Review: "Thermal conductivity of firn at Lomonosovfonna, Svalbard, derived from subsurface temperature measurements" by Marchenko et al.

General comments

In the paper the authors retrive profiles of thermal conductivity of near-surface snow/firn in Svalbard as optimization parameters from a comparison of the numerical solution of the heat equation with thermistor (chain) measurements buried over several years. The optimization is restricted to the dry zone.

In my opinion, using intermediate complexity (few parameter) models to constrain physical properties from measurements via optimization is often more conclusive over a mere comparison with more complex (many parameter) models. Even though this method is not new, the work is apparently sound and the authors come to a clear and important conclusion, namely that common density based parametrizations are insufficient for the thermal conductivity in near-surface firn. While I would immediately agree on this finding on physical grounds (see comments), I don't fully understand how the assumption that ρ and k are used as *independent* optimization parameters (which is certainly not true) may affect this finding. A little more discussion seems required here. This and other things are listed in the comments below.

But overall, the manuscript is well written and the methods are thoroughly described such that the paper warrants publication after revisions have been made.

Henning Löwe

Specific comments

(p2/l10): In literature there is some inconsistency about *effective*: In the context of upscaling (e.g. Calonne 2011) "effective" is used in the sense of macroscopic, even if conduction only. Sometimes "effective" is used when the mix of conductive and phase-change processes is meant.

(p2/l29): Another discussion of uncertainties can be found in [2].

(p3/l10): Maybe also [1] should be mentioned due to the similarity to the present work. The extension to wet snow might be relevant in the future.

(p3/l6): We have shown in [3] how the structural anisotropy can be quantitatively utilized to correct the bias/scatter in density-based parametrizations for k. The parametrization has been confirmed for tundra snowpacks with strong variability in structural anisotropy [4]. In a nutshell, if snow or firn is subject to temperature gradient (TG) metamorphism the structure is reorganized with almost no or little changes in density but with an increase in structural anisotropy that causes e.g. an increase in thermal conductivity. For near surface snow in perennial snowpacks the anisotropy stems from TG metamorphism from the persistent temperature gradient in the top part from the pentration of the annual heat wave into the firn. Anisotropy in near-surface arctic or antarctic firn is well discussed, e.g. in [5]. From this point of view the results obtained in the present paper are consistent with reported influences of structural anisotropy on physical properties in snow/firn.

In principle the optimization method suggested here could be readily utilized to infer the anisotropy parameter Q directly by plugging the parametrization $k(\rho, Q)$ from [3] into the heat equation (3) and subsequently optimizing over ρ and Q instead of ρ and k. This would also heal the fact that k and ρ are (erroneously) treated as uncorrelated optimization parameters, while, in contrast, density and anisotropy can be effectively regarded as two independent geometrical features of the microstructure (values for Q should likely lie in the range $[0.33, 0.45]$) It must be kept in mind though that the parametrization from [3] *must* fail somewhere at very high snow/firn densities, but the limit of validity has not been explored yet. While a comprehensive analysis in this direction is certainly beyond the scope of the paper, a simple test in this direction could help to provide some confidence, that the main conclusion is not affectd by existing correlations between k and ρ

(p7/l4): Either the functional should be denoted by $F_{\tau,\varrho}$ instead of $F_{\tau,\varsigma}$ or otherwise measured densities denoted by ς (check throughout)

(p7/l14): The statement that there is no data on error estimates of density measurements in snow/firn is a too brave.

(p7/l15): I think at least a quick sensitiviy should be made (i guess that has been done anyway) on the value of γ. This is directly related to the assumed accurcacy of density measurements.

(p8/l18): $\rightarrow$ ... and A is defined as...

(p13/l6): either replace "reduction" by "increase" or "wrong" by "right"?

(p13/l27): Maybe use `$\bar{A}_{ij}$` instead of `$\bar{A_{ij}}$`, the latter looks like superscript minus...

(p13/l30): Just a thought: The different error pattern is probably an effect of the high variability in the stratigraphy in 2014. From the exactly known *stationary* solution of Eq (3) one can infer that the temperature at a particular location involves the harmonic mean of the entire conductivity profile, which is highly affected by the vertical variability. A similar mathematical structure likely governs the *transient* problem used for the optimization, so I would expect that (synthetically) decreasing the fluctuations in the density profile from 2014 (e.g. using a running mean of measured densities for the optimization) would lead to a different spatio-temporal uncertainty pattern.

(p12): It might be interesting to illustrate the behavior of the cost function, which lives on high-dimensional space, in the vicinity of the minimum, e.g. as a function on a 2d subspace of density and conductivity for one location in space.

(p15/l27): In my opinion, the assumed uncertainty of 1kg/m^3 is unrealistically low anyway. (See comment above on γ) BTW: The method section should maybe include a quick recap on how densities were actually measured from the firn core

References

[1] Nicolsky et al, Using in-situ temperature measurements to estimate saturated soil thermal properties by solving a sequence of optimization problems, The Cryosphere, 1, 41-58, 2007

[2] Riche, F., Schneebeli, M. (2010). Microstructural change around a needle probe to measure thermal conductivity of snow. Journal of Glaciology, 56(199), 871-876. doi:10.3189/002214310794457164

[3] Löwe, H., Riche, F., and Schneebeli, M.: A general treatment of snow microstructure exemplified by an improved relation for thermal conductivity, The Cryosphere, 7, 1473-1480, https://doi.org/10.5194/tc-7-1473-2013, 2013.

[4] Gouttevin, I., Langer, M., Löwe, H., Boike, J., Proksch, M., and Schneebeli, M.: Observation and modelling of snow at a polygonal tundra permafrost site: spatial variability and thermal implications, The Cryosphere, 12, 3693-3717, https://doi.org/10.5194/tc-12-3693-2018, 2018.

[5] Fujita, S., Hirabayashi, M., Goto-Azuma, K., Dallmayr, R., Satow, K., Zheng, J., and Dahl-Jensen, D.: Densification of layered firn of the ice sheet at NEEM, Greenland, J. Glaciol., 60, 905921, doi:10.3189/2014JoG14J006, 2014.