# Review of Sensitivity of centennial mass loss projections of the Amundsen basin to the friction law by Brondex et al.

Thomas Kleiner

## Summary

It is very clear already from the early days of ice flow modelling, that the knowledge about subglacial conditions and the sliding relation is crucial for the understanding of ice sheet dynamics. This has been nicely elaborated in the previous study by Brondex et al. (2017) based on an idealised geometry set-up. In this study the authors conduct prognostic simulations for the Amundsen Sea Embayment (ASE) of about 100 years each to assess the relative sensitivity of grounded ice area and volume above flotation to the chosen friction law and initialisation strategy. The applied numerical flow model solves the two-dimensional shallow shelf approximation (SSA) for the stress balance. Model initial states are computed by means of a control method, where basal shear stresses and the viscosity are adjusted to minimise the misfit between modelled and observed surface velocities. A published data set of simulated ice sheet temperatures is used to account for the temperature dependency, initially. Within the prognostic simulations a synthetic perturbation of ice shelf basal melt rates is used to asses the model response to the given friction laws.

The paper is well written and well structured, and certainly deserves to be published. Nonetheless I would hope for a round of revisions to address my major concerns and the list of issues (below).

## Major concerns and suggestions

Unfortunately, the authors do not conclude about the most appropriate friction low for the ASE study site. In addition to that, the discussion of the results is based on the numerical flow modelling only. Constraints on the friction law based on observational data are entirely ignored, but should be included to gain the scientific outcome of this study. The ASE is probably the area in Antarctica that is covered by observations the most (e.g. Rippin et al., 2011; Smith et al., 2013; Brisbourne et al., 2017). If discussed with respect to observations, the study could already be a significant contribution "on getting a better understanding of the physical processes at play at the ice/bedrock interface in order to constrain the form of the friction law which needs to be used in models."

I have overlooked this several times, but the rheology parameters $A_0$ and $Q$, given in Table 1, are very uncommon. Especially the pre-exponential factor $A_0$ for temperatures above -10° C differs by several magnitudes from the commonly used Paterson and Budd (1982) parameterization of the Arrhenius Law Eq. (4). I have checked Elmer-Ice (Gillet-Chaulet et al., 2012), ISSM source code and PISM source code. They all use very similar values as in EISMINT (Payne et al., 2000). With the reported values in this study, the viscosity would be much larger (see Figure 1) and thus, the friction law more important. Given the specific ice rheology in

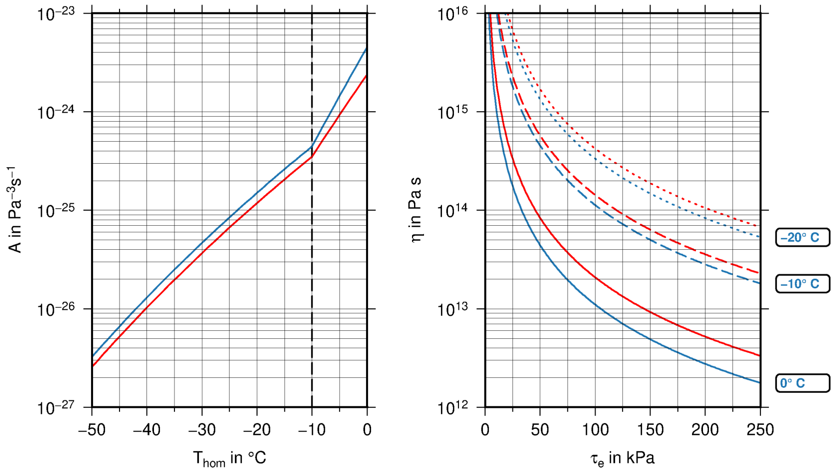


Figure 1: Rate factor $A$ as a function of the pressure corrected temperature ($T_{\mathrm{hom.}}$) and the effective viscosity $\eta(A, \tau_\mathrm{e})$ with the effective stress $\tau_\mathrm{e}$ for the Paterson and Budd (1982) rheology (blue) and Brondex et al. (red).

this study, I have strong doubts, whether the results can be transferred to other models. If these values are not just different (wrong?) in the table, I would highly recommend to re-run the experiments with a more common set of parameters.

## Detailed, line-by-line comments/suggestions

**P. 1, L. 3:** "Amundsen Sea Embayement" → "Amundsen Sea Embayment"

**P. 1, L. 16:** "to the oceans" → "to the ocean"

**P. 1, L. 21:** "trustworthy" consider "accurate/reliable"

**P. 1, L. 22:** "subcentennial timescales" → "sub-centennial timescales"

**P. 1, L. 25:** "a long standing problem" → "a long-standing problem"

**P. 2, L. 19:** "geometry and velocity field" → "geometry and (the) surface velocity field"

**P. 2, L. 23:** "~~Yet,~~ Adhalgeirsdottir et al. (2014) have shown . . . "

**P. 2, L. 24:** Consider "initial state of the model" instead of "model initial state"

**P. 2, L. 27–28:** "Our work being based on a schematic perturbation scenario, the results . . . of the ASE to SLR." This sentence appears to be incomplete.

**P. 3, L. 1?:** "two-dimensionnal" → "two-dimensional"

**P. 3, L. 1?:** Consider "shelfy-stream approximation (SSA)" instead of or in addition to "shallow shelf approximation (SSA)" here, because of the basal shear stresses. This is widely used in the literature for MacAyeal's equations (e.g. in Morlighem et al., 2010).

**P. 3, Eqns. (1,2):** Although formal correct I would recommend to rewrite Eq. 1 with $\bar{\eta}$ as the vertically averaged effective viscosity with units Pa s (instead of integrated; units: Pa s m). Thus,

$$4\frac{\partial}{\partial x}\left(H\bar{\eta}\frac{\partial u}{\partial x}\right)+\ldots, \quad (1)$$

with

$$\bar{\eta}=\frac{1}{H}\int_{z_\mathrm{b}}^{z_\mathrm{s}}\ldots\,dz. \quad (2)$$

**P. 4, L. 1:** "$\ldots\eta_0$ is the viscosity given by ..."
It is very misleading to call $\eta_0$ a viscosity, because it is obviously not (see units, e.g. in your Fig. 4). The equations (2) and (3) are correct and also how they are applied is correct, but your $\eta_0$ is only a substitution for the temperature dependent contribution to the viscosity, thus

$$XXX=\frac{1}{2}A^{-1/n}=\frac{1}{2}B, \quad (3)$$

where $A$ is the rate factor depending on the temperature relative to the temperature melting point and $B$ is the associated rate factor (Greve and Blatter, 2009, p. 56). I am specifically asking for a better name and symbol for $XXX$.

**P. 4, L. 3:** Although $A$ is called "fluidity parameter" already in Brondex et al. (2017) consider to use the commonly used term "rate factor" instead (Gillet-Chaulet et al., 2012; Gagliardini et al., 2013). Consider to use $T'$ instead of $T$ to account for the different meaning. Please state clearly, if you have used the temperature or the pressure corrected temperature from Van Liefferinge and Pattyn (2013).

**P. 4, Eqns. (5–7):** The "-" signs in front of $\tau_{\mathrm{b,x}}$ and $\tau_{\mathrm{b,y}}$ appear to be wrong in Eq. (1) with this notation of the different friction laws. Consider to use $\tau_\mathrm{b}=\ldots$ as in Brondex et al. (2017, Eqns. (1–3)).

**P. 5, L. 6:** "where $a_\mathrm{s}$ is the meteoric accumulation rate applied to the top surface of the whole domain and $a_\mathrm{b}$ ..."
Use "surface mass balance" for $a_\mathrm{s}$ as on page 6 line 9. I would suggest something like "where $a_\mathrm{s}$ is the surface mass balance $a_\mathrm{b}$ ~~applied to the top surface of the whole domain~~ ...". It should be stated that basal melt is ignored for the grounded part of the ice and why in another sentence.

**P. 5, Eq. (13):** "$\bar{\eta}$" → "$H\bar{\eta}$" with $\bar{\eta}$ being the average effective viscosity. See also P. 4, L. 1 above.

**P. 5, L. 25:** Although this can be guessed from the figures, it should be stated that the calving front is not evolving.

**P. 6, L. 30–33:** Why is the Budd law only applied to one of the inferred states?

**P. 6, L. 33:** "one of the inferred state" → "one of the inferred states"?

**P. 7, Table 1:** The numbers for the pre-exponential factors $A_0$ and and activation energies $Q$ for 'warm' and 'cold' ice are very unexpected. See the Major concerns and suggestions section.

**P. 7, L. 6:** "ice temperature map"
I am not sure what this means. The word map suggests something two-dimensional for me, but the temperature is used for the rate factor $A$ and thus $\eta_0$ within the integral of Eq. (2). May "three-dimensional temperature field/distribution" fits better.

If the temperature is a three-dimensional field, than it is not clear what is shown as map in your figure 4g.

**P. 7, L. 7:** "a reference ~~viscosity~~ field" and rename $\eta_{0,\mathrm{ref}}$ as mentioned above (P. 4, L. 1).

**P. 7, L. 6:** The temperature field from Van Liefferinge and Pattyn (2013), based on the model of Pattyn (2010) is a very important part for this study. Therefore, the methods used to get this field should be summarised within a few sentences. Which data set is applied here (ensemble mean, one specific ensemble member)?

**P. 7, L. 8:** "on each node~~s~~ of a regular grid"

**P. 7, L. 18:** "wether" → "whether"

**P. 8, L. 2–4:** "Indeed, several model states ... adjusting ~~rather~~ the basal shear stress or ~~rather~~ the viscosity."

**P. 8, L. 5:** "we construct three inferred states — denoted $I_{SV}$, $I_{R_{\gamma,100}}$ and $I_{R_{\gamma,1}}$ — by means of the control method"
At this place the inferred states are introduced by names and the reader needs to continue reading until page 9, line 17 for the explanation of $I_{R_{\gamma,100}}$ and $I_{R_{\gamma,1}}$. This might be unavoidable as a number of equations must be presented first. Nevertheless, I missed the explanation of the subscript 'SV' in $I_{SV}$ until the end of the document.

**P. 9, L. 3-4:** Consider to move "respectively" further to the end of the sentence: "... which are related to the linear Weertman law coefficient and the viscosity, respectively, as follows:", but this is personal preference only.

**P. 9, L. 32:** "occurence" → "occurrence"

**P. 10, L. 21-22:** "except for the Budd law for which the identification has been done only for the case $I_{R_{\gamma,1}}$"
Why?

**P. 10, L. 23:** "at every grounded node~~s~~ covered with ice"

**P. 10, L. 24:** "which are ice free" → "which are ice-free"

**P. 11, L. 5:** "which are ice free" → "which are ice-free"

**P. 12, L. 11:** "local adjust~~e~~ment of viscosity"

**P. 12, L. 18:** "the inversion algorithm~~n~~"

**P. 12, L. 23:** "has already been showed" → "has already been shown"

**P. 12, L. 21–25:** "It is also this same mechanism ... Borstad et al., 2012, 2013)." Although damage could play a role, I am not convinced of this argument.

I think, the shear margins are just not well enough resolved in the velocity field that has been simulated in the study by Van Liefferinge and Pattyn (2013, 5 km horizontal resolution). Unfortunately, ice flow velocities are not presented in Van Liefferinge and

Pattyn (2013) or Pattyn (2010). The basal drag in an ice stream is usually low, thus the lateral drag at the shear margins balances the ice stream's driving stress. Similar to the condition at an ice sheets base, the drag leads to deformation of ice (strain) and thus strain heating. As the viscosity depends on temperature the viscosity decreases (see e.g. Bondzio et al., 2017). This is supported by your figure 4 panel g, where no viscosity variations across the shear margins of PIG near the GL are visible.

The cited literature is only related to 'damage' in ice shelves (Larsen B and C) and not appropriate for the conditions in the ASE.

**P. 13, L. 1:** "loosing" → "losing"

**P. 13, L. 10:** "at every grounded node~~s~~"

**P. 13, L. 19:** "whithin" → "within"

**P. 13, L. 31:** "the relative differences ~~on~~ in the velocity field"?

**P. 14, L. 1:** "the gaussian integration" → "the Gaussian integration"

**P. 15, L. 19:** "is primary controlled by" → "is primarily controlled by"

**P. 15, L. 23:** "significantly different than the" → "significantly different from the"
My preference.

**P. 15, L. 23:** "$z_f = \ldots$, constitutes the thickness above flotation."
This is only true for grounded ice. Consider to show the flotation altitude (red line in Fig. 9) only for the grounded part.

**P. 16, L. 7:** "Dotson ice shelve" → "Dotson Ice Shelf"

**P. 16, L. 8:** "viscosiy" → "viscosity"

**P. 16, L. 11:** "tens of degrees celsius" → "tens of degrees Celsius"

**P. 16, L. 12:** "temperature map" See comment above (P. 7, L. 6).

**P. 17, L. 2:** "showing a highest contribution" → "showing the highest contribution"

**P. 17, L. 28:** "leading to important retreat of the GL" → "leading to an/the important retreat of the GL"

**P. 17, L. 30:** "occurence" → "occurrence"

**P. 17, L. 33:** "solid black line in bottom left panel of Fig. 9" → "solid black line in the bottom left panel of Fig. 9"

**P. 18, L. 23:** "parameters are uncertains" → "parameters are uncertain"

**P. 18, L. 24:** "viscosity is not inferred but ~~simply~~ deduced"

**P. 18, L. 24:** "ice temperature maps" See above.

**P. 18, L. 31:** "equals ~~to~~ the value of" or "is equal to the value of"

**P. 19, L. 1–2:** Consider to rearrange the sentence (personal preference only). E.g. "This procedure induces significant but very localised discrepancies between the recomputed velocity field and the reference velocity field used for the identification, in particular within ice shelves." or "... particularly within ice shelves."

**P. 19, L. 4–16:** The authors state very clear at the beginning (P. 2, L. 28), that "…the results presented here should not be considered as actual projections of the future contribution of the ASE to SLR." Consider to choose other terms to replace "projections" within this and other parts of the text.

**P. 19, L. 14:** "constain" → "constrain"

**P. 24, Fig. 4:** I think, maps of the basal shear stress $|\tau_b|$ are required in addition to the stress ratios presented in (d,e,f) for the three inferred states. This would allow to compare your inversion with other modelling studies conducted in this area (e.g. Joughin et al., 2009; Morlighem et al., 2010) and observational data.

A large portion of your model domain appears white in the panels a–c indicating that observed velocities are not available here. This is not so easy to see in Rignot et al. (2011), but in Mouginot et al. (2014, Fig. 1). Please explain how do you conduct the inversion in these areas. It is not clear, how the features in the panels d–f can be explained, given the extensive data gap in a–c.

**P. 25, Fig. 5:** I can't see any difference between a,b and c. The tiny little areas in between the green and grey areas appear all just red. The zoom in area should be marked in one of the figures for the whole ASE.

**P. 27, Fig. 8:** The coloured lines should be slightly thicker.

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