Neutral equilibrium and forcing feedbacks in marine ice sheet modelling

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Abstract.

Poor convergence with resolution of ice sheet models when simulating grounding line migration has been known about for over a decade. However, some of the associated numerical artifacts remain absent from the published literature.

In the current study we apply a Stokes-flow finite element marine ice sheet model to idealised grounding line evolution experiments. We show that with insufficiently fine model resolution, a region containing multiple steady state grounding line positions exists, with one steady state per node of the model mesh. This has important implications for the design of perturbation experiments used to test convergence of grounding line behaviour with resolution. Specifically, the design of perturbation experiments can be under-constrained, potentially leading to a "false positive" result. In this context a false positive is an experiment that appears to achieve convergence when in fact the model configuration is not close to its converged state. We demonstrate a false positive: an apparently successful perturbation experiment (i.e. reversibility is shown) for a model configuration that is not close to a converged solution. If perturbation experiments are to be used in the future, experiment design should be modified to provide additional constraints to the initialisation/spin up requirements.

This region of multiple locally stable steady state grounding line positions has previously been mistakenly described as neutral equilibrium. This distinction has important implications for understanding the impacts of discretizing a forcing feedback involving grounding line position and basal friction. This forcing feedback can not, in general, exist in a region of neutral equilibrium, and could be the main cause of poor convergence in grounding line modelling.

1 Introduction

Strongly resolution dependent behaviour when implementing grounding line movement (sometimes referred to as grounding line migration) in a marine ice sheet model was identified by Vieli and Payne (2005) and was further characterised as a convergence problem by subsequent studies (Durand et al., 2009; Goldberg et al., 2009; Gladstone et al., 2010a, b, 2012). Some models incorporating a moving grid that explicitly tracks grounding line position do not appear to exhibit this poor convergence (Vieli and Payne, 2005). Various forms of mesh refinement help to address the problem, though very high resolution is still needed (Goldberg et al., 2009; Cornford et al., 2013), and special treatments of the grid cell or element containing the grounding line can also improve convergence (Pollard and DeConto, 2009; Gladstone et al., 2010b; Gagliardini et al., 2016).

This problem has also been decribed as neutral equilibrium (Durand et al., 2009; Pattyn et al., 2006) in modelling studies. This terminology may follow from earlier studies in which it was proposed that real marine ice sheet systems may exhibit neutral equilibrium (Hindmarsh, 2006). Although these theories are no longer accepted (Schoof, 2007), unconverged model behaviour at coarse resolution is still sometimes referred to as neutral equilibrium (Durand et al., 2009).

5 Most of the studies cited above use a Weertman sliding relation (Weertman, 1957). More recent studies (Leguy et al., 2014; Tsai et al., 2015; Gladstone et al., 2017) suggest that the convergence issues may be to some extent mitigated by use of sliding relations incorporating a dependence on effective pressure at the bed. However, irrespective of sliding law, similar convergence issues may arise due to a step change in basal melting at the grounding line (Gladstone et al., 2017).

In the current study we employ a flowline Stokes-flow model with Weertman sliding and no basal melting (Section 2) to
10 further characterise the nature of this grounding line convergence issue (Section 3). We explore implications for design of computer experiments (Section 3.1) and for the issue of neutral equilibrium (Section 4).

2 Flowline modelling

Our aim is to provide a model configuration in which convergence with resolution is not achieved (i.e. our resolution is too coarse for self-consistent model behaviour), and explore the nature of the grounding line problems. Our setup is similar to that
15 of the original Marine Ice Sheet Model Intercomparison Project (Pattyn et al., 2012).

We use the ice dynamic model (IDM) Elmer/Ice (Gagliardini et al., 2013). The Stokes equations for a viscous fluid with non-linear rheology are solved using the finite element method over a two-dimensional flowline domain (one vertical and one horizontal dimension) in which lateral drag is parameterised (Gagliardini et al., 2010) according to channel width, W, and a contact problem is solved to determine the evolving grounding line position (Favier et al., 2012).

20 The rheology follows Glen's law (Glen, 1952; Paterson, 1994) with viscosity calculated using a temperature dependent Arrhenius law (Gagliardini et al., 2013; Paterson, 1994). A constant uniform temperature of -15 C is used in all simulations.

The linear down sloping bedrock, b, is given in m relative to sea level by

$$b = 500 - 0.005 \times x, \tag{1}$$

where x is distance from the inland boundary.

25 The horizontal component of the velocity is set to zero at the inland boundary, and an ocean pressure condition applied at the ice front and under the floating ice shelf. We take ice density to be $910 \ \mathrm{kgm^{-3}}$ and water density to be $1000 \ \mathrm{kgm^{-3}}$.

A spatially uniform net surface accumulation flux, a, is used, and this value is varied between simulations.

The basal friction or shear stress, τ_b, acts opposite to the direction of flow and has magnitude (Weertman, 1957)

$$\tau_b = C u_b^{\frac{1}{3}} \tag{2}$$

30 where u_b is the sliding velocity and C is a friction coefficient. C is set to $0.02417 \ \mathrm{MPam^{-\frac{1}{3}}a^{\frac{1}{3}}}$ for all simulations in the current study.
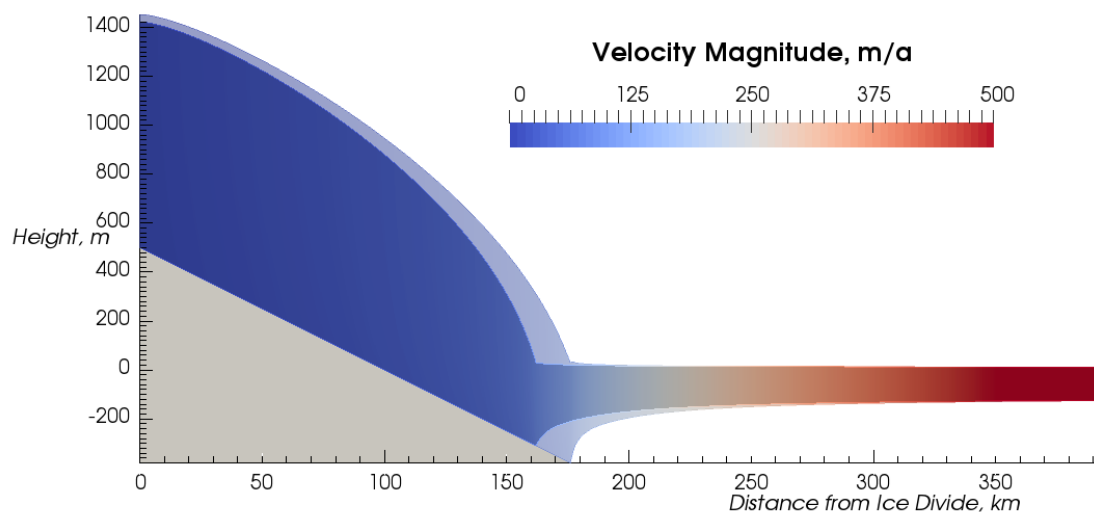
Figure 1. Ice sheet profiles after 13ka at the end of the advance/retreat simulations described in Section 2.1. The simulations with an advance forcing of $a = 0.7 \mathrm{ma}^{-1}$ (solid colour) and $a = 1.7 \mathrm{ma}^{-1}$ (semi-transparent) are shown. These states provide the initial state for perturbations experiments P1 and P2 respectively (Section 2.2).

The simulations carried out for the current study are described below. They comprise grounding line advance simulations followed by grounding line retreat simulations (Section 2.1). In some cases further perturbation experiments are then carried out (Section 2.2). These simulations were all carried out with a horizontally uniform element size of $1 \, \mathrm{km}$. Typical steady state profiles for this model setup are shown in Figure 1.

2.1 Advance/retreat experiments

The advance simulations are spun-up from a uniform slab of 100m thickness. They comprise $7 \, \mathrm{ka}$ of evolution with a different net accumulation forcing for each simulation. Values range from $0.2 \, \mathrm{ma}^{-1}$ to $2.0 \, \mathrm{ma}^{-1}$ (the full set of values used is given in the Figure 2 legend).

The advance simulations are followed immediately by "retreat" simulations, which in some (but not all) cases exhibit grounding line retreat. These simulations continue from the final states of the advance simulations. They all use a net accumulation forcing of $a = 0.2 \, \mathrm{ma}^{-1}$ and are run for a further $6 \, \mathrm{ka}$.
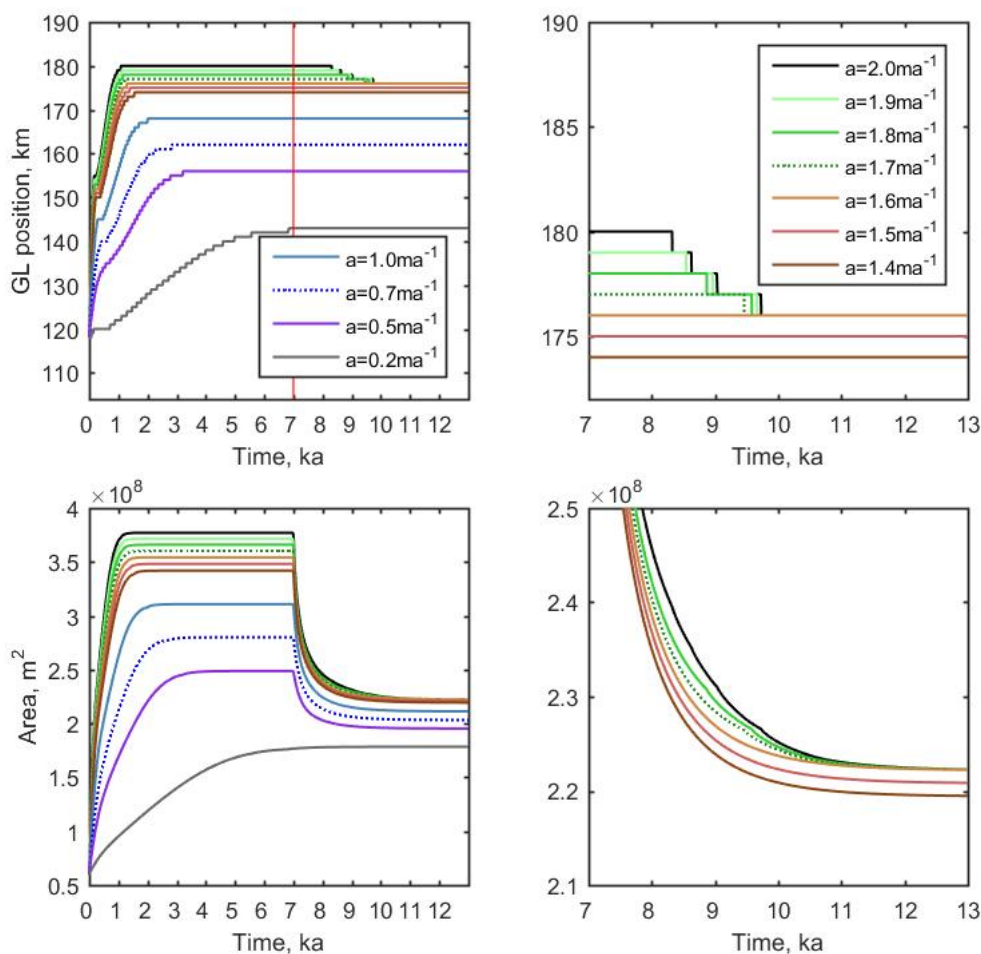
Figure 2. Advance/retreat simulations (described in Section 2.1). Evolution of (top panel) grounding line position for all simulations and (lower panel) ice area. Area in this case is the flowline equivalent of ice volume for a 3D ice sheet, and can also be interpreted as volume per unit width of the glacier. The right hand subplots show detail of a subset of the retreat simulations. The red vertical line indicates the forcing change at 7ka when the simulations switch from advance to retreat. The legend shows the accumulation rates prescribed during the advance phase, while for the retreat accumulation was $0.2 \ \mathrm{ma}^{-1}$. The dotted lines are the cases also shown in Figure 1 and provide the initial state for the P1 and P2 experiments (described in Section 2.2).

2.2 Perturbation experiments

Starting from the final states of two of the advance/retreat simulations, (i.e. after 13ka total simulation time) we carried out two perturbation simulations. P1 starts from the final state of the simulation that used an advance forcing of $a = 0.7 \ \mathrm{ma}^{-1}$ and P2 starts from the final state of the simulation that used an advance forcing of $a = 1.7 \ \mathrm{ma}^{-1}$. These advance/retreat simulations are

shown with a dotted line in Figure 2 and their final states (which form the initial states for P1 and P2) are shown in Figure 1. Note that although P1 and P2 start from approximate steady states, and in both cases the steady state was approached with $a = 0.2 \ \mathrm{ma}^{-1}$, these steady states are distinct. The forcing for the perturbation experiments P1 and P2 is identical apart from initial state. They are run for 1 ka with $a = 2.0 \ \mathrm{ma}^{-1}$ followed by 4 ka with $a = 0.2 \ \mathrm{ma}^{-1}$. The perturbation experiments P1

5 and P2 are shown in Figure 3.

We also carried out a small perturbation experiment, PS. PS starts from the same state as P1. It is identical to P1 except that the magnitude of the perturbed forcing is $a = 0.71 \ \mathrm{ma}^{-1}$.

3 Multiple steady states

Figure 2 summarises the evolution over time of the advance/retreat simulations. Given that 7ka years is sufficient to approach
10 steady state (we did not impose a test for steady state but informally note that no grounding line change occurred during the last few ka years in nearly all advance simulations), retreat should occur after 7ka years for all simulations in which a was initially greater than $0.2 \ \mathrm{ma}^{-1}$. This is due to the uniqueness of stable ice sheet configurations on a linear down sloping bed, demonstrated for a "shelfy-stream" approximation by Schoof (2007). However, as also seen with a "shelfy-stream" model (Gladstone et al., 2010a), multiple steady states exist as a model artefact. In common with the advance simulations, 6 ka
15 years of retreat is not sufficient to achieve an exact steady state in all simulations, but rates of change are low in all simulations after 6 ka years of retreat.

The multiple steady states that exist after 13 ka are almost certainly numerical artifacts, with the underlying system having only one viable steady state. Similar studies have shown that the size of this region decreases with finer resolution (Gladstone et al., 2010a). The region containing steady state grounding line positions in the current study spans from $x = 143$ km to
20 $x = 176$ km. We propose that the model is capable of exhibiting as many viable steady state grounding line positions as there are mesh nodes within this region. We tested this hypothesis near the seaward end of the region by implementing small increments in a between advance simulations. Specifically we obtained a final grounding line position on every node from $x = 174$ km to $x = 180$ km for the advance simulations and from $x = 174$ km to $x = 176$ km for the retreat simulations.

The volume evolution plots indicate a reduction in volume for all retreat simulations (except the simulation which advanced
25 under $a = 0.2 \ \mathrm{ma}^{-1}$ forcing), even for simulations showing no grounding line movement. Simulations ending the retreat phase with the same grounding line position (several simulations end at 176 km) have the same final volume. Simulations with a more landward final grounding line position have a lower final volume.

3.1 Implications for experiment design

Here we consider perturbation experiments P1 and P2, both of which adhere to a typical perturbation design and both of which
30 experience identical forcing during the experiment. Perturbation experiments are common in IDM studies and intercomparison projects (e.g. Pattyn et al. (2006, 2012, 2013); Favier et al. (2012)). The premise is that an initial spinup procedure results in an IDM in steady state. A forcing perturbation is applied, causing change, and then removed. The analysis then considers

whether or not reversibility has been demonstrated (i.e. whether the IDM state returned to its post spinup state after the forcing perturbation was reset). However, the existence of multiple steady state grounding line positions means that the requirement to start in steady state is not sufficient to constrain the initial (post-spinup) state.

The outputs of the perturbation experiments are shown in Figure 3. Although both experiments adhere to typical perturbation

5   experiment design, and are both subject to the same perturbation, P2 shows full reversibility and P1 does not. The outcomes in terms of reversibility are opposites, resulting directly from the choice of initial state.

We now consider an example of this vulnerability in design of perturbation experiments from the published literature. Pattyn et al. (2006) investigated the role of transition zones in grounding line modelling. The transition zone is a region immediately upstream of the grounding line over which the stress state changes from a grounded regime (in which high basal shear stress

10   approximately balances gravitational driving stress) to a floating regime (where basal shear stress is zero and longitudinal stress in the ice balances a low gravitational driving stress).

Pattyn et al. (2006) used a spinup procedure that resulted, for most of their simulations, in retreat of the grounding line as steady state was approached. This suggests (but does not prove) that the end of the spinup resulted in a steady state grounding line position located at the seaward end of the region of multiple steady states, analogous to our experiment P2 (Figure 3).

15   These simulations did demonstrate reversibility. However, their simulation with a short prescribed grounding line transition zone involved no movement of the grounding line as steady state was approached. This suggests (but again does not prove) that the end of the spinup resulted in a steady state grounding line position somewhere within the region of multiple steady states, analogous to our experiment P1 (Figure 3), see Figure 4 of Pattyn et al. (2006). This simulation did not demonstrate reversibility. Thus the result of Pattyn et al. (2006) that a longer transition zone results in better reversibility may be an artefact

20   of their experiment design rather than a robust result.

4   Neutral equilibrium

An equilibrium state (or steady state) of a system is a state that does not change unless the forcing changes. In the context of IDM grounding line simulations, this means that neither the forcing applied to the domain nor the ice sheet configuration are changing over time. Such steady states are typically obtained through long simulations in which forcing is kept constant and

25   the state of the simulated ice sheet gradually stops evolving as equilibrium is approached.

It is important to clarify different types of equilibria for the following discussion. Consider the example of a ball at rest (i.e. in equilibrium) under gravitation on a solid surface. The ball is then subjected to a perturbation: it is moved along the surface then left only under gravitation. Different types of equilibrium may be illustrated by considering the behaviour of the ball after the perturbation has been removed.

30   An equilibrium state where the perturbation results in the system tending to return to the original state is a stable equilibrium (e.g. Figure 4a - the ball rolls back down to the original position).

An equilibrium state where the perturbation results in the system tending to move further from the original state is an unstable equilibrium (not shown, but consider a ball on a hill - it will continue rolling away from the summit after begin given a small push in any direction).

An equilibrium state where the perturbation results in the system remaining in the new state is termed neutral equilibrium
5 (e.g. Figure 4b - the ball may be at rest anywhere on the flat region).

Figure 4c illustrates a system with multiple locally stable equilibria within a confined region. A large perturbation will result in the ball finding a new equilibrium position, but a small perturbation will result in a return to the original position.

These types of behaviour for a ball under gravity have analogies for a marine ice sheet system. Schoof (2007) demonstrated the existence of a single stable equilibrium for a marine ice sheet on a downward (in the ice flow direction) sloping bed. Schoof
10 (2007) also demonstrated the existence of an unstable equilibrium on an upward sloping bed, though this may not always be the case in the presence of high lateral drag (Katz and Worster, 2010; Gudmundsson et al., 2012). As mentioned in Section 1, neutral equilibrium in real world marine ice sheet systems is no longer considered plausible, but multiple stable equilibria could exist as a function of bedrock geometry Schoof (2007).

IDM studies using different models have demonstrated that multiple steady states can exist in models where the system
15 being modelled should exhibit a single stable equilibrium (Durand et al., 2009; Gladstone et al., 2010a). This has been referred to as neutral equilibrium (Durand et al., 2009), and here we consider the distinction between a region of neutral equilibrium and a region of multiple locally stable steady states. We argue that IDMs exhibit a region containing multiple locally stable equilibria (similar to Figure 4c) and not a region of neutral equilibrium.

Figure 5 shows output from experiment PS, the small perturbation experiment. The perturbation, although not sufficient to
20 cause a change in grounding line position (Figure 5a), is sufficient to cause a shift in model state, as evidenced by the change in total ice volume (Figure 5c). However, the forcing reset results in a return to the original state. This behaviour indicates a locally stable steady state rather than a region of neutral equilibrium. This argument against marine IDMs exhibiting neutral equilibria may appear to be a matter of semantics, but there are important implications toward understanding the nature of the grounding line convergence problem, discussed in Section 5.

5 Forcing feedback

Figure 6 shows in more detail the evolution of total basal friction during the advance phase of perturbation experiments P1 and P2. The spikes in total friction correspond to advance of the grounding line by a single element. These features are characterised by an instananeous increase of total friction followed by a rapid decrease and an ensueing gradual increase. The spikes can be explained as follows: An instantaneous increase in basal friction results from a grounding line advance due to the increased
30 contact area. This increase reduces the sliding velocity, causing the rapid decrease in total friction. This is followed by a more gradual return to the longer term trend.

This is a model discretization of what should be a continuous feedback: incremental grounding line advance should cause incremental increase in total basal friction, causing an incremental slowing and thickening. This positive feedback (which

we refer to as the friction force feedback) between grounded extent and total friction is continuous in the underlying system being simulated, but heavily discretised in the model due to basal friction reaching a peak at the grounding line. The modelled flux across the grounding line must be higher than that of the system it attempts to represent in order to compensate for the missing basal friction immediately downstream of the grounding line due to the discretisation. Specifically, the PS experiment

5 (Figure 5) demonstrates that even an increase in modelled volume and total friction force of several tens of percent may not be sufficient to cause a single element of grounding line advance. This understanding could not have been attained if the region of multiple locally stable steady states was viewed as a region of neutral equilibrium, because a neutral equilibrium can have no positive feedback between forcing and state (except in the vanishingly low probability case of an exactly compensating mechanism).

10 We postulate that this discretisation of a continuous feedback is the main cause of numerical artifacts and poor convergence with resolution in grounding line modelling. This is consistent with the finding that sliding relations incorporating a strong dependency on effective pressure at the bed show far better convergence with resolution (Gladstone et al., 2017). This is due to the basal friction approaching zero at the grounding line, so that the advance or retreat of the grounding line by a single element will not have a significant impact on total basal friction.

15 Considering several published sliding relations that feature a dependence on effective pressure, it should be noted that the hybrid sliding relations of Gagliardini et al. (2007) and Tsai et al. (2015) typically feature steep basal friction gradients over a transition zone near the grounding line (Brondex et al., 2017) and so may not exhibit such good convergence as the sliding relation of Budd et al. (1979, 1984). It should also be noted that improved convergence is not a valid reason to choose one sliding relation over another: physical realism should be the deciding factor. A final note is that this issue is not fundamentally

20 specific to the equations solved for ice flow, so while the simulations carried out here use Elmer/Ice to solve the Stokes equations, the same principles should apply in other IDMs that implement some kind of sliding relation.

It might be thought that a special treatment of the grid cell or element containing the grounding line, such that the grounding line position within the cell or element can be represented, would resolve this problem of discretising the friction force feedback. However, the grounding line parameterisations introduced by Gladstone et al. (2010b) (a study featuring numerous

25 different parameterisations implemented in a flowline shelfy-stream model) still show a strong non-linear behaviour correlated to grid cell grounding line advance in the evolution of model state (See Gladstone et al. (2010b) Figures 3, 4 and 6). Gladstone et al. (2010b) also find multiple steady states exist, with one steady state grounding line position per grid cell, although the steady state position is not constrained to lie on a grid point. Thus grounding line parameterisations do not necessarily resolve the problem of a discretised (or highly non-linear on a grid cell scale) friction forcing feedback.

6 Conclusions

The established poor convergence of many marine ice sheet models regarding grounding line movement is characterised by a region of multiple locally stable states. Our results demonstrate that this is not, as has been previously claimed, a neutral equilibrium.

This region of steady states implies that perturbation experiments, such as are often used in model intercomparison projects, can have a hitherto unrecognised dependence on initial conditions. Thus the size of the region of multiple locally stable steady states may be a more useful metric for assessing convergence of modelled grounding line movement than advance only simulations, retreat only simulations, or reversibility. If perturbation experiments are used in future tests of convergence of

5   grounding line behaviour, we advocate that the spinup method must also be prescribed, as a simple requirement for steady state is not in general sufficient to constrain the experiment design.

This poor convergence is not only a result of inherent difficulties in respresenting a spatial step change in basal drag across the grounding line, but is also due to a temporal forcing feedback involving grounding line movement and basal shear stress.

These results apply to marine ice sheet models using a Weertman sliding relation, and may be less relevant to sliding

10   relations incorporating a smoother transition in basal drag across the grounding line, such as through a dependence on height above buoyancy.

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References

Brondex, J., Gagliardini, O., Gillet-Chaulet, F., and Durand, G.: Sensitivity of grounding line dynamics to the choice of the friction law, Journal of Glaciology, 63, 854–866, doi:10.1017/jog.2017.51, 2017.

Budd, W., Keage, P. L., and Blundy, N. A.: Empirical studies of ice sliding, Journal of Glaciology, 23, 157–170, 1979.

5 Budd, W., Jenssen, D., and Smith, I.: A 3-dimensional time-dependent model of the West Antarctic Ice-Sheet, Annals of Glaciology, 5, 29–36, 1984.

Cornford, S. L., Martin, D. F., Graves, D. T., Ranken, D. F., Le Brocq, A. M., Gladstone, R. M., Payne, A. J., Ng, E., and Lipscomb, W. H.: Adaptive mesh, finite volume modeling of marine ice sheets, Journal of Computational Physics, 232, 529–549, doi:10.1016/j.jcp.2012.08.037, 2013.

10 Durand, G., Gagliardini, O., de Fleurian, B., Zwinger, T., and Le Meur, E.: Marine ice sheet dynamics: Hysteresis and neutral equilibrium, Journal of Geophysical Research-Earth Surface, 114, doi:10.1029/2008JF001170, 2009.

Favier, L., Gagliardini, O., Durand, G., and Zwinger, T.: A three-dimensional full Stokes model of the grounding line dynamics: effect of a pinning point beneath the ice shelf, The Cryosphere, 6, 101–112, doi:10.5194/tc-6-101-2012, http://www.the-cryosphere.net/6/101/2012/, 2012.

15 Gagliardini, O., Durand, G., Zwinger, T., Hindmarsh, R. C. A., and Le Meur, E.: Coupling of ice-shelf melting and buttressing is a key process in ice-sheets dynamics, Geophysical Research Letters, 37, doi:10.1029/2010GL043334, l14501, 2010.

Gagliardini, O., Zwinger, T., Gillet-Chaulet, F., Durand, G., Favier, L., de Fleurian, B., Greve, R., Malinen, M., Martín, C., Råback, P., Ruokolainen, J., Sacchettini, M., Schäfer, M., Seddik, H., and Thies, J.: Capabilities and performance of Elmer/Ice, a new generation ice-sheet model, Geoscientific Model Development Discussions, 6, 1689–1741, doi:10.5194/gmdd-6-1689-2013, http://www.

20 geosci-model-dev-discuss.net/6/1689/2013/, 2013.

Gagliardini, O., Brondex, J., Gillet-Chaulet, F., Tavard, L., Peyaud, V., and Durand, G.: Brief communication: Impact of mesh resolution for MISMIP and MISMIP3d experiments using Elmer/Ice, The Cryosphere, 10, 307–312, doi:10.5194/tc-10-307-2016, http://www.the-cryosphere.net/10/307/2016/, 2016.

Gagliardini, O., Cohen, D., Raback, P., and Zwinger, T.: Finite-element modeling of subglacial cavities and related friction law, Journal Of

25 Geophysical Research-Earth Surface, 112, doi:10.1029/2006JF000576, 2007.

Gladstone, R., Lee, V., Vieli, A., and Payne, A.: Grounding Line Migration in an Adaptive Mesh Ice Sheet Model, Journal of Geophysical Research-Earth Surface, 115, doi:10.1029/2009JF001615, 2010a.

Gladstone, R., Payne, A., and Cornford, S.: Parameterising the grounding line in flow-line ice sheet models, The Cryosphere, 4, 605–619, doi:10.5194/tc-4-605-2010, 2010b.

30 Gladstone, R., Payne, A. J., and Cornford, S. L.: Resolution requirements for grounding-line modelling: sensitivity to basal drag and ice-shelf buttressing, Annals Of Glaciology, 53, 97–105, doi:10.3189/2012AoG60A148, 2012.

Gladstone, R. M., Warner, R. C., Galton-Fenzi, B. K., Gagliardini, O., Zwinger, T., and Greve, R.: Marine ice sheet model performance depends on basal sliding physics and sub-shelf melting, The Cryosphere, 11, 319–329, doi:10.5194/tc-11-319-2017, http://www.the-cryosphere.net/11/319/2017/, 2017.

35 Glen, J. W.: Experiments on the deformation of ice, Journal of Glaciology, 2, 111–114, 1952.

Goldberg, D., Holland, D. M., and Schoof, C.: Grounding line movement and ice shelf buttressing in marine ice sheets, Journal of Geophysical Research-Earth Surface, 114, doi:10.1029/2008JF001227, 2009.

Gudmundsson, G. H., Krug, J., Durand, G., Favier, L., and Gagliardini, O.: The stability of grounding lines on retrograde slopes, Cryosphere, 6, 1497–1505, doi:10.5194/tc-6-1497-2012, 2012.

Hindmarsh, R. C.: The role of membrane-like stresses in determining the stability and sensitivity of the Antarctic ice sheets: back pressure and grounding line motion, Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences,

5   364, 1733–1767, doi:10.1098/rsta.2006.1797, http://rsta.royalsocietypublishing.org/content/364/1844/1733, 2006.

Katz, R. F. and Worster, M. G.: Stability of ice-sheet grounding lines, Proceedings of the Royal Society A-Mathematical Physical and Engineering Sciences, 466, 1597–1620, doi:10.1098/rspa.2009.0434, 2010.

Leguy, G. R., Asay-Davis, X. S., and Lipscomb, W. H.: Parameterization of basal friction near grounding lines in a one-dimensional ice sheet model, The Cryosphere, 8, 1239–1259, doi:10.5194/tc-8-1239-2014, http://www.the-cryosphere.net/8/1239/2014/, 2014.

10   Paterson, W.: The physics of glaciers, Pergamon, Oxford, third edn., 1994.

Pattyn, F., Huyghe, A., De Brabander, S., and De Smedt, B.: Role of transition zones in marine ice sheet dynamics, Journal of Geophysical Research-Earth Surface, 111, doi:10.1029/2005JF000394, 2006.

Pattyn, F., Schoof, C., Perichon, L., Hindmarsh, R. C. A., Bueler, E., de Fleurian, B., Durand, G., Gagliardini, O., Gladstone, R., Goldberg, D., Gudmundsson, G. H., Huybrechts, P., Lee, V., Nick, F. M., Payne, A. J., Pollard, D., Rybak, O., Saito, F., and Vieli, A.: Results of the

15   Marine Ice Sheet Model Intercomparison Project, MISMIP, Cryosphere, 6, 573–588, doi:10.5194/tc-6-573-2012, 2012.

Pattyn, F., Perichon, L., Durand, G., Favier, L., Gagliardini, O., Hindmarsh, R. C. A., Zwinger, T., Albrecht, T., Cornford, S., Docquier, D., Furst, J. J., Goldberg, D., Gudmundsson, G. H., Humbert, A., Huetten, M., Huybrechts, P., Jouvet, G., Kleiner, T., Larour, E., Martin, D., Morlighem, M., Payne, A. J., Pollard, D., Rueckamp, M., Rybak, O., Seroussi, H., Thoma, M., and Wilkens, N.: Grounding-line migration in plan-view marine ice-sheet models: results of the ice2sea MISMIP3d intercomparison, Journal Of Glaciology, 59, 410–422,

20   doi:10.3189/2013JoG12J129, 2013.

Pollard, D. and DeConto, R. M.: Modelling West Antarctic ice sheet growth and collapse through the past five million years, Nature, 458, 329–332, doi:10.1038/nature07809, 2009.

Schoof, C.: Ice sheet grounding line dynamics: Steady states, stability, and hysteresis, Journal of Geophysical Research-Earth Surface, 112, doi:10.1029/2006JF000664, 2007.

25   Tsai, V. C., Stewart, A. L., and Thompson, A. F.: Marine ice-sheet profiles and stability under Coulomb basal conditions, Journal of Glaciology, 61, 205–215, doi:doi:10.3189/2015JoG14J221, http://www.ingentaconnect.com/content/igsoc/jog/2015/00000061/00000226/art00001, 2015.

Vieli, A. and Payne, A.: Assessing the ability of numerical ice sheet models to simulate grounding line migration, Journal of Geophysical Research-Earth Surface, 110, doi:10.1029/2004JF000202, 2005.

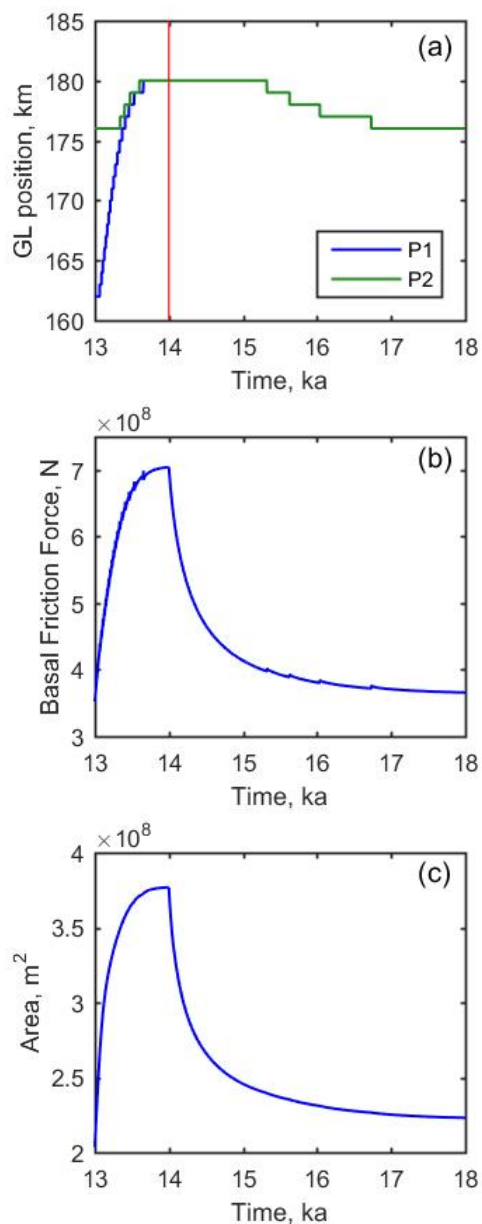30   Weertman, J.: On the sliding of glaciers, J. Glaciol., 3, 33–38, 1957.

Figure 3. Evolution of (a) grounding line position, (b) total basal friction (this is the basal shear stress integrated over the grounded region) and (c) ice area (the flowline equivalent of volume) for perturbation experiments. Both perturbation experiments P1 and P2 are shown in (a), and the difference in initial states is clearly visible. (b) and (c) show only P1. Both P1 and P2 were run for 1 ka with accumulation rate $a = 0.2 \, \mathrm{ma}^{-1}$ followed by 4 ka with $a = 0.2 \, \mathrm{ma}^{-1}$.
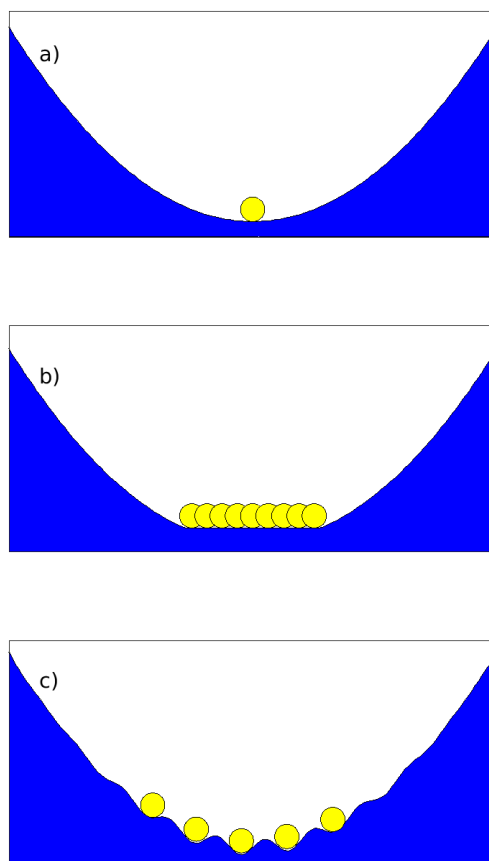
Figure 4. A simple idealised system of a ball under gravitation used to demonstrate types of equilibria: (a) one stable equillibrium is present and the ball will always tend to return to this; (b) a region of neutral equilibrium is present (a region of flat surface), and the ball will remain in equilibrium anywhere on this surface; (c) multiple locally stable equilibria exist, and the ball will tend to roll downhill to the nearest.
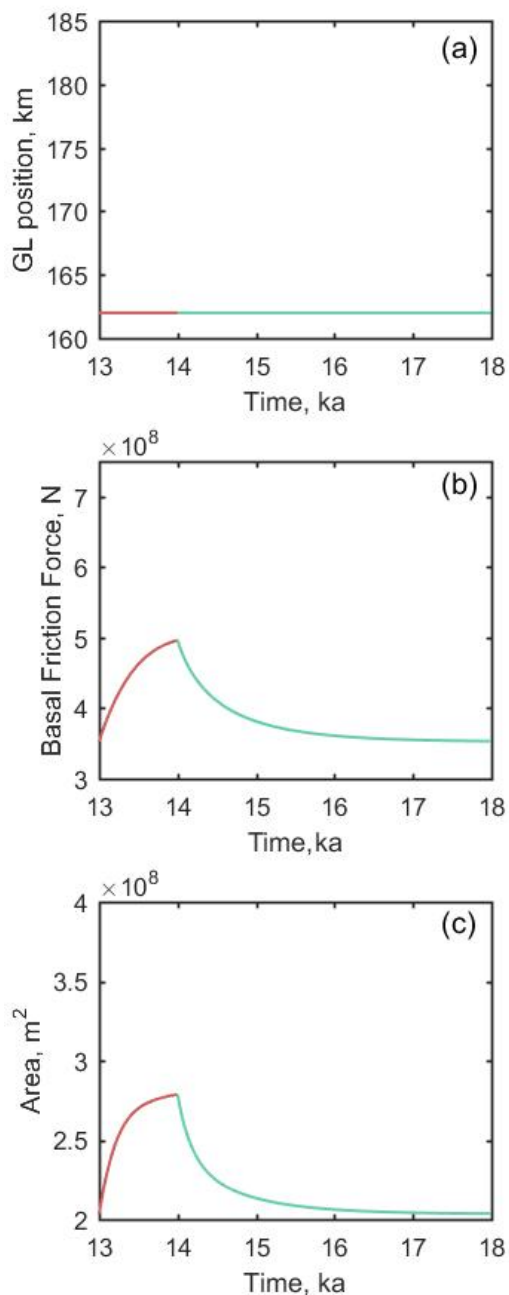
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Figure 5. Evolution of (a) grounding line position, (b) total basal friction, and (c) ice volume for the small perturbation experiment PS.PS is identical to P1 except that the magnitude of the perturbed forcing for the first 1 ka is $a = 0.71 \text{ ma}^{-1}$.
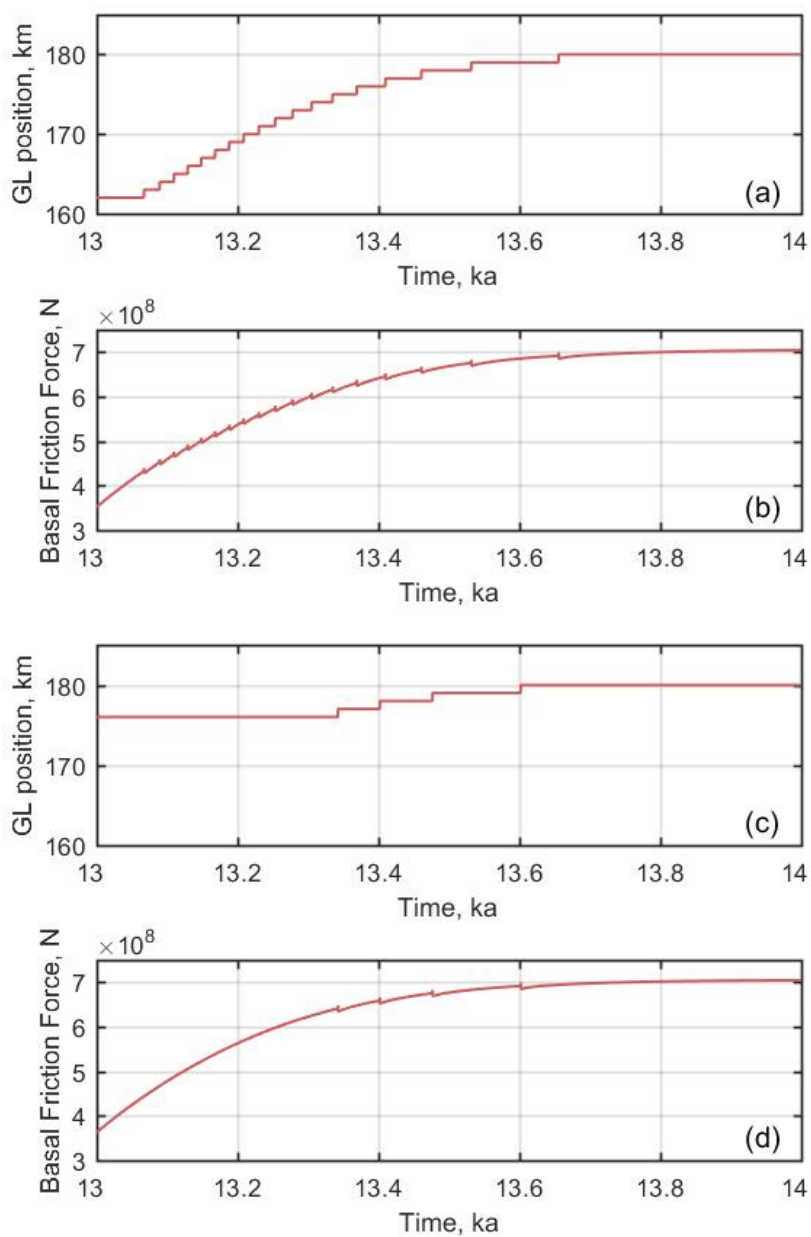
Figure 6. A closer look at the advance phase of the perturbation experiments. Evolution of (a) grounding line position for P1, (b) total basal friction for P1, (c) grounding line position for P2, (d) total basal friction for P2.