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Interactive comment on "An analysis of instabilities and limit cycles in glacier-dammed reservoirs" *by* Christian Schoof

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This paper investigates the mathematical properties of a conduit model (R-channel + one linked-cavity) when the upper boundary condition is a reservoir of fixed surface area and recharge rate. It looks at reservoir sizes corresponding a range from large glacier dammed lakes to moulins. It finds that for a given reservoir size that there are two stable regimes: one when the reservoir drains through the linked-cavity (i.e. recharge is low) and the other, when recharge is high, it drains through a moulin-like configuration. In-between the reservoir drainage is unstable and in fact periodic (for most situations), i.e. lake outburst floods. I think the paper nicely illustrates and investigates the range of behaviours to be expected from ice dammed reservoirs. Whilst this range of behaviours (leaking lake, out-bursting lake, and moulin-like) has been known,

it has not yet been described quantitatively; certainly not with this mathematical rigour. Thus this paper is a significant step forward. However, the paper is a bit on the technical side for a glaciological paper. This is nicely illustrated by the brutal subsection of the Discussion (Sec 5.2), where the un-expecting reader – suffering from formula-overload already – is again presented with a wall of maths. And all this "discussion" does not actually lead to any further notable points of Discussion. I recommend to publish this MS after Sec 5.2 has been banned to the supplement (or another publication) and the comments detailed below are addressed.

I would like to retain some version of the material in sec 5.2 here as opposed to offloading it to another paper, since such a paper would be rather stunted. The purpose of 5.2 was two-fold: first, the stability analysis leans heavily on that already presented in section 3.1, so keeping it in the same paper (overall, be that in the main text or supplementary material) makes sense to me. Secondly, the earlier paper by Schoof et al (2014) provides the main motivation for the present paper, and there we discuss not only outburst floods from localized reservoirs but also the effect of spatially extended storage, so it is natural to tie together the analysis of the mechanisms invovled in causing unsteady flow. I would propose a solution where most of section 5.2 goes into the supplementary material but I still sketch the main insights (mostly in words) in the discussion section, tying this better into the rest of the discussion.

The different triggering mechanism should probably be discussed a bit further. Of interest is in particular that many lakes drain with different triggering mechanisms from outburst to outburst (e.g. Grimsvötn 1996 vs other years, Gorner Lake (Huss et al., 2007)). In the case of Gorner Lake, no observations can predict the triggering mechanism.

This I am happy to do.

Further, Huss et al. (2007) also show that Gorner Lake can indeed leak before drainage. This should be mentioned alongside Fisher (1973).

Ditto

The paper by Kessler and Anderson (2004) should be discussed further, both in the Introduction and Discussion as it uses also a conduit model (linked-cavity + R-channel) and applies it to a lake drainage (their section 4.2). For instance, they also see the pre-drainage leakage.

Ditto

The model used has a single cavity, but it could also be used with many cavities in parallel. The supp. of Schoof (2010) does this. Mention briefly what the impact would be, I suspect it would only be quantitative.

Again, I am happy to do that. Things get a touch more complicated here and ultimately reveal the limitation of a 'lumped' model in the sense that a spatially extended (or 2D network model) predicts the expected formation of a single channel, whereas multiple identical conduits in parallel as in Schoof (2010) rather strangely predict that the stable limit cycle involves all conduits eventually behaving identically: the channelizing instability that causes one conduit to 'win' apparently does not work fast enough in the simplified, lumped-conduits-in-parallel world the comment refers to. Exactly why I have not figured out yet, but I can do a little further work to elaborate on this - most likely also to be banished to the supplementary material.

The fact that moulins are "small reservoirs" is only mentioned really late in the MS. Could/should this be mentioned earlier?

Sure

It is not clear to me why Appendix A is there but most other extra calculations are in the supplement, in particular as the more detailed calculation of Appendix A is also in the supplement.

Appendix A is there to make the paper a little more self-contained. I felt (and this perhaps aligns with referee # 1?) that the other material in the supplementary material is

really designed to fill in detail that a reader familiar with basic theory of dynamical systems could reconstruct, should they really want to try (in the terms of the other review, this is material that I'm including to help out the more "glaciological" reader, which a "mathematical" reader would probably not need). Appendix A is a brief summary of the material that goes beyond that level - it provides a brief sketch of results in the main paper that I expect would not be totally straightforward to reconstruct for a "mathematical" reader without undue effort.

Please run a spell checker over the MS!

Yes. This seems to be a common complaint, my apologies!

P1, L7: delete "a"

sure

P1, L16: mention that a lake drainage can also terminate when the lake is completely empty. This should be mentioned at a later stage as well, stating that this is not not relevant for this MS (the bed is flat).

Indeed; I was focusing on the "other", lower bound on N (zero) because it plays a role in initiation, instead of on the uppper (lake empty) bound because it aligns more closely with the main thrust of the paper, but fair point.

P1, L20 write: "magnitude and timing of the flood." As for hazard prevention knowing the timing is probably equally important to the magnitude.

Ok

P2, L13: "directly directed" is awkward

will omit "directly"

Eq 1a: state that the pressure dependence of the melting point is neglected

sure

Eq 1c: I find this equation strange. For vo (and vc) no separate equation vo = vo (S) is added either. Thus be consistent and just write q(S, Ψ) in Eq 1a & b. Similarly in all later equations.

The reason for doing this is in the form stated is that q is a variable, not a function (ie there is a $\partial q/\partial x$, which doesn't make sense if q is a function (not of x) — the inconsistency is rather in using the same letter for variable and function. I will change this.

P2: would it make sense to somewhere define what a "conduit", a "channel" and a "cavity" is? For example on P7,L4 "conduit" is used signalling the use of the vo term again. So an unexpected reader may trip there without a clear definition.

Sure, I will elaborate on this to the effect that a conduit is a generic drainage element, while channels and cavities are really equilibria of a conduit that have different qualitative properties. My mistake was probably being careless about the use of the word "channel" (e.g. p. 4, line 11) — I will now flag at the start of section 2 that I will use "conduit" consistently (using channel-like and cavity-like to describe situations in which there are dominant balances that are like those in classical channel and cavity models), and make sure that I actually follow that terminology consistently.

P4, L5: write "background hydraulic potential"

sure

P4, L24: "large lakes"

yes

P4, L28: This paragraph confuses me. Is this not obvious? If not, be explicit what is odd. If it is obvious, delete.

Nothing is odd here, but "obvious" is probably in the eye of the beholder. I put this in for the reader not as familiar with the theory of outburst floods

P5, L1: "in general"

indeed

Figure 0: A figure depicting the used geometry would be helpful.

Ok

P6, L28: "model"

yes

P7, L21: these "reservoirs" are, e.g., moulins. Why not state this here?

will do; there is of course the case of very large reservoirs too, which are probably not realistic but theoretically can be stabilized, so I don't want to make a one-to-one correspondence with moulins here.

P8 (on this page line numbers are messed up), L5+2: q(SĬD, NĬD)

indeed

P8, first un-numbered Eq: this should be v_o not v_0 . Or is $v_0 = v_o(S)$? If so state.

This is a simple typo.

P9, L24-29: this describes again a moulin

I am happy to mention moulins here, subject to the same caveat as above

Fig 1: replace "lake" with "reservoir"

ok

Fig 1: parenthesis missing after "3.3"

indeed

P11, L11: again v_o

correct

P12, L22–23: the "immediately" needs to be weakened here. According to fig. 3: lake is empty and starts filling at the point (70,10), then the lake is filling again but S still drops from 10m2 to âĹij0.

This is true, but the it takes a very short amount of *time* (rather than distance in phase space) for that gap in phase space to be covered. I will say 'almost immediately'

Fig.3: Split the second sentence at the "and"

Good idea

Fig.4: could the plotted Vp be added as horizontal lines to Fig. 6?

No doubt they can.

Fig.4: I don't understand what the line style "solid dashed" is supposed to be. I think the unstable periodic should be described as "dotted coloured"

Indeed. I have no idea what I was thinking. "Dotted coloured" it is.

Fig.4: "insets"

yes

Fig.5: zoom to relevant q_{in} values

Yes, I will get rid of the blank space on the left

Fig.6: it is not clear to me what is meant with "as well as in a small strip to the right of the right-hand branch of the red curve." nor is this intriguing strip ever mentioned in the text. Clarify. Maybe a zoomed inset?

I will clarify. This is the small range of q_{in} values in figure 4 where a limit cycle co-exists with a stable equilibrium (and an unstable periodic solution), see e.g. the inset in figure 4b.

Eq 16: should be "âĹij"

yes

P19, L16: "the its", delete "the"

yes

Fig 7: "(b), 10 km"

yes

Fig 7: "plotted in black" needs to be more specific. "plotted as black line"

indeed, will change

Fig. 8/9, P32, L5: In view of this model behaviour and the potentially unstable numerics, some words should be said about the employed numerical methods (spatial discretisation, time-stepper). Yes, the code is provided but this should be in the text.

I will make clear that the numerics are described in Schoof et al (2014, appendix)

Fig 9: What is $S_{\mathcal{R}}$?

It's an idiotic legacy notation error on my part; it should say S.

P23, L8: I don't see: "amplitude slowly grows" in fig 10

Ok, "slowly" may be an overstatement. "Grows and saturates" may be better

P25, L1: "reservoir" instead of "lake"

thanks for spotting this

P26: twice wrong reference to 2012 instead of 2014

yes

P26, L6: "results", P26, L21: "a spatial"

indeed

P27, L11: $\Re$ needs to be defined

Ok; would plain “Re” be ok as “real part”

P30, L5–9: remove if Sec 5.2 is removed

see above

Supplement: Excellent, that the code is published! Two things: (1) There should be a README in each zip file, stating at least which script needs to be run to produce which figures. (2) I would suggest to add a licence to each zip-file (preferably an approved open source licence, the BSD-licence is popular with Matlab files https://opensource.org/licenses/BSD-3-Clause). Then it is clear under which conditions the code can be used.

Will do

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