Reply to Anonymous Referee # 1 comments for the manuscript: Impact of frontal ablation on the ice thickness estimation of marine-terminating glaciers in Alaska by Beatriz Recinos et al.

We would like to thank the reviewer for taking the time to read our manuscript and give us insightful comments on the methodology, as these comments led us to formulate new experiments, to improve the code and the manuscript. We agree that this important point was missing from our manuscript and will provide a new section better explaining our method and our objectives.

Here we present a point-by-point response (given in normal font) to the related issues made by the reviewer (given in *italics*). Additionally to this response, we have prepared a **Python Notebook**, where we explain the new experiments and elaborate on the iterative procedure implemented in the Open Global Glacier Model (OGGM) to find a frontal ablation flux. We think that the notebook addresses best the issues raised by the reviewer: our answer is self-contained but is based on these simulations. Interested readers can try the experiments themselves by visiting **the interactive version of the notebook**.

RC: Convergence of iterative procedure.
Looking at your iterative calibration procedure from the perspective of an optimisation, I wonder what target quantity is minimised. In other words, what is the reason for this procedure to converge or reach the stopping criterion. Starting from an initial thickness guess, you infer $F_{calving}$ *and update the temperature sensitivity* μ^* *in the surface mass balance equation (5). Then, you re-run the reconstruction and get an updated frontal thickness value. From all involved equations, I cannot see a good reason why the following updates should produce values with a gradually reducing relative differences. A reason for non-divergence is that the thickness update involves a polynomial relation with an exponent smaller than one. Yet even if convergence is reached, I wonder about the physical meaning of this specific solution. Please do not misunderstand me here, but I really think that this is an important point with serious implications for the expedience of your approach. I unfortunately do not have a good suggestion for a useful target quantity or another potential quick fix. To convince the reader about the functionality of this calibration procedure, I think you have to expand the article by another section, which will elaborate on the stability and the convergence behaviour for a few test cases. I am particularly interested in figures showing the iterative changes in the frontal thickness. Is it monotonous or are the over-shootings. The latter seems unlikely considering the underlying equations. An interesting test would be to check what happens if you started from a too large thickness value (for a well-studied glacier). I would expect an even higher calving flux and thus a further increase in ice thickness. I ultimately miss a relation which counter-balances a steady increase during the iterations. In general, you should assure that the final thickness profile does not depend on the initial thickness guess. Another informative analysis would be to see what happens if the stopping criterion is ignored and you continue the iterations for 100 or even 1000 steps. Do the relative differences in the frontal thickness decrease further? This would be a requirement for the introduction of the suggested stopping criterion. To put my whole concern in simple words: by introducing a calving flux in the mass budget, you have to reduce the amount of necessary melt (for a balanced situation) This reduction further increase the necessary calving flux in each iteration. To break this run-away cycle, you need another physically motivated relation that penalises either low melt values, high calving fluxes or high frontal thicknesses. Such a counter-balance effect might already be at work by the underlying functional dependencies but without a clear physical motivation.*

AR: To answer this issue we have followed the reviewer's suggestion and are preparing a new section in our revised manuscript with experiments that illustrate the converges of the parameterisation. This new section will be based on the notebook linked above, where we experiment with the LeConte Glacier (RGI60-01.03622). Note that it is possible to test any another marine-terminating glacier as well (see the **interactive version** of the notebook).

We use a simple calving law borrowed from Oerlemans and Nick (2005), which relates frontal ablation $F_{calving}$ to the frontal ice thickness H_f, the water depth d and the terminus width w:

$$F_{calving} = kH_f dw$$

with $F_{calving}$ in km^3 yr^{-1}, k a calibration parameter (default 2.4 yr^{-1}) and d the water depth calculated as:

$$d = H_f - E_t$$

where E_t is the free board.

As explained in our manuscript, ice conservation methods applied to tidewater glaciers *must* take into account this mass-flux at the terminus, otherwise the ice thickness is underestimated. In fact, the default OGGM ice thickness inversion procedure assumes an ice flux of zero at the terminus.

In Fig. 1, we examine how this frontal ablation flux would change if we increase the terminus ice thickness, while keeping the free board fixed (the free board is the only variable we know "with certainty", from the DEM surface elevation at the terminus).

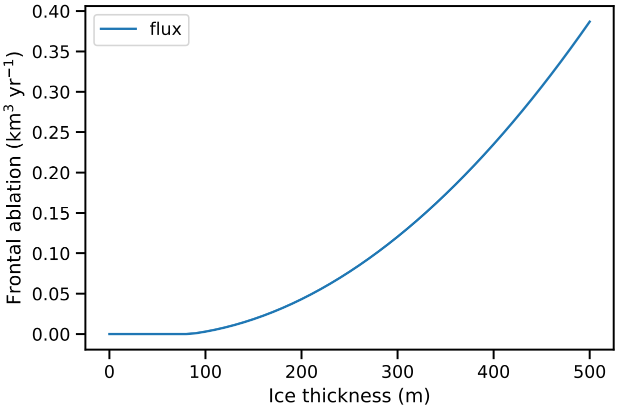

Figure 1: Frontal ablation flux computed by the calving law as a function of the terminus ice thickness.

The flux remains equal to zero as long as the frontal ice thickness is not thick enough to reach water, after which the water depth is positive and calving occurs. The calving flux varies with H_f as a polynomial of degree 2.

We are unaware of the *real* value for the calving flux at this glacier. But from here, we can make some very coarse assumptions:

- the Oerlemans and Nick calving law is perfectly exact
- the tuning parameter k is known
- our glacier is in equilibrium (mass-conservation inversion in OGGM)

- ice deformation at the glacier terminus follows Glen's flow law

Under these assumptions, we set up a new experiment where we compute a frontal ablation flux (from the calving law) for a range of prescribed frontal ice thickness (see Fig. 2), then give this flux back to the OGGM inversion model, which will use this flux to compute the frontal ice thickness according to the physics of ice flow (see the **OGGM documentation** or our manuscript for more information). Fig. 2 shows that there is a unique value for the frontal thickness (H_f) that complies with **both** the calving law and the ice thickness inversion model of OGGM.

We already know that the calving law relates the ice thickness to the flux with a root of degree two (blue curve of Fig. 2). But for the the orange curve in Fig. 2, it is Glen's flow law, which relates the ice thickness to the flux with a 5th degree root (assuming $n = 3$). Showing the reason why there is one (and only one) non-zero solution to the problem of finding a calving flux; a flux which is compatible with both the calving law and the physics of ice deformation (under our simplified framework).

Note that changing Glen's deformation parameter A or adding sliding does not change the problem: we will still solve a polynomial degree 5 in OGGM, **with a new term in degree 3** (see green curve in Fig. 2).

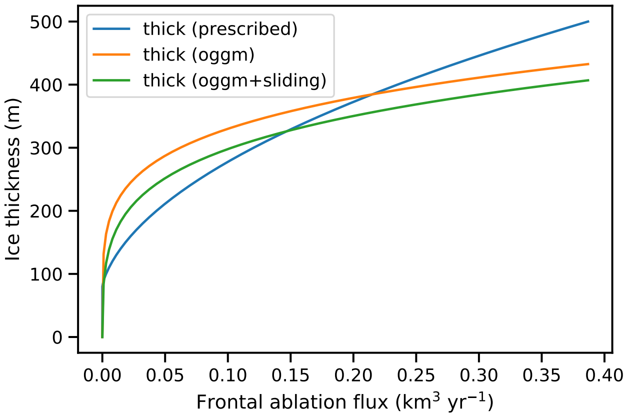

Figure 2: Ice thickness per frontal ablation flux calculated at each iteration. The orange line is the flux calculated by prescribing a thickness and using the calving law. The blue line is the flux calculated by OGGM using the iterative procedure. The green line is the flux calculated by OGGM using the iterative procedure and adding a sliding velocity.

There are several ways to find this "optimal" calving flux (or optimum frontal ice thickness), where mass-conservation inversion and the calving law are compatible. In OGGM, we implement an iterative procedure converging to this value in a few iterations (see Fig. 3).

The procedure starts with an initial water depth of 1 m (arbitrary choice that might change in the next version of our manuscript), then iteratively feeds the calving flux back to the mass-conservation inversion function of OGGM, which adapts the mass-balance model to cope with a non-zero ice flux at the front. In order to do that, the temperature sensitivity of the glacier μ^* has to be reduced (per construction, the original μ^* is defined such that the flux at the front is zero). Convergence is reached when the OGGM flux equals the calving law flux within 0.1%. Thanks to the uniqueness of the solution, the method always converges regardless of the starting water depth (Fig. 4).

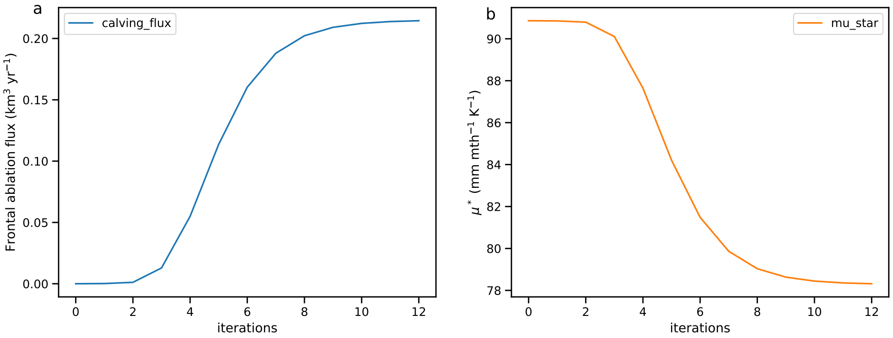

Figure 3: Right: Frontal ablation flux at each iteration. b: Temperature sensitivity μ^* of the glacier at each iteration.

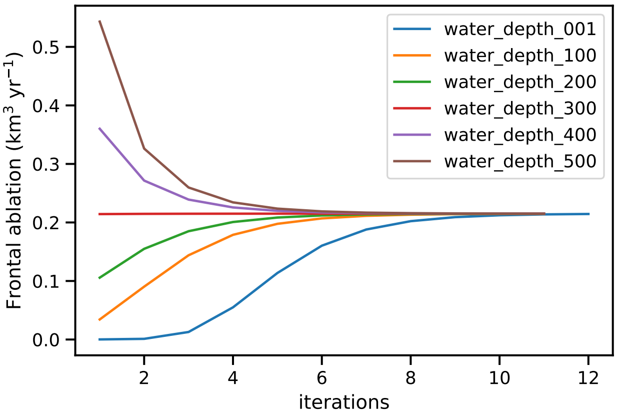

Figure 4: Frontal ablation flux calculated from different starting points. The different colors represent different water depths at the beginning of the iteration procedure.

However, for some glaciers the calving flux given by the calving law is larger than a flux that can be explained by climate alone, i.e. even without melt, the computed flux is larger than the total accumulation over our glacier. This can happen for several reasons:

- precipitation is underestimated
- the flux is overestimated because of uncertainties in k and the terminus geometry
- the equilibrium assumption is not valid

During these conditions our iterative search can "overshoot". Fig. 5 simulates this case where we set an unrealistically large calving parameter (k equal to 10 yr^{-1} in this experiment). If this happens during the iteration, OGGM is going to set μ^* to zero and compute the corresponding flux (the maximal physically possible value).

RC: Manuscript structure *The structure of the manuscript is not very clear and only after reading all of the results, I finally got my head around the overall strategy to set up the method. A major drawback is that the calibration of the proportionality factor k in the calving relation with respect to available regional estimates of the frontal ablation is presented rather late in the text. I think that a calibration section will be very useful at the end of the methodology (P9).*

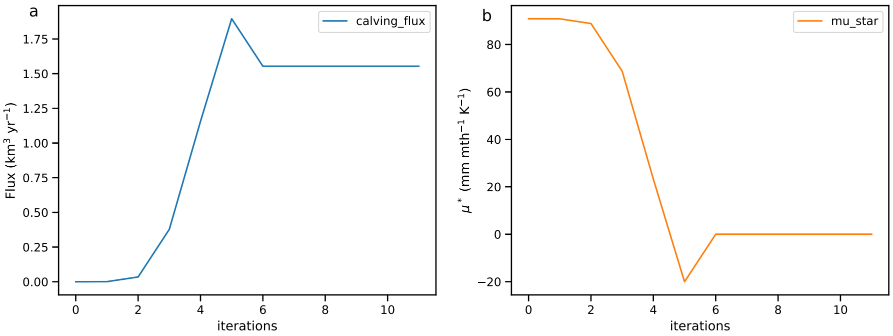

Figure 5: a: Frontal ablation flux per iteration. b: Temperature sensitivity μ^* per iteration. With a k value of 10 yr^{-1}.

This section can also serve to explain that you will use two variants of the model: one with sliding and another without.

AR: We would like to clarify that the calibration of the proportionality factor k with respect to available regional estimates of frontal ablation does not take part in the iteration procedure described in section 3.4. During the iteration procedure and in most of the study experiments, we keep k constant with a value of 2.4 yr^{-1}. We only modify k to match regional estimates during the sensitivity experiments (see section 4.4 and 4.5) with the intention of showing that regardless of the parameter configuration chosen, we can still estimate the relative part of the ice volume that is missed when ignoring frontal ablation in the inversion. We consider that this part of the text does not belong on the methods section, since our optimum goal is to make OGGM independent of frontal ablation observations. Hopefully, the clarification of the iterative procedure above helps clarify this point, too.

RC: Specific comments.
Suggestions for the iterative calibration procedure.
A. Initial thickness
Concerning the first two steps in your iterative process (P8L19-22), you determine an initial guess for the calving flux, by assuming a frontal ice thickness which is 1m higher than the surface elevation. I think that it will be beneficial to use the flotation criterion here, making an assumption on the ocean water density. This criterion is simple to implement, it will give a larger first guess and it will therefore speed up your convergence.
B. A-priori limits
The flotation criterion for the frontal ice thickness also provides a lower bound Hminto the "real" frontal thickness value. The reason is that most tidewater glacier will likely be thicker and firmly grounded. An upper bound for the frontal thickness (Hmax) is given by integrating the accumulation field. This will provide the maximum ice flux possible along the glacier. Alternatively, you could integrate the SMB above the ELA. This will give smaller maximum flux values (these will however dependent on μ^*). The maximum flux can then be translated into an upper bound for the frontal thickness value (Hmax) via Eqs. (8-10). A conflict between the upper and lower bound (Hmin > Hmax), will indicate inconsistencies in the climatology and thereby give useful information.
C. Stopping criterion
The stopping criterion is chosen to be an absolute flux value. In this way, the stopping criterion is easier to be reached for small glaciers with overall lower flux values. I do not think that this is a desirable behaviour and it was not communicated as a deliberate decision. I would therefore

suggest that you define the threshold as a fraction of the annually received precipitation volume. If this should not be feasible, you could use a constant values that scales either with glacier area or the terminus width.

AR: For the issues addressed here, please refer to the first section of the reply: convergence of iterative procedure.

In addition to the changes mentioned above, modifications to the code were needed in order to correct a bug found in the iterative procedure which now makes the result of the parameterisation compatible with mass conservation. The changes implemented can be found **here**.

We will add an explanation of all the modifications to the code in our revised manuscript.

Our main result of the ice volume underestimation when ignoring frontal ablation is still the same after this correction.

We thank you for your comments again and for helping us to improve our method. We will do our best to clarify those points that caused confusion in our manuscript.

Reference:

Oerlemans, J. and Nick, F.: A minimal model of a tidewater glacier, Annals of Glaciology, 42, 1–6, https://doi.org/10.3189/172756405781813023, 2005.