The Cryosphere Discuss.,
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Interactive comment on "Brief communication: Sampling c-axes distributions from the eigenvalues of ice fabric orientation tensors" *by* Martin Rongen

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Dear Referee,

Thank you for your timely and detailed review. Please find the responses to the issues raised in-line with your review comments below:

This brief communication presents a sampling method to build discrete c-axis distribution for given eigenvalues of the second-order orientation tensor using a superposition of girdle and single maximum fabric given as a Watson distribution.

Even if the proposed method is certainly interesting, the glaciology context is clearly missing. A number of previous works have already been done on that subject and are not mentioned in this paper. For example:

- in Gagliardini et al. (2009), a comprehensive list of the PDFs that have been proposed in the literature to describe polycrystalline fabric is given and they are all compared in their capability of representing observed fabrics.
- the form proposed in this paper was already proposed as a good possible representation of ice fabric by Lliboutry in 1993.
- Gillet-Chaulet et al. (2005), with the same objective as in the current paper, have presented a method to construct a discrete fabric for given eigenvalues of the second orientation tensor assuming a parameterised PDF derived from an analytical solution and capable to describe directly orthotropic fabrics (without the need of superposition of two PDFs restricted to transversally isotropic fabrics as in the proposed approach).

In other words, there are clearly missing references to give an appropriate context of what have already been done on that subject in glaciology (and certainly other than the three listed here).

> While this brief communication article was intended as a minimalistic methods description, I appreciate that some context about previous work in the field is needed. As the referee may have recognized, my primary occupation is not in glaciology, so I am thankful for

the provided references. Working through them I see the following context:

Gagliardini et al. (2009) summarizes a number of commonly used PDFs, including the original work by Lliboutry. It shall be mentioned to provide general context. The Watson distribution is not explicitly mentioned in this publication.

The Lliboutry (1993) paper proposes the use of a Fisherian distribution and of a distribution given by $f_\nu(\theta) = \nu \cos^{\nu-1}\theta$. It does not mention a Watson distribution or combinations of Watson distributions and as such does not include the form proposed in this paper.

The Gillet-Chaulet et al. (2005) paper is very interesting and indeed pursues the same objective. Yet in using a conjugate gradient method to converge from an initial (arbitrary) c-axis distribution to a distribution resulting in the desired eigenvalues is computationally inefficient. In the application for which the presented method was developed (photon propagation through a birefringent polycrystal), millions of c-axes need to be generated on the fly, rendering the method described by Gillet-Chaulet et al. inapplicable.

Branching out from the provided references, the Kennedy et al. (2013, doi: 10.3189/2013JoG12J159) publication is a nice example of a previous use of a single Watson distribution to generate crystal distributions. The innovation in the presented manuscript shall be understood as the proper analytic combination of two Watson distributions, so to yield arbitrary eigenvalues of the second order orientation tensor."

Based also on additional comments from the other referee, the following section shall be added to the introduction:

> "Probability density distribution functions (PDFs) previously used to sample c-axes have for example been summarized in Gagliardini et al. (2009). These include most prominently the Fisher distribution and its restricted form as proposed for use in glaciology by Lliboutry (1993). Both distributions are only applicable when describing a single maximum and can not be used for axially symmetric girdle fabrics. A single Watson distribution, to describe either a girdle or a unimodal fabric, has already been successfully used for example by Kennedy et al. (2013). Gillet-Chaulet et al. (2005) presented a conjugate gradient method which, starting from an arbitary c-axis distribution, in-time converges to a distribution which can describe any orthotropic fabric. While generally applicable, the employed method is computationally inefficient and as such undesirable for the intended purpose. As a result we present a method based on the combination of a vertical girdle and a single maximum Watson distributions, which is computationally efficient and can reproduce arbitrary eigenvalues of the second order orientation tensor."

In addition the introduction of the Watson distribution in section 3, page 3 shall be extended to:

Of the presented PDFs the Watson (1965) distribution, as also for example used by Kennedy et al. (2013), seems most applicable for our case as ...

Over the three figures, Fig. 1 is from Woodcock (1977) and Fig. 2 is from Duncan Campbell (from his github). Did the author get the authorization to replicate these figures in his paper?

Thank you for pointing out this issue. I shall discuss the two figures with the editor and replace or drop them.

Regarding the result of the method, I don't really understand why the uniform fabric presented in Fig. 3a has not $S_1 = S_2 = S_3 = 1/3$ exactly. This should be possible? It is surprising that it is for the simplest fabric (uniform) that there is the largest differences between the input and output eigenvalues.

The fabric input which is to be recovered in Figure 3a is setup to not be perfectly uniform, but to be $\ln(S1/S2) = \ln(S2/S3) = 0.1$, which is very closely recovered. Attached please find a figure which indeed reproduces an ideal uniform fabric. As it seems the chosen example for Figure 3a has lead to some confusion, the pure uniform fabric shall be adopted for the new manuscript revision.

As discussed at the end of the paper, it seems that the assumption that a natural fabric can be described by the superposition of a purely girdle and a purely unimodal Watson distribution is a bit strong and doesn't work especially for not textured fabric (i.e. fabric close to a uniform distribution).

> When aiming for a perfectly uniform distribution ($S_1 = S_2 = S_3 = 1/3$) the k-parameter calculation for the the Watson distribution (equations (3) and (4) will yield exactly 0 in both cases. And the sampled distribution will be exactly uniform in the limit of infinite statistics. Also see the answer above and the attached figure.

I would have like to see an inverse approach showing how a real fabric can be described using the superposition of two transversally isotropic PDFs, as done in Gagliardini et al. (2009).

> To be honest I do not see much added value from such a test. The limitations of the methods should be clear at that point and the result of the comparison (as for example done in Figure 2 of Gagliardini et al. (2009)) will primarily depend on the chosen dataset (as the functional form of the PDF is fixed).

Minor remarks:

- page 1, line 8: polycrystalline ice will most

  This will be changed to the referee's recommendation.

- page 2, line 11: I don't understand the "strict" ordering (eigenvalues can be all equal for a uniform distribution). It should write $S_1 \geq S_2 \geq S_3$.

  They will never be numerically identical in any natural system.

But to also include this edge case, I will change this to the referee's recommendation.

- page 2, line 24: the citation should be Voigt (2017). Donald is the first name, not the family name. Same at other places and in the references section.

  Indeed. This will be corrected.

Gagliardini O., F. Gillet-Chaulet and M. Montagnat, 2009. A Review of Anisotropic Polar Ice Models: from Crystal to Ice-Sheet Flow Models. In "Physics of Ice Core Records II", Supplement Issue of Low Temperature Science, Vol. 68, December 2009.

Gillet-Chaulet F., O. Gagliardini, J. Meyssonnier, M. Montagnat and O. Castelnau,2005. A user-friendly anisotropic flow law for ice-sheet modelling. J. of Glaciol., 51(172), p. 3-14.

L. Lliboutry. Anisotropic, transversely isotropic non linear viscosity of rock ice and rheological parameters inferred by homogenization. Int. J. Plast., 9:619–632, 1993.

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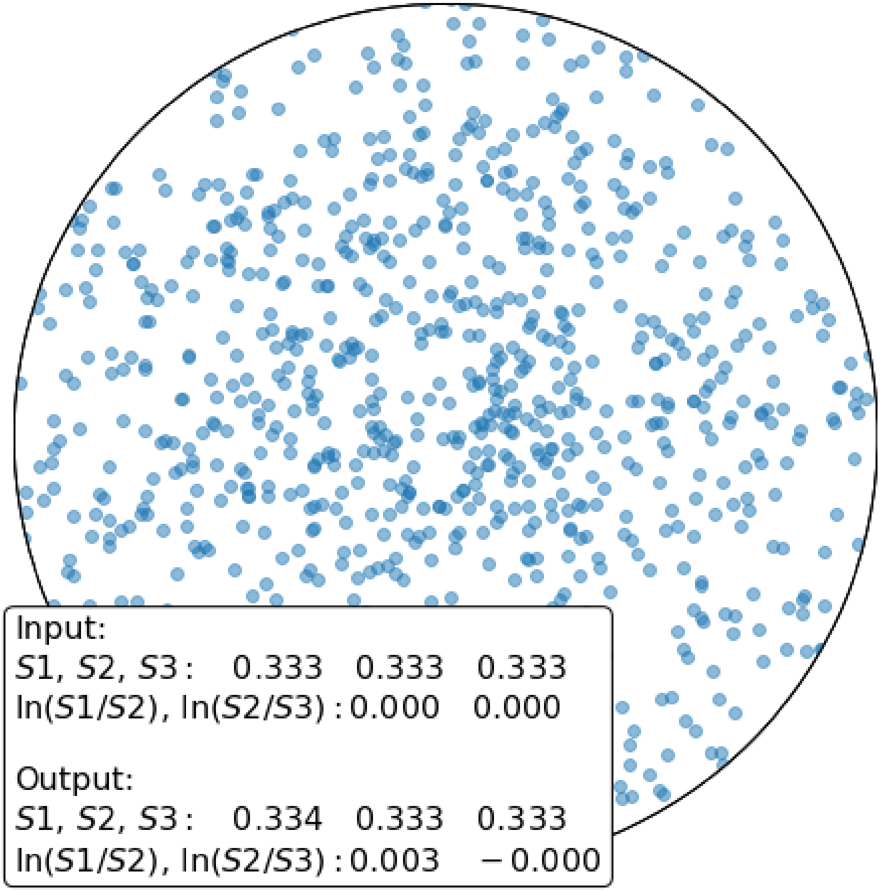

Fig. 1.

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Discussion paper