

Response to Reviewer 2

We would like to thank the Reviewer for useful comments that helped us to significantly improve the manuscript.

MAJOR COMMENTS:

Reviewer: *Applying adjoint methods especially in the sea ice model is a difficult work and the readers are keen to see if there really are some advances on this field. The analytic differentiation as reported by the authors, the damping term and even the codes should be publicly accessible at least from the appendix or the supplement materials, though there is something still not clear for me, but they are not.*

Reply: Following the Reviewer's request, we now provide a detailed description of the TL model in the Appendix A. We provide the description of the part of the ADJ model as well. The full adjoint model operator is the transposed to the TL model operator and can be derived easily.

Reviewer: *the manuscript is not well-prepared. It seems to be a draft on its first version. The context is little bit tedious on some unnecessary parts from my feeling and ignores too many details that, however, should be elaborated. I guess the co-authors even did not really go through the manuscript, let alone help to improve the text. Too many small grammar mistakes that, however, can be easily corrected by grammar check in MS word or spelling check if you use the Latex! All your citation styles in the text should be also taken care of.*

Reply: The revised manuscript is thoroughly corrected. We also strongly apologize for the misspelling issue. We current version of the manuscript was checked by two of our native English co-authors.

MINOR COMMENTS:

Reviewer: *L24: About abbreviations such as SIM, SI, LFI:, I indeed find it does not improve but reduce the readability of the text.*

Reply: The number of abbreviations was reduced significantly. In particular we removed abbreviations SIM, SI and LFI from the revised version of the manuscript. .

Reviewer: *L33: As above, the citation style. Add '.' after 'al'.*

Reply: Corrected.

Reviewer: *L35: 'the sea ice rheology is defined by ...' needs to be rephrased. I think the word 'defined' is not proper*

Reply: Corrected. **Line 39**

Reviewer: *L56-58: Better to remove this paragraph. I did not find any connection with the context. The stochastic parameters are locally varied, but this is actually another story when stochastic effects are considered.*

Reply: The paragraph was removed.

Reviewer: L90, 95: what are the div and tr? Please state clearly in the text!.

Reply: The notation is clarified (**line 118** of the revised manuscript). Eq. (6) was reformulated in terms of the trace operator only to remove the necessity of introducing the determinant.

Reviewer: L117: Please break this long sentence into shorter ones and elaborate how did you deal with the analytic differentiation of the equation in the appendix. I also wonder how is the Δ , which is highly non-linear, be processed.

Reply: The sentence is rephrased. The new Appendix A describes the numerical scheme and the TLA codes structure in much more detail.

Reviewer: L130: About the 'TL code', since the model is not such complicated, please make all your 'TL code' publicly accessible for better reproducibility for the community.

Reply: The current version of the manuscript includes a more detailed description of the TLA models. Full adjoint code can be easily derived by transposition of the operator of the TL model. We now make a detailed outline of the respective procedure in the Appendix. Regarding the public access, the NRL regulations imply that the codes could be obtained only after filing an official request in the NRL security system.

Reviewer: L144: Regarding the 'spatial spectrum', it's not clear that what kind of spatial spectrum you refer to.

Reply: We meant the local spectrum of the sea ice thickness (SIT) component of the state vector in the direction orthogonal to the ridges. The clarifying correction has been made (**line 168-180**).

Reviewer: L148: Regarding the 'Newtonian friction term', please implicitly show the equation and the damping time scales that you used

Reply: Now described in the Appendix A (**equations A18-A20**). The damping scales are given can be found in lines **182-183** of the current version of the manuscript.

Reviewer: Figure 1: Please consider to use dotted line. it can obviously show how many experiments you did.

Reply: Done.

Reviewer: Section 2.3: I would significantly simplify this section, since only the observation errors are used. You do not need to introduce all these. When I read this section, I was thinking about the experiments are dealing with the real observations. But actually, I think for the ideal experiments, these observation errors only set a reference.

Reply: We do not completely agree with the Reviewer, because specifying observational errors is critical for correct formulation of the OSSEs. In particular, the weights of various model-data misfit terms in the cost function are inversely proportional to the errors of the respective observations. Therefore, we specify error levels similar to those in the real observations and provide detailed estimates of the errors in satellite products that are widely used in sea ice data assimilation systems. We underline this on the first two lines of this section (**Line 206-207**).

Reviewer: L224: the assimilation window is really short. I just wonder if the experiment results show sensitivities on the assimilation window.

Reply: We now put more emphasis in articulating the objectives of the study focused on the improvement of the short-term ice forecasts (**lines 96-100, 485-494**) in the ice pack and near the ice edge. These regions are subject to variability at time scales of several days, so 3-day

data assimilation window looks reasonable. Additional experiments with longer (5-day) window demonstrated similar results.

Reviewer: L232: what is ' ω '?

Reply: The sentence text has been changed to remove ω and improve the clarity of presentation.

Reviewer: L235: what is 'DAS'?

Reply: Abbreviation removed.

Reviewer: Section 3.1: the configuration of the experiment is not clear. For example, is the initial SIC condition symmetric? It seems not from Figure 2a. How is the boundary condition? And in the text, the authors should explain why the spatial distribution of the polynya is not symmetric over the y-axis. Coriolis effects or the initial condition effects?

Reply: The initial conditions for SIC and SIT were symmetric. Emergence of the noise-like features after 3 days of integration and some asymmetry for the day 0 in the previous version of the manuscript were due to several reasons:

- SIC/SIT initial conditions along the northern and southern boundaries had the form of a narrow tongue (1 grid point wide and 3 grid points long).
- Dispersive properties of the Lax-Wendorff advection scheme.
- Effects of the Matlab function PCOLOR utilized for plotting. This function inherently use cubic spline which produce some artificial "noisy" features.

The polynya is non-symmetric due to the Coriolis force. We added respective comments (see **lines 259-256, 265-266**) in the revised version of the manuscript and slightly modified initial SIC and SIT in this experiment:

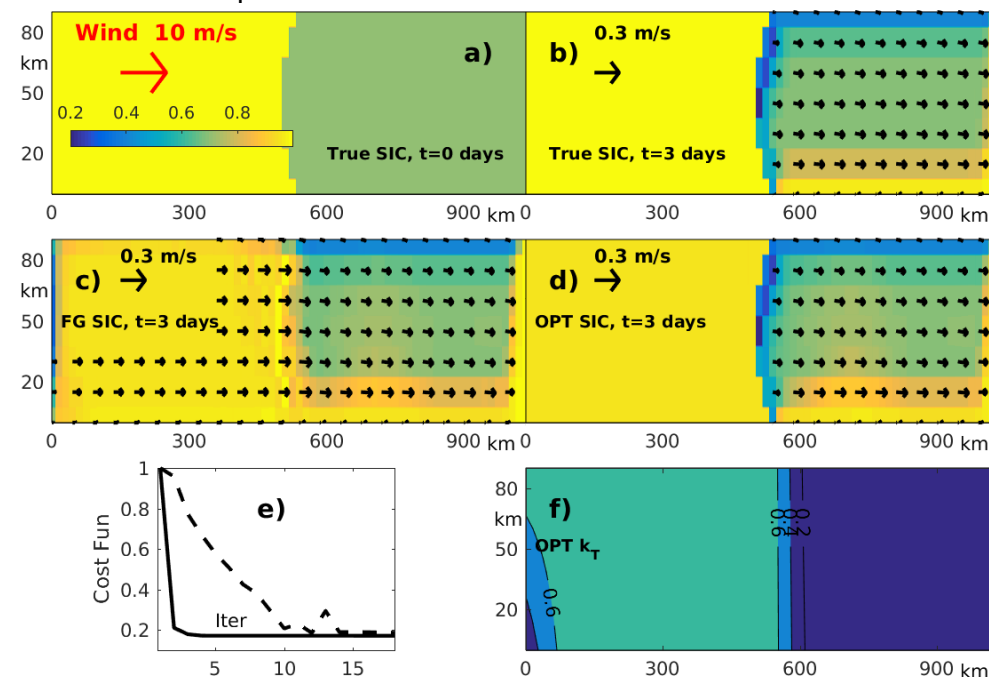


Figure 1 (new figure 4)

Reviewer: L275: GYRE-0/W. Elaborate the meanings of the abbreviations

Reply: The meaning of these abbreviation is explained now in **lines 239-242**.

Reviewer: L278: *It's not clear why you use such weird initial SIC distribution*

Reply: We use the same kind of the sin function as we used in our previous publication Stroh et al, (2019), but two-dimensional. It allows to have regions with high/low concentration and thickness simultaneously.

Reviewer: L293: *Please say clearly how you mimic the realistic observations, just their magnitude?*

Reply: Our “synthetic” observations is a sum of the observations derived from “true” solution plus some noise. The magnitude of noise had the realistic values discussed in Section 2.3. Some additional clarification was added to the manuscript (**Lines 345-349**)

Reviewer: L310: *I wonder whether you optimize C_{rh} first then the initial conditions, you could get the same conclusion.*

Reply: In strongly nonlinear inversions uniqueness of the solution cannot be guaranteed, because the cost function may have multiple minima and the optimized solution in this case depends on the first guess values of the control variables and the initial descent direction. In our case, finding a physically sensible first guess control vector is a necessity, which was realized in the form of three-step minimization.

Reviewer: L338: *'the western part agrees well with true e distribution'. Actually, only part of. I think the authors could just say something like “ show parts of agreement”. L343: “and therefore has a minor rheological impact of the sea ice dynamics”. That is not the case, as most parts in fig 5b still have $SIC \geq 0.95$.*

Reply: We meant a decrease of the impact in the regions with $SIC < 0.8$ where the RPs are very difficult to reconstruct from surface observations of SIC, SIT and SIV. Design of the PIZ experiments had the major incentive to have a closer examination of observability of RPs in pack ice. The respective clarification has been made (**lines 395-400**).

Reviewer: L414: *the authors never defined RMSE*

Reply: Corrected (**lines 493-494**)

Reviewer: L431: *I do not know why it is worth to address the realistic observation errors are used.*

Reply: We consider that our final goal is to develop the 4Dvar data assimilation system for the sea ice model which will be capable to retrieve rheological parameters from *realistic* observations. Because of that we are trying to underline, that using available SIV/SIC observations with realistic error bars and SIT observations with twice smaller (0.3m) errors than currently available, can be successfully utilized for this purpose. We suggest that that accurate SIT observation are already available for some moorings and will be available from MOSAiC and from the future satellites.

Reviewer: *Conclusions: it's currently too long. Please try to simplify what you really want to say.*

Reply: Actually, this section contained *both* “Conclusions and Discussion”, so we changed the title accordingly. Other reviewers recommended to include more discussion here, so the section was expanded.

Reviewer: *Spelling/language corrections (lines 66, 101, 110, 194, 292, 307, 318, 368, 388).*

Reply: All corrected.