Budgets

Energy Budget

Energy fluxes in ISBA-DEB are the same as those in ISBA, with the exception of an additional energy flux for melting the glacier (M_{ice}). The energy budget of the debris is

$$\int_{z=z_b}^{z=0} c_g \frac{\partial T_g}{\partial t} dz = \Phi_g + p_{sn} G_{gn}{}^{*} + (1 - p_{sn})(R_n - H - LE) - M_{ice} \tag{11}$$

* In the model, G_{gn}, the net ground flux, is represented by $(G_{gn} + G_{n,corr})$. The additive term is a numerical correction term that also serves to incorporate the effects of a disappearing snowpack on the energy budget.

The left hand side of equation 11 represents the change in total energy stored with the debris, integrated vertically through all k debris layers (total N_g layers). This is a function of the volumetric heat capacity ($c_{g,k}$, J/m^{3}K), layer thickness (Δz), time (t), and temperature ($T_{g,k}$, K).

$$\int_{z=z_b}^{z=0} c_g \frac{\partial T_g}{\partial t} dz = \sum_{k=1}^{N_g} \frac{c_{g,k} \Delta z_k}{\Delta t} \left(T_{g,k}^{t+\Delta t} - T_{g,k}^{t} \right) \tag{12}$$

Φ_g (W/m^2) gives the total latent heat flux due to phase changes in the debris (i.e. freezing and thawing). p$_{sn}$ gives the fraction of the model grid cell comprised of snow such that the heat conduction flux at the snow-debris interface (“ground," G_{gn}, W/m^2) is multiplied by this fraction and the net radiative, sensible, and latent heat fluxes (R_n, H, and LE, respectively) at the snow-free debris surface are multiplied by the share of the surface that is snow-free ($1 - p_{sn}$).

Incorporating snow gives a total energy budget that contains the enthalpy (i.e. heat content) of the snowpack, including the enthalpy added by any snowfall during the timestep. When combining the energy budgets of debris and snow, the G_{gn} term cancels.

The daily average energy budget for winter is dependent on the absence (Figure S1a) or presence (Figure S1b) of snow and varies from that of summer (Figure S1c).

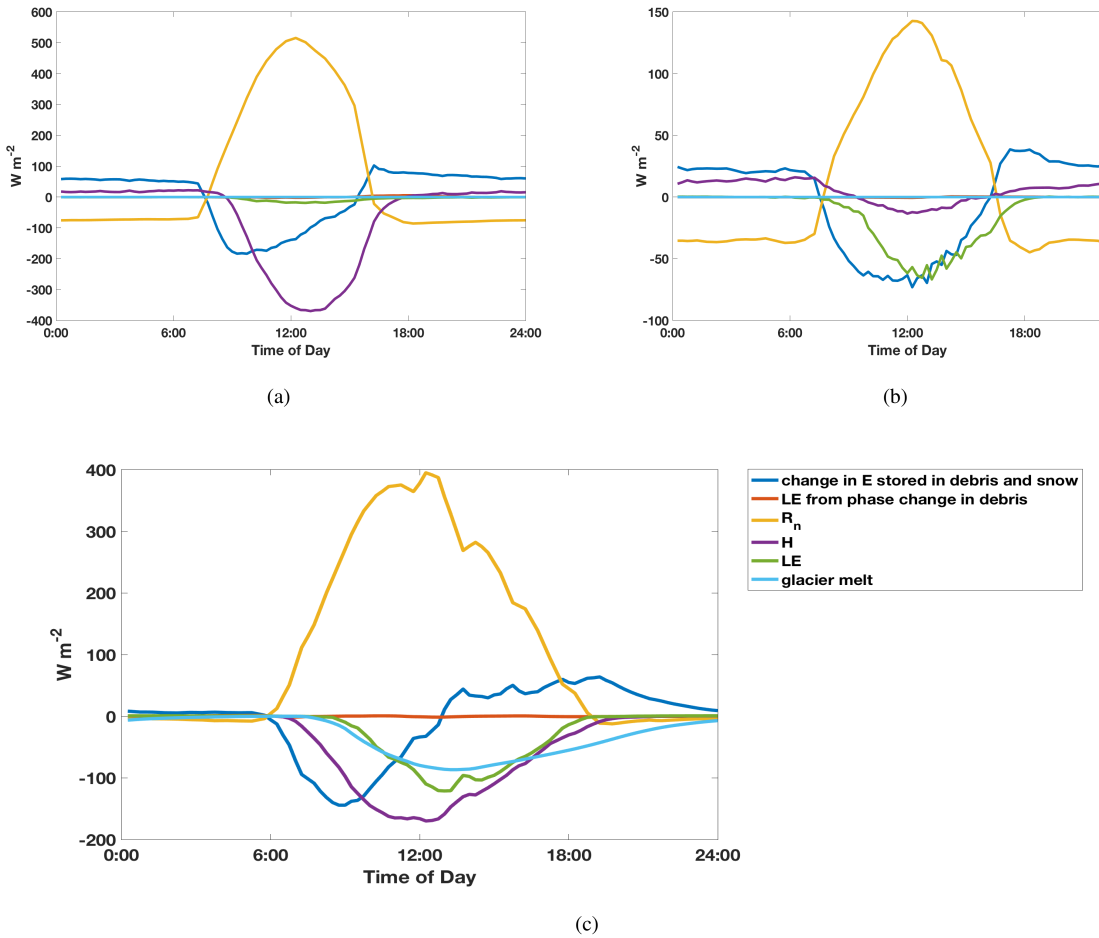

Figure S1. Daily average winter energy fluxes when ground is (a) bare and (b) snow-covered and daily average summer energy fluxes (c), using the convention that flux is positive when directed toward the surface. Bare winter surface (a) is 40 days 1 January – 2 February 2014, snow-covered winter surface (b) is 40 days 16 February – 26 March 2013, and summer surface (c) is 1 – 31 July 2013. Note that in both (a) and (b), glacier melt never occurs; in all three periods depicted, phase change within the debris does not supply a flux. The y-axis of (b) is considerably less than that of (a), which is explained by the increased albedo of snow relative to debris (the net shortwave radiation is much lower when a snow cover is present). Time of Day is given in local Nepal time.

Water Budget

Similar to the energy budget, the water budget in ISBA-DEB is identical to the one in ISBA but includes the additional M_{ice}. The change in water stored in debris, $\frac{\partial W_g}{\partial t}$, is

$$\frac{\partial W_g}{\partial t} = Q_s + \frac{M_{ice}}{L_m} + (1 - p_{sn})\, P_r - D_r - R - (1 - p_{sn}) \left(\frac{LE_g}{L_v} + \frac{LE_{gu}}{L_s} \right) - E_{n,corr} \tag{13}$$

Here, P_r is rain falling on the bare debris, Q_s is snowmelt entering the debris, p_{sn} is the fraction of the debris gridcell covered by snow, LE_g is evaporation from the debris without snow cover, LE_{gi} is the sublimation from the debris without snow cover, R is runoff, and D_r is the drainage (included only for completeness; $D_r = 0$ in ISBA-DEB). $E_{n,corr}$, similar to $G_{n,corr}$ above, is a numerical correction term to ensure a closed mass budget when a snowpack entirely vanishes within a timestep. L_v, L_s, and L_m are the latent heats of vaporization, sublimation, and fusion, respectively, as listed in Table A1. The total water content in the debris is

$$W_g = \rho_w \sum_{k=1}^{N_g} (w_{g\,,k} + w_{gi\,,k})\, \Delta z_k \tag{14}$$

where w_g and w_{gi} are volumetric liquid water and liquid-water-equivalent volumetric ice content in the debris, respectively, and ρ_w is the density of liquid water.

Incorporating snow into equation 13 gives

$$\frac{\partial W_n}{\partial t} + \frac{\partial W_g}{\partial t} = \frac{M_{ice}}{L_m} + P_s + P_r - D_r - R - (1 - p_{sn}) \left(\frac{LE_g}{L_v} + \frac{LE_{gf}}{L_s} \right) - p_{sn} \left(\frac{LE_{sl}}{L_v} + \frac{LE_s}{L_s} \right) \tag{15}$$

Terms not previously defined are P_s, the snowfall rate in water equivalent, and the evaporation and sublimation from the snow: LE_{sl} and LE_s. The total water content of the snow is

$$W_s = \sum_{k=1}^{N_s} W_{s,k}, \tag{16}$$

where N_s is the number of snow layers and $W_{n,k}$ the snow water equivalent (SWE) for each snow layer k. Q_s and $E_{n,corr}$ do not appear in equation 15 because they cancel when adding the snow water budget to equation 13.

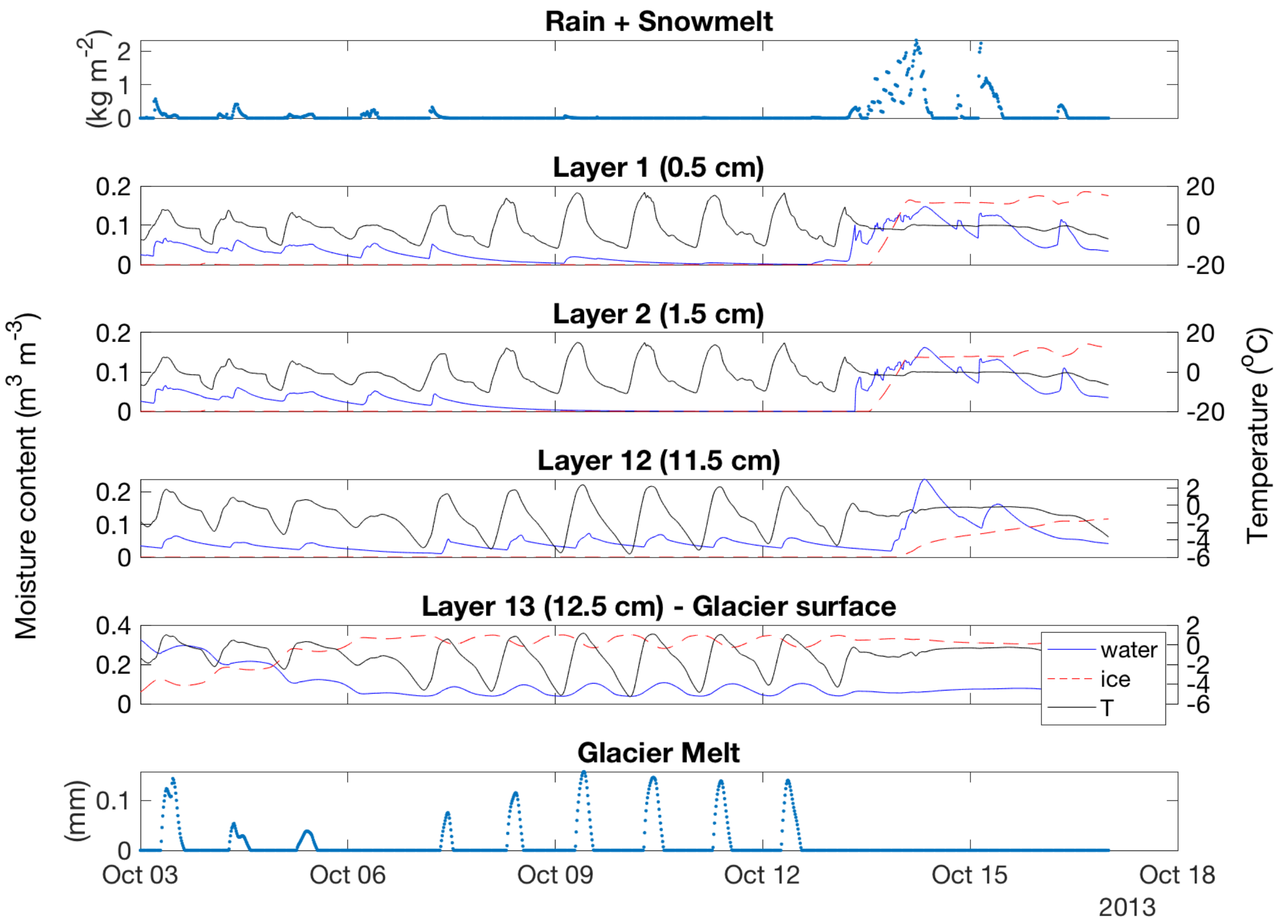

Figure S2. Time-evolving water content, ice content, and temperature of the top two and bottom two layers of debris, shown during a 2-week period when water enters the debris both at its surface (from rain and snowmelt, top panel) and at its base (from glacier melt, bottom panel). The phase of the moisture changes as a function of temperature. Note that the moisture and temperature y-scales vary between the layers.

Sensitivity

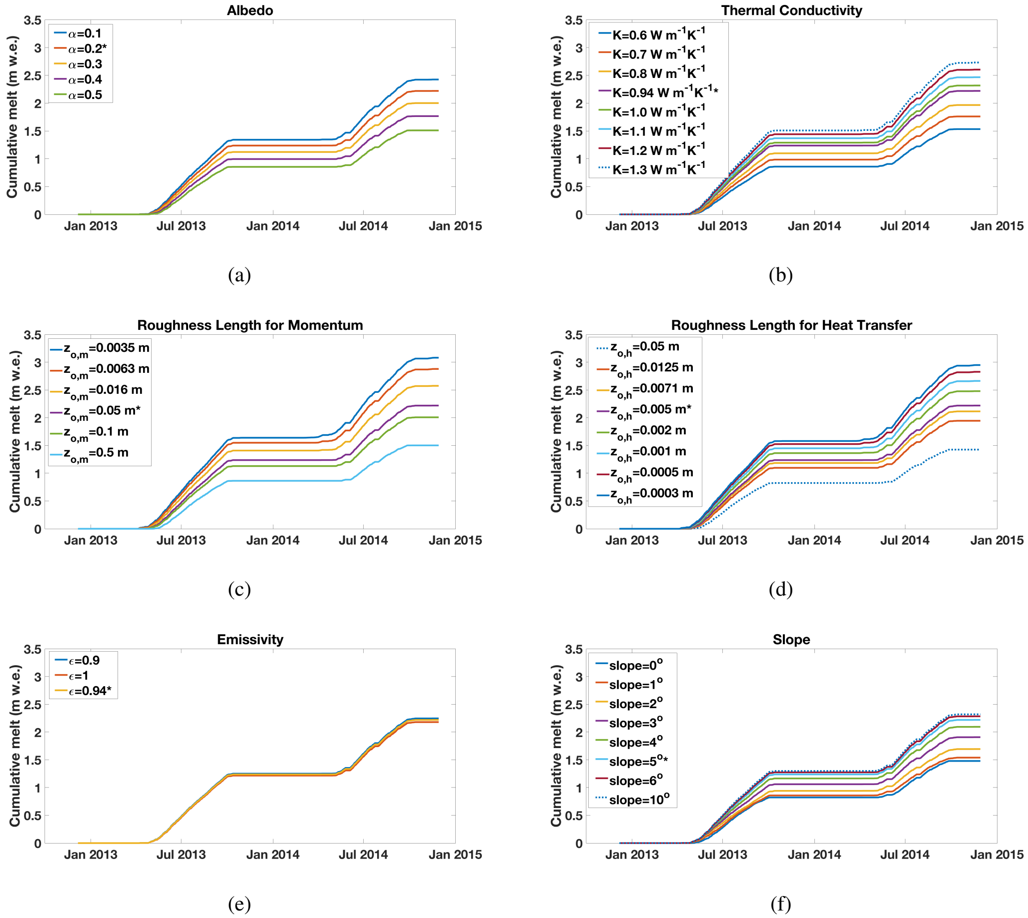

Figure S3. Cumulative glacier melt over the December 2012 – November 2014 period, run with different values of key parameters to test ISBA-DEB's sensitivity. An asterisk in legends indicates values used in the run demonstrating model behavior (Section 6.1), and sensitivities are quantified relative to melt simulated with these values in Table 3.