Manuscript prepared for The Cryosphere Discuss.
with version 2014/09/16 7.15 Copernicus papers of the LaTeX class copernicus.cls.
Date: 18 June 2019

On the multi-fractal scaling properties of sea ice deformation

Pierre Rampal[1], Véronique Dansereau[1], Einar Olason[1], Sylvain Bouillon[3], Timothy Williams[1], Anton Korosov[1], and Abdoulaye Samaké[2]

[1]Nansen Environmental and Remote Sensing Centre and Bjerknes Centre for Climate Research, Bergen, Norway
[2]Université de Bamako, Bamako, Mali
[3]Numéca, Louvain La Neuve, Belgium

Correspondence to: Pierre Rampal (pierre.rampal@nersc.no)

Abstract

In this paper, we evaluate the neXtSIM sea ice model with respect to the observed scaling invariance properties of sea ice deformation in the spatial and temporal domains. Using an Arctic set-up with realistic initial conditions, state-of-the-art atmospheric reanalysis forcing and geostrophic currents retrieved from satellite data, we show that the model is able to reproduce the observed properties of these scaling in both the spatial and temporal domains over a wide range of scales and ~~, for the first time,~~ in particular their multi-fractality. The variability of these properties during the winter season are also captured by the model. We also show that the simulated scaling exhibit a space-time coupling, a suggested property of brittle deformation at geophysical scales. The ability to reproduce the multi-fractality of these scaling is crucial in the context of downscaling model simulation outputs to infer sea ice variables at the sub-grid scale, and also has implication in modeling the statistical properties of deformation-related quantities such as lead fractions, and heat and salt fluxes.

1 Introduction

The fact that sea ice deformation maps look similar at different scales, with highly localized deformation features intersecting with a wide range of intersection angles ~~(Hutchings et al., 2005; W~~ (e.g., Hutchings et al., 2005; Wang, 2007; Hutter et al., 2019), suggests scale-invariance in the spatial domain (Erlingsson, 1988). We note that scale-invariance in space is also observed in sea ice for other deformation-related quantities, such floe sizes ~~(Rothrock and Thorndike, 1984; Matsushita,1985) and keel profiles (Rothrock and Thorndike, 1980)~~(Rothrock and and keel profiles (Rothrock and Thorndike, 1980). Comprehensive datasets of sea ice drift are now available at different spatial and temporal resolutions, from 50 m/10 min (Oikkonen et al., 2017), 400 m/2 days (Thomas et al., 2004, 2007, 2009) to 5-10 km/3 days (Kwok, 2001; Stern and Moritz, 2002). Analyses of these datasets have confirmed the presence of scale-invariance and, in particular, has confirmed that sea ice deformation is highly localized in both space and time.

In the spatial domain, deformation is observed to be concentrated along quasi one-dimensional, so-called kinematic linear linear kinematic features (LKFs) organized in "web-like arrays" (Kwok et al., 1998; Thomas et al., 2007) that can be clearly identified over a wide range of space scales (Thomas et al., 2007; Linow and Dierking, 2017)(Thomas et al., 2007; Lino . Sea ice deformation also appears to be a self-similar, highly localized process in the time domain. Isolated, short-duration deformation fracturing events of various levels of intensity occur over a wide range of frequencies. These events also sustain larger-scale deformation, maintaining the LKFs "active" for many days Coon et al. (2007)Coon et al. (2007). The reorganization and formation of new LKFs occur in response to changes in the large scale atmospheric forcing (Kwok, 2001), and permanent deformation with high deformation rates in the ice pack is mainly synchronous with high winds events (Oikkonen et al., 2017).

A quantitative indication of *scale-invariance* in sea ice deformation is given by the shape of the distribution of deformation rate invariants , such as the shear , divergenceand (i.e. shear and divergence) and of the total deformation rates, which we refer to here as $\dot{\varepsilon}$. These probability density functions (P) have indeed been shown to be "heavy-tailed", i.e., dominated by extreme valuesthat are out of the Gaussian basin of attraction, following a power law decay of the type

$$P(\dot{\varepsilon}) \sim \dot{\varepsilon}^{-\gamma}, \qquad (1)$$

where γ is an exponent larger than 1 that depends on the spatial and time scale considered (Lindsay and Stern, 2003; Marsan et al., 2004; Rampal et al., 2008; Hutchings et al., 2011; Bouillon and Rampal, 2015b). This important characteristic expresses scale-invariance, as it is impossible from a power law distribution to determine the scale of a given deformation even by comparing the relative number of deformation events of different sizes.

Localization in the time and space domain is revealed by scaling analysis of the deformation rate invariants. In such analysis, deformation rates are estimated at different spatial and temporal scales, by such methods as "coarse-graining" . The *mean* deformation rate, estimated by (see section 3 for more details). Estimated using coarse-graining analysis (e.g., Lindsay and Stern, 2003; Marsan et al., 2004; Bouillon and Rampal, 2015b; Rampal et al.,

~~, or~~ (e.g., Lindsay and Stern, 2003; Marsan et al., 2004; Bouillon and Rampal, 2015b) or dispersion analysis of pair of buoys ~~dispersion analysis (Rampal et al., 2008), have~~ (Rampal et al., 2008), the *mean* sea ice deformation rate has been shown to vary with the spatial scale, L, and temporal scale of observation, T, as

$$\langle \dot{\varepsilon} \rangle \sim L^{-\beta}, \tag{2}$$

$$\langle \dot{\varepsilon} \rangle \sim T^{-\alpha}, \tag{3}$$

respectively, hence following a power law. The scaling exponents, β and α are both equal or greater than zero and quantify the degree of localization of the deformation. In the space domain, $\beta = 0$ characterizes the homogeneous deformation of an elastic solid or viscous fluid, i.e., a deformation that does not depend on the spatial scale, while $\beta = 2$, i.e. the topological dimension for the 2D-like sea ice cover, corresponds to a single ~~linear fracture~~ "point" concentrating all of the deformation in an otherwise undeformed material (Rampal et al., 2008). Conversely, in the time domain, $\alpha = 0$ corresponds to a homogeneous deformation and a single, temporally isolated deformation event corresponds to the limit of $\alpha = 1$ (Rampal et al., 2008). This scaling has been shown to hold over a very wide range of space and time scales (Rampal et al., 2008; Oikkonen et al., 2017; Weiss, 2017), with that α and β larger than 0, even for time scales on the order of the winter season and for space scales on the order of the length of the Arctic basin. This result indicates the absence of a characteristic time and/or space scale for the mean sea ice deformation over these scales and, as a consequence, that sea ice deformation can not be ~~approximated~~ assumed homogeneous over time/space scales relevant for ~~the Arctic Ocean~~Arctic system simulations.

The fact that sea ice deformation is characterized by ~~extreme events out of the Gaussian basin of attraction~~ heavy-tailed statistical distribution, i.e. dominated by extreme events, also indicates that the mean (moment of order 1) is not a sufficient quantity to describe ~~fully~~ the distribution of deformation rates at a given time/space scale~~and therefore not a sufficient basis for temporal and scaling analyses~~. Higher moments of the distribution of deformation rates, such as the variance (order 2) and skewness (order 3), should ~~therefore~~ indeed also be explored to ~~describe the entire~~ better describe the distribution and the associated process of sea ice deformation, and considered in temporal and scaling analyses as proposed in this study.

While the value of the scaling exponents β and α for the first moment (the mean) describes the rate at which the magnitude of deformation events decreases with the scale of observation, it is the change in the value of β and α with respect to the moment q of the distribution that ~~indeed~~ indicates how the temporal and spatial localization itself changes with the *magnitude* of deformation events. This change can be described by so-called structure functions of the form

$$\beta(q) = aq^2 + bq, \tag{4}$$

$$\alpha(q) = cq^2 + dq, \tag{5}$$

$$\tag{6}$$

in space and time respectively. In the case of a linear structure function, i.e. ~~, no curvature , the amount of localization of large and small deformation events is the same and the~~ no curvature or equivalently $a = 0$ or $c = 0$, the scaling is said to be *mono-fractal*. ~~For~~ In the case where both coefficients a and b or c and d ~~> 0, the functions are quadratic and convex. The~~ are positive the structure functions are convex, meaning that the higher order moments of the distribution therefore increase much faster than the lower order moments with decreasing scale of observation. ~~This indicates that~~ In other words, *large* deformation events are *more localized* in time and space than smaller events~~. This effect is stronger as the curvature is higher and in itself is the definition of multifractal *heterogeneity* and~~ , corresponding to the definition of a *multi-fractal* scaling (e.g., Kolmogorov, 1962; Lovejoy and Sche . Note that in the literature multifractality is also called *intermittency* when present in the time dimension and *heterogeneity* when present in the spatial dimension. The largest the curvature, the stronger the *degree of multifractality* of the scaling.

Spatial scaling analysis of sea ice deformation retrieved from radar or buoy drift data ~~have shown~~ show a clear multi-fractal scaling expressed by a power law scaling of the first, second, and third moments, ranging from the resolution of the data up to hundreds of km (e.g., Marsan et al., 2004; Rampal et al., 2008; Hutchings et al., 2011; Bouillon and Rampal, 2015a). Recently, Weiss and Dansereau (2017) have suggested, based on the combination of all available data, including the ones of Oikkonen et al. (2017), that this multi-fractality also holds in the time domain, over a period of 3 to 160 days. We note that multi-fractality in space has also been observed for open water densities (Weiss and Marsan, 2004) and lead fractions (Olason et al., 2019), and in time for shear stress amplitudes (Weiss and Marsan, 2004) and principal stress directions (Weiss, 2008).

These properties of sea ice deformation imply that observations of these quantities available at large scales can be statistically related, i.e, downscaled, to the same quantities at smaller, unresolved scales. In the case of model simulations, *downscaling* of outputs could be particularly valuable to infer quantities at the sub-grid and/or sub-time-step scale. In this context, the capability to reproduce mono- versus the multi-fractality of these properties becomes very important. Indeed, if one was to estimate the distribution of a variable at the sub-grid scale based on a model that would not reproduce the observed multi-fractality, but only a mono-fractality, then the downscaled distribution of this variable would greatly underestimate extreme values.

~~Despite the~~ The multi-fractal behaviour of sea ice has been the subject of a large number of interesting studies ~~on the subject and the numerous hypothesis of its significance for sea ice rheology (e.g., Weiss and Dansereau, 2017), no numerical model was yet shown to be able to reproduce the~~ and is hypothesized to be of significant importance for sea-ice rheology (e.g., Weiss and Dansereau, 2017). Bouillon and Rampal (2015a) and Rampal et al. (20 showed that previous versions of neXtSIM were capable of reproducing the spatial scaling and multi-fractal behaviour of ~~sea icedeformation in *both* the space and time domains.~~ the ice, with a very weak temporal scaling reported by Rampal et al. (2016). Spreen et al. (2017) and Bouchat and Tremblay (2017) have used some scaling analysis to investigate their respective viscous-plastic models, without going into the full details of a multi-fractal analysis or considering the temporal scaling. Hutter et al. (2018) on the other hand does a full multi-fractal analysis of the spatial and temporal scaling in a viscous-plastic model. Their work shows that with a model running at ∼1 km resolution they can reproduce reasonably good spatial scaling and multi-fractality down to the 10 km scale and up to 200 km; it is not shown how well the scaling holds down to the actual model resolution and their spatial scaling does not hold beyond the 200 km scale. They report inconsistent temporal scaling with a reasonably good temporal scaling when considering the full domain (where they don't report on multi-fractality), and no temporal scaling in the region covered by the EGPS data they compare to. They also only report temporal scaling for up to 30 days. Hutter et al. (2019) appear to improve on these results, but as this paper is still under review further detailing of their results is premature.

The observed self-similarity and multi-fractality in the deformation and related characteristics of sea ice actually poses great challenges to the development of sea ice models, in particular in the continuum framework. On the one hand, the momentum and evolution equations for sea ice properties are based on *mean* variables. On the other hand, however, the observed multi-fractality in sea ice deformation implies that there is not a clear separation of scales between the strain rate due to mesoscale (50-100 m) heterogeneities in the ice (leads, ridges, etc.) and the strain rates at 10 to 100 km scale. Consequently, no scale appears appropriate to homogenize sea ice motion and thereby define a mean velocity or deformation rate for model resolution ranging from 50 m to 100 km.

In the absence of a characteristic space/time scale for the sea ice deformation ~~and with the knowledge that localization goes beyond the space/time resolution of typical regional and global models,~~ perhaps the best a continuum framework for sea ice modelling can do is to ~~localize the deformation at the smallest available or~~ correctly reproduce the statistics of deformation from the smallest scales resolved (the *nominal* ~~*scale.*~~ *scale*) to the largest scale, i.e. from the resolution of the grid in space and the model time step in time, to the size of the Arctic basin and the time scale of a season. This is one of the ~~major objective~~ motivation in developing neXtSIM, the numerical model used in the current study. ~~Models with high localization capabilities are otherwise essential in the view of allowing an accurate representation of sea ice-ocean-atmosphere interactions , in both the contexts of short-term and climate predictions~~Such localization at the nominal scale is the most faithful representation of the discontinuous nature of sea ice possible in a continuum model.

Knowing the importance of essentially discontinuous features, such as leads, for atmosphere–ocea interaction modulated by sea ice (e.g. Smith, 1974; Kozo, 1983; Esau, 2007; Marcq and Weiss, 20 , we can expect the effect of using an ice model which localizes features at the nominal scale to be essential for improving the representation of this interactions in a coupled system.

This paper consists in the last step in validating neXtSIM against sea ice deformation statistics. While previous work have shown that the model reproduces the observed scaling of sea ice deformation (Bouillon and Rampal, 2015a; Rampal et al., 2016) in *space*, the temporal scaling and multi-fractality of both types of scaling have not yet been demonstrated for this model. The comparison performed here is based on satellite observations of sea ice deformation and winter-long simulations over the Arctic Ocean.

~~The first part of the paper~~ Section 2.1 and 2.2 discuss the recent developments of neXtSIM, the simulation setup and the observations~~(Section 2). The second part~~ . Section 3 describes the methodology used to perform the multi-fractal scaling analyses on both the model and observational data~~(Section 3)~~. Results of these analyses are presented in Section 4 and discussed in Section 5.

2 Model and observations

2.1 Model and simulation setup

neXtSIM is a finite elements sea ice model that uses a moving Lagrangian mesh. Its original dynamical component was based on the Elasto-Brittle (EB) mechanical framework of ~~(Amitrano et al., 1999)~~Amitrano et al. (1999), first implemented in the context of sea ice by Girard et al. (2011) to account for brittle fracturing processes and the associated spatial localization of deformation. This framework was later adapted by Bouillon and Rampal (2015b) and Rampal et al. (2016) for long-term simulations of the Arctic sea ice cover including thermodynamical processes and advection, using a Lagrangian treatment of the equations of motion and a dynamical remeshing scheme. Year-long simulations were presented in Rampal et al. (2016) and evaluated with respect to sea ice area, extent, volume, drift, and deformation. ~~In particular~~For example, the simulated deformation rates were demonstrated to be in good agreement with observations on the basis of their scaling properties *in space*.

However, the Elasto-Brittle model does not, by definition, include a physical mechanism for *irreversible* deformations, as it is based on a strictly linear-elastic constitutive law. It therefore cannot represent the transition between the small, elastic deformations associated with the fracturing of the ice cover and the permanent, potentially large, post-fracturing deformation that dissipates internal stresses. It is therefore not suited to represent the dynamical behavior of a fractured ice cover over long (>day) time scales and cannot represent fully the properties of sea ice deformation *in time*.

The recent Maxwell-Elasto-Brittle (MEB) rheology addresses this limitation of the EB framework by including a mechanism for the relaxation of internal stresses that depends on the degree of fracturing of the sea ice cover (Dansereau et al., 2016). It is implemented in the current version of neXtSIM, which is used for this study. Another addition to the model is the introduction of a three-thickness-categories scheme that represents explicitly the thin and newly-formed ice. The other model components (thermodynamics, slab ocean, etc.) remain unchanged relative to the version presented by Rampal et al. (2016).

All of the relevant equations ~~entering~~ of the current version of neXtSIM are presented in the ~~Appendix~~ appendices (Sections A1 for the dynamical core and A2 for the three-thickness-categories scheme and sea ice thermodynamics). The numerics (spatial and temporal discretizations, advection scheme and numerical solvers) are the same as described in Rampal et al. (2016).

The initial mesh is generated in pre-processing over a pan-Arctic region by using the mesh generator presented in Remacle and Lambrechts (2016) with a prescribed mean resolution (i.e. mean length of the ~~vertices~~ edges of the triangular elements) of 10 km. The coasts are defined from the Global Self-consistent, Hierarchical, High-resolution Geography Database[1]. The domain is restricted to the central Arctic by putting open boundaries on

[1]GSHHS_f_L1.shp, downloaded from https://www.ngdc.noaa.gov/mgg/shorelines/data/gshhg/latest/gshhg-shp-2.3.5-1.zip, accessed 1 February 2017

the lines cutting the Bering Strait from (-166.0 , 67.7 N) to (-170.7, 65.7 N) and cutting the Canadian Arctic Archipelago from (-59.0, 76.7 N) to (-121.0 ,69.5 N) and on the 2-segments line cutting the Greenland and Barents and Kara Seas by joining the coordinates (-19.0, 77.0N), (11.0, 73.0 N), (22.0, 72.9 N), (43.9, 76.1 N), (75.4, 75.7 N) and (88.5, 73.6 N). We checked that using a larger domain with open conditions much further from the zone of interest does not impact the results presented in this paper.

The atmospheric forcing consists of the applied 10 m wind velocity, the 2 m air temperature, the mixing ratio, the mean sea level pressure, the total precipitation amount and snow fraction, and the incoming short-wave and long-wave radiation. All of these quantities are provided as three-hourly means and on a 30 km spatial resolution grid from the atmospheric state of the Arctic System Reanalysis[2] (Bromwich et al., 2016).

The ice-ocean surface stress is computed from monthly ocean surface geostrophic currents derived as in Armitage et al. (2017) from the Arctic sea surface height data obtained from altimeters by Armitage et al. (2016). The provided fields surface height fields have a hole of missing data around the North Pole that we filled using a linear interpolation between the northernmost available points and their mean. A smoother is applied to the ocean velocity components in the filled area to avoid spurious oscillations due to the interpolation method. The final ocean currents forcing is at a spatial resolution of 50 km. The slab ocean salinity and temperature are nudged towards the daily sea surface temperature and salinity data provided as daily means and at 12.5 km spatial resolution over the Arctic region by the TOPAZ4 reanalysis[3] (Sakov et al., 2012) with a nudging time scale equal to 30 days. TOPAZ4 is a coupled ocean and sea ice data assimilation system for the North Atlantic and the Arctic that is based on the HYCOM ocean model and the ensemble Kalman filter data assimilation method using 100 dynamical members. A 23-year reanalysis has been completed for the period 1991–2013 and is the multi-year physical product in the Copernicus Marine Environment Monitoring Service. The ocean depth, H, used for the basal stress

[2]https://rda.ucar.edu/datasets/ds631.0, ASRv1 30-km, formerly ASR final version, Byrd Polar Research Centre/The Ohio State University. Accessed 15 April 2015

[3]available at http://marine.copernicus.eu/services-portfolio/access-to-products/

parametrization comes from the 1 arc-minute ETOPO1 global topography[4] (Amante and Eakins, 2009).

Our reference simulation starts on November 15th, 2006. The level of damage of the ice cover (see Appendix appendix A1) is initially set to zero where sea ice is present. Initial sea ice concentration and thickness are set from a combination of the TOPAZ4 reanalysis, and the OSISAF climate data record (Tonboe et al., 2016) and ICESAT[5] Kwok et al. (2009) datasets respectively.

2.2 Satellite observations

We use the Lagrangian displacement data produced by the RADARSAT Geophysical Processor System (RGPS) as described in Kwok et al. (1998). This dataset covers the Western Arctic for the period 1996–2008 and provides trajectories of sea ice "points" initially located on a 10 km regular grid (http://rkwok.jpl.nasa.gov/radarsat/lagrangian.html). The positions of these points are updated when two successive SAR images are available. The time interval between two updates is typically 3 days. For the present analysis we use the data covering the winter period 2006-12-03 to 2007-04-30 from the reprocessed RGPS Lagrangian displacement product, so-called RGPS Image Pair Product, introduced and used in Bouillon and Rampal (2015b) (section 2.2).

3 Methodologies for scaling analysis

Scaling analyses of sea ice deformation can be performed with two approaches: the a so-called coarse-graining method as in Marsan et al. (2004) and buoy dispersion analysis e.g. Marsan et al. (2004) or buoys dispersion method as in Rampal et al. (2008) (using pairs of buoys) or in Oikkonen et al. (2017) (using triplets of buoys). We use the coarse-graining

[4] available at https://www.ngdc.noaa.gov/mgg/global/

[5] available at https://icdc.cen.uni-hamburg.de/1/daten/cryosphere/seaicethickness-satobs-arc.html

~~approach in this study. It is applied on velocity gradients computed at the resolution of the trajectory dataset, using~~ a similar method as Oikkonen et al. (2017) for this study, i.e. computing velocity gradients from the trajectories of triplets of points. The nominal resolution for a scaling analysis is defined as the square root of the surface area of the polygon considered. For example, the minimal spatial resolution that can be achieved with the RGPS dataset ~~is about 7.5 km when using~~ when using the 3-sided polygons obtained from ~~Delaunay triangulation.~~ a Delaunay triangulation is about 7.5 km. This also set the nominal spatial resolution of the analysis presented in this study.

Drifters in the model are seeded at the location of the RGPS grid points as of December 3, 2016. The RGPS grid for this initialization is undeformed and the points are regularly spaced by 10 km. The positions of the simulated drifters are updated at each model time step until the end of the simulation or until the ice concentration drops to zero (through melting or opening of a lead). Both the RGPS and simulated trajectories are filtered for the presence of coasts, with a proximity threshold of 100 km. Only the trajectories ~~that are common to~~ spanning the same time periods in both the simulation and RGPS dataset are considered in the calculation of the deformation and their statistics. This selection lead to discarding about 1% only of the total trajectory dataset, and does not affect the results of the analyses presented in this paper. However, we apply this selection in order to make our comparison between model and observations as much consistent and clean as possible.

Triplets of drifting points are defined as the result of Delaunay triangulation of the initial positions of the tracked RGPS points, which ensures that the associated polygons are independent, i.e., non-overlapping. The exact same triplets of points are considered in the model for the analysis, meaning that we follow the exact same set of triplets of trajectories (or triangles) in the model and in the observations. The polygons after initiation are defined by the positions of their three nodes at any given time. We stress the fact that the simulated trajectories are not reinitialized every 3 days to match the RGPS positions; only the initial positions are identical between the RGPS and the model trajectories.

~~Coarse-graining in space is obtained by performing Delaunay triangulations on the~~ To perform a spatial scaling analysis of sea ice deformation, one needs to consider triplets of points with different spacing, i.e. different sizes of polygons. In order to obtained sets of polygons of different surface areas, we perform successive Delaunay triangulation through the clouds of points defined by the initial positions of the RGPS points, using increasingly sub-sampled cloud of initial RGPS drifter positions clouds of these points. Each set of contiguous polygons obtained using this process will be is associated to a spatial scale, L, defined as the mean of the square root of the polygon surface areas . The obtained from the triangulation, i.e from 7.5 to 580 km in this study. We note that due to the finite size of the Arctic basin and the largest spatial scale of 580 km considered here, the number of triplets available for the statistical analyses decreases by a factor (570/7.5)^2 as the space scale increases from 7.5 to 580 km. Coarse-graining in time is performed by considering To perform a temporal scaling analysis of sea ice deformation, one also need to consider the positions of triplets of drifters separated by a time different times T. The number of available triplets for our analysis in the time domain therefore also decreases as the time scale increasesconsidered increases due to the finite time covered by our simulation (about 5 months) which is constrained by the fact that we wish to limit our analysis to the winter period, i.e. from early December to mid-April.

For each available polygon, the total deformation rate is calculated as:

$$\dot{\varepsilon}_{tot} = \sqrt{\dot{\varepsilon}^2_{shear} + \dot{\varepsilon}^2_{div}} \quad (7)$$

where $\dot{\varepsilon}_{shear}$ and $\dot{\varepsilon}_{div}$ are the two invariantsinvariant, shear and divergence respectively, of the deformation rate. These invariants invariant are estimated using a contour integral calculation as follows: The spatial derivatives of the components u and v of the velocity calculated at a given time scale T are obtained by calculating the contour integrals as in Kwok et al. (2008) and Bouillon and Rampal (2015b) around the boundary of each polygon

Discussion Paper
Discussion Paper
Discussion Paper

associated to a given space scale L:

$$u_x = \frac{1}{A}\oint u dy \tag{8}$$

$$u_y = -\frac{1}{A}\oint u dx \tag{9}$$

$$v_x = \frac{1}{A}\oint v dy \tag{10}$$

$$v_y = -\frac{1}{A}\oint v dx, \tag{11}$$

where A is the encompassed area of the polygon equal to L^2. For example, u_x is approximated by:

$$u_x = \frac{1}{A}\sum_{i=1}^{n}\frac{1}{2}(u_{i+1}+u_i)(y_{i+1}-y_i), \tag{12}$$

where $n = 3$ and subscript $n+1 = 1$. The shear rates $\dot{\varepsilon}_{shear}$ and divergence rates $\dot{\varepsilon}_{div}$ are then computed as:

$$\dot{\varepsilon}_{shear} = \sqrt{(u_x - v_y)^2 + (u_y + v_x)^2}, \tag{13}$$

$$\dot{\varepsilon}_{div} = u_x + v_y. \tag{14}$$

The distribution of total deformation rates is constructed from each given coupled space/time scale (L, T), and their first 3 moments are calculated as $\langle \dot{\varepsilon}^q_{tot} \rangle$ where $q = 1, 2, 3$ is the moment order.

Below we discuss some issues that are inherent to the data and ~~coarse-graining~~ our method and their impact in terms of the robustness of the statistics calculated here.

– With time, the triangular elements can become too distorted, in which case their length scale, L, is poorly defined. Applying a test for distortion based on the smallest angle of the polygons and discarding the most distorted ones was found to affect the results in terms of the *slope* of the scaling, and the goodness of the fit of the power law fit of the scaling. Hence here we discard from the analysis the polygons having a minimum angle of 30 degrees or less.

- The RGPS trajectories are not sampled at regular time intervals, ~~as the modelis~~contrary to the model, due to the irregular interval between two satellite orbits. The mean sampling is of about 3 days, and 90% of trajectories are sampled with a frequency between 2.5 and 3 days. ~~We found that using different~~ Because sea ice deformation depends on the time scale (see results of section 4.2) one should make sure to use similar sampling times for the observations and the model ~~affect the comparison results~~when computing and comparing deformation rates estimates. To deal with this issue, we performed a sub-sampling of the RGPS trajectory dataset using a nearest-neighbour interpolation of the original positions at ~~3-days intervals~~3-day intervals, but only when the RGPS drifter's position is available within plus or minus 6 hours around the interpolation target time. The positions simulated by the model~~are~~ , that are outputted every 3 hours from midnight to midnight each day, are taken to match the sub-sampled RGPS time series obtained as described above.

- The 3-days RGPS sampling additionally places a lower bound on the time scales one can explore when comparing the simulated and observed deformation rates. In the present analysis, we therefore ~~chose to not explore smaller time scales~~ restrict ourselves to time scales equal or greater than 3 days.

We find ~~on the whole~~ that the relative number of available polygons is what has the largest impact on the deformation statistics. Some facts therefore need to be kept in mind when performing a scaling analysis over a *finite* period of time. In the time domain, in particular, this entails that sea ice deformation is better sampled, i.e., more triplets are available, for the early than for the late part of the period. In the present case, the computed statistics are therefore more representative of early than late winter. This effect is even more important for the larger time scales: polygons separated by small time scales T will indeed approximately sample the entire period while for large time scales, more polygons will be available at the beginning than at the end of the period.

4 Results

Figure **??** 1 shows the maps of the 3-days shear total, shear and absolute deformation rates simulated by the model and estimated from the RGPS data at the same locationsand for the same . Note that to obtain a better spatial coverage, these maps are showing all simulated or observed deformation rates for the period of 7 days centered on 4 February 2007. The cumulative probability probability density functions of the simulated and observed shear total, shear and absolute divergence deformation rates from the snapshots of Figure **??** 1 are shown on Figure **??**. Both 2. All distributions exhibit a power-law tail, with almost identical slopes of about -3, similar to what e.g., Marsan et al. (2004) found in their study. This , and with a remarkable agreement between the model and the observations for each invariant of the deformation. From the statistical point of view, this implies that one needs to consider higher moments than the mean and standard deviation of the distributions to fully describe the statistics of the sea ice deformation process (Sornette, 2006). In the scaling analysis presented in the following sections, we thus systematically calculate the 3 first moments of the distributions of deformation rates.

4.1 Spatial scaling analysis

Figure 3 (left panel) shows the winter mean of the spatial scaling analysis for the observations and model calculated for a $T = 3$ days temporal scale. We found that both model and observations statistics are following power-laws. As suggested in Stern et al. (2018) , we We use logarithmically spaced bins and applied an ordinary least squared square method to the binned data in log-log space to get reasonably accurate estimate of the power-law fits . The mean (Stern et al., 2018). The deformation rates are very well captured by the model at all scales. The second and third across scales. However, the first, second moments of the distributions are , however, slightly underestimated slightly overestimated by the model compared to the observationsfor scales lower than about 40 km, whatever the spatial scale considered. For example, at the nominal scale of 7.5 km, the second and third moments are underestimated first and second moments are overestimated by a factor of 2 and 3 respectively compared to the observations. This may be due to one or several of the following factors: (1) inaccuracies in the atmospheric forcing (2) our choice of mechanical parameters values and (3) the value of the atmospheric drag coefficient (e.g. Bouchat and Tremblay, 2017). It is especially important to note that the simulated deformation rate has not been tuned with respect to every the MEB mechanical parameters in the present simulations. We consider such tuning to be out of the scope of this study, which focuses on the ability to reproduce the observed scaling (exponents of the power laws) and, in particular, their multi-fractal property (non-linearity of the structure function). The simulated and observed structure functions (i.e. the dependence of the scaling exponents to the moment order) $\beta(q)$ are , however, equal within their margin of error (Figure 3 , shown in Figure 3 (right panel). The error spatial scaling obtained for both the model and the RGPS are clearly multi-fractal, as their structure functions can be both approximated by a quadratic function as defined by equation (4) with coefficients $a = 0.11$ and 0.13, respectively. These values, corresponding to the curvature of the structure functions, are very close to those reported in previous studies ($a = 0.13$ - 0.14; Marsan et al., 2004; Rampal et al., 2016). The bars are estimated from the minimal and maximal local scaling exponent values, as in Bouillon and Rampal (2015a) and correspond to *upper-bound* estimates.

In both cases, the scaling is clearly multi-fractal, as no linear function can be contained within the error bars. Instead, both structure functions are obtained by applying a quadratic fit to the data (in the least squared sense) as defined by equation . The good agreement between the observed and modelled structure functions This good agreement is a relevant indication that the scaling in the simulated deformation fields is consistent with that observed between 7.5 and 580 km.

A Using successive and contiguous *snapshots* throughout the winter, a time-series of the value of the scaling exponent for the mean spatial scaling exponent β obtained for the successive and contiguous *snapshots* throughout the winter is shown on Figure **??** (left

~~panel)~~mean deformation rates ($q = 1$) is calculated, and plotted on Figure 4. It shows that the spatial scaling exponent varies between -0.1 and -0.34. These exponents are in good agreement with the 1-month running means of the scaling exponents calculated by Stern and Lindsay (2009) for the entire period covered by the RGPS dataset (1996-2008). The scaling exponent for the mean is about ~~0.2~~ -0.2 on average over the whole winter period for the simulated and observed total deformation rates, which is also the value found by Marsan et al. (2004) for a snapshot of deformation rates calculated over a 3-days period centred on 6 November 1997. We note also that the model reproduces well the observed variability of the scaling exponent throughout the whole period. ~~A time-series of the value of the curvature (parameter a in equation) is also calculated for that period (Figure **??**, right panel). It shows that the curvature values fluctuate within the range 0.03-0.13. The value of the curvature, corresponding to the level of multi-fractality of the scaling and indicating the degree of heterogeneity of the deformation fields, is about 0.07 on average for the model and about 0.08 for the observations over the winter period analyzed here.~~

We further characterize the properties of the spatial scaling for both the model and observations by exploring its dependence on the temporal scale, T. We find that the estimated spatial scaling exponent, β, decreases with increasing T~~(Figures **??** and **??**~~, although this behavior is only obvious for the moments of order 2 and 3 (Figures 5 and 6, left panels). This ~~is the signature of the~~ seems to correspond to the existence of space-time coupling of the scaling properties of sea ice deformation~~, originally suggested in Rampal et al. (2008) and~~. This property was originally suggested by Rampal et al. (2008) from the result of their scaling analysis of buoy pairs dispersion, and was further explained in Marsan and Weiss (2010) as being a possible characteristic of brittle deformation at geophysical scales. ~~This property is for~~ To our knowledge, this is the first time ~~shown~~ such coupling is obtained from a sea ice model simulation ran at such relatively coarse spatial resolution. The origin of this coupling has been previously proposed to be linked to the complex correlation patterns related to chain triggering of ice-quakes (Marsan and Weiss, 2010). Further study is, however, needed to explore this hypothesis, which is out of the scope of this paper.

We also note a decrease of the multi-fractal character of the spatial scaling (i.e. the curvature of $\beta(q)$) when increasing the time scales from $T = 3$ to $T = 96$ days (Figures ?? and ??5 and 6, right panels). For the model, we transition from a multi-fractal to a mono-fractal scaling, while for the observations the scaling remains clearly multi-fractal at all temporal both the model and the observations, we observe the multi-fractality property is present for all scales considered in this study although a decrease of the degree of multi-fractality is observed as the scale increases. The curvature values are decreasing from 0.085 to nearly zero 0.115 to 0.054 for the model and from 0.16 to 0.09 0.13 to 0.063 for the observations following a power-law (Figure ??). The 7). While the general behaviour of decreasing the degree of multi-fractality of the spatial scaling as the time scale increases is thus captured by the model, but the model fails at keeping the multi-fractal signature at the largest scales. This may come from the fact that the highest deformation events are too evenly distributed over the Arctic region in the simulation compared to we note that the degree of multi-fractality of the deformation is systematically underestimated by the model compared to the observations. This could either be attributed to inaccurate position or lacking of extreme events in the observations. The atmospheric forcing, or to an inadequate or insufficiently tuned parameterization of the damage healing in the model. In any case, the reason for this discrepancy should be further explored but is out of scope of the present paper.

4.2 Temporal scaling analysis

The results of the temporal scaling analysis for $L = 7.5$ km is shown on Figure 8 (left panel). We see a robust and very similar power-law scaling for the two first moments ($q = 1, 2$) for both the model and observations between $T = 3$ days (i.e., the temporal resolution of the observations) and $T = 96$ days. In previous studies based on drifting buoy trajectories whose positions are sampled at higher frequency, it has been suggested that the temporal scaling for the mean total deformation holds down to 1 hour (Hutchings et al., 2011). A recent study based on very high resolution ship radar measurements has demonstrated that it holds down even to 10 min (Oikkonen et al., 2017). Here, we obtain a perfect agreement between the slope (about -0.3) of the temporal scaling for the mean deformation rates estimated in this recent studyby Oikkonen et al. (2017), and that estimated from the RGPS data and the present model simulations(gray, dark and cyan top curves in the left panel of Fig. 8). .

We note, however, that the third moment of the distributions are slightly underestimated by the model at all time scales. This means that the proportion of extreme deformation events compared to lower ones is too small or that their values are too low in the simulation. This may come from the inaccuracy of the relatively coarse (30 km) atmospheric reanalysis we use to force our model and that is known to poorly resolve the most *extreme* low pressure systems, a common shortcoming of all the available global or regional atmosphere reanalysis to date. Another explanation could be the fact that we have not tuned the MEB rheology parameters to reproduce the proportion of extreme deformation events versus the lower ones. In this rheology, the coupling between the damage and the mechanical behavior of sea ice is non-linear and it is therefore expected that varying parameter values can change the proportion of the simulated extreme events, i.e., the skewness of the distribution of deformation rates.

As in the spatial domain, the temporal scaling is found to be multi-fractal for the model and observations, and the match is virtually perfect. The quadratic functions $\alpha(q)$ gives curvature values of 0.11 remarkably good. The curvature values (i.e. the coefficient c in equation 5 are 0.67 for the model and 0.13 0.62 for the observations, the exact same value as the one found by Weiss and Dansereau (2017) (figure 1), despite the fact that they analyzed a different period (winter 1996-1997). This seems to argue that the multi-fractality . This suggests that the multi-fractal character of the temporal scaling is stronger than the spatial scaling and possibly a robust property of sea ice deformation, at least in the winter time, independent of the observed change in sea ice cover state and the associated shift of its dynamical regime during the period 1996-2006 (e.g., Rampal et al., 2009a, b). We note that the values of curvature of the structure functions obtained here cannot be directly compared to the one reported in Weiss and Dansereau (2017) since in their paper the authors are plotting the normalized moments $\langle \dot{\varepsilon}^q_{tot} \rangle^{1/q}$ of the distribution versus the temporal scale instead of the actual moments $\langle \dot{\varepsilon}^q_{tot} \rangle$ as we do here.

We also investigate the dependence of the temporal scaling on the spatial scale of observation, L, for both the model and RGPS data (Figures ~~?? and ??~~9 and 10, left panels). We find that the scaling exponent, α, decreases with L. Similar to the spatial scaling analysis performed in Section 4.1, we find here the signature of a space-time coupling in the scaling properties of sea ice deformation. The multi-fractal character of the temporal scaling holds at all the spatial scales considered here ($L = 7.5$ to ~~$L = 360$~~ $L = 580$ km), and is similar in the model and observations (Figures ~~?? and ??~~9 and 10, right panels). The curvature values are going from ~~0.11 down to 0.015~~ 0.67 down to 0.37 for the model and from ~~0.13 to 0.01~~ 0.63 to 0.35 for the observations following a power-law (Figure ~~??~~11). The decrease in the degree of multi-fractality of the temporal scaling as the space scale increases as seen in the observations is remarkably well captured by the model.

5 Discussion

Our statistical analyses have shown that the neXtSIM model reproduces correctly the distribution of sea ice deformation rates, its scaling properties in both the space and time domains and its multi-fractal behavior. In particular, it is the first time that multi-fractality in the time domain is shown to be reproduced in a sea ice model.

The MEB rheology was developed with the aim of improving the representation of the physics of sea ice continuum models by including the ingredients hypothesized by Weiss and Dansereau (2017) to ~~be the cause of the~~ possibly play an important role in the emergence of multi-fractal heterogeneity and intermittency of sea ice deformation. This hypothesis is based on the analysis of observational data that have highlighted the existence of multi-fractality of sea ice deformation in space and time (Rampal et al., 2008; Bouillon and Rampal, 2015b; Weiss and Dansereau, 2017) ~~as well as on the close analogy with other systems~~ and on a close and arguably sound analogy that can be made with other large scale solids sharing these properties such as the Earth crust ~~as proposed originally~~

~~in Weiss et al. (2009)~~(Weiss et al., 2009). According to Weiss and Dansereau (2017) the ingredients ~~required~~ are: a *threshold mechanism* for brittle fracturing, some *disorder* that represents the natural heterogeneity of the material at the mesoscale, *long-range elastic interactions* within the ice cover that promote avalanches of fracturing events through a cascading mechanism, post-fracturing relaxation of elastic stresses through *viscous-like relaxation*, and a *slow restoring/healing mechanism* of the sea ice mechanical properties. We argue that the results obtained here are ~~an important step towards the confirmation of this hypothesis~~at least showing that a model including these ingredients can indeed reproduce some aspects of sea ice dynamics complexity.

We show here that the spatial scaling of sea ice deformation simulated in a realistic setup by neXtSIM holds down to the nominal resolution of the mesh, a result that is in agreement with previous analyses of the MEB model in idealized simulations (Dansereau et al., 2016) and realistic ones (Rampal et al., 2016). It means that neXtSIM does not need to be run at higher spatial resolution in order to ~~resolve the presence of linear kinematic features (~~reproduce the observed scalings, as e.g.~~,~~ Hutter et al. (2018) do when running at about 1 km resolution in order to resolve sea ice deformation at scale of about 10 km~~).~~. Localizing the deformation at the nominal model resolution also means that related quantities, such as ridges, leads, and linear kinematic features should be better resolved, although this is not investigated directly here. We note that using a Lagrangian mesh then helps preserving such features, once formed, but plays no role in their formation.

We show also that this spatial localization and the multi-fractal character of the simulated mean sea ice deformation is resolution-independent in this setup. This is what is shown on figure 12. However, and despite the fact that the scaling remains multi-fractal when neXtSIM runs at coarser resolutions (e.g., 15 or 30 km), the level of multi-fractality is decreasing with decreasing resolution. Indeed, the second and third moments of deformation rates from the 15 and 30 km runs differ from the results obtained from the 7.5 km run (figure 12, right panel), which suggests an underestimation of extreme deformation events at the smaller spatial scales with increasing model resolution. Nevertheless, the representation of multi-fractality at all resolutions implies that neXtSIM could be adequately used to explore a wider range of space-time scales than that covered by the currently available observations of the global Arctic. In particular, it could allow to "zoom in" and explore the statistical properties of sea ice deformation at the sub-satellite observations scales, which are of increasing interests for both regional to global climate modelling and operational forecasting. A model that could otherwise not represent the multi-fractal character of sea ice deformation and would only reproduce a mono-fractal scaling would greatly underestimate extreme deformation events and their impact on sea ice conditions at such scales like e.g., the presence or not of leads and ridges.

A model that allows reproducing sea ice deformation and its scaling properties down to its nominal resolution does not preclude the need for appropriate sub-grid scale parametrizations. On the contrary, we believe that physically sound parametrizations are indeed required and that the knowledge of the distribution of deformation rates at the the sub-grid scale made possible by neXtSIM could be highly valuable in terms of informing these parametrizations. An appropriate sub-grid scale parametrization links the deformation simulated at the scale of the grid cell with the scale at which deformation really occurs within the ice cover, which is the size of individual leads and ridges.

We moreover argue that, as sea ice deformation is strongly tied to other model variables, such as drift, lead fraction and thickness distribution. A , a proper simulation of these variables is a necessary prerequisite to using models for investigating various coupled ocean–ice–atmosphere processes, and their impact on their immediate vicinity and on the polar climate system. For example, the accuracy of neXtSIM in reproducing the observed statistical properties of sea ice deformation as demonstrated in this paper is thought to go hand-in-hand with its capability in representing the observed properties of lead fraction. This is the subject of a concurrent study presented in Olason et al. (2019)parallel study that is about to be submitted.

6 Conclusions

In this study we have compared the deformation rates simulated by neXtSIM to those derived from RGPS observations by comparing their distributions and how these distributions scale in time and space. The conclusions of our analysis are:

- The neXtSIM model reproduces well the first, second and third moments of the statistical distribution of observed sea ice deformation rates and how it scales in space and time. In particular, this is the first time the observed scaling invariance in the temporal domain (i.e. intermittency) of sea ice deformation is shown to be reproduced by a model on a realistic Pan-Arctic setup over such a large range of scales.
- Sea ice deformation rates calculated over a temporal scale of 3 days scale in space from the scale of the model (mesh resolution)/observations up to about 700 km in a multi-fractal manner.
- Sea ice deformation rates calculated over a spatial scale of 7.5 km scale in time over the range 3 days–3 months in a multi-fractal manner.
- A space-time coupling in the scaling properties of sea ice deformation is ~~for the first time~~ shown to be reproduced by ~~a~~ the model. This suggests that neXtSIM could be a proper tool to study the physical meaning and origin of this coupling, in the context of brittle deformation of geophysical solids.
- The simulated mean sea ice deformation rates and their associated scaling invariance characteristics are resolution-independent, i.e., when running the neXtSIM model at resolutions of 7.5, 15 or 30 km. The most extreme deformation events may be missed however if running at coarser resolutions, i.e. the second and third order moments may be underestimated compared to the high-resolution run.
- As the mono versus multi-fractal character of the scaling of deformation rates is the discriminating factor for the heterogeneity and intermittency of the deformation, we

suggest that a multi-fractal scaling analysis should be a prerequisite could be considered as a meaningful validation step before further analyzing sea ice model outputs that could be influenced by sea ice dynamics.

– The good agreement between the model and observations motivates the use of neXtSIM as a tool to further investigate physical processes that are highly sensitive to sea ice deformation.

Appendix A: Model description

This section presents the dynamical and thermodynamical components of neXtSIM. The wave-in-ice module implemented by Williams et al. (2017) is not included here. Prognostic sea ice variables are listed in Table 1 and all parameter values are listed in table 2.

A1 Dynamical core

The evolution equation for sea ice velocity comes from vertically integrating the horizontal sea ice momentum equation as follows:

$$\rho_i H \frac{D\boldsymbol{u}}{Dt} = \nabla \cdot \left(H\boldsymbol{\sigma}\right) + \boldsymbol{\tau}_a + \boldsymbol{\tau}_w + \boldsymbol{\tau}_b - \rho_i H \left(f\boldsymbol{k} \times \boldsymbol{u} + g\nabla\eta\right). \tag{A1}$$

The parameter ρ_i is the ice density, H is the mean ice thickness per unit grid cell area, $\boldsymbol{\sigma}$ is the sea ice internal stress tensor, $\boldsymbol{\tau}_a$, $\boldsymbol{\tau}_w$ and $\boldsymbol{\tau}_b$ are the surface wind, ocean and basal stresses, respectively, and are defined as in Rampal et al. (2016). The parameter f is the Coriolis frequency, $\boldsymbol{k}$ is the upward pointing unit vector, g is the acceleration due to gravity and η is the ocean surface elevation. In the region with only thin ice or with thick ice thick-ice thickness lower than a given threshold (defining our ice edge), the momentum equation is replaced by a Laplacian equation so that the velocity linearly decreases from the ice edge to the nearest coast (see Samaké et al. (2017)). The additional ice pressure term introduced in Rampal et al. (2016) is not included here.

Following Dansereau et al. (2016), the evolution equation for the internal stress takes the form of the Maxwell constitutive law:

$$\frac{D\boldsymbol{\sigma}}{Dt} + \frac{\boldsymbol{\sigma}}{\lambda} = E\boldsymbol{K} : \dot{\boldsymbol{\varepsilon}}(\boldsymbol{u}) \tag{A2}$$

where λ is the relaxation time for the stress, E, is the elastic modulus and $\dot{\boldsymbol{\varepsilon}}$, the strain rate tensor, is defined as ~~the rate of strain tensor~~

$$\dot{\boldsymbol{\varepsilon}}(\boldsymbol{u}) = \frac{1}{2}\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right). \tag{A3}$$

Plane stresses conditions are assumed and the stiffness tensor $\boldsymbol{K}$ reads

$$\begin{bmatrix}(\boldsymbol{K}:\boldsymbol{\varepsilon})_{11}\\(\boldsymbol{K}:\boldsymbol{\varepsilon})_{22}\\(\boldsymbol{K}:\boldsymbol{\varepsilon})_{12}\end{bmatrix} = \frac{1}{1-\nu^2}\begin{pmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{pmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix} \tag{A4}$$

where ν is Poisson's ratio.

As in Dansereau et al. (2016), both the elastic modulus, E, and the relaxation time are functions of the ice concentration, A, and the level of damage, d. The level of damage is a scalar, grid-scale variable that represents the density of fractures at the sub-grid scale. Its value is 0 for an undamaged and 1 for a "completely" damaged material, which we note is the opposite convention compared to Dansereau et al. (2016). The elastic modulus is a linear function of d:

$$E(A,d) = E_0(1-d)f(A), \tag{A5}$$

where E_0 is the undamaged elastic modulus and $f(A)$ introduces a dependence on the ice concentration via the following exponential function:

$$f(A) = \mathrm{e}^{\underline{c^*(1-A)}\,-c^*(1-A)}, \tag{A6}$$

where c^* is the ice compactness parameter introduced by Hibler (1979). As in Dansereau et al. (2016), the relaxation time is a power function of d:

$$\lambda(d) = \lambda_0(1-d)^{\alpha-1}, \tag{A7}$$

where λ_0 is its undamaged value and α is a constant exponent greater than 1. Here, we use the values ~~$\alpha = 4$~~ $\alpha = 5$ and $\lambda_0 = 10^7 s$ (115 days) (as in the realistic Maxwell-EB simulations of Dansereau et al., 2017) to ensure that the relaxation of stresses is virtually zero over an undamaged ice cover but is significant when the ice is damaged.

The evolution of the damage is controlled by the location of the predicted stress state relative to the failure envelope, which as in Rampal et al. (2016) is defined in terms of the principal stress components

$$\sigma_1 = -\frac{\sigma_{11}+\sigma_{22}}{2} + \sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2 + \sigma_{12}^2} \tag{A8}$$

$$\sigma_2 = -\frac{\sigma_{11}+\sigma_{22}}{2} - \sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2 + \sigma_{12}^2}, \tag{A9}$$

with the convention that compressive stresses are positive.

Here, the envelope combines a Mohr-Coulomb failure criterion and a maximum tensile and compressive stress. The three criteria are given by

$$\sigma_1 - q\sigma_2 \leq \sigma_c g(H) \quad \text{(Mohr-Coulomb criterion)}, \tag{A10}$$

$$-\frac{\sigma_1+\sigma_2}{2} \leq \sigma_{T\,\mathrm{max}} g(H) \quad \text{(tensile stress criterion)}, \tag{A11}$$

$$\frac{\sigma_1+\sigma_2}{2} \leq \sigma_{N\,\mathrm{max}} g(H) \quad \text{(compressive stress criterion)}, \tag{A12}$$

where $q = \left[\left(\mu^2+1\right)^{1/2} + \mu\right]^2$, $\sigma_c = \frac{2c}{\left[\left(\mu^2+1\right)^{1/2} - \mu\right]}$, μ is the internal friction coefficient, c is the cohesion, $\sigma_{T\,\mathrm{max}}$ is the maximal tensile strength and $\sigma_{N\,\mathrm{max}}$ the maximum compressive strength (see table 2). The cohesion, c, is scaled as a function of the model spatial resolution, as described in Bouillon and Rampal (2015a).

When one of the damage criteria is met, d is modified ~~according to~~ by multiplying $(1-d)$ with Ψ, or

$$\cancel{1-d'=}\, d \leftarrow 1-\Psi(1-d), \tag{A13}$$

where

$$\Psi = \begin{cases} \frac{\sigma_c}{\sigma_1 - q\sigma_2} & \text{if } \sigma_1 - q\sigma_2 > \sigma_c \\ \frac{2\sigma_{T\,\mathrm{max}}}{-\sigma_1+\sigma_2} & \text{if } -\frac{\sigma_1+\sigma_2}{2} > \sigma_{T\,\mathrm{max}} \\ \frac{2\sigma_{N\,\mathrm{max}}}{\sigma_1+\sigma_2} & \text{if } \frac{\sigma_1+\sigma_2}{2} > \sigma_{N\,\mathrm{max}}. \end{cases} \tag{A14}$$

Healing is included here to represent the counteracting effect of refreezing of water within leads on the level of damage of the ice cover. It is implemented via a constant term in the damage evolution equation:

$$\frac{Dd}{Dt} = \frac{(1-d)(1-\Psi)}{T_d} - \frac{1}{T_h}, \tag{A15}$$

where T_h is the characteristic time for healing and T_d, the characteristic time for damaging (Dansereau et al., 2016).

A2 Ice thickness redistribution and thermodynamics

neXtSIM includes ~~the~~ a multi-category model inspired from Stern and Rothrock (1995), i.e. considering 3 ~~ice categoriessuggested by Stern and Rothrock (1995)~~categories: thick ice, thin ice and open water. In our implementation the thin ice is only newly formed ice, so ice will only be transferred from the thin-ice category to thick ice, but not in the reverse direction. In addition, we don't apply additional open water source terms, and nor do we use the formulation of Gray and Morland (1994) to keep the ice concentration less than 1. (We simply redistribute ice and snow volume if this occurs.) Thin ice is described by its concentration, A_t, and volume per unit area, H_t, and snow volume per unit area, $h_{s,t}$. Thick ice is described by the concentration, A, ~~and volume per~~ volume per unit area, H, and snow volume per unit area, h_s. We assume that the thin ice has no mechanical strength and simply follows the motion of the surrounding thick ice.

Note the total ice concentration and volume per unit area are $A+A_t$ and $H+H_t$, and the total snow volume per unit area is $h_s+h_{s,t}$.

Thin ice thickness is considered to be uniformly distributed with thickness $h_t=H_t/A_t$ required to be between $h_{min}=5$~~cm and $h_{max}=50$ cmso that the volume per unit area is bounded between $H_{min}=Ah_{min}$ and $H_{max}=A\frac{h_{min}+h_{max}}{2}$.~~ cm and $h_{max}=27.5$ cm. The evolution equations for A, H, h_s, A_t~~and~~, H_t and $h_{s,t}$ have the following form:

$$\frac{D\phi}{Dt}=-\phi\nabla\cdot\boldsymbol{u}+\Psi_\phi+S_\phi, \tag{A16}$$

where $\frac{D\phi}{Dt}$ is the material derivative that is defined for any scalar ~~and vector as~~ as

$$\frac{D\phi}{Dt}=\frac{\partial\phi}{\partial t}+(\boldsymbol{u}\cdot\nabla)\phi. \tag{A17}$$

Here $\nabla\cdot\boldsymbol{u}$ is the divergence of the horizontal velocity, Ψ_ϕ a sink/source term due to ridging, and S_ϕ a thermodynamical sink/source term. Volume conservation is imposed by setting ~~$\Psi_H=-\Psi_{Ht}$ and~~ $\Psi_H=-\Psi_{H_t}$ and $\Psi_{h_s}=-\Psi_{h_{s,t}}$ and an additional constraint is that ~~$A_h+A\le 1$~~$A_t+A\le 1$.

The evolution of A, H, A_t and H_t is computed following three main steps (variables updated in each step are denoted with a prime):

1. Advection: The scheme solves the equation:

$$\frac{D\phi}{Dt}=-\phi\nabla\cdot\boldsymbol{u}, \tag{A18}$$

for each conserved scalar quantity (A, H, A_t, H_t, etc.). For this paper, we use the purely Lagrangian scheme presented in Rampal et al. (2016). After this step the concentration could be larger than 1.

2. Mechanical redistribution: The scheme imposes the limit $A_t + A \leq 1$ on the total ice area by following those steps:

   (a) Compute the new open water concentration as:

$$A_0 = \max(0, 1 - A - A_t),; \tag{A19}$$

   a source term for the open water could be added here (as in Stern and Rothrock, 1995) to represent sub-grid scale sub-grid-scale convergence/divergence.

   (b) Compute the new thin ice concentration as:

$$A_t{}^{n+1\prime} = \max(0, \min(1, 1 - A - A_0)) \tag{A20}$$

   (c) Compute the transfer of thin ice if $A_t^{n+1} < A_t$ $A_t' < A_t$ by setting:

$$H_t{}^{n+1\prime} = H_t \frac{A_t^{n+1}}{A_t} \frac{A_t'}{A_t} \tag{A21}$$

$$h_{s,t}' = h_{s,t} \frac{A_t'}{A_t} \tag{A22}$$

$$H^{n+1\prime} = H + (H_t - H_t{}^{n+1\prime}) \tag{A23}$$

$$h_s' = h_s + (h_{s,t} - h_{s,t}') \tag{A24}$$

$$\Delta A = \frac{A_t - A_t^{n+1}}{\zeta} \frac{A_t - A_t'}{\zeta}. \tag{A25}$$

   where Here, we have transferred ice and snow volume from thin to thick ice in a conservative manner, but we will not try to conserve ice area: ζ is an aspect ratio parameter set to 10. (tuned to 10) which causes ridging to preferentially increase ice thickness over ice area.

(d) Compute the new thick ice concentration as:

$$A^{n+1\prime} = \max(0, \min(1, 1 - A_t^{n+1\prime} - A_0 + \Delta A)) \tag{A26}$$

(e) Apply more ridging if $(A^{n+1} + A_t^{n+1}) > 1$ by setting $A^{n+1} = 1 - A_t^{n+1}$ $(A' + A'_t) > 1$ by setting $A' = 1 - A'_t$.

3. Growth/melt: The source/sink terms from the thermodynamics are computed by applying the zero-layer Semtner (1976) vertical thermodynamics to the new ice category and that of Winton (2000) for the thick ice, as if the thickness was uniform and equal to H/A for the thick ice and H_t/A_t for the thin ice. Freezing of open water is computed as in Rampal et al. (2016) such that heat loss from the ocean that would cause super cooling is redirected to ice formation. The newly formed ice is transferred to the thin ice category and is assumed to have a thickness equal to h_{min}. The transfer from the thin ice to the thick ice and the lateral melting of thin ice is computed by applying the bounding limit H_{min} and H_{max} h_{max} — if $h_t > h_{max}$, then we update the variables as

follows:

$$h'_t = h_{max}, \tag{A27}$$

$$A'_t = \frac{h_{max} - h_{min}}{h_t - h_{min}} A_t, \tag{A28}$$

$$H'_t = A'_t * h'_t, \tag{A29}$$

$$h'_{s,t} = \frac{A'_t}{A_t} h_{s,t}, \tag{A30}$$

$$H' = H - (H'_t - H_t), \tag{A31}$$

$$A' = A - (A'_t - A_t), \tag{A32}$$

$$h'_s = h_s - (h'_{s,t} - h_{s,t}). \tag{A33}$$

The form of the reduction in thin ice concentration in (A28) is a little arbitrary, but we wanted to allow the possibility of the thin ice completely changing into thick ice at some point.

Author contributions. This work is the result of a long-term team effort at the Nansen Centre in Norway carried by Sylvain Bouillon, Einar Oláson, Pierre Rampal, Abdoulaye Samaké and Timothy Williams to develop the new sea ice model `neXtSIM` and include the Maxwell-Elasto-Brittle (MEB) rheology of Dansereau et al. (2016). PR led the research and performed the analyses presented in this paper; VD and PR led the writing with contributions from EO, SB and TW; AS implemented the parallel C++ version of the model code described in Samaké et al. (2017), with support from SB and EO; SB and VD implemented the MEB rheology.

Acknowledgements. This manuscript was prepared thanks to the financial support of the Research Council of Norway (RCN) through the *FRASIL* project (grant no. 263044). However, the development of the neXtSIM model has been supported since 2013 through several projects from the RCN, in particular the *SIMECH* (grant no. 231179), and the *NEXTWIM* (grant no. 244001) projects. TOTAL E&P is also thanked for their continuous support over the period 2013-2017 through the *KARA* project. And finally, we thank Jerome Weiss and David Marsan for the fruitful discussions.

References

Amante, C. and Eakins, B. W.: ETOPO1 Global Relief Model converted to PanMap layer format, doi:10.1594/PANGAEA.769615, http://doi.pangaea.de/10.1594/PANGAEA.769615, 2009.

Amitrano, D., Grasso, J. R., and Hantz, D.: From diffuse to localised damage through elastic interaction, Geophys. Res. Lett., 26, 2109–2112, 1999.

Armitage, T. W. K., Bacon, S., Ridout, A. L., Thomas, S. F., Aksenov, Y., and Wingham, D. J.: Arctic sea surface height variability and change from satellite radar altimetry and GRACE, 2003?2014, Journal of Geophysical Research: Oceans, 121, 4303–4322, doi:10.1002/2015JC011579, http://dx.doi.org/10.1002/2015JC011579, 2016.

Armitage, T. W. K., Bacon, S., Ridout, A. L., Petty, A. A., Wolbach, S., and Tsamados, M.: Arctic Ocean surface geostrophic circulation 2003–2014, The Cryosphere, 11, 1767–1780, doi:10.5194/tc-11-1767-2017, https://www.the-cryosphere.net/11/1767/2017/, 2017.

Bouchat, A. and Tremblay, B. L.: Using sea-ice deformation fields to constrain the mechanical strength parameters of geophysical sea ice, Journal of Geophysical Research: Oceans, 122, 5802–5825, 2017.

Bouillon, S. and Rampal, P.: On producing sea ice deformation data sets from SAR-derived sea ice motion, The Cryosphere, 9, 663–673, doi:10.5194/tc-9-663-2015, http://www.the-cryosphere.net/9/663/2015/, 2015a.

Bouillon, S. and Rampal, P.: Presentation of the dynamical core of neXtSIM, a new sea ice model, Ocean Modelling, 91, 23–37, 2015b.

Bromwich, D. H., Wilson, A. B., Bai, L.-S., Moore, G. W. K., and Bauer, P.: A comparison of the regional Arctic System Reanalysis and the global ERA-Interim Reanalysis for the Arctic, Quarterly Journal of the Royal Meteorological Society, 142, 644–658, doi:10.1002/qj.2527, http://dx.doi.org/10.1002/qj.2527, 2016.

Coon, M., Kwok, R., Levy, G., Pruis, M., Schreyer, H., and Sulsky, D.: Arctic Ice Dynamics Joint Experiment (AIDJEX) assumptions revisited and found inadequate, J. Geophys. Res., 112, C11S90, 2007.

Dansereau, V., Weiss, J., Saramito, P., and Lattes, P.: A Maxwell elasto-brittle rheology for sea ice modelling, The Cryosphere, 10, 1339–1359, doi:10.5194/tc-10-1339-2016, https://www.the-cryosphere.net/10/1339/2016/, 2016.

Dansereau, V., Weiss, J., Saramito, P., Lattes, P., and Coche, E.: Ice bridges and ridges in the Maxwell-EB sea ice rheology, The Cryosphere, 11, 2033–2058, 2017.

Erlingsson, B.: Two-dimensional deformation patterns in sea ice, J. Glaciol., 34, 1988.

Esau, I. N.: Amplification of turbulent exchange over wide Arctic leads: Large-eddy simulation study, Journal of Geophysical Research: Atmospheres, 112, n/a–n/a, doi:10.1029/2006JD007225, http://dx.doi.org/10.1029/2006JD007225, d08109, 2007.

Girard, L., Bouillon, S., Weiss, J., Amitrano, D., Fichefet, T., and Legat, V.: A new modelling framework for sea ice mechanics based on elasto-brittle rheology, Ann. Glaciol., 52(57), 123–132, 2011.

Gray, J. and Morland, L.: A two-dimensional model for the dynamics of sea ice, Philosophical Transactions of the Royal Society of London. Series A: Physical and Engineering Sciences, 347, 219–290, 1994.

Hibler, W. D.: A dynamic thermodynamic sea ice model, J. Phys. Ocean., 9, 817–846, 1979.

Hutchings, J. K., Heil, P., and Hibler, W. D.: Modeling linear kinematic features in sea ice, Month. Weath. Rev., 133, 3481–3497, 2005.

Hutchings, J. K., Roberts, A., Geiger, C. A., and Richter-Menge, J.: Spatial and temporal characterization of sea-ice deformation, Ann. Glaciol., 52, 360–368, 2011.

Hutter, N., Losch, M., and Menemenlis, D.: Scaling Properties of Arctic Sea Ice Deformation in a High-Resolution Viscous-Plastic Sea Ice Model and in Satellite Observations, Journal of Geophysical Research: Oceans, 117, 1038–16, 2018.

Hutter, N., Zampieri, L., and Losch, M.: Leads and ridges in Arctic sea ice from RGPS data and a new tracking algorithm, The Cryosphere, 13, 627–645, 2019.

Kolmogorov, A. N.: A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number, Journal of Fluid Mechanics, 13, 82–85, 1962.

Kozo, T. L.: Initial model results for Arctic mixed layer circulation under a refreezing lead, Journal of Geophysical Research: Oceans, 88, 2926–2934, doi:10.1029/JC088iC05p02926, http://dx.doi.org/10.1029/JC088iC05p02926, 1983.

Kwok, R.: Deformation of the Arctic Ocean sea ice cover between November 1996 and April 1997: a qualitative survey, Solid Mechanics And Its Applications, 94, 315–322, 2001.

Kwok, R., Schweiger, A., Rothrock, D., Pang, S. S., and Kottmeier, C.: Sea ice motion from satellite passive microwave imagery assessed with ERS SAR and buoy motions, J. Geophys. Res., 103, 8191–8214, 1998.

Kwok, R., Hunke, E. C., Maslowski, W., Menemenlis, D., and Zhang, J.: Variability of sea ice simulations assessed with RGPS kinematics, J. Geophys. Res., 113, doi:10.1029/2008JC004783, 2008.

Kwok, R., Cunningham, G. F., Wensnahan, M., Rigor, I., Zwally, H. J., and Yi, D.: Thinning and volume loss of the Arctic Ocean sea ice cover: 2003–2008, Journal of Geophysical Research: Oceans, 114, doi:10.1029/2009JC005312, c07005, 2009.

Lindsay, R. W. and Stern, H. L.: The RADARSAT geophysical processor system: Quality of sea ice trajectory and deformation estimates, J. Atm. Ocean.Tech., 20, 1333–1347, 2003.

Linow, S. and Dierking, W.: Object-Based Detection of Linear Kinematic Features in Sea Ice, Remote Sensing, 9, doi:10.3390/rs9050493, http://www.mdpi.com/2072-4292/9/5/493, 2017.

Lovejoy, S. and Schertzer, D.: Scale, Scaling and Multifractals in Geophysics: Twenty Years on, in: Nonlinear Dynamics in Geosciences, pp. 311–337, Springer, New York, NY, New York, NY, 2007.

Marcq, S. and Weiss, J.: Influence of sea ice lead-width distribution on turbulent heat transfer between the ocean and the atmosphere, The Cryosphere, Volume 6, Issue 1, 2012, pp. 143-156, 6, 143–156, 2012.

Marsan, D. and Weiss, J.: Space/time coupling in brittle deformation at geophysical scales, Earth Planet. Sci. Lett., 296, 353–359, 2010.

Marsan, D., Stern, H. L., Lindsay, R., and Weiss, J.: Scale dependence and localization of the deformation of Arctic sea ice, Phys. Rev. Lett., 93, 178 501, 2004.

Matsushita, M.: Fractal Viewpoint of Fracture and Accretion, Journal of the Physical Society of Japan, 54, 857–860, doi:10.1143/JPSJ.54.857, 1985.

Oikkonen, A., Haapala, J., Lensu, M., Karvonen, J., and Itkin, P.: Small-scale sea ice deformation during N-ICE2015: From compact pack ice to marginal ice zone, Journal of Geophysical Research: Oceans, 122, 5105–5120, doi:10.1002/2016JC012387, http://dx.doi.org/10.1002/2016JC012387, 2017.

Olason, E., Rampal, P., and Dansereau, V.: On the statistical properties of sea ice lead fraction and heat fluxes in the Arctic, submitted to The Cryosphere Discussion, XXX, 2019.

Rampal, P., Weiss, J., Marsan, D., Lindsay, R., and Stern, H.: Scaling properties of sea ice deformation from buoy dispersion analysis, Journal of Geophysical Research: Oceans, 113, doi:10.1029/2007JC004143, http://dx.doi.org/10.1029/2007JC004143, c03002, 2008.

Rampal, P., Weiss, J., and Marsan, D.: Positive trend in the mean speed and deformation rate of Arctic sea ice, 1979-2007, Journal of Geophysical Research: Oceans, 114, doi:10.1029/2008JC005066, http://dx.doi.org/10.1029/2008JC005066, c05013, 2009a.

Rampal, P., Weiss, J., Marsan, D., and Bourgoin, M.: Arctic sea ice velocity field: General circulation and turbulent-like fluctuations, Journal of Geophysical Research: Oceans, 114, C10 014, 2009b.

Rampal, P., Bouillon, S., Ólason, E., and Morlighem, M.: neXtSIM: a new Lagrangian sea ice model, The Cryosphere, 10, 1055–1073, doi:10.5194/tc-10-1055-2016, http://www.the-cryosphere.net/10/1055/2016/, 2016.

Remacle, J.-F. and Lambrechts, J.: Fast and Robust Mesh Generation on the Sphere ? Application to Coastal Domains, Procedia Engineering, 163, 20 – 32, doi:http://dx.doi.org/10.1016/j.proeng.2016.11.011, http://www.sciencedirect.com/science/article/pii/S187770581633315X, 25th International Meshing Roundtable, 2016.

Rothrock, D. A. and Thorndike, A. S.: Geometric properties of the underside of sea ice, J. Geophys. Res., 85, 3955–3963, 1980.

Rothrock, D. A. and Thorndike, A. S.: Measuring the sea ice floe size distribution, Journal of Geophysical Research: Oceans, 89, 6477–6486, doi:10.1029/JC089iC04p06477, https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JC089iC04p06477, 1984.

Sakov, P., Counillon, F., Bertino, L., Lisæter, K. A., Oke, P., and Korablev, A.: TOPAZ4: An ocean sea ice data assimilation system for the North Atlantic and Arctic, Ocean Sci., 8, 633–662, doi:10.5194/osd-9-1519-2012, 2012.

Samaké, A., Rampal, P., Bouillon, S., and Ólason, E.: Parallel implementation of a Lagrangian-based model on an adaptive mesh in C++: Application to sea-ice, Journal of Computational Physics, 350, 84–96, 2017.

Semtner, A. J.: A model for the thermodynamic growth of sea ice in numerical investigations of climate, J. Phys. Ocean., 6, 379–389, 1976.

Smith, J.: Oceanographic investigations during the AIDJEX lead experiment, AIDJEX Bull., 27, 125–133, 1974.

Sornette, D.: Power Law Distributions, pp. 93–121, Springer Berlin Heidelberg, Berlin, Heidelberg, doi:10.1007/3-540-33182-4_4, https://doi.org/10.1007/3-540-33182-4_4, 2006.

Spreen, G., Kwok, R., Menemenlis, D., and Nguyen, A. T.: Sea-ice deformation in a coupled ocean–sea-ice model and in satellite remote sensing data, The Cryosphere, 11, 1553–1573, 2017.

Stern, H. L. and Lindsay, R. W.: Spatial scaling of Arctic sea ice deformation, Journal of Geophysical Research: Oceans, 114, n/a–n/a, doi:10.1029/2009JC005380, http://dx.doi.org/10.1029/2009JC005380, c10017, 2009.

Stern, H. L. and Moritz, R. E.: Sea ice kinematics and surface properties from RADARSAT synthetic aperture radar during the SHEBA drift, Journal of Geophysical Research: Oceans, 107, SHE 14–1–SHE 14–10, doi:10.1029/2000JC000472, http://dx.doi.org/10.1029/2000JC000472, 8028, 2002.

Stern, H. L. and Rothrock, D. A.: Open water production in Arctic sea ice: Satellite measurements and model parameterizations, J. Geophys. Res., 100, 20 601–20 612, 1995.

Stern, H. L., Schweiger, A. J., Zhang, J., and Steele, M.: On reconciling disparate studies of the sea-ice floe size distribution, Elementa Science of the Anthropocene, 6, 1–16, 2018.

Thomas, M., Kambhamettu, Chandra, and Geiger, C. A.: Discontinuous Non-Rigid Motion Analysis of Sea Ice using C-Band Synthetic Aperture Radar Satellite Imagery, in: Computer Vision and Pattern Recognition Workshop, 2004. CVPRW ’04. Conference on, p. 24, 2004.

Thomas, M., Kambhamettu, C., and Geiger, C. A.: Near-real time application of SAR-derived sea ice differential motion during APLIS ice camp 2007, Remote Sensing and . . . , 2007.

Thomas, M., Kambhamettu, C., and Geiger, C. A.: Mapping of large magnitude discontinuous sea ice motion, SIGSPATIAL Special, 1, 45–50, 2009.

Tonboe, R. T., Eastwood, S., Lavergne, T., Sørensen, A. M., Rathmann, N., Dybkjær, G., Pedersen, L. T., Høyer, J. L., and Kern, S.: The EUMETSAT sea ice concentration climate data record, The Cryosphere, 10, 2275–2290, 2016.

Wang, K.: Observing the yield curve of compacted pack ice, Journal of Geophysical Research: Oceans, 112, n/a–n/a, doi:10.1029/2006JC003610, http://dx.doi.org/10.1029/2006JC003610, c05015, 2007.

Weiss, J.: Intermittency of principal stress directions within Arctic sea ice, Phys. Rev. E, 77, 056 106, 2008.

Weiss, J.: Exploring the “solid turbulence” of sea ice dynamics down to unprecedented small scales, Journal of Geophysical Research: Oceans, 122, 6071–6075, 2017.

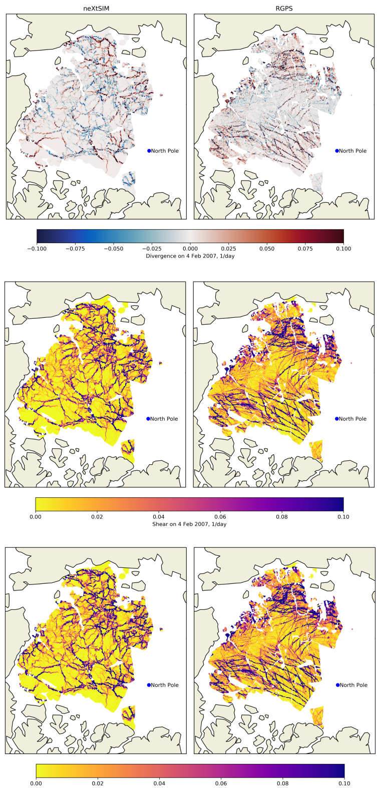
Weiss, J. and Dansereau, V.: Linking scales in sea ice mechanics, Philosophical Transaction Royal Society A: Mathematical, Physical and Engineering Sciences, 375, 20150 352, doi:10.1098/rsta.2015.0352, http://rsta.royalsocietypublishing.org/content/375/2086/20150352, 2017.

Weiss, J. and Marsan, D.: Scale properties of sea ice deformation and fracturing, CR Phys., 5, 735–751, 2004.

Weiss, J., Marsan, D., and Rampal, P.: Space and Time Scaling Laws Induced by the Multiscale Fracturing of The Arctic Sea Ice Cover, in: IUTAM Symposium on Scaling in Solid Mechanics, edited by Borodich, F., vol. 10 of *Iutam Bookseries*, pp. 101–109, Springer Netherlands, doi:10.1007/978-1-4020-9033-2_10, http://dx.doi.org/10.1007/978-1-4020-9033-2_10, 2009.

Williams, T. D., Rampal, P., and Bouillon, S.: Wave-ice interactions in the neXtSIM sea-ice model, The Cryosphere Discussions, 2017, 1–28, doi:10.5194/tc-2017-24, https://www.the-cryosphere-discuss.net/tc-2017-24/, 2017.

Winton, M.: A reformulated three-layer sea ice model, J. Atm. Ocean.Tech., 17, 525–531, 2000.



neXtSIM
RGPS
North Pole
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
Divergence on 4 Feb 2007, 1/day
0.00
0.02
0.04
0.06
0.08
0.10
Shear on 4 Feb 2007, 1/day
Total deformation on 4 Feb 2007, 1/day

Figure 1. Divergence, shear and total sea ice deformation rates in per day (top to bottom), as simulated by the model (left column) and observed from satellite (right column). The deformation rates are calculated over a time scale of 3 days. In order to get a better spatial coverage, we show all the deformation rates calculated within the period of 7 days centred on 4 February 2007. The model field is masked to match spatially with the RGPS data coverage

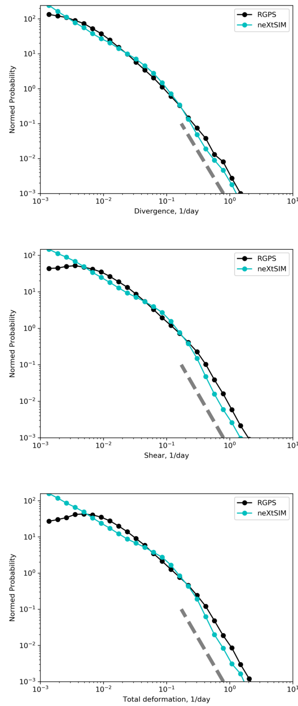


Figure 2. ~~Cumulative probability~~ Probability density functions of the absolute divergence, shear and total deformation rates ~~shown~~ showed on the maps of figure ~~??~~ 1 for the model (cyan) and the RGPS observations (black). The deformations are calculated over a time scale of 3 days, and a spatial scale of 7.5 km (mean of the squared root of triangle's surface areas and for which the deformations are calculated). Power law fits of the tails of the distributions for the model and the RGPS observations and for each invariant give similar exponents ranging from -2.9 and -3.2. The dashed line is shown for reference and corresponds to a power-law with an exponent equal to -3.

~~Shear sea ice deformation rate in per day, as simulated by the model (left) and observed from satellite (right). The deformation is calculated over a time scale of 3 days, for the period of 7 days centred on 4 February 2007~~

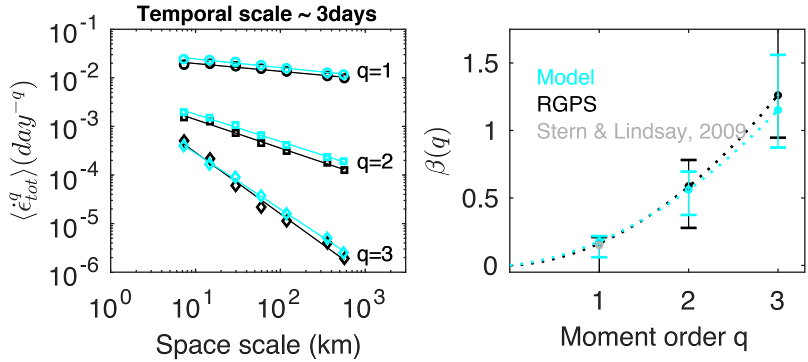


Figure 3. Spatial scaling analysis of the observed (black) and simulated (blue) total deformation rate calculated over a time scale of 3 days from the motion of the same triplets in the model ~~than in~~ and the RGPS dataset. Left panel: ~~Power law fits $\langle\dot{\varepsilon}^q\rangle \sim L^{-\beta(q)}$ for the moments~~ Moments $\langle\dot{\varepsilon}^q_{tot}\rangle$ of order $q = 1, 2$ and 3 of the distributions of the ~~shear~~ total deformation rate ~~$\dot{\varepsilon}$~~ $\dot{\varepsilon}_{tot}$ calculated at ~~different spatial~~ a temporal scale of 3 days and space scales ~~L are shown as~~ varying from 7.5 to 580 km. The solid lines indicate the associated power-law scaling $\langle\dot{\varepsilon}^q_{tot}\rangle \sim L^{-\beta(q)}$. Right panel: Corresponding structure functions $\beta(q)$ for both the model and observation, where β indicates the exponent of the power laws fits ~~,~~ and q is the moment order~~are shown as dashed lines~~. The ~~error~~ bars ~~are estimated~~ indicate the deviation from the ~~minimal~~ power law as they correspond to the minimum and ~~maximal local scaling exponent~~ maximum power-law exponents obtained for two successive spatial scales as in Bouillon and Rampal (2015a) and ~~thus correspond to upper-bound estimates~~can be viewed as an estimation of the goodness of the fit.

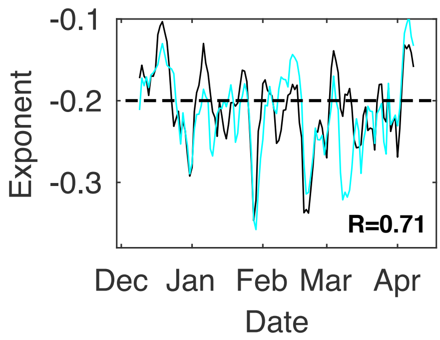


Figure 4. Time series of the power spatial scaling exponents for the mean total deformation (lefti.e. $q = 1$) calculated for individual snapshots and at a temporal scale of the curvature of the structure function (right) T = 3 days for the model (cyan) and the RGPS observations (black). The dashed line is shown only for reference and corresponding to the value of 0.2 reported in Marsan et al. (2004) for the 3-day deformation calculated over a period centered on 6 November 1997.

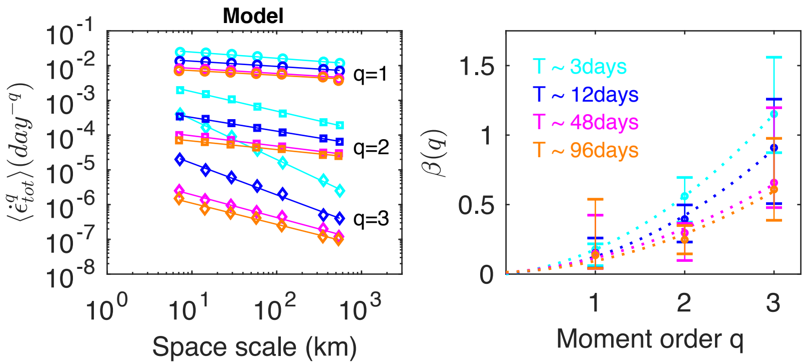


Figure 5. Same as Figure 3 but for the model at various temporal scales.

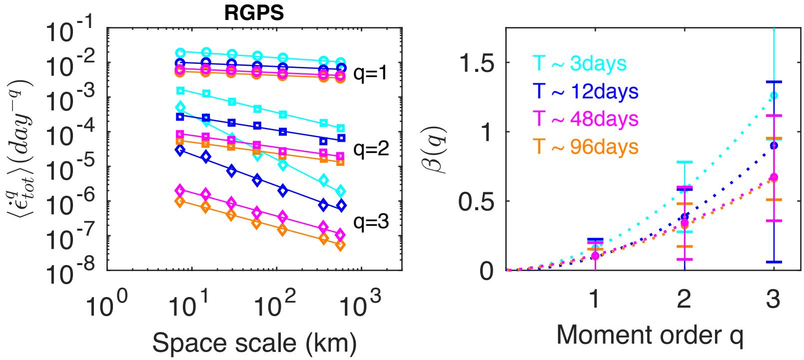


Figure 6. Same as Figure 3 but for the observations at various temporal scales.

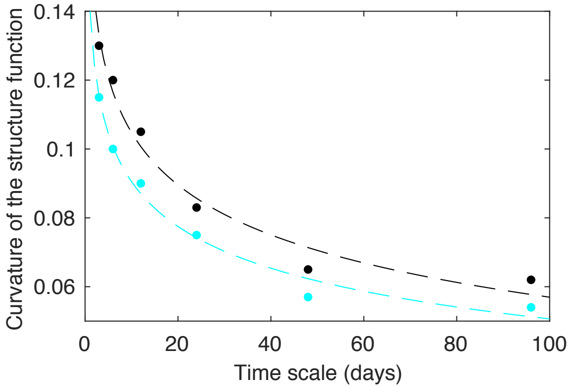


Figure 7. Curvature of the structure function as a function of the time scale T for the model (cyan dots) and the RGPS observations (black dots). The dashed lines are power-law fits (in the least-squared sense) through the data.

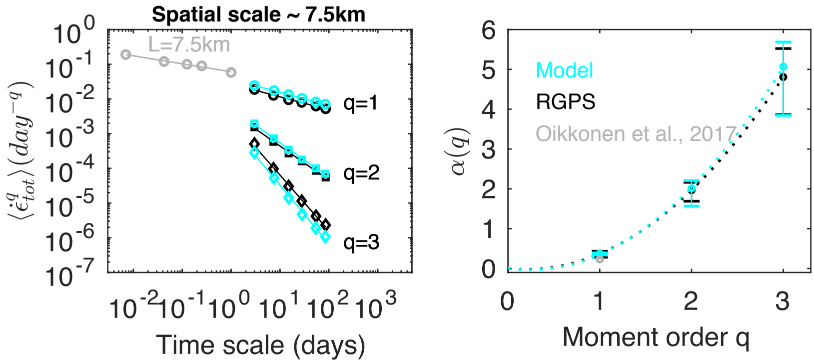


Figure 8. Temporal scaling analysis of the ~~observed (black) and simulated (blue)~~ total deformation rate derived from the motion of the same triplets with initial surface area of 7.5 ~~squared~~ km for the Model (cyan) and the RGPS observations (black). Left panel: ~~Normalized moments~~ Moments $\langle \dot{\varepsilon}^q_{tot} \rangle$ of order $q = 1, 2$ and 3 of the distributions of the total deformation rate $\dot{\varepsilon}_{tot}$ calculated at a spatial scale of 7.5 km and time scales varying from 3 to 100 days~~for the observations and 3 hours to 100 days for the model~~. The solid lines indicate the associated power-law scaling $\langle \dot{\varepsilon}^q_{tot} \rangle \sim t^{-\alpha(q)}$. ~~The dashed lines are extrapolation for~~ Grey dots correspond to the ~~smallest~~ mean total deformation rates obtained by Oikkonen et al. (2017) at a same spatial scale of 7.5 km and for time scales ranging from 3 hours to 1 day. Right panel: Corresponding structure functions $\alpha(q)$ for both model and ~~observation~~ RGPS observations where α indicates the exponent of the power laws fits, and q is the moment order. The ~~error~~ bars ~~are estimated~~ indicate the deviation from the ~~minimal and maximal local scaling exponent~~ power law as ~~in Bouillon and Rampal (2015a) and thus~~ they correspond to ~~upper-bound estimates~~the minimum and maximum power-law exponents obtained for two successive temporal scales and can be viewed as an estimation of the goodness of the fit.

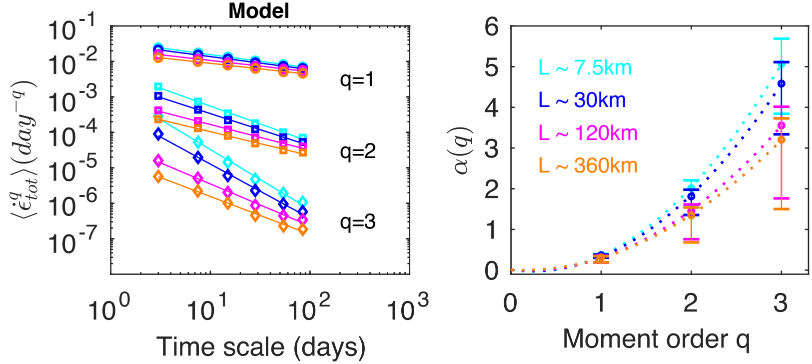


Figure 9. Same as Figure 8 but for the model at various spatial scales.

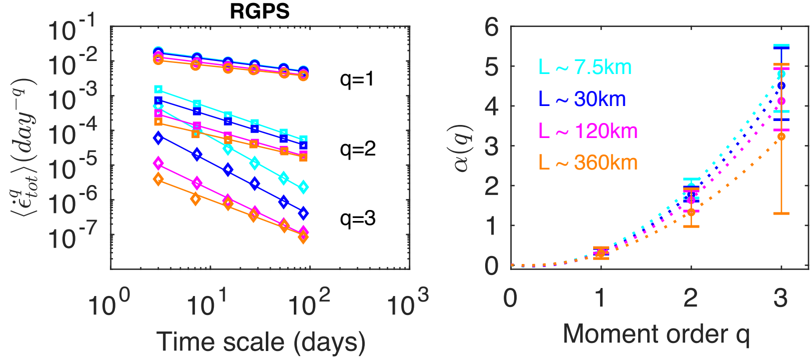


Figure 10. Same as Figure 8 but for the RGPS observations at various spatial scales.

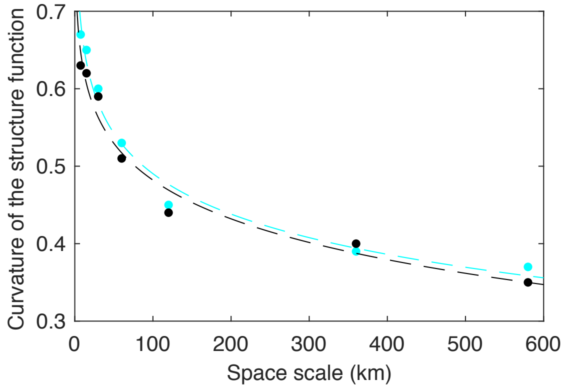


Figure 11. Curvature of the structure function as a function of the space scale L for the model (cyan dots) and the RGPS (black dots). The dashed lines are power-law fits (in the least-squared sense) to the data.

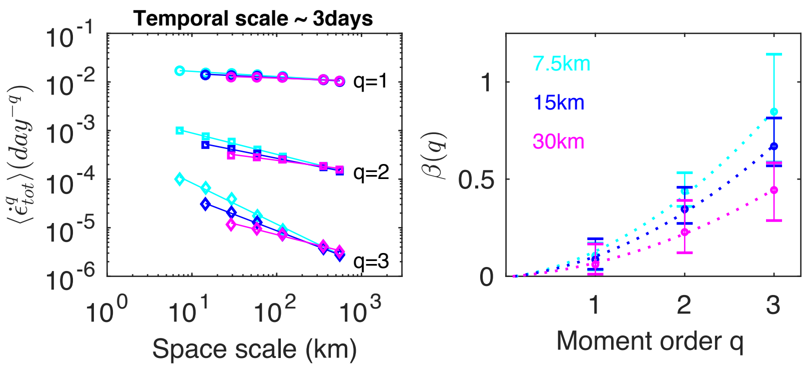


Figure 12. Spatial scaling analysis of the simulated deformation derived from the motion of triplets over a time scale of $T = 3$ days in 3 different model runs, at 7.5, 15 and 30 kilometers resolution respectively. Left panel: Normalized moments Moments $\langle \dot{\varepsilon}^q_{tot} \rangle$ of order $q = 1, 2$ and 3 of the distributions of the total deformation rate $\dot{\varepsilon}_{tot}$ calculated at a temporal scale of 3 days and for spatial scales varying from 7.5 to 580 kilometers. The solid lines indicate the associated power-law scaling $\langle \dot{\varepsilon}^q_{tot} \rangle \sim L^{-\beta(q)}$ as in figure 3. Right panel: Corresponding structure functions $\beta(q)$ where β indicates the exponent of the power-law fits, and q is the moment order. The error bars are estimated indicate the deviation from the minimal power law as they correspond to the minimum and maximal local scaling exponent maximum power-law exponents obtained for two successive spatial scales as in Bouillon and Rampal (2015a) and thus correspond to upper-bound estimates and can be viewed as an estimation of the goodness of the fit.

Table 1. List of variables used in neXtSIM.

Symbol	Name	Meaning	Unit
H	sea ice thickness	volume of ice per unit area	m
h_s	snow thickness	volume of snow per unit area	m
A	sea ice concentration	surface of ice per unit area	-
H_t	thin sea ice thickness	volume of ice per unit area	m
$h_{s,t}$	snow thickness on thin ice	volume of snow per unit area	m
A_t	thin sea ice concentration	surface of ice per unit area	-
d	sea ice damage	0=undamaged, 1=completely damaged ice	-
$\boldsymbol{u}$	sea ice velocity	horizontal sea ice velocity	m s^{-1}
$\boldsymbol{\sigma}$	sea ice internal stress	planar internal stress	N m^{-2}

Table 2. Parameters used in the model with their values for the simulation at 7.5 km resolution used for this study.

Symbol	Meaning	Value	Unit
ρ_a	air density	1.3	kg m^{-3}
c_a	air drag coefficient	4.9×10^{-3}	-
θ_a	air turning angle	0	degree
ρ_w	water density	1025	kg m^{-3}
c_w	water drag coefficient	5.5×10^{-3}	-
θ_w	water turning angle	25	degrees
ρ_i	ice density	917	kg m^{-3}
ν	Poisson coefficient	0.3	-
μ	internal friction coefficient	0.7	-
E_0	undamaged elastic modulus	50.0	MPa
Δx	mean distance between mesh nodes	10	km
Δt	time step	200	s
c	cohesion	25	kPa
$\sigma_{N\,\mathrm{min}}$	tensile strength	-21	kPa
$\sigma_{N\,\mathrm{max}}$	compressive strength	75	kPa
c^*	compactness parameter	~~−20~~ 20	-
α	damage exponent	~~4~~ 5	-
λ_0	undamaged relaxation time	10^7	s
T_d	characteristic time for damaging	20	s

Anonymous Referee #1
Received and published: 28 February 2019

General comments

The manuscript applies a scaling and multi-fractal analysis to sea-ice deformation fields simulated with neXtSIM and derived from RGPS satellite observations. It shows that the spatial and temporal scaling of the observed sea-ice deformation is well reproduced by the model. The paper also explores the multi-fractality and spatio-temporal coupling of the scaling, but whether these behaviors are significant is debatable considering the very large error bars on the values compared.

The manuscript is generally well written but the wording used is often too strong or too conclusive for what the results are showing. I think the paper would be greatly improved if the authors would include a discussion of the error to support their affirmations. I also disagree with the choice of method for analyzing the RGPS data and would also like more information on how the scaling analysis is performed. For these reasons, I recommend that is this paper be reconsidered after major revisions and I provide more details about points to be addressed in the revisions below.

Dear reviewer, thank you for your in-depth review of our manuscript, and all your suggestions to improve it. We hope your numerous concerns about the methodology will be cleared after reading our responses and the modification we made to our manuscript. We think these concerns were mostly our fault and we thank you for having spotted incoherency or inaccuracy in our wording.

The review process also allowed us to find a bug in our analysis script, which mainly impacted our results of the space-time coupling of sea ice deformation that we propose to look through an actual change in the degree of multi-fractality of the scaling with respect to the temporal or spatial scale considered. We therefore re-performed the whole analysis and generated new figures with updated results.

Also, we found that our values of curvature of the structure functions calculated from the temporal scaling were incorrect.

Please find below our answers (in red) to your comments/suggestions/questions.

Major points to be addressed:

1. I have reservations about the choices of methods used to analyze the data in this study. By using a nearest-neighbour interpolation on the RGPS trajectories, the authors artificially set all initial temporal scales in the RGPS data to 3 days, although they vary strongly from a few minutes to up to 10 days. A filtering that keep the original RGPS temporal scales, for example, keeping only the trajectories that are updated more or less every 3 days, would be more appropriate. As it is now, it is not clear to me that the temporal scaling and spatio-temporal coupling the authors are reporting here is not an artifact of the method used.

Moreover, it would be necessary to include more details on the scaling (or "coarse- graining") procedure used in this study so that the results can be reproduced by others. As of now, it is also unclear what are the effects of using a sub-sampling of the trajectories (as I understood is done) instead of spatio-temporal averaging as usually done for the scaling analysis. The differences in the method and also the justification for choosing a different method need to be clearly stated.

2. I am also not convinced of the significance of the multi-fractality and spatio-temporal coupling behavior that the authors affirm is present in the results. The error bars on the values used to infer these behaviors are sometimes used to confirm that values overlap and therefore are equal, but are elsewhere ignored to affirm that the values are different. Also, the "error bars" as defined in this study rather represent the goodness of fit on the data, than an actual error on the values calculated. Proper error estimates are needed to claim that the multi-fractality or the coupling are significant.

3. The discussion of scaling in sea-ice models is limited to (Maxwell) Elasto-Brittle (EB/MEB) papers. There are several studies using the viscous-plastic (VP) rheology that are worth mentioning. Especially, the claim that for the first time spatio-temporal scaling is shown for a model, is not true (see Fig. 7 in Hutter et al., 2018 - see reference below).
4. The conclusion is mostly a summary of what the neXtSIM model is capable of rather than overall conclusion that can be drawn from the presented work for other studies or model development. The overall conclusion that is drawn (that is, that multi-fractal scaling analysis should be the prerequisite validation step before analysis of any other variable that might be related to sea-ice dynamics) is clearly application-dependent and needs to be modified. I would wish that the authors come up with a conclusion that is more useful for the scientific community than just promoting the model. There is clearly enough good material in the paper to do so.
5. I would appreciate if the model physics and the configuration of the simulation (e.g. wind forcing, rheology parameters, Lagrangian grid, etc.) would be separated more carefully when drawing conclusions from the simulated results. It feels like when there is agreement between the model and observations, the authors attribute this agreement to their choice of model physics, while if the model results disagree with the observations, the authors note that it could be due to the model configuration. Maybe a change in wording would help to reduce this impression.

Below are more specific comments to help the authors address the general comments above.
=== SPECIFIC COMMENTS

Page 2:

- Line 16: There is a new paper about filtering LKFs in the entire RGPS data- set that also shows are wide range of intersection angles of LKFs (Hutter et al., 2019, see References below) and is worth to citing here.

Reference added.

Page 3:

- Line 4: Again, please add Hutter et al. (2019) (Linow and Dierking only studied 10 RGPS snapshots, whereas in the above mentioned paper, the LKF length was studied for the entire RGPS data-set).

Reference added.

- Line 7: "These events also sustain deformation, maintaining the LKFs “active” for many days Coon et al. (2007)" This is not clear. If the deformations are of short duration, how are they responsible for "sustaining" deformation rates over many days? Please clarify.

There was indeed a typo in the previous sentence: We wanted to say “short-duration *fracturing* events of…” instead of “short-duration *deformation* events of…”. The text has been changed accordingly.

- Line 15: To me, it is the distribution that can be in or out of the Gaussian basin of attraction, not the values themselves. I would remove "that are out of the Gaussian basin of attraction" or rewrite something like "...i.e., dominated by extreme values and out of the Gaussian basin of attraction"

Yes, we agree. The text has been changed following your suggestion, i.e. removing “that are out of the Gaussian basin of attraction”.

- Line 25: Please shortly explain what a "coarse-graining" method is.

We prefer to refer to the methodology section of the manuscript.

Page 4:

- Line 6: I agree that beta=0 is homogeneous deformations. But if one imagines the scaling exponent to be something similar as the fractal dimension (see Weiss, 2003), then beta=1 would correspond to deformations concentrated in one line, and for beta=2, all deformations would be localized in one single point. This is also what one would expect from averaging in two spatial dimensions. Please clarify.

Yes this is correct. We thank the reviewer for spotting this typo. We changed the text accordingly: “In the space domain, $\beta = 0$ characterizes the homogeneous deformation of an elastic solid or viscous fluid, i.e., a deformation that does not depend on the spatial scale, while $\beta = 2$, i.e. the topological dimension for the 2D-like sea ice cover, corresponds to a single "point" concentrating all of the deformation in an otherwise undeformed material (Rampal et al. 2008)”.

- Lines 21-24: Two paragraphs above you state that the distribution of the sea-ice deformation rates is out of the Gaussian basin of attraction (i.e. the decay has a slope <= 3). For those power-law distributions it is known that the higher moments (variance, skewness, etc.) do not converge, due to the presence of extreme values, to a real number but to infinity. Please clarify how those ill defined moments help better describe the distribution?

The comment of the reviewer is interesting. We understand the confusion we may bring here by expressed ourselves the way we did, and in particular saying that the deformation distribution was “out of the Gaussian attraction basin”. This is actually wrong. The distribution of sea ice deformation are indeed heavy-tailed, dominated by extreme values, but with a slope close to 3 (actually slightly larger), meaning that they are still in the Gaussian attraction basin, but requires us to consider higher order moments (in our study up to q=3) than just the mean to be describe them and the associated process of sea ice deformation. In our case both variance and skewness can therefore be defined, and are considered for the scaling analysis along with the mean. In order to correct this mistake and clarify this point as suggested by the reviewer, the text has been changed to the following:

“The fact that sea ice deformation is characterized by heavy-tailed statistical distribution, i.e. dominated by extreme events, also indicates that the mean (moment of order 1) is not a sufficient quantity to describe the distribution of deformation rates at a given time/space scale. Higher moments of the distribution of deformation rates, such as the variance (order 2) and skewness (order 3), should indeed also be explored to better describe the distribution and the associated process of sea ice deformation, and considered in temporal and scaling analyses as proposed in this study.”

Page 5:

- Line 5: "the amount of localization of large and small deformation events is the same" How do you define the localization? I thought the scaling exponents are quantifying the "degree of localization" of deformation (see page 4, line 5)?

Here, even if the curvature is zero, the scaling exponents are still changing linearly with the moment q, so that they are different and then I don’t think you can say that the "amount of localization of large and small deformation events is the same".

Yes! you are right indeed. Thanks for spotting this mistake. The sentence “the amount of localization of large and small deformation events is the same" was put here by mistake. We therefore removed this sentence.

- Line 10-11: Please add references for this.

We added two references: Lovejoy and Schertwer (2007) and Kolmogorov (1962).

The definitions of heterogeneity and intermittency are confusing to me... Isn't saying that a field shows high localization in space/time (ie. beta or alpha > 0) the same as saying that the field is heterogeneous/intermittent?

Not really in fact. The definition of "heterogeneity" and "intermittency" are often related to the characteristics and degree of localization of the process in space and time respectively, i.e. related to the exponent of the structure function.

So, as we said in the manuscript, in the literature the terms spatial "heterogeneity" and temporal "intermittency" are often used when we are in the presence of a field/ or process that can be modeled by *"log-normal cascade"* associated with a multifractal scaling (not mono-fractal). However, we do acknowledge that these definitions of "heterogeneity" and "intermittency" are not consistent throughout the literature and different research communities (for example the one focusing on the study of highly non-linear (e.g. chaotic) dynamical systems differs from the one focusing on geophysical fluid turbulence), which we agree can be a source of confusion.

We changed the text as follows in order to make these definitions more clear:

"In the case of a linear structure function, i.e. no curvature or equivalently a=0 or c=0, the scaling is said to be *mono-fractal*.

In the case where both coefficients a and b or c and d are positive the structure functions are convex, meaning that the higher order moments of the distribution therefore increase much faster than the lower order moments with decreasing scale of observation. In other words, *large* deformation events are *more localized* in time and space than smaller events, corresponding to the definition of a *multi-fractal* scaling (e.g. Kolmogorov 1962, Lovejoy and Schertzer 2007). Note that in the literature multifractality is also called *intermittency* when present in the time dimension and *heterogeneity* when present in the spatial dimension. The largest the curvature (i.e. a or c > 0), the stronger the *degree of multifractality* of the scaling."

Why does a field need to have a (quadratic) change in the localization exponent with the different moments q to define it as heterogeneous/intermittent? What if the structure function is cubic? Please clarify.

As said above, this is a matter of definition, but one may indeed say that a field can exhibit different "*degrees of heterogeneity"* or *"degree of intermittency"* depending on the value of the structure function exponent. So the structure function does not need to be quadratic, but at least diverge from a linear function.

Note that there is another way to look at the degree of multifractality, which is to calculate the moment scaling function defined as $K(q)=C(q^{alpha}-q)/(alpha-1)$. In the framework of the theory of *multiplicative cascade processes*, the exponent alpha (which is related to the so-called "Levy exponent of the generator") is lower or equal to 2. The special case when alpha is equal to 2 (the scaling is then strongly *multifractal*) in fact corresponds to the so-called *"log-normal"* multiplicative cascades (Kolmogorov, 1962), whereas the case when the exponent is equal to 1 (the scaling is then *monofractal*) corresponds to the so-called "*beta-model"* (Frisch et al., 1978).

- Line 20 - "(Olason et al., 2019)" I couldn't find this paper in the Cryosphere Discussion.

This is correct, as this paper is only about to be submitted. We therefore removed this reference and replace it with a conference talk where these results were presented.

- Line 24,25: Downscaling of the modeled sea-ice deformation might be performed if one characterizes the scaling exponents, however, Spreen et al. (2017) have shown that the dependence of the scaling exponent on the sea-ice concentration and thickness has non-trivial effects on the "scaled" deformation rates, so I would suppose that only if you knew the distribution of A and h at a subgrid-scale (which we don't) could you actually perform such a downscaling.

Thanks for this interesting comment. Spreen et al. used a different model with a different dependence of deformation on concentration and thickness. So their results are, a priori, not applicable in our case. Their paper also showed a poor reproduction of deformation scaling and does not explore the multi-fractality of their solution. We make a point of underlining the importance of multi-fractality in the paragraph in question so given that Spreen et al don't consider multi-fractality at all makes it even more difficult to see how their results can be of importance here. Finally, we don't mention concentration and thickness here at all - only deformation. It is true that down-scaling thickness, and concentration (or more generally the thickness and flow size distributions) would be of considerable interest, but this is outside the scope of the paper and we feel that a discussion of this potential application would be distracting for the reader.

Page 6:

- Lines 3-6: It would be worth to include a short summary of studies that used scaling analysis to evaluate sea-ice deformation in simulations and their findings (so far only EB studies are mentioned). For example, Hutter et al. (2018) have shown that the VP rheology can reproduce the observed spatial scaling and also multifractal characteristics in the spatial domain (i.e. quadratic spatial structure function at very high resolution. Please see also Spreen, et al. (2017) and Bouchat and Tremblay (2017).

We now include such a summary.

- Lines 16-19: Please elaborate on why this is needed. What does it mean "to localize the deformation at the nominal scale"?

We've rewritten the paragraph so it should be clear that "localizing the deformation at the nominal scale" refers to reproducing the statistics of deformation from the smallest scales resolved by the model (i.e. the grid spacing and the time step). We have also elaborated on why we expect such localization to be essential to further improve the simulation of ocean-atmosphere interactions modulated by the presence of sea ice.

Page 9:

- Line 2: Please provide the spatial and temporal resolution of the forcing used.

Horizontal spatial resolutions of the forcing used (here 30km for the atmosphere, 50km for the geostrophic current and 12.5km for the ocean temperature and salinity) are now mentioned in the text.

Page 10:

- Line 12: "We use the coarse-graining approach..." Since you already men-
tion both approaches, please justify why you haven chosen the first one.
- Line 13: Why do you choose triplets? It is known that the boundary definition error (Lindsay & Stern, 2003) is larger for lower number of vertices. You have chosen the minimal number of vertices and thus the highest uncertainty. Why? Do you use the smoothing filter as suggested by Bouillon and Rampal (2015) in your analysis to compensate this effect? If not, please mention why not and how you deal with the uncertainties introduced by choosing triangles instead of rectangles as done originally in RGPS.

We indeed chose to consider triplets because it was the best compromise between the number of samples (or triangles) we could consider in our analysis, while still getting robust power law fits through the moments of the distributions. Using triangles has more uncertainties than using quadrangles if the absolute values of deformation rates are of interest (as mentioned already in Bouillon and Rampal (2015)). Here however, we focus on the scaling properties of sea ice deformation rates and on the comparison of a model with observations. So what matters is to make sure one applies the same methodology on both to derive sea ice deformation rates, which we do by considering and tracking over the entire winter season the exact same set of triplets in the model and the RGPS dataset. Note also that using triangle allows to decrease the actual minimum spatial scale considered in the scaling analysis from 10km to 7.5km.

We found that applying the filtering suggested in our previous study is therefore not needed for the sake of comparing model and observations as done in this study.

That being said, we do agree that using quadrangles would be a possible way to go, but this would reduce the number of samples in the distributions strongly, and therefore would imply to consider more than one winter dataset to perform this analysis, which we consider to be beyond the goal of the present study.

- Line 15: The value of 7.5 km is an average for the triangulation over the 2006/2007 season? Otherwise, it is not clear to me how you get that number analytically. Please specify.

7.5 km is the number obtained when averaging the square root of surface area of the triangles obtained from the delaunay triangulation through the initial (i.e. on December 3 2006) RGPS drifter's position.

- Line 25: Do you use the same triangulation for both the RGPS and model trajectory sets?

Yes, the exact same one, meaning that we follow the exact same set of triplets of trajectories (or triangles) in the model and in the observations, defined from the Delaunay triangulation performed on the initial set of RGPS/model drifter points. We added a sentence in the text to clarify this.

Also, how do you handle the different streams of the original RGPS Lagrangian ice motion dataset?

This specific points are explained in Bouillon and Rampal (2015), see section 3, 3rd paragraph. We now refer to this paper in the text, since repeating the whole procedure here would be too long and may contribute losing the reader.

Page 11:

- Line 5-8: Not clear. What is the "subsampled cloud" of positions? How do you select the triangles to add up to a certain spatial scale? Please add details of the subsampling procedure!
- Line 9: "The number of triplets available for the statistical analyses decreases as the space scale increases." Why? Because you use a filter to discard larger scales if they are not filled up to a certain percentage? Please clarify.
- Line 9-11: "Coarse-graining in time..." Are you re-sampling the trajectories at larger time intervals and then computing new estimates of the strain rates at these larger time scales, instead of averaging multiple 3-day strain rates together? To be consistent with the spatial scaling analysis as it is usually done (e.g. as in Marsan et al. 2004), the strain rates should be averaged and not re-sampled.
- Line 10: "The number of available triplets also decreases as the time scale increases."

Again, please indicate why this is the case.
- Line 18: "...around the boundary of each polygon associated to a given space scale L" Again, it seems like you are saying that you are re-calculating the strain rates at different scales instead of averaging multiple triangles of the original data set together to add up to a certain time/spatial scale. If you are really recalculating the strain rates instead of averaging them, then please show what are the effects of doing this vs averaging on the scaling analysis, as I don't recall other studies that have used this method. Or maybe simply re-calculate your strain rates by averaging the triangles instead.

We realized that the reviewer misunderstood our methodology, and we acknowledge that the confusion may have come from the fact that we said we were using a so-called "coarse-graining" method in our study in order to explore the scaling properties of sea ice deformation. Our statement was wrong. We in fact use the "buoy's" dispersion method, using triplets, as in Oikkonnen et al. 2017. This consists in mapping the Arctic with contiguous triangles of different sizes (i.e. 7.5; 15; 30; 60 ... 700 km that corresponding to the spatial scales we want to consider for the scaling analysis) and to look at how the

triangles with the same given size deform over different time period (i.e. from 3 days to 100 days in our study). The above mentioned set of triangles are defined once, at the starting time of the RGPS trajectories, i.e. 2 December 2006 in our case, and then are followed throughout the winter season.
So, in order to address all the above comments (related to page 11), we rephrased some parts of the first paragraph of section 3 that describes our methodology, trying to make it more clear. We also send the reader to Oikkonen et al. 2017 for more details on the methodology.

Page 12:
- Line 1: The actual area A of the polygons will differ slightly from the nominal scale L^2. Do you filter the polygons for their area to match the given nominal length scale?
No, see the previous comment

Also, are you using the definition of A with the summation around the vertices of the polygons, e.g. as in Linsday and Stern (2003)? Please specify.
Yes, we do.

I am also confused about which polygons are which... If you are always grouping triangles that have a mean length scale of Li = 7.5 km to make new polygons at different length scales, wouldn't the average length scale defined as "the mean of the square root of the polygon surface areas", as written on page 11, i.e. what I understand as L = (L1 + L2 + L3 + ... + Ln)/ n (with Li, i=1,...,n being the length scale of the individual triangles), also equal about 7.5 km regardless of the number of triangles you average together? I feel like it would make more sense to define the spatial scale L of the new polygons (i.e. the aggregation of triangles) as the square root of the sum of all the triangle areas in the polygon, i.e. L = sqrt(A1 + A2 + A3 +...) where Ai is the area of the i-th triangle that is averaged together. Or maybe I just don't understand your scaling procedure. Please clarify.
See our comment above. The polygons at a larger scale L2>L1 are NOT the aggregation of smaller triangles of scale L1. They are independent triangles, defined once from the triangulation performed through the points corresponding to a subset of the original RGPS points positions. So the surface areas of the L2-triangles are directly derived from the summation around their vertices, e.g. as in Lindsay and Stern (2003), the spacing between their vertices being just larger compared to the L1-triangles.

Page 13:
- Line 1: I strongly disagree with this procedure. A nearest neighbour interpolation will artificially set all initial temporal scales in RGPS data to 3 days, although they vary strongly from a few minutes to up to 10 days. Why do you not use the original temporal scale of the observations for the scaling analysis?
How much of the method is therefore responsible for the temporal or spatio-temporal scaling you are showing afterwards?
Wer agree with the reviewer that an interpolation like the one he/she thinks we have performed would be physically wrong and likely impact the results.
In fact, we suspect here a misunderstanding of the reviewer of what we meant by "interpolating" the RGPS data, probably due to our fault, i.e. using inaccurate wording in our submitted manuscript. We in fact applied a nearest neighbor "interpolation" at a regular 3 days frequency only when a RGPS observation is available within + or - 6 hours around the target time. All the RGPS position records that are not satisfying this condition are discarded, which does not mean that the entire trajectory is discarded, but truncated instead. Therefore, the smallest temporal scales for which deformation rates are calculated from the RGPS data is 2.5 days. Or in other words, the smallest temporal scale considered for the scaling

analysis of the RGPS data is not an exact time i.e. 3-days but rather a temporal bin between 2.5 and 3.5 days.
From the model simulation, positions of the drifters are saved every 6 hours (from midnight to midnight). Therefore, the above-mentioned problem does not exist in this case. Our method allows for an accurate comparison (over a quasi equal time window) of the simulated and observed deformation rates at a given temporal scale T.
The changed the text of the manuscript, which now reads: “The RGPS trajectories are not sampled at regular time intervals, contrary to the model, due to the irregular interval between two satellite orbits. The mean sampling is of about 3 days, and 90% of trajectories are sampled with a frequency between 2.5 and 3 days. Because sea ice deformation depends on the time scale (see results of section \ref{sec:temporal_scaling}) one should make sure to use similar sampling times for the observations and the model when computing and comparing deformation rates estimates. To deal with this issue, we performed a sub-sampling of the RGPS trajectory dataset using a nearest-neighbour interpolation of the original positions at 3-day intervals, but only when the RGPS drifter's position is available within plus or minus 6 hours around the interpolation target time. The positions simulated by the model, that are outputted every 3 hours from midnight to midnight each day, are taken to match the sub-sampled RGPS time series obtained as described above.”

- Lines 10-15: Because trajectories are eventually removed from your analysis by filtering? Or why else is this the case?
No, this is only because we analyse one single winter period (2006-2007), and we therefore face with a limited number of synchronous trajectories that are forming the triplets we then track over time and use for the deformation calculations. We acknowledge though that using all the years covered by the RGPS dataset would resolve the potential lack of robustness of the statistics. However, we found it was not necessary to do so in order to obtain significant and robust power law scaling of the simulated/observed deformation rates.

- Line 17: "the 3-day shear [...] for the same period of 7 days" How do you get the strain rates on a 7-day period if they are the 3-day strain rates?
It seems like there is a misunderstanding here. What we do here is to pick up all the calculated 3-day strain rates that are observed/or simulated within the time period of 7 days, and we plot them all together to build the map shown on figure 1, assuming they are synchronous in time. This assumption is not strictly exact but allows to improve the spatial coverage of the data shown on the map, and thus for illustration at least, improve visually the field of deformation rates shown on the map.
We modified the text as follows “Note that to obtain a better spatial coverage, these maps are showing all simulated or observed deformation rates for the period of 7 days centered on 4 February 2007” and changed the caption accordingly.

- Line 19: Technically, what you are showing is not the cumulative probability distribution (CDF), but the complementary cumulative distribution function (CCDF), i.e. the probability of having a value greater than a given strain rate. Please correct.
We now show PDFs instead
Also, why choose to show the CCDF instead of the PDF as in previous studies? It would be interesting to show here the PDFs of shear and divergence since it is the first time it would be shown for the MEB rheology in this configuration.
We now plot the distributions as PDFs as suggested by the reviewer for the total, shear and absolute divergence rates.

- Line 20-21: Please discuss the fact that the probability distribution for your model is always greater than that of RGPS. What does this imply?

The distributions of total deformation rates that we show in figure 2 suggest that our model rather slightly under-estimate the largest values compared to the RGPS.

- Line 21: If we assume a power law probability distribution function (PDF) that goes like P(x) -> xˆ{-alpha}, then the CDF (or CCDF) would decay like C(x) -> xˆ{-alpha+1}. Hence, if you find a slope of -3 for the CCDF of both your model and RGPS, it means that the PDFs for both data sets decay with a slope of -4, which according to Sornette (2006), implies that the PDFs slowly converge to Gaussian distributions (or that they are in the "Gaussian basin of attraction") and therefore your argument following in the text does not hold... Please address this.

This is correct. We made a mistake in comparing the slope of the tails of the CCDF to -3, which is supposed to be meaningful value when looking at PDFs, not CCDFs. As said in our response above, we now show in figure 2 the PDFs instead of the CCFD, and we therefore still compare the slope of the tail with the power law with a -3 exponent.

We corrected the text accordingly and removed any mention to the Gaussian basin of attraction. However, we still state that higher order moments than the mean (variance and skewness) can be calculated and are necessary to better describe the distributions of sea ice deformation rates. We stop at q=3 though, because a transition is observed around q_{critic}=2.5 to 3: indeed, because the PDF decays as x^{-3}, moments of order $q>q_{critic}$ diverge (Schertzer and Lovejoy, J. Geophys. Res. **92**, 9693, 1987)

Page 14:

- Line 3: Stern et al. (2018) suggest to use Maximum Likelihood Estimators to determine power-law exponents and test those with a goodness-of-the-fit test (Clauset et al., 2009).

We probably wrongly expressed ourselves here. Indeed, Stern et al. (2018) recommend the use of MLE as the best solution to estimate power-law exponents, but also they also say that already using binned data in log-log space allows to get “reasonably accurate estimates”. We therefore changed the text in the revised manuscript to make this clearer as follows: “We use logarithmically spaced bins and applied an ordinary least square method to the binned data in log-log space to get reasonably accurate estimate of the power-law fits (Stern et al., 2018)”.

- Lines 18-19: Defined as in Bouillon and Rampal (2015), these bars are rather representing the "goodness of the linear fit" rather than an actual error on the values you are comparing. It would be much more useful (in terms of comparing the model to observations) to compute the error on your observed and simulated deformation rates given the known error on the trajectory positions (see for example Lindsay and Stern, 2003) and then the ensuing error on your scaling analysis. Only then can you conclude that the structure functions are "equal within their margin of error".

Yes, you are right that this would allow us to be more conclusive. This is something we will probably do in the future when performing such analysis. Also, one may note that putting “error” bars on the structure functions is something very rarely done in the literature. As long as the compared curves are very close to each other and/or showing clear deviation from the linear model, these “error” bars. We removed the term error bars and replace it with just saying that “the plotted *bars* represent the goodness of the power law fit”.

- Line 21: "... the scaling is clearly multi-fractal, as no linear function can be contained within the error bars." I can pass a line through the origin and through all the "error bars" for the model values. Please remove.

Your remark is valid. However, please note that we discovered a bug in our analysis script calculating the deformation rates and the moments for the model. The updated figure 3 now shows the results after correction. Our statement about saying that no line can pass through the origin and through all the “error bars” is consistent with this new result. But as commented by the reviewer above, the “error bars” are not derived statistically and should rather be called “bars” representing upper-bound estimates of the goodness of the power law fits. So we decided to remove “as no linear function can be contained within the error bars” from our sentence to be more accurate.

- Line 22: "applying a quadratic fit" Please provide the quadratic fit parameters for both RGPS and the model, either here or on the figure.
Good suggestion. Done.

Page 15:
- Line 12-13: Mean curvatures of 0.07 and 0.08 seem quite low to consider this a "clear" signature of multi-fractality... Again, it would be necessary to have the error on these values to know if it is significant or not.
This very low values obtained for the curvature of the structure functions were in fact due to the mistake we found in our analysis script and that we mentioned above already, affecting the spatial scaling in particular. We find that the values of the curvature are now 0.11 and 0.13 (averaged over the winter period) for the model and the RGPS respectively. This means that the model is still lower on average compared to the RGPS. We chose to remove the plot showing the time series of the curvature values as we realized that it does not add any substantial information to the paper.
We note also that such fairly low values of curvatures for multi-fractality of a field in the spatial domain are typical in geophysics (e.g. in the range 0.05-0.15 for the wind, cloud radiances, topography...), although it can be much larger (0.25-0.7 for rain and turbulent fluxes) (See e.g. Lovejoy, S., and D. Schertzer (2007), Scale, Scaling and Multifractals in Geophysics: Twenty Years on, in *Nonlinear Dynamics in Geosciences*, vol. 59, pp. 311–337, Springer, New York, NY)

- Line 16: "beta decreases with increasing T" This is not very clear from figures 5 and 6... In fact, from the right panel in figure 5, it looks more like beta is increasing with increasing T for q=1. Please add a log-log plot of beta vs T for the different values of q for both RGPS and the model, similar to what is done in figures 5 and 7 in Hutter et al. (2018).
The dependance is now clearer on our new results. We do agree though that the scaling exponents for the mean are very weakly dependent on the temporal scale considered.

- Line 20: "This property is for the first time shown from a sea ice model simulation." This is not true. See Figure 7 in Hutter et al. (2018).
Yes, indeed Hutter et al. (2018) are showing some coupling as well. We in fact meant that this coupling is obtained from sea ice model simulation ran at a relatively coarse resolution. We changed the sentence as follows: “To our knowledge, this is the first time such coupling is obtained from a sea ice model simulation ran at such relatively coarse spatial resolution.”

Moreover, is this coupling really significant, as the "error bars" overlap for all temporal scales (for each moment respectively)? If you say that the structure functions for both RGPS and the model are equal in Figure 3, then I would also say they are equal here in Figure 5 for all time scales, and we therefore cannot conclude to a significant coupling.

We think that our new results are now showing more clearly that this coupling is significant, especially if we consider as the reviewer suggests that our “error bars” should not be considered as statistical error bars, but more as upper bound estimates of the goodness of the power fits.

Page 16:

- Lines 3-4: A few more words might be helpful here to understand this offset in the model curvature: Is MEB leading to damage in the ice cover everywhere and, therefore, evenly distributed events with no preferred regions of deformation?

The offset in the model curvature is not readily explained. Indeed we state in lines 5-6: “The reason for this discrepancy should be further explored but is out of scope of the present paper”. To underline this, and to make clearer the last statement of the paragraph we have removed the sentence “This may come from the fact that the highest deformation events are too evenly distributed over the Arctic region in the simulation compared to the observations”.

- Line 20: "This means that the proportion of extreme deformation events compared to lower ones is too small or that their values are too low in the simulation." The CCDF for the shear deformations in Figure 1 actually shows that the probability of having larger deformations is higher in the model than for RGPS, no? If you show the PDFs of shear and divergence, it would probably help to clarify this.

We now show the distributions of total deformation rates in Figure 2 and these show that the model actually under-estimate the largest values, which is consistent with the statement above that the reviewer is picking up: “This means that the proportion of extreme deformation events compared to lower ones is too small or that their values are too low in the simulation.”. The previous inconsistency spotted by the reviewer was in fact due to the fact that we were looking at the shear deformation rates distribution in Figure 1, whereas the scaling analysis is performed on the total deformation rates.

Page 17:

- Line 12-15: It is not clear from Figures 9 and 10 that alpha is decreasing for increasing L. Please add a plot of alpha vs L for q=1,2,3. Again, the question of whether this coupling is significant if all alpha(q=1,2) lie within the errorbars of alpha(q=1,2) arises.

We think our new results show this more clearly now. It is however true that the coupling is not as strong for the mean, compared to the second and third moments.

- Line 23: "reproduces correctly the distribution of sea ice deformation rates" Please show PDFs of shear and divergence to affirm this.

We now show the PDF of shear and divergence as well as the total deformation rates. We think it shows clearly enough that the model reproduces correctly the distribution of all invariant of sea ice deformation. We kept the sentence in the revised manuscript.

Page 18:

- Line 9: "a threshold mechanism" Is that the damage parametrization?

Yes, the fact of relating the large scale motion and deformation of the ice to a variable (here the damage) which evolves at the scale of one single element of the mesh in a highly non-linear way only and only when a criterion is fulfilled (i.e. sort of step function/discontinuous behavior).

- Line 14: You show that your model reproduces some of the observed scaling characteristics, but you have not shown that it does because your model includes the "ingredients" mentioned above. The configuration of the model (i.e. forcing, strength parameters, etc.) as well as the Lagrangian mesh instead

of an Eulerian grid also have the potential for generating/influencing these behaviors, and it is not clear yet to which model parametrization or configuration ingredients these behaviors are due.
Yes, we have not shown causality between including these ingredients in the MEB rheological framework and the reproduced scaling characteristics. So we rephrased this paragraph to avoid confusing the reader and to not let him/her think that we say these ingredients are the only way to generate such scaling properties..

That said, we would like to stress that a quite large community of geophysicists have been working on the deformation of the Earth crust over the last 30 years, on its scaling invariance properties and their physical origin. Today's understanding in this community is that the ingredients we list here, and that the MEB framework is using, are i) playing a role in the observed Earth crust dynamics, and ii) are all making physical sense as they can be related to the brittle nature of a geophysical solid, where earthquakes/icequakes allow the system to release energy when a critical internal stress state is reached, where the associated elastic waves allow to redistribute this energy within the system and over long distances, where this redistribution results in propagation of fractures associated with local "mechanical damage" of the material that is keeping the memory of this critical process in the system over long time scales, etc… .
So part of these scaling are of course probably inherited from e.g. the atmospheric/oceanic forcing (although this has never been proved so far), but the large range of spatial and temporal scales over which these scaling are observed for the sea ice (down to a couple of minutes according to Oikkonnen et al. 2017) let us think that, at least at the small time scales, some of the ingredients mentioned in our paper and present in the MEB rheology may play a role if not to be the main source.
What we therefore aim at with this study (and more generally with the development of neXtSIM) is to show that a rheological framework including meaningful (in the physical sense) ingredients in a context of geophysical solid dynamics can reproduce some non-trivial characteristics of the observed sea ice dynamics, and therefore that these may be reproduced in the model for reasons that are mechanically relevant.

- Line 16: "the spatial scaling [...] holds down to the nominal resolution of the mesh" and just after "It means that neXtSIM does not need to be run at higher spatial resolution in order to resolve the presence of linear kinematic features..." I am not sure that the first sentence justifies the second one... For example, Spreen et al. (2017) and Bouchat and Tremblay (2017), both show that VP models at 9 km and 10 km can also have a spatial scaling that "holds down" to 10 km (ie. the nominal resolution), but you would still need to run the model at higher resolution if you want to resolve finer structures in the sea-ice fields because the models are represented on Eulerian grids. In the case of your model, you might better resolve LKFs because you are using a Lagrangian mesh, which represents discontinuities more accurately, not necessarily because of the scaling of deformations.
We don't agree that the examples of Spreen et al. (2017) and Bouchat and Tremblay (2017) show that their scaling holds down to the nominal resolution. In the case of Spreen et al they show very weak scaling, with exponent of -0.08 +/- 0.05 for winter time shear for the 9 km resolution run. Their figure 8 can even be interpreted such that there is even less, or no scaling from 10 km to about 100 or 200 km and a stronger scaling after that. The results of Bouchat and Tremblay suffer much the same problem, with very weak scaling and a large scatter of the points. Bouchat and Tremblay don't provide error or uncertainty estimates for their exponents, but one could again argue that there's very weak scaling in the 10 to 100 km range and a slightly stronger scaling above that. The point here is that with such weak scaling as Spreen et al and Bouchat and Tremblay have it is very difficult to see if it flattens out towards the nominal scale or not. What is worse, one could argue that their scaling actually starts flattening out around 100 km, not when nearing 10 km. The only publications showing a reasonably strong scaling with a model

other than neXtSIM is that of Hutter et al (2018, 2019), but unfortunately the authors chose not to show how their model scales below the 10 km scale, even if the model is run at a much higher resolution.

The papers of Spreen et al and Brouchat and Tremblay aside, the point here is that a correct scaling down to the nominal resolution means that the correct statistical behaviour at a length scale of 10 km is reproduced when the model runs at a 10 km resolution. So in order to study the statistics at 10 km we don't need to run the model at e.g. 1 km. This is irrespective of the advection scheme as it relates strictly to the formation of features, not their preservation over longer time. Indeed, Girard et al. (2011) ran their model for only 72 hours with no advection. It was not accurate to say that since the spatial scaling holds down to the nominal resolution then we can better resolve linear kinematic features, however, and we have modified the text to make clearer the distinction between reproducing the statistical behaviour at the nominal scale (which we do) and reproducing related features (such as LKFs - this is not explored here).

- Line 20: Add reference to Hutter et al. (2018)? You seem to be indirectly referring to this study.
Done

- Lines 21-28: This should be moved to the results section.
We argue that these lines should stay where they are. Lines 28 of page 18 to 8 of page 19 are a discussion of the results. Lines 21-28 are indeed more descriptive, but they nonetheless bring together various elements of our results necessary for the discussion that follows. Lines 21-28 could as such be moved to the results section, but lines 18-8 would then not make any sense. To make this point clearer we have made lines 21 to 8 a paragraph of their own.

Page 20:
- Line 12-13: I disagree. See comment for Page 15, Line 20.
Fine, if you want. We removed "for the first time" in the sentence.

- Lines 21-25: This is a too strong statement that depends a lot on what the model is used for. There are applications where heterogeneity and intermittency of deformation are important (i.e. regional and short range forecasting of ice conditions) but there are also larger scale applications where other parameters are more relevant. Either remove this statement or give specific application areas where this is needed.
We slightly rephrased this statement to make it weaker. Here is how it reads now: "As the mono versus multi-fractal character of the scaling of deformation rates is the discriminating factor for the heterogeneity and intermittency of the deformation, we suggest that a multi-fractal scaling analysis could be considered as a meaningful validation step before further analyzing sea ice model outputs that could be influenced by sea ice dynamics."

Page 22:
- Line 10: In Dansereau et al. (2016), d=1 for undamaged and d=0 for completely damaged ice. Please indicate that you use the reverse definition.
Done

Page 23:
- Lines 11-13: g(H) is not defined.
- Equation (A13) and (A14): The prime variables have not been defined. in (A14), shouldn't it be sigma_1_prime and sigma_2_prime instead?

g(H) is superfluous here and we've removed it. The primed variables were supposed to indicate values after updating the damage, but this was neither clear nor correctly done. We've reformulated the text to make it clearer and remove the primed variables.

Page 24:

- As it seems that the implementation of this 3-thickness categories differs from Stern and Rothrock (1995), I would like to have a bit more details on how it is done/defined and how it is different from what was suggested Stern and Rothrock (1995).

Yes, there are some differences. Things in common:

- 3-layer model
- Thin ice is ridged first, before thick ice.

Differences:

- No additional open water source terms.
- Thick ice cannot return to being thin ice - our thin ice is perhaps more correctly described as young ice.
- We do not use the formulation of Gray & Morland (1994) to keep our total ice concentration <= 1, but only redistribute ice volume if concentration > 1.

-For example, please explain the addition of the divergence term in the evolution equations and a term for ridging for the thin ice category as well.

The divergence term is fairly standard, $\partial\phi/\partial t + \partial/\partial x(u\phi) + \partial/\partial y(v\phi) = D\phi/Dt + \phi(\partial u/\partial x + \partial v/\partial y)$; in our Lagrangian framework it represents the fact that ϕ should increase if the area of the mesh element decreases and vice versa.

The thin ice ridging term is covered by (A16) and implicitly by the mechanical redistribution procedure.

- Please also clarify if A and H are the total ice concentration and volume per unit area? i.e. A = A_thin + A_thick and H = H_thin + H_thick?

We clarified this in the appendix and table 2: total conc = A + A_t, total volume per unit area = H+H_t, total snow volume per unit area = h_s+h_{s,t}.

- Lines 16-18: "Thin ice thickness is considered to be uniformly distributed between hmin and hmax", do you mean linearly distributed? Why does that put a maximum bound on the total ice volume per area? Maybe here it should be "H_t_min = A_t * h_min" and "H_t_max = A_t * (h_min+h_max)/2" instead?

The reviewer is correct we originally meant to say it was linearly distributed. However we have since realised that there is a simpler, equivalent, formulation with a uniform distribution and added some text and some equations to elaborate on the thermodynamic transfer of ice from thin to thick ice category.

Page 26:

- Equation (A23): Please explain why you introduce this delta_A variable and what is the purpose of the "aspect ratio parameter" zeta, and what a value of 10 implies.

Beginning with the other variables, we treat ice and snow volume in a conservative manner. ΔA will be used to increase the concentration of thick ice A in a similar way, but non-conservatively: by increasing A by an amount less than $A_t - A'_t$ (through the parameter ζ), we are preferring to increase the absolute thickness of the thick ice more than it would be if area was conserved - the factor of extra increase is approximately $1 + (1 - \frac{1}{\zeta})\frac{A_t - A'_t}{A + A_t - A'_t}$.

- Line 8: Shouldn't more ridging also affect the value of H^{n+1}?

No, the ice volume should not change (only the absolute thickness would increase).

Figure 1: - I would switch for PDFs and also add a panel with divergence distributions.
We prefer to keep showing the Probability of exceedance (or CCDF) as it makes the comparison of the tails of the distribution clearer, as in Marsan et al., 2004. Note that we have now switched figure 1 and figure 2 (the fields of total deformation rates first in figure 1 and the corresponding distributions in figure 2), following the suggestion of Reviewer 2.

Figure 2: - Please also show the divergence fields.
We are now showing the **total** deformation rates for the model and the RGPS, instead of the **shear** deformation rates. For consistency reasons with the rest of the paper presenting statistics and analysis for the total deformation rates, as well as for simplicity, we chose to keep considering and showing the total deformation rates here, as throughout the paper.
Both the captions and the text are now corrected accordingly.

Figure 3: - The left panel y-axis shows the scaling for eps_tot, however, in the caption it is written that you are showing the scaling and the structure for the shear deformation rate... Please show the scaling and the structure function for eps_tot instead.
Thanks for spotting this. It was in fact a typo in the caption. What we showed in this figure was already for the total deformation rates.

Figures 5 & 6: - You could group these two figures for ease of comparison between the model and RGPS observations.
We tried this but it was at the cost of lack of visibility. We therefore chose to separate these two figures.

Figure 8:
- Caption, Line 3: Normalized moments have not been defined in the text.
Indeed. We are not showing the normalized moments, but the moments. It was just a typo in the caption. This is now corrected.

Figures 9 & 10: - You could group these two figures for ease of comparison between the model and RGPS observations.
Same answer as for the figure 5 & 6 (see above)

== TECHNICAL CORRECTIONS

Page 2:
- Line 7: - Delete "for the first time"
Done.

- Line 16-17: "e.g.," should come before enumerating the references.
Corrected

Page 3:
- Line 2: Replace "kinematic linear features" with "Linear Kinematic Features"
Done

- Line 6: Delete "levels of"
Done

- Line 7: "Coon et al., 2007" should be in parenthesis
Corrected

- Line 9: Add a coma after (Kwok, 2001), ie: "(Kwok, 2001), and permanent..."
Done

- Line 13: Delete "such as" and replace with "... of the deformation rate invariants (i.e. shear and divergence) and of the total deformation rates, which..."
Done

- Line 26: I think there is a part missing in this sentence. Maybe add "applied to observed deformation fields derived from satellite imagery" before "(e.g. Lindsay..." or something like that?
We changed the sentence with the following one: "Estimated using coarse-graining analysis (e.g. Lindsay 2003, Marsan et al, 2004, Bouillon and Rampal 2015b) or dispersion analysis of pair of buoys (Rampal et al. 2008), the *mean* sea ice deformation rate has been shown to vary with the spatial scale, *L*, and temporal scale of observation, *T*,..."

- Line 27: Replace "or pair of buoys dispersion analysis" with "or by dispersion analysis of pair of buoys"
Done (See previous response)

Page 4:
- Line 4: Add "The scaling exponents..."
Done

- Line 9-10: Rewrite "...to a homogeneous deformation, and alpha=1 to a single, temporally isolated deformation event."
We instead rewrote the sentence as follows: "In the space domain, beta = 0 characterizes the homogeneous deformation of an elastic solid or viscous fluid, i.e., a deformation that does not depend on the spatial scale, while beta = 2, i.e. the topological dimension for the 2D-like sea ice cover, corresponds to a single "point" concentrating all of the deformation in an otherwise undeformed material (Rampal et al., 2008)"

- Line 16: "approximated" –> "modeled as"
We changed with "assumed" instead
and "relevant for Arctic system simulations"?
Done

- Line 18: Delete "out of the Gaussian basin of attraction" (see specific comment for Page 3, Line 14).
Done (see which changes we applied in our response to the corresponding "specific comment" of the reviewer listed above)

- Line 25: Replace "beta" with "the scaling exponents beta and alpha"
Done

- Line 27: Delete "indeed"
Done

Page 5:

- Line 4: "... linear structure function, i.e., no curvature, ..." replace with "... linear structure function, i.e. no curvature or equivalently a=0 or b=0, ..."

Done

- Line 6: Replace "For both coefficients..." with "In the case where both coefficients..."

Done

- Line 7: Add "therefore" between "distribution" and "increase", i.e "...of the distribution therefore increase..."

Done

- Line 12-13: Replace "have shown" with "show"

Done

Page 6:

- Line 7: "... in the deformation and related characteristics of sea ice" Not clear. Please reformulate.

Sorry, we could not find which sentence the reviewer is referring to.

- Lines 21-23: This sentence is not clear. Please reformulate.

We reformulated the sentence as follows: "In the absence of a characteristic space/time scale for the sea ice deformation and with the knowledge that the scaling invariance holds beyond the space/time nominal resolution of typical regional and global model's grids, perhaps the best a continuum framework for sea ice modelling can do is to correctly reproduce the statistics of deformation from the smallest available (or *nominal*) scales that can be resolved, i.e. at the resolution of the grid in space and for the model time step in time, to the largest scales as possible, i.e. the size of the Arctic basin and the time scale of a season."

Page 7:

- Line 1: Replace "the first part of the paper" with "Section 1 and section 2 of the paper" and delete "(section 2)" in line 2.

Done

- Line 2: Replace "The second part" with "Section 3" and remove "(Section 3)" in line 4.

Done

- Line 10: Replace "(Amitrano et al., 1999)" with "Amitrano et al. (1999)"

Done

- Line 17: Rewrite "In particular, it was shown that the simulated deformation rates..."

Done

- Line 18: Add "... in space only".

We disagree with the reviewer here. A temporal scaling was also performed in this paper (see figure 12) showing good agreement of the temporal scaling exponent.

Page 8:

- Line 8: Replace "entering" with "of"

Done

- Line 9: Replace "Appendix" with "appendices"

Done

- Line 15: Replace "length of the vertices" with "distance between the vertices"

We rather changed for: "i.e. mean length of the edges of the triangular elements"

- Line 24: Remove "the applied", and all of "the" in front of the quantities enumerated
Done

Page 10:
- Line 1: Replace "displacement" with "ice motion"
Done

Page 12:
- Line 18: Add "... 30 degrees or less".
Done
- Line 19: Replace "as the model is" with "contrary to the model"
Done
- Line 23: "affect" should be "affects"
Done
- Line 23: "sub- sampling" Do you mean interpolation?
Yes. We slightly change the text with the following sentence: "The positions simulated by the model are taken to match the sub-sampled RGPS time series obtained as described above."

Page 13:
- Line 6: "we therefore chose to..." You do not chose, you can't go below 3 days given that this is the smallest time scale you have for your dataset.
Yes. We changed the sentence with: "we therefore restrict ourselves to time scales equal or greater than 3 days".
- Line 7: Remove "on the whole"
Done
- Line 17: Maybe relabel Figure 2 to Figure 1 since you are referring to it first?
Yes. Done
- Line 17: Replace "3-days" with "3-day"
Done

Page 14:
- Line 2: Add "... spatial scaling analysis for a T= 3 days temporal scale..."
Done.
- Line 12: Add "our choice of mechanical parameters values (e.g. Bouchat and Tremblay, 2017)"
Reference added.
- Line 22: Replace "a quadratic fit to the data (in the least squared sense)" with "a least-square quadratic fit to the data"
Done
- Line 25: Add "... simulated deformation fields is consistent..."
Done
- Line 27: Add "... the value of the spatial scaling exponent beta..."
Done
- Line 27: "for the mean obtained for the successive and contiguous snapshots throughout the winter" This is not clear. The mean = mean deformations , i.e. q=1? Please rewrite.
Yes, you understood correctly. The sentence has been rewritten as follows: "Using successive and contiguous *snapshots* throughout the winter, a time-series of the value of the spatial scaling exponent beta obtained for the mean deformation rates is calculated, and shown on Figure…".

Page 15:

- Line 1: Replace "the scaling exponent varies" with "the spatial scaling exponent varies"

Done

- Line 4: Replace "for the mean" with "for the mean deformation rates (i.e. q=1)"

Done

- Line 5: Add "... which is also the value..."

Done

- Line 10: Add "...that the observed and simulated curvature values..."

Done

- Line 21: "The origin of this coupling has been previously proposed to be linked to the complex correlation patterns related to chain triggering of ice-quakes." Please add reference for this.

We added the Marsan and Weiss (2010) reference at this exact location in the text:

Marsan, D., and J. Weiss (2010), Space/time coupling in brittle deformation at geophysical scales, *Earth Planet. Sci. Lett.*, *296*(3-4), 353–359, doi:10.1016/j.epsl.2010.05.019.

- Line 24: Add "... the multi-fractal character of the spatial scaling (i.e. the curvature of beta(q)) for both RGPS and the model when..."

Done

Page 16:

- Line 9: Remove "robust" and "very similar" since you then discuss how it differs for the q=3.

We instead changed the text to make clear the scaling are very similar for the two first moments of the distributions:

"We see a robust and very similar power-law scaling for the two first moments ($q=1,2$) for both the model and observations…"

And we kept the adjective "robust" in the sentence because the power law fits are indeed very "robust" in a statistical sense.

- Line 16: Replace "in this recent study" with "by Oikkonen et al. (2017)"

Done

- Lines 17-18: Remove "(gray, dark and cyan top curves in the left panel of Fig. 8)"

Done

Page 17:

- Line 4: "virtually perfect" please change to a more sober wording. For example, the values for q=2 are not "perfectly" matching.

We changed the wording as suggested, with "remarkably good".

- Line 4: Rewrite "The curvature of the quadratic functions alpha(q) are 0.11 for..."

Done

- Line 7: "This seems to argue that..." Weird wording. Please rephrase.

Yes indeed. We changed the beginning of this sentence with "This suggests that…"

Page 19:

- Line 18: Replace "." after "distribution" by a coma, and change "A proper..." for "a proper..."

Done

- Line 24: Replace "concurrent" with "parallel"?

Done

Page 20:

- Line 2: Add "from RGPS observations..."

Done

- Line 13: Remove "for the first time" and "by a model"

We followed reviewer's suggestion here, and removed "for the first time".

However, we guess that the reviewer is making reference to the paper by Hutter et al. 2018, which is indeed showing a coupling of the scaling in space and time. But we would like to stress that this coupling is "very weak" as the authors acknowledge themselves in this paper, which may be the consequence of the very weak scaling reproduced by their model in the temporal domain.

Page 21:

- Line 16: Replace "thick ice thickness" with "thick-ice thickness"

Done

Page 22:

- Line 15: Why not write -c* with c* =20 as done in Hibler (1979)? And you could put (A6) back in (A5) to save space.

We followed the reviewer suggestion here, changing c* to -c* in equation A6. We however prefer to keep A5 and A6 separated.

Figure 1:

- Please add legend in the figure for ease of comparison - Add the fit exponents on the figure or in the caption, for both model and RGPS.

Done

- Caption, Line 2: Add "...and the RGPS observations"

Done

Figure 2:

- There seems to be a plotting issue since some of the triangles are touching the land boundaries (e.g. on the Alaskan coast), but you mention in the manuscript that you filter out trajectories that are 100 km or closer to land. Please correct.

We indeed filter out the trajectories that are 100 km or closer to the land, but only for the scaling analyses we performed. However, this figure is mostly for illustration, and therefore we show the whole dataset available from the RGPS (except if the triangles are too deformed/large), and masked the model data to match spatially those observations. We added the following sentence in the caption: "The model field is masked to match spatially with the RGPS data coverage."

- Please add "RGPS" and "Model" on top of the panels.

Done

- Caption: Add that the green lines are the model's open boundaries.

There removed the green lines from the figure.

Figure 3:

- Caption, Line 2: Replace "than in the RGPS dataset" with "and RGPS dataset"

Done

- Caption, Line 7, "local scaling exponents" Not clear. Use a similar wording as in Bouillon and Rampal (2015).

We changed the caption accordingly using the same wording as in Bouillon et al. 2015, as suggested.

- Caption: Use the same wording for the caption as for Figure 8 (with the suggested corrections).
Done

Figure 4:
- Please add legend in the figure for ease of comparison
Done
- Caption, Line 1: Replace "power scaling exponents" with "spatial scaling exponents for the average total deformation (i.e. q=1)"
Done
- Caption: Add something like "calculated for individual snapshots, i.e. at a temporal scale of T = 3 days"
Done

Figure 5:
- Please use same y-axis for left panels in Figures 3,5,6,8,9,10,12 for ease of comparison. - Please use same y-axis for right panels in Figures 3,5,6,12
We thought about doing as the reviewer suggested and tried it when preparing the manuscript but although it eases the comparison between figures, it also significantly degrade the visibility of each individual figure. We thus prefer to keep the axes as they are.

Figure 7:
- Caption: Add "..for the RGPS observations..."
Done.

Figure 8:
- Caption, Line 3: Replace "distributions of the deformation rate" with "distributions of the total deformation rate"
Done
- Caption, Line 4: Switch "for the observations" with "for the model" later in the sentence, and indicate that the values for T=3hrs to 1 day are taken from Oikkonen et al. (2017) in the caption as well.
Done
- Caption, Line 6: Remove "The dashed lines are extrapolation for the smallest scales" There are no dashed lines.
Done
- Caption, Line 7: Replace "observation" with "RGPS observations"
Done

Figure 10:
- Caption: Add "..for the RGPS observations..."
Done

==

REFERENCES

Bouchat, A., and B. Tremblay (2017), Using sea-ice deformation fields to constrain the mechanical strength parameters of geophysical sea ice, J. Geophys. Res. Oceans, 122, doi:10.1002/2017JC013020.
Clauset, A, Shalizi, CR and Newman, MEJ. 2009. Powerlaw distributions in empirical data. SIAM Rev 51(4): 661–703, https://doi.org/10.1137/070710111
Hutter, N., Zampieri, L., and Losch, M.: Leads and ridges in Arctic sea ice from RGPS data and a new tracking algorithm, The Cryosphere, 13, 627-645, https://doi.org/10.5194/tc-13-627-2019, 2019

Hutter, N., Losch, M., & Menemenlis, D. (2018). Scaling properties of arctic sea
ice deformation in a highâĂŘresolution viscousâĂŘplastic sea ice model and in satellite observations. Journal of Geophysical Research: Oceans, 123, 672–687. https://doi.org/10.1002/2017JC013119
Lindsay, R.W. and H.L. Stern, 2003: The RADARSAT Geophysical Pro- cessor System: Quality of Sea Ice Trajectory and Deformation Estimates. J. Atmos. Oceanic Technol., 20, 1333–1347, https://doi.org/10.1175/1520- 0426(2003)020<1333:TRGPSQ>2.0.CO;2
Sornette, D.: Power Law Distributions, pp. 93–121, Springer Berlin Heidelberg, Berlin, Heidelberg, doi:10.1007/3-540-33182-4_4, https://doi.org/10.1007/3-540-33182-4_4, 2006.
Spreen, G., Kwok, R., Menemenlis, D., and Nguyen, A. T.: Sea-ice deformation in a coupled ocean–sea-ice model and in satellite remote sensing data, The Cryosphere, 11, 1553-1573, https://doi.org/10.5194/tc-11-1553-2017, 2017.
Weiss, J. Surveys in Geophysics (2003) 24: 185. https://doi.org/10.1023/A:1023293117309

Anonymous Referee #2

Received and published: 6 March 2019

General comments

This paper aims at validating the basic behavior of deformation rate in the neXtSIM sea ice model which is based on the Maxwell-Elasto-Brittle rheology, focusing on the scaling properties in space and time. The model domain was the whole Arctic Ocean and the coarse graining method was used for scaling analysis with the drifters' data in the model. For validation data, the Lagrangian displacement data produced from RADARSAT Geophysical Processor System (RGPS) were used. Through scaling analysis, it was shown that the multi-fractal properties can be reproduced for the first time in the numerical sea ice model. Besides, the statistical properties of the first, second, and third moments of deformation rates at temporal scales of 3 days to 96 days and spatial scales of 7.5 km to 700 km were shown to be mostly consistent with the observations. In conclusion, since the fundamental properties were validated, they suggest that the neXtSIM model could be used as a proper tool to further study the physical meaning of the processes related to deformation. Considering that it is still a big challenge to reproduce the rapid thinning trend of ice thickness distribution in the Arctic Ocean in the numerical sea ice model and the need to improve the deformation processes in the model has been recognized for a long time, the topic of this paper is timely, and the results of this paper will provide insightful implications. Overall, I feel that this paper is an elaborate and nice work, and this approach is indispensable to improve our understanding of the dynamic behavior of sea ice. Therefore, I believe this work will contribute to the development of sea ice dynamics, especially for the parameterization of the model, related to the deformation.

First of all, we would like to thank the reviewer for this very positive evaluation and minor, though important/well spotted comments/points. Please find below (in red) our answers.

My comments, which might come from the lack of my knowledge about mathematics, are limited to minor points as follows:

1) Regarding the description of exponents, α and β (Eqs. 4 and 5), could you please explain more about why these exponents can be expressed as a quadratic equation of the moment parameter (q)? To be honest, I could not follow the subsequent paragraph (P5L4-11) completely. To my understanding, multi-fractal means the geometric properties that contain various dimensions of fractals. If this is correct, why can the curvature of the exponents as a function of q be an indicator of multi-fractal which discriminates from mono-fractal? In my mind, if I could accept this concept, the manuscript would have become much more understandable to me.

In order to make the description of the concepts of mono versus multi-fractal scaling hopefully more easy to follow by the reviewer or any other reader of our article, we slightly reformulated the paragraph picked up by the reviewer as follows:

"In the case of a linear structure function, i.e., no curvature, the amount of localization of large and small deformation events is the same and the scaling is said to be *mono-fractal*.

When both coefficients *a* and *b* or *c* and *d* are positive the structure functions are quadratic and convex, meaning that the higher order moments of the distribution increase much faster than the lower order moments with decreasing scale of observation. In other words, *large* deformation events are *more localized* in time and space than smaller events, corresponding to the definition of a *multi-fractal* scaling. Note that in the literature multifractality is also called *intermittency* when present in the time dimension and *heterogeneity* when present in the spatial dimension. The largest the curvature of the structure function, the stronger the *degree of multifractality* of the scaling."

2) Regarding the methodology of analysis, it is stated that you used the coarse-graining approach (P10L12). Is this the method described after P11L12? If so, it might make it readable when you insert "(shown later)" at the end of the sentence (P10L12).
No, we apologize for this confusion but we made a mistake saying that we used a coarse-graining approach. We instead used a buoy dispersion method, using triplets. This has been corrected in the revised version of our manuscript.

Besides, regarding the statement, "Only the trajectories that are common to both the simulation and RGPS dataset are considered in the calculation of the deformation and their statistics" (P10L22-24), I am a bit concerned whether this approach might affect the results by setting a bias in the calculation. I mean the data consistent with observations might have preferentially selected. If you can add some description about how much fraction of data were discarded by this method and show that this selection did not affect the result significantly, it would be appreciated.
Very few portions of the trajectories are discarded by this method, i.e. representing only about 2% of the total dataset. This selection does not affect in any case the results obtained, but was made in order to make our comparison between model and observations as much consistent and clean as possible. We slightly changed the sentence picked up by the reviewer by the following one:
"Only the trajectories spanning the same time periods in both the simulation and RGPS dataset are considered in the calculation of the deformation and their statistics. This selection lead to discarding about 1% only of the total trajectory dataset, and does not affect the results of the analyses presented in this paper. However we apply this selection in order to make our comparison between model and observations as much consistent and clean as possible."

3) Regarding the interpretation of the scaling analysis (Fig.5&6), it is stated that "We find that the estimated spatial scaling exponent, β, decreases with increasing T (Figure 5 and 6, left panels)" (P15L15-16). To my understanding, β corresponds to the slopes of the graphs. As far as looking at the left panels, however, the slopes appear not to be significantly different for all the values of T (3 days to 96 days) at least for q = 1. When looking at right panels, there certainly be a decreasing trend with the increase of T for q = 2 and 3. Thus, unless there is a physical meaning in the decreasing trend of β with the increase of T, it might be one idea to focus on the decrease of the multifractality of the spatial scaling with the increase of T. The similar discussion may apply for the last paragraph in section 4.2 (P17L11-21).
You are right. The statement of this sentence should have been more accurate. We changed it to the following one in the revised manuscript:
"We find that the estimated spatial scaling exponent, β, decreases with increasing T, although this behavior is only obvious for the moments of order 2 and 3"
We also lowered tone the next statement we made in the original manuscript by reformulating the text as follows:
"This seems to correspond to the existence of space-time coupling of the scaling properties of sea ice deformation. This property was originally suggested in Rampal(2008) from the result of their scaling analysis of buoy pairs dispersion, and was further explained in Marsan(2010) as being a possible characteristic of brittle deformation at geophysical scales."
Besides the fact that a dedicated and more throughout analysis is deserved to conclude on this point, we decided to keep mentioning this result in the revised manuscript because (i) we do not know of any other sea ice modeling study showing such result, (ii) we know this property has been already observed and documented in previous studies focusing on Earth crust dynamics, which are likely to be an analogue of sea ice dynamics. The text in the revised manuscript is now as follows:

“To our knowledge, this is the first time such result is shown from a sea ice model simulation. The origin of this coupling has been previously proposed to be linked to the complex correlation patterns related to chain triggering of ice-quakes. Further study is however needed to explore this hypothesis, which is out of the scope of this paper.

Besides, the additional description about the physical implications of the decrease of the multi-fractality would be appreciated if it is possible.
We guess that the reviewer is referring here to the fact that the model does not reproduce the observed heterogeneity of the sea ice deformation at large time scale. As we mentioned in the manuscript, this means that the largest deformation events are too evenly distributed over the Arctic basin in the model compared to the observations, and therefore the spatial “localization” is lost when considering statistics over large temporal window. One could blame the atmospheric forcing to not represent properly the extremes, or at least the presence and trajectories of polar lows in the Arctic region. Or one could also relate this to the healing mechanism (applied on the damage variable in the model) we currently use in the MEB rheology and that may be inadequately parameterized or tuned.
In order to stress these point more clearly, we added the following text in the revised manuscript:
“This could either be attributed to inaccurate position or lacking of extreme events in the atmospheric forcing, or to an inadequate or insufficiently tuned parameterization of the damage healing in the model.”

Specific comments:
*(P2L19-20) “Rothrock and Thorndike, 1984; Matsushita, 1985” & “Rothrock and Thorndike, 1980” are missing in the reference lists.
Thank you for spotting this. There are now in the reference list
*(P3L7-8) “Coon et al. (2007)” should be “(Coon et al., 2007)”
Corrected.
*(P12L18) Is there any meaning in the selection of 30 degrees?
The value itself has been taken arbitrarily so that every triangle selected for the analysis is not too much “distorted”, in the sense that when calculating the deformation of these triangles at a given spatial scale L=sqrt(surface area of the triangle), the homogeneity assumption we make about the deformation is actually making sense.
*(Figure 1&2) Considering the order of appearance in the manuscript, it would be preferable to exchange Figure 1 and 2.
You are right. We changed the order of these two figures.
*(P15L4) I think “0.2” should be “-0.2”.
Well spotted. This has been corrected.
*(Figure 8) It is stated that “The dashed lines are extrapolation for the smallest scales” in the caption. However, I could not see the dashed lines. Besides, “L=7.5km”, which appears in the upper left corner of the figure, is misleading. Please take it if not necessary.
Absolutely correct. This sentence mentioning the dash lines has been removed from the caption.

That is all. Faithfully yours.