

Response to reviewers:
On the suitability of the Thorpe-Mason model for
calculating sublimation of saltating snow

Varun Sharma, Francesco Comola and Michael Lehning

August 31, 2018

A note to all reviewers

Please see the additional document found in the “comments_to_all.pdf” file

Response to Reviewer # 1

Opening Remarks:

We would like to thank Reviewer #1 for his/her detailed critique of the submitted manuscript and for asking different clarifications and questions. Broadly stating, the following additions and/or corrections were made to the article in response to Reviewer #1’s comments:

- We have enlarged Section 2 with a more detailed description of the dynamics of the heat and mass transfer from a solitary ice-grain and made clear the approximations entailed.
- A new figure is added that shows the evolution of particle diameter and temperature in Experiment I.
- A visual representation of one of the LES performed in Experiment III is provided to make clear, the sort of LES that have been performed.
- The supplementary material has been updated with 5 additional figures detailing various results from the LES.
- The caveats and limitations of the current LES model setup have been more explicitly mentioned in the updated manuscript and a few future directions of research have been listed in the expanded concluding section of the manuscript.

A: Scientific Concerns

- *A.1 : P1 L8 (and throughout): This does not appear to be a perturbation in a functional sense as you are not perturbing a system. This is more like a sensitivity analysis, changing initial conditions. There is ambiguity in this phrasing as a perturbation of 1 K can mean strictly a temperature difference of 1 K (which I believe you mean) or adding 1 K to the difference. I would suggest replacing perturbation so as to not mislead the reader into thinking*

they will be reading a manuscript using perturbation theory.

Response A.1: We thank the reviewer for this comment and agree that perhaps using the word “perturbation” is misleading. We modify the text as follows:

With a small temperature difference of 1 K between the air and the snow surface, the errors due to the TM model are already as high as 100% with errors increasing for larger temperature differences.

- **A.2 : P2 L1:** *There are actually three modes of transport, including creep.*

Response A.2: We agree with the comment and the text has been modified as follows:

Aeolian transport of snow can be classified into three modes, namely, creeping, saltation and suspension. Creeping consists of heavy particles rolling and sliding along the surface of the snowpack either due to form drag or bombardment due to impacting particles.

- **A.3 : P3 L20:** *Is this truly a representative illustration? What is the ventilation rate of that bottle?*

Response A.3: This analogy was used only to highlight the fact that there is a possibility of deposition of vapor on saltating ice grains. This possibility has never been explored and/or accounted for in the existing models that only assume a uni-directional exchange of water mass from the ice grain to the atmosphere (unless there is super-saturation of the atmosphere, which is usually not allowed in atmospheric models). On a beer bottle or anything from a refrigerator, we see a reverse process of extraction of vapor from the atmosphere onto the material and the atmosphere does not need to be super-saturated for this !

In terms of ventilation rate, if we consider the Reynolds’ number of a beer bottle of diameter 5 centimeters with a ice grain of diameter 200 microns, there is approximately two order of magnitude of difference. To have the same Reynolds number, $|\vec{u}_{rel}|^{icegrain} \approx 250|\vec{u}_{rel}|^{beer\ bottle}$. Thus, for a typical relative velocity between a saltating ice grain and air of 5 m/s, the beer bottle’s relative velocity needs to be only 0.02 m/s for the same Reynolds number and exchange coefficients. This is entirely plausible. Thus, even in terms of Reynolds numbers, the beer bottle analogy works !

- **A.4 : P4 L1:** *Thats very true!*

Response A.4: *Skipped*

- **A.5 : P4 L15-17:** *This is a very important change of sign. What is the explanation for the initial change from deposition to sublimation for the colder-than-air particles? Was this a period where the particle actually warmed up? Gained mass? Was the air surrounding the particle cooling through latent or sensible heat? It is very exciting that this information is finally available! This concept is overlooked throughout the paper. You have a wonderfully extensive data set. Explain whether or not sublimation is the only transfer of energy to your saltating particles. Please explain whether or not (and why) the particles actually warm in your simulations. Is there no thermodynamic feedback on the systems in section 2? Does Sigma star never change with time? Why or why not? Do values in Fig 1b,e actually affect the change in the ambient or near-particle air, or did the model assume these energy exchanges were “advected” away?*

Response A.5: We thank the reviewer for posing several critical points in this question. These questions strike at the heart of the message of the paper and thus it is extremely important for us to make sure that we are able to get our message across to the readers.

In the first set of experiments in Experiment I, the particle as well as the air have the same temperature of 263.15 K. However, the air is not saturated and thus there is a diffusion of mass from the ice grain to the air as described by Equation 2. However, since the temperature of the ice grain is the same as the air, there is no heat transfer. The initial energy for the sublimation must then come from the internal energy of the ice grain. The internal energy is nothing but the heat energy stored in the ice grain as represented by the grain temperature. As the internal energy of the ice grain is consumed, its temperature decreases and as soon as this happens, heat transfer between the ice grain and the air commences. After a transient period, an equilibrium condition is achieved where the particle temperature becomes constant and all the energy necessary for sublimation comes directly from the atmosphere.

The Thorpe-Mason model neglects the initial consumption of internal energy for sublimation and instead assumes that all the energy for sublimation comes from the atmosphere. In fact, the Thorpe-Mason model, by means of further approximations, does not consider particle temperature at all ! In this manuscript we show that for ice-grains in saltation, it is important to take into account, the ice-grain temperature and its evolution.

Returning to Experiment I, in the second part, we vary the initial temperature of the ice-grain with the ice grain being warmer or colder than the surrounding air. Here, the interpretation become slightly more difficult. In the case where the particle is colder than the air, there is both the warming of the particle as well as deposition. The particle gains energy both from convective heat transfer (second term in the RHS of Eq 1) as well as gains mass (Eq 2). At a certain point in time however, the particle becomes warm enough (though still colder than air), that it begins to sublimate.

Note that the temperature (T_{Air}) and saturation (represented by σ_*) of the air surrounding the ice-grain does not change and all mass or energy gain/loss of quantities in the air as assumed to be advected way. We justify this because we considering the dynamics of a solitary ice grain, subjected to relatively strong air motions. A helpful image is to imagine a special hair-dryer blowing air onto a 200 micron ice grain. However, in the LES experiments in Section 3, all the feedbacks are taken into account.

Thus our motivation for Section 2 was to simply highlight the fact that particle temperature, and the coupled heat and mass exchange dynamics are important to account for, instead of the approximate solution presented by the Thorpe-Mason approach, particularly for the

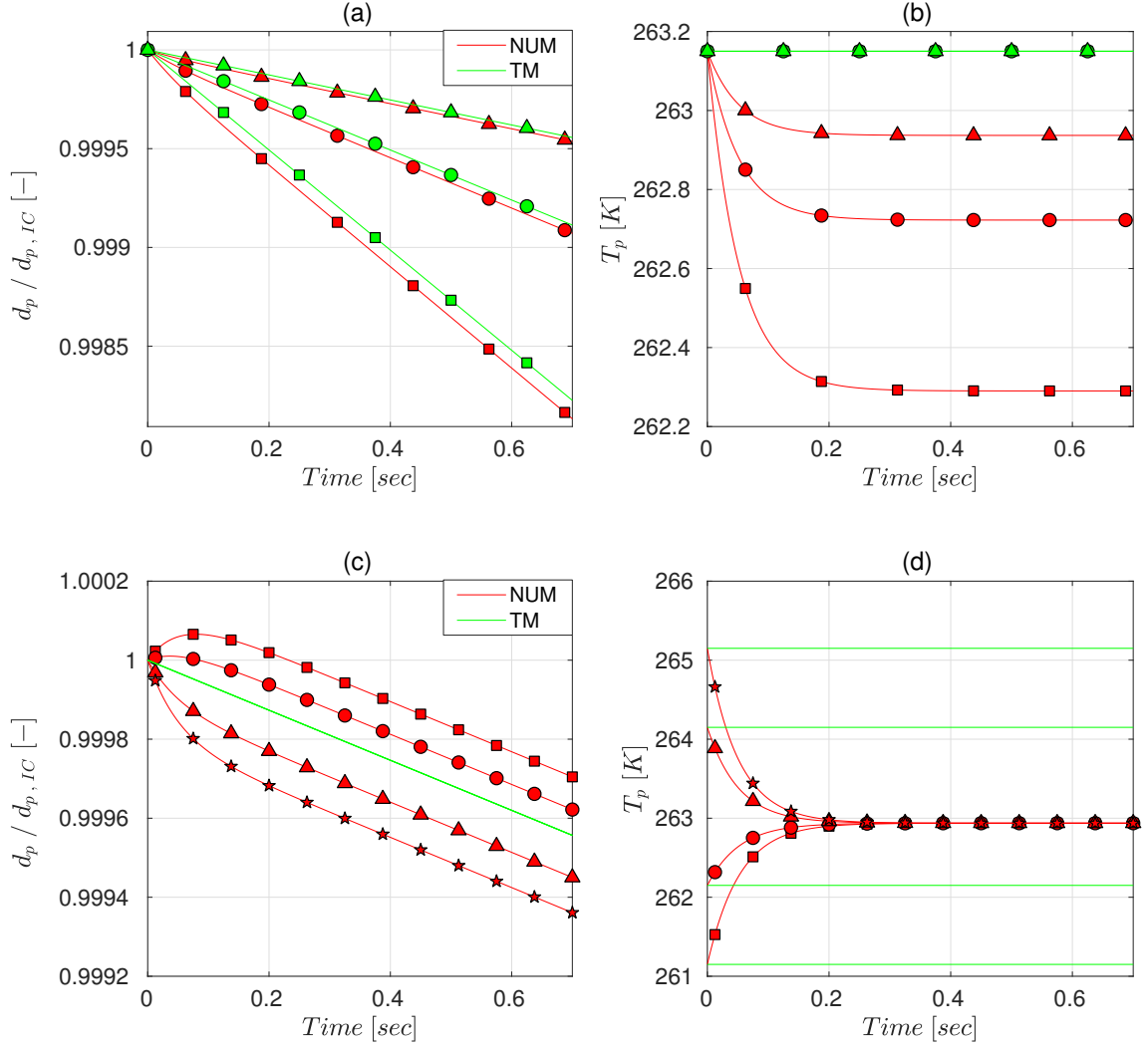


Figure 1: NUM and TM solutions for a particle of $200 \mu\text{m}$ diameter in different environmental conditions. **Experiment I-A:** Evolution of particle (a) diameter and (b) temperature; $T_{p,IC} - T_{Air} = 0$, $\sigma_* = 0.8$ (squares), 0.9 (circles), 0.95 (triangles). **Experiment I-B:** (c-d) same as (a-b) with $\sigma_* = 0.95$; $T_{p,IC} - T_{Air} = -2 \text{ K}$ (squares), -1 K (circles), 1 K (triangles), 2 K (stars). Note that the particle diameters are normalized by the initial diameter of the particle ($d_{p,IC}$).

short time-scales that we are interested in.

In response to the points raised in A.5, we have decided to update Section 2 to be more explicit about the nature of the simulations performed and the simplifications of the experiments. We have split Figure 1 of the original manuscript into two independent figures (a figure each for Experiment I and II) so that the plots are more clear and add an additional figure (Figure 1 in this document) to describe the evolution of particle diameter and temperatures in the different experiments. The new figure is added below for reference. The change in the text can be seen in the updated manuscript with the differences highlighted.

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- **A.6 : P4 L21-24:** *This is an interesting idea. However, there appears to be some serious assumptions used to reach this conclusion. Please clarify the following: Did you assume there is no ventilation or sublimation of particles*

when they are in contact with the surface, and is it assumed that the wind speed is constant across all heights of the trajectory of a saltating particle? Admittedly, there is certainly a connection between relaxation times and residence times, but it would increase the quality of the paper to convey either what assumptions are necessary to make the conclusions from Figure 2a to be truthful?

Response A.6: We thank the reviewer for raising a pertinent point here and giving us a chance to clarify. Firstly, we do not make a conclusive statement as evidenced by the use of the word “likely”. Since there is no actual data on particle temperatures measured in wind tunnels or in the field at present, we cannot make a conclusive statement and more research is needed. Thus it is only a conjecture at present. The results in Experiments I and II however are well-correlated to those from LES data in Experiment III and IV and thus there is credible support for this idea.

It is true that we do not take into account, the particle temperature and sublimation while it is at rest at the surface. The heat and mass transfer from the particle to the air begins only once it is lifted from the surface (either aerodynamically or due to splash entrainment). Secondly, it is indeed true that the wind speed is not constant across all heights of the trajectory of the saltating particle. This is the reason why we compute the relaxation time from relative velocities ranging from 0 to 10 m/s. These would correspond to the upper and lower bounds of the relaxation time for particle heat and mass transfer dynamics. We compare the mean and median residence times of the saltating particles to this “range” of relaxation times (the shaded region in Figure 2a in the original manuscript) rather than a single value of relaxation time.

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- **A.7 : P4 L21:** *Relaxation time is great, but what about the time that $Err_{Q,M}$ goes to zero (Fig 1f)? This value seems equally as important, as it appears to be a lower bound on the timeframe in which stationary wind/transport conditions are required to allow all the numerical errors to cancel out. This paper would benefit from an exploration (surface plot) of relaxation time over the parameters (σ_* , $T_P - T_{Air}$), and supplement Fig 1 a-c very well.*

Response A.7: This is great observation by the reviewer. However, we would like to point out that the errors in the cumulative heat and mass output in Figure 1c and 1f go to zero “very slowly” and in fact does not go to zero within typical saltation residence times. The quantity of relaxation time as we have defined is a far more robust measure to identify from simulations. It is also a more conservative measure as any particle with residence time lower than the relaxation time will, by definition, be lower than the measure proposed by the reviewer.

As far as the exploration of the relaxation time over the parameters goes, we did in fact do this exploration. However, it was found that the relaxation time depends only on the particle diameter and the relative velocity between the particle and the air. This is shown in Figure 2a (in the original manuscript) in the shaded region.

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- **A.8 : P4 L27:** *Can you speculate as to what is the physical (or numerical) meaning of this scaling relationship? Or is this a purely empirical finding?*

Response A.8: Following the work described in this manuscript, we explored this interesting relationship a bit further and we have reasons to believe that this quantity can actually be derived directly from equations (1) and (2) of the manuscript. This derivation is not yet complete and we leave it for future publications.

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- **A.9 : P4 L31 Fig 1g-l: Please expand the negative range of $T_P - T_{Air}$. There are environments where fohn events can bring dramatic changes of temperature up to 10C over only a few hours!**

Response A.9: A similar comment was raised Reviewer #2 and so we have increased the range in the updated figure to -5 K to 5 K. Figure 1(g-l) in the previous manuscript are Figure 3(a-f) in the revised manuscript.

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- **A.10 : P5 L20: Look at parameterization**

Response A.10: This has been updated in the revised manuscript.

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- **A.11 : P5 L26: It is unclear to me how using this stationary flow is fundamentally different from your steady state model. Was the LES used because it is a more sophisticated framework in which to calculate these fluxes? Besides the evolution of friction velocity is Figure S2, I am afraid I have missed the point of using such a complex tool to solve some PDEs.**

Response A.11: There are two principal reasons for using the LES. Firstly, we wanted to find out about the residence time of typical saltating ice grains. This information is not available in literature and so we decided to perform LES of a turbulent channel flow with a erodible snow surface as the lower “wall” of the flow. The surface acts as a source or sink of particles with simple stochastic models to account for different entrainment and deposition processes. The transport of particles is modeled by solving the equations of motion for each particle individually once the particle is eroded and is air-borne. The LES methodology for aeolian transport is well established and has been validated in the past. We realize that we have not cited past works in this section and have rectified this oversight.

The second motivation is in fact directly related to a previous comment by the reviewer (A.5). Unlike Experiment I and II, the air surrounding the particle (thinking from the frame of reference of the particle) is continuously evolving with different wind speed, temperature and humidity values. How do the two different approaches for computing sublimation (TM and NUM) compare in this scenario with complete feedback between air and ice grains ? This is question we answer in Section 3 using LES.

Within the LES context, by stationary turbulent flow, we intended to say that the logarithmic profile of the velocity is achieved and the time-averaged turbulent statistics (or Reynolds averaged statistics) are horizontally homogeneous and steady and vary only in the vertical direction. The wall-bounded channel flow that we simulate still has a significant shear in the vertical direction (as expected in the wall-bounded shear flows).

The LES also allows for simulating vertical gradients of temperature and humidity as would exist in nature. The vertical mixing of these scalars allows the sublimation of saltating ice grains to continue as dry air from aloft is continuously mixing downwards into the saltation layer. A detailed analysis of the heat and moisture budgets in the presence of the saltating ice grains will be presented in a future publication.

However, this and other comments have led us to believe that we have perhaps not motivated the use of LES sufficiently in the submitted manuscript, or described the LES in sufficient detail. Even though we go into great detail about the LES and the setup in the supplementary material, we expand the section 3.1 in the revised manuscript.

Additionally we submit a movie (Supplementary Video M1) illustration of the simulation we perform to make it clear the kind of LES we have performed.

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- **A.12 : P5 L32: *Why was this not discussed in the previous experiments?***

Response A.12: The initial condition for particle was indeed discussed in the previous experiments but this was perhaps not clear due to lack of proper notation (no mention of $T_{p,IC}$ in Section 2 for example). In the revised Section 2, we explicitly state that we are imposing $T_{p,IC}$ in the experiments in Section 2 as well.

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- **A.13 : P6 L3: *Please stop calling this realistic saltation of snow. The parameterizations and assumptions necessary to run this numerical model make this statement misleading. Please rephrase as LES simulation of saltation or something similar.***

Response A.13: We have removed the word “realistic” from the sentence. Adding “LES simulation of saltation” does not seem to be appropriate as Experiment III is purely about using LES. The entire sentence now reads as follows:

The principle aims of Experiment III are to firstly quantify particle residence times (PRT) and their dependence on wind speeds and relative humidities and secondly, compute the differences in the heat and mass output between the NUM and the TM approaches during saltation of snow with complete feedback between the air and the particles.

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- **A.14 : P6 L5: *These varying friction velocities are referred to as “low medium and high wind speeds” in line 33. What wind speeds were necessary for these values? Friction velocities do a poor job of representing turbulence in even subtly complex terrain, and as saltation is a drag-driven process, at the very least, mean wind speeds should be included in the manuscript, and extensive time series of turbulence statistics (Turbulence Intensity, TKE, shear stress, etc.) in the supplementary material. As this research is conducted to benefit those that models in natural settings, and those natural setting will be much more turbulent than the LES, and that turbulence is what is driving the ventilation rates, more information about the model is needed.***

Response A.14: We thank the reviewer to bringing to our attention this fact. The **TO BE DONE !**

- **A.15 : P6 L5:** *Where is “the surface” defined for this stress calculation? And how is that surface defined? How can that be reconciled with the fact there is windpumping into the snow pack? Or is this a Reynolds-stress-based value?*

Response A.15: In terms of the forces that the surface imparts to the overlying fluid, the surface is treated as a rough wall. The roughness is parameterized using a roughness length (z_0) = $10^{-5}m$. This approach does not account for the windpumping into the snowpack. We mention this in the revised text.

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- **A.16 : P6 L15:** *Why a different range of temperatures than Section 2?*

Response A.16: The range of temperatures in Experiment II has now been increased to -5K to +5K.

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- **A.17 : Fig 2a:** *Redo the plots so it is clear what is happening. I cannot understand anything from 200 μm to 1000 μm . The diameter plot markers appear somewhat logarithmically. Try plotting with a $\log x$ scale? And why do the residence time measurements become more sparse at smaller particle sizes? Please redo the symbols as they are confusing and inconsistent, or eliminate them altogether.*

Response A.17: We thank the reviewer for pointing out lack of clarity in Figure 2. This figure is the most essential part of the paper and thus, it is extremely important for us to make sure that it is well understood by our readers.

- We have now restricted the figure to 600 μm . There are not enough particles larger than 600 μm and thus the statistics are noisy.
- The x-axis of the figure is indeed logarithmic. We have added this information in the figure’s caption.
- The markers were added only for differentiating and labeling the different trend-lines. Not all data-points have been marked.
- As mentioned in the submitted manuscript at P5 L21-22: *The snow surface consists of particles with a log-normal size distribution with a mean particle diameter of 200 μm and standard-deviation of 100 μm .* The particle size distribution (PSD) imposed on the surface comes from previous studies of modeling of saltation of snow. The PSD constrains the particle diameters that are air-borne and undergo transport. Also, we use a continuous spectrum and thus, when calculating statistics of mean and median residence times, we use a fixed bin size of 25 microns. As Figure 2a has a logarithmic x-axis, the measurements appear to be sparse at the lower range of the diameters.

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- **A.18 : P6 L23:** *There is no dependence on σ ?*

Response A.18: No, the relaxation time $\tau_{relaxation}$ does not depend on σ_* . This is one of the remarkable results of Section 2 and we now make this point more explicitly in the revised manuscript.

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- **A.19 : P6 L29:** *Please elaborate why the values of mass loss are wrong. It appears in Fig 1c,f that the cumulative errors go to zero over time. Why is this no longer the case with LES?*

Response A.19: Once again, we feel that we could have perhaps done a better job in explaining the relationship between Experiments I/II and Experiments III/IV.

The cumulative errors in Figure 1c,f *tend* towards zero but for a solitary ice grain. In the LES, a particle, original resting at the surface, is made air-borne (either due to aerodynamic entrainment or splashing), makes multiple hops across the snow surface, where it rebounds from the surface, and ultimately comes to rest, i.e, it impacts the surface and does not rebound. In the LES, there are many thousands of particles that go through this cycle during the course of the simulation. Since models parameterizing the erosion and deposition of the particles are stochastic, particles in saltating have a range of hops, distance traveled and residence times. Additionally there are a range of particle diameters present in the flow. We track the residence time of each particle, and calculate statistics of mean and median residence time as a function of diameter.

It is found that the smaller grains (with diameters less than 150 microns) have “on average” residence times that are longer than the relaxation time. Thus for these particles only, the cumulative errors averaged over multiple particles, will indeed *tend* to zero. The LES also have particles (with diameters greater than 225 microns) that have residence times “on average” larger than the plausible values of the relaxation time. Thus, for only these particles, the cumulative errors of mass and heat output will **not** go to zero. Summing all these errors for all the particles in the flow, the total error is non-zero. In fact Figure 3 shows precisely this error and it is found to range from 28% to as high as 40 % in Experiment III.

Thus, the LES are not performed for a single ice grain, with different simulations for different particle diameters. The LES is performed of a turbulent channel flow with an erodible snow surface consisting of a distribution of particle diameters at the lower wall. The ice grains enter and exit the flow at the surface according to models governing the erosion and deposition mechanisms. The supplementary movie M1 will aid in making this point clear.

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- **A.20 : P7 L1:** *Please rephrase “larger-scale turbulence statistics.” It unclear to me how any “larger-scale turbulence” can be represented in a $6\times 6\times 6$ meter box. Is this not an increase in mean windspeed?*

Response A.20: By “larger-scale turbulence statistics”, we meant to say that the dynamics of the heavy particles to be invariant to different flow speeds. We simplify the statement as follows:

This means that the dynamics of the heavier particles are unaffected by different wind speeds simulated in Experiment III.

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- **A.21 : P7 L3-8:** *This is a very interesting finding! This suppression of vertical motions and how the model responds should be elaborated on! A comparison of the vertical turbulence statistics amongst the experiments is necessary as*

they all assume uniform initial air temperature (P6 L5 comment). How does vertical mixing in the LES deal with this over time? Logic would imply that this same suppression of vertical mixing could also be caused by a colder snow surface temperature and increased stability. Why have you disregarded particle surface temperature in your PRT experiments? Would this effect be found if a temperature gradient as found in nature were present, or would the numerical effect be overwhelmed by the near-surface temperature gradients? As it stands, this statement cannot stand alone and the conclusion needs more development and supporting data/plots would be very beneficial.

Response A.21: We agree with the reviewer that this is indeed an interesting finding. We have added an entire section in the supplementary material providing a preliminary analysis of this phenomenon by showing the vertical profiles of the vertical buoyancy flux. However, as we explain in the “comments_to_all.pdf” document, this is an ancillary result that is not directly related to the core message of the paper. The role of buoyancy in mediating aeolian transport is a very interesting and as-of-yet unexplored topic. We are in fact working on this topic currently and hope to present results focusing on this topic in the coming months.

Coming to the additional questions posed by the reviewer, we answer them as follows:

- Accounting for surface temperature is not likely to have a major impact on the stability of the atmosphere in strong snow drift events that we are considering. Whether the snow is sublimating on the surface, or during transport, both processes are going to result in stable stratification of the atmosphere. However, the amount of sublimation and the resulting cooling is much more from the particles in air, in comparison to those lying on the surface. In our simulation, where we have fully developed saltation/snow transport, the effect of the sublimation, and stability due to surface sublimation is likely to be negligible in comparison to the corresponding effect emerging from particles in the air. Note that we have stably stratified air in our simulations as well. Just that the stability emerges due to sublimation of particles in the air and not on the surface. We agree that in intermittent snow transport conditions, the surface boundary condition will become important. This is a matter for further exploration.
- This effect would indeed be found if there is a temperature gradient present. Note that only the initial condition for temperature is fixed at 263.15 K. The temperature in the LES evolves with time and the atmosphere does become stably stratified.
- We stress again the fact that this, although an interesting result, is only ancillary to the core message of the paper and we stress upon this point more in the concluding section of the paper.

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- **A.22 : P7 L3-8** *These are very small particles, can they be considered in “Suspension?” Obviously, there is a full spectrum of motions, but approximately where have other researches been separating saltation from suspension on Fig 2a? This would be very informative as the paper by nature is a saltation study.*

Response A.22: We present results only for particles that saltate. There are indeed a few particles in “suspension”, i.e, particles that once leaving the surface, never deposit during the course of the simulation. But the number of such particles is an order of magnitude lower than those that saltate. Residence times are thus computed only for particles that leave and return to the surface.

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- **A.23 : P7 L12: *Very exciting finding!***

Response A.23: We agree !

- **A.24 : P7 L18: *What is field scale?***

Response A.24: We have removed this phrase in the revised manuscript.

- **A.25 : P7 L21: *Can anything be said about the low end of the friction velocity domain where intermittent transport dominates? Would TM over or underestimate in that case?***

Response A.25: No, intermittent transport is a very interesting phenomenon where a lot more research is required to simulate it properly. The initial friction velocities are chosen such that we have “fully-developed” saltation. Having said that, the TM would still underestimate the mass lost by the solid ice phase due to sublimation but the underestimation will be lower than those found in Experiment III.

- **A.26 : Fig 3: *Where have the particle diameters gone? What distribution of sizes are you using?***

Response A.26: The particle size distribution (PSD) is imposed as described on P5 L21-22: *The snow surface consists of particles with a log-normal size distribution with a mean particle diameter of 200 μm and standard-deviation of 100 μm .* As mentioned earlier, we have now added a figure with the PSD in the revised manuscript.

Fig 3 shows the “total” mass lost due to sublimation - from all the particles that have undergone sublimation during the simulation.

- **A.27 : P8 L22: *Not perturbations.***

Response A.27: We have replaced “temperature perturbations” with “temperature differences” in the revised manuscript.

B: Technical Concerns

- *B.1 : P1 L7 Please specify: snowpack surface temperature, snow particle surface temperature?*

Response B.1: In the revised manuscript, the temperature of the snowpack surface temperature and the air flow is specified (as 263.15 K).

- *B.2 : EQ3 What is “d”? d_p ?*

Response B.2: Yes, it is indeed d_p . This error has been corrected in the revised manuscript.

- *B.3 : P2 L24 “Saturation σ_* ...” ? Do you mean sigma is saturation?*

Response B.3: In fact, σ_* is the rate of saturation (or saturation-rate). The corresponding line is corrected in the revised manuscript as:
saturation-rate (σ_*) = $\rho_{w,\infty}/\rho_s(T_{a,\infty})$.

- *B.4 : P2 L27: Add space after sentence end.*

Response B.4: The corresponding line has been corrected in the revised manuscript.

- *B.5 : P5 L12: “an erodible”*

Response B.5: Appropriate corrections have been made in the revised manuscript.

Response to reviewers:
On the suitability of the Thorpe-Mason model for
calculating sublimation of saltating snow

Varun Sharma, Francesco Comola and Michael Lehning

August 31, 2018

A note to all reviewers

Please see the additional document found in the “comments.to_all.pdf” file

Response to Reviewer # 2

Opening Remarks:

We thank Reviewer #2 for his/her critique of the submitted manuscript. We have updated the manuscript based on the advice received and provide in the following section, a point-by-point response to the questions posed and the clarifications sought.

Based on Reviewer #2's comments, we have added an entire section in the supplementary material providing vertical profiles of some mean and turbulent quantities for some of the LESs performed.

A: Concerns in the main text

- ***A1 : P.2, L.5: Perhaps it will be a good idea to refer the previous sublimation simulations in suspension. e.g., Xiao et al., 2000, An intercomparison among four models of blowing snow, Boundary-Layer Meteorology, 97, 109-135.***

Response A.1: We have now added a host of references in this section. The updated sentences read as follows:

This is true for both field studies (Mann et al., 2000), where sublimation losses are calculated using measurements, usually at the height of $O(1\text{ m})$, and in mesoscale modeling studies (Xiao et al., 2000; Dry and Yau, 2002; Groot Zwaaftink et al., 2011; Vionnet et al., 2014),

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- ***A.2 : P2., L.9: “recent studies using high-resolution large-eddy simulations” is the reference really use LES? I could not confirm in that paper.***

Response A.2: We re-checked the reference and it is indeed correct that Dai and Huang, 2014 did not use LES but rather a RANS type simulation. We have edited the text to read as follows:

However, recent studies using high-resolution steady-flow, Reynolds-averaged Navier-Stokes (RANS) type simulations (Dai and Huang, 2014) claim that sublimation losses in the saltation layer are not negligible,

- **A.3 : P.2, L.24:** *“saturation σ_* ” does it mean rate of saturation?*

Response A.3: We agree with the view that confusion that using the word “saturation” may cause. We have edited all references to σ_* as *saturation-rate*.

- **A.4 : P.2, L.25:** *“Groot Zwaaftink et al. 2011, 2014 should be Groot Zwaaftink et al. 2011”. In Groot Zwaaftink et al. (2014), mass loss due to sublimation are neglected in the calculation.*

Response A.4: This is indeed true and we have corrected this mistake.

- **A.5 : P.2, L.27:** *“Vionnet et al., 2014).In” should be “Vionnet et al., 2014). In” (please add a space).*

Response A.5: This edit has been made in the revised manuscript.

- **A.6 : P.3, L.9:** *The units should be given in Roman type, I think.*

Response A.6: All units have to updated to Roman type in the revised manuscript as well as the supplementary material.

- **A.7 : P.4, L.3:** *Is the first-order scheme sufficient in the calculation?*

Response A.7: Yes, given the extremely small time-step of 50 microseconds, the first-order scheme is indeed sufficient. We mention the time-step in the revised version of manuscript as follows:

For the NUM approach, Eqs. (1) and (2) are solved in a coupled manner using a simple first-order finite-differencing scheme for time-stepping with a time-step of 50 μ s.

- **A.8 : P.5, L.12:** *“a erodible” should be “an erodible”.*

Response A.8: This has been corrected in the revised version of the manuscript.

- **A.9 : P.6, L.15:** *Do you have any specific reason for the different temperature conditions (-5;dT;5 in EX. IV, -2.5;dT;2.5 in EX. II).*

Response A.9: We have updated the temperature range in EX. II to be between -5 and 5K. The corresponding figures as well as the associated text have been updated as well.

- **A.10 : P.7, L.7-8.:** *“low initial saturation results in more sublimation and cooling near the surface, resulting in suppression of vertical motions.” This is interesting indeed. Could you show the modification of the vertical profiles (temperature, sublimation rate, wind speed etc.) to illustrate these processes?*

Response A.10: Please refer to the section “ A note to all reviewers” with regards to this question.

- **A.11 : P.7, L.18:** *“at a field scale” specifically, during realistic saltation of snow?*

Response A.11: We have modified this line to read as follows in the revised manuscript: *We can directly assess the implications of differences in grain-scale sublimation between the two approaches on total mass loss rates during saltation at larger spatial scales as simulated using LES in Experiments III and IV.*

- **A.12 : P.8, L.2:** *“here).” should be “here.” (an unnecessary parenthesis).*

Response A.12: We have corrected this in the revised manuscript.

B: Concerns in the supplementary material

- **B.1 : P.2, L.26 (S9):** *“ln” should be given in Roman type.*

Response B.1: We have corrected this in the revised supplement.

- ***B.2 : P.3, L.27: I think parenthesis is missing around the reference.***

Response B.2: We have corrected this - by removing the reference to Groot Zwaaftink et al., 2014.

- ***B.3 : P.3, L.29: σ_{d_p} is undefined, I think.***

Response B.3: This line is now as follows:
characterized by the mean, $\langle d_p \rangle$ and standard deviation, σ_{d_p}

- ***B.4 : P.4, L.2-4: Could you include the relevant references (Clifton and Lehn-ing (2008) ?).***

Response B.4: We have added this reference in the revised supplement.

- ***B.5 : P.4, L.16-17: Is the rebound angle the same as that for sand? I think Kok and Renno (2009) obtained the results for sand. Do you hypothesis the angle is similar to sand saltation?***

Response B.5: This is indeed true, but a previous work (Nalpanis et al. 1993) made wind tunnel experiments with different granular media, including sand and snow and found that the saltation geometries, ejection and rebound angles are invariant. We cite this work in the revised supplement.

- ***B.6 : P.4, L.19-20: “dislodge additional additional particles” should be “dis-lodge additional particles”.***

Response B.6: This has been corrected in the updated supplement.

- ***B.7 : P.7, L.23: Is the first-order scheme adequate for the computation in this study?***

Response B.7: Yes - considering that we use an extremely small time-step of 50 microseconds, it is adequate. Additionally, a higher order method would require additional memory.

- ***B.8 : P.8. “Time step” All the elements (fluid, particle, and scalars) have the same timestep?***

Response B.8: Yes - all equations are progressed in time using the same time-step.

- ***B.9 : P.9, L.5: “It shows that that once” should be “It shows that once”?***

Response B.9: This has been corrected in the updated manuscript.

Response to reviewers:
On the suitability of the Thorpe-Mason model for
calculating sublimation of saltating snow

Varun Sharma, Francesco Comola and Michael Lehning

August 31, 2018

A note to all reviewers

Apart from addressing the points raised by each of the two reviewers individually, we felt that it might be pertinent to write to both the reviewers together especially in view of the fact that the most outstanding issue raised by both is essentially the same - the need to present more data.

At the outset, we would like to thank the reviewers for mostly positive remarks regarding the quality of the submitted manuscript and the core message of the paper, i.e., questioning the well-established, Thorpe-Mason (TM) model for computing sublimation of snow particles in the atmosphere. We are further grateful to their detailed comments regarding the study and clarifications sought. We feel that this has genuinely improved the manuscript and help us convey our ideas more clearly to the prospective reader.

Coming to the common concern raised by both the reviewers, both the reviewers felt that we need to present more data, especially from the large-eddy simulations (LES) in Section 3 and used for Experiments III and IV. As would be apparent from the length of the submitted manuscript, we intended to be extremely focused on the core message - challenging the Thorpe-Mason model - and presented only those results that we felt were *most* relevant to support our result and make it clear to the community that this is a worthwhile challenge to the existing models and estimates for sublimation. We write in the originally submitted quite explicitly that the large-eddy simulations are, in the context of the current study, used for two purposes only; (a) to find the residence time of saltating particles and (b) see if the results from the extremely idealized grain-scale simulations of sublimation are relevant during saltation. These two aspects are covered by Figures 2 and 3 in the original manuscript.

We agree with Reviewer #1 that we need to at least state the flow speed in the different u_* cases and we have added them in the revised manuscript. However, we humbly state that presenting vertical profiles of various mean and turbulent quantities, although quite interesting, would make the article lose its focus and move the discussion into areas that are far from the core message of the paper. This is especially true because we have 30 simulations in total. Given the immense size of the data-set, presenting it in any which way would certainly make the paper unwieldy, without perhaps adding much to core message of the paper.

Are the vertical profiles of different variables interesting? Certainly. Indeed, we are currently working towards a manuscript analyzing the LES results using mean and turbulent kinetic energy, heat and moisture budgets. The present round of reviews has in fact motivated us even further

about the importance of such an analysis for the community. However, the depth of the analysis necessitates a separate paper. In view of our opinion on this issue we state the following points:

- It seems to us that for both the reviewers, the interest in analyzing vertical profiles from the LES simulations was triggered by a hypothesis presented in the paper to explain the role of increased sublimation and the associated cooling and stabilization of the saltation layer in lowering the residence time of lighter (smaller) particles as opposed to the heavier (larger) particles. This hypothesis is indeed confirmed in our data and can be explained by looking at the total vertical buoyancy flux. However, presenting this quantity would require presenting a host of other associated quantities. Most importantly, this point by itself, although quite interesting and a major motivation for a lot of our current work, is not of principal interest for the core message of the paper. We have remolded the text in the revised manuscript accordingly and touched upon this point in the concluding section of the paper.
- If the reviewers feel quite strongly that this data is absolutely necessary in the current paper, we hope that it is indeed agreeable to the reviewers that it is sufficient to add it to the supplement (as suggested by Reviewer #1). We attempted to add it to the main portion of the manuscript but it was really hard to maintain the flow of the paper and not distract the reader into issues far from the core message of the paper. Thus, the reviewers will find, in what follows, the new additional analysis that we hope can be added to supplement.

REPRODUCED FROM THE REVISED SUPPLEMENT

S3.1 Vertical profiles of mean and turbulent quantities

In this section, vertical profiles extracted from the large-eddy simulations in Experiment III are shown to provide additional context for the simulations performed. Note that a detailed analysis of the vertical profiles is out of the scope of the present study and will be presented in a future publication. All the profiles presented below are time-averaged as well as averaged in the horizontal (periodic) directions.

In Fig. 1, the velocity magnitude for the low (UL), medium (UM) and high wind (UH) cases are presented. Influence of initial relative humidity (the RL, RM and RH variants) was not found to be important and thus not presented. Before the snow surface is allowed to be eroded, a fully developed channel flow is allowed to develop. This can be seen in Fig. S2 in the previous section. This also implies the formation of the logarithmic velocity profile as can be seen in Fig. 1. Once the snow surface is allowed to erode, snow transport begins and the particles in the flow cause enhanced drag in the flow. This causes the velocity profile to change with an overall deceleration of the flow. The wind speeds before saltation begins at a height of 1 m above the surface are 11 m/s, 16.33 m/s and 21.86 m/s for the UL, UM and UH cases respectively. Once the snow transport is fully-developed (i.e., when the total mass of snow in the air is constant in Fig. S1), the corresponding wind speeds have reduced to 8.771 m/s (-20%), 11.34 m/s (-30%) and 12.98 m/s (-40%) respectively for the three cases.

The snow drift density is the mass of snow per unit volume present in the air and is shown for the UL-, UM- and UH- cases. Once again, the RL, RM and RH invariants of each of these cases are not found to be significantly different from each other and are thus not presented. Two time points chosen lie during the transient period where increasing snow mass is being entrained into air (profile at 10 seconds) and during fully-developed or steady state snow transport (profile at 240 seconds). The profiles are qualitatively as well as quantitatively (order of magnitude comparison) similar to previous works (see, for example, Gordon et al. 2009). As expected, the amount of

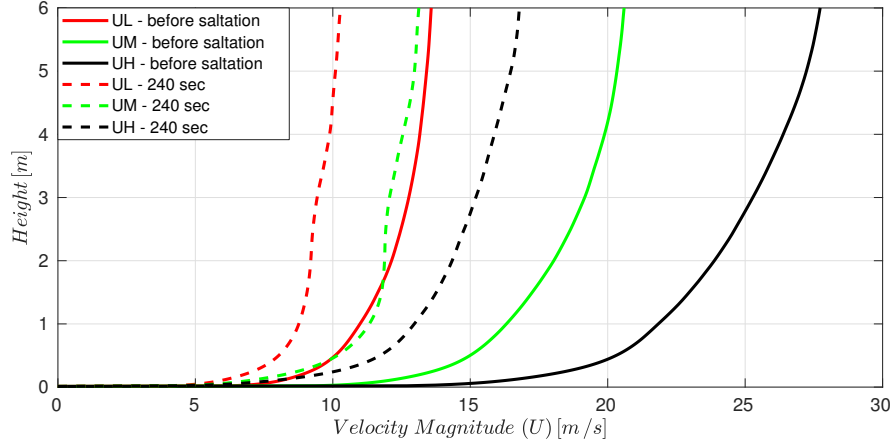


Figure 1: Vertical profiles of mean velocity magnitude for the low (UL), medium (UM) and high wind (UH) cases in Experiment III. For each case, profiles before commencement of saltation and 240 seconds after saltation begins are shown.

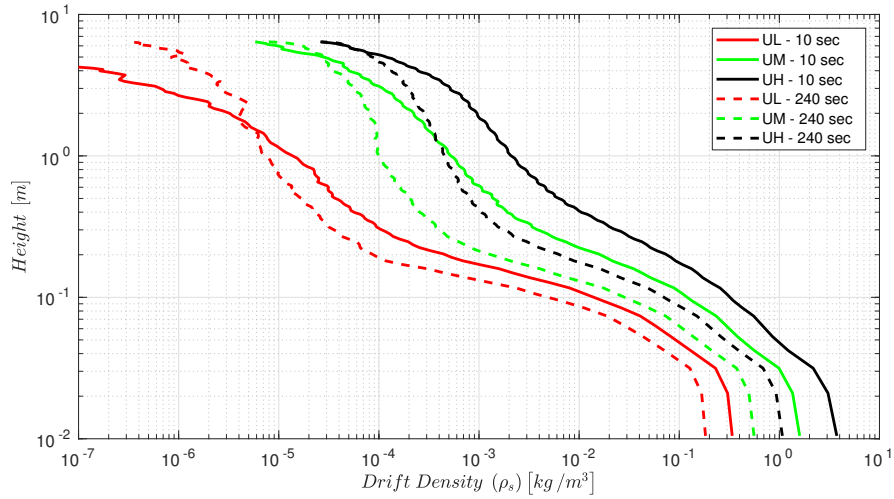


Figure 2: Vertical profiles of snow drift density for the low (UL), medium (UM) and high wind (UH) cases in Experiment III. For each case, profiles 10 seconds and 240 seconds after commencement of saltation are shown.

mass in the air increases with increasing wind speed and is found to be concentrated in the lowest 10 centimetres of air above the surface.

Figure 3 presents an inter-comparison of the evolving thermodynamic state of the air computed using either the NUM or the TM approach, with subfigures a,b and c showing vertical profiles of temperature, specific humidity and relative humidity respectively. Only one of the nine cases in Experiment III, namely the UL-RL case is chosen for illustration. Recall that as per our LES setup, the only source or sink of heat and mass in the atmosphere is through interaction with the particles. First, let's focus on subfigure c which shows the relative humidity (R.H) profiles at 3 different times after saltation begins, along with the initial condition for R.H, which in the UL-RL case was fixed at 30% in the entire domain. As time progresses, the R.H in the air increases due to cooling as well as larger amounts of water vapor, both due to sublimation of particles aloft. The profiles on the extreme right of subfigure c, which are extracted 1000 seconds after the start of saltation are similar for both the NUM and TM approaches with the air is close to saturation in both the cases. In the profiles at earlier time-steps, the R.H is higher near the surface and decreases with height as expected. The near surface air reaches a high saturation-rate (90%) within 100 seconds after saltation begins, but it takes almost 900 seconds more to reach saturation. This can be explained by turbulent mixing which continuously supplies dry air from aloft to the near-surface region.

While Fig. 3c, shows qualitatively a similar behavior for both the NUM and TM approaches as far as R.H evolution is concerned, we have shown in the main text that the TM approach underestimates the mass flux due to sublimation as compared to the NUM approach (see Figure 4 in the main article). The reason for this is the difference in the total cooling of the air between the two cases. This can be observed in the temperature profiles in Fig. 3a. For the TM approach, the cooling is much stronger, with the final temperature being 260.3 K, 2.85 K lower than initial air temperature of 263.15 K. On the other hand, for the final air temperature for NUM approach is 262.4 K, almost 2 K warmer than the TM case. The dynamics of the evolution of air temperature are much more complicated in the NUM case due to the inter-play between the thermodynamics of the air as well as the particles. Further work is needed to establish proper thermodynamic constraints on the coupled air-particle system. Ultimately, the results in Experiment I and II show that even for a solitary ice-grain, the TM approach under-predicts the mass sublimated in comparison to the NUM approach for exactly the same environmental conditions. This is reflected in the profiles of specific humidity in subfigure c. The NUM approach, at each of the three time-steps chosen shows higher flux as compared to the TM approach.

Vertical profiles of streamwise $\left(\sqrt{u'u'}\right)$, cross-stream $\left(\sqrt{v'v'}\right)$ and vertical $\left(\sqrt{w'w'}\right)$ velocity fluctuations are shown in Fig. 4 for the UL-RL case before and during saltation. The TKE is highest near the surface and decreases with distance from the surface. Interestingly, during snow transport, each of the TKE components show a decrease as compared to their respective value before snow transport, upto a height on approximately 2 m above the surface. Above this height, the TKE components actually show an increase.

In the final figure in this section, we compare profiles of vertical buoyancy fluxes $\left(\overline{w'b'} = \left(g/\langle\theta_v\rangle_{xy}\right)\overline{w'\theta'_v}\right)$ from three cases, UL-RL, UL-RM and UL-RH, from Experiment III. The three subfigures show profiles for three different times after beginning of saltation. The vertical buoyancy flux is an important quantity as it is a term of the budget equation for vertical velocity fluctuations $\left(\sqrt{w'w'}\right)$. For each simulation case, the buoyancy flux decreases as time progresses. The UL-RL case is also found to have the largest magnitude of buoyancy flux close to the surface in each of the time-steps shown, followed by UL-RM and finally UL-RH, which has the least buoyancy flux amongst the three cases. Note that this is negative buoyancy flux and thus, in terms of vertical velocity fluctuations, the -RL, -RM and the -RH cases have increasing vertical fluctuations in that order. This could potentially explain the results in Fig. 3a in the main text, where the lighter particles show increasing residence times in the order -RL, -RM and -RH. Further exploration of role of

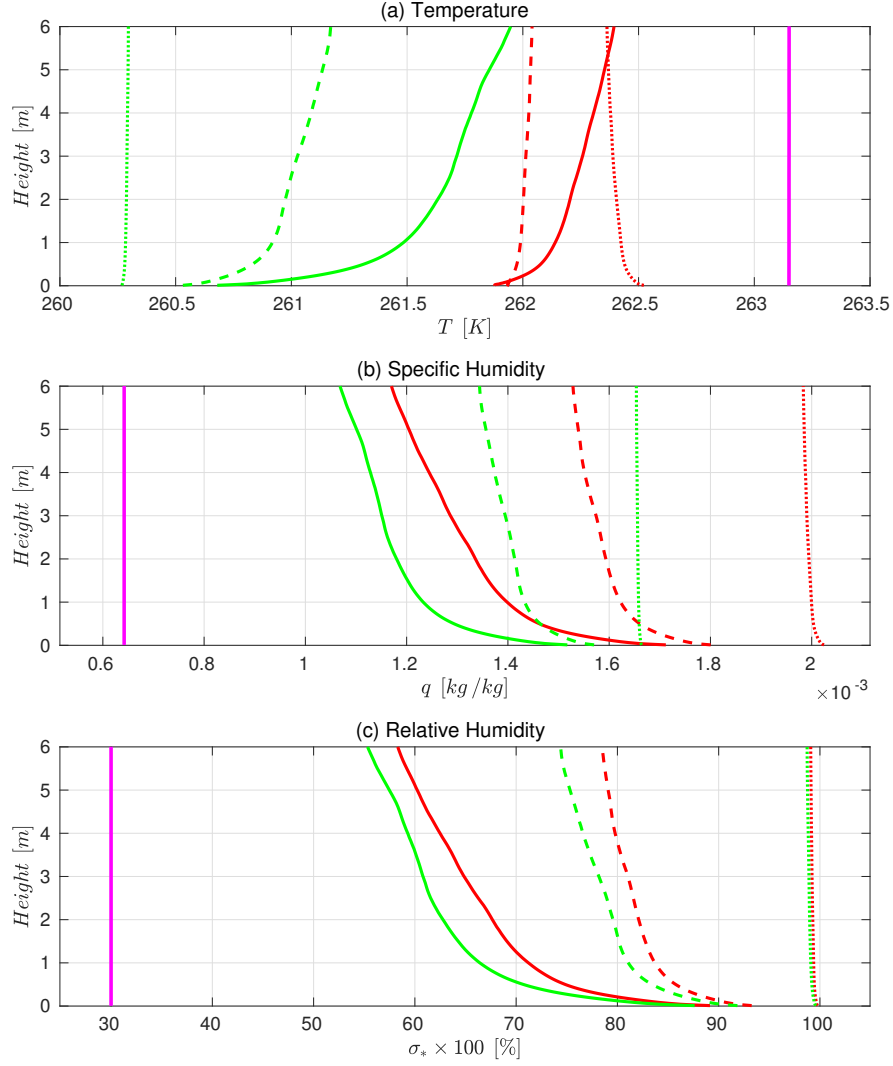


Figure 3: Intercomparison between the NUM (redlines) and TM (green lines) approaches for calculating sublimation of saltating snow in the UL-RL case in Experiment III. The magenta line is the initial condition for temperature, specific humidity and relative humidity. In all the subfigures, the solid, broken and dotted lines are profiles extracted 100, 240 and 1000 seconds after the commencement of saltation respectively.

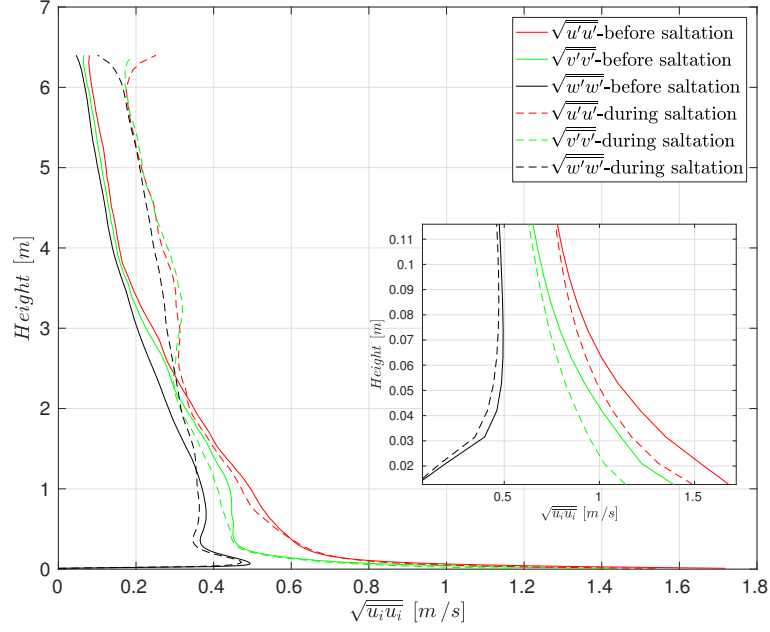


Figure 4: Vertical profiles of the three different constituents of the turbulent kinetic energy before and during saltation (240 seconds after saltation begins) for the case UL-RL in Experiment III.

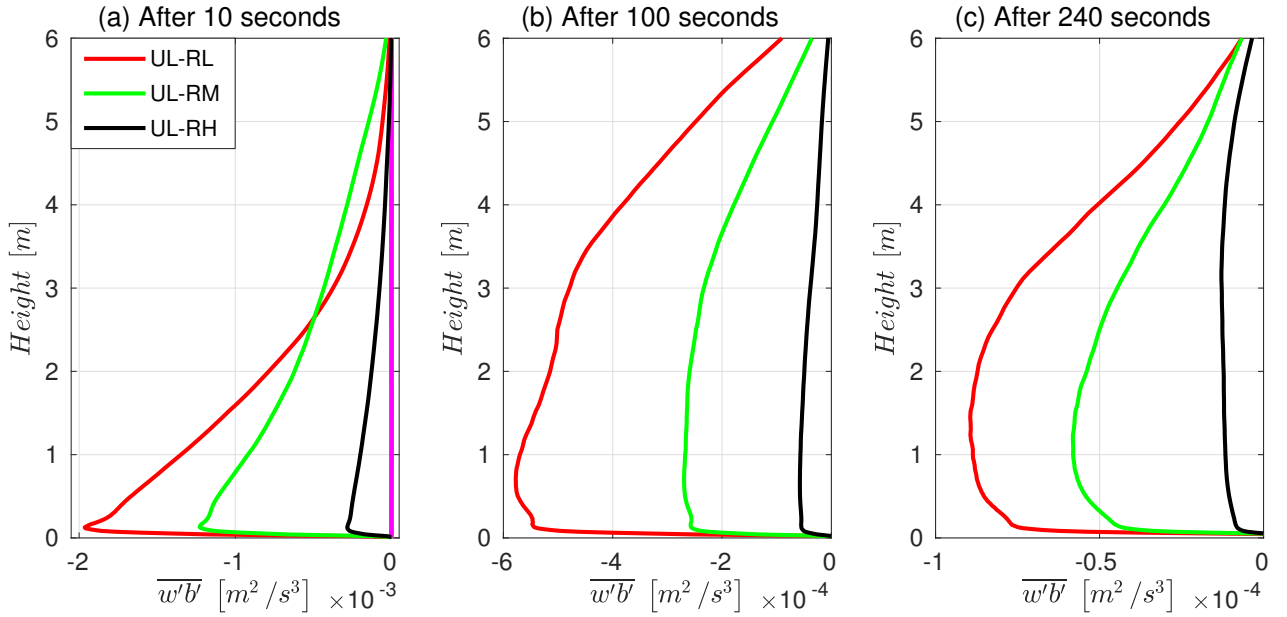


Figure 5: Vertical buoyancy fluxes for three cases, UL-RL, UL-RM, UL-RH at different times after the commencement of saltation.

buoyancy in affecting saltation dynamics is left for future work.

REPRODUCED FROM THE REVISED SUPPLEMENT

On the suitability of the Thorpe-Mason model for Calculating Sublimation of Saltating Snow

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Abstract. The Thorpe and Mason (TM) model for calculating the mass lost from a sublimating snow grain is the basis of all existing small and large-scale estimates of drifting snow sublimation and the associated snow mass balance of polar and alpine regions. We revisit this model to test its validity for calculating sublimation from saltating snow grains. It is shown that numerical solutions of the unsteady mass and heat balance equations of an individual snow grain reconcile well with the steady-state solution of the TM model, albeit after a transient regime. Using large-eddy simulations (LES), it is found that the residence time of a typical saltating particle is shorter than the period of the transient regime, implying that using the steady state solution might be erroneous. For scenarios with equal [\[..¹\] initial air and particle temperatures of 263.15 K](#), these errors range from 26% for low-wind [\[..²\] low saturation-rate](#) conditions to 38% for high-wind [\[..³\] high saturation-rate](#) conditions. With a small temperature [\[..⁴\] difference](#) of 1 K between the air and the snow [\[..⁵\] particles](#), the errors due to the TM model are already as high as 100% with errors increasing for larger temperature [\[..⁶\] differences](#).

Copyright statement. TEXT

1 Introduction

Sublimation of drifting and blowing snow has been recognized as an important component of the mass budget of polar and alpine regions (Liston and Sturm, 2004; van den Broeke et al., 2006; Lenaerts et al., 2012; Vionnet et al., 2014). Field observations and [\[..⁷\] modeling](#) efforts focused on Antarctica have highlighted the fact that precipitation and sublimation losses are the dominant terms of the mass budget in the katabatic flow region as well as the coastal plains (van den Broeke et al., 2006). Even though precipitation is challenging to measure accurately, methods to measure it exist, for example, using radar (Grazioli et al.,

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2017) or snow depth change (Vögeli et al., 2016). In comparison, sublimation losses are even harder to measure and can only be calculated implicitly; using measurements of wind speed, temperature and humidity. Thus, in regions where sublimation loss is a dominant term of the mass balance, it is also a major source of error. This error ultimately results in errors in the mass accumulation of ice on Antarctica, which is a crucial quantity for understanding sea-level rise and climate change (Rémy and Frezzotti, 2006; Rignot et al., 2011; Lenaerts et al., 2012).

Aeolian transport of snow can be classified into [\[.8\]](#)three modes, namely, [creeping](#), saltation and suspension. [Creeping consists of heavy particles rolling and sliding along the surface of the snowpack either due to form drag or bombardment due to impacting particles.](#) Saltation consists of particles being transported along the surface via short, ballistic trajectories with heights [mostly](#) less than 10 [\[.9\]](#)cm and involves mechanisms of aerodynamic entrainment along with rebound and splashing of ice grains (Doorschot and Lehning, 2002; Comola and Lehning, 2017). Suspension on the other hand refers to transport of small ice grains at higher elevations and over large distances without contact with the surface. Current calculations of sublimation losses are largely restricted to losses from ice grains in suspension. This is true for both field studies ([Mann et al., 2000](#)), where sublimation losses are calculated using measurements, usually at the height of [\[.10\]](#) $\mathcal{O}(1\text{ m})$, and in mesoscale [\[.11\]](#)modeling studies (Xiao et al., 2000; Déry and Yau, 2002; Groot Zwaafink et al., 2011; Vionnet et al., 2014), where the computational grids and time-steps are too large to resolve flow dynamics at saltation length and time scales. Mass loss in the saltation layer is hard to measure and is neglected based on the justification that the saltation layer is saturated. However, recent studies using high-resolution [\[.12\]](#)steady-flow, Reynolds-averaged Navier-Stokes (RANS) type simulations ([Dai and Huang, 2014](#)) claim that sublimation losses in the saltation layer are not negligible, particularly for wind speeds close to the threshold velocities for aeolian transport, wherein a majority of aeolian snow transport occurs via saltation rather than suspension.

The coupled heat and mass balance equations of a single ice particle immersed in turbulent flow are

$$c_i m_p \frac{dT_p}{dt} = L_s \frac{dm_p}{dt} + \pi \mathcal{K} d_p (T_{a,\infty} - T_p) \mathcal{N}u, \quad (1)$$

$$\frac{dm_p}{dt} = \pi \mathcal{D} d_p (\rho_{w,\infty} - \rho_{w,p}) \mathcal{S}h, \quad (2)$$

where, m_p , T_p and d_p are the mass, temperature and diameter of the particle respectively that vary with time, c_i is the specific heat capacity of ice, L_s is the latent heat of sublimation, $\mathcal{K}$ is the thermal conductivity of moist air and $\mathcal{D}$ is the mass diffusivity of water vapor in air. Transfer of heat and mass is driven by differences of temperature and vapor density between the particle surface (T_p , $\rho_{w,p}$) and the surrounding fluid ($T_{a,\infty}$, $\rho_{w,\infty}$) [\[.13\]](#). [The vapor density at the surface of the ice particle is considered to be the saturation vapor density for the particle temperature. The transfer mechanisms are](#) enhanced by turbulence, the effect of which is [\[.14\]](#)parameterized by the Nusselt ($\mathcal{N}u$) and Sherwood ($\mathcal{S}h$) numbers respectively. $\mathcal{N}u$ and

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Sh are related to the relative speed ($|\mathbf{u}_{rel}|$) between the air and the particle via the particle Reynolds number (Re_p) as

$$Re_p = \frac{d|\mathbf{u}_{rel}|}{\nu_{air}}; Nu = 1.79 + 0.606 Re_p^{1/2} Pr^{1/3}; Sh = 1.79 + 0.606 Re_p^{1/2} Sc^{1/3}, \quad (3)$$

where ν_{air} is the kinematic viscosity of air and Pr and Sc are the Prandtl and Schmidt numbers respectively.

Thorpe and Mason (1966) solved the above coupled Eqs. (1) and (2) by, (a) neglecting the thermal inertia of the ice particle, thus effectively stating that all the heat necessary for sublimation is supplied by the air, and (b) considering the temperature difference between the particle and surrounding air to be *small*, thereby allowing for Taylor series expansion of the Clausius-Clapeyron equation and neglecting higher-order terms, resulting in their formulation for the mass loss term as,

$$\frac{dm_p}{dt} = \pi d_p (\sigma_* - 1) \left/ \left(\frac{L_s}{\mathcal{K} T_{a,\infty} Nu} \left(\frac{L_s M}{R T_{a,\infty}} - 1 \right) + \frac{1}{\mathcal{D} \rho_s(T_{a,\infty}) Sh} \right) \right., \quad (4)$$

where $\rho_s(T_{a,\infty})$ is the saturation vapor density of air surrounding the particle, [\[..¹⁵\]saturation-rate](#) $\sigma_* = \rho_{w,\infty}/\rho_s(T_{a,\infty})$, M is the molecular weight of water and R is the universal gas constant. The above formulation has been used extensively to [\[..¹⁶\]analyze](#) data collected in the field (Mann et al., 2000), wind tunnel experiments (Wever et al., 2009), and numerical simulations of drifting and blowing snow [\[..¹⁷\]\(D  ry and Yau, 2002; Groot Zwaaftink et al., 2011; Vionnet et al., 2014\)](#). In the [\[..¹⁸\]modeling](#) studies, the mass loss term is computed using Eq. (4) and is added, with proper [\[..¹⁹\]normalization](#), to the advection-diffusion equation of specific humidity while the latent heat of sublimation multiplied by the mass loss term is added to the corresponding equation for temperature (Groot Zwaaftink et al., 2011).

Two observations motivated us to investigate the suitability of the TM model for sublimation of saltating snow particles. Firstly, the TM model assumes that all the energy required for sublimation is supplied by the air. This assumption was tested by Dover (1993) who compared the potential rates of cooling of particles with that of the surrounding air due to sublimation. Using scale analysis, Dover (1993) formulated the quantity $\xi = 6\rho_{air}c_{p,air} \left/ \pi\rho_i c_i \overline{d_p}^3 N \right.$, where $\overline{d_p}$ is the mean particle diameter, N is the particle number density, ρ_i is the density of ice, and showed that for $\xi \gg 1$, it can be accurately considered that the heat necessary for sublimation comes from the air. For standard values for an ice particle in suspension, [\[..²⁰\] \$\overline{d_p} = 50 \mu m\$](#) and $N \sim \mathcal{O}(10^6)$, this condition is easily met ($\xi \sim \mathcal{O}(10^3)$). However, if we input values typical for saltation, i.e. [\[..²¹\] \$\overline{d_p} = 200 \mu m\$](#) and $N \sim \mathcal{O}(10^8)$, $\xi \sim \mathcal{O}(1)$, and the condition is not met. Thus, for sublimation of saltating particles, it is important to consider the thermal inertia of the particles. A similar conclusion was reached in other [\[..²²\]modeling](#) studies on topics of heat and mass exchange between disperse particulate matter in turbulent flow such as small water droplets in heat exchangers (Russo et al., 2014) and sea-sprays (Helgans and Richter, 2016).

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Secondly, Eq. (4) computes mass loss as being directly proportional to σ_* and neglects the temperature difference between the particle and air. Eq. (4) thus predicts a mass loss even in extremely high [\[..²³ \]saturation-rate](#) conditions, whereas immediate deposition of water vapor would occur on a particle even slightly colder than the air. Indeed, some field experiments have reported deposition as opposed to sublimation which was expected, on the basis of the measured under-saturation of the environment, particularly near coastal polar regions (Sturm et al., 2002). A simple everyday observation illustrates this fact clearly; There is immediate deposition of [\[..²⁴ \]vapor](#) and formation of small droplets on the surface of a cold bottle of beer even in room conditions with moderate humidity!

Motivated by the observations described above, in this [\[..²⁵ \]article](#), we describe four numerical experiments where we compare differences between the fully numerical and the Thorpe and Mason (1966) solutions (referred to as NUM and TM approaches respectively). In Experiment I and II, we numerically solve Eqs. (1) and (2) and compare the results with Eq. (4) for physically plausible values of a saltating ice particle. Results of these tests are presented in Sect. 2. High-resolution large-eddy simulations (LES) of [the atmospheric surface layer with](#) saltating snow are performed for a range of environmental conditions to compute the differences between the NUM and TM approaches in realistic wind-driven saltating events. These results are presented in Sect. 3. A summary of the [\[..²⁶ \]article](#) is made in Sect. 4

2 Comparison between NUM and TM solutions: EXPERIMENT I and II

We consider an [\[..²⁷ \]idealized](#) scenario where a [solitary](#) spherical ice particle is held still in a turbulent air flow with constant mean speed, temperature and under-saturation. [The evolution of the mass, diameter and temperature of the ice particle is calculated using both the NUM and TM models and an inter-comparison is made.](#) This scenario is similar to the wind-tunnel study performed by Thorpe and Mason (1966) who measured mass loss of solitary ice grains suspended on fine [\[..²⁸ \]fibers](#). [In this scenario, we consider that the heat and mass transfer between the ice particle and the air changes the mass and temperature of the particle only while the mass and energy anomalies in the air are rapidly advected and mixed away. This implies that the environmental conditions subjected to the ice particle remain constant.](#) While it can be expected that the environmental conditions will vary along the trajectory of a ice particle undergoing saltation or suspension, it is nevertheless useful to perform this analysis as it reveals important characteristics [\[..²⁹ \]about](#) the heat and mass evolution of a ice particle during sublimation and about the [approximations used to derive the](#) TM model.

For the NUM approach, Eqs. (1) and (2) are solved [in a coupled manner](#) using a simple first-order finite-differencing scheme [for time-stepping with a time-step of 50 \$\mu\$ s.](#) For the TM approach, Eq. (4) is used with a similar numerical setup as [for the NUM approach. In the TM approach, particle temperature is not considered and the mass and energy transfer is](#)

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determined only by air temperature and saturation-rate. The initial particle diameter ^[.30] $(d_{p,IC})$ is $200\mu m$ and the air-flow temperature is ^[.31] $263.15 K$ for both the NUM and TM approaches. We use a constant air speed of ^[.32] $5ms^{-1}$ resulting in $Re_p = 80$, $Nu = 6.7$ and $Sh = 6.5$ (using Eq. (3)). The values used here are typical of a saltating ice particle.

In Experiment I, we study the heat and mass output from a sublimating ice grain as a function of time. In the first case,
 5 Experiment I-A, we consider the effect of three different values of air-flow ^[.33]saturation-rate ($\sigma_* = 0.8, 0.9$ and 0.95) on differences between NUM and TM solutions. ^[.34]The NUM approach requires specification of the initial condition for the particle temperature ($T_{p,IC}$). In Experiment I-A, ($T_{p,IC}$) is taken to be the same as the air-flow temperature for the NUM approach^[.35], i.e, $263.15 K$. Results for Experiment I-A are shown in Fig. 1(a-c), with subfigure (a) showing the mass output rate, F_M and subfigure (b) showing the heat output rate, F_Q . Note that in this figure and subsequent figures, $+(-)$ signifies mass
 10 and heat gained (lost) by the air. Since we keep the temperature and under-saturation of the air constant, the solutions of the TM approach are “steady-state” solutions with constant heat and mass transfer rates as seen in Fig. 1a and b. On the other hand, since the NUM approach solves the coupled equations that consider the evolution of the particle temperature, the heat and mass transfer rates evolve with time.

It can be seen that the NUM solutions ^[.36]initially evolve with time and reconcile with the steady-state TM solutions after
 15 a transient regime of about 0.3 seconds. Since the initial temperature of the particle is the same as the air, there is no heat transfer between the air and the particle (see the second term of the R.H.S of Eq. 1) initially. Thus, all heat transfer rates are initially zero for the NUM case in Fig. 1b. The under-saturation of the air forces mass transfer from the ice particle to air and the energy for the phase change comes from the internal energy of the ice particle. This causes the particle temperature to drop (see Fig. 2 below). With the particle now colder than the air, heat transfer from the air to the
 20 particle commences and ultimately, the energy for sublimation comes entirely from the heat extracted from the air. The initial dynamics of the heat and mass transfer are completely neglected by the TM approach. In subfigure (c), the errors ($Err(t) = \left(\int_0^t F_{NUM} dt / \int_0^t F_{TM} dt - 1 \right) * 100$) for mass, Err_M and heat, Err_Q are shown. The errors reduce dramatically with time (for example, 15% at 0.3 seconds) and interestingly do not depend on the ^[.37]saturation-rate of the air-flow.

In the following case, ^[.38]Experiment I-B, similar simulations as in Experiment I-A are performed, but with $\sigma_* = 0.95$
 25 while the initial temperature difference between the particle and the air is varied as ^[.39] $T_{p,IC} - T_{Air} = -2, -1, 1, 2 K$. The results are shown in Fig. 1(d-f). It is interesting to note that ^[.40]for each of the four cases considered, the TM solution predicts sublimation of the particle (consistent with ^[.41] $\sigma_* < 1$, see numerator of R.H.S of Eq. 3). On the other hand, for

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cases with colder particles, the NUM solutions show that there is initially deposition on the particle, along with larger values of heat absorbed from the air. Correspondingly, in the cases with particles being warmer than the air, the mass loss is much higher in the NUM solution than that computed by the TM solution while the heat ^[.42] gained by the particle is also much higher. These higher differences are reflected in the Err_M and Err_Q curves in subfigure (f) where errors are found to be an order of magnitude higher than those in subfigure (c).

We define relaxation time ($\tau_{relaxation}$) as the time required for the NUM solution to reconcile with the TM solution. The importance of this quantity lies in the fact that if the residence time of a saltating ice grain in air is shorter than $\tau_{relaxation}$, the TM approach is likely to be erroneous and the NUM approach would be required. It is intuitive that $\tau_{relaxation}$ increases with d_p on account of increasing inertia and decreases with $|\mathbf{u}_{rel}|$ due to more vigorous heat and mass transfer. Experiment I was repeated for values of d_p and $|\mathbf{u}_{rel}|$ ranging between ^[.43] $(50 - 1000 \mu m)$ and $(0 - 10 ms^{-1})$ respectively. The upper-bound of the wind-speed range is quite high and it is extremely unlikely to find ^[.44] $|\mathbf{u}_{rel}| > 10 ms^{-1}$ in naturally-occurring aeolian transport. Numerical results indeed confirm our intuition and it is found that for any given value of $|\mathbf{u}_{rel}|$, $\tau_{relaxation}$ is found to be $\propto d_p^\alpha$, where $\alpha (\sim 1.65)$. Furthermore, $\tau_{relaxation}$ decreases monotonically with increasing $|\mathbf{u}_{rel}|$ for a given value of d_p . For ^[.45] $d_p = 200 \mu m$, the plausible values of $\tau_{relaxation}$ are found to lie between 0.28 and 1.5 seconds (for $|\mathbf{u}_{rel}| = 10$ and 0 ^[.46] ms^{-1} respectively). Interestingly, $\tau_{relaxation}$ is not found to depend on saturation-rate of air and the difference between the initial particle temperature and air. Plots of $\tau_{relaxation}$ are highly relevant to discussion in Sect. 3 and presented there.

In Fig. 2, evolution of particle diameter (d_p) and temperature (T_p) is presented with subfigures (a) and (b) respectively describing the evolution for simulations in Experiment I-A with (c) and (d) being the corresponding results from Experiment I-B. In Experiment I-A, the particle diameters reduce linearly with time for both the NUM and TM approaches with the more shrinking (or in other words, sublimation) in the NUM solutions. More interesting is the evolution of the particle temperature, where the particle undergoes significant cooling due to sublimation and ultimately achieves a constant temperature. For example, in the case for $\sigma_* = 0.8$, the particle temperature is ultimately 0.85 K lower than the initial particle temperature of 263.15 K. Note that for the TM approach, particle temperature is of no consequence and it is shown simply for reference.

Following results of Experiment I, in Experiment II, we explore the parameter space of ^[.47] $(\sigma_*, T_{p,IC} - T_{Air})$ and compute the total mass ($M = \int_0^t F_M dt$) and total heat ($Q = \int_0^t F_Q dt$) output by a sublimating ice grain for a finite time of $t = 0.5$ seconds. Results shown in Fig. ^[.48] 3 subfigures a and b provide a comparison of the total mass lost using the NUM and TM solutions respectively and the corresponding error is shown in subfigure ^[.49] c. Similar figures are presented for the total heat

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lost/gained by the air in subfigures ([..⁵⁰]d-f). The inclusion of the inertial terms essentially causes the contours to be sloped for the NUM solution while the TM solutions do not depend on [..⁵¹] $T_{p,IC} - T_{Air}$ as expected. The error between the NUM and TM solutions are accentuated at high [..⁵²]saturation-rate regimes, with errors larger than 30 % for $\sigma_* > 0.8$.

In summary, Experiments I and II highlight the fact that during the sublimation of an ice grain, there exists a finite, well-defined transient regime before the NUM solutions match the steady-state TM solutions. Furthermore, the NUM and TM solutions diverge rapidly with slight temperature differences between the particle and the air and with increasing σ_* (which is a cause of concern since in snow-covered environments, the air usually is highly saturated). The results described above prompt an interesting question: are the residence times of saltating ice particles comparable to $\tau_{relaxation}$? We use large-eddy simulations to answer this question in the following section.

10 3 Large-eddy Simulations of Saltating Snow

3.1 Experiment III and IV: Simulation Details

To further understand the implications of the differences between the NUM and the TM approach, we performed LES of the atmospheric surface layer with [..⁶⁰]an erodible snow surface as the lower boundary. We describe here only the main details of the LES that are relevant to our discussion [..⁶¹]with full model description along with equations [..⁶²]presented in Supplementary Material S1. The LES solves filtered Eulerian equations for momentum, temperature and specific humidity on a computational domain of [..⁶³]6.4 m $\times$ 6.4 m in the horizontal directions with vertical extent of the domain being 6.4 m as well. The snow surface, which constitutes the lower boundary of the computational domain, consists of spherical snow particles with a log-normal size distribution with a mean particle diameter of 200 μm and standard-deviation of 100 μm . The coupling between the erodible snow-bed and the atmosphere is modeled through statistical models for aerodynamic entrainment (Anderson and Haff, 1988), splashing and rebounding of particle grains (Kok and Renno, 2009), which have been updated recently by Comola and Lehning (2017) to include the effects of cohesion and heterogeneous particle sizes. The use of these models essentially allows for overcoming the immense computational cost of resolving individual grain-to-grain interactions and allow us to consider the snow-surface as a *bulk* quantity rather than a collection of millions of individual snow particles. Once the ice grains are in the flow[..⁶⁴], their equations of motion are solved in the Lagrangian frame of reference with only gravitational and turbulent form drag forces included. Since the particle velocities are known, $|\mathbf{u}_{rel}|$ is calculated explicitly and used to compute Re_p , Nu and Sh . The horizontal boundaries of the domain are periodic

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and the lower boundary condition (LBC) for velocity uses flux ^[.65]parameterizations based on Monin-Obukhov similarity theory, additionally corrected for flux partition between fluid and particles between the wall and the first flow grid point (Raupach, 1991; Shao and Li, 1999). The LBC for scalars (temperature and specific humidity) are flux-free and thus the only source/sink of heat and water ^[.66]vapor in the simulations is through the interaction of the flow with the saltating particles.

5 ^[.67]All simulations are performed on a grid of 64 x 64 x 128 grid points with a uniform grid in the horizontal directions and a stretched grid in the vertical. A stationary turbulent flow is allowed to first develop, following which, the snow surface is allowed to be eroded by the air. All physical constants and parameters along with additional details of the numerical setup are provided in Supplementary Material S2.

For the TM approach, Eq. (4) is used to compute the specific humidity and (by multiplying with the latent heat of sublimation) 10 heat forcing due to each ice grain in the flow. On the other hand, for the NUM approach, Eqs. (1) and (2) are solved and only the turbulent transfer of heat between the air and the particle (second term in R.H.S of Eq. (1)) acts as a heat forcing on the flow. An implication of the NUM approach is that the particle temperature evolves during the ice-grain's motion and this necessitates providing an initial condition for the particle temperature ($T_{p,IC}$).

The principle aims of Experiment III are to firstly quantify particle residence times (PRT) and their dependence on wind 15 speeds and relative humidities and secondly, compute the differences in the heat and mass output between the NUM and the TM approaches during ^[.68]saltation of snow with complete feedback between the air and the particles. PRT is defined as the total time the particle is air-borne and in motion, including multiple hops across the surface. Note that the PRT is not computed for particles in suspension, i.e, particles that stay aloft and never return to the surface. Towards this goal, simulations are performed, each with a combination of initial surface stress, ^[.69] $u_* \in \{0.4, 0.6, 0.8\} \text{ ms}^{-1}$ and initial saturation-rate, 20 $\sigma_* \in \{0.3, 0.6, 0.9\}$. These values are classified as low (L), medium (M) and high (H) and ^[.70]correspond to wind speed at 1 m height above the surface of 11 m/s, 16.3 m/s and 21.8 m/s respectively. Note that during fully developed snow transport, the particles in the air impart drag on the flow causing the flow to decelerate. The wind speeds at 1 m during fully developed saltation are 8.77 m/s, 11.34 m/s and 13 m/s respectively. The simulations are named as $U\alpha\text{-}R\beta$, where $(\alpha, \beta) \in \{L, M, H\}$. Each combination is simulated independently for the NUM and TM approaches resulting in a total of 25 eighteen simulations. Experiment III is limited to simulating the *usual* case where the initial air temperature ($T_{Air,IC}$) is the same as $T_{p,IC}$.

Experiment IV is aimed at exploring the implications of differences between the two approaches in cases where $T_{Air,IC}$ is significantly different from ^[.71] $T_{p,IC}$. Such conditions can occur in nature during events such as marine-air intrusions,

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katabatic winds, spring-season saltation events and winter flows over sea-ice floes, where significant temperature differences between the air and snow-surface are likely. We repeat the low wind case of Experiment III with $u_* = 0.4 \text{ ms}^{-1}$ and choose the initial saturation-rate to be 0.95, motivated by results in Experiment II where errors were found to increase with increasing saturation-rate. Simulations (named as UL-T(γ), where $T_{Air,IC} - T_{p,IC} = \gamma$) are performed once again for each of two approaches with $\gamma \in \{\pm 1 \text{ K}, \pm 2.5 \text{ K}, \pm 5 \text{ K}\}$ resulting in a total of twelve simulations. In all simulations performed for Experiments III and IV, $T_{p,IC} = 263.15 \text{ K}$. It is important to note that the initial condition for particle temperature ($T_{p,IC}$) is fixed throughout the simulation period, which essentially means that surface temperature is kept constant. This is consistent with the imposed zero flux of heat at the surface. This imposition will be justified *a posteriori* in the following section.

3.2 Results

In this section, results from the LESs performed for Experiments III and IV are presented. Note that only the relevant results are presented, namely (a) particle residence times as a function of particle diameters and different forcing setups and (b) differences between the NUM and TM approaches for calculating average mass and heat transfer rates during saltation. Other results, for example, vertical profiles of mean and turbulent quantities, although interesting are relegated to the supplementary material as their analysis is out of scope of the current work. Additionally, a video illustration (Supplementary Movie M1) of an LES is provided to help visualize and frame the context of the simulations performed.

3.2.1 Particle Residence Times versus $\tau_{relaxation}$

As mentioned in the concluding lines of the Sect. 2, the principle quantity of interest is the PRT of saltating ice grains. In Fig. 4a, the mean and median PRT of five different simulations of Experiment III are shown as a function of the particle diameter. Additionally, values of $\tau_{relaxation}$ computed in Experiment I for wind speeds ranging from 0 to 10 ms^{-1} are also shown in the shaded region. Recall that the shaded region represents all the plausible values of $\tau_{relaxation}$ in naturally-occurring aeolian transport. As examples, $\tau_{relaxation}$ trends for 3 wind speeds, 0, 1 and 10 ms^{-1} are shown and the power-law dependence can clearly be seen. It is found that $\tau_{relaxation}$ is comparable to the PRT of saltating grains with diameters between 125 and 225 μm . For 200 μm , the mean PRT is found to be 0.6 seconds while the median PRT

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is 0.2 seconds, which is outside the range of admissible values of $\tau_{relaxation}$. For particles larger than 225 ^[.83] μm , the PRTs are an order of magnitude smaller than plausible values of $\tau_{relaxation}$ and therefore the TM model is likely to provide wrong values of mass loss. On the other hand, lighter particles with diameters smaller than 100 ^[.84] μm have much longer PRTs and the TM model is therefore valid. This proves that while the TM model is applicable for a majority of particles in suspension, it is likely to cause errors for particles in saltation.

Results presented in Fig. ^[.85] 4a provides two additional insights. Firstly, it is quite interesting to note that particles larger than 100 ^[.86] μm have the same mean PRT irrespective of low, medium or high wind speeds. This means that the dynamics of the heavier particles are unaffected by ^[.87] different wind speeds simulated in Experiment III, which is consistent with the notion of self-organized saltation, which has recently been shown by Paterna et al. (2016). For particles smaller than 100 ^[.88] μm , the mean PRTs increases with wind speed. Secondly, the initial ^[.89] saturation-rate does not seem to effect the PRT statistics for medium and high wind conditions and the UM- and UH- curves for different R values overlap (this is the reason only five PRTs are shown in Fig. ^[.90] 4a). In these cases turbulence is sufficient to rapidly mix any temperature anomaly due to sublimation throughout the surface layer. On the other hand, in low wind conditions (UL), low initial ^[.91] saturation-rate results in more sublimation and cooling near the surface, resulting in suppression of vertical motions. This is reflected in the mean PRTs of particles smaller than 75 ^[.92] μm , which decrease with decreasing initial σ_* . ^[.93] Even though this an interesting result, we leave further exploration of this phenomenon for a future study with some preliminary analysis provided in Supplementary Material S3.

The PRT distributions are found to be quasi-exponential with long tails, thus resulting in large differences in mean and median PRTs shown in Fig. ^[.94] 4a. These distributions are also strongly dependent on the particle diameter. As an illustration, in Fig. ^[.95] 4b, cumulative distributions of PRTs are shown for four particle diameters along with the corresponding range of plausible $\tau_{relaxation}$ values. For the mean particle diameter of 200 ^[.96] μm , we find that between 65% to 85% of particles have PRTs shorter than $\tau_{relaxation}$, whereas for the 75 μm particles, at most 30% particles lie below the maximum $\tau_{relaxation}$ threshold. This reinforces the fact that applying the steady-state TM solution to sublimating ice-grains in saltation could be potentially erroneous.

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3.2.2 Differences in total mass loss between NUM and TM models

We can directly assess the implications of differences in grain-scale sublimation between the two approaches on total mass loss rates during saltation at [..⁹⁷]larger spatial scales as simulated using LES in Experiments III and IV. In Fig. 5, we compare the total 15-min averaged rate of mass loss computed in all cases in Experiment III (subfigure a) and Experiment IV (subfigure c) using the NUM and the TM approaches with corresponding errors shown in subfigures b and d respectively. Recalling the adopted convention of +(-) as gain(loss) of flow quantities, it can be seen in Experiment III, that sublimation increases with u_* and decreases with σ_* . The errors on the other hand increase with increasing values of both u_* and σ_* . The increase in error with u_* is mostly due to the fact that an increase in u_* proportionally increases the total mass entrained by air (see Supplementary Fig. S1). The increase in error with increasing σ_* is in accordance with analysis done in Experiment II (see Fig. 3([..⁹⁸]c,f)) where it was shown that the NUM and TM solutions diverge with increasing [..⁹⁹]saturation-rate. The least error, 26% is found for case UL-RL (i.e., $u_* = 0.4$, $\sigma_* = 0.3$) while the largest error, 38% is found for UH-RH ($u_* = 0.8$, $\sigma_* = 0.9$). Overall, for all the simulation combinations, the NUM approach computes larger mass-loss than the TM approach.

Experiment IV highlights the effect of temperature difference between particle and air on sublimation. As shown in Fig. 5c, the mass output is found to be negative (deposition) for the NUM solutions when the air is warmer than the particles (i.e., cases UL-T(γ) with $\gamma > 0$). This is contrary to the TM solutions which indicate sublimation. In cases with $\gamma < 0$, the NUM approach shows a much higher sublimation rate than the TM solutions. This occurs firstly due to higher vapor pressure at the grain surface that results in enhanced vapor transport and secondly because the warmer particles heat the surrounding air via sensible heat exchange, causing the relative humidity to decrease. Errors increase dramatically from an already high 100% for UL-T(+1) to 800% for UL-T(+5). Simulations performed for medium and high wind cases in Experiment IV showed even higher errors, similar to results in Experiment III and are shown here[..¹⁰⁰].

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4 Discussion and Conclusion

In this [..¹⁰²]article, we revisit the Thorpe and Mason (1966) model used to calculate sublimation of drifting and blowing snow and check its validity for saltating ice grains. We highlight the fact that solutions to unsteady heat and mass transfer equations (NUM solutions) converge to the steady-state TM model solutions after a *relaxation* time, denoted as $\tau_{relaxation}$ that has a power-law dependence on the particle diameter and is inversely proportional to the relative wind speed. Through extensive LESs of snow saltation, we compute the statistics of the PRTs as a function of their diameters and find them to be comparable to $\tau_{relaxation}$. This helps explain the difference between mass output when using the NUM model to the TM approach, also computed during the same LESs. The NUM approach computes higher sublimation losses ranging from 26% in low-wind, low [..¹⁰³]saturation-rate conditions to 38% in high-wind, high [..¹⁰⁴]saturation-rate conditions. Another set of numerical experiments explore the role of temperature differences between particle and air temperature in inducing differences between NUM and TM solutions. We find the effect to be extremely dramatic with errors of 100% for a temperature difference of 1 K with increasing errors for larger temperature perturbations. In general, the two solutions are found to diverge rapidly as the [..¹⁰⁵]saturation-rate tends towards 1. The results showing differences of mass output between the NUM and TM approaches in the LESs in Experiments III and IV, with complete feedback between particles and the air are thus shown to be closely correlated to the results from extremely idealized simulations of heat and mass transfer from a solitary ice grain in Experiments I and II.

[..¹⁰⁶]The LES results do come with a few important caveats. Firstly, the temperature and specific humidity fluxes at the surface are neglected. In other words, particles lying on the surface are considered to be dormant and do not exchange heat or mass with the air. A corollary to neglecting the scalar fluxes at the surface is that the initial condition for temperature of the particles entering the flow is fixed. This may be justified by considering that during drifting and blowing snow events, the friction velocity at the surface drops dramatically. This fact has been observed in both in experiments (Walter et al., 2014) and in our current LESs (see Supplementary Fig. S2). This implies that direct turbulent exchange between the surface and air is curtailed and instead, the dominant exchange occurs between air-borne particles and the air. In fully-developed snow transport events, this is most likely to be true and only in intermittent snow-transport events will the surface fluxes be relatively important. This is nevertheless an important assertion that shall be more closely examined in future studies involving a full surface energy balance model, where the evolving temperature of the saltating ice grains, prior to deposition is taken into account while calculating snow-surface temperatures.

Further work is required to make concrete improvements to modeling of sublimation of saltating snow, especially in large-scale models that do not explicitly resolve saltation dynamics. One potential approach is to modify the Monin-Obukhov based lower boundary conditions for heat and moisture to account for particle temperature during blowing snow events. An ancillary outcome of this study is the discovery that buoyancy can affect the dynamics of lighter snow particles

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(with diameters less than $75\text{ }\mu\text{m}$) and decrease their residence times. Investigating this phenomenon requires a detailed analysis of turbulent structure within the saltation layer and is left for future publications.

In conclusion, analogous to the role played by saltating grains in efficient momentum transfer to the underlying granular bed, the NUM approach can be considered as an efficient transfer of heat and mass between the flow and the underlying snow surface, albeit with a closer physical relationship between the thermodynamics of the snow surface and that of the air. ¹⁰⁷ Thus, along with momentum balance of blowing snow particles, particle temperature and its thermal balance must also be taken into account.

Acknowledgements. We acknowledge the support of the Swiss National Science Foundation (Grant n. 200021-150146) ¹⁰⁸ and the Swiss Super Computing Center (CSCS) for providing computational resources (Project: s633) ¹⁰⁹. We additionally thank Marco Giometto for providing the original version of the LES code and Hendrik Huwald, Tristan Brauchli, Annelen Kahl and Celine Labouesse for illuminating discussions and important suggestions in improving the manuscript. No new data were used in producing this manuscript.

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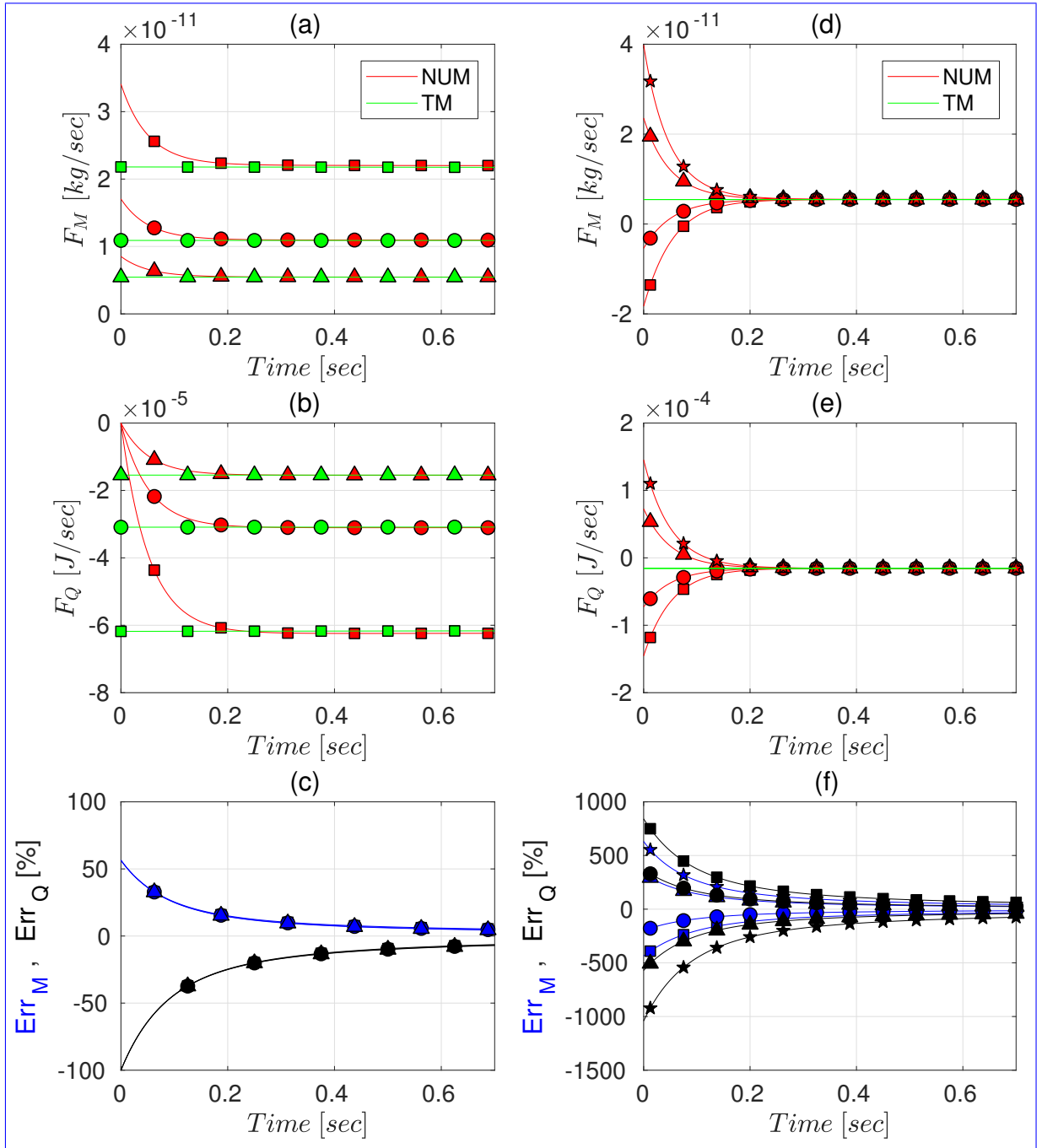


Figure 1. TM and NUM solutions for a particle of $200\ \mu\text{m}$ diameter in different environmental conditions. [..⁵³] **Experiment I-A:** (a) Rate of mass and (b) heat output with (c) corresponding errors; [..⁵⁴] $T_{p,IC}$ –[..⁵⁵] $T_{a,\infty} = 0$, [..⁵⁶] $\sigma_* = 0.8$ (squares), 0.9 (circles), 0.95 (triangles). **Experiment I-B:** (d-f) same as (a-c) with $\sigma_* = 0.95$; [..⁵⁷] $T_{p,IC}$ –[..⁵⁸] $T_{Air} = -2\ \text{K}$ (squares), $-1\ \text{K}$ (circles), $1\ \text{K}$ (triangles), $2\ \text{K}$ (stars). [..⁵⁹]

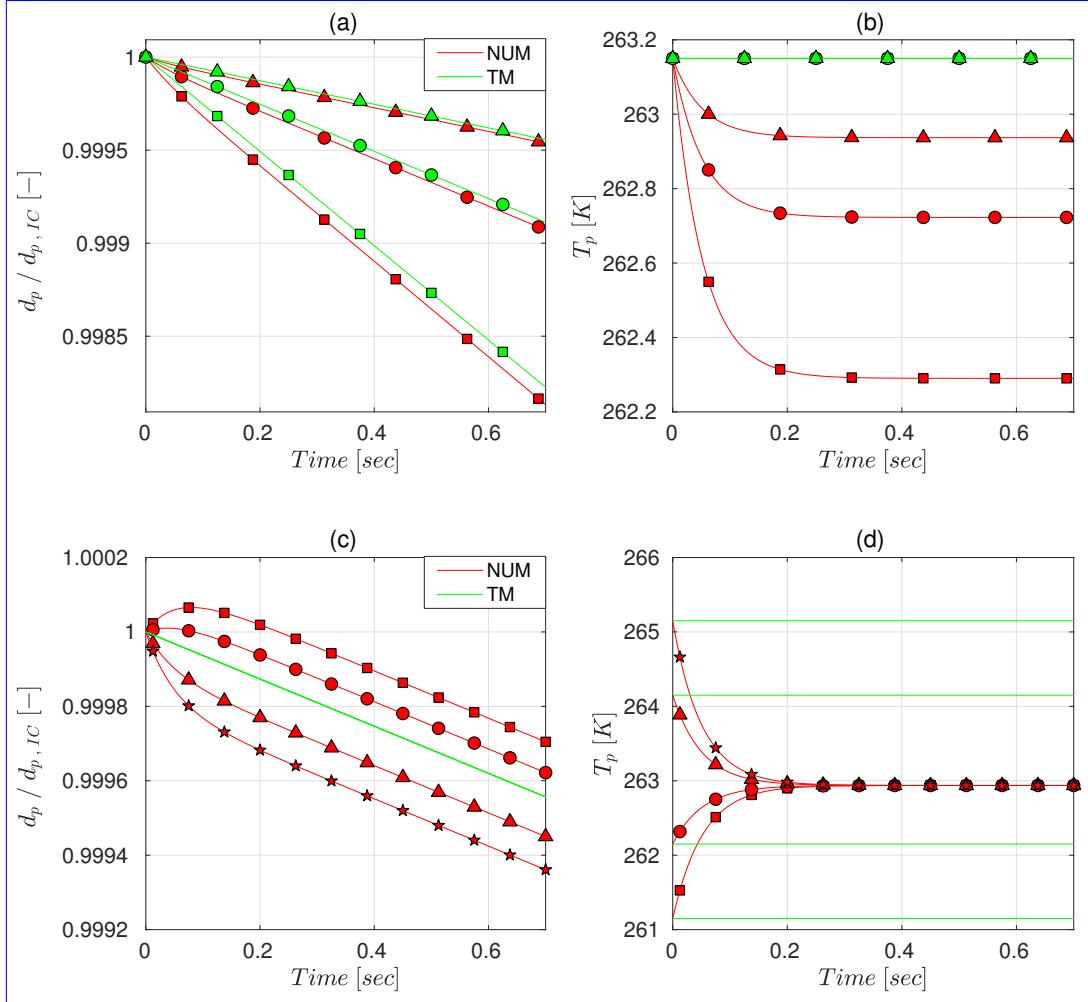


Figure 2. TM and NUM solutions for a particle of 200 μm diameter in different environmental conditions. **Experiment I-A:** Evolution of particle (a) diameter and (b) temperature; $T_{p,IC} - T_{Air} = 0$, $\sigma_* = 0.8$ (squares), 0.9 (circles), 0.95 (triangles). **Experiment I-B:** (c-d) same as (a-b) with $\sigma_* = 0.95$; $T_{p,IC} - T_{Air} = -2$ K (squares), -1 K (circles), 1 K (triangles), 2 K (stars). Note that the particle diameters are normalized by the initial diameter of the particle ($d_{p,IC}$).

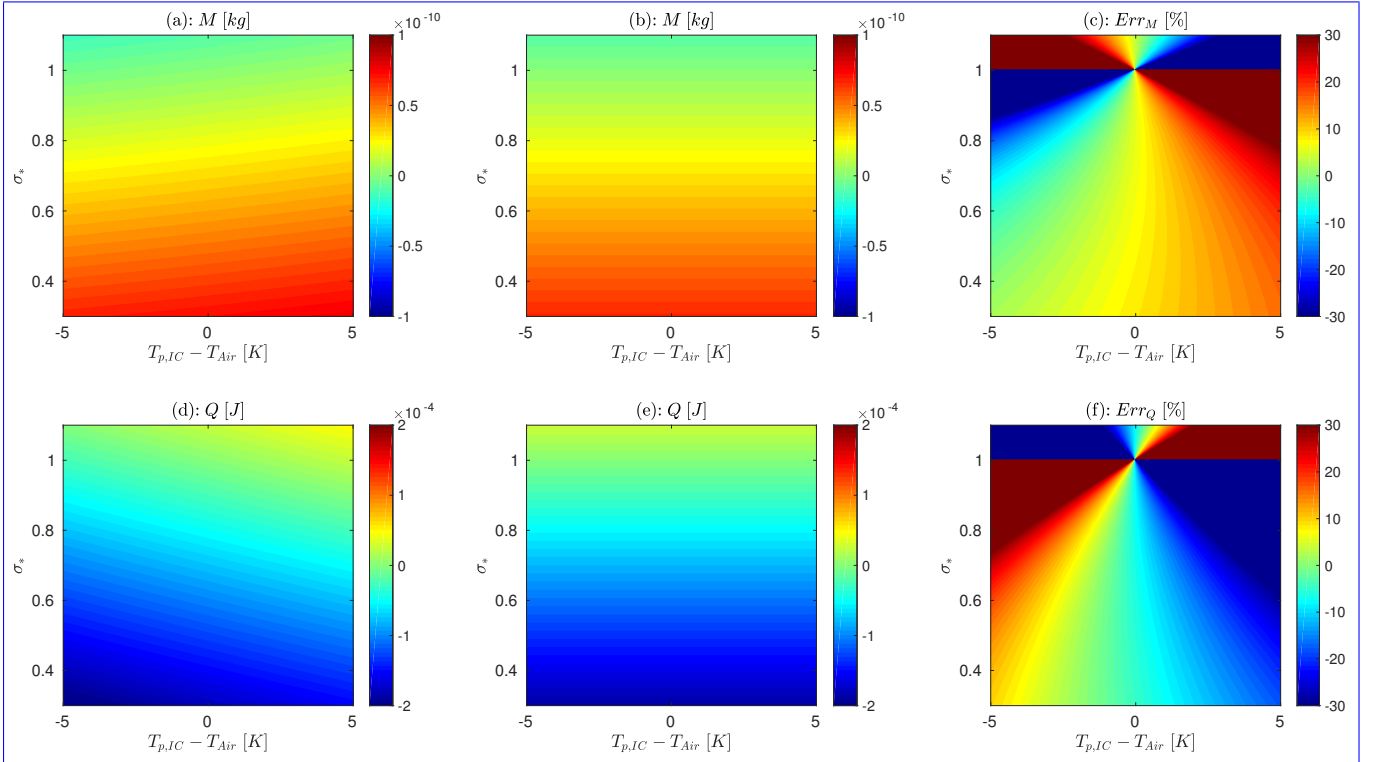


Figure 3. TM and NUM solutions for a particle of $200 \mu m$ diameter in different environmental conditions. **Experiment II:** Total mass output during 0.5 seconds by the (a) NUM and (b) TM solutions with (c) corresponding error for $\{0.3 \leq \sigma_* \leq 1.1, -5 K \leq T_p - T_{Air} \leq 5 K\}$. Similar plots for total heat output presented in (d-f).

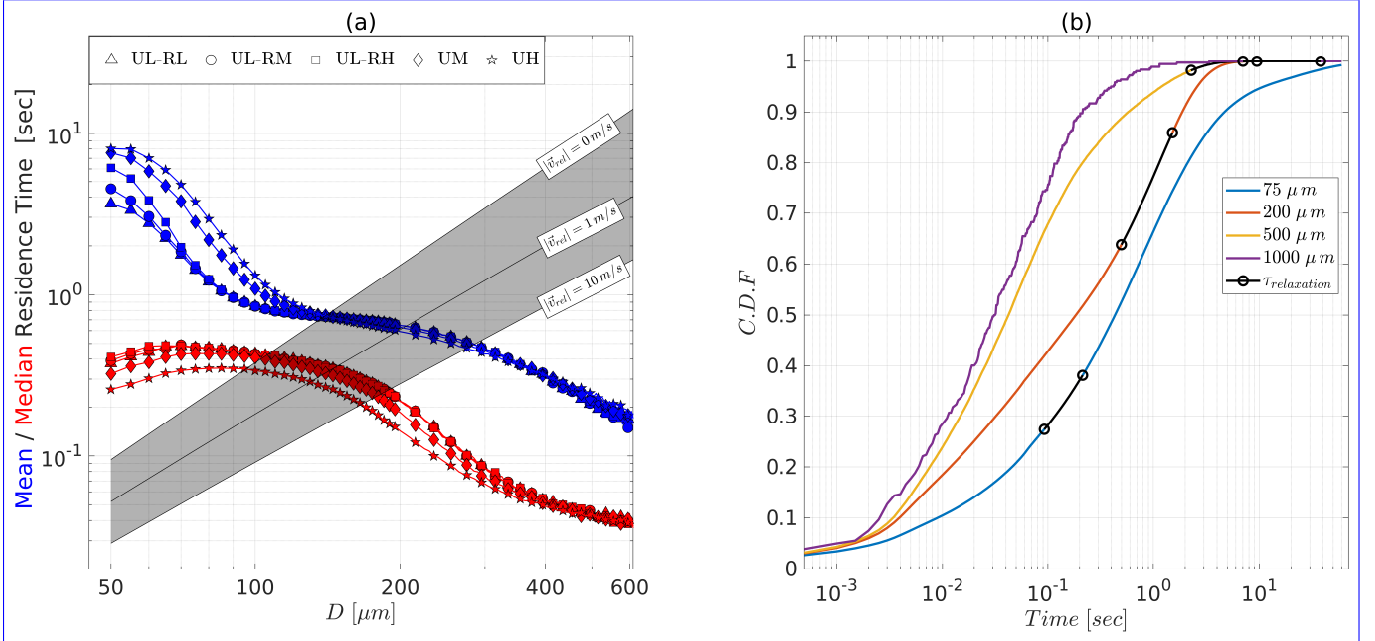
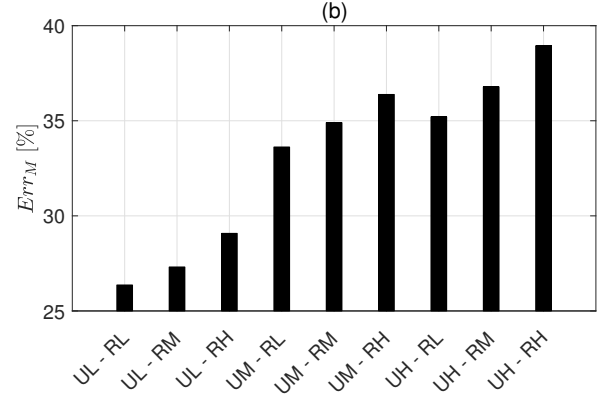
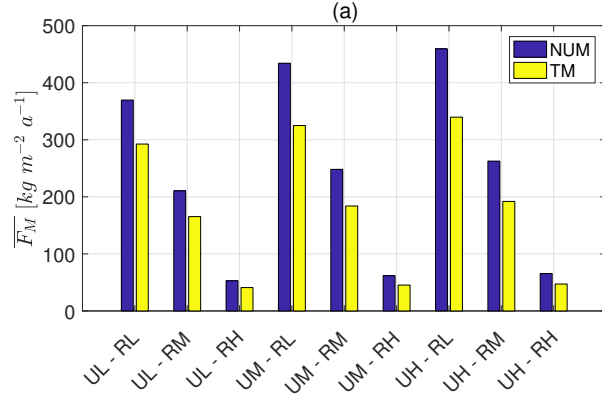


Figure 4. (a) Mean and median particle residence time (PRT) as a function of particle diameter. The plausible values of $\tau_{relaxation}$ are represented by the shaded region with trends for three values of $|u_{rel}|$ shown by straight lines. **Note that the horizontal axis is logarithmic.** (b) Cumulative Distribution Functions of PRTs for four particle diameters along with range of plausible $\tau_{relaxation}$ values marked by overlying black curves

EXPERIMENT - III



EXPERIMENT - IV

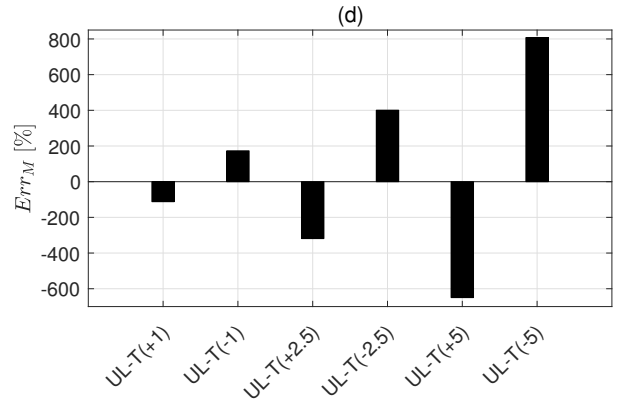
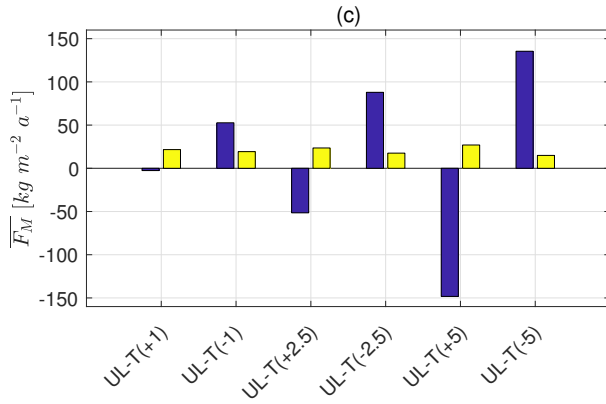


Figure 5. Experiment III: (a) Average rate of mass loss during 15 minutes of saltation, (b) Error between NUM and TM solutions. Corresponding plots for Experiment IV in (c) and (d) respectively. Note that the units used for rate of mass loss are kilograms per unit area per unit year.