

1 Reviewer #1 (R. Gladstone)

The paper addresses the impact of schemes for handling sub-shelf melting in elements containing a section of grounding line in ice sheet models. The main points are that fairly strict resolution requirements might need to be imposed in order to provide a converged result in the presence of high melt rates near the grounding line, and that applying melt over partially grounded elements when resolution is not sufficiently fine is likely to give an overestimate of retreat rates and mass loss for a retreating grounding line. The paper is clearly written, the experiments simple and to the point, and the figures show the scientific content very clearly. This is a useful and timely contribution to the ice sheet modelling community. Anyone carrying out marine ice sheet modelling needs to be aware of the main points made by this paper.

No advance experiments were carried out. Of course one cannot conduct advance experiments by starting from a steady state without melting and then imposing melting, but starting from a steady state with high melting and then reducing the melt rate is perfectly viable. If we consider the grounding line convergence issue due to the basal resistance change across the grounding line, some models/parameterisations give better convergence in advance and some in retreat. There may be a similar issue with the melt problem. If you look at the bottom left panel of Fig 5 of Gladstone et al 2017 (also TC) you can see that we observed worse convergence in advance than in retreat in the presence of basal melting with no parameterisation (albeit with a different sliding relation to the ones used here). Of course in the current climate we're more interested in retreat than advance but temporary advance could occur as part of a larger retreat pattern (see also Jong et al TCD 2017 (now accepted for TC), and Torsten Albrecht's work on overshoot (work from last year, not sure if it is published yet, but you can contact him if you want to know more)). The addition of advance experiments would enhance this paper, but then again the paper is worth publishing as is and needs to be brought to the attention of other modellers. So I do not have a strong preference whether advance experiments should be added at this point. But at least add some brief discussion of the implications. Why does Weertman sliding with no melt parameterisation show such wonderful convergence? Perhaps it would be terrible in advance?

We thank Rupert Gladstone for his detailed review and constructive comments. The question of numerical treatment needed to accurately represent grounding line advance is indeed an important one that deserves being investigated as well. One complication is that as melt needs to be applied to grow the glacier during the initial steady-state in this case, the initial state is going to be impacted by the choice of melt parameterization. For this reason, we decided to add a paragraph in the discussion to discuss this question and refer to previous studies that are mentioned here. We discuss the convergence of Weertman NMP further down.

Why should experiments with the Tsai sliding law show less sensitivity to melt

parameterisations than experiments with the Weertman sliding law? Is it something to do with the different geometries? Weertman having thicker ice and steeper slopes close to the grounding line?

We think that when a Weertman sliding law is used, as the slopes are steeper close to the grounding line, the thinning is more localized to the area just upstream of the ice shelf, while Tsai friction favors rapid spread of thinning further inland of the glacier.

Specific comments

Page 1

Line 4. add → adds

Done

Line 19. Yields → leads?

Done

Page 2 Line 16. “impact to” → “impact on”

Done

Line 21. “except if specified” → “except where specified”

Done

Line 32. Can you state what temperature this corresponds to?

This corresponds to a temperature of about -9°C. Added

Page 3 Line 10. The connectivity is between the subglacial hydrologic system and the ocean. Just saying “bed” allows the possibility of a dry bed, which cannot support sliding.

Clarified

Page 4 Lines 8-11. This means the initial steady state is always approached through advance. This is a good design for retreat perturbations (I also have a paper coming out in TCD in the next few days that discusses multiple steady states and design of grounding line experiments). But you have not said how steady state is defined. You state that all spin up simulations are at least 50ka, which is good and should suffice for a robust starting point, but can you also add a statement about steady state, e.g. “ $dVAF/dt$ is less than $xx \text{ km}^3/a$ in all cases” or similar? [edit: I see you discuss Exp0 further down to look closer at steady state. So ignore my comment about quantifying steady state here, but

add a line like “Achievement of steady-state is analysed through Experiment 0 below”]

Done

Page 5 Lines 12-17. Note that “-Xm below sea level” contains a double negative (because the minus sign and the word “below” imply direction) and technically would mean Xm above sea level. You should say “Xm below sea level” or “-Xm relative to sea level (where X is positive in the upward direction)”.

The double negative was removed.

Lines 12-17. Please add the equation for this melt parameterisation, since you’ve shown one for the other parameterisation.

We added the melt equation for Experiment 2.

Page 6 Line 2. “resolution” → “resolutions”

Done

Line 2. Those numbers don’t look right to me, looking at the plot. Did you get the Tsai and Weertman laws mixed up in either the text of the plot?

The numbers are indeed the right numbers (the confusion probably comes from the most advanced position being on the right panel). We added a table detailing the initial conditions (grounding line position and volume above floatation) to clarify that.

Line 5. “small oscillations with minimal amplitude” → “small oscillations”

Done

Page 7 Lines 2-4. This is quite similar to our TC paper (Gladstone et al 2017): the first melt param used here (your exp 1) is similar to the water column scaling we used. Scaling the melt to zero as the GL is approached reduces the resolution dependency.

We added a sentence referencing this study in the discussion.

Line 6. “the type sub-element” → “the sub-element” or “the type of sub-element”

Done

Line 7. “a mass loss” → “mass loss”

Done

Page 8 Line 5. “why” → “whereas”

Done

Lines 6-7. Experiment names are repeated when they are clearly supposed to be different experiments.

We replaced the names with the right experiment names.

Page 9 line 11. I think the point here is not that the melt rates are small generally, but that they approach zero as the grounding line is approached (due to the water column scaling).

This is correct (and what is stated in the previous sentence), we clarified this sentence.

Page 10 line 13 to page 11 line 5. Do you think this problem is specific to the finite element method? Steph Cornford essentially predicted these results based on theoretical reasoning a couple of years ago (this was in a conversation, don't think he published anything like this). He said he would expect any parameterisation that allows melt on the last grounded grid point to overestimate retreat and to give worse convergence than a scheme that only applied melt to the first floating grid point. He mainly uses finite difference or finite volume methods.

This is indeed not specific to the finite element method, and we should expect similar results with other numerical methods. We added a sentence to generalize the results to other methods in the discussion.

Page 11 Line 13. “even other” → “even over”

Done

Page 12 Lines 11-17. Well, yes, but such processes could well mean that the melt parameterisations are actually closer to reality than NMP, though of course parameterising a tidal grounding zone should also not be resolution dependent.

We think this highlights that we need to better understand what happens close to the grounding line and in very shallow water columns, with the complexity of adding tides. However, guessing what would happen in the presence of tides is well beyond the scope of this manuscript.

Line 17. Isn't there a PISM paper that does exactly that ? using the grounding zone concept as a justification for inaccuracy... not sure if it is constructive to point the finger by citing it though...

We think parameterization of melt “at the grounding line” is an important point to study to raise awareness in the community and avoid redoing the same mistakes, not to emphasize what was not done wrong in the past.

Lines 18-23. One of the main points is similar to Gladstone et al 2017 (also TC), which used a Stokes flow model: that the convergence is worse, and resolution requirements are stricter, for the case of high melt close to the grounding line. The importance of vertical shear probably depends on choice of sliding law vertical shear should have a larger impact when using Weertman than with one of the sliding relations featuring a grounded transition zone.

We added a reference to the similar conclusion in Gladstone et al. (2017).

Line 28. “large amount of” → “large”

Done

line 29. “for a Weertman and a Tsai sliding laws”

Done

lines 29 and 31. Please indicate roughly what “large” and “small” mean here, for the benefit of people who just look at the pictures and read the conclusion!

Done

Figures. Fig 4. Right panel y-axis label. Minor formatting issue. Large gap in km^2 .

Done

Fig 4. I presume the Tsai purple line is hidden behind the green one? i.e. perfect convergence at 250m? You should state this in the caption or readers might think the purple line is missing. I find it confusing switching between Figures 5 and 7 because the colours have completely different meaning. Could you manage different line types in Fig 7 instead of different colours? Or if you want to stick with colours, can you make the colours different from those in Fig 5? I instinctively see the blue line in Fig 7 and think “ah that’s the 2km resolution”...

Done: added comment in the caption to say that purple and green line are superimposed. Also changed the colors for figures 7 and 8.

What I am missing from all Figures is a way to compare the converged result across different melt parameterisations. Of course the converged result should be the same across all melt parameterisations for a given sliding law. But this

is hard to compare in Fig 5 because they all have separate sub plots, and Fig 7 shows relative differences. I’m not sure the best way to show this, perhaps a new figure or just a table... and of course it may be complicated by the oscillations in the Weertman case. I don’t view this as essential, just “would be nice”.

We added two tables detailing the ice loss for experiments 1 and 2 to provide an easy comparison. We show ice mass loss values (similar to figures 5 and 6) and not total ice as the oscillations in the initial steady-state and different initial values between the resolutions could be confusing.

I found myself wanting to look at Fig 7 while looking at Fig 5. Maybe you should swap around Figs 6 and 7 and refer to the convergence plot a bit earlier in the text? Just a thought I am not insisting on this.

We decided to keep the order of the figures, as it was consistent with the reasoning in our results and discussion.

2 Reviewer #2 (D. Martin)

This work explores the convergence and accuracy characteristics of a set of choices in representing subshelf melting near marine ice sheet grounding lines. Since grounding lines often fall in the interior of computational grid cells, modelers are presented with a decision on how to represent subshelf melting in partially-grounded cells. One can either restrict melt in the model to cells which are entirely floating, include full representations of subshelf melting in all partially- and fully-grounded cells, or use some sort of scheme which reduces the model melt in partially-grounded cells to account for the fact that such cells are only partially floating. Existing model results in the literature employ the full range of these approaches, with unknown impacts on the model projections.

In this work, the authors employ four schemes to represent melt near grounding lines: (1) a scheme in which melt forcing is only applied to fully-floating cells (their “NMP”), (2) a scheme in which melt is applied fully to all cells which are even partially floating (FMP), and (3) two schemes which attempt to represent a subgrid-scale distribution of the melt forcing (in which forcing will tend to zero as the floating area in the cell tends to zero) (“SMP”). They apply these choices to an idealized marine ice stream problem (MISMIP+) with two melt parameterizations designed to test two different regimes. The experiments are carried out over a range of model resolutions, designed to examine the convergence behavior of each scheme. They find that schemes which apply melting to partially-grounded cells (both the “FMP” and “SMP”) tend to over-represent ice sheet response, particularly at coarse resolutions, while the no-melt (“NMP”) scheme tends to under-predict ice sheet response, while also displaying better accuracy and convergence properties. Therefore, their conclusion is that one should use schemes which don’t apply melt to partially-grounded cells, particularly at the coarse resolutions typically used for full-ice-sheet-scale models.

Given the importance of subshelf-melt forcing to marine ice sheet dynamics and its relevance to projections of ice sheet contributions to sea level rise, and the fact that many studies predict large melt rates near the grounding lines, this work is a very important step toward understanding how to incorporate subshelf melt into modeling efforts in an accurate way. The paper itself is well-constructed, clearly-written, and was a pleasure to read. The authors present a convincing explanation of their results, and their conclusion is well-supported by their results. I fully support publication, after a few minor fixes.

We thank Dan Martin for his detailed review and insightful comments.

Specific notes: 1. (p1, line 5): “which ultimately add...” add → adds

Done

2. (p2, line 20): It would be nice if you would also specify boundary conditions here to give a better feel for the problem without having to look up the citation for those unfamiliar with the Asay-Davis paper.

Done

3. Figure 3 (and accompanying text): It would be helpful to see a convergence plot like figures 7 and 8 for the steady-state initial condition (or the results of Experiment 0) in order to see how the model itself is converging independent of the melt behavior (to better place the melt convergence results into context).

Figure 7 and 8 show the convergence of the change in volume above floatation, and there is no change in volume above floatation in Experiment 0 (as it is just to test the steady-state). To be able to compare the initial steady-states, we added a table (Table 1) with the initial volumes above floatation and grounding line positions, as well as two tables showing the change in volume above floatation for experiments 1 and 2 so that the numbers can be easily compared.

4. Experiments 1 and 2: How far does the GL (centerline) retreat in these experiments? It’s useful to have some context relative to the coarse mesh spacing. (i.e. if it’s only retreating $O(10)$ 2km cells, that could be relevant, particularly in terms of how smooth the NMP and FMP parameterizations would appear to the ice sheet (since they’re discontinuous in time, while the SMP ones are continuous as the GL retreats)).

The grounding line retreat varies between 33 and 75 km, depending on the sliding law, melt experiment and melt parameterization. We added a couple of sentences in the results to discuss this retreat.

5. (Figure 4): It's apparent that the 2km results aren't even in the asymptotic convergence regime. (just an observation, which would be clearer if there was a figure like I suggest in (3) above)

Thanks for pointing this out.

6. (p7, line 6): "type sub-element" → "type of sub-element"

Done

7. (p7, line 7): "with a mass loss" → "with mass loss"

Done

8. (p7, line 10): Are the different experiments all converging to the same solution? I think they are, but you never actually say that, and it's hard to tell exactly from the figures given their size)

We added two tables as suggested by reviewer 1 so that it's easier to see if the solutions converge towards the same state.

9. (p8, line 5): "why the difference" → "while the difference"

Done

10. (Figures 5 and 6): I find the mesh-independence of the Weertman NMP case surprising, particularly for experiment 2, since you're potentially omitting a fair bit of melt near the GL. Any idea why this case looks that way?

We were also surprised by this result, so we ran the simulations several times and the results are robust. We don't have a detailed explanation, but we think this is caused by the Weertman sliding law not being dependent on the effective pressure, so the thinning is limited to the ice shelf and does not propagate far inland. As the friction remains unchanged in all cases (except where the grounding line retreats), the changes in driving stress between the different resolutions are similar enough to cause similar changes in the simulations.

11. (p.10, line 3): "overestime" → "overestimate"

Done

12. (p.10, line 8): "mass by a factor" → "mass loss by a factor"

Done

13. (p.10, line 9): "the grounding" → "the grounding line"

Done

14. (Figures 7 and 8): I *think* you're referencing each scheme to its own 125 m result, which is problematic given that you observe that not all of them are fully resolved at 125 m for experiment 2 (this will tend to underestimate the real error being made here for those cases, since figures 7-8 imply that all of the models have error which goes to 0 at 125 m.). I'd suggest instead referencing them all to the same baseline result. Since the 125 m NMP run appears to be converged, I'd suggest using that result as the baseline value to compare all of the other results to. Or, you could run a single 62.5m NMP run (which should be very close to the 125 NMP) and use *that* as the baseline solution. Regardless, you should clarify what the reference choice is.

We do reference each scheme to its 125 m resolution result. We think this has a limited impact, as the results between the different are not very different at this resolution for experiment 1 (see table 2). For experiment 2 (see table 3), the results between NMP and the other schemes are large enough that it is not entirely clear if we have reached a perfect convergence at 125 m. We added some clarification in the figure captions.

15. (Figures 7 and 8): It might help to mention that you're plotting the abs(error) in these plots, which winds up being the negative of the actual difference for the NMP case. (I spent some time trying to figure out why the NMP line in the Experiment 1 Tsai figure was above the FMP line, until I remembered the sign difference)

Agreed, we clarified the caption.

16. (p. 11, line 5): This conclusion likely holds broadly for any model which applies melt forcing over the entire cell (including finite-difference and finite-volume approaches), not just simply the ISSM FEM model. For example, I'd expect the finite-volume BISICLES to behave the same way.

We generalized the conclusion.

17. (p. 11, line 8): guaranty → guarantee

Done

18. (p. 11, line 13): “even other” → “even over”

Done

19. (p. 12, line 6): This is an important point which can't be repeated enough. You could cite Cornford et al (2016) here, which also makes the point about the necessity to quantify or clarify the effects of mesh resolution; both this work

and that one provide a template for how to go about doing it (mesh convergence study).

Good point, we added references to previous studies who mention the importance of quantifying mesh resolution.

References: 20. (p. 14, line 11). My name is spelled incorrectly

Sorry we misspelled your name. Fixed

21. (p. 14, line 29). “West Antarctica”

Done

22. (p. 15, line 3): “andvvan” $\rightarrow$ “and V van”

Done

Representation of basal melting at the grounding line in ice flow models

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Abstract. While a lot of attention has been given to the numerical implementation of grounding lines and basal friction in the grounding zone, little has been done about the impact of the numerical treatment of ocean-induced basal melting in this region. Several strategies are currently being employed in the ice sheet modeling community, and the resulting grounding line dynamics may differ strongly, which ultimately ~~add~~adds significant uncertainty to the projected contribution of marine ice sheets to sea level rise. We investigate here several implementations of basal melt parameterization on partially floating elements in a finite element framework, based on the Marine Ice Sheet-Ocean Model Intercomparison Project (MISOMIP) setup: (1) melt applied only to entirely floating elements, (2) melt applied over the entire elements that are crossed by the grounding line, and (3) melt integrated partially over the floating portion of a finite element using two different sub-element integration methods. All methods converge towards the same state when the mesh resolution is fine enough. However, (2) and (3) will systematically overestimate the rate of grounding line retreat in coarser resolutions, while (1) converges faster to the solution in most cases. The differences between sub-element parameterizations are exacerbated for experiments with large melting rates in the vicinity of the grounding line and for a Weertman sliding law. As most real-world simulations use horizontal mesh resolutions of several hundreds of meters at best, and large melt rates are generally present close to the grounding lines, we recommend using (1) to avoid overestimating the rate of grounding line retreat.

1 Introduction

Basal melt under floating ice tongues is important as it is one of the main factors driving the current increase in ice discharge in West Antarctica (e.g. Pritchard et al., 2012). Changes in basal melt impact ice shelf thickness, and thinning leads to a reduction of ice shelf buttressing, thereby leading to an acceleration of the ice streams feeding it. This acceleration is responsible for the dynamic thinning of the ice upstream of the grounding line, eventually leading to grounding line retreat, which ~~yields~~causes to a further increase in ice speed, and therefore ice discharge. Accurate representation of ice shelf ocean-induced melt in ice flow models is therefore critical. This remains an active field of research as observations of basal melt remain scarce, and new parameterizations are starting to emerge (Lazeroms et al., 2018; Reese et al., 2017).

Over the past decade, the ice sheet modeling community has made tremendous progress in terms of representation of grounding line dynamics in ice sheet models. Model intercomparisons have shown that lateral stress and high mesh resolution (below

2 km) in the grounding zone are required to accurately capture the behavior of the grounding line (Pattyn et al., 2012, 2013). New sub-element parameterizations of grounding line position and the representation of basal friction in partially floating elements showed promising results for both flow band and plan view models (Pattyn et al., 2006; Gladstone et al., 2010; Seroussi et al., 2014a; Feldmann et al., 2014), as they relaxed the mesh resolution requirements in this region. These studies, however, are all based on ideal geometries and completely ignored basal melt under floating ice (i.e., no melt is applied under floating ice). In reality, melt can be strong, especially in the vicinity of the grounding line, where it can reach ~ 100 m/yr (Dutrieux et al., 2013; Rignot et al., 2013; Berger et al., 2017). Several studies have showed that for the same melt parameterization, the choice of numerical implementation of melt has a strong impact on model results for both projections of the West Antarctic Ice Sheet (Cornford et al., 2016; Arthern and Williams, 2017) and idealized glaciers (Gladstone et al., 2017). This problem has however not been fully investigated or quantified yet, and it remains unclear what parameterizations should be employed in partially floating elements.

We investigate these questions here by using different numerical implementations of basal melting in partially floating elements and two friction laws on a setup similar to the Marine Ice Sheet-Ocean Model Intercomparison Project (MISOMIP) (Asay-Davis et al., 2016). We first summarize the model setup and detail the four different parameterizations of basal melt in elements partially floating and partially grounded. We then describe the experiments used to test these parameterizations. We present the results, discuss their impact ~~to model~~ on the modeling of grounding line evolution and conclude on the relevance of using sub-element parameterizations of ocean-induced melt under ice shelves.

2 Model

We use the Ice Sheet System Model (ISSM, Larour et al., 2012) to simulate the ice flow of an idealized case representative of outlet glaciers in West Antarctica (Asay-Davis et al., 2016). The model setup is identical to the one described in Asay-Davis et al. (2016) that we briefly summarize here. All the parameters are identical to their description, except ~~if~~ where specified otherwise.

The experiments simulate a glacier in a marine terminating confined valley, with a bedrock lying between -720 and 350 m as shown in Fig. 1a. The accumulation is uniform over the domain and set to 0.3 m/yr. Basal melting is applied under floating ice, with a different magnitude depending on the experiments. The model domain extends between 0 and 640 km, and between 0 and 80 km in the x and y direction, respectively. Boundary conditions are a no slip condition at $x = 0$ km, a free-slip condition at $y = 0$ and $y = 80$ km, and a fixed ice front at $x = 640$ km. This domain is discretized using a triangular mesh with resolutions of 2 km, 1 km, 500 m, 250 m, and 125 m resulting in meshes with a number of elements varying from 28,000 to 1,745,000. All mesh resolutions are spatially uniform, except in the case of the 125 m resolution mesh for which the model resolution is 125 m only in the portion of the domain located between $x = 300$ km and $x = 600$ km (i.e. where we expect to see the grounding line), the resolution is otherwise 1 km for $x < 200$ km, and 500 m for the rest of the domain.

The two-dimensional Shelfy-Stream Approximation (MacAyeal, 1989) is used as an approximation of the full-Stokes equations to solve the stress balance equations and the grounding line position is determined assuming hydrostatic equilibrium. The

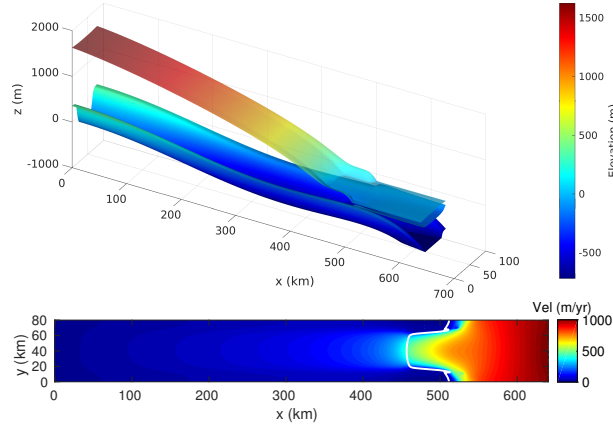


Figure 1. Model domain and initial steady-state geometry for the 125 m resolution mesh with a Weertman sliding law. (a) Bedrock elevation and initial steady-state ice surface and basal elevation (Note the vertical exaggeration). (b) Initial steady-state velocity (in m/yr). The white line shows the initial grounding line position.

ice rheology is spatially uniform in the domain and follows Glen’s flow law with a rate factor, A , equal to $2.0 \times 10^{-17} \text{ Pa}^{-3} \text{ yr}^{-1}$, equivalent to an ice temperature of about -9°C . We test here two different friction laws. The first one is a power sliding law, following Weertman (1957):

$$\boldsymbol{\tau}_b = -\beta^2 \|\mathbf{u}_b\|^{1/m-1} \mathbf{u}_b \quad (1)$$

- 5 with $\boldsymbol{\tau}_b$ the basal stress, $\mathbf{u}_b$ the basal velocity vector, $m = 3$ and β^2 the friction coefficient uniform in space and equal to $1.0 \times 10^4 \text{ Pa m}^{-1/3} \text{ yr}^{1/3}$. This friction law induces a sharp discontinuity in basal friction at the grounding line that is not realistic and not appropriate for problems investigating grounding line evolution, but remains nevertheless widely used in the community (Brondex et al., 2017).

The second sliding law is a modified power law designed to prevent the basal traction to exceed a fraction of the effective
10 pressure, proposed by Tsai et al. (2015):

$$\boldsymbol{\tau}_b = -\min\left(\alpha^2 N, \beta^2 \|\mathbf{u}_b\|^{1/m}\right) \|\mathbf{u}_b\|^{-1} \mathbf{u}_b \quad (2)$$

with $\alpha^2 = 0.5$ and N the effective pressure at the ice base, assuming a perfect connectivity of the bed-subglacial hydrologic system with the ocean.

The representation of basal friction at the grounding line is the same in all experiments, and follows the SEP2 parameteriza-
15 tion of Seroussi et al. (2014a). It has been shown that this parameterization is satisfactory to capture grounding line dynamics, as it converges faster to the solution as the mesh resolution increases compared to other methods.

In this study, we use the same methodology as Seroussi et al. (2014a), but apply it to sub-element melting parameterizations in elements partially floating and partially grounded. Fig. 2 shows the four different parameterizations adopted in this study. In

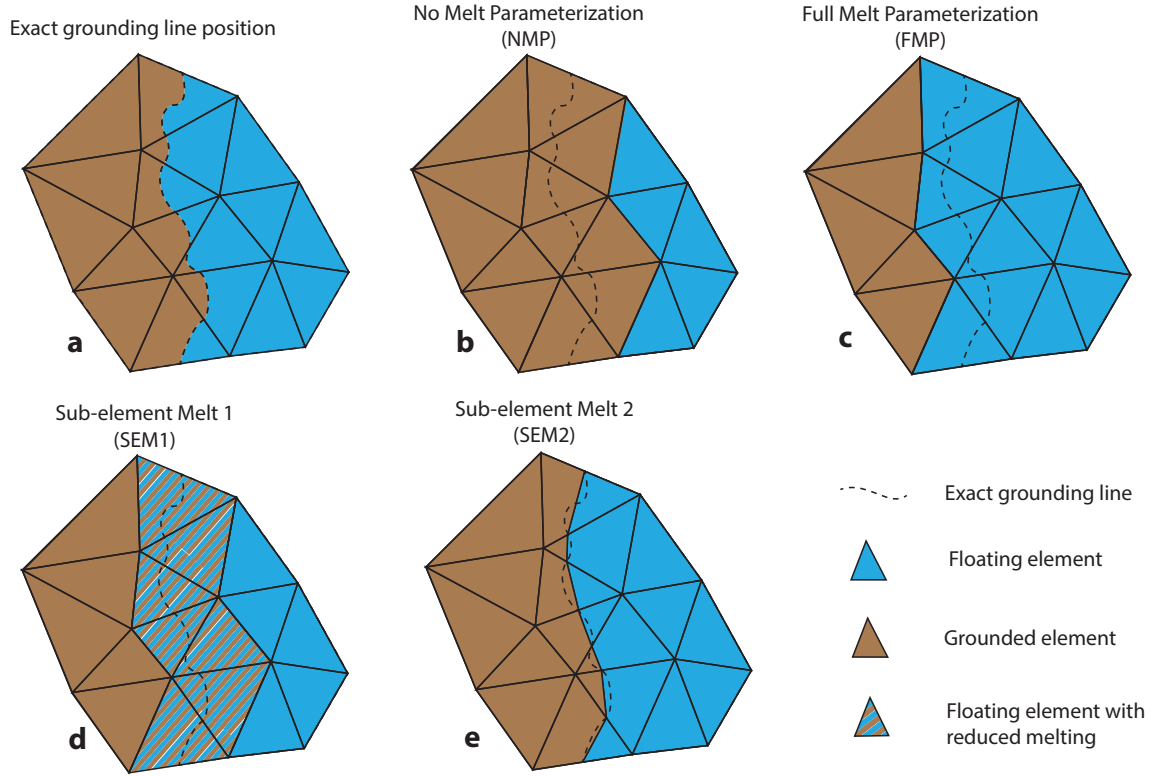


Figure 2. Grounding line discretization. Grounding line exact location (a), No Melt Parameterization (NMP, b), Full Melt Parameterization (FMP, c), Sub-Element Melt 1 (SEM1, d), and Sub-Element Melt 2 (SEM2, e)

the case of the ‘Full Melt parameterization’ (FMP), melt is applied everywhere over all partially floating elements, regardless of the exact position of the grounded line, while in the ‘No Melt Parameterization’ (NMP), there is no melt applied to any area of the partially floating elements. The last two cases use a sub-element parameterization. In the ‘Sub-Element Melt 1’ (SEM1), melt is applied to the entire area of partly floating element, but the magnitude of the melt is reduced by the fraction area of the floating ice in the element, so that the total melt applied is proportional to the floating ice area. In the ‘Sub-Element Melt 2’ (SEM2) parameterization, the ocean induced melt rate is integrated exactly over the floating part of the element in the mass transport equation, so that melt rate is only applied to the floating part of the element.

Testing two sliding laws, four melt parameterizations and five mesh resolutions results in a total of 40 different configurations. The same experiments are performed on each of these configurations.

3 Experiments

We first run every configuration to a steady-state ice stream without any melt. The initial ice thickness is equal to 1 m and the ice stream grows over several tens of thousands of years (at least 50,000 years) in response to surface mass balance accumulation, while no basal melting is applied under floating ice. This steady-state is therefore independent of the sub-element basal melt parameterization applied. Convergence of the solution to the steady-state is discussed in the analysis of Experiment 0 in section 4.

Starting from this steady-state, three transient experiments with varying ice shelf basal melting conditions are performed for a period of 100 years. In Experiment 0, no basal melting is applied under floating ice, similar to the steady-state initialization of the model. Experiment 0 is therefore mainly designed to check the initial steady-state. Basal melting is applied under floating ice in Experiment 1 and Experiment 2, and we assess the impact of the melt parameterization, model resolution and sliding laws on the glacier evolution. Experiment 1 is similar to the MISMP+ *Ice1r* experiment in Asay-Davis et al. (2016): basal melting varies spatially and represents a balance between the latent heat of melting and a parameterized ocean turbulent heat flux:

$$m_i = \Omega \tanh\left(\frac{H_c}{H_{c0}}\right) \max(z_0 - z_d, 0) \quad (3)$$

with Ω a coefficient equal to 0.2 yr^{-1} , H_c the water column thickness, z_d the ice shelf basal elevation, z_0 the depth above which the melt rate is equal to zero (100 m), and H_{c0} a constant equal to 75 m (see also equations (12)-(17) in Asay-Davis et al. (2016) for the derivation of this parameterization).

Experiment 2 is based on a basal melt under floating ice that varies linearly with depth, with a maximum melt magnitude of 30 m/yr in the deepest part, where the ice base is at or below ~~-500~~ -500 m below sea level, and that linearly decreases to 0 m/yr melt for ice base equal to ~~-50~~ -50 m below sea level. There is therefore no melt when the ice base is above ~~-50~~ -50 m below sea level:

$$m_i = \begin{cases} 0 \text{ m/yr}, & \text{if } z_d > -50 \text{ m} \\ -1/15(z_d + 50) \text{ m/yr}, & \text{if } -500 < z_d < -50 \text{ m} \\ 30 \text{ m/yr}, & \text{if } z_d < -500 \text{ m} \end{cases} \quad (4)$$

with z_d the ice shelf basal elevation. This experiment simulates ice shelves resting in warm waters, similarly to what has been observed in the Amundsen or Bellingshausen sea areas (e.g. Dutrieux et al., 2013; Rignot et al., 2013) and used in previous modeling experiments (e.g. Favier et al., 2014; Joughin et al., 2014; Seroussi et al., 2014b, 2017).

Experiments 0, 1 and 2 are all run for 100 years. We use the following convention to refer to the different experiments. For the steady-state (SS) and Experiment 0, names are as follows: EXP_sliding_resolution, where EXP is the number of the experiment (SS or EXP0), the sliding refers to the sliding law (Weertman or Tsai), and ‘resolution’ is the mesh resolution (2 km, 1 km, 500 m, 250 m, or 125 m), e.g., EXP0_Weertman_250m. For Experiment 1 and Experiment 2, the names are

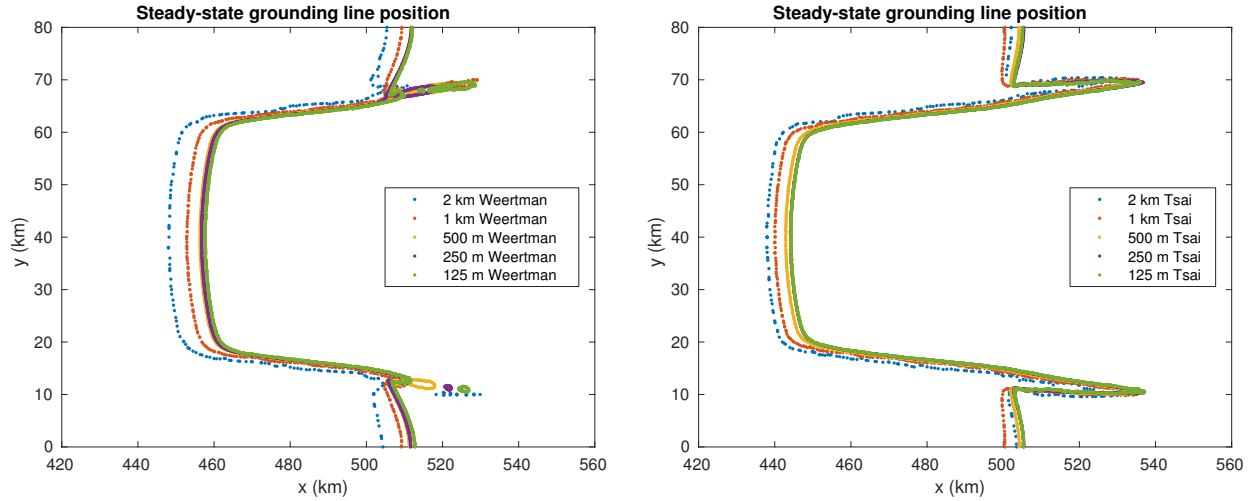


Figure 3. Steady-state grounding line positions for the Weertman (left) and Tsai (right) friction law for the 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple) and 125 m (green) mesh resolutions

similar: EXP_sliding_resolution_SEM, except that we add SEM, the sub-element melt parameterization at the grounding line (NMP, FMP, SEM1 or SEM2), e.g., EXP1_Weertman_250m_SEM1, as the results of these simulations now depend on the sub-element melt parameterization adopted.

4 Results

- 5 Figure 1 shows the initial steady-state configuration for SS_Weertman_125m. Its geometry is shown in Fig. 1a, and the velocity and grounding line in Fig. 1b. The grounding line position varies between 458 km in the centerline of the glacier and 528 km on its sides; the ice velocity is maximum at the ice front, reaching 1012 m/yr. This configuration is comparable to previous results based on the same geometry (Gudmundsson et al., 2012; Gudmundsson, 2013; Asay-Davis et al., 2016). The mesh resolution and the type of basal sliding law both impact the grounding line position as shown in Fig. 3. The grounding line position on the
- 10 glacier centerline varies between 438 km for SS_Tsai_2km and 458 km for SS_Weertman_125m, with a larger spread between the different ~~resolution-resolutions~~ for the Tsai friction law (9.6 km) than for the Weertman friction law (6.2 km) (Fig. 3 and Tab. 1).

- Experiment 0 is mostly designed to ensure that the model has reached a steady-state, as no melt is applied, similar to the initial steady-state. The ice mass above floatation (Fig. 4a) remains constant over the 100-year simulation for the 10 configurations,
- 15 while the grounded ice area (Fig. 4b) experiences small oscillations with ~~minimal-amplitudes~~small oscillations, especially for the Weertman sliding law. Such oscillations, that average to zero change in the grounded area over time, have been noted by Asay-Davis et al. (2016) and are orders of magnitude smaller than the changes simulated in Experiment 1 and Experiment 2. Figure 4 confirms that sub-kilometer resolution is needed to accurately capture the grounding line positions, similarly to

Table 1. Steady-state grounding line position in the glacier centerline and volume above floatation (VAF)

<u>Friction law</u>	<u>Resolution</u>	<u>GL($y = 40$ km)</u>	<u>VAF (Gt)</u>
<u>Weertman</u>	<u>2 km</u>	<u>448.0 km</u>	<u>46327</u>
<u>Weertman</u>	<u>1 km</u>	<u>452.8 km</u>	<u>47044</u>
<u>Weertman</u>	<u>500 m</u>	<u>456.3 km</u>	<u>47540</u>
<u>Weertman</u>	<u>250 m</u>	<u>456.6 km</u>	<u>47674</u>
<u>Weertman</u>	<u>125 m</u>	<u>456.7 km</u>	<u>47737</u>
<u>Tsai</u>	<u>2 km</u>	<u>437.9 km</u>	<u>44996</u>
<u>Tsai</u>	<u>1 km</u>	<u>440.0 km</u>	<u>45238</u>
<u>Tsai</u>	<u>500 m</u>	<u>442.9 km</u>	<u>45700</u>
<u>Tsai</u>	<u>250 m</u>	<u>444.1 km</u>	<u>45899</u>
<u>Tsai</u>	<u>125 m</u>	<u>444.1 km</u>	<u>45889</u>

what has been suggested by previous studies (e.g., Vieli and Payne (2005); Gladstone et al. (2010); Pattyn et al. (2012, 2013); Feldmann et al. (2014); Seroussi et al. (2014a)). The difference in modeled volume ([see table 1](#)) between the 1 km and 500 m models is 1.02% and 1.05%, and the difference in grounded area is 0.61% and 0.62% respectively for the Weertman and Tsai friction laws. Differences between models at 500 m, 250 m, and 125 m resolution are all well below 1% (the curves for SS_Tsai_125m and SS_Tsai_250m are superimposed on Fig. 4). By comparison, the difference in volume above floatation and grounded area between the two friction laws at 125 m resolution is respectively of 3.9% and 1.6%.

Experiment 1 simulates the evolution of the glacier when ocean induced melt is applied under floating ice. The equation that governs the melt rate in this experiment provides limited melt close to the grounding line, as the water column thickness becomes smaller (see Eq. 3). Figure 5 shows the evolution of the ice volume above floatation for this experiment for the different sub-element melt parameterizations, mesh resolutions and the two friction laws. The volume above floatation lost ([see also table 2](#)) varies between 4140 Gt and 6690 Gt for the EXP1_Weertman_2km_NMP and EXP1_Tsai_2km_FMP scenarios respectively. Experiments performed with the Tsai friction law show a larger mass loss (between 5480 and 6690 Gt over the 100-year period) than the ones performed with a Weertman friction law (between 4140 and 5410 Gt). The impact of the sub-element melt parameterization adopted, however, is more pronounced in the case of Weertman sliding law. The Tsai sliding law shows similar results for all sub-element parameterizations if the mesh resolution is 1 km and under, suggesting that any sub-element melt parameterization can be adopted in this case. Results performed at 2 km resolution all overestimate the mass loss, except when the NMP is adapted, which underestimate the mass loss. If the Weertman sliding law is applied, the results are strongly dependent on both the **type**-sub-element parameterization and the mesh resolution. SEM1, SEM2, and FMP behave very similarly, with **a**-mass loss being reduced as the resolution increases (from ~ 5400 Gt at 2 km resolution to ~ 4150 Gt at 250 m resolution). The difference between the runs becomes smaller as the mesh resolution increases, but the results are within 5%

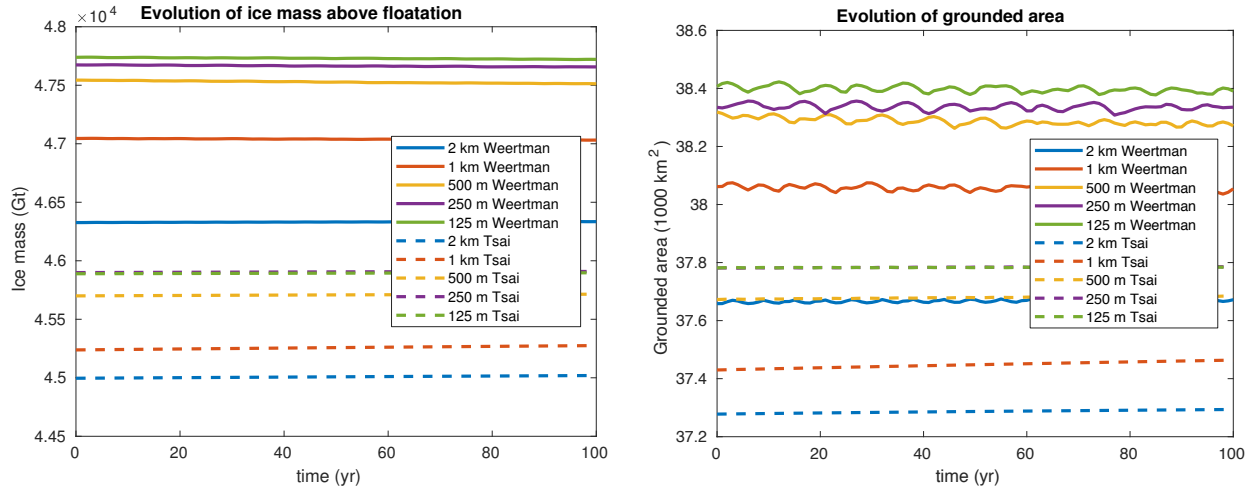


Figure 4. Evolution of ice volume above floatation (left) and grounded area (right) for Experiment 0 (steady-state case with no melt applied). Solid and dashed lines represent simulations with Weertman and Tsai friction respectively for resolutions of 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple), and 125 m (green). Results for 250 m and 125 m resolutions are superimposed for the Tsai friction.

Table 2. Change in volume above floatation (Δ VAF in Gt) in Experiment 1 for the Weertman (left) and Tsai (right) friction laws

<u>Melt Parameterization (Weertman)</u>					<u>Melt Parameterization (Tsai)</u>				
<u>Resolution</u>	<u>NMP</u>	<u>FMP</u>	<u>SEM1</u>	<u>SEM2</u>	<u>Resolution</u>	<u>NMP</u>	<u>FMP</u>	<u>SEM1</u>	<u>SEM2</u>
<u>2 km</u>	<u>-4137</u>	<u>-5411</u>	<u>-5210</u>	<u>-5304</u>	<u>2 km</u>	<u>-5480</u>	<u>-6692</u>	<u>-6504</u>	<u>-6576</u>
<u>1 km</u>	<u>-4272</u>	<u>-4724</u>	<u>-4637</u>	<u>-4673</u>	<u>1 km</u>	<u>-6127</u>	<u>-6454</u>	<u>-6394</u>	<u>-6417</u>
<u>500 m</u>	<u>-4246</u>	<u>-4359</u>	<u>-4331</u>	<u>-4340</u>	<u>500 m</u>	<u>-6261</u>	<u>-6333</u>	<u>-6318</u>	<u>-6324</u>
<u>250 m</u>	<u>-4225</u>	<u>-4252</u>	<u>-4244</u>	<u>-4246</u>	<u>250 m</u>	<u>-6293</u>	<u>-6315</u>	<u>-6304</u>	<u>-6305</u>
<u>125 m</u>	<u>-4196</u>	<u>-4221</u>	<u>-4213</u>	<u>-4215</u>	<u>125 m</u>	<u>-6294</u>	<u>-6307</u>	<u>-6309</u>	<u>-6311</u>

of the results obtained with a resolution of 125 m only for resolutions below 500 m. The NMP presents a completely different behavior, with results almost identical for all mesh resolutions for the Weertman sliding law (less than 150 Gt variation after 100 years). The runs relying on NMP underestimate the mass change for the Tsai friction law, with 650 Gt less mass loss for the EXP1_Tsai_2km_NMP compared to EXP1_Tsai_1km_NMP. During the experiment, the grounding line retreat in the centerline of the glacier varies between 40 and 55 km depending on the mesh resolution and the melt parameterization for the Weertman sliding law, and between 55 and 70 km for the Tsai sliding law, with larger retreats for the FMP, SMP1 and SMP2 at coarse resolution, and smaller retreats for FMP, SMP1 and SMP2 at fine resolution and NMP.

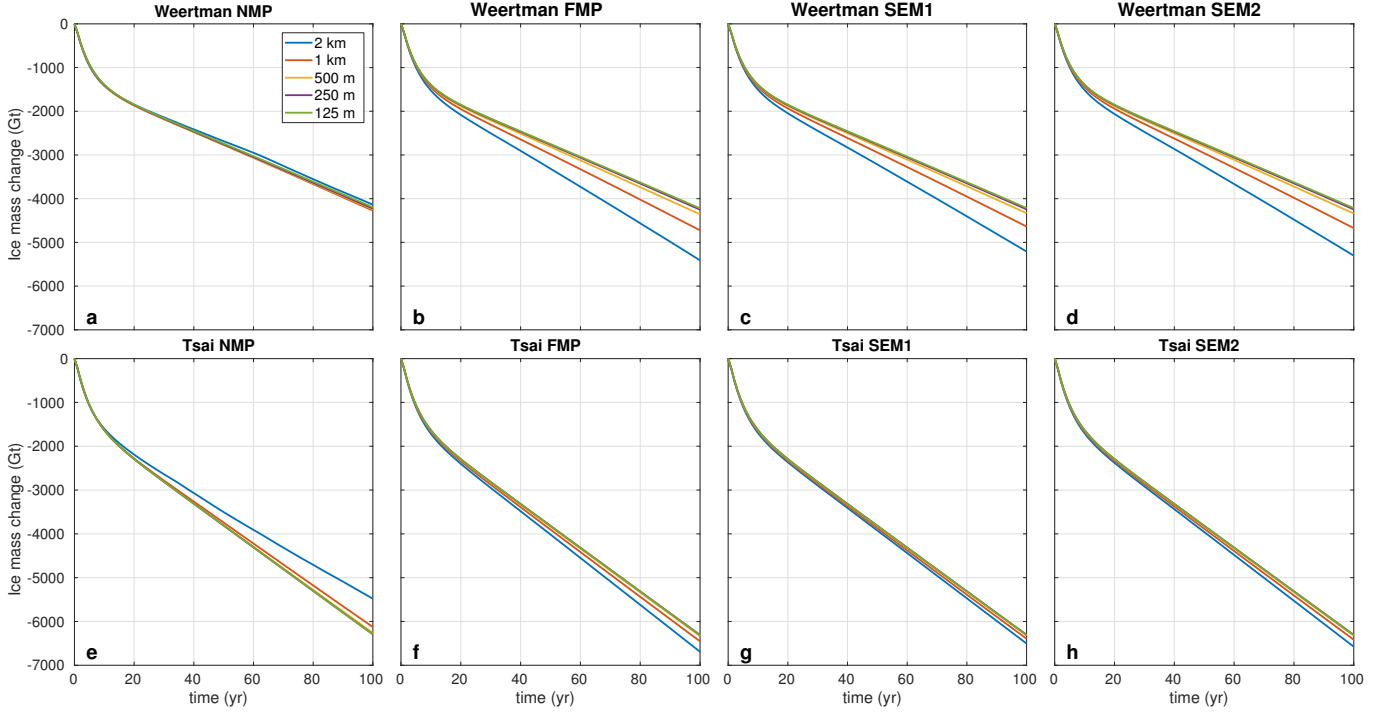


Figure 5. Evolution of ice volume above flotation in Experiment 1 for the NMP (a and e), FMP (b and f), SEM1 (c and g), and SEM2 (d and h), for the Weertman (a-d) and Tsai (f-h) friction laws. Each plot represents the evolution for the 5 mesh resolutions: 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple), and 125 m (green).

Table 3. Change in volume above flotation (Δ VAF in Gt) in Experiment 2 for the Weertman (left) and Tsai (right) friction laws

<u>Melt Parameterization (Weertman)</u>					<u>Melt Parameterization (Tsai)</u>				
<u>Resolution</u>	<u>NMP</u>	<u>FMP</u>	<u>SEM1</u>	<u>SEM2</u>	<u>Resolution</u>	<u>NMP</u>	<u>FMP</u>	<u>SEM1</u>	<u>SEM2</u>
<u>2 km</u>	<u>-4132</u>	<u>-6536</u>	<u>-5672</u>	<u>-5644</u>	<u>2 km</u>	<u>-4943</u>	<u>-7585</u>	<u>-6614</u>	<u>-6533</u>
<u>1 km</u>	<u>-4130</u>	<u>-5895</u>	<u>-5235</u>	<u>-5188</u>	<u>1 km</u>	<u>-5150</u>	<u>-7060</u>	<u>-6365</u>	<u>-6284</u>
<u>500 m</u>	<u>-4120</u>	<u>-5289</u>	<u>-4824</u>	<u>-4775</u>	<u>500 m</u>	<u>-5374</u>	<u>-6469</u>	<u>-6034</u>	<u>-5976</u>
<u>250 m</u>	<u>-4130</u>	<u>-4890</u>	<u>-4565</u>	<u>-4523</u>	<u>250 m</u>	<u>-5474</u>	<u>-6112</u>	<u>-5846</u>	<u>-5808</u>
<u>125 m</u>	<u>-4115</u>	<u>-4748</u>	<u>-4464</u>	<u>-4428</u>	<u>125 m</u>	<u>-5510</u>	<u>-6038</u>	<u>-5812</u>	<u>-5783</u>

In Experiment 2, a large ice shelf melt rate of up to 30 m/yr is applied under the ice shelf, including close to the grounding line. Figure 6 ~~shows~~ and table 3 ~~show~~ the results of this experiment for the different sub-element parameterizations, mesh resolutions, and the two sliding laws. The overall mass loss is similar to Experiment 1 and varies between 4110 Gt and 7590

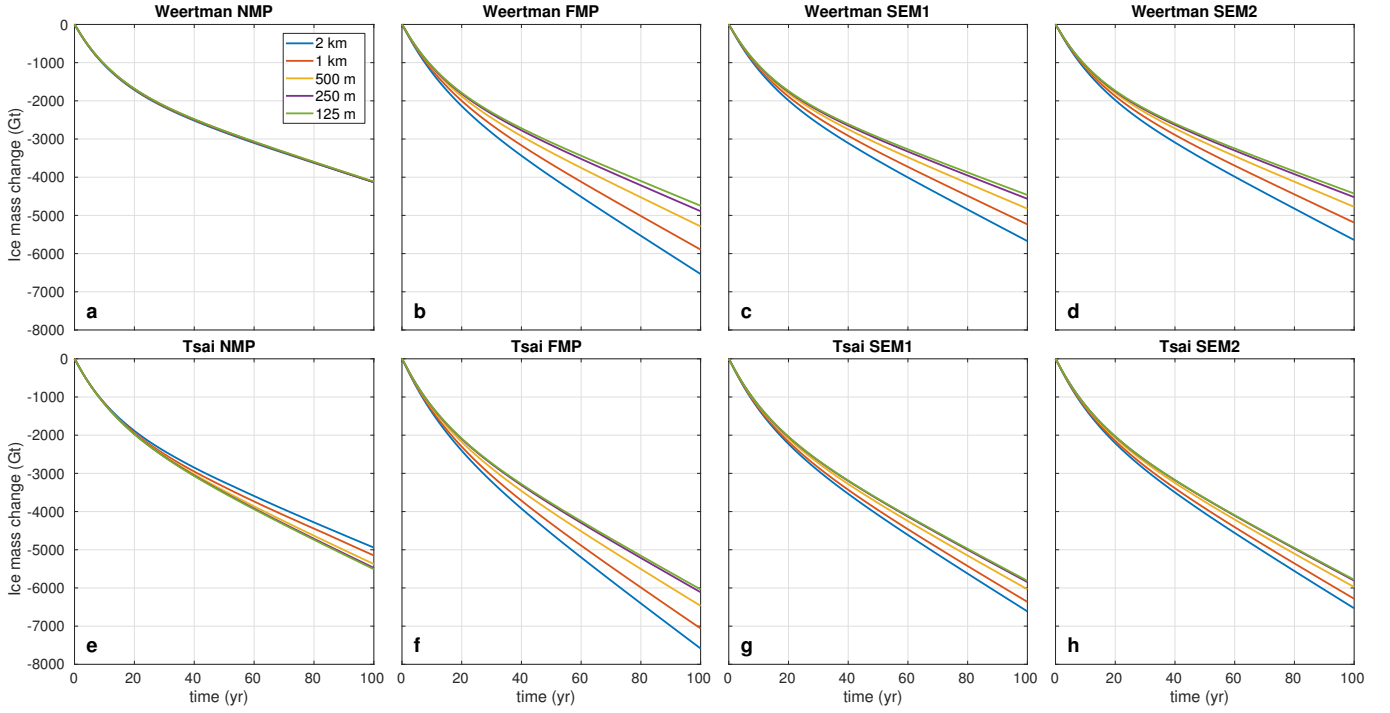


Figure 6. Evolution of ice volume above floatation in Experiment 2 for the NMP (a and e), FMP (b and f), SEM1 (c and g), and SEM2 (d and h), for the Weertman (a-d) and Tsai (f-h) friction laws. Each plot represents the evolution for the 5 mesh resolutions: 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple), and 125 m (green).

Gt for EXP1_Weertman_250m_NMP and EXP1_Tsai_2km_FMP scenarios respectively, with a larger ice loss for the Tsai friction law overall. The impact of mesh resolution and sub-element parameterization is more pronounced than in Experiment 1. At 2 km resolution, the difference in mass loss varies by 45% and 42% between NMP and FMP for both the Weertman and Tsai sliding laws, respectively. This spread is reduced as the mesh resolution increases, but a 125 m resolution is not sufficient to have similar results for NMP and FMP (14% and 9% difference between NMP and FMP at 125 m resolution for the Weertman and Tsai friction laws), suggesting that not all parameterizations have fully converged despite the level of mesh resolution. The SEM1 and SEM2 results are intermediate between FMP and NMP and behave similarly in all cases. Figure 6 also shows that NMP is by far the least sensitive to mesh resolution for the Weertman sliding law, with, e.g., a mass change of only 20 Gt between EXP2_Weertman_2km_NMP and EXP2_Weertman_250m125m_NMP, ~~why~~ whereas the difference reaches ~~+210-1216~~ Gt between EXP2_Weertman_2km_SEM1 and EXP2_Weertman_2km125m_SEM1, and ~~+710-1790~~ Gt between EXP2_Weertman_250m2km_FMP and EXP2_Weertman_250m125m_FMP. Results performed with the two sub-element melt parameterizations show a reduced dependence on mesh resolution. This improvement is not sufficient, however, to have accurate results with relatively coarse mesh resolutions. The impact of mesh resolution and sub-element melt

parameterization is more pronounced with the Weertman than the Tsai sliding friction law. Similarly to what was observed for Experiment 1, experiments performed with the Tsai friction law show less sensitivity to sub-element parameterization and mesh resolution than the Weertman friction law, except for NMP simulations that experience a mass loss reduced by 570 Gt over 100 years for the EXP2_Tsai_2km_NMP compared to the EXP2_Tsai_125m_NMP. The difference in ice loss after 100 years between EXP2_Tsai_2km_FMP and the EXP2_Tsai_125m_FMP and between EXP2_Tsai_2km_SEM1 and EXP2_Tsai_125m_SEM1 is respectively reduced by 1000 and 800 Gt. During this experiment, the grounding line retreat in the centerline of the glacier varies between 33 and 63 km depending on the mesh resolution and the melt parameterization for the Weertman sliding law, and between 42 and 75 km for the Tsai sliding law, with larger retreats for the FMP, SMP1 and SMP2 at coarse resolution, and smaller retreats for NMP and FMP, SMP1 and SMP2 at fine resolution.

10 5 Discussion

The results presented in this study show that the impact of sub-element melt parameterization and mesh resolution is different for the Weertman and Tsai friction laws. Models relying on Weertman sliding laws are more sensitive to the mesh resolution and the type of sub-element melt parameterization than when a Tsai sliding law is employed. These conclusions are in agreement with the ones of Gladstone et al. (2017) on a flowline case. Figures 7 and 8 show the convergence of results with mesh resolution for the four sub-element mesh parameterizations. For the Weertman sliding law, the results vary by less than 2.0% for all the mesh resolutions regardless of the melt applied when NMP is used. Results using SEM1, SEM2, and FMP vary by at least one order of magnitude more, demonstrating that these parameterizations are not satisfying in this case. When a Tsai sliding law is used, the results vary depending on the amount of sub-ice shelf melt close to the grounding line. When ~~small-melt-rates~~ melt rates converging towards zero close to the grounding line are applied, SEM1 and SEM2 converge slightly faster than FMP and NMP, and results within 5% of the 125 m resolution runs can be obtained for all sub-element parameterizations for mesh resolutions of 1 km or less. When large melt rates are applied close to the grounding line (Experiment 2), NMP converges the fastest but the behavior of SEM1 and SEM2 is close to NMP, with NMP underestimating the mass loss, while SEM1 and SEM2 ~~overestimate~~ overestimate it. In all cases, SEM1 and SEM2 results are almost identical (similarly to what was observed for sub-element parameterization of basal friction, see Seroussi et al. (2014a)) and are intermediate between NMP and FMP. Differences between mass loss produced with NMP and FMP can be as large as about 50% for 2 km mesh resolution (see Fig. 6). This difference is reduced as the mesh resolution increases, but remains larger than 10% even at 125 m resolution (see Fig.6) for large melt rates. Using the FMP never produces the best convergence of results and overestimates the mass loss by a factor of two in several cases, it should therefore be avoided. NMP shows the least dependence on mesh resolution, except for small melt rates close to the grounding line and a Tsai friction law (Fig. 7 and 8).

To explain this behavior, one needs to look at the numerical implementation of the equations that are affected by melt. The ocean induced melt is only present as a right-hand side term in the mass transport equation:

$$\frac{\partial H}{\partial t} = -\nabla \cdot H \bar{\mathbf{v}} + \dot{a} - m_i \quad (5)$$

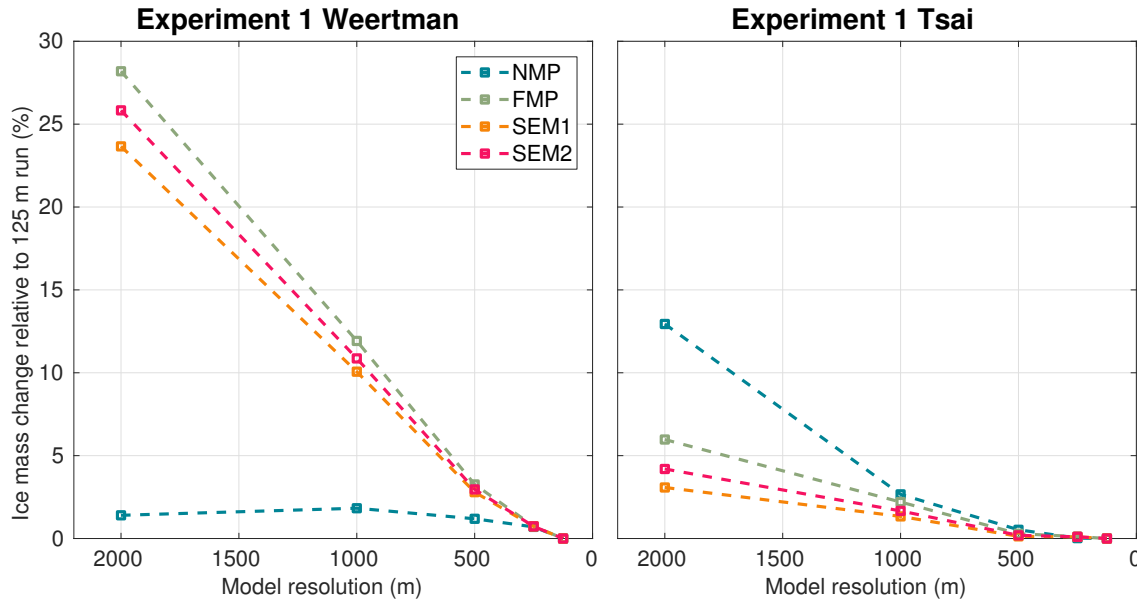


Figure 7. Convergence of ice volume above floatation at the end of Experiment 1 as a function of mesh resolution and Absolute error relative to the corresponding 125 mesh resolution results (same friction law and melt parameterization scheme) for the Weertman (a) and Tsai (b) friction laws for the NMP (blue), FMP (redgreen), SEM1 (yelloworange), and SEM2 (purplered) sub-element melt parameterizations.

where H is the ice thickness, $\bar{v}$ is the depth average ice velocity, $\dot{a}$ is the surface mass balance. With the finite element method, H is assumed to be a sum of nodal functions, and integrating basal melt, m_i , over partially floating elements will lead to a thinning at the grounded nodes of these elements that is inherent to the finite element method. In other words, applying melt in partially floating elements will induce a thinning upstream of the grounding line that is purely numerical, and the grounding line retreat will therefore be systematically overestimated. Using the no melt parameterization, no numerical thinning is applied to the grounded nodes of partially floating elements. Additional experiments, not shown here, confirm that even with a perfectly static marine ice sheet system (i.e., $\mathbf{v} = \mathbf{0}$ at all time), the grounding line will artificially retreat, except for the NMP, which confirms that it is the numerically correct way of treating basal melting in partially floating elements or cells, regardless of the numerical method adopted.

- Unlike what has been recommended for sub-element parameterizations of basal friction at the grounding line (e.g., Pattyn et al., 2006; Vieli and Payne, 2005; Feldmann et al., 2014; Seroussi et al., 2014a), using a sub-element *melt* parameterization does therefore not guaranty-guarantee an improvement compared to simulations that do not include such implementations, and does not necessarily relax the requirements of mesh resolutions. This is especially true when large melt rates are applied in the vicinity of the grounding line and for the Weertman sliding law. Many simulations in the Amundsen Sea Sector of West Antarctica (e.g. Favier et al., 2014; Joughin et al., 2014; Seroussi et al., 2014b) applied large melt rates in this region, consistently with observations (Dutrieux et al., 2013). A previous model study performed with NMP and SEM1 on this region

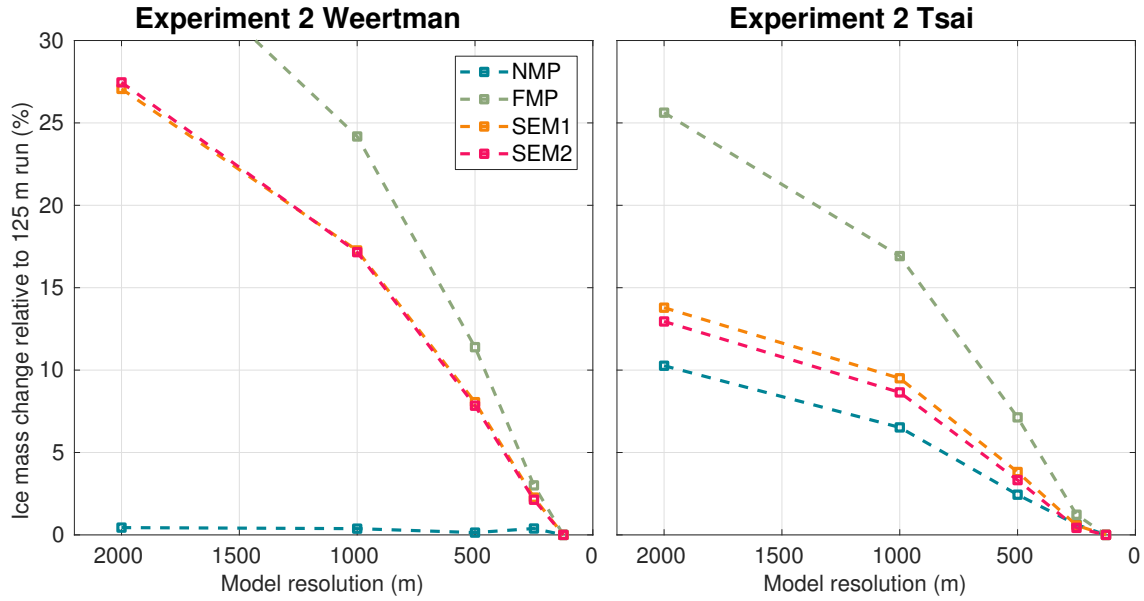


Figure 8. Convergence of ice volume above flotation at the end of Experiment 2 as a function of mesh resolution and Absolute error relative to the corresponding 125 mesh resolution results (same friction law and melt parameterization scheme) for the Weertman (a) and Tsai (b) friction laws for the NMP (blue), FMP (redgreen), SEM1 (yelloworange), and SEM2 (purplered) sub-element melt parameterizations.

showed extreme differences even other-over 100 years, and a potential collapse of Thwaites Glacier in less than 100 years for large melt rate scenarios (Arthern and Williams, 2017). Our study sheds light on this problem, as the SEM1 was probably under-resolved, leading to an overestimation of grounding line retreat.

In this study, we only considered mesh resolutions that are 2 km or less. However, large scale simulations of the Antarctic ice sheet typically rely on significantly coarser resolutions (e.g., Golledge et al., 2015; DeConto and Pollard, 2016; Pollard et al., 2015), especially when performing long term simulations. In this case, using the FMP, SEM1, and SEM2 will always lead to large overestimates in the amount of mass loss and even collapse of entire regions if large melt rates are applied close to the grounding line or if experiment scenarios include large melt rates in these regions, for both Weertman and Tsai sliding laws. Quantifying As mentioned in previous studies (e.g., Cornford et al., 2016; Gladstone et al., 2017), quantifying the impact of mesh resolution on model results is therefore extremely important in this case in order to provide reliable estimates of uncertainties in ice sheet mass loss over the coming decades and centuries. This is especially important when simulating the collapse of marine terminating glaciers resting on retrograde bed slope that are sensitive to the Marine Ice Sheet Instability (MISI, Weertman (1974)), as such an instability would be potentially simulated several centuries too early if ice shelf melt rates are applied on partially floating elements (Arthern and Williams, 2017; Golledge et al., 2015).

The results presented here were all performed on simulations that experience grounding line retreat and no grounding line advance. As most glaciers around the world are experiencing sustained retreat in response to climate change, cases of

grounding line advance are less common. The numerical scheme or resolution needed to correctly reproduce grounding line advance are however different than those needed to accurately capture grounding line retreat: Gladstone et al. (2017) showed that convergence was even worse in the case of grounding line advance. Convergence tests are even more critical to perform in such a case.

5 Grounding lines are constantly migrating, not only on long time scales due to changes in oceanic or atmospheric conditions, but also over short time scales with tides (e.g. Gudmundsson, 2007; Le Meur et al., 2014; Padman et al., 2018). Observations show that melting in the grounding zones is complex and tidal motion probably involves complex melt rate patterns changing on tidal time scales as grounding line advances and retreats, and tidal flexure pumps ocean water in the grounding zone (Walker et al., 2013). This process could lead to more complicated patterns than the ones used in this study, assuming that the ice shelf
10 is in hydrostatic equilibrium. However, such processes remain poorly understood, additional studies are required to better evaluate them, and should not be used as a justification for numerical model inaccuracy.

 All the simulations performed in this study are based on the two-dimensional SSA. We expect, however, the results to be qualitatively similar for other stress balance approximations that determine the grounding line position based on the hydrostatic equilibrium, as melt rates in partially floating elements are treated in a similar way regardless of the stress balance approximation.
15 Using a Stokes flow line model, Gladstone et al. (2017) demonstrate a similar greater dependence of model results when large melt rates are applied close to the grounding line and the need for stricter resolution requirements. Simulations performed with three dimensional higher-order (Pattyn, 2003) or L1L2 (Hindmarsh, 2004) models should however generally experience lower changes in these cases, as previous studies showed that SSA models tend to respond more quickly than models including vertical shear (Pattyn et al., 2013; Pattyn and Durand, 2013).

20 6 Conclusions

 In this study we investigate the impact of the numerical implementation of ice shelf melt rates immediately downstream of the grounding line. We compare several sub-element parameterizations that (1) do not apply any melt over partially floating elements, (2) apply basal melt over the entire partially floating elements, or (3) apply some melt over partially floating elements. Simulations are performed with different mesh resolutions for two experiments with small and large ~~amount of~~ melt rates close
25 to the grounding line, and for a Weertman and a Tsai sliding laws. Our results demonstrate that, for limited melt rates in the order of 1 m/yr close to the grounding line, all sub-element melt parameterizations behave similarly for resolutions lower than 1 km and 500 m respectively for the Tsai and Weertman friction laws. For large melt rates in the order of 30 m/yr just downstream of the grounding line, however, models based on varying resolutions and sub-element melt rates behave differently. Both (2) and (3) overestimate the mass loss and resolutions well below 500 m are needed, while (1) shows a behavior that is less
30 dependent on the mesh resolution. These results were performed using the finite element method, but can be extrapolated to other numerical methods, such as the finite element and finite volume methods. As continental scale simulations of Antarctica typically use resolutions of several kilometers in the grounding line region, we therefore recommend models not to apply ice shelf melt rates in partially floating elements and to carefully assess the impact of mesh resolution on their simulation results.

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[for their constructive comments that improved the clarity of the paper.](#)

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Representation of basal melting at the grounding line in ice flow models

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Abstract. While a lot of attention has been given to the numerical implementation of grounding lines and basal friction in the grounding zone, little has been done about the impact of the numerical treatment of ocean-induced basal melting in this region. Several strategies are currently being employed in the ice sheet modeling community, and the resulting grounding line dynamics may differ strongly, which ultimately adds significant uncertainty to the projected contribution of marine ice sheets to sea level rise. We investigate here several implementations of basal melt parameterization on partially floating elements in a finite element framework, based on the Marine Ice Sheet-Ocean Model Intercomparison Project (MISOMIP) setup: (1) melt applied only to entirely floating elements, (2) melt applied over the entire elements that are crossed by the grounding line, and (3) melt integrated partially over the floating portion of a finite element using two different sub-element integration methods. All methods converge towards the same state when the mesh resolution is fine enough. However, (2) and (3) will systematically overestimate the rate of grounding line retreat in coarser resolutions, while (1) converges faster to the solution in most cases. The differences between sub-element parameterizations are exacerbated for experiments with large melting rates in the vicinity of the grounding line and for a Weertman sliding law. As most real-world simulations use horizontal mesh resolutions of several hundreds of meters at best, and large melt rates are generally present close to the grounding lines, we recommend using (1) to avoid overestimating the rate of grounding line retreat.

1 Introduction

Basal melt under floating ice tongues is important as it is one of the main factors driving the current increase in ice discharge in West Antarctica (e.g. Pritchard et al., 2012). Changes in basal melt impact ice shelf thickness, and thinning leads to a reduction of ice shelf buttressing, thereby leading to an acceleration of the ice streams feeding it. This acceleration is responsible for the dynamic thinning of the ice upstream of the grounding line, eventually leading to grounding line retreat, which causes to a further increase in ice speed, and therefore ice discharge. Accurate representation of ice shelf ocean-induced melt in ice flow models is therefore critical. This remains an active field of research as observations of basal melt remain scarce, and new parameterizations are starting to emerge (Lazeroms et al., 2018; Reese et al., 2017).

Over the past decade, the ice sheet modeling community has made tremendous progress in terms of representation of grounding line dynamics in ice sheet models. Model intercomparisons have shown that lateral stress and high mesh resolution (below

2 km) in the grounding zone are required to accurately capture the behavior of the grounding line (Pattyn et al., 2012, 2013). New sub-element parameterizations of grounding line position and the representation of basal friction in partially floating elements showed promising results for both flow band and plan view models (Pattyn et al., 2006; Gladstone et al., 2010; Seroussi et al., 2014a; Feldmann et al., 2014), as they relaxed the mesh resolution requirements in this region. These studies, however, are all based on ideal geometries and completely ignored basal melt under floating ice (i.e., no melt is applied under floating ice). In reality, melt can be strong, especially in the vicinity of the grounding line, where it can reach ~ 100 m/yr (Dutrieux et al., 2013; Rignot et al., 2013; Berger et al., 2017). Several studies have showed that for the same melt parameterization, the choice of numerical implementation of melt has a strong impact on model results for both projections of the West Antarctic Ice Sheet (Cornford et al., 2016; Arthern and Williams, 2017) and idealized glaciers (Gladstone et al., 2017). This problem has however not been fully investigated or quantified yet, and it remains unclear what parameterizations should be employed in partially floating elements.

We investigate these questions here by using different numerical implementations of basal melting in partially floating elements and two friction laws on a setup similar to the Marine Ice Sheet-Ocean Model Intercomparison Project (MISOMIP) (Asay-Davis et al., 2016). We first summarize the model setup and detail the four different parameterizations of basal melt in elements partially floating and partially grounded. We then describe the experiments used to test these parameterizations. We present the results, discuss their impact on the modeling of grounding line evolution and conclude on the relevance of using sub-element parameterizations of ocean-induced melt under ice shelves.

2 Model

We use the Ice Sheet System Model (ISSM, Larour et al., 2012) to simulate the ice flow of an idealized case representative of outlet glaciers in West Antarctica (Asay-Davis et al., 2016). The model setup is identical to the one described in Asay-Davis et al. (2016) that we briefly summarize here. All the parameters are identical to their description, except where specified otherwise.

The experiments simulate a glacier in a marine terminating confined valley, with a bedrock lying between -720 and 350 m as shown in Fig. 1a. The accumulation is uniform over the domain and set to 0.3 m/yr. Basal melting is applied under floating ice, with a different magnitude depending on the experiments. The model domain extends between 0 and 640 km, and between 0 and 80 km in the x and y direction, respectively. Boundary conditions are a no slip condition at $x = 0$ km, a free-slip condition at $y = 0$ and $y = 80$ km, and a fixed ice front at $x = 640$ km. This domain is discretized using a triangular mesh with resolutions of 2 km, 1 km, 500 m, 250 m, and 125 m resulting in meshes with a number of elements varying from 28,000 to 1,745,000. All mesh resolutions are spatially uniform, except in the case of the 125 m resolution mesh for which the model resolution is 125 m only in the portion of the domain located between $x = 300$ km and $x = 600$ km (i.e. where we expect to see the grounding line), the resolution is otherwise 1 km for $x < 200$ km, and 500 m for the rest of the domain.

The two-dimensional Shelfy-Stream Approximation (MacAyeal, 1989) is used as an approximation of the full-Stokes equations to solve the stress balance equations and the grounding line position is determined assuming hydrostatic equilibrium. The

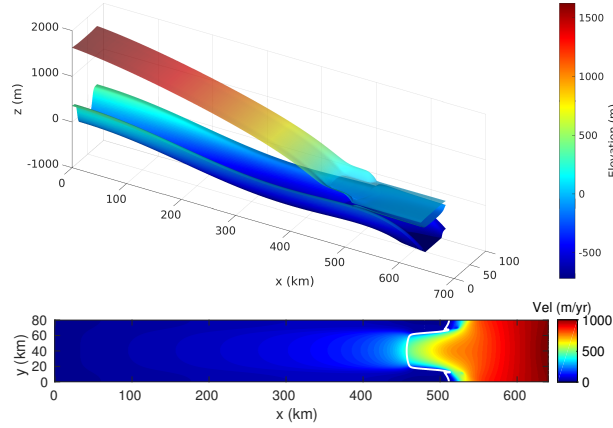


Figure 1. Model domain and initial steady-state geometry for the 125 m resolution mesh with a Weertman sliding law. (a) Bedrock elevation and initial steady-state ice surface and basal elevation (Note the vertical exaggeration). (b) Initial steady-state velocity (in m/yr). The white line shows the initial grounding line position.

ice rheology is spatially uniform in the domain and follows Glen’s flow law with a rate factor, A , equal to $2.0 \times 10^{-17} \text{ Pa}^{-3} \text{ yr}^{-1}$, equivalent to an ice temperature of about -9°C . We test here two different friction laws. The first one is a power sliding law, following Weertman (1957):

$$\boldsymbol{\tau}_b = -\beta^2 \|\mathbf{u}_b\|^{1/m-1} \mathbf{u}_b \quad (1)$$

- 5 with $\boldsymbol{\tau}_b$ the basal stress, $\mathbf{u}_b$ the basal velocity vector, $m = 3$ and β^2 the friction coefficient uniform in space and equal to $1.0 \times 10^4 \text{ Pa m}^{-1/3} \text{ yr}^{1/3}$. This friction law induces a sharp discontinuity in basal friction at the grounding line that is not realistic and not appropriate for problems investigating grounding line evolution, but remains nevertheless widely used in the community (Brondex et al., 2017).

The second sliding law is a modified power law designed to prevent the basal traction to exceed a fraction of the effective
 10 pressure, proposed by Tsai et al. (2015):

$$\boldsymbol{\tau}_b = -\min\left(\alpha^2 N, \beta^2 \|\mathbf{u}_b\|^{1/m}\right) \|\mathbf{u}_b\|^{-1} \mathbf{u}_b \quad (2)$$

with $\alpha^2 = 0.5$ and N the effective pressure at the ice base, assuming a perfect connectivity of the subglacial hydrologic system with the ocean.

The representation of basal friction at the grounding line is the same in all experiments, and follows the SEP2 parameteriza-
 15 tion of Seroussi et al. (2014a). It has been shown that this parameterization is satisfactory to capture grounding line dynamics, as it converges faster to the solution as the mesh resolution increases compared to other methods.

In this study, we use the same methodology as Seroussi et al. (2014a), but apply it to sub-element melting parameterizations in elements partially floating and partially grounded. Fig. 2 shows the four different parameterizations adopted in this study. In

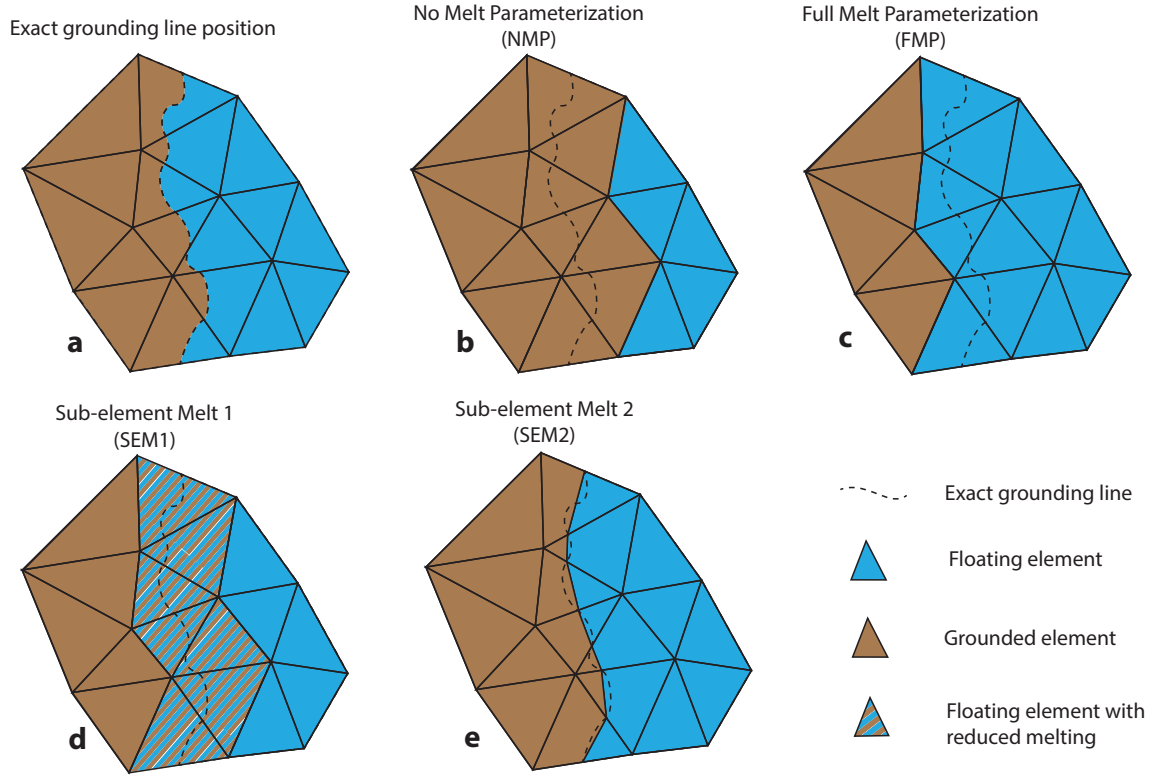


Figure 2. Grounding line discretization. Grounding line exact location (a), No Melt Parameterization (NMP, b), Full Melt Parameterization (FMP, c), Sub-Element Melt 1 (SEM1, d), and Sub-Element Melt 2 (SEM2, e)

the case of the ‘Full Melt parameterization’ (FMP), melt is applied everywhere over all partially floating elements, regardless of the exact position of the grounded line, while in the ‘No Melt Parameterization’ (NMP), there is no melt applied to any area of the partially floating elements. The last two cases use a sub-element parameterization. In the ‘Sub-Element Melt 1’ (SEM1), melt is applied to the entire area of partly floating element, but the magnitude of the melt is reduced by the fraction area of the floating ice in the element, so that the total melt applied is proportional to the floating ice area. In the ‘Sub-Element Melt 2’ (SEM2) parameterization, the ocean induced melt rate is integrated exactly over the floating part of the element in the mass transport equation, so that melt rate is only applied to the floating part of the element.

Testing two sliding laws, four melt parameterizations and five mesh resolutions results in a total of 40 different configurations. The same experiments are performed on each of these configurations.

3 Experiments

We first run every configuration to a steady-state ice stream without any melt. The initial ice thickness is equal to 1 m and the ice stream grows over several tens of thousands of years (at least 50,000 years) in response to surface mass balance accumulation, while no basal melting is applied under floating ice. This steady-state is therefore independent of the sub-element basal melt parameterization applied. Convergence of the solution to the steady-state is discussed in the analysis of Experiment 0 in section 4.

Starting from this steady-state, three transient experiments with varying ice shelf basal melting conditions are performed for a period of 100 years. In Experiment 0, no basal melting is applied under floating ice, similar to the steady-state initialization of the model. Experiment 0 is therefore mainly designed to check the initial steady-state. Basal melting is applied under floating ice in Experiment 1 and Experiment 2, and we assess the impact of the melt parameterization, model resolution and sliding laws on the glacier evolution. Experiment 1 is similar to the MISMIP+ *Ice1r* experiment in Asay-Davis et al. (2016): basal melting varies spatially and represents a balance between the latent heat of melting and a parameterized ocean turbulent heat flux:

$$m_i = \Omega \tanh\left(\frac{H_c}{H_{c0}}\right) \max(z_0 - z_d, 0) \quad (3)$$

with Ω a coefficient equal to 0.2 yr^{-1} , H_c the water column thickness, z_d the ice shelf basal elevation, z_0 the depth above which the melt rate is equal to zero (100 m), and H_{c0} a constant equal to 75 m (see also equations (12)-(17) in Asay-Davis et al. (2016) for the derivation of this parameterization).

Experiment 2 is based on a basal melt under floating ice that varies linearly with depth, with a maximum melt magnitude of 30 m/yr in the deepest part, where the ice base is at or below 500 m below sea level, and that linearly decreases to 0 m/yr melt for ice base equal to 50 m below sea level. There is therefore no melt when the ice base is above 50 m below sea level:

$$m_i = \begin{cases} 0 \text{ m/yr,} & \text{if } z_d > -50 \text{ m} \\ -1/15(z_d + 50) \text{ m/yr,} & \text{if } -500 < z_d < -50 \text{ m} \\ 30 \text{ m/yr,} & \text{if } z_d < -500 \text{ m} \end{cases} \quad (4)$$

with z_d the ice shelf basal elevation. This experiment simulates ice shelves resting in warm waters, similarly to what has been observed in the Amundsen or Bellingshausen sea areas (e.g. Dutrieux et al., 2013; Rignot et al., 2013) and used in previous modeling experiments (e.g. Favier et al., 2014; Joughin et al., 2014; Seroussi et al., 2014b, 2017).

Experiments 0, 1 and 2 are all run for 100 years. We use the following convention to refer to the different experiments. For the steady-state (SS) and Experiment 0, names are as follows: EXP_sliding_resolution, where EXP is the number of the experiment (SS or EXP0), the sliding refers to the sliding law (Weertman or Tsai), and ‘resolution’ is the mesh resolution (2 km, 1 km, 500 m, 250 m, or 125 m), e.g., EXP0_Weertman_250m. For Experiment 1 and Experiment 2, the names are similar: EXP_sliding_resolution_SEM, except that we add SEM, the sub-element melt parameterization at the grounding line

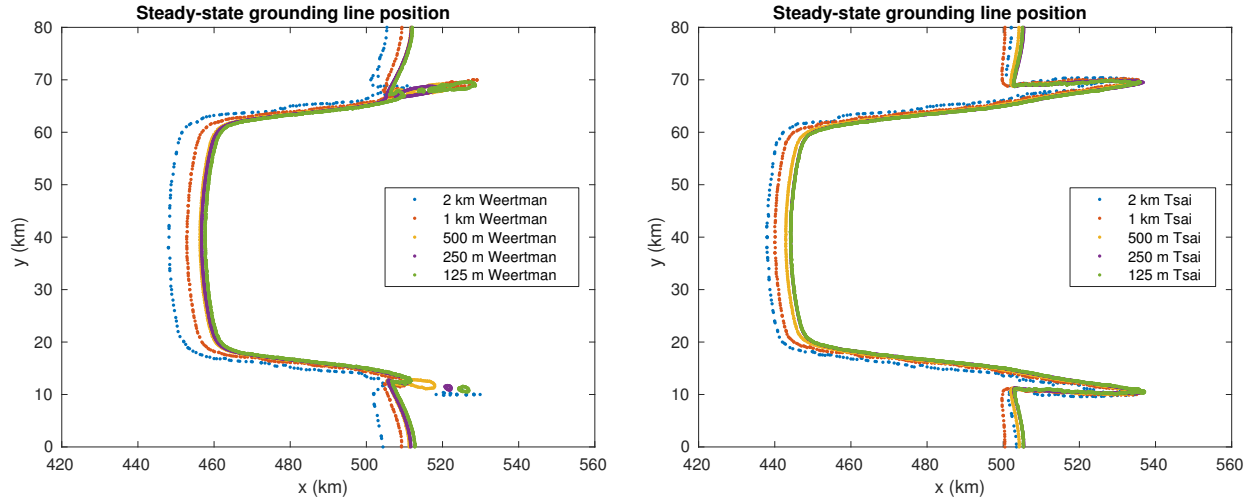


Figure 3. Steady-state grounding line positions for the Weertman (left) and Tsai (right) friction law for the 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple) and 125 m (green) mesh resolutions

(NMP, FMP, SEM1 or SEM2), e.g., EXP1_Weertman_250m_SEM1, as the results of these simulations now depend on the sub-element melt parameterization adopted.

4 Results

Figure 1 shows the initial steady-state configuration for SS_Weertman_125m. Its geometry is shown in Fig. 1a, and the velocity and grounding line in Fig. 1b. The grounding line position varies between 458 km in the centerline of the glacier and 528 km on its sides; the ice velocity is maximum at the ice front, reaching 1012 m/yr. This configuration is comparable to previous results based on the same geometry (Gudmundsson et al., 2012; Gudmundsson, 2013; Asay-Davis et al., 2016). The mesh resolution and the type of basal sliding law both impact the grounding line position as shown in Fig. 3. The grounding line position on the glacier centerline varies between 438 km for SS_Tsai_2km and 458 km for SS_Weertman_125m, with a larger spread between the different resolutions for the Tsai friction law (9.6 km) than for the Weertman friction law (6.2 km) (Fig. 3 and Tab. 1).

Experiment 0 is mostly designed to ensure that the model has reached a steady-state, as no melt is applied, similar to the initial steady-state. The ice mass above floatation (Fig. 4a) remains constant over the 100-year simulation for the 10 configurations, while the grounded ice area (Fig. 4b) experiences small oscillations with small oscillations, especially for the Weertman sliding law. Such oscillations, that average to zero change in the grounded area over time, have been noted by Asay-Davis et al. (2016) and are orders of magnitude smaller than the changes simulated in Experiment 1 and Experiment 2. Figure 4 confirms that sub-kilometer resolution is needed to accurately capture the grounding line positions, similarly to what has been suggested by previous studies (e.g., Vieli and Payne (2005); Gladstone et al. (2010); Pattyn et al. (2012, 2013); Feldmann et al. (2014); Seroussi et al. (2014a)). The difference in modeled volume (see table 1) between the 1 km and 500 m models

Table 1. Steady-state grounding line position in the glacier centerline and volume above floatation (VAF)

Friction law	Resolution	GL($y = 40$ km)	VAF (Gt)
Weertman	2 km	448.0 km	46327
Weertman	1 km	452.8 km	47044
Weertman	500 m	456.3 km	47540
Weertman	250 m	456.6 km	47674
Weertman	125 m	456.7 km	47737
Tsai	2 km	437.9 km	44996
Tsai	1 km	440.0 km	45238
Tsai	500 m	442.9 km	45700
Tsai	250 m	444.1 km	45899
Tsai	125 m	444.1 km	45889

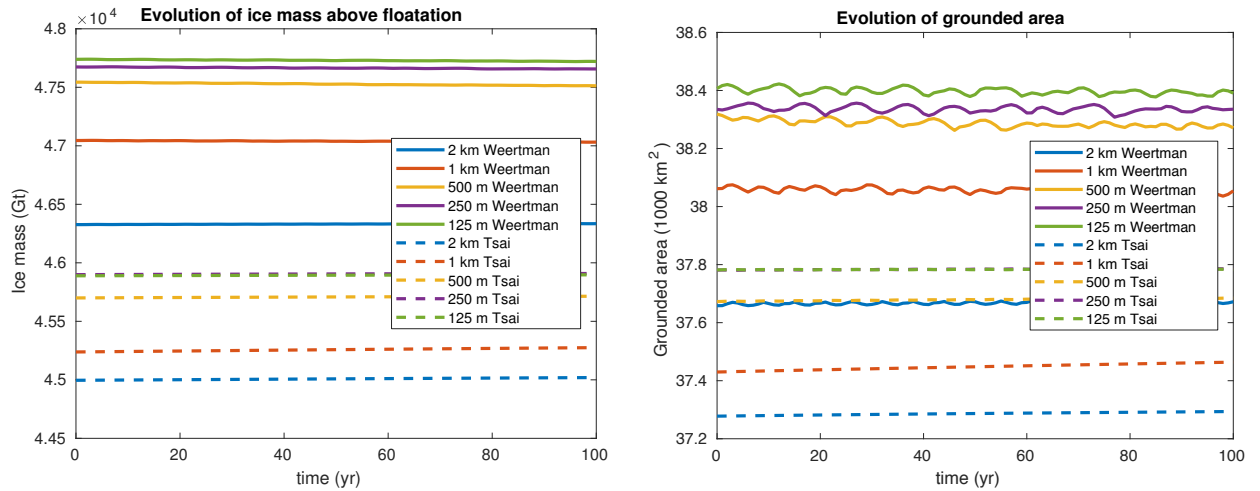


Figure 4. Evolution of ice volume above floatation (left) and grounded area (right) for Experiment 0 (steady-state case with no melt applied). Solid and dashed lines represent simulations with Weertman and Tsai friction respectively for resolutions of 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple), and 125 m (green). Results for 250 m and 125 m resolutions are superimposed for the Tsai friction.

is 1.02% and 1.05%, and the difference in grounded area is 0.61% and 0.62% respectively for the Weertman and Tsai friction laws. Differences between models at 500 m, 250 m, and 125 m resolution are all well below 1% (the curves for SS_Tsai_125m and SS_Tsai_250m are superimposed on Fig. 4). By comparison, the difference in volume above floatation and grounded area between the two friction laws at 125 m resolution is respectively of 3.9% and 1.6%.

Table 2. Change in volume above floatation (Δ VAF in Gt) in Experiment 1 for the Weertman (left) and Tsai (right) friction laws

Melt Parameterization (Weertman)					Melt Parameterization (Tsai)				
Resolution	NMP	FMP	SEM1	SEM2	Resolution	NMP	FMP	SEM1	SEM2
2 km	-4137	-5411	-5210	-5304	2 km	-5480	-6692	-6504	-6576
1 km	-4272	-4724	-4637	-4673	1 km	-6127	-6454	-6394	-6417
500 m	-4246	-4359	-4331	-4340	500 m	-6261	-6333	-6318	-6324
250 m	-4225	-4252	-4244	-4246	250 m	-6293	-6315	-6304	-6305
125 m	-4196	-4221	-4213	-4215	125 m	-6294	-6307	-6309	-6311

Experiment 1 simulates the evolution of the glacier when ocean induced melt is applied under floating ice. The equation that governs the melt rate in this experiment provides limited melt close to the grounding line, as the water column thickness becomes smaller (see Eq. 3). Figure 5 shows the evolution of the ice volume above floatation for this experiment for the different sub-element melt parameterizations, mesh resolutions and the two friction laws. The volume above floatation lost (see also table 2) varies between 4140 Gt and 6690 Gt for the EXP1_Weertman_2km_NMP and EXP1_Tsai_2km_FMP scenarios respectively. Experiments performed with the Tsai friction law show a larger mass loss (between 5480 and 6690 Gt over the 100-year period) than the ones performed with a Weertman friction law (between 4140 and 5410 Gt). The impact of the sub-element melt parameterization adopted, however, is more pronounced in the case of Weertman sliding law. The Tsai sliding law shows similar results for all sub-element parameterizations if the mesh resolution is 1 km and under, suggesting that any sub-element melt parameterization can be adopted in this case. Results performed at 2 km resolution all overestimate the mass loss, except when the NMP is adapted, which underestimate the mass loss. If the Weertman sliding law is applied, the results are strongly dependent on both the sub-element parameterization and the mesh resolution. SEM1, SEM2, and FMP behave very similarly, with mass loss being reduced as the resolution increases (from $\sim$ 5400 Gt at 2 km resolution to $\sim$ 4150 Gt at 250 m resolution). The difference between the runs becomes smaller as the mesh resolution increases, but the results are within 5% of the results obtained with a resolution of 125 m only for resolutions below 500 m. The NMP presents a completely different behavior, with results almost identical for all mesh resolutions for the Weertman sliding law (less than 150 Gt variation after 100 years). The runs relying on NMP underestimate the mass change for the Tsai friction law, with 650 Gt less mass loss for the EXP1_Tsai_2km_NMP compared to EXP1_Tsai_1km_NMP. During the experiment, the grounding line retreat in the centerline of the glacier varies between 40 and 55 km depending on the mesh resolution and the melt parameterization for the Weertman sliding law, and between 55 and 70 km for the Tsai sliding law, with larger retreats for the FMP, SMP1 and SMP2 at coarse resolution, and smaller retreats for FMP, SMP1 and SMP2 at fine resolution and NMP.

In Experiment 2, a large ice shelf melt rate of up to 30 m/yr is applied under the ice shelf, including close to the grounding line. Figure 6 and table 3 show the results of this experiment for the different sub-element parameterizations, mesh resolutions, and the two sliding laws. The overall mass loss is similar to Experiment 1 and varies between 4110 Gt and 7590 Gt for

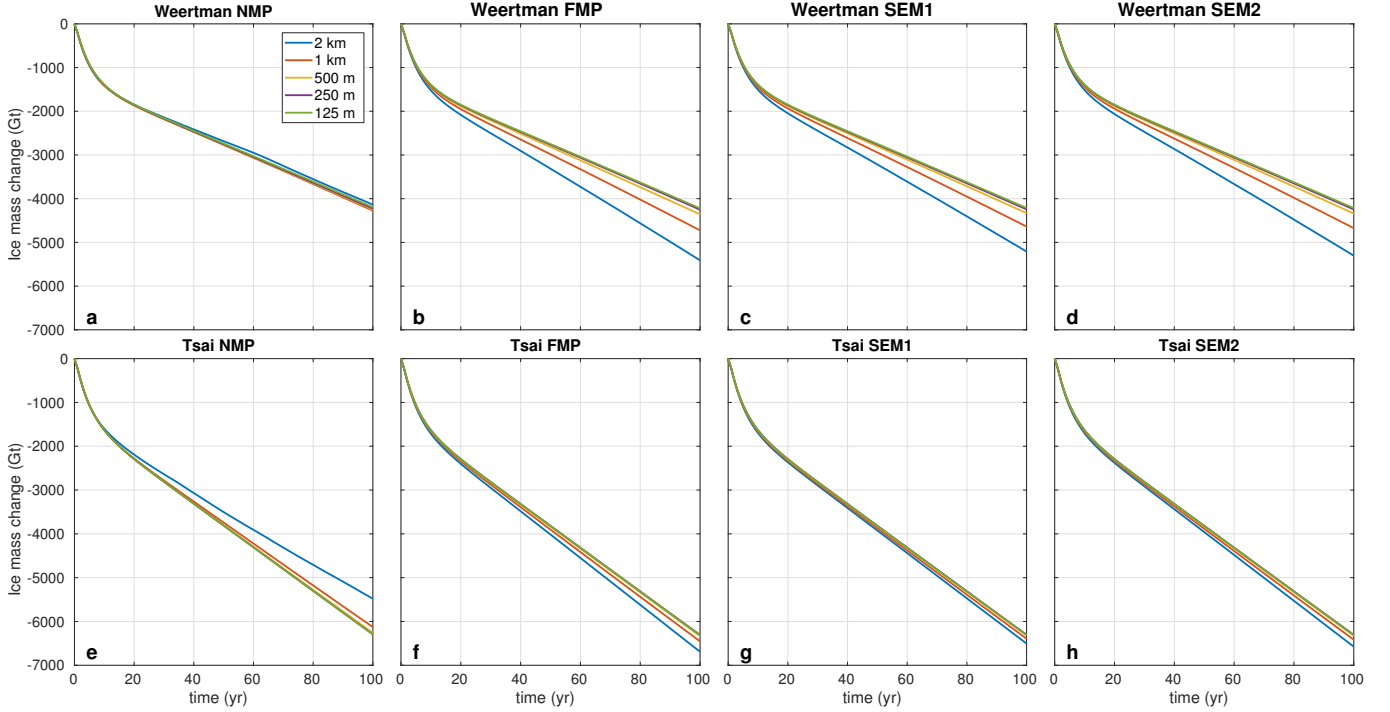


Figure 5. Evolution of ice volume above floatation in Experiment 1 for the NMP (a and e), FMP (b and f), SEM1 (c and g), and SEM2 (d and h), for the Weertman (a-d) and Tsai (f-h) friction laws. Each plot represents the evolution for the 5 mesh resolutions: 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple), and 125 m (green).

Table 3. Change in volume above floatation (Δ VAF in Gt) in Experiment 2 for the Weertman (left) and Tsai (right) friction laws

Melt Parameterization (Weertman)					Melt Parameterization (Tsai)				
Resolution	NMP	FMP	SEM1	SEM2	Resolution	NMP	FMP	SEM1	SEM2
2 km	-4132	-6536	-5672	-5644	2 km	-4943	-7585	-6614	-6533
1 km	-4130	-5895	-5235	-5188	1 km	-5150	-7060	-6365	-6284
500 m	-4120	-5289	-4824	-4775	500 m	-5374	-6469	-6034	-5976
250 m	-4130	-4890	-4565	-4523	250 m	-5474	-6112	-5846	-5808
125 m	-4115	-4748	-4464	-4428	125 m	-5510	-6038	-5812	-5783

EXP1_Weertman_250m_NMP and EXP1_Tsai_2km_FMP scenarios respectively, with a larger ice loss for the Tsai friction law overall. The impact of mesh resolution and sub-element parameterization is more pronounced than in Experiment 1. At 2 km resolution, the difference in mass loss varies by 45% and 42% between NMP and FMP for both the Weertman and Tsai

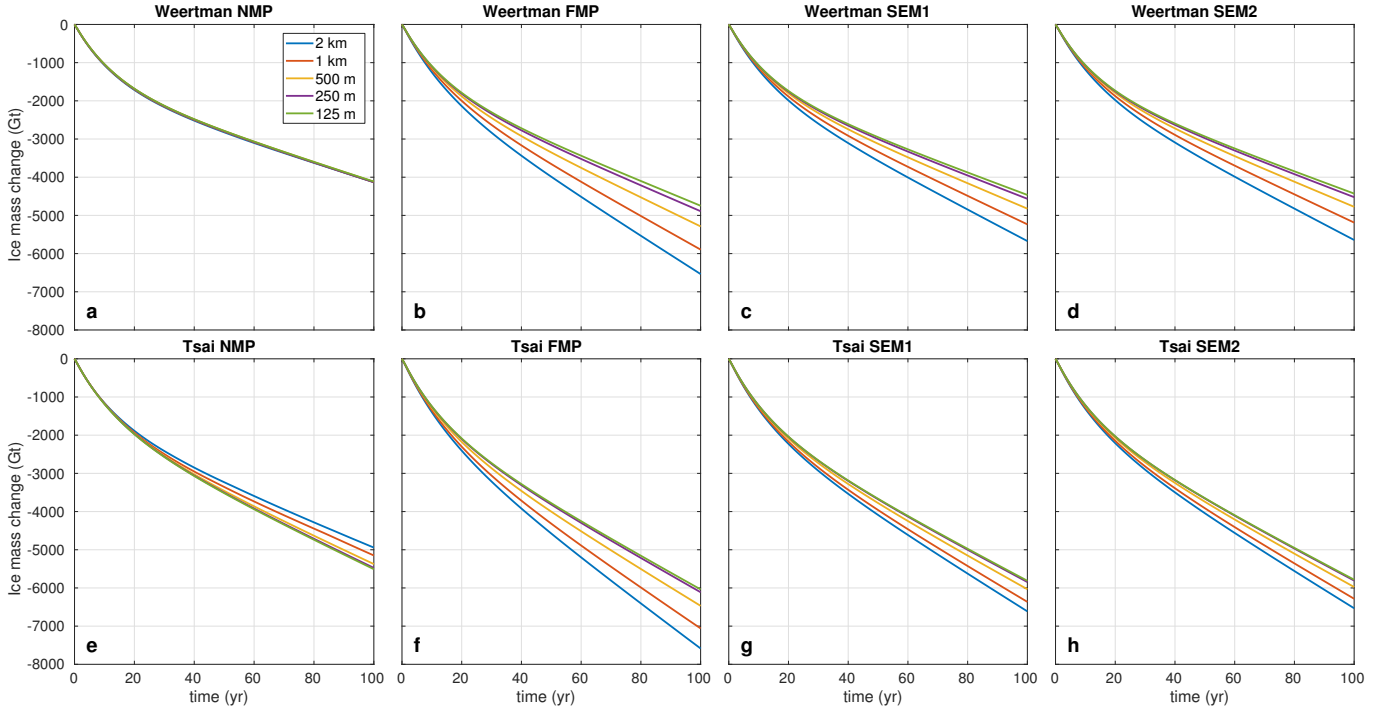


Figure 6. Evolution of ice volume above floatation in Experiment 2 for the NMP (a and e), FMP (b and f), SEM1 (c and g), and SEM2 (d and h), for the Weertman (a-d) and Tsai (f-h) friction laws. Each plot represents the evolution for the 5 mesh resolutions: 2 km (blue), 1 km (red), 500 m (yellow), 250 m (purple), and 125 m (green).

sliding laws, respectively. This spread is reduced as the mesh resolution increases, but a 125 m resolution is not sufficient to have similar results for NMP and FMP (14% and 9% difference between NMP and FMP at 125 m resolution for the Weertman and Tsai friction laws), suggesting that not all parameterizations have fully converged despite the level of mesh resolution. The SEM1 and SEM2 results are intermediate between FMP and NMP and behave similarly in all cases. Figure 6 also shows that NMP is by far the least sensitive to mesh resolution for the Weertman sliding law, with, e.g., a mass change of only 20 Gt between EXP2_Weertman_2km_NMP and EXP2_Weertman_125m_NMP, whereas the difference reaches 1216 Gt between EXP2_Weertman_2km_SEM1 and EXP2_Weertman_125m_SEM1, and 1790 Gt between EXP2_Weertman_2km_FMP and EXP2_Weertman_125m_FMP. Results performed with the two sub-element melt parameterizations show a reduced dependence on mesh resolution. This improvement is not sufficient, however, to have accurate results with relatively coarse mesh resolutions. The impact of mesh resolution and sub-element melt parameterization is more pronounced with the Weertman than the Tsai sliding friction law. Similarly to what was observed for Experiment 1, experiments performed with the Tsai friction law show less sensitivity to sub-element parameterization and mesh resolution than the Weertman friction law, except for NMP simulations that experience a mass loss reduced by 570 Gt over 100 years for the EXP2_Tsai_2km_NMP

compared to the EXP2_Tsai_125m_NMP. The difference in ice loss after 100 years between EXP2_Tsai_2km_FMP and the EXP2_Tsai_125m_FMP and between EXP2_Tsai_2km_SEM1 and EXP2_Tsai_125m_SEM1 is respectively reduced by 1000 and 800 Gt. During this experiment, the grounding line retreat in the centerline of the glacier varies between 33 and 63 km depending on the mesh resolution and the melt parameterization for the Weertman sliding law, and between 42 and 75 km for the Tsai sliding law, with larger retreats for the FMP, SMP1 and SMP2 at coarse resolution, and smaller retreats for NMP and FMP, SMP1 and SMP2 at fine resolution.

5 Discussion

The results presented in this study show that the impact of sub-element melt parameterization and mesh resolution is different for the Weertman and Tsai friction laws. Models relying on Weertman sliding laws are more sensitive to the mesh resolution and the type of sub-element melt parameterization than when a Tsai sliding law is employed. These conclusions are in agreement with the ones of Gladstone et al. (2017) on a flowline case. Figures 7 and 8 show the convergence of results with mesh resolution for the four sub-element mesh parameterizations. For the Weertman sliding law, the results vary by less than 2.0% for all the mesh resolutions regardless of the melt applied when NMP is used. Results using SEM1, SEM2, and FMP vary by at least one order of magnitude more, demonstrating that these parameterizations are not satisfying in this case. When a Tsai sliding law is used, the results vary depending on the amount of sub-ice shelf melt close to the grounding line. When melt rates converging towards zero close to the grounding line are applied, SEM1 and SEM2 converge slightly faster than FMP and NMP, and results within 5% of the 125 m resolution runs can be obtained for all sub-element parameterizations for mesh resolutions of 1 km or less. When large melt rates are applied close to the grounding line (Experiment 2), NMP converges the fastest but the behavior of SEM1 and SEM2 is close to NMP, with NMP underestimating the mass loss, while SEM1 and SEM2 overestimate it. In all cases, SEM1 and SEM2 results are almost identical (similarly to what was observed for sub-element parameterization of basal friction, see Seroussi et al. (2014a)) and are intermediate between NMP and FMP. Differences between mass loss produced with NMP and FMP can be as large as about 50% for 2 km mesh resolution (see Fig. 6). This difference is reduced as the mesh resolution increases, but remains larger than 10% even at 125 m resolution (see Fig.6) for large melt rates. Using the FMP never produces the best convergence of results and overestimates the mass loss by a factor of two in several cases, it should therefore be avoided. NMP shows the least dependence on mesh resolution, except for small melt rates close to the grounding line and a Tsai friction law (Fig. 7 and 8).

To explain this behavior, one needs to look at the numerical implementation of the equations that are affected by melt. The ocean induced melt is only present as a right-hand side term in the mass transport equation:

$$\frac{\partial H}{\partial t} = -\nabla \cdot H \bar{\mathbf{v}} + \dot{a} - m_i \quad (5)$$

where H is the ice thickness, $\bar{\mathbf{v}}$ is the depth average ice velocity, $\dot{a}$ is the surface mass balance. With the finite element method, H is assumed to be a sum of nodal functions, and integrating basal melt, m_i , over partially floating elements will lead to a thinning at the grounded nodes of these elements that is inherent to the finite element method. In other words, applying melt

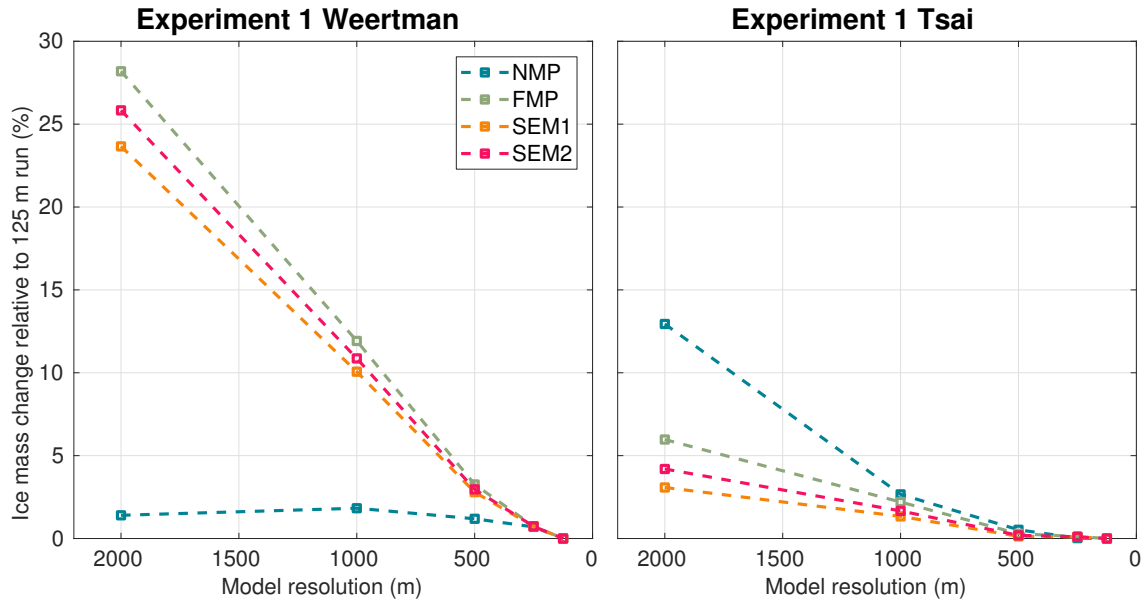


Figure 7. Convergence of ice volume above floatation at the end of Experiment 1 as a function of mesh resolution. Absolute error relative to the corresponding 125 mesh resolution results (same friction law and melt parameterization scheme) for the Weertman (a) and Tsai (b) friction laws for the NMP (blue), FMP (green), SEM1 (orange), and SEM2 (red) sub-element melt parameterizations.

in partially floating elements will induce a thinning upstream of the grounding line that is purely numerical, and the grounding line retreat will therefore be systematically overestimated. Using the no melt parameterization, no numerical thinning is applied to the grounded nodes of partially floating elements. Additional experiments, not shown here, confirm that even with a perfectly static marine ice sheet system (i.e., $\mathbf{v} = \mathbf{0}$ at all time), the grounding line will artificially retreat, except for the NMP, which confirms that it is the numerically correct way of treating basal melting in partially floating elements or cells, regardless of the numerical method adopted.

Unlike what has been recommended for sub-element parameterizations of basal friction at the grounding line (e.g., Pattyn et al., 2006; Vieli and Payne, 2005; Feldmann et al., 2014; Seroussi et al., 2014a), using a sub-element *melt* parameterization does therefore not guarantee an improvement compared to simulations that do not include such implementations, and does not necessarily relax the requirements of mesh resolutions. This is especially true when large melt rates are applied in the vicinity of the grounding line and for the Weertman sliding law. Many simulations in the Amundsen Sea Sector of West Antarctica (e.g. Favier et al., 2014; Joughin et al., 2014; Seroussi et al., 2014b) applied large melt rates in this region, consistently with observations (Dutrieux et al., 2013). A previous model study performed with NMP and SEM1 on this region showed extreme differences even over 100 years, and a potential collapse of Thwaites Glacier in less than 100 years for large melt rate scenarios (Arthern and Williams, 2017). Our study sheds light on this problem, as the SEM1 was probably under-resolved, leading to an overestimation of grounding line retreat.

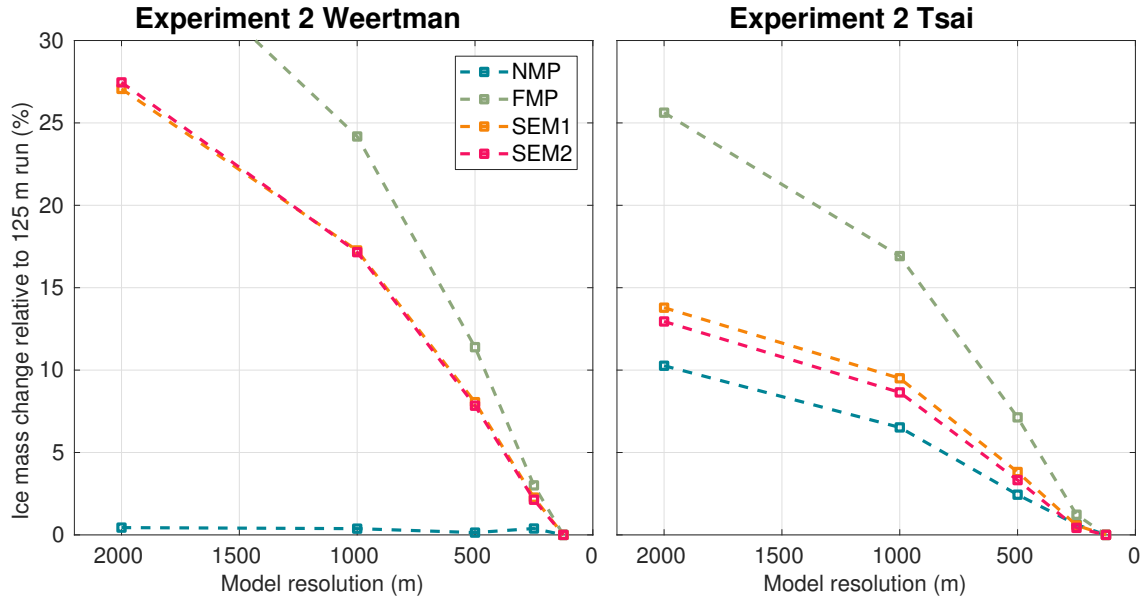


Figure 8. Convergence of ice volume above floatation at the end of Experiment 2 as a function of mesh resolution. Absolute error relative to the corresponding 125 mesh resolution results (same friction law and melt parameterization scheme) for the Weertman (a) and Tsai (b) friction laws for the NMP (blue), FMP (green), SEM1 (orange), and SEM2 (red) sub-element melt parameterizations.

In this study, we only considered mesh resolutions that are 2 km or less. However, large scale simulations of the Antarctic ice sheet typically rely on significantly coarser resolutions (e.g., Golledge et al., 2015; DeConto and Pollard, 2016; Pollard et al., 2015), especially when performing long term simulations. In this case, using the FMP, SEM1, and SEM2 will always lead to large overestimates in the amount of mass loss and even collapse of entire regions if large melt rates are applied close to the grounding line or if experiment scenarios include large melt rates in these regions, for both Weertman and Tsai sliding laws. As mentioned in previous studies (e.g., Cornford et al., 2016; Gladstone et al., 2017), quantifying the impact of mesh resolution on model results is therefore extremely important in this case in order to provide reliable estimates of uncertainties in ice sheet mass loss over the coming decades and centuries. This is especially important when simulating the collapse of marine terminating glaciers resting on retrograde bed slope that are sensitive to the Marine Ice Sheet Instability (MISI, Weertman (1974)), as such an instability would be potentially simulated several centuries too early if ice shelf melt rates are applied on partially floating elements (Arthern and Williams, 2017; Golledge et al., 2015).

The results presented here were all performed on simulations that experience grounding line retreat and no grounding line advance. As most glaciers around the world are experiencing sustained retreat in response to climate change, cases of grounding line advance are less common. The numerical scheme or resolution needed to correctly reproduce grounding line advance are however different than those needed to accurately capture grounding line retreat: Gladstone et al. (2017) showed

that convergence was even worse in the case of grounding line advance. Convergence tests are even more critical to perform in such a case.

Grounding lines are constantly migrating, not only on long time scales due to changes in oceanic or atmospheric conditions, but also over short time scales with tides (e.g. Gudmundsson, 2007; Le Meur et al., 2014; Padman et al., 2018). Observations show that melting in the grounding zones is complex and tidal motion probably involves complex melt rate patterns changing on tidal time scales as grounding line advances and retreats, and tidal flexure pumps ocean water in the grounding zone (Walker et al., 2013). This process could lead to more complicated patterns than the ones used in this study, assuming that the ice shelf is in hydrostatic equilibrium. However, such processes remain poorly understood, additional studies are required to better evaluate them, and should not be used as a justification for numerical model inaccuracy.

All the simulations performed in this study are based on the two-dimensional SSA. We expect, however, the results to be qualitatively similar for other stress balance approximations that determine the grounding line position based on the hydrostatic equilibrium, as melt rates in partially floating elements are treated in a similar way regardless of the stress balance approximation. Using a Stokes flow line model, Gladstone et al. (2017) demonstrate a similar greater dependence of model results when large melt rates are applied close to the grounding line and the need for stricter resolution requirements. Simulations performed with three dimensional higher-order (Pattyn, 2003) or L1L2 (Hindmarsh, 2004) models should however generally experience lower changes in these cases, as previous studies showed that SSA models tend to respond more quickly than models including vertical shear (Pattyn et al., 2013; Pattyn and Durand, 2013).

6 Conclusions

In this study we investigate the impact of the numerical implementation of ice shelf melt rates immediately downstream of the grounding line. We compare several sub-element parameterizations that (1) do not apply any melt over partially floating elements, (2) apply basal melt over the entire partially floating elements, or (3) apply some melt over partially floating elements. Simulations are performed with different mesh resolutions for two experiments with small and large melt rates close to the grounding line, and for a Weertman and a Tsai sliding laws. Our results demonstrate that, for limited melt rates in the order of 1 m/yr close to the grounding line, all sub-element melt parameterizations behave similarly for resolutions lower than 1 km and 500 m respectively for the Tsai and Weertman friction laws. For large melt rates in the order of 30 m/yr just downstream of the grounding line, however, models based on varying resolutions and sub-element melt rates behave differently. Both (2) and (3) overestimate the mass loss and resolutions well below 500 m are needed, while (1) shows a behavior that is less dependent on the mesh resolution. These results were performed using the finite element method, but can be extrapolated to other numerical methods, such as the finite element and finite volume methods. As continental scale simulations of Antarctica typically use resolutions of several kilometers in the grounding line region, we therefore recommend models not to apply ice shelf melt rates in partially floating elements and to carefully assess the impact of mesh resolution on their simulation results.

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