestimated direct near-IR albedo and  
31 overestimated direct near-IR heating persistent across all two-stream models for solar  
32 zenith angles  $> 75^\circ$ . These results are incorporated in a hybrid model SNICAR-AD,  
33 which can now serve as a unified solar radiative transfer model for snow in ESM land,  
34 land ice, and sea-ice components.

Author  
Deleted: Icepack

Author  
Deleted: Icepack

## 1. Introduction

Snow cover on land, land ice, and sea ice, modulates the surface energy balance of [mid and high latitudes](#) of the Earth, principally because even a thin layer of snow [can](#) greatly increase the surface albedo. Integrated over the solar spectrum, the broadband albedo of opaque snow ranges from 0.7 – 0.9 (e.g., Wiscombe and Warren 1980; Dang et al., 2015). In contrast, the albedo of other natural surfaces is smaller: 0.2, 0.25, and 0.5-0.7 for damp soil, grassland, and bare multi-year sea ice, respectively (Perovich 1996; Liang et al., 2002; Brandt et al., 2005; Bøggild et al., 2010). [The accumulation, evolution, and depletion of snow cover modify the seasonal cycle of surface albedo globally. In particular, snow over sea ice absorbs more solar energy and begins to melt in the spring, which forms melt ponds that bring the sea-ice albedo to as low as 0.15 to further accelerate ice melt \(Light et al., 2008, 2015\).](#) An accurate simulation of the shortwave radiative properties of snowpack is therefore crucial for spectrally partitioning solar energy and representing snow-albedo feedbacks across the Earth system. Unfortunately, computational demands and coupling architectures often constrain representation of snowpack radiative processes in Earth System Models (ESMs, [please refer to Table 1 for all acronyms used in this work](#)) to relatively crude approximations such as two-stream methods (Wiscombe and Warren, 1980, Toon et al., 1989). In this work, we intercompare two-stream methods widely used in snow models and then introduce a new parameterization that significantly reduces their snowpack reflectance and heating biases at large zenith angles, to produce more realistic behavior in polar regions.

Snow albedo is determined by many factors including the snow grain radius, the solar zenith angle, cloud transmittance, light-absorbing particles, and the albedo of underlying ground if snow is optically thin (Wiscombe and Warren, 1980; Warren and Wiscombe, 1980); it also varies strongly with wavelength since the ice absorption coefficient varies by 7 orders of magnitudes across the solar spectrum (Warren and Brandt, 2008). At visible wavelengths (0.2 - 0.7  $\mu\text{m}$ ), ice is almost non-absorptive [such](#) that the absorption of visible energy by snowpack is mostly due to the light-absorbing particles (e.g. black carbon, organic carbon, mineral dust) that were incorporated during ice nucleation in clouds, scavenged during precipitation, or slowly sedimented from the atmosphere by gravity (Warren and Wiscombe, 1980, 1985; Doherty et al., 2010, 2014, 2016; Wang et al., 2013; Dang and Hegg 2014). As snow becomes shallower, visible photons are more likely to penetrate through snowpack and get absorbed by darker underlying ground. At near-infrared (near-IR) wavelengths (0.7 – 5  $\mu\text{m}$ ), ice is much more absorptive [that](#) the

Author

Deleted: large parts

Author

Deleted: and

76 snow albedo is lower than the visible albedo. Larger ice crystals form a lower albedo  
77 surface than smaller ice crystals hence aged snowpacks absorb more solar energy.  
78 Photons incident at smaller solar zenith angles are more likely to penetrate deeper  
79 vertically and be scattered in the snowpack until being absorbed by the ice/the underlying  
80 ground/absorbing impurities, which also leads to a smaller snow albedo. To compute the  
81 reflected solar flux, spectrally resolved albedo must be weighted by the incident solar  
82 flux, which is mostly determined by solar zenith angle, cloud cover and transmittance,  
83 and column water vapor. Modeling the solar properties of snowpacks must consider the  
84 spectral signatures of these atmospheric properties.

85

86 Several parameterizations have been developed to compute the snow solar properties  
87 without solving the radiative transfer equations and some are incorporated into ESMs or  
88 regional models. Marshall and Warren (1987) and Marshall (1989) parameterized snow  
89 albedo in both visible and near-IR bands as functions of snow grain size, solar zenith  
90 angle, cloud transmittance, snow depth, underlying surface albedo, and black carbon  
91 content. Marshall and Oglesby (1994) used this in an ESM. Gardner and Sharp (2010)  
92 parameterized the all-wave snow albedo with similar inputs. This was incorporated into  
93 the regional climate model RACMO

94 (<https://www.projects.science.uu.nl/iceclimate/models/racmo.php>) to simulate snow  
95 albedo in glaciated regions like Antarctica and Greenland (Munneke et al., 2011). Dang  
96 et al., (2015) compute snow albedo as functions of snow grain radius, black carbon  
97 content, and dust content for visible and near-IR bands and 14 narrower bands used in the  
98 rapid radiative transfer model (RRTM, Mlawer and Clough, 1997). Their  
99 parameterization can also be expanded to different solar zenith angles using the zenith  
100 angle parameterization developed by Marshall and Warren (1987). Aoki et al., (2011)  
101 developed a more complex model based on the offline snow albedo and a transmittance  
102 look-up table. This can be applied to multilayer snowpack to compute the snow albedo  
103 and the solar heating profiles as functions of snow grain size, black carbon and dust  
104 content, snow temperature, and snowmelt water equivalent. These parameterizations are  
105 often in the form of simplified polynomial equations, which are especially suitable to  
106 long-term ESM simulations that require less time-consuming snow representations.

107

108 More complex models that explicitly solve the multiple scattering radiative transfer  
109 equations have also been developed to compute snow solar properties. Flanner and  
110 Zender (2005) developed the SNow Ice and Aerosol Radiation model (SNICAR) that  
111 utilizes two-stream approximations (Wiscombe and Warren 1980; Toon et al., 1989) to

Author  
Deleted: and

113 predict heating and reflectance for multi-layer snowpack. They implemented SNICAR in  
 114 the Community Land Model (CLM) to predict snow albedo and vertically-resolved solar  
 115 absorption for snow-covered surfaces. Before SNICAR, CLM prescribed snow albedo  
 116 and confined all solar absorption to the top snow layer (Flanner and Zender 2005). Over  
 117 the past decades, updates and new features have been added to SNICAR to consider more  
 118 processes such as black carbon/ice mixing states (Flanner et al., 2012) and snow grain  
 119 shape (He et al., 2018b). Concurrent with the development of SNICAR, Briegleb and  
 120 Light (2007) improved the treatment of sea-ice solar radiative calculations in Community  
 121 Climate System Model (CCSM). They implemented a [different two-stream scheme with  
 122 delta-Eddington approximation and adding-doubling technique \(hereafter, dEdd-AD\) that](#)  
 123 [allows CCSM to compute bare/ponded/snow-covered sea ice albedo and solar absorption](#)  
 124 [profiles of multi-layer sea ice. Before these improvements, the sea-ice albedo was](#)  
 125 [computed based on surface temperature, snow thickness, and sea-ice thickness using](#)  
 126 [averaged sea ice and snow albedo, dEdd-AD has been adopted by](#) the sea-ice physics  
 127 library Icepack (<https://github.com/CICE-Consortium/Icepack/wiki>), [which is](#) used by the  
 128 Los Almos Sea Ice Model CICE (Hunke et al., 2010) and [Model for Prediction Across](#)  
 129 [Scales Sea Ice](#) MPAS-seaice (Turner et al., 2018). CICE itself is used in numerous global  
 130 and regional models.

131  
 132 [SNICAR and dEdd-AD](#) solve the multiple scattering radiative transfer equations and  
 133 provide much improved solar radiative representations for the cryosphere, though their  
 134 separate development and implementation created an artificial divide for snow  
 135 simulation. In ESMs that utilize both SNICAR and [dEdd-AD](#), such as the Community  
 136 Earth System Model (CESM, <http://www.cesm.ucar.edu/>) and the Energy Exscale Earth  
 137 System Model (E3SM, previously known as ACME, <https://e3sm.org/>), the solar  
 138 radiative properties of snow on land and snow on sea ice are computed separately via  
 139 SNICAR and [dEdd-AD](#). As a result, the same snow in nature has different solar radiative  
 140 properties such as reflectance depending on which model represents it. These differences  
 141 are model artifacts that should be eliminated so that snow has consistent properties across  
 142 the Earth system.

143  
 144 In this paper, we evaluate the accuracy and biases of three two-stream [models listed in](#)  
 145 [Table 2](#), including the algorithms used in SNICAR and [dEdd-AD](#), at representing  
 146 reflectance and heating. [In Sections 2-4, we describe the radiative transfer algorithms and](#)  
 147 [calculations performed in this work. The results and model inter-comparisons are](#)  
 148 [discussed in Section 5. In Section 6, we introduce a parameterization to reduce the](#)

Author  
 Deleted: two-stream delta-Eddington method

Author  
 Deleted: that

Author  
 Deleted: This method has carried into

Author  
 Deleted:

Author  
 Deleted: that

Author  
 Deleted: comprises the column physics

Author  
 Deleted: .

Author  
 Deleted: The shortwave methods in SNICAR and in CICE

Author  
 Deleted: CICE/MPAS-seaice

Author  
 Deleted: CICE/MPAS-seaice

Author  
 Deleted: algorithms

Author  
 Deleted: described

Author  
 Deleted: in Section 2 and

Author  
 Deleted: 1

Author  
 Deleted: Icepack



165 | [simulated albedo and heating bias for solar zenith angles larger than 75°](#). In Section 7, we  
166 | [summarize the major differences of algorithm implementations between SNICAR and](#)  
167 | [dEdd-AD in ESMs](#). We use these results to develop and justify a unified surface  
168 | shortwave radiative transfer method for all Earth system model components in the  
169 | cryosphere, [presented in Section 8](#).

170

## 171 | **2. Radiative Transfer Model**

172

173 | In this section, we summarize the three two-stream models and the benchmark DISORT  
174 | model with 16-streams. These algorithms are well documented in papers by Toon et al.,  
175 | (1989), Briegleb and Light (2007), Jin and Stamnes (1994), and Stamnes et al. (1988).  
176 | Readers interested in detailed mathematical derivations should refer to those papers. We  
177 | only include their key equations to illustrate the difference among two-stream models for  
178 | discussion purposes.

179

### 180 | 2.1 SNICAR [in land models CLM and ELM](#)

181 | [SNICAR is implemented as the default snow shortwave radiative transfer scheme in](#)  
182 | [CLM and E3SM land model \(ELM\)](#). It adopts the two-stream algorithms and the rapid  
183 | solver developed by Toon et al., (1989) to compute the solar properties of multi-layer  
184 | snowpacks. These two-stream algorithms are derived from the general equation of  
185 | radiative transfer in a plane parallel media:

186

$$187 \quad \mu \frac{\partial I}{\partial \tau}(\tau, \mu, \Phi) = I(\tau, \mu, \Phi) - \frac{\varpi}{4\pi} \int_0^{2\pi} \int_{-1}^1 P(\mu, \mu', \phi, \phi') I(\tau, \mu', \Phi') d\mu' d\phi' - S(\tau, \mu, \Phi)$$

188

189

190

191

192

193

194

195

196

(1)

192 | where  $\arccos(\mu)$  and  $\Phi$  are zenith angle and azimuth angle,  $\varpi$  is single-scattering albedo.  
193 | On the right-hand side, the three terms are intensity at optical depth  $\tau$ , internal source  
194 | term due to multiple scattering, and external source term  $S$ . For a purely external source  
195 | at solar wavelengths  $S$  is:

196

$$197 \quad S = \frac{\varpi}{4} F_s P(\mu, -\mu_0, \phi, \phi_0) \exp\left(\frac{-\tau}{\mu_0}\right)$$

198

199

200

(2)

199 | where  $\pi F_s$  is incident solar flux,  $\mu_0$  is the incident direction of the solar beam. Integrating  
200 | equation (1) over azimuth and zenith angles yields the general solution of two-stream

Author

Deleted: .

202 approximations (Meador and Weaver, 1980). The upward and downward fluxes at optical  
203 depth  $\tau$  of layer  $n$  can be represented as:

204

205

$$206 \quad F_n^+ = k_{1n} \exp(\Lambda_n \tau) + \Gamma_n k_{2n} \exp(-\Lambda_n \tau) + C_n^+(\tau) \quad (3a)$$

207

$$208 \quad F_n^- = \Gamma_n k_{1n} \exp(\Lambda_n \tau) + k_{2n} \exp(-\Lambda_n \tau) + C_n^-(\tau) \quad (3b)$$

209

210

211 where  $\Lambda_n$ ,  $\Gamma_n$ ,  $C_n$  are known coefficients determined by the two-stream method, incident  
212 solar flux, and solar zenith angle; whereas  $k_{1n}$  and  $k_{2n}$  are unknown coefficients  
213 determined by the boundary conditions. For an N-layer snowpack, the solutions for  
214 upward and downward fluxes are coupled at layer interfaces to generate 2N equations  
215 with 2N unknown coefficients  $k_{1n}$  and  $k_{2n}$ . Combining these equations linearly generates  
216 a new set of equations with terms in tridiagonal form that enables the application of a fast  
217 tri-diagonal matrix solver. With the solved coefficients, the upward and downward fluxes  
218 are computed at different optical depths (Equations 3a and 3b) and eventually the  
219 reflectance, transmittance, and absorption profiles of solar flux for any multilayer  
220 snowpack.

221

222 SNICAR itself implements all three two-stream algorithms in Toon et al., (1989):  
223 Eddington, Quadrature, and Hemispheric-mean. In [practical simulations](#), it utilizes the  
224 Eddington and Hemispheric-mean approximations to compute the visible and near-IR  
225 snow properties, respectively (Flanner et al., 2007). In addition to their algorithms,  
226 SNICAR implements [the Delta-transform](#) of the fundamental input variables asymmetry  
227 factor ( $g$ ), single-scattering albedo ( $\omega$ ), and optical depth ( $\tau$ ) to account for the strong  
228 forward scattering in snow (Equations 2 (a)-(c), Wiscombe and Warren, 1980).

229

230 2.2. [dEdd-AD in sea ice models](#) Icepack, CICE, and MPAS-seaice

231 Icepack, CICE, and MPAS-seaice use the same [shortwave radiative scheme dEdd-AD](#)  
232 developed and documented by Briegleb and Light (2007). Sea ice is divided into multiple  
233 layers to first compute the single-layer reflectance and transmittance using two-stream  
234 delta-Eddington solutions to account for the multiple scattering of light within each layer  
235 (Equation set 50, Briegleb and Light, 2007), [where the name “delta” implies dEdd-AD](#),  
236 implements the Delta-transform to account for the strong forward scattering of snow and

Author
Deleted: solar
Author
Deleted: treatment
Author
Deleted: In the following discussions, we will refer to this method as CICE since it is more widely used.
Author
Deleted: ), where
Author
Deleted: the
Author
Deleted: CICE

245 | sea ice (Equations 2 (a)-(c), Wiscombe and Warren, 1980). The [single-layer](#) direct albedo  
246 and transmittance are computed by equations:

247

$$248 \quad R(\mu_{0,n}) = A_n \exp\left(\frac{-\tau}{\mu_{0,n}}\right) + B_n(\exp(\varepsilon_n \tau) - \exp(-\varepsilon_n \tau)) - K_n \quad (4a)$$

249

$$250 \quad T(\mu_{0,n}) = E_n + H_n(\exp(\varepsilon_n \tau) - \exp(-\varepsilon_n \tau)) \exp\left(\frac{-\tau}{\mu_{0,n}}\right) \quad (4b)$$

251

252 where coefficients  $A_n$ ,  $B_n$ ,  $K_n$ ,  $E_n$ ,  $H_n$ , and  $\varepsilon_n$  are determined by the single-scattering

253 | albedo ( $\varpi$ ), asymmetry factor ( $g$ ), optical depth ( $\tau$ ), and angle of [the](#) incident beam at

254 layer  $n$  ( $\mu_{0,n}$ ). Following the delta-Eddington assumption, simple formulas are available

255 for the single-layer reflectance and transmittance under both clear sky (direct flux,

256 equations 4a and 4b) and overcast sky (diffuse flux) conditions, however, the formula

257 derived by applying diffuse-flux upper boundary conditions sometimes yields negative

258 albedos (Wiscombe 1977). To avoid the unphysical values, diffuse reflectance  $\bar{R}$  and

259 transmittance  $\bar{T}$  of a single layer are computed by integrating the direct reflectance  $R(\mu)$

260 and transmittance  $T(\mu)$  over the incident hemisphere assuming isotropic incidence:

261

$$262 \quad \bar{R} = 2 \int_0^1 \mu R(\mu) d\mu \quad (5a)$$

263

$$264 \quad \bar{T} = 2 \int_0^1 \mu T(\mu) d\mu \quad (5b)$$

265

266 This is the same as the method proposed by Wiscombe and Warren (1980, their equation

267 5). In practice, eight Gaussian angles are implemented to perform the integration for

268 every layer.

269

270 | The [computed](#), [single-layer](#) reflectance and transmittance of direct and diffuse

271 components are then combined to account for the inter-layer scattering of light to

272 compute the reflectance and transmission at every interface (Equation set 51, Briegleb

273 and Light, 2007), and eventually the upward and downward fluxes (Equation set 52,

274 Briegleb and Light, 2007). These upward and downward fluxes at each optical depth are

275 then used to compute the column reflectance and transmittance, and the absorption

276 profiles for any multilayered media, such as snowpacks on land and sea ice.

277

278 In nature, a large fraction of sea ice is covered by snow during winter. As snow melts

279 away in late spring and summer, it exposes bare ice, and melt ponds form on the ice

Author

Deleted: se

281 surface. Such variation of sea-ice surface types requires the shortwave radiative transfer  
 282 model to be flexible and capable of capturing the light refraction and reflection.  
 283 Refractive boundaries exist where air (refractive index  $m_{re} = 1.0$ ), snow (assuming snow  
 284 as medium of air containing a collection of ice particles,  $m_{re} = 1.0$ ), pond (assuming pure  
 285 water,  $m_{re} = 1.33$ ), and ice (assuming pure ice,  $m_{re} = 1.31$ ) are present in the same sea-ice  
 286 column. The general solution of delta-Eddington, and the two-stream algorithms used in  
 287 SNICAR are not applicable to such non-uniformly refractive layered media. To include  
 288 the effects of refraction, Briegleb and Light (2007) modified the adding formula at the  
 289 refractive boundaries (i.e. interfaces between air/ice, snow/ice, air/pond). The reflectance  
 290 and transmittance of the adjacent layers above and below the refractive boundary are  
 291 combined with modifications to include the Fresnel reflection and refraction of direct and  
 292 diffuse fluxes (Section 4.1, Briegleb and Light, 2007). [dEdd-AD](#) can thus be applied to  
 293 any layered media with either uniform (e.g., snow on land) or non-uniform (e.g., snow on  
 294 sea ice) refractive indexes.

296 In this paper, we [apply dEdd-AD to](#) snowpacks that can be treated as uniform refractive  
 297 media such as the air/snowpack/land columns assumed in SNICAR [for model evaluation](#).  
 298 An ideal radiative treatment for snow should, however, keep the potential to include  
 299 refraction for further applications to snow on sea ice or ice sheets. Therefore, besides  
 300 these two widely used algorithms in Icepak and SNICAR, we evaluate a third algorithm  
 301 (section 2.3) that can be applied to layered media with either uniform or non-uniform  
 302 refractive indexes.

### 304 2.3. two-stream discrete-ordinate algorithm (2SD)

305 A refractive boundary also exists between the atmosphere and the ocean, and models  
 306 have been developed to solve the radiative transfer problems in the atmosphere-ocean  
 307 system using the discrete-ordinate technique (e.g. Jin and Stamnes, 1994; Lee and Liou,  
 308 2007). Similar to the two-stream algorithms of Toon et al., (1989) used in SNICAR, Jin  
 309 and Stamnes (1994) also developed their algorithm from the general equation:

$$311 \mu \frac{\partial I}{\partial \tau}(\tau, \mu) = I(\tau, \mu) - \frac{\omega}{4\pi} \int_{-1}^1 P(\tau, \mu, \mu') I(\tau, \mu') d\mu' - S(\tau, \mu) \quad (6)$$

313 Equation (6) is the azimuthally integrated version of equation (1). However, for vertically  
 314 inhomogeneous media like the atmosphere-ocean or sea ice, the external source term  
 315  $S(\tau, \mu)$  is different. Specifically, for the medium of total optical depth  $\tau^a$  above the

Author  
 Deleted: This adding-doubling delta-Eddington method

Author  
 Deleted: .

Author  
 Deleted: focus on

320 refractive interface, one must consider the contribution from the upward beam reflected  
 321 at the refractive boundary (second term on the right-hand side):

322

$$323 \quad S^a(\tau, \mu) = \frac{\omega}{4\pi} F_s P(\tau, -\mu_0, \mu) \exp\left(\frac{-\tau}{\mu_0}\right) + \frac{\omega}{4\pi} F_s R(-\mu_0, m) P(\tau, +\mu_0, \mu) \exp\left(\frac{-(2\tau^a - \tau)}{\mu_0}\right)$$

324

325 (7)

326

327 where  $R(-\mu_0, m)$  is the Fresnel reflectance of radiation and  $m$  is the ratio of the  
 328 refractive indices of the lower to the upper medium. For the medium below the refractive  
 329 interface, one must account for the Fresnel transmittance  $T(-\mu_0, m)$  and modify the  
 330 angle of beam travel in media b:

331

$$332 \quad S^b(\tau, \mu) = \frac{\omega}{4\pi} \frac{\mu_0}{\mu_{0n}} F_s T(-\mu_0, m) P(\tau, -\mu_0, \mu) \exp\left(\frac{-\tau^a}{\mu_0}\right) \exp\left(\frac{-(\tau - \tau^a)}{\mu_{0n}}\right)$$

333 (8)

334 | where  $\mu_{0n}$  is the cosine zenith angle of refracted beam incident at angle  $\mu_0$  above [the](#)  
 335 refractive boundary, by Snell's law:

336

$$337 \quad \mu_{0n} = \sqrt{1 - (1 - \mu_0^2)/m^2}$$

338 (9)

339 For uniformly refractive media like snow on land, one can just set the refractive index  $m_{re}$   
 340 equal to 1 for every layer. In this case, the Fresnel reflectance  $R(-\mu_0, m)$  is 0 in equation  
 341 (7), the Fresnel transmittance  $T(-\mu_0, m)$  is 1 in equation (8), and  $\mu_{0n}$  equals to  $\mu_0$ : the  
 342 two source terms  $S^a(\tau, \mu)$  and  $S^b(\tau, \mu)$  become the same and equal to the source term of  
 343 homogenous media given in equation (2).

344

345 For two-stream approximations of this method, analytical solutions of upward and  
 346 downward fluxes are coupled at each layer interface to generate 2N equations with 2N  
 347 unknown coefficients for any N-layer stratified column. The solutions of two-stream  
 348 algorithms and boundary conditions for homogenous media are well documented  
 349 (Sections 8.4 and 8.10 of Thomas and Stamnes, 1999). Despite the extra source terms,  
 350 these 2N equations can also be organized into a tridiagonal matrix similar to the method  
 351 of Toon et al. (1989) used in SNICAR. Flexibility and speed therefore make this two-  
 352 stream discrete-ordinate algorithm (hereafter, 2SD) a potentially good candidate for long-

353 term Earth system modeling. In this work, we only apply 2SD to snowpack and note that  
354 it can be applied to any uniformly or non-uniformly refractive media like snow on land or  
355 sea ice, with the Delta-transform implemented to fundamental optical variables  
356 (Equations 2 (a)-(c), Wiscombe and Warren, 1980).

357

#### 358 2.4 16-stream DISORT

359 Besides the mathematical technique, the accuracy and speed of radiative transfer  
360 algorithms depend on the number of angles used for flux estimation in the upward and  
361 downward hemispheres. SNICAR, dEdd-AD, and 2SD use one angle to represent upward  
362 flux and one angle to represent downward flux, hence they are named two-stream  
363 algorithm. Lee and Liou (2007) use two upward and two downward streams. Jin and  
364 Stamnes (1994) documented the solutions for any even number of streams. The  
365 computational efficiency of these models is lower than that of two-stream models while  
366 their accuracy is better. To quantify the accuracy of the three two-stream algorithms for  
367 snow shortwave simulations, we use the 16-stream DIScrete-Oordinate Radiative Transfer  
368 model (DISORT) as the benchmark model (<http://llab.phy.stevens.edu/disort/>) (Stamnes  
369 et al., 1988).

370

### 371 3. Input for radiative transfer models

372 In this work, we focus on the performance of two-stream algorithms for pure snow  
373 simulations. The inputs for these three models are the same: single-scattering properties  
374 (SSPs, i.e. single-scattering albedo  $\omega$ , asymmetry factor  $g$ , extinction coefficient  $\sigma_{ext}$ ) of  
375 snow determined by snow grain radius  $r$ , snow depth, solar zenith angle  $\theta$ , solar incident  
376 flux, and the albedo of underlying ground (assuming Lambertian reflectance of 0.25 for  
377 all wavelengths). A Delta-transform is applied to fundamental input optical variables for  
378 all simulations (Equations 2 (a)-(c), Wiscombe and Warren, 1980).

379

380 In snow, photon scattering occurs at the air-ice interface, and the absorption of photons  
381 occurs within the ice crystal. The most important factor that determines snow shortwave  
382 properties is the ratio of total surface area to total mass of snow grains, aka “the specific  
383 surface area” (e.g. Matzl and Schneebeli, 2006, 2010). The specific surface area ( $\beta$ ) can  
384 be converted to a radiatively effective snow grain radius  $r$ :

385

$$386 \beta = 3 / (r \rho_s) \quad (10)$$

387

Author

Deleted: The algorithms used in

Author

Deleted: Icepack

Author

Deleted: speed

Author

Deleted: slower

392 where  $\rho_{ice}$  is the density of pure ice,  $917 \text{ kg m}^{-3}$ . Assuming the grains are spherical, the  
 393 SSPs of snow can thus be computed using Mie theory (Wiscombe, 1980) and ice optical  
 394 constants (Warren and Brandt, 2008). In nature, snow grains are not spherical, and many  
 395 studies have been carried out to quantify the accuracy of such spherical representations  
 396 (Grenfell and Warren, 1999; Neshyba et al., 2003; Grenfell et al., 2005). In recent years,  
 397 more research has been done to evaluate the impact of grain shape on snow shortwave  
 398 properties (Dang et al., 2016; He et al., 2017, 2018ab), and they show that non-spherical  
 399 snow grain shapes mainly alter the asymmetry factor. Dang et al., (2016) also point out  
 400 that the solar properties of a snowpack consisting of non-spherical ice grains can be  
 401 mimicked by a snowpack consisting of spherical grains with a smaller grain size by  
 402 factors up to 2.4. In this work, we still assume the snow grains are spherical, and this  
 403 assumption does not qualitatively alter our evaluation of the radiative transfer algorithms.  
 404

405 | The input SSPs of snow grains are computed using Mie theory at a fine spectral  
 406 resolution for a wide range of ice effective radius  $r$  from 10 to  $3000 \mu\text{m}$  that covers the  
 407 possible range of grain radius for snow on Earth (Flanner et al., 2007). The same spectral  
 408 SSPs were also used to derive the band-averaged SSPs of snow used in SNICAR. Note  
 409 Briegleb and Light (2007) refer to SSPs as inherent optical properties.  
 410

#### 411 **4. Solar spectra used for the spectral integrations**

412 | In climate modeling, snow albedo computation at a fine spectral resolution is expensive  
 413 and unnecessary. Instead of computing spectrally resolved snow albedo as shown in  
 414 | [Figure 1](#), wider-band solar properties are more practical. For example, CESM and E3SM  
 415 aggregate the narrow RRTMG bands used for the atmospheric radiative transfer  
 416 simulation into visible ( $0.2 - 0.7 \mu\text{m}$ ) and near-IR ( $0.7 - 5 \mu\text{m}$ ) bands. The land model and  
 417 sea-ice model thus receive visible and near-IR fluxes as the upper boundary condition,  
 418 and return the corresponding visible and near-IR albedos to atmosphere model. In  
 419 practice, these bands are also partitioned into direct and diffuse components. Therefore, a  
 420 practical two-stream algorithm should be able to simulate the direct visible, diffuse  
 421 visible, direct near-IR and diffuse near-IR albedos and absorptions of snow accurately.  
 422

423 The band albedo  $\alpha$  is an irradiance-weighted average of the spectral albedo  $\alpha(\lambda)$ :

$$424 \alpha = \frac{\int_{\lambda_1}^{\lambda_2} \alpha(\lambda) F(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} F(\lambda) d\lambda} \quad (11)$$

426  
 427

428 In this work, we use the spectral irradiance  $F(\lambda)$  generated by the atmospheric DISORT-  
429 based Shortwave Narrowband Model (SWNB2) (Zender et al., 1997; Zender, 1999) for  
430 typical clear-sky and cloudy-sky conditions of mid-latitude winter as shown in [Figure](#)  
431 [2\(a\)](#). The total clear-sky down-welling surface flux at different solar zenith angles are  
432 also given in [Figure 2\(b\)](#).

433  
434

## 435 5. Model Evaluation

### 436 5.1 Spectral albedo and reflected solar flux

437 The spectral reflectance of pure deep snow computed using two-stream models and 16-  
438 stream DISORT are shown in [Figure 1](#). The snow grain radius is  $100\ \mu\text{m}$  - a typical grain  
439 size for fresh new snow. For clear sky with direct beam source (left column), all three  
440 two-stream models show good accuracy at visible wavelengths ( $0.3 - 0.7\ \mu\text{m}$ ), and within  
441 this band, the snow albedo is large and close to 1. As wavelength increases, the albedo  
442 diminishes in the near-IR band. two-stream models overestimate snow albedo at these  
443 wavelengths, with maximum biases of 0.013 (SNICAR and [dEdd-AD](#)) and 0.023 (2SD)  
444 within wavelength  $1 - 1.7\ \mu\text{m}$ . For cloudy-sky cases with diffuse upper boundary  
445 conditions, [dEdd-AD](#) reproduces the snow albedo at all wavelengths with the smallest  
446 absolute error ( $< 0.005$ ), SNICAR and 2SD both overestimate the snow albedo with  
447 maximum biases  $> 0.04$  between  $1.1-1.4\ \mu\text{m}$ .

448

449 In both sky conditions, the errors of snow albedo are larger at near-IR wavelengths  
450 ranging from  $1.0-1.7\ \mu\text{m}$ , while the solar incident flux peaks at  $0.5\ \mu\text{m}$  then decrease as  
451 wavelength increases. The largest error in reflected flux is within the  $0.7-1.5\ \mu\text{m}$  band for  
452 SNICAR and 2SD, as shown in the 3<sup>rd</sup> row of [Figure 1](#). [dEdd-AD](#) overestimate the direct  
453 snow albedo mostly at wavelengths larger than  $1.5\ \mu\text{m}$  where the error in reflected flux is  
454 almost negligible.

455

### 456 5.2 Broadband albedo and reflected solar flux

457 Integrated over the visible and near-IR wavelengths, the error in band albedos computed  
458 using two-stream models for different cases are shown in [Figure 3-6](#).

459

460 [Figure 3](#) shows the error in direct band albedo for fixed snow grain radius of  $100\ \mu\text{m}$  with  
461 different snow depth and solar zenith angles. As introduced in [Section 2](#), SNICAR and  
462 [dEdd-AD](#) both use delta-Eddington method to compute the visible albedo. They  
463 overestimate the visible albedo for solar zenith angles smaller than  $50^\circ$  by up to 0.005,

Author  
Deleted: Figure 2

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: s

Author  
Deleted: Figure 2

Author  
Deleted: CICE

Author  
Deleted: CICE



471 and underestimate it for solar zenith angles larger than 50° by up to -0.01. 2SD produces  
472 similar results for the visible band but at a larger solar zenith angle threshold of 75°. In  
473 the near-IR band, SNICAR and 2SD overestimate the snow albedo for solar zenith angles  
474 smaller than 70°, beyond this, the error in albedo increases by up to -0.1 as solar zenith  
475 angle increases. [dEdd-AD](#) produces a similar error pattern with a smaller solar zenith  
476 angle threshold at 60°. As snow ages, its average grain size increases. For typical old  
477 melting snow of grain radius 1000 μm (Figure 4), two-stream models produce similar  
478 errors of direct albedo in all bands. For snow consisting of smaller grain size, two-stream  
479 models produce larger errors for visible albedo. Integrating over the entire solar band, the  
480 three two-stream models evaluated show similar error patterns for direct albedo.

481

482 For a fixed solar zenith angle of 60°, the error of direct albedo for different snow depth  
483 and snow grain radii are shown in Figure 5. SNICAR and [dEdd-AD](#) underestimate the  
484 visible albedo in most scenarios, while 2SD overestimates the visible albedo for a larger  
485 range of grain radius and snow depth. All three two-stream models tend to overestimate  
486 the near-IR albedo except for shallow snow with large grain radius; the error of 2SD is  
487 one order of magnitude larger than that of SNICAR and [dEdd-AD](#).

488

489 Figure 6 is similar to Figure 5, but shows the diffuse snow albedo. In the visible band,  
490 SNICAR and [dEdd-AD](#) generate similar errors in that they both underestimate the albedo  
491 as snow grain size increases and snow depth decreases. 2SD overestimates the albedo  
492 with a maximum error of around 0.015. In the near-IR, two-stream models tend to  
493 overestimate snow albedo, while the magnitude of biases produced by SNICAR and 2SD  
494 are one order larger than that of [dEdd-AD](#) with the maximum error of 0.035 generated by  
495 SNICAR. As a result, the all-wave diffuse albedos computed using [dEdd-AD](#) are more  
496 accurate than those computed using SNICAR and 2SD.

497

498 Figures 7, 8 and 9 show the errors in reflected shortwave flux caused by snow albedo  
499 errors seen in Figures 3, 4, and 6. In general, two-stream models produce larger errors in  
500 reflected direct near-IR flux (Figure 7 and 8), especially with the 2SD model: the  
501 maximum overestimate of reflected near-IR flux is 6-8 Wm<sup>-2</sup> for deep melting snow with  
502 solar zenith angle < 30°. Errors in reflected direct visible flux are smaller (mostly within  
503 ±1 Wm<sup>-2</sup>) for all models in most scenarios, and become larger (mostly within ±3 Wm<sup>-2</sup>) as  
504 snow grain size increases to 1000 μm if computed using 2SD. As shown in Figure 9, for  
505 diffuse flux with solar zenith angle of 60° at TOA, SNICAR and [dEdd-AD](#) generate

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

513 small errors in reflected visible flux (mostly within  $\pm 1 \text{ Wm}^{-2}$ ), while 2SD always  
514 overestimates reflected visible flux by up to  $5 \text{ Wm}^{-2}$ . In the near-IR, SNICAR and 2SD  
515 overestimate reflected flux by as much as  $10\text{-}12 \text{ Wm}^{-2}$ ; the error in reflected near-IR flux  
516 produced by [dEdd-AD](#) is much smaller, mostly within  $\pm 1 \text{ Wm}^{-2}$ .

517  
518 In general, [dEdd-AD](#) produces the most accurate albedo and thus reflected flux for both  
519 direct and diffuse components. SNICAR is similar to [dEdd-AD](#) for its accuracy of direct  
520 albedo and flux, yet generates large error for [the](#) diffuse component. 2SD tends to  
521 overestimate snow albedo and reflected flux in both direct and diffuse components and  
522 shows the largest errors among three two-stream models. [These relatively small  
523 differences between algorithms may still yield large impact on snowpack. For example,  
524 compared to dEdd-AD, SNICAR and 2SD overestimate the diffuse albedo by  \$\sim 0.015\$  for  
525 melting snow \(Figure 6\). In Greenland, the daily averaged downward diffuse solar flux  
526 from May to September is  \$200 \text{ W/m}^2\$ , and the averaged cloud cover fraction is 80%  
527 \(Figure 6, Dang et al., 2017\). In this case, SNICAR and 2SD overestimate the reflected  
528 solar flux by  \$2.4 \text{ W/m}^2\$  per day – the amount of energy otherwise enough to melt 10 cm of  
529 snow water equivalent from May to September. \[dEdd-AD also remediates self-  
530 compensating spectral biases \\(where visible and Near-IR biases are of opposite signs\\)  
531 present in the other schemes. Those spectral biases do not affect the broadband fluxes  
532 like the diffuse biases, but they nevertheless degrade proper feedbacks between snow/ice  
533 reflectance and heating.\]\(#\)”](#)

534

### 535 5.3 Band absorption of solar flux

536 Figure 10 shows absorption profiles of shortwave flux computed using the 16-stream  
537 DISORT model, with errors in absorbed fractional solar flux computed using two-stream  
538 models. The snowpack is 10-cm deep and is divided into 5 layers, each 2-cm thick. The  
539 snow grain radius is set to  $100 \mu\text{m}$ . The figure shows fractional absorption for snow  
540 layers 1-4 and the underlying ground with [an](#) albedo of 0.25.

541

542 As shown in the first column of Figure 10, for new snow with [a](#) radius of  $100 \mu\text{m}$ , most  
543 solar absorption occurs in the top 2-cm snow layer, where roughly 10% and 15% of  
544 diffuse and direct near-IR flux are absorbed and dominate the solar absorption within [the](#)  
545 snowpack. In the second layer (2-4 cm), the absorption of solar flux is less than 1% and  
546 gradually decreases within the interior layers. The underlying ground absorbs roughly 2%  
547 of solar flux, mostly visible flux that penetrates the snowpack more efficiently. As snow

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: Note that the final errors of snow albedo and reflected solar flux are the weighted sum of direct and diffuse components, and their weights are largely determined by cloud cover fraction (e.g. Figure 6, Dang et al., 2017), which we do not address explicitly in this paper.

Author  
Deleted: ,

Author  
Deleted:

560 ages and snow grain grows, photons penetrate deeper into the snowpack. For typical old  
561 melting snow with a radius of 1000  $\mu\text{m}$ , most solar absorption still occurs in the top 2-cm  
562 snow layer, where roughly 20% and 14% of diffuse and direct near-IR flux are absorbed.  
563 The second snow layer (2-4 cm) absorbs more near-IR solar flux by roughly 2%. More  
564 photons can penetrate through the snowpack, and results in a high fractionally absorption  
565 by the underlying ground, especially for the visible band. As snow depth increase, the  
566 ground absorption will decrease for both snow radii.

567  
568 Comparing to 16-stream DISORT, two-stream models underestimate (overestimate) the  
569 column solar absorptions for new (old) snow, especially for the surface snow layer and  
570 the underground. Overall, dEdd-AD gives the most accurate absorption profiles among  
571 the three two-stream models, especially for new snow.

### 573 6. Correction for direct albedo for large solar zenith angles

574  
575 It has been pointed out in previous studies that the two-stream approximations become  
576 poor as solar zenith angle approaches  $90^\circ$  (e.g. Wiscombe 1977, Warren 1982). As shown  
577 in Figures 3 and 4, all three two-stream models underestimate the direct snow albedo for  
578 large solar zenith angles. In the visible band, when the snow grain size is small, the error  
579 in direct albedo is almost negligible (Figure 3); while as snow ages and snow grains  
580 become larger, the error increases yet remains low if the snow is deep (Figure 4). In the  
581 near-IR, the biases of albedo are also larger for larger snow grain radii. For a given snow  
582 size, the magnitudes of such biases are almost independent of snow depth, and mainly  
583 determined by the solar zenith angle. In general, the errors of all-wave direct albedo are  
584 mostly contributed by the errors of near-IR albedo, especially for optically thick  
585 snowpacks (i.e., semi-infinite), because the errors of direct albedo in the visible are  
586 negligible compared with those in the near-IR. To improve the performance of two-  
587 stream algorithms, we develop a parameterization that corrects the underestimated near-  
588 IR snow albedo at large zenith angles.

589  
590 Figure 11 shows the direct near-IR albedo and fractional absorption of 2-meter thick  
591 snowpacks consisting of grains with radius 100  $\mu\text{m}$  and 1000  $\mu\text{m}$ , computed using two-  
592 stream algorithms and 16-stream DISORT. For solar zenith angles  $> 75^\circ$ , two-stream  
593 models underestimate snow albedo and overestimate solar absorption within snowpack,  
594 mostly in the top 2-cm of snow, and the differences among three two-stream models are  
595 small. In Section 5, we have shown that dEdd-AD produces the most accurate snow

Author  
Deleted: are able to

Author  
Deleted: layer.

Author  
Deleted: CICE

Author  
Deleted: 1997

Author  
Deleted: still

Author  
Deleted: a

602 [albedo in general, with anticipated wide application of dEdd-AD, we develop the](#)  
 603 [following parameterization to adjust its low biases in computed near-IR direct albedo.](#)

604

605 We define and compute  $R_{75+}$  as the ratio of direct semi-infinite near-IR albedo computed  
 606 using 16-stream DISORT ( $\alpha_{16-DISORT}$ ) to that computed using [dEdd-AD](#) ( $\alpha_{dEdd-AD}$ ). This  
 607 ratio is shown in Figure 11 (c) and can be parameterized as a function of snow grain  
 608 radius ( $r$ , unit in meter) and the cosine of incident solar zenith angle ( $\mu_0$ ), as shown in  
 609 Figure 11(c):

610

$$611 \quad R_{75+} = \frac{\alpha_{16-DISORT}}{\alpha_{dEdd-AD}} = c_1(\mu_0) \log_{10}(r) + c_0(\mu_0) \quad (12)$$

612

613 where coefficients  $c_1$  and  $c_0$  are polynomial functions of  $\mu_0$ , as shown in Figure 11(d):

614

$$615 \quad c_1(\mu_0) = 1.304\mu_0^2 - 0.631\mu_0 + 0.086 \quad (13a)$$

$$616 \quad c_0(\mu_0) = 6.807\mu_0^2 - 3.338\mu_0 + 1.467 \quad (13b)$$

617

618 Since two-stream models always underestimate snow albedo,  $R_{75+}$  always exceeds 1  
 619 (Figure 11c). We can then adjust the direct near-IR snow albedo ( $\alpha_{dEdd-AD}$ ) and direct  
 620 near-IR solar absorption ( $F_{abs,dEdd-AD}$ ) by snow computed using [dEdd-AD](#) with ratio  
 621  $R_{75+}$ :

622

$$623 \quad \alpha_{dEdd-AD}^{adjust} = R_{75+} \alpha_{dEdd-AD} \quad (14a)$$

624

$$625 \quad F_{abs,dEdd-AD}^{adjust} = F_{abs,dEdd-AD} - (R_{75+} - 1) \alpha_{dEdd-AD} F_{nir} \quad (14b)$$

626

627 where  $F_{nir}$  is the direct near-IR flux. This adjustment reduces the error of near-IR albedo  
 628 from negative 2-10% to within  $\pm 0.5\%$  for solar zenith angles larger than  $75^\circ$ , and for  
 629 grain radii ranging from 30-1500  $\mu\text{m}$  (Figure 12). Errors in broadband direct albedo are  
 630 therefore also reduced to  $< 0.01$ . The direct near-IR flux absorbed by the snowpack  
 631 decreases after applying this adjustment.

632

633 [When the solar zenith angle exceeds  \$75^\circ\$ , our model adjusts the computed direct near-IR](#)  
 634 [albedo  \$\alpha\_{dEdd-AD}\$  by the ratio  \$R\_{75+}\$ , following equations 12-14a and reduces direct near-](#)

Author  
 Deleted: CICE...Edd-AD ( $\alpha_{CICE}$ ) ... [1]

Author  
 Deleted: CICE

Author  
 Deleted: CICE...Edd-AD) and direct near-I ... [2]

Author  
 ... [3]

Author  
 ... [4]

Author  
 Formatted ... [5]

655 [IR absorption following equation 14b](#). If snow is divided into multiple layers, we assume  
656 all decreased near-IR absorption ( $2^{\text{nd}}$  term on the [right-hand side](#), equation 14b) is  
657 confined within the top layer. This assumption is fairly accurate for the near-IR band,  
658 since most [absorption occurs at the surface of snowpack](#) (Figures 10 and 11). [As](#)  
659 [discussed previously, this parameterization is developed based on albedo computed using](#)  
660 [dEdd-AD. For models that do not use dEdd-AD but SNICAR and 2SD, the same](#)  
661 [adjustment still applies given the small differences of near-IR direct albedo computed](#)  
662 [using two-stream models \(Figure 11\). For models that adopt other radiative transfer](#)  
663 [algorithms it is best for the developers to examine their model against a benchmark](#)  
664 [model such as 16-stream DISORT or two-stream models discussed in this work before](#)  
665 [applying this correction.](#)

667 It is important to note that although the errors of direct near-IR albedos are large for large  
668 solar zenith angles, the absolute error in reflected shortwave flux is small (Figures 7 and  
669 8) as the down-welling solar flux reaches snowpack decreases as solar zenith angle  
670 increases (Figures 1(b)). However, such small biases in flux can be important [to high](#)  
671 [latitudes where the solar zenith angle remains large for many days in late winter and early](#)  
672 [spring.](#)

## 674 7. Implementation of snow radiative transfer model in Earth system models

676 ESMs often use broader band-averaged SSPs of snow and aerosols for computational  
677 efficiency, rather than using brute-force integration of spectral solar properties across  
678 narrower bands (per equation 11). Besides using different radiative transfer  
679 approximations, SNICAR and [dEdd-AD](#) also adopt different methods to derive the band-  
680 averaged SSPs of snow for different band schemes.

682 In SNICAR, snow solar properties are computed for 5 bands: one visible band (0.3 -  
683 0.7 $\mu\text{m}$ ), and four near-IR bands (0.7 - 1  $\mu\text{m}$ , 1 - 1.2  $\mu\text{m}$ , 1.2 - 1.5  $\mu\text{m}$ , and 1.5 - 5  $\mu\text{m}$ ).  
684 The solar properties of four subdivided near-IR bands are combined by fixed ratios to  
685 compute the direct/diffuse near-IR snow properties. These two sets of ratios are derived  
686 offline based on the incident solar spectra of typical of mid-latitude winter for clear and  
687 cloudy-sky conditions clear sky and cloudy sky, respectively (Figure 1(a)).

689 The band-averaged SSPs of snow grains are computed following the Chandrasekhar  
690 Mean approach (Thomas and Stamnes, 1999, their Equation 9.27; Flanner et al., 2007).

Author  
Deleted: In practice,

Author  
Deleted: if

Author  
Deleted: right

Author  
Deleted: direct IR

Author  
Deleted: at

Author  
Deleted: We have implemented this parameterization in MPAS-seaice to quantify its impact on polar climate, though these experiments are beyond the scope of the present paper.

Author  
Deleted: CICE

702 Specifically, spectral SSPs of snow grains are weighted into bands according to surface  
 703 incident solar flux typical of mid-latitude winter for clear and cloudy sky conditions. In  
 704 addition, the single-scattering albedo  $\varpi(\lambda)$  of ice grains are also weighted by the  
 705 hemispheric albedo  $\alpha(\lambda)$  of an optically thick snowpack:

706

$$707 \quad \bar{\varpi}(\bar{\lambda}) = \frac{\int_{\lambda_1}^{\lambda_2} \varpi(\lambda) F(\lambda) \alpha(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} F(\lambda) \alpha(\lambda) d\lambda} \quad (15a)$$

$$708 \quad \bar{g}(\bar{\lambda}) = \frac{\int_{\lambda_1}^{\lambda_2} g(\lambda) F(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} F(\lambda) \alpha(\lambda) d\lambda} \quad (15b)$$

$$709 \quad \bar{\sigma}_{ext}(\bar{\lambda}) = \frac{\int_{\lambda_1}^{\lambda_2} \sigma_{ext}(\lambda) F(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} F(\lambda) \alpha(\lambda) d\lambda} \quad (15c)$$

710

711 Two sets of snow band-averaged SSPs are generated for all grain radii, suitable for direct  
 712 and diffuse light, respectively. For each modeling step and band, SNICAR is called twice  
 713 to compute the direct and diffuse snow solar properties.

714

715 | In [dEdd-AD](#), the snow-covered sea ice properties are computed for 3 bands: one visible  
 716 band (0.3 – 07  $\mu\text{m}$ ), and two near-IR bands (0.7 – 1.19  $\mu\text{m}$  and 1.19 – 5  $\mu\text{m}$ ). The solar  
 717 properties of these two near-IR bands are combined using ratios  $w_{nir1}$  and  $w_{nir2}$  for 0.7-1  
 718 .19  $\mu\text{m}$  and 1.19-5  $\mu\text{m}$ , depending on the fraction of direct near-IR flux  $f_{nir}$ :

719

$$720 \quad w_{nir1} = 0.67 + 0.11 * (1 - f_{nir}) \quad (16a)$$

$$721 \quad w_{nir2} = 1 - w_{nir1} \quad (16b)$$

722

723 The band SSPs of snow are derived by integrating the spectral SSPs and the spectral  
 724 surface solar irradiance measured in the Arctic under mostly clear sky.

725

$$726 \quad \bar{\varpi}(\bar{\lambda}) = \int_{\lambda_1}^{\lambda_2} \varpi(\lambda) F(\lambda) d\lambda \quad (17a)$$

$$727 \quad \bar{g}(\bar{\lambda}) = \int_{\lambda_1}^{\lambda_2} g(\lambda) F(\lambda) d\lambda \quad (17b)$$

$$728 \quad \bar{\sigma}_{ext}(\bar{\lambda}) = \int_{\lambda_1}^{\lambda_2} \sigma_{ext}(\lambda) F(\lambda) d\lambda \quad (17c)$$

729

Author  
 Deleted: CICE

731 In addition, the band-averaged single-scattering albedo  $\varpi(\bar{\lambda})$  is also increased to  $\varpi(\bar{\lambda})'$   
732 until the band albedo computed using averaged SSPs matches the band albedo  $\bar{\alpha}$  within  
733 0.0001, where  $\bar{\alpha}$  is:

$$735 \quad \bar{\alpha} = \int_{\lambda_1}^{\lambda_2} \alpha(\lambda) F(\lambda) d\lambda \quad (18)$$

736  
737 [dEdd-AD](#) adopts this single set of band SSPs for both direct and diffuse computations. In  
738 practice, the physical snow grain radius  $r$  is adjusted to a radiatively equivalent radius  $r_{eqv}$   
739 based on the fraction of direct flux in the near-IR band ( $f_{nidr}$ ):

$$741 \quad r_{eqv} = (f_{nidr} + 0.8(1 - f_{nidr}))r \quad (19)$$

742  
743 This  $r_{eqv}$  and the corresponding snow SSPs are then used in the radiative transfer  
744 calculation. The computed direct and diffuse solar properties alone are less accurate,  
745 while the combined all-sky broadband solar properties agree with SNICAR (Briegleb and  
746 [Light, 2007](#)). As a result, for each modeling step and band, [dEdd-AD](#) radiative transfer  
747 subroutine is called only once to compute both the direct and diffuse snow solar  
748 properties simultaneously.

749  
750 SNICAR and [dEdd-AD](#) also use different approaches to avoid numerical singularities. In  
751 SNICAR, singularities occur when the denominator of term  $C_n^\pm$  in equation (3) equals to  
752 zero (i.e.,  $\gamma^2 - 1/\mu_0^2 = 0$ ), where  $\gamma$  is determined by the approximation method and SSPs  
753 of snow, and  $\mu_0$  is the cosine of the solar zenith angle (Equations 23 and 24, Toon et al.,  
754 1989). When such a singularity is detected, SNICAR will shift  $\mu_0$  by + 0.02 or -0.02 to  
755 obtain physically realistic radiative properties. In the [dEdd-AD](#) algorithm, singularities  
756 arise only when  $\mu_0 = 0$  (Equation 4). Therefore, in practice, for  $\mu_0 < 0.01$ , [dEdd-AD](#)  
757 computes the sea-ice solar properties for  $\mu_0 = 0.01$  to avoid unphysical results.

## 759 **8. Discussion: a unified radiative transfer model for snow, sea ice, and land ice.**

760  
761 Based on the inter-comparison of three two-stream algorithms and their implementations  
762 in ESMS, we formulated the following surface shortwave radiative transfer  
763

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: CICE

769 recommendations for an accurate, fast, and consistent treatment for snow on land, land  
770 ice, and sea ice in ESMs:

771

772 First, the two-stream delta-Eddington adding-doubling algorithm by Briegleb and Light  
773 (2007) is unsurpassed as a radiative transfer core. The evaluation in Section 5 shows that  
774 this algorithm produces the least error for snow albedo and solar absorption within  
775 snowpack, especially under overcast skies. This algorithm applies well to both uniformly  
776 refractive media such as snow on land, and to non-uniformly refractive media, such as  
777 bare/snow-covered/ponded sea ice and bare/snow-covered land ice. Numerical  
778 singularities occur only rarely (when  $\mu_0 = 0$ ) and are easily avoided in model  
779 implementations. Among the three two-stream algorithms discussed here, [dEdd-AD](#) is  
780 also the most efficient one as it takes only  $\sim 2/3$  of the time of SNICAR and 2SD to  
781 compute solar properties of multi-layer snowpacks.

782

783 Second, any two-stream cryospheric radiative transfer model can incorporate the  
784 parameterization described in Section 6 to adjust the low bias of direct near-IR snow  
785 albedo and high bias of direct near-IR solar absorption in snow, for solar zenith angles  
786 larger than  $75^\circ$ . These biases are persistent across all two-stream algorithms discussed in  
787 this work, and should be corrected for snow-covered surfaces. Alternatively, adopting a  
788 4-stream approximation would reduce or eliminate such biases, though at considerable  
789 expense in computational efficiency.

790

791 Third, a cryospheric radiative transfer model should prefer physically based  
792 parameterizations that are extensible and convergent (e.g., with increasing spectral  
793 resolution) for the band-averaged SSPs and size distribution of snow. Although the  
794 treatments used in SNICAR and [dEdd-AD](#) are both practical since they both reproduce  
795 the narrowband solar properties with carefully derived band-averaged inputs as discussed  
796 in Section 7, the snow treatment used in SNICAR is more physically based and  
797 reproducible since it does not rely on subjective adjustment and empirical coefficients as  
798 used in [dEdd-AD](#). Specifically, the empirical adjustment to snow grain radius  
799 implemented in [dEdd-AD](#) may not always produce compensating errors. For example, in  
800 snow containing light-absorbing impurities such adjustment may also lead to biases in  
801 aerosol absorption since the albedo reduction caused by light-absorbing particles does not  
802 linearly depend on snow grain radius (Dang et al., 2015). For further model development  
803 incorporating non-spherical snow grain shapes (Dang et al., 2016; He et al., 2018ab),  
804 such adjustment on grain radius may fail as well. Moreover, SNICAR computes the snow

Author

Deleted: y

Author

Deleted: the CICE radiative core

Author

Deleted: CICE

Author

Deleted: CICE

Author

Deleted: CICE



810 properties for four near-IR bands, which helps capture the spectral variation of albedo  
811 ([Figure 1](#)) and therefore better represents near-IR solar properties. It is also worth noting  
812 that unlike the radiative core of [dEdd-AD](#), SNICAR is actively maintained with  
813 numerous modifications and updates in the past decade (e.g. Flanner et al., 2012; He et  
814 al., 2018b). Snow radiative treatments that follow SNICAR conventions for SSPs may  
815 take advantage of these updates. Note that any radiative core that follows SNICAR SSP  
816 conventions must be called twice to compute diffuse and direct solar properties,  
817 respectively.

818

819 Fourth, a surface cryospheric radiative transfer model should flexibly accommodate  
820 coupled simulations with distinct atmospheric and surface spectral grids. Both the 5-band  
821 scheme used in SNICAR and the 3-band scheme used in [dEdd-AD](#) separate the visible  
822 from near-IR spectrum at 0.7  $\mu\text{m}$ . This boundary aligns with the Community  
823 Atmospheric Model's original radiation bands (CAM; Neale et al., 2012), though not  
824 with the widely used Rapid Radiative Transfer Model (RRTMG; Iacono et al., 2008)  
825 which places 0.7  $\mu\text{m}$  squarely in the middle of a spectral band. A mismatch in spectral  
826 boundaries between atmospheric and surface radiative transfer schemes can require an  
827 ESM to unphysically apportion energy from the straddled spectral bin when coupling  
828 fluxes between surface and atmosphere. The spectral grids of surface and atmosphere  
829 radiation need not be identical so long as the coarser grid shares spectral boundaries with  
830 the finer grid. In practice maintaining a portable cryospheric radiative module such as  
831 SNICAR requires a complex offline toolchain (Mie solver, spectral refractive indices for  
832 air, water, ice, and aerosols, spectral solar insolation for clear and cloudy skies) to  
833 compute, integrate, and rebin SSPs. Aligned spectral boundaries between surface and  
834 atmospheric would simplify the development of efficient and accurate radiative transfer  
835 for the coupled Earth system.

836

837 Last, it is important to note that, although we only examine the performance of the [dEdd-](#)  
838 [AD](#) for pure snow in this work, this algorithm can be applied to the surface solar  
839 calculation of all cryospheric components with or without light-absorbing particles  
840 present. First, Briegleb and Light (2007) proved its accuracy for simulating ponded/bare  
841 sea-ice solar properties against observations and a Monte Carlo radiation model. Second,  
842 In CESM and E3SM, the radiative transfer simulation of snow on land ice is carried out  
843 by SNICAR with prescribed land ice albedo. Adopting [dEdd-AD radiative core](#) in  
844 SNICAR will permit these ESMs to couple the snow and land ice as a non-uniformly  
845 refractive column for more accurate solar computations since bare/snow-covered/ponded

Author  
Deleted: Figure 2  
Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted:   
Author  
Deleted: CICE  
Author  
Deleted: adding-doubling algorithm

Author  
Deleted: the CICE  
Author  
Deleted: adding-doubling core

854 land ice is physically similar to bare/snow-covered/ponded sea ice, and the latter is  
855 already treated well by [dEdd-AD](#), radiative transfer core. Third, adding light-absorbing  
856 particles in snow will not change our results qualitatively. Both [dEdd-AD](#) and SNICAR  
857 simulate the impact of light-absorbing particles (black carbon and dust) on snow and/or  
858 sea ice using self-consistent particle SSPs that follow the SNICAR convention (e.g.,  
859 [Flanner et al., 2007](#); [Holland et al. 2012](#)). These particles are assumed to be either  
860 [internally or externally mixed with snow crystals; the combined SSPs of mixtures \(e.g.](#)  
861 [Appendix A of Dang et al., 2015\)](#) are then used as the inputs for radiative transfer  
862 calculation. The adoption of [dEdd-AD](#) radiative transfer algorithm in SNICAR, and the  
863 implementation of SNICAR snow SSPs in [dEdd-AD](#) enables a consistent simulation of  
864 the radiative effects of light-absorbing particles in the cryosphere across ESM  
865 components.

866  
867 In summary, this inter-comparison and evaluation has shown multiple ways that the solar  
868 properties of cryospheric surfaces can be improved in the current generation of ESMs.  
869 [We have merged these findings into a hybrid model SNICAR-AD, which is primarily](#)  
870 [composed of the radiative transfer scheme of dEdd-AD, 5-band snow/aerosol SSPs of](#)  
871 [SNICAR, and the parameterization to correct for snow albedo biases when solar zenith](#)  
872 [angle exceeds 75°. This hybrid model can be applied to snow on land, land ice, and sea](#)  
873 [ice to produce consistent shortwave radiative properties for snow-covered surfaces across](#)  
874 [the Earth system. With the evolving and further understanding of snow and aerosol](#)  
875 [physics and chemistry, the adoption of this hybrid model will obviate the effort to modify](#)  
876 [and maintain separate optical variable input files used for different model components.](#)

877  
878 [SNICAR-AD is now implemented in both the sea-ice \(MPAS-seaice\) and land \(ELM\)](#)  
879 [components of E3SM. More simulations and analyses are underway to examine its](#)  
880 [impact on E3SM model performance and simulated climate. The results are however](#)  
881 [beyond the scope of this work and will be thoroughly discussed in a future paper.](#)

## 882 9. Conclusions

883 [In this work, we aim to improve and unify the solar radiative transfer calculations for](#)  
884 [snow on land and snow on sea ice in ESMs by evaluating the following two-stream](#)  
885 [radiative transfer algorithms: the two-stream delta-Eddington adding-doubling algorithm](#)  
886 [dEdd-AD](#) implemented in sea-ice model Icepack/CICE/MPAS-seaice, the two-stream  
887 delta-Eddington and two-stream delta-Hemispheric-Mean algorithms implemented in  
888 snow model SNICAR, and a two-stream delta-Discrete-Ordinate algorithm. Among these  
889 three models, the [dEdd-AD](#) produces the most accurate snow albedo and solar absorption

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Formatted: None, Font:(Default) Times, Underline color: Blue, Font color: Black

Author  
Deleted: .

Author  
Deleted: CICE

Author  
Deleted: CICE

Author  
Deleted: will enable

Author  
Deleted: n

Author  
Deleted: We have adopted these recommendations in a hybrid model SNICAR-AD, implemented in MPAS-seaice and E3SM Land Model (ELM), to examine the response of climate to this improved and unified cryospheric surface radiation treatment in future E3SM studies. .

Author  
Deleted: .

Author  
Deleted: two-stream delta-Eddington adding-doubling algorithm

907 (Section 5). All two-stream models underestimate near-IR snow albedo and overestimate  
908 near-IR absorption when solar zenith angles are larger than 75°, which can be adjusted by  
909 a parameterization we developed (Section 6). We compared the implementations of  
910 radiative transfer cores in SNICAR and [dEdd-AD](#) (Section 7) and recommended a  
911 consistent [and hybrid](#) shortwave radiative [model SNICAR-AD](#) for snow-covered surfaces  
912 across ESMs (Section 8). Improved treatment of surface cryospheric radiative properties  
913 in the thermal infrared has recently been shown to remediate significant climate  
914 simulation biases in Polar Regions (Huang et al., 2018). It is hoped that adoption of  
915 improved and consistent treatments of solar radiative properties for snow-covered  
916 surfaces as described in this study [will further remediate simulation biases in snow-](#)  
917 [covered regions.](#)  
918

Author  
**Deleted:** CICE

Author  
**Deleted:** treatment

Author  
**Deleted:** (i.e. the hybrid model SNICAR-AD)

Author  
**Deleted:** Polar Regions.

924 **Data availability.** The data and models are available upon request to Cheng Dang  
925 (cdang5@uci.edu). SNICAR and [dEdd-AD](#) radiative transfer core can be found at  
926 <https://github.com/E3SM-Project/E3SM>.  
927

928 **Competing interests.** The authors declare that they have no conflict of interest.  
929

930 **Acknowledgments.** The authors thank Prof. Stephen G. Warren and Prof. Qiang Fu for  
931 [insightful](#) discussions on radiative transfer algorithms. The authors thank Dr. Adrian  
932 Turner for instructions on installing and running MPAS-seaice. [The authors thank Dr.](#)  
933 [David Bailey and one anonymous reviewer for their careful reading and insightful](#)  
934 [suggestions.](#) This research [is](#) supported as part of the Energy Exascale Earth System  
935 Model (E3SM) project, funded by the U.S. Department of Energy, Office of Science,  
936 Office of Biological and Environmental Research, with funding number [DE-SC0012998](#).  
937

### 938 References

- 939 Aoki, T., Kuchiki, K., Niwano, M., Kodama, Y., Hosaka, M. and Tanaka, T.: Physically  
940 based snow albedo model for calculating broadband albedos and the solar heating  
941 profile in snowpack for general circulation models. *Journal of Geophysical Research*,  
942 116, D11114, <https://doi.org/10.1029/2010JD015507>, 2011.
- 943 Bøggild, C.E., Brandt, R.E., Brown, K.J. and Warren, S.G.: The ablation zone in  
944 northeast Greenland: ice types, albedos and impurities. *Journal of Glaciology*, 56,  
945 101-113, pp.101-113, <https://doi.org/10.3189/002214310791190776>, 2010.
- 946 Brandt, R.E., Warren, S.G., Worby, A.P. and Grenfell, T.C.: Surface albedo of the  
947 Antarctic sea ice zone. *Journal of Climate*, 18(17), pp.3606-3622.  
948 <https://doi.org/10.1175/JCLI3489.1> 2005.
- 949 Briegleb, P. and Light, B.: A Delta-Eddington multiple scattering parameterization for  
950 solar radiation in the sea ice component of the community climate system model,  
951 2007.
- 952 Dang, C. and Hegg, D.A.: Quantifying light absorption by organic carbon in Western  
953 North American snow by serial chemical extractions. *Journal of Geophysical*  
954 *Research*, 119(17), pp.10-247. <https://doi.org/10.1002/2014JD022156>, 2014.
- 955 Dang, C., Fu, Q. and Warren, S.G., 2016. Effect of snow grain shape on snow albedo.  
956 *Journal of the Atmospheric Sciences*, 73(9), pp.3573-3583.  
957 <https://doi.org/10.1175/JAS-D-15-0276.1>, 2016.
- 958 Dang, C., Brandt, R.E. and Warren, S.G.: Parameterizations for narrowband and  
959 broadband albedo of pure snow and snow containing mineral dust and black carbon.  
960 *Journal of Geophysical Research: Atmospheres*, 120(11), pp.5446-5468,  
961 <https://doi.org/10.1002/2014JD022646>, 2015.
- 962 Dang, C., Warren, S.G., Fu, Q., Doherty, S.J., Sturm, M. and Su, J.: Measurements of  
963 light-absorbing particles in snow across the Arctic, North America, and China:  
964 Effects on surface albedo. *Journal of Geophysical Research: Atmospheres*, 122(19),  
965 pp.10-149. <https://doi.org/10.1002/2017JD027070>, 2017.

Author  
Deleted: CICE

Author  
Deleted: e

Author  
Deleted: helpful

Author  
Deleted: was

Author  
Deleted: DOE BER ESM (

Author  
Deleted: )

- 972 Doherty, S.J., Hegg, D.A., Johnson, J.E., Quinn, P.K., Schwarz, J.P., Dang, C. and  
973 Warren, S.G.: Causes of variability in light absorption by particles in snow at sites in  
974 Idaho and Utah. *Journal of Geophysical Research: Atmospheres*, 121(9), pp.4751-  
975 4768. <https://doi.org/10.1002/2015JD024375>, 2016.
- 976 Doherty, S.J., Warren, S.G., Grenfell, T.C., Clarke, A.D. and Brandt, R.E.: Light-  
977 absorbing impurities in Arctic snow. *Atmospheric Chemistry and Physics*, 10(23),  
978 pp.11647-11680. <https://doi.org/10.5194/acp-10-11647-2010>, 2010.
- 979 Doherty, S.J., Dang, C., Hegg, D.A., Zhang, R. and Warren, S.G.: Black carbon and other  
980 light-absorbing particles in snow of central North America. *Journal of Geophysical*  
981 *Research: Atmospheres*, 119(22), pp.12-807, <https://doi.org/10.1002/2014JD022350>,  
982 2014.
- 983 Flanner, M.G. and Zender, C.S.: Snowpack radiative heating: Influence on Tibetan  
984 Plateau climate. *Geophysical Research Letters*, 32(6).  
985 <https://doi.org/10.1029/2004GL022076>, 2005.
- 986 Flanner, M.G., Zender, C.S., Randerson, J.T. and Rasch, P.J.: Present-day climate forcing  
987 and response from black carbon in snow. *Journal of Geophysical Research:*  
988 *Atmospheres*, 112(D11). <https://doi.org/10.1029/2006JD008003>, 2007.
- 989 Flanner, M.G., Liu, X., Zhou, C., Penner, J.E. and Jiao, C.: Enhanced solar energy  
990 absorption by internally-mixed black carbon in snow grains. *Atmospheric Chemistry*  
991 *and Physics*, 12(10), pp.4699-4721. <https://doi.org/10.5194/acp-12-4699-2012>, 2012.
- 992 Gardner, A.S. and Sharp, M.J.: A review of snow and ice albedo and the development of  
993 a new physically based broadband albedo parameterization. *Journal of Geophysical*  
994 *Research: Earth Surface*, 115(F1). <https://doi.org/10.1029/2009JF001444>, 2010.
- 995 Grenfell, T.C., Neshyba, S.P. and Warren, S.G.: Representation of a nonspherical ice  
996 particle by a collection of independent spheres for scattering and absorption of  
997 radiation: 3. Hollow columns and plates. *Journal of Geophysical Research:*  
998 *Atmospheres*, 110(D17). <https://doi.org/10.1029/2005JD005811>, 2005.
- 999 He, C., Liou, K.N., Takano, Y., Yang, P., Qi, L. and Chen, F.: Impact of grain shape and  
1000 multiple black carbon internal mixing on snow albedo: Parameterization and radiative  
1001 effect analysis. *Journal of Geophysical Research: Atmospheres*, 123(2), pp.1253-  
1002 1268. <https://doi.org/10.1002/2017JD027752>, 2018a.
- 1003 He, C., Flanner, M.G., Chen, F., Barlage, M., Liou, K.N., Kang, S., Ming, J. and Qian,  
1004 Y.: Black carbon-induced snow albedo reduction over the Tibetan Plateau:  
1005 uncertainties from snow grain shape and aerosol–snow mixing state based on an  
1006 updated SNICAR model. *Atmospheric Chemistry and Physics*, 18, pp.11507-11527.  
1007 <https://doi.org/10.5194/acp-18-11507-2018>, 2018b.
- 1008 He, C., Takano, Y., Liou, K.N., Yang, P., Li, Q. and Chen, F.: Impact of Snow Grain  
1009 Shape and Black Carbon–Snow Internal Mixing on Snow Optical Properties:  
1010 Parameterizations for Climate Models. *Journal of Climate*, 30(24), pp.10019-10036.  
1011 <https://doi.org/10.1175/JCLI-D-17-0300.1>, 2017.
- 1012 [Holland, M.M., Bailey, D.A., Briegleb, B.P., Light, B. and Hunke, E.: Improved sea ice](#)  
1013 [shortwave radiation physics in CCSM4: The impact of melt ponds and aerosols on](#)

- 1014 | [Arctic sea ice. Journal of Climate, 25\(5\), pp.1413-1430. https://doi.org/10.1175/JCLI-](https://doi.org/10.1175/JCLI-D-11-00078.1)  
1015 | [D-11-00078.1, 2012.](https://doi.org/10.1175/JCLI-D-11-00078.1)
- 1016 | Huang, X., Chen, X., Flanner, M., Yang, P., Feldman, D. and Kuo, C.: Improved  
1017 | representation of surface spectral emissivity in a global climate model and its impact  
1018 | on simulated climate. *Journal of Climate*, 31(9), pp.3711-3727.  
1019 | <https://doi.org/10.1175/JCLI-D-17-0125.1>, 2018.
- 1020 | Hunke, E. C., Lipscomb, W. H., Turner, A. K., Jeffery, N., and Elliott, S.: CICE: the Los  
1021 | Alamos Sea Ice Model Documentation and Software User's Manual Version 4.1 LA-  
1022 | CC-06-012. T-3 Fluid Dynamics Group, Los Alamos National Laboratory 675. 2010.
- 1023 | Iacono, M.J., Delamere, J.S., Mlawer, E.J., Shephard, M.W., Clough, S.A. and Collins,  
1024 | W.D.: Radiative forcing by long-lived greenhouse gases: Calculations with the AER  
1025 | radiative transfer models. *Journal of Geophysical Research: Atmospheres*, 113(D13),  
1026 | <https://doi.org/10.1029/2008JD009944>, 2008.
- 1027 | Jin, Z. and Stamnes, K.: Radiative transfer in nonuniformly refracting layered media:  
1028 | atmosphere-ocean system. *Applied Optics*, 33(3), pp.431-442.  
1029 | <https://doi.org/10.1364/AO.33.000431>, 1994.
- 1030 | Kuipers Munneke, P., Van den Broeke, M.R., Lenaerts, J.T.M., Flanner, M.G., Gardner,  
1031 | A.S. and Van de Berg, W.J.: A new albedo parameterization for use in climate  
1032 | models over the Antarctic ice sheet. *Journal of Geophysical Research: Atmospheres*,  
1033 | 116(D5). <https://doi.org/10.1029/2010JD015113>, 2011.
- 1034 | Lee, W.L. and Liou, K.N.: A coupled atmosphere-ocean radiative transfer system using  
1035 | the analytic four-stream approximation. *Journal of the Atmospheric Sciences*, 64(10),  
1036 | pp.3681-3694. <https://doi.org/10.1175/JAS4004.1>, 2007.
- 1037 | Liang, S., Fang, H., Chen, M., Shuey, C.J., Walthall, C., Daughtry, C., Morisette, J.,  
1038 | Schaaf, C. and Strahler, A.: Validating MODIS land surface reflectance and albedo  
1039 | products: Methods and preliminary results. *Remote sensing of environment*, 83(1-2),  
1040 | pp.149-162. [https://doi.org/10.1016/S0034-4257\(02\)00092-5](https://doi.org/10.1016/S0034-4257(02)00092-5), 2002.
- 1041 | [Light, B., Grenfell, T. C. and Perovich, D.K.: Transmission and absorption of solar](https://doi.org/10.1029/2006JC003977)  
1042 | [radiation by Arctic sea ice during the melt season. Journal of Geophysical Research:](https://doi.org/10.1029/2006JC003977)  
1043 | [Oceans 113, no. C3. https://doi.org/10.1029/2006JC003977, 2008.](https://doi.org/10.1029/2006JC003977)
- 1044 | [Light, B., Perovich, D.K., Webster M.A., Polashenski, C., and Dadic, R.: Optical](https://doi.org/10.1002/2015JC011163)  
1045 | [properties of melting first-year Arctic sea ice. Journal of Geophysical Research:](https://doi.org/10.1002/2015JC011163)  
1046 | [Oceans 120, no. 11: 7657-7675. https://doi.org/10.1002/2015JC011163, 2015.](https://doi.org/10.1002/2015JC011163)
- 1047 | Marshall, S. and Oglesby, R.J.: An improved snow hydrology for GCMs. Part 1: Snow  
1048 | cover fraction, albedo, grain size, and age. *Climate Dynamics*, 10(1-2), pp.21-37.  
1049 | <https://doi.org/10.1007/BF00210334>, 1994.
- 1050 | Marshall, S.E.: A Physical Parameterization of Snow Albedo for Use in Climate Models,  
1051 | NCAR Cooperative Thesis 123, National Center for Atmospheric Research,  
1052 | Boulder, CO, 175 pp. 1989.

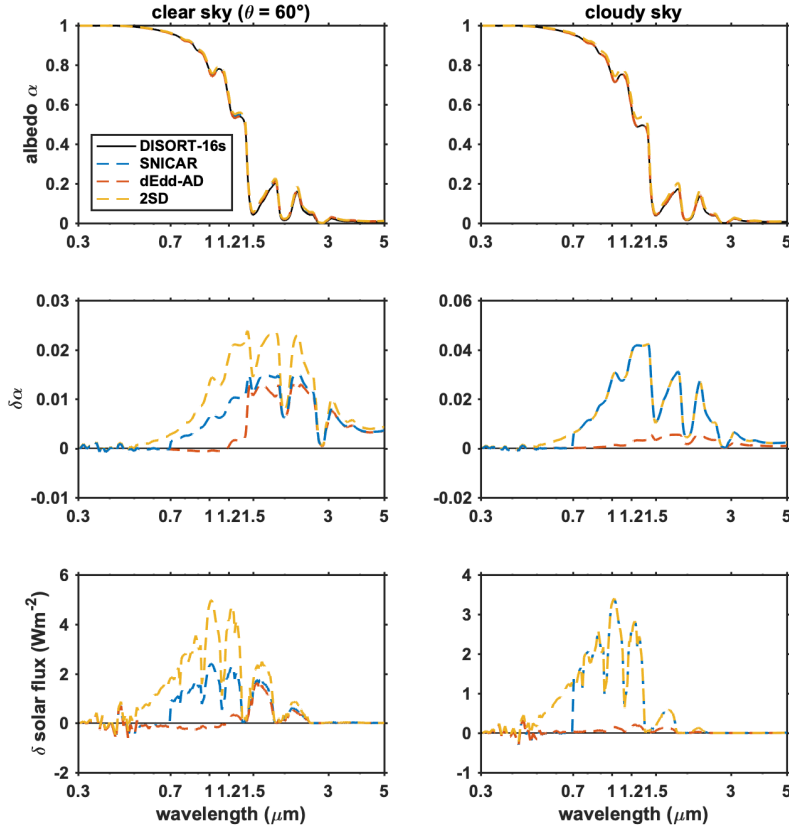
- 1053 Matzl, M. and Schneebeli, M.: Measuring specific surface area of snow by near-infrared  
1054 photography. *Journal of Glaciology*, 52(179), pp.558-564.  
1055 <https://doi.org/10.3189/172756506781828412>, 2006.
- 1056 Matzl, M. and Schneebeli, M.: Stereological measurement of the specific surface area of  
1057 seasonal snow types: Comparison to other methods, and implications for mm-scale  
1058 vertical profiling. *Cold Regions Science and Technology*, 64(1), pp.1-8.  
1059 <https://doi.org/10.1016/j.coldregions.2010.06.006>, 2010.
- 1060 Meador, W.E. and Weaver, W.R.: Two-stream approximations to radiative transfer in  
1061 planetary atmospheres: A unified description of existing methods and a new  
1062 improvement. *Journal of the Atmospheric Sciences*, 37(3), pp.630-643.  
1063 [https://doi.org/10.1175/1520-0469\(1980\)037<0630:TSATRT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1980)037<0630:TSATRT>2.0.CO;2), 1980.
- 1064 Mlawer, E.J. and Clough, S.A.: On the extension of rapid radiative transfer model to the  
1065 shortwave region. In *Proceedings of the 6th Atmospheric Radiation Measurement*  
1066 *(ARM) Science Team Meeting, US Department of Energy, CONF-9603149, 223–*  
1067 *226. 1997.*
- 1068 Neale, Richard B., Chen, C.-C., Gettelman, A., Lauritzen, P. H., Park, S., Williamson, D.  
1069 L., Conley, A. J., Garcia, R., Kinnison, D., Lamarque, J. F., and Marsh, D.:  
1070 Description of the NCAR community atmosphere model (CAM 5.0). NCAR Tech.  
1071 Note NCAR/TN-486+ STR 1, no. 1: 1-12. 2010.
- 1072 Neshyba, S.P., Grenfell, T.C. and Warren, S.G.: Representation of a nonspherical ice  
1073 particle by a collection of independent spheres for scattering and absorption of  
1074 radiation: 2. Hexagonal columns and plates. *Journal of Geophysical Research:*  
1075 *Atmospheres*, 108(D15). <https://doi.org/10.1029/2002JD003302>, 2003.
- 1076 Perovich, D. K.: The optical properties of sea ice (No. MONO-96-1). COLD REGIONS  
1077 RESEARCH AND ENGINEERING LAB HANOVER NH. 1996.
- 1078 Stamnes, K., Tsay, S.C., Wiscombe, W. and Jayaweera, K.: Numerically stable algorithm  
1079 for discrete-ordinate-method radiative transfer in multiple scattering and emitting  
1080 layered media. *Applied optics*, 27(12), pp.2502-2509.  
1081 <https://doi.org/10.1364/AO.27.002502>, 1988.
- 1082 Thomas, G.E. and K. Stamnes (1999), *Radiative transfer in the atmosphere and ocean.*  
1083 Cambridge University Press.
- 1084 Toon, O. B., McKay, C. P., Ackerman, T. P., and Santhanam, K.: Rapid calculation of  
1085 radiative heating rates and photodissociation rates in inhomogeneous multiple  
1086 scattering atmospheres, *J. Geophys. Res.*, 94(D13), 16,287–16,301.  
1087 <https://doi.org/10.1029/JD094iD13p16287>, 1989.
- 1088 Turner, A.K., Lipscomb, W.H., Hunke, E.C., Jacobsen, D.W., Jeffery, N., Ringler, T.D.  
1089 and Wolfe, J.D.: MPAS-Seaice: a new variable resolution sea-ice model. *J. Adv.*  
1090 *Model Earth Sy.*, in preparation., 2018.
- 1091 Wang, X., Doherty, S.J. and Huang, J.: Black carbon and other light-absorbing  
1092 impurities in snow across Northern China. *Journal of Geophysical Research:*  
1093 *Atmospheres*, 118(3), pp.1471-1492. <https://doi.org/10.1029/2012JD018291>, 2013.

- 1094 Warren, S. G.: Optical properties of snow. *Reviews of Geophysics* 20, no. 1: 67-89.  
1095 <https://doi.org/10.1029/RG020i001p00067>,1982.
- 1096 Warren, S.G. and Wiscombe, W.J.: A model for the spectral albedo of snow. II: Snow  
1097 containing atmospheric aerosols. *Journal of the Atmospheric Sciences*, 37(12),  
1098 pp.2734-2745.[https://doi.org/10.1175/1520-](https://doi.org/10.1175/1520-0469(1980)037<2734:AMFTSA>2.0.CO;2)  
1099 0469(1980)037<2734:AMFTSA>2.0.CO;2, 1980
- 1100 Warren, S.G. and Brandt, R.E.: Optical constants of ice from the ultraviolet to the  
1101 microwave: A revised compilation. *Journal of Geophysical Research: Atmospheres*,  
1102 113(D14). <https://doi.org/10.1029/2007JD009744>, 2008.
- 1103 Warren, S.G. and Wiscombe, W.J.: Dirty snow after nuclear war. *Nature*, 313(6002),  
1104 p.467. <https://doi.org/10.1038/313467a0>, 1985.
- 1105 Wiscombe, W. J.: The delta-Eddington approximation for a vertically inhomogeneous  
1106 atmosphere, NCAR Tech. Note TN-140\_STR [NTIS P6270618]. 1977.
- 1107 Wiscombe, W.J.: Improved Mie scattering algorithms. *Applied optics*, 19(9), pp.1505-  
1108 1509. <https://doi.org/10.1364/AO.19.001505>,1980.
- 1109 Wiscombe, W.J. and Warren, S.G.: A model for the spectral albedo of snow. I: Pure  
1110 snow. *Journal of the Atmospheric Sciences*, 37(12), pp.2712-2733.  
1111 [https://doi.org/10.1175/1520-0469\(1980\)037<2712:AMFTSA>2.0.CO;2](https://doi.org/10.1175/1520-0469(1980)037<2712:AMFTSA>2.0.CO;2), 1980.
- 1112 Zender, C.S.: Global climatology of abundance and solar absorption of oxygen collision  
1113 complexes. *Journal of Geophysical Research: Atmospheres*, 104(D20), pp.24471-  
1114 24484. <https://doi.org/10.1029/1999JD900797>, 1999.
- 1115 Zender, C.S., Bush, B., Pope, S.K., Bucholtz, A., Collins, W.D., Kiehl, J.T., Valero, F.P.  
1116 and Vitko Jr, J.: Atmospheric absorption during the atmospheric radiation  
1117 measurement (ARM) enhanced shortwave experiment (ARESE). *Journal of*  
1118 *Geophysical Research: Atmospheres*, 102(D25), pp.29901-29915.  
1119 <https://doi.org/10.1029/97JD01781>, 1997.
- 1120



1121 | [Figure 1](#). The spectral albedo of pure snow computed using 16-stream DISORT,  
 1122 | SNICAR, [dEdd-AD](#), and 2SD models, for clear-sky (direct beam at solar zenith angle  
 1123 | 60°) and cloudy-sky conditions in the left and right panels, respectively. The top panels  
 1124 | show spectral albedo. The middle panels show the difference ( $\delta\alpha = \alpha_c - \alpha_{16}$ ) in spectral  
 1125 | albedos computed using [the](#) two-stream model ( $\alpha_c$ ) and 16-stream DISORT ( $\alpha_{16}$ ). The  
 1126 | bottom panels show the difference of reflected spectral flux given  $\delta\alpha$ . The snowpack is  
 1127 | set to semi-infinite deep with grain radius of 100  $\mu\text{m}$ .

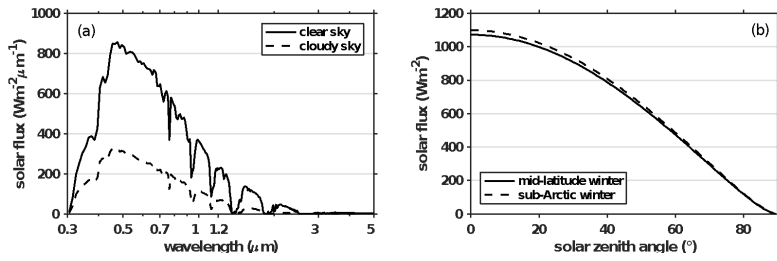
Author  
 Deleted: CICE



1128  
 1129  
 1130  
 1131 |

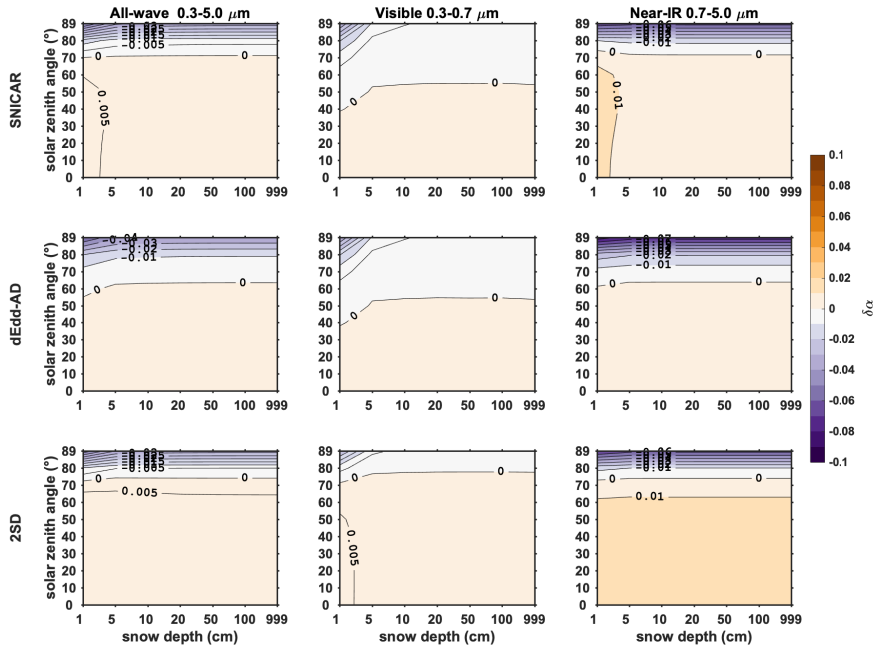
1133  
1134  
1135  
1136  
1137  
1138  
1139

Figure 2. Spectral and total down-welling solar flux at surface computed using SWNB2 for (a) standard clear-sky and cloudy-sky atmospheric profiles of mid-latitude winter assuming solar zenith angle is  $60^\circ$  at the top of the atmosphere, and for (b) standard clear sky profiles of mid-latitude and sub-Arctic winter with different incident solar zenith angles.



1140  
1141  
1142

1143 Figure 3. The difference in direct snow albedo ( $\delta\alpha = \alpha_s - \alpha_{16}$ ) computed using two-stream  
 1144 models ( $\alpha_s$ ) and using 16-stream DISORT model ( $\alpha_{16}$ ), for various snow depths and solar  
 1145 zenith angles, with snow grain radius of  $100 \mu\text{m}$ . From the top to the bottom rows are  
 1146 results of two-stream models SNICAR, [dEdd-AD](#), and 2SD. From the left to the right  
 1147 columns are albedo differences of all-wave, visible, near-IR bands.

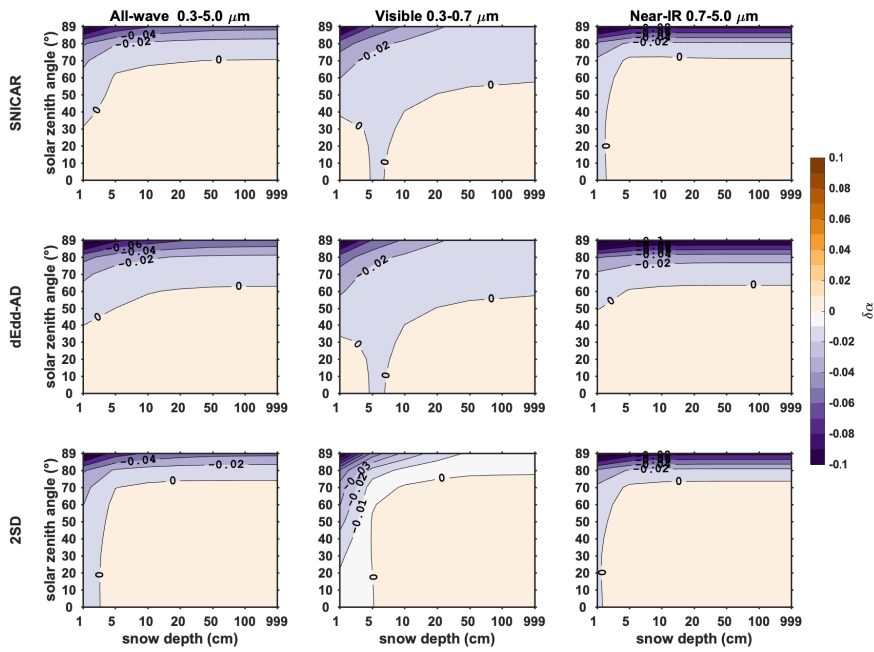


Author  
 Deleted: CICE

1148  
 1149  
 1150

1152  
1153  
1154  
1155

Figure 4. The difference in direct snow albedo ( $\delta\alpha = \alpha_s - \alpha_{16}$ ) computed using two-stream models ( $\alpha_s$ ) and using 16-stream DISORT model ( $\alpha_{16}$ ) for various snow depths and solar zenith angles, with snow grain radius of  $1000 \mu\text{m}$ .

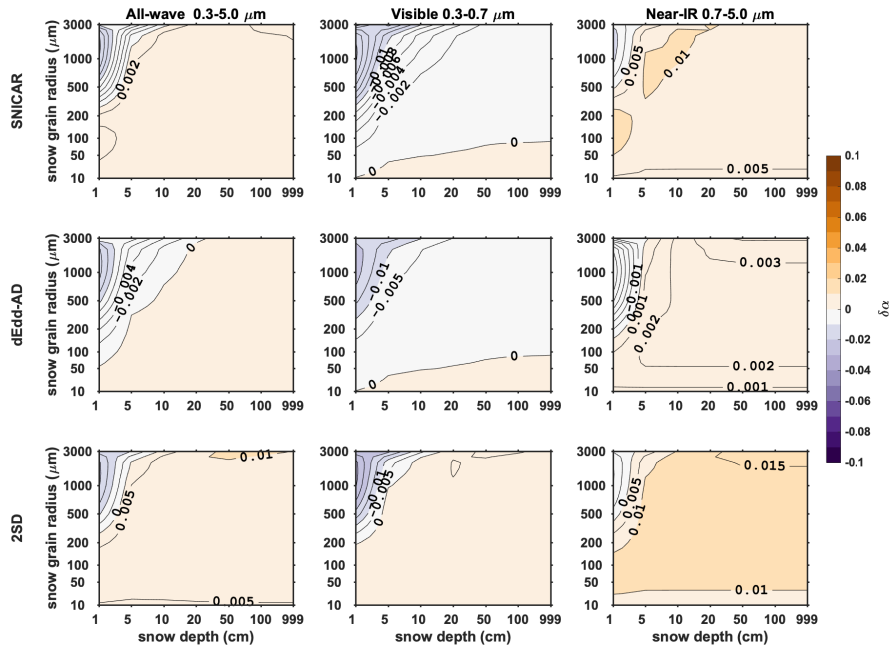


1156  
1157  
1158

Author  
~~Deleted:~~ The same to Figure 3  
Author  
~~Deleted:~~ , but for snow grain radius of

1161  
1162  
1163  
1164

Figure 5. The difference in direct snow albedo ( $\delta\alpha = \alpha_s - \alpha_{16}$ ) computed using two-stream models ( $\alpha_s$ ) and using 16-stream DISORT model ( $\alpha_{16}$ ), for various snow depths and snow grain radii, with solar zenith angle of  $60^\circ$ .



1165  
1166

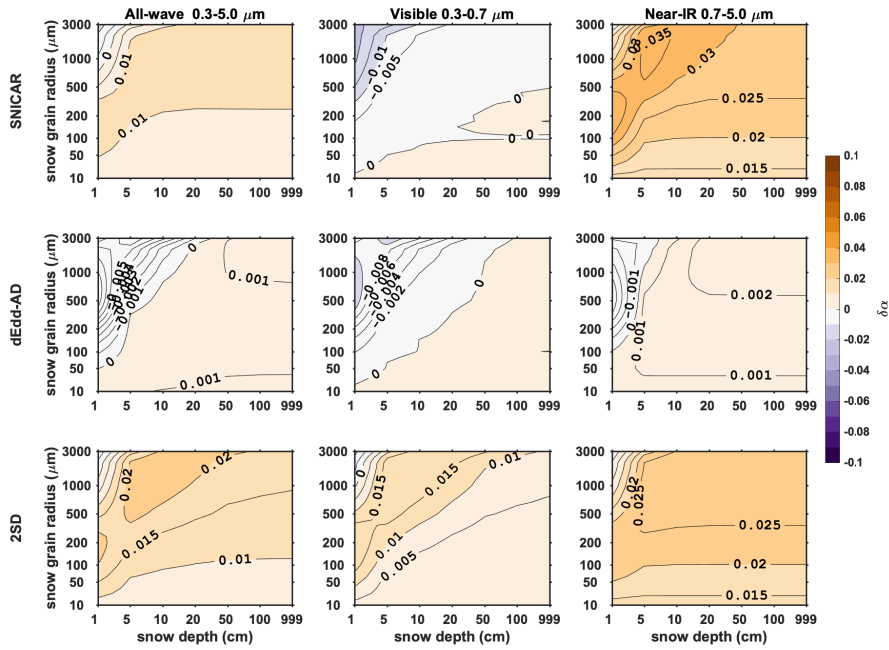
Author

Deleted: The same to Figure 3, but for fixed

Author

Deleted: and different snow grain radii

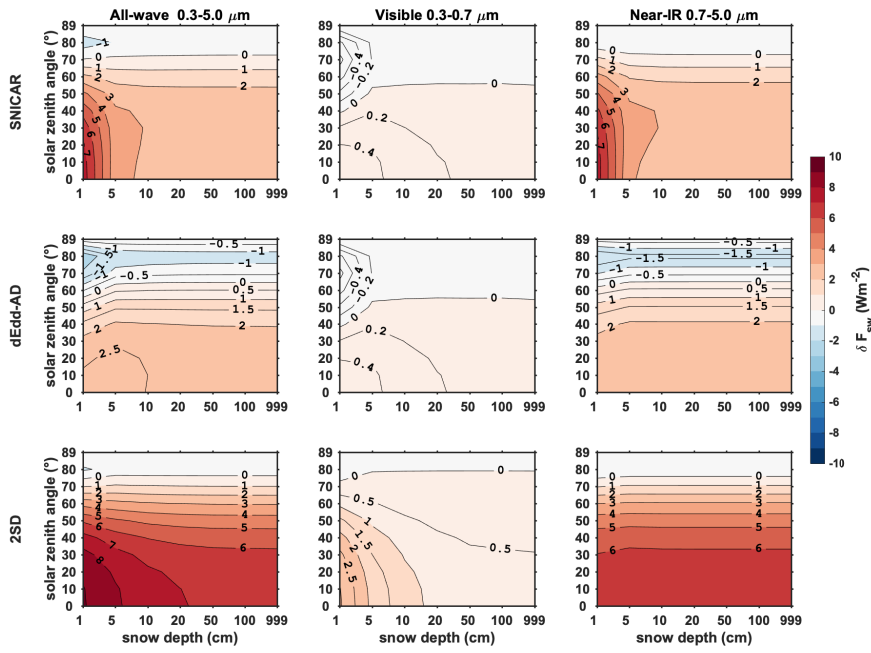
1169 | Figure 6. The difference in diffuse snow albedo ( $\delta\alpha = \alpha_s - \alpha_{16}$ ) computed using two-  
 1170 | stream models ( $\alpha_s$ ) and using 16-stream DISORT model ( $\alpha_{16}$ ), for various snow depths  
 1171 | and snow grain radii, with solar zenith angle of  $60^\circ$  at the top of the atmosphere.  
 1172



Author  
 Deleted: The same to Figure 5, but for diffuse snow albedo with different snow grain radii.

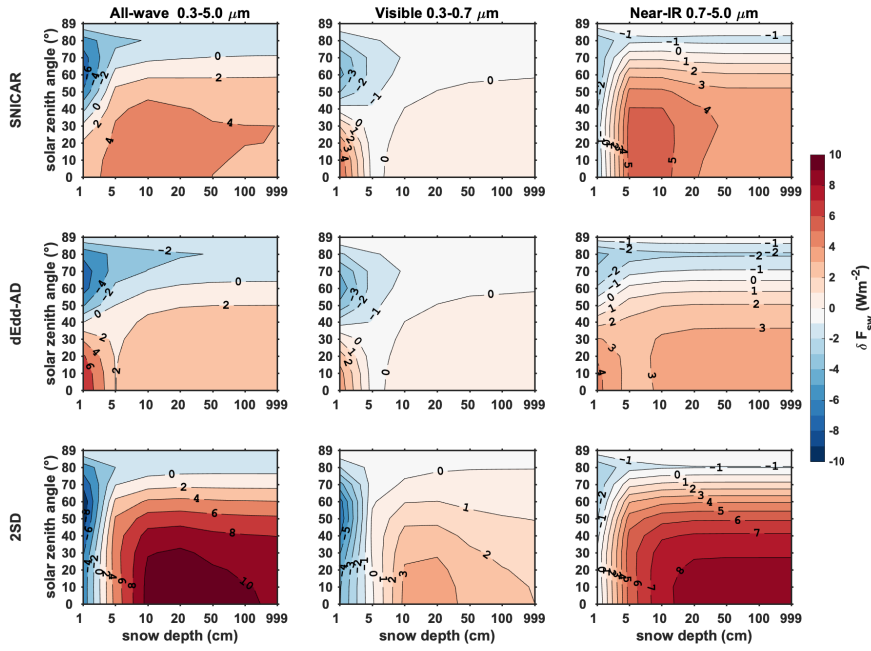
1173

1177 Figure 7. Error in reflected direct solar flux given albedo errors shown in Figure 3.



1178  
1179

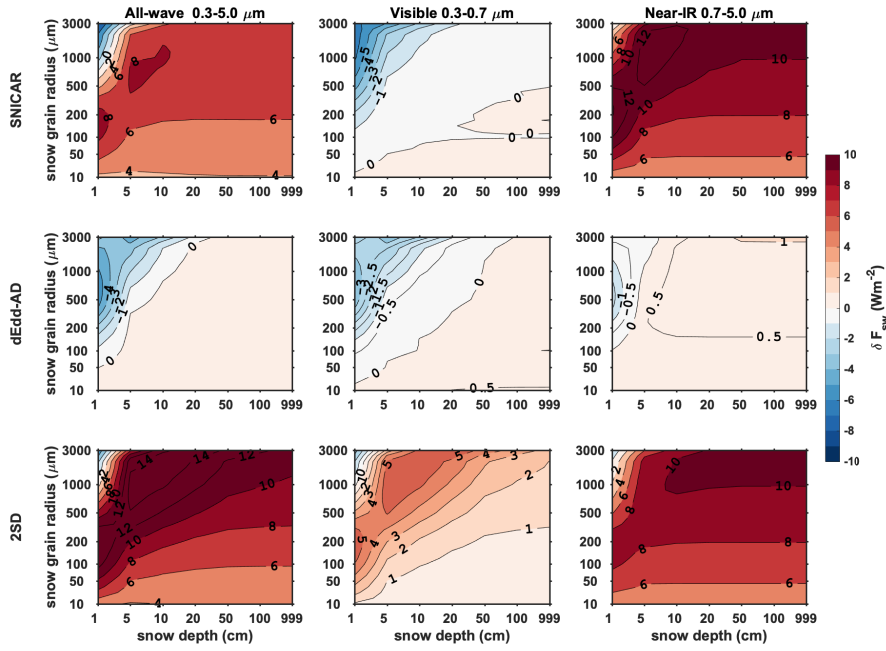
1180 Figure 8. Error in reflected direct solar flux given albedo errors shown in Figure 4.



1181  
1182  
1183

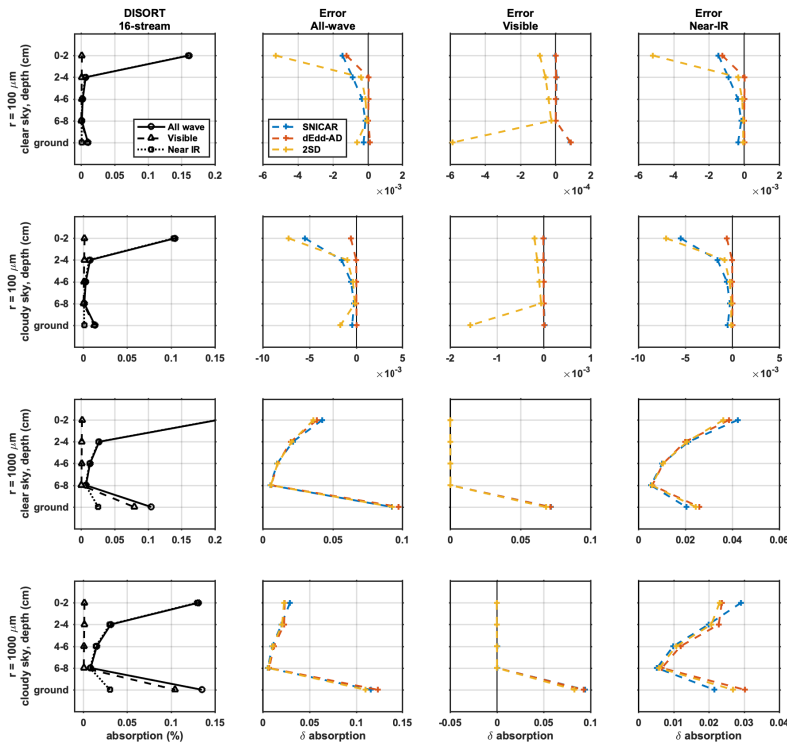


1184 Figure 9. Error in reflected diffuse solar flux given albedo errors shown in Figure 6.



1185  
1186  
1187

1188 Figure 10. Comparison of light-absorption profiles derived from two-stream models and  
 1189 | 16-stream DISORT. The left-most column shows fractional band absorptions computed  
 1190 using 16-stream DISORT. The right three panels show the errors of all-wave, visible, and  
 1191 near-IR fractional absorptions calculated using two-stream models. The top and bottom  
 1192 panels are for clear-sky and cloudy-sky conditions (solar zenith angle of  $60^\circ$ ),  
 1193 | respectively. The snowpack is 10 cm deep, and is divided evenly into five 2-cm thick  
 1194 layers, for new snow ( $r = 100 \mu\text{m}$ ) and old snow ( $r = 1000 \mu\text{m}$ ). The layers 1-4 represent  
 1195 the top four snow layers (top 8 cm), and layer 5 represents underlying ground with albedo  
 1196 of 0.25.

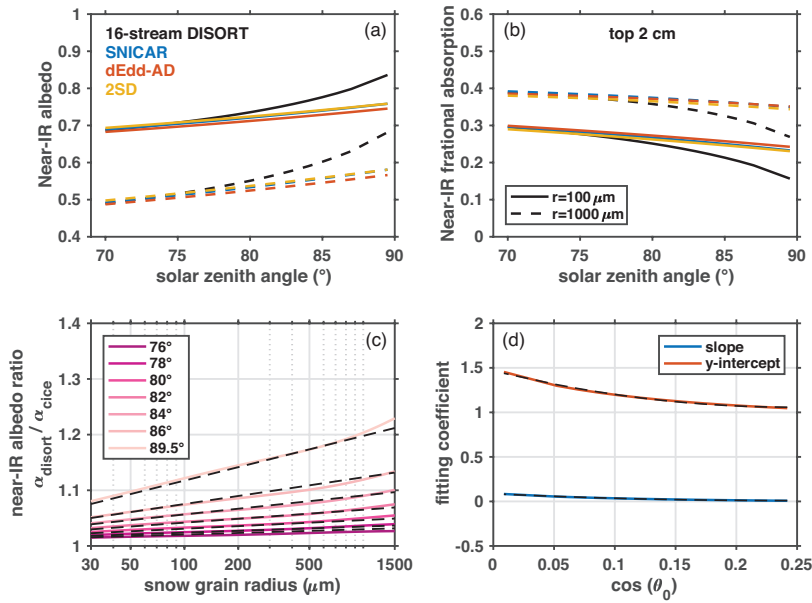


Author  
 Deleted: .

1197  
 1198

1200  
1201  
1202  
1203  
1204  
1205  
1206  
1207

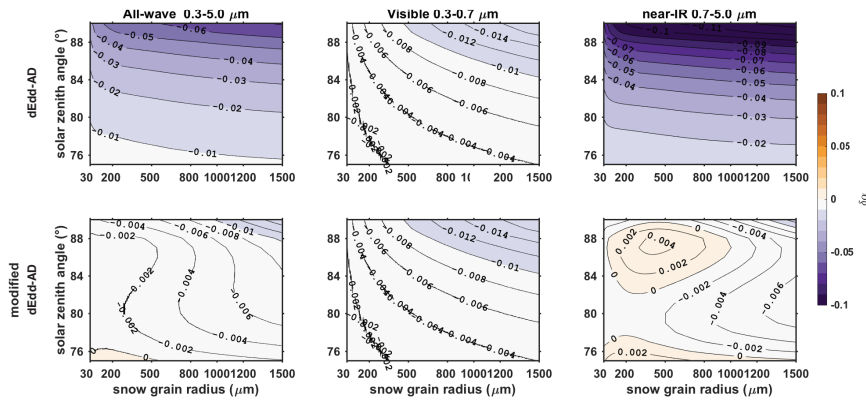
Figure 11. (a) Direct near-IR snow albedo and (b) near-IR fractional absorption by top 2-cm snow of a 2-m thick snowpack, for solar zenith angles larger than  $70^\circ$  and snow grain radii of  $100\ \mu\text{m}$  and  $1000\ \mu\text{m}$ . (c) The ratios of near-IR albedo computed using CICE to that computed using 16-stream DISORT for different solar zenith angles. These ratios are parameterized as linear functions of the logarithmic of snow grain radius. The slopes and y-intercepts are shown in (d). The black dashed curves in figures (c) and (d) are fitting values computed using parameterization discussed in Section 5.



1208  
1209  
1210  
1211

1212 | Figure 12. Error in semi-infinite snow albedo computed using [dEdd-AD](#), before (top row)  
 1213 and after (bottom row) incorporating corrections for near-IR albedo, for different solar  
 1214 zenith angles and snow grain radii.  
 1215

Author  
 Deleted: CICE



1216  
 1217  
 1218  
 1219

1221 Table 1. Acronyms used in this paper and their references.  
 1222  
 1223

ESM/ESMs	Earth System Models	
E3SM	Energy Exscale Earth System Model	Global climate model, previously know as ACME, <a href="https://e3sm.org/">https://e3sm.org/</a>
CESM	Community Earth System Model	Global climate model, <a href="http://www.cesm.ucar.edu/">http://www.cesm.ucar.edu/</a>
CCSM	Community Climate System Model	Global climate model, <a href="http://www.cesm.ucar.edu/models/ccsm4.0/">http://www.cesm.ucar.edu/models/ccsm4.0/</a>
RACMO	Regional Atmospheric Climate Model	Regional model, <a href="https://www.projects.science.uu.nl/iceclimate/models/racmo.php">https://www.projects.science.uu.nl/iceclimate/models/racmo.php</a>
CAM	Community Atmospheric Model	Atmospheric model, Neale et al., 2012
ELM	E3SM land model	Land model of E3SM, <a href="https://e3sm.org/model/e3sm-model-description/v1-description/">https://e3sm.org/model/e3sm-model-description/v1-description/</a>
CLM	Community land model	Land model of CESM, <a href="http://www.cesm.ucar.edu/models/clm/">http://www.cesm.ucar.edu/models/clm/</a>
MPAS-seaice	Model for Prediction Across Scales Sea Ice	Sea-ice model of E3SM, Turner et al., 2018
CICE	Los Almos Sea Ice Model	Sea-ice model of CESM, Hunke et al., 2010
RRTM	Rapid Radiative Transfer Model	Standalone column radiative transfer model, Mlawer and Clough, 1997, <a href="http://rtweb.aer.com/rrtm_frame.html">http://rtweb.aer.com/rrtm_frame.html</a>
RRTMG	Rapid Radiative Transfer Model for GCM components	Modified RRTM for GCM application, Iacono et al., 2008, <a href="http://rtweb.aer.com/rrtm_frame.html">http://rtweb.aer.com/rrtm_frame.html</a>
DISORT	DIScrete-Odinate Radiative Transfer model	Standalone column radiative transfer model, <a href="http://llab.phy.stevens.edu/disort/">http://llab.phy.stevens.edu/disort/</a> , Stamnes et al., 1988
SWNB2	Shortwave Narrowband Model	Standalone column radiative transfer model, Zender et al., 1997; Zender, 1999
SNICAR	SNow ICe and Aerosol Radiative module	Snow module used in ELM and CLM, Flanner and Zender, 2005; Toon et al., 1989

dEdd-AD	Two-stream delta-Eddington Adding-Doubling radiative transfer algorithm	Sea-ice radiative transfer core in MPAS-seaice and CICE, Briegleb and Light, 2007
2SD	Two-Stream Discrete ordinate radiative transfer algorithm	Radiative transfer algorithm tested in this work, Jin and Stamnes, 1994
SNICAR-AD	SNICAR – Adding Doubling	Hybrid snow/sea-ice radiative transfer model, Section 8
SSP/SSPs	Single-Scattering Properties	Single-scattering albedo $\varpi$ , asymmetry factor $g$ , extinction coefficient $\sigma_{ext}$
near-IR	Near Infrared band	Wavelengths of 0.7 - 5 $\mu\text{m}$

1224  
1225 |

1226  
 1227  
 1228  
 1229  
 1230

Table 2. Two-stream radiative transfer algorithms evaluated in this work, including algorithms that are currently implemented in Earth System Model CESM and E3SM.

ESM Component	Land	Sea Ice	
Model	SNICAR	<a href="#">dEdd-AD</a>	2SD
Radiative transfer approximation	two-stream $\delta$ -Eddington (visible) $\delta$ -Hemispheric-mean (near-IR)	two-stream $\delta$ -Eddington	two-stream $\delta$ -Discrete-ordinate
Treatment for multi-layered media	matrix inversion	adding-doubling	matrix inversion
Fresnel reflection/refraction	no	yes	yes
Number of bands implemented in ESMs	5 bands (1 visible, 4 near-IR)	3 bands (1 visible, 2 near-IR)	
Applies to	snow	bare/ponded/snow-covered sea ice, and snow	bare/ponded/snow-covered sea ice, and snow

Author  
 Deleted: 1

Author  
 Deleted: CICE/MPAS-seaice

1231