

We thank the two reviewers for their insightful remarks. In response to these remarks, we have made a number of major changes in the manuscript. We first describe the major changes and then respond to individual comments. The revised manuscript and supplementary material are attached below.

Major Changes to the manuscript

1. In our original manuscript, we measured velocities from common offset GPR records by isolating diffractions through Plane Wave Destruction filtering, migrating the filtered data through a suite of constant velocities, and then computing the Varimax norm by isolating individual diffractions or groups of diffractions. This approach required defining the region of interest, viewing the migrated image that corresponded to the peak varimax value, and evaluating the quality of the velocity pick by inspecting the migrated image.

In our revised manuscript, we compute the varimax norm in sliding windows that encompass the entire time section and have with a fixed width of ~10 meters. The result is a continuous set of velocity estimates along the GPR profile. Spurious picks are suppressed by smoothing the results in x. The revised method does not require inspection of the migrated images.

2. In our original manuscript, we presented two field data sets with independent snow density, depth, and SWE measurements. In the revised we include four additional field data sets with coincident manual observations. These data are included in our validation of the method, results from individual data sets as well as our manual snow density measurements are included in a supplementary materials file.
3. There was some confusion regarding our analysis of radar attenuation our dry snow assumption. Since the main focus of our paper is to estimate the radar velocity and thus the density of dry snow and the attenuation discussion detracts from this focus, we have reduced this discussion to one paragraph and moved it to *Section 2.7 Estimating SWE*.

4. We determined that the second synthetic data set, which was used to test the ability of the method to resolve lateral velocity changes, was redundant and we have removed it from the revised manuscript. Although the remaining synthetic data set has a constant snow velocity, the changing snow depths cause the migration velocity to vary along the profile (Figure 5, revised manuscript).

Response to reviewer comments:

Reviewer #1

(Reviewer Comments in italics)

General Comments:

In this paper, the authors attempt to apply established techniques from the exploration seismology literature to the problem of measuring Snow Water Equivalent (SWE) using constant offset GPR with the stated goal of simplifying that process while obtaining reliable results. The manuscript goes on to detail the application of these complicated methods to a few lines of field data with exceedingly limited success at producing results and processing flows that are neither reliable or simple. Given these results (as presented and described in the body of the manuscript itself) the concluding statements that “the processing flow that we presented in this paper proved to be an efficient way to measure radar velocities within seasonal snow” (Line 508) and “the method requires less processing time than visually scanning each migrated image and could make GPR a more attractive tool for estimating SWE at the watershed scale” (Line 509) are un-supported.

In response to your general, specific and technical comments, we have made a number of major changes in the manuscript.

1. Most significantly, we have eliminated the need for examining isolated diffractions in the GPR image. In the revised manuscript, we now compute the Varimax Norm in ~10-meter-wide sliding windows that span the entire time section. The result is a continuous set of varimax curves along the GPR profile and choosing the peak of each curve provides a

continuous estimate of the migration velocity along the profile. Although some of these curves provide spurious velocity estimates, smoothing the velocity picks in the horizontal direction reduces the influence of spurious velocity picks.

2. Our revised manuscript includes the results of six field GPR data sets, four pits, and 86 probed depth observations.
3. To qualitatively assess the efficiency of the method, we include an example of the processing time required for one 100 meter long GPR profile on a 2016 MacBook Pro with a 2 GHz processor. Lines (727-734):

“One of our main goals was to produce a processing flow that allows for the rapid processing of common offset GPR data with minimal user interaction. The two most time computationally expensive parts of the processes are the migrations and the varimax calculations. As an example, on a 2016 MacBook Pro with a 2GHz processor, for the ~ 100-meter-long Line 4, performing 51 migrations takes approximately 5 minutes, the varimax calculation takes about half as long, and the PWD filtering takes a few seconds. The most time-consuming part of the process is picking the arrival times of snow surface and ground surface reflections.”

Specific Comments:

The paper states that the “primary purpose of this study is to simplify the process of measuring GPR velocity in seasonal snow and obtain reliable SWE estimates” (Line 444) yet it accomplishes neither:

We admit that the word “simplify” was the wrong word to use. The goal of our study is to develop an efficient method for obtaining GPR velocities in seasonal, and thus snow densities, from common-offset GPR data. Additionally, we sought to minimize the amount of human interpretation throughout the process. In the revised manuscript, this sentence reads (lines 664-666):

“The primary purpose of this study is to develop an efficient processing flow for measuring GPR velocity and thus snow density SWE from common-offset data that requires a minimum amount of human interpretation.”

Comparisons between our GPR derived estimates of depth, density, and SWE are summarized in Table 2 and in lines 434 to 442 of the revised manuscript. Aside from Line 3, which was located 1.5 meters off of Pit 2 and measured a shallower snowpack than observed in the pit, our GPR derived density and SWE estimates agree with manual measurements within 13% for density and 8% for SWE.

In terms of processes, simplification, the adaptation of Fomel's seismic approach (Line 133) and the NSE metric (Line 477) are gratuitously complicated, fail due to data quality issues (Line 230), and are not justified given the limited data set. There is no case made that the claims of efficiency (e.g. Line 221, Line 508) address actual measurement and processing bottlenecks or are even accurate. Further, once implemented these techniques still require frequent and substantial manual interventions (Lines 230, 377, 447, 460, and 462) culminating in the authors' suggestion that wavelength scale "point diffractors suitable for this type of analysis can be scouted for ahead of time during summer months or on aerial photographs" (Lines 499). Traditional GPR methods are much simpler than this (e.g. do not require advance scouting or aerial observation).

The results presented in our revised manuscript include six field data sets, 4 snow density pits, and 86 probed depth measurements. Utilizing our revised processing flow, we get similar results to those presented in the original manuscript, but with more data and without intervention.

Our results are summarized in Table 2 and in lines 700-708 of the revised manuscript:

"To validate the method, we compared estimated snow densities, depths, and SWE to observations made in four snow pits and to 86 probed snow depth measurements. The results are summarized in Table 2 and in Figure 9. If we exclude the two obvious outliers (Figure 10a), the RMS error for our depth predictions for the remaining 88 depth observations is 12% of the mean snowdepth observation. The RMS error for snow density and SWE relative to the mean observed values are 15% and 18%. Averaging the velocities across the entire line (Figure 10 red crosses) reduce the difference between predicted and observed depth values to an RMS error of 9%, suggesting that lateral variations in snow velocity are minimal. Averaging the velocities across the entire line reduces the RMS errors for density and SWE to 8% and 10%, respectively.
"

We have removed the statement about scouting for point diffractors and instead comment on the high likelihood of encountering such objects in mountain watersheds (Lines 755-761):

“The data presented in this paper contained an abundance of diffractions located near the soil/ground interface allowing an average velocity for the entire snowpack to be obtained. These events are likely due to small-scale variations in surface topography, rocks, and/or vegetation along the ground surface, which may not be present in all environments. However, we note that mountain watersheds free of vegetation, small undulations in surface topography, and surface rocks are probably rare. Thus, the method may be useful in many regions where seasonal snowpacks exist.”

In terms of reliable SWE estimation, the authors show that the presented approach “does not allow us to confidently differentiate between dry snow and moist snow” (Line 350). Further, authors show that the collected data does not exhibit the frequency dependent attenuation upon which the entire approach depend (Line 318 and Line 331) and even though “within uncertainty bounds there is no resolvable frequency change” (Line 440) the authors go on to use that uncertainty speculate that “there may be up to a 36 MHz shift” and then estimate a volumetric water content from it (Line 442) even though nothing suggests the existence of such a shift beyond the authors’ assumption and assertion it should exist (Line 318). This is inappropriate and, as written, likely invalidates the results and conclusions of the paper. Further, the lack of observed frequency dependence is likely due to an actual lack of frequency dependence in radar attenuation in ice for frequencies below 1 GHz (Gudmandsen, 1971). This must be addressed. At the very least, the authors’ statement “averaging mean snow-densities from manual observations may be a better strategy” (Line 489) makes it clear that the stated goal to “obtain reliable SWE estimates” (line 144) was not met.

We apologize for the confusion regarding radar attenuation in wet snow and its relationship to volumetric water content. We agree that in ice (or dry snow) the signal attenuation is negligible. It is the presence of water in the snowpack and the high contrast between the dielectric properties of dry snow and water that cause the attenuation of the GPR signal (see Bradford et al., 2009). Since the

attenuation coefficient radar waves propagating in water is a linear function of frequency (Turner and Siggins, 1994), we expect to see a systematic decrease in higher frequency energy that is represented by a systematic decrease in the mean frequency content of the signal.

We recognize that our discussion of radar signal attenuation was a distraction from the main point of our paper and we have reduced the discussion of signal attenuation to one paragraph and moved it to **Section 2.7 Estimating SWE**. In addition, we have not used this analysis to report the maximum volumetric water content within our data resolution.

The revised discussion of radar attenuation (Lines 417-432) reads:

“In this paper, we are primarily concerned with measuring radar velocities and we assume that our data measure the properties of dry snow. The real part of the dielectric constant for water (~ 80) is much larger than that of snow ($\sim 1.5 - 2$) and the imaginary part, which describes the attenuation of the signal, is non-negligible (Bradford et al., 2009). The dry snow assumption can be tested from the data by analyzing the attenuation properties of the snowpack (Bradford et al., 2009). The attenuation coefficient for radar waves in water is frequency-dependent (i.e. Turner and Siggins, 1994), with the higher frequencies attenuating more rapidly than the lower frequencies because they go through more cycles per distance traveled. When liquid water is present in the snow, the ground reflection will have a lower mean frequency content than a reference event (the snow reflection for the snowmobile collected data and the direct arrival for the skier-pulled data). To test the dry snow assumption, we calculate the maximum local instantaneous frequency (Fomel, 2007) within a time window surrounding the event of interest then average this value across all of the traces in the GPR image. The standard deviation provides an estimate of the measurement uncertainty. We note that at 500 MHz, a small shift in frequencies results in a non-negligible volumetric water content..”

Technical Corrections:

Line 107: Add an explanation and citation for “targets that have lateral dimensions that approximate the wavelength of the signal”.

The diffraction literature makes reference to objects that are of similar dimension to the signal wavelength as well as to the first Fresnel zone. In consideration of our analysis depicted in Figure 1, we suggest that the Fresnel zone is more appropriate for our purposes. Lines 129-132 read:

“In this paper, we are specifically interested in targets that have lateral dimensions that are less

than the Fresnel zone. These objects scatter energy in all directions and appear on the raw GPR image as hyperbolic events, called diffractions (Landa and Keydar, 1998).“

Citation:

Landa, E. and S. Keydar.: Seismic monitoring of diffraction images for detection of local heterogeneities, *Geophysics*, 63, 1998.

Lines 122: Why mention the 800 MHz data if it's not used in the analysis? Consider removing.

We agree with this comment and the revised manuscript does not mention the 800 MHz data.

Line 202: This entire paragraph has a level of detail that seems inappropriate for a journal manuscript. Consider dropping or revising.

We think that this paragraph may be useful for readers who are unfamiliar with migration and have kept it in the revised manuscript.

Line 235: Provide a justification for the “equally likely” claim.

Thank you for this comment. We have justified our uncertainty estimate by comparing images migrated at different velocities to the corresponding varimax value. We found that images that were visually indistinguishable corresponded to varimax values that were greater than approximately 95% of the maximum. (See Figure 2 of the revised manuscript). Because the migrated images were indistinguishable to the human eye, we interpret these migration velocities to be valid velocity estimates and use them as upper and lower bounds on the true velocity.

Line 251: The units of the left and right side of this equation do not match.

Thank you for pointing out this error. The equation should be:

$$V_{snow} = \sqrt{\frac{V_{mig}^2 t_{soil} - V_{air}^2 t_{snow}}{t_{soil} - t_{snow}}}$$

and we have fixed this in the revised manuscript.

Line 318: Add an explanation and citation for “the coefficient increases with increased frequency” and address the fact that, in ice and below 1 GHz it does not (Gudmandsen, 1971).

We have added a citation and explanation. In the revised manuscript Lines 422-432 read:

“The attenuation coefficient for radar waves in water is frequency-dependent (i.e. Turner and Siggins, 1994), with the higher frequencies attenuating more rapidly than the lower frequencies because they go through more cycles per distance traveled. When liquid water is present in the snow, the ground reflection will have a lower mean frequency content than a reference event (the snow reflection for the snowmobile collected data and the direct arrival for the skier-pulled data). To test the dry snow assumption, we calculate the maximum local instantaneous frequency (Fomel, 2007) within a time window surrounding the event of interest then average this value across all of the traces in the GPR image. The standard deviation provides an estimate of the measurement uncertainty. We note that at 500 MHz, a small shift in frequencies results in a non-negligible volumetric water content.”

Citation:

Turner, G., and A. F. Siggins.: Constant Q attenuation of subsurface radar pulses, *Geophysics*, 59, 1994.

Line 155, 404, and others: Justify uncertainties and provide basis throughout the manuscript for uncertainties that are currently just asserted.

Lines 341-343:

“We use the upper and lower bounds on our velocity estimates to compute upper and lower bounds on all subsequent calculations.”

Lines 401-403

"To propagate our velocity uncertainty estimates through the dix equation, we assign a travel-time uncertainty of 0.2 ns to our travel-time observations and use Eq. 5 along with our velocity uncertainty estimates to compute upper and lower bounds on the snow velocity."

Reviewer # 2

The paper by St. Clair and Holbrook describes a novel approach for determining the snow-water-equivalent (SWE). I have read the well-written paper with great interest, and I judge it a useful contribution for the Cryosphere community. Before the paper can be accepted, I suggest that the authors address the following issues.

1. There are inconsistencies and errors with the units. I suggest that they should use mks units throughout the entire manuscript.

In the revised manuscript we use meters for units of distance (and depth) and SWE, m/ns for radar velocity and kg/m³ for snow density.

2. It is often referred to Line 7 and Line 19. Without a map showing these profiles and/or a table describing the characteristics of the profiles, these references are not helpful. Since there seem to be only lines 7 and 19 discussed, it would make sense to rename them to line 1 and line 2.

We have provided tables to describe each field data set. Table 1 summarizes snowpits 1-4. Table 2 describes the GPR data and reports data of acquisition, the acquisition mode (ski or snowmobile), the type of independent observations we compared our results to, and the difference between GPR estimated values and manually measured values.

3. It is my understanding that the methodology works only for dry snow and that determining the liquid water content from the GPR data was not successful. These two facts should be stated more explicitly.

We have made this more explicit in several parts of the revised manuscript:

Lines 99-101:

"Since our primary goal is to develop a method for quick velocity estimations, we assume that the snow we are measuring is dry."

Lines 417-418;

"In this paper, we are primarily concerned with measuring radar velocities and we assume that our data measure the properties of dry snow."

4. Figures 10 and 11 indicate that the GPR estimated depths are systematically smaller compared with the probe depths. This should be discussed.

After reprocessing the data with the new approach, there is not a systematic error in the depth predictions.

5. Further discussion is required on the basic assumption of the approach that the scattering bodies can be considered as a point diffractor. I guess that the scattering occurs mostly at sizeable boulders. Therefore, I am not convinced that the point scatterer assumption is really justified. The diffraction hyperbola of a finite sized scattering body is expected to appear wider in a GPR profile, which would likely result in an over- estimation of the snow velocity. Consequently, one would thus rather overestimate the snow depth, but, as mentioned in point 4, the opposite seems to be the case.

Thank you for this comment. We have evaluated the effects of non-finite point diffractors, or diffracting objects that approach the dimension of the Fresnel zone. Figure 1 illustrates the effects of a lateral diffractor dimension and curvature on the peak value of the varimax norm. We also include the following discussion:

Lines 286-305:

"To assess possible errors in the migration velocity analysis, we applied our workflow to a synthetic data set generated from diffractors of varying size. The Fresnel radius is given by $R_f = \sqrt{\frac{z\lambda}{2}}$ (Sheriff, 1980) where z is depth and λ is the dominant wavelength. Figure 1 shows the effect of such an event on V . We created five synthetic diffractions with migration a migration velocity of 0.24 m/ns. The first four (Figure 1a) correspond to rectangular objects at 1 meter depth with horizontal dimensions 0.1, 0.2, 0.3 and 0.4 meters, and thickness of 0.03 m and the fifth corresponds to a circular object with a radius of 0.4 meters (close to R_f for the 500 MHz

ricker wavelet used to generate the diffractions). The corresponding varimax curves for the windows shown in Figure 1a are plotted in Figure 1b. The V curves are peaked at 0.24 m/ns for all of the rectangular diffractors, with flatter (less well-resolved) peaks as the horizontal dimension of the diffracting object increases, suggesting a larger uncertainty in the velocity estimate. The peak V value for the circular diffractor is at 0.268 m/ns, indicating that curved objects with lateral dimensions close to the size of the Fresnel zone may continue to focus at velocities higher than their true velocity. Finally, Figure 1c shows the V curve for the entire image, peaked at the correct velocity of 0.24 m/ns. This analysis suggests that the peak V value will correspond to the correct velocity if the majority of the diffractions correspond to objects much than R_f ..”

Lines 709-726:

“The greatest potential for systematic error in this analysis is the presence diffracting objects whose dimensions exceed the radius of the first Fresnel zone. The field data offer the opportunity to evaluate the influence of diffractor size on velocity estimates. Line 1, for example, shows four prominent diffractions between 50 and 70 meters. The Varimax norm has a maximum value at 0.256 m/ns, which is the velocity that focuses the two leftmost diffractions (Figure 6c). The diffractions on the right are clearly not focused because they are caused by an object (most likely a log) with a radius greater than the first Fresnel zone. Because the leftmost two have a higher amplitude than the others, they have the largest influence on the varimax value. Thus, although there are clearly events in the field data that have the potential to give erroneous results, our results suggest that reliable velocity estimates can be achieved so long as the majority of the diffracted energy is related to objects that can be considered point diffractors.

.”

6. Some of the figures are lacking proper axis labeling.

In our original manuscript many of the plots did not have axes labels, if the plots immediately above or adjacent to them had the same axis. The lower and leftmost plots had the axes labels on them. In the revised manuscript, all plots have axes labels.

7. It seems that in Figures 7 and 9 only portions of the GPR profiles are shown. This should be clarified.

In the revised manuscript, all figures depicting data show the entire data set.

1 Measuring snow water equivalent from common offset GPR records through migration

2 velocity analysis

3

4 James St. Clair^{1,2} and W. Steven Holbrook^{1,3}

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27 **Abstract**

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28 Many mountainous regions depend on seasonal snowfall for their water resources.

29 Current methods of predicting the availability of water resources rely on ~~long-term~~ relationships between stream discharge and snow pack monitoring at isolated locations, which

30 ~~the~~ Deleted: relationship

31 are less reliable during abnormal snow years. Ground-penetrating-radar (GPR) has been shown

32 to be an effective tool for measuring snow water equivalent (SWE) because of the close

33 relationship between snow density and radar velocity. However, the standard methods of

34 measuring radar velocity can be time consuming. Here we apply a migration focusing method

35 originally developed for extracting velocity information from diffracted energy observed in

36 zero-offset seismic sections to the problem of estimating radar velocities in seasonal snow from

37 common-offset GPR data. Diffractions are isolated by plane-wave-destruction filtering and the

38 optimal migration velocity is chosen based on the varimax norm of the migrated image. We

39 then use the radar velocity to estimate snow density, depth, and SWE. The GPR-derived SWE

40 estimates are within ~~6%~~ Deleted: 3 of manual SWE measurements when the GPR antenna is coupled to

41 the snow surface and ~~3-21%~~ Deleted: 18 of the manual measurements when the antenna is mounted on

42 the front of a snowmobile ~0.5 meters above the snow surface.

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54 **1. Introduction**

55 Many regions of the world are critically dependent on seasonal snowfall for their water
56 resources; accurate estimates of how much water is stored in **mountain landscapes** are
57 necessary to manage this resource. In the United States, a large network of SNOTEL sites
58 provide continuous information about snow depth, density, and snow water equivalent that are
59 used to make water availability predictions (Serreze et. al., 1999). While these sites provide
60 valuable information at **a** site, scaling these point measurements up for basin or grid scale
61 estimates can be challenging (Molotch and Bales, 2005). Currently, these data are used to
62 develop empirical relationships between SWE and nearby stream discharge. These predictions
63 are most accurate during average years and may be not reliable during abnormal years (Bales et
64 al., 2006), thus there is a need to develop new and reliable methods for estimating SWE at a
65 basin scale.

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66 Several previous studies have demonstrated that Ground-Penetrating-Radar (GPR) can
67 be used to measure SWE (e.g. Bradford et al., 2009, Tiuri et al., 1984, Holbrook et al. 2016).
68 Tiuri et al. (1984) showed that at microwave frequencies, the real part of the dielectric constant
69 for dry snow, which governs the velocity, is almost completely determined by the bulk density
70 of snow. However, when liquid water is present, both the real and imaginary parts are needed
71 to determine the volumetric water content of the snow. The complex dielectric constant can
72 be measured by analyzing both the velocity and attenuation characteristics of the snow
73 (Bradford et al., 2009). In the simplest case of dry snow, bulk density can be estimated directly
74 from radar velocity. Snow depth can be measured from the two-way travel time of the radar

80 pulse between the snow surface and the ground surface and the velocity. SWE can then be
81 calculated as the product of snow density and snow height.

82 Velocity measurements can be made from the surface in several ways. Common-
83 midpoint gathers (CMP), where the distance between transmitting and receiving antennas is
84 steadily increased about a central location, provide highly accurate measurements; the two-
85 way travel-time to subsurface reflectors ~~is~~ a function of offset and velocity. Collecting CMP's
86 requires separable antennas, and it can be time-consuming to both collect and process these
87 data. Common-offset antennas, where both the transmitting and receiving antennas are
88 housed in the same unit at a fixed offset, allow large amounts of data to be collected with
89 minimal effort. Measuring the velocity from common offset data can be ~~achieved through~~
90 calibration from measured snow depths, modeling diffraction hyperbolae travel-times, ~~or~~
91 migration focusing analysis.

92 In this paper, we apply the migration velocity analysis (MVA) presented by Fomel (2007)
93 to the problem of estimating radar velocities, and thus snow density and SWE, from 500 MHz
94 common-offset GPR images. After testing the method on ~~a~~ synthetic data ~~set~~, we estimate
95 SWE from ~~six~~ field data sets. The first ~~two~~ data ~~sets were~~ collected by pulling the GPR along the
96 snow surface, and the ~~remaining four were~~ collected with the GPR antenna mounted on the
97 front of a snowmobile. ~~To validate the method, we compare snow depth, density and SWE~~
98 ~~estimates to measurements made in pits and probed depth observations along the profiles.~~
99 ~~Since our primary goal is to develop a method for quick velocity estimations, we assume that~~
100 ~~the snow we are measuring is dry. The GPR-derived estimates agree with manual SWE~~
101 ~~measurements within the estimated uncertainties.~~

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118 **2. Methods**

119 GPR surveys utilize high-frequency, broadband electromagnetic signals. The signal is
120 generated at the transmitting antenna and propagates in three dimensions at velocity given by
121 $v = c/\sqrt{\kappa'}$, where c is the speed of light in a vacuum and κ' is the real part of the dielectric
122 constant. Signal attenuation is frequency-dependent and can be approximated as $\alpha \approx$
123 $\sqrt{\frac{\mu_0 \kappa''}{\kappa'}}$ ω , where μ_0 is the magnetic permeability of free space and κ'' is the imaginary
124 component of the dielectric constant (Bradford, 2007). While both κ' and κ'' are frequency
125 dependent, within the typical frequency range utilized for GPR studies, only κ'' exhibits strong
126 variations with frequency; in dry snow $\kappa'' \approx 0$ (Bradford et al., 2009).

127 When a GPR signal encounters a boundary between subsurface materials with
128 contrasting dielectric constants, some of the energy is reflected back and recorded by a
129 receiving antenna. In this paper, we are specifically interested in targets that have lateral
130 dimensions that are less than the Fresnel zone. These objects scatter energy in all directions
131 and appear on the raw GPR image as hyperbolic events, called diffractions (Landa and Keydar,
132 1998), whose shape depends on the depth of the object and the velocity of the overlying media
133 (i.e. Claerbout, 1985). The velocity information contained in diffractions can be extracted by
134 fitting hyperbolic curves to the data or by migrating the image until the hyperbola is collapsed
135 to a point or "focus." The latter process is called migration velocity analysis (MVA). In this
136 paper, we follow an approach described by Fomel (2007) and develop a semi-automated MVA
137 program in Matlab for the purpose of measuring radar velocities in seasonal snow. The
138 processing flow consists of three steps: 1. Separate diffractions from reflections through

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150 Plane-Wave-Destruction (PWD), 2. Migrate the filtered images at a range of potential
151 velocities, and 3. Use the varimax norm as a measure of diffraction focusing to pick velocities.

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153 **2.1 Data Acquisition**

154 **2.1.1 GPR data**

155 During February and March 2015, we collected GPR, snow density, and snow-depth data
156 in the Medicine Bow Mountains, SE Wyoming. The GPR data were acquired with a Mala pulse
157 radar system with a center frequencies of 500 MHz. The data were collected in two ways. In
158 one configuration (Lines 1 and 2), we mounted the GPR antenna in a plastic sled and pulled it
159 behind a skier. The unit was set to fire continuously at a rate of 20 traces per second and the
160 sample interval on each trace was 0.3223 ns. In the other configuration (Lines 3, 4, 5 and 6) the
161 antennas were mounted on an aluminum frame attached to the front of a Polaris RMK 600
162 snowmobile. The unit was set to fire at a rate of 100 traces per second and the sample interval
163 was 0.3181 ns. Mounting the GPR antenna in front of the snowmobile allows us to measure
164 undisturbed snow as well as providing a snow-surface reflection, which can be used to analyze
165 the attenuation properties of the snow (Bradford et al., 2009). In both cases, we kept track of
166 our position with a Trimble R8 GPS unit that recorded our location at 1-second intervals.

167

168 **2.1.2 Snow depth and density data**

169 To validate our snow density and velocity estimates from the GPR data, we manually
170 measured snow depth and densities. (Table 1). On Lines 1, 2, 3 and 4 we dug snow pits and
171 located them with a handheld Trimble GPS unit. To measure snow densities, we used a 0.001

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Deleted: In this paper, we only present 500 MHz data because the lower frequencies show higher amplitude and more continuous ground reflections and produces better results when separating reflections from diffractions. ... [2]

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Deleted: Line 7, we used a probe to measure snow depths at 5-meter intervals along the profile

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193 cubic meter, wedge-shaped snow sampler and a scale that is accurate within 5-10 grams. We
194 made snow density measurements at 10 cm intervals in the sidewall of the snow-pits starting
195 from the snow surface and continuing to the ground. Pit locations were chosen based on the
196 presence of diffractions near the snow/ground interface after viewing the GPR images in the
197 field. On lines 4, 5, and 6 we measured snow depth at regular intervals with a probe.

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198 Probed depth measurements are subject to uncertainties due to uneven ground and
199 deviations in probe angle. We estimate our depth measurements to be accurate within +/- 5
200 cm. Snow density observations are subject to over and under sampling and we assign an
201 uncertainty of +/- 5 g/cm³. We calculate the average density for each pit profile assigning each
202 snow density observation to a 10 (+/-1) cm column of snow and performing a weighted sum.
203 Propagating the uncertainties through the averaging process yields uncertainty estimates of 10-
204 14 % of the averaged value, consistent with uncertainty estimates for snow pit density
205 measurements reported by Conger and McClung (2009).

206

207 **2.2 Pre-Processing the GPR data**

208 Prior to MVA we use MATGPR R3 (Tzanis, 2010) to apply several basic processing steps
209 to the GPR data including: 1. Reset trace to time-zero, 2. Trim time window, 3. Interpolate
210 traces to equal spacing using the GPS data, 4. Bandpass filter from 100 to 1000 MHz, 5. median
211 filter to remove antenna ringing, and 6. Scale the amplitudes by t^2 .

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218 **2.3 Plane-Wave-Destruction**

219 Plane wave destruction (PWD) is a predictive filtering method designed to suppress
 220 events in a seismic or GPR record having a particular dip (Claerbout, 1992; Fomel, 2002). The
 221 GPR image is modeled as the local superposition of plane waves described by the differential
 222 equation (Fomel, 2002):

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224
$$\frac{dP}{dx} - \sigma \frac{dP}{dt} = 0 \quad (1)$$

225

226 where $P(x, t)$ is the wave-field and $\sigma(x, t)$ is the local dip. Equation 1 provides the means for
 227 predicting a trace in the GPR image from its neighbor as a function of local dip. Fomel's (2002)
 228 three-point filter ~~is derived from~~ this equation:

229

230
$$C(\sigma) = \frac{\frac{(1+\sigma)(2+\sigma)}{12} - \frac{(1-\sigma)(2-\sigma)}{12}}{\frac{(2+\sigma)(2-\sigma)}{6} - \frac{(1-\sigma)(2-\sigma)}{6}} \quad (2)$$

231

232 where σ is the local dip and the filtering is accomplished by convolving (2) with the GPR image.

233 The goal is to suppress continuous reflections that have small dips (such as snow layering and
 234 the ground surface) compared to the steeply dipping diffraction limbs.

235 To estimate local dips, we make an initial guess σ_0 for the dip and solve the set of
 236 equations

237

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Deleted: Since we do not know the local dips, we use the
 stencil in equation 2 to estimate them directly from the
 data.

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245
$$\begin{pmatrix} \mathbf{C}'(\sigma_0) \mathbf{d} \\ \varepsilon \mathbf{D} \end{pmatrix} \Delta\sigma = \begin{pmatrix} -\mathbf{C}(\sigma_0) \mathbf{d} \\ 0 \end{pmatrix} \quad (3)$$

246

247 for $\Delta\sigma$. Here, $\mathbf{C}(\sigma)$ denotes the convolution of the filter with the data (\mathbf{d}), $\mathbf{C}'(\sigma)$ is the
 248 derivative of the filter with respect to σ ($\mathbf{C}'(\sigma)\mathbf{d}$ is a diagonal matrix), \mathbf{D} is the gradient
 249 operator, and ε is a weighting parameter that controls the smoothness of the estimated dip
 250 field. Imposing smoothness constraints on the dip field estimate ensures stability in the solution
 251 and helps target the reflections in the image, since they generally show higher amplitudes and
 252 are more laterally continuous than the diffractions we seek to preserve. The estimated dip field
 253 is then used to filter the data.

254 **2.4 Migration**

255 Migration is the process that moves reflected and diffracted energy in a seismic or GPR

256 record to its true location in the subsurface (i.e. Claerbout, 1985). The quality of the migration

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257 process depends on the accuracy of the velocity estimate. When the correct migration velocity

258 is chosen, diffraction hyperbolas will collapse to a compact “focus.” With too low a velocity, the

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259 hyperbola will only be partially collapsed, while a velocity that is too high will cause the

260 hyperbola to be mapped into a “smile”.

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261 For the MVA analysis, we migrate the entire image through a suite of velocities (0.19 to

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262 0.29 m/ns in increments of 0.002 m/ns) using MATGPR’s implementation of the Stolt algorithm

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263 (Stolt, 1955). The Stolt algorithm performs the migration in the frequency wave-number

264 domain and is computationally efficient. To reduce computational time, we modified the code

265 to perform all the migrations in one function call so that the forward Fourier transform is only

266 performed once.

273

274 **2.5 Velocity Picking**

275 After PWD filtering and migrating the data through the suite of velocities, the next task
276 is to use a focusing indicator to pick the image that is optimally focused. Following Fomel
277 (2007), we use the varimax norm (V):

278

279
$$V = \frac{N \sum_{i=1}^N s_i^4}{(\sum_{i=1}^N s_i^2)^2} \quad (4)$$

280

281 where s_i is the amplitude of the i th sample and N is the number of samples included in the
282 calculation. V is a measure of the “simplicity” of a signal (Wiggins, 1978). Since the simplest
283 possible signal is a spike and the optimal migration velocity will map hyperbolas to the most
284 compact “focus”, the maximum V value will correspond to the image migrated with the optimal
285 velocity.

286 To assess possible errors in the migration velocity analysis, we applied our workflow to a
287 synthetic data set generated from diffractors of varying size. The Fresnel radius is given by $R_f =$
288 $\sqrt{\frac{z\lambda}{2}}$ (Sheriff, 1980) where z is depth and λ is the dominant wavelength. Figure 1 shows the
289 effect of such an event on V. We created five synthetic diffractions with migration a migration
290 velocity of 0.24 m/ns. The first four (Figure 1a) correspond to rectangular objects at 1 meter
291 depth with horizontal dimensions 0.1, 0.2, 0.3 and 0.4 meters, and thickness of 0.03 m and the
292 fifth corresponds to a circular object with a radius of 0.4 meters (close to R_f for the 500 MHz
293 ricker wavelet used to generate the diffractions). The corresponding varimax curves for the

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[... [3]]

297 windows shown in Figure 1a are plotted in Figure 1b. The V curves are peaked at 0.24 m/ns for
298 all of the rectangular diffractors, with flatter (less well-resolved) peaks as the horizontal
299 dimension of the diffracting object increases, suggesting a larger uncertainty in the velocity
300 estimate. The peak V value for the circular diffractor is at 0.268 m/ns, indicating that curved
301 objects with lateral dimensions close to the size of the Fresnel zone may continue to focus at
302 velocities higher than their true velocity. Finally, Figure 1c shows the V curve for the entire
303 image, peaked at the correct velocity of 0.24 m/ns. This analysis suggests that the peak V value
304 will correspond to the correct velocity if the majority of the diffractions correspond to objects
305 much than R_f :

306 We choose to compute V ~~in sliding windows that span the entire time section and have~~
307 ~~a user-defined width. Computing V in this way allows us to incorporate many diffraction events~~
308 ~~and maximize the likelihood that the bulk of the diffractions satisfy the point diffractor~~
309 ~~assumption. Moreover, sliding windows offer the potential to capture lateral variability in snow~~
310 ~~density.~~

311 After ~~computing V for the entire data set, we choose the maximum V value in each~~
312 ~~window to get an estimate of the migration velocity. Noise in the filtered image, large~~
313 ~~diffracting objects, or a lack of diffractions, may cause the peak of the Vnorm to correspond to~~
314 ~~an incorrect velocity. To reduce the influence of erroneous velocity picks, we smooth the picks~~
315 ~~in the lateral direction with a boxcar averaging filter the same width as the sliding window.~~

316 ~~We use the shape of the upper portion of the V curve to estimate uncertainties in the~~
317 ~~velocity pick. Comparing the Vnorm curves for synthetic diffractions as well as those from our~~
318 ~~data, we find that Vnorm values that are greater than 95% of the peak value correspond to~~

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339 migrated images that are indistinguishable to the human eye (Figure 2). We therefore obtain
340 upper and lower bounds on our velocity estimate by finding the minimum and maximum
341 velocities with V_{norm} values equal to 95% of the maximum. We use the upper and lower
342 bounds on our velocity estimates to compute upper and lower bounds on all subsequent
343 calculations.

344

345 2.6 Dix Equation

346 The migration velocity is the RMS velocity of all of the material between the GPR
347 antenna and the diffractor. When the GPR antenna is in contact with the snow and the
348 diffractor is located at the base of the snow, we interpret the migration velocity to be the
349 average velocity of the snow across the width of the diffraction hyperbola. When the GPR unit
350 is mounted on the front of the snowmobile, the signal must pass through the air between the
351 antenna and the snow-surface so that the migration velocity is higher than that of the snow. To
352 find the snow-velocity from these data, we use the Dix equation (Dix, 1955):

353

354
$$V_{snow} = \sqrt{\frac{V_{mig}^2 t_{soil} - V_{air}^2 t_{snow}}{t_{soil} - t_{snow}}} \quad (5)$$

355

356 where velocity subscripts refer to the migration velocity, the velocity in air, and the velocity
357 within the snowpack and time subscripts refer to the two-way travel-times of the snow surface
358 and soil surface reflections.

359 The Dix equation contains two important assumptions. First, the velocity of the snow
360 must be approximately constant over the width of the hyperbola and second, the half-width of

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yields uncertainty estimates of +/- 0.005-0.01 m/ns, which is
comparable to the 0.005 m/ns reported in studies that rely
on picking

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(Bradford and Harper, 2005).

Deleted: $\sqrt{\frac{V_{mig}^2 t_{soil} - V_{air}^2 t_{snow}}{t_{soil} - t_{snow}}}$,

369 the hyperbola should be small compared to the depth of the diffractor ($x \ll z$). The diffractions
370 in our data sets are approximately 4 to 5 meters wide; thus we assume that any lateral
371 variations in snow density occur on a larger scale than this. If the second assumption is not
372 valid, then the Dix velocity will be higher than the true velocity, resulting in a density estimate
373 that is too low. The snow depths in our data range from ~1-2 meters, which is comparable to
374 the half-width of the hyperbolas.

375 To determine the minimum snow depth that satisfies the $x \ll z$ assumption, we traced
376 rays from point diffractors at depths ranging from 0 to 5 meters through a 0.23 m/ns snowpack,
377 representing a snow density of 0.358 g/cm³ (see section 2.7), with a 0.5 meter thick air layer
378 between the snow surface and the receiver positions (Figure 3). For each resulting travel-time
379 curve, we obtained nine different estimates of the migration velocity by performing a least-
380 squares fit to the travel-time data and successively reducing the widths of the hyperbolas from
381 10 to 2 meters in 1 meter increments. Using the Dix equation, we obtained estimates of the
382 snow velocity as a function of diffractor depth and hyperbola width (Figure 4). The velocity
383 estimates made with the Dix equation approach the true velocity as the diffractor depth
384 increases and the hyperbola width decreases. For hyperbolas that are 4 to 5 meters wide (the
385 average width that we observe in our data), the Dix velocity is within 2 percent of the true
386 velocity when the diffractors are about 1.5 meters deep, 5 percent when the diffractors are
387 about 1 meter deep, and 10 percent or greater when the diffractors are 0.5 meters deep. We
388 conclude that the use of the Dix is justified for diffractors buried deeper than 1.5 meters
389 beneath the snow surface.

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394 Although the results of this analysis are only valid for travel-time modeling, the $x \ll z$
395 assumption may be less severe for migration focusing analysis (see section 3.1). Diffraction
396 amplitudes decrease with increasing horizontal distance from the diffractor location, thus the
397 traces closest to the diffractor have the greatest contribution to the final image, suggesting that
398 the Dix equation may give adequate results for diffractors that are less than 1.5 meters deep
399 when velocities are estimated from MVA (we test this with our ~~synthetic data set in section~~
400 3.1).

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401 To propagate our velocity uncertainty estimates through the Dix equation, we assign a
402 travel-time uncertainty of 0.2 ns to our travel-time observations and use Eq. 5 along with our
403 velocity uncertainty estimates to compute upper and lower bounds on the snow velocity.

404

405 2.7 Estimating SWE

406 To estimate SWE from the radar data, we need to know the depth of the snow and the
407 snow density ($SWE = z_{snow}\rho_{snow}$). The depth can be found by picking the two-way travel-
408 time of the ground reflection and, if applicable, the snow-surface reflection and then using the
409 velocity estimate to convert time to depth. Using Eq. 1, we convert radar velocity to dielectric
410 constant ($\nu = c/\sqrt{\kappa'}$) and estimate the density of dry snow with the empirical relationship
411 (Tiuri et al., 1984):

412

$$413 \kappa'_d = 1 + 1.7\rho + 0.7\rho^2, \quad (6)$$

414

415 where κ'_d is the dielectric constant and ρ is the density of dry snow.

417 In this paper, we are primarily concerned with measuring radar velocities and we
418 assume that our data measure the properties of dry snow. The real part of the dielectric
419 constant for water (~ 80) is much larger than that of snow ($\sim 1.5 - 2$) and the imaginary part,
420 which describes the attenuation of the signal, is non-negligible (Bradford et al., 2009). The dry
421 snow assumption can be tested from the data by analyzing the attenuation properties of the
422 snowpack (Bradford et al., 2009). The attenuation coefficient for radar waves in water is
423 frequency dependent (i.e. Turner and Siggins, 1994), with the higher frequencies attenuating
424 more rapidly than the lower frequencies, because they go through more cycles per distance
425 traveled. When liquid water is present in the snow, the ground reflection will have a lower
426 mean frequency content than a reference event (the snow reflection for the snowmobile
427 collected data and the direct arrival for the skier-pulled data). To test the dry snow assumption,
428 we calculate the maximum local instantaneous frequency (Fomel, 2007) within a time window
429 surrounding the event of interest then average this value across all of the traces in the GPR
430 image. The standard deviation provides an estimate of the measurement uncertainty. We note
431 that at 500 MHz, a small shift in frequencies results in a non-negligible volumetric water
432 content.

434 3. Data and Results

435 Snow depth, density and SWE estimates for all of our GPR profiles and pits are summarized in
436 Tables 1 and 2. Here we discuss the processing and describe results for a synthetic data set and
437 two representative field data sets.

438

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486 3.1 Synthetic test

487 As a first test on the reliability of migration focusing analysis for reconstructing radar

488 velocities, we performed the analysis on ~~a synthetic data set~~ generated with REFLEX software.

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489 The synthetic data ~~set was~~ generated using a 500 MHz Kuepper wavelet sampled at 0.0332 ns

490 and traces are 0.01 meters apart.

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491 The ~~synthetic~~ model is 50 meters long and consists of a 0.5 meter thick layer of air

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492 overlying a 0.24 m/ns ~~layer of snow~~ (corresponding to a density of 0.29 g/cc), with depths that

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493 range from 0.5 to 5.7 meters. Beneath the snow is a 0.10 m/ns layer representative of soil.

494 Along the snow/soil interface there are 16 diffractors buried at depths ranging from 0.5 to 5.7

495 meters. The purpose of this data set (Figure 5a) was to test the performance of the Dix

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496 equation on velocities estimated from the MVA analysis ~~and, since the migration velocity~~497 ~~changes as a function of snow depth, to see if we can resolve lateral variations in velocity.~~

498 After applying the PWD filter, the ground reflection was adequately suppressed (Figure

499 5b). We migrated the filtered image at 0.002 m/ns intervals from 0.18 to 0.28 m/n and

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500 measure the optimal migration velocity for each diffractor by computing V ~~in an 8-meter-wide~~501 ~~sliding window~~ (Figure 5c). We use the Dix equation to convert the migration velocities to the

Deleted: 4c) within small windows centered over the apex of the hyperbola (Figure 4b).

502 velocity of the snow layer, (Figure 5d). The average of all snow velocity measurements is 0.241

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503 m/ns with a standard deviation of 0.002 m/ns.

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504 There is no systematic relationship between the velocities recovered and the depth of

505 the diffractor (Figure 5d). The shallowest diffractor was at ~0.5 m depth and the recovered

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506 velocity was 0.232 m/ns. The greatest differences between recovered and true velocities were

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507 for diffractors at depths of 0.5, 1.5, and 2.2 and 2.3 meters. Here the recovered velocities were

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524 0.232, 0.247 0.247, and 0.247 m/ns. The shallowest observation underestimates the true
525 velocity, which is the opposite of the effect predicted by our travel-time modeling (Section 2.6,
526 Figure 4). The observations for diffractors between 1.5 and 2.3 meters all overestimate the true
527 velocity by approximately the same amount. We conclude that the Dix equation is appropriate
528 for snowdepths of 0.5 meters and greater.

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529 Although the snow in this synthetic model has a constant velocity, the migration velocity
530 changes as a function of the snow depth due to the changing proportions of air and snow in the
531 total travel path. Where the snow is shallow, the velocities are highest and where the snow is
532 deep, the velocities are low. That the method is capable of resolving lateral velocity variations
533 in this synthetic example is evident in Figure 5c, where the picked velocities are negatively
534 correlated with snowdepth.

Deleted: The second model is 10 meters long with a snow layer that ranges from 1.9 to 2.7 meters thick with a velocity that increases from 0.257 m/ns at x=0, to 0.262 m/ns at x = 10 meters. There are seven diffractors along the soil/snow interface. The primary purpose of this data set (Figure 6a) was to see whether this method could resolve a lateral change in velocity. ... [9]

535 536 3.2 Ski-pulled GPR data

537 We collected two GPR profiles in the skier -pulled configuration on February 25, 2015, in
538 below-freezing conditions. A representative line, Line 1 (Figure 6) is 74 meters long and shows
539 an abundance of diffractions along the snow/ground interface, likely a result of small boulders,
540 and a few isolated diffractions within the snowpack, most likely small trees, bushes or logs.
541 After interpolation to equal spacing, trace spacing was 0.362 m. Since the antenna was coupled
542 to the snow, we compare the average frequency of the direct wave to that of the soil reflection
543 to determine whether there is any liquid water present in the snowpack. The average
544 frequency of the direct arrival for every trace in the image along Line 1 is 410 MHz with a
545 standard deviation of 10 MHz and the average frequency of the soil reflection across the whole

Deleted: Line 19 is a 74 meter long, skier pulled data set
Deleted: The data show
Deleted: or bushes (Figure 7a).

566 line is 457 MHz with a standard deviation of 42 MHz. The soil reflection appears to have a
567 higher frequency content than the reference frequency, perhaps due to thin-layer "tuning"
568 effects. Since we do not observe a decrease in frequency with travel time, we infer that there
569 was no liquid water present in the snow on this day.

570 After the PWD filtering step we are left with many diffractions along the ground surface
571 and a few isolated events within the snowpack (Figure 6b). We compute V in 10-meter-wide
572 sliding windows and pick the velocity that corresponds to the peak value of V (Figure 5d, blue
573 line). After smoothing these picks (Figure 6d, red line) we obtain velocities between 0.237 and
574 0.276 m/ns, with an average uncertainty of 0.01 m/ns, corresponding to densities of 313 to 145
575 kg/m³. It is unlikely that the snow density is as low as 145 kg/m³, and the velocity
576 measurements that yield such unlikely results are confined to the region between x ~30 -55
577 meters. Either the diffractors along this part of the line are all too large to meet our point
578 diffractor assumption, or the noise levels in the image are higher than the signal.

579 Excluding the picks between x=30 and 55 meters, we estimate snow densities between
580 193 and 311 kg/m³, with an average density of 274 kg/m³. Notably, the low-density
581 estimates are from the part of the profile near x = 55 to 65 meters where a prominent set of
582 mid-snow diffractors exist. The two-way travel time to the tops of these diffractors is ~7.414 ns,
583 which at the observed migration velocity of 0.256 m/ns yields a depth estimate of ~0.95
584 meters. Thus, this snow density estimate of 193 kg/m³ corresponds to the upper 0.95 meters of
585 snow. Estimated snow depths, densities and SWE along the entire profile are shown in Figure 7.

586 We measured snow density and depth in Pit 1 located at 68 meters along the Line 1
587 (Figure 7). The snow pit showed a depth of 1.33 meters and an average density of 300 +/- 40

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Deleted: Velocities on this line range from 0.23 to 0.25 m/ns with an average uncertainty of +/- 0.01 m/ns. Estimated snow-depths range from 1.6 to 1.9 meters with an average uncertainty of +/- 0.07 m. Estimated snow densities range from 0.23 to 0.36 g/cc with an average uncertainty of +/- 0.07 g/cc. Estimated SWE ranges from 0.3 to 0.5 meters with an average uncertainty of 0.08 meters (Figure 8). .

... [10]

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601 kg/m³ resulting in a SWE measurement of 0.40 +/- 0.07 meters. GPR derived estimates at the
602 pit location are: snow depth = 1.28 +/- 0.06 meters, density = 288 +/- 50 kg/m³, SWE = 0.37 +/-
603 0.07 meters. The average density of the upper 0.95 meters of snow in this pit is 190 kg/m³ (Fig
604 S1), which is very close to the value estimated from the GPR data between x = 55 and x = 65
605 meters.

606

607 **3.3 Snowmobile-Mounted GPR data**

608 ~~We collected four GPR profiles in the snowmobile-mounted configuration between Feb~~
609 ~~25 and March 17, 2015. Here we discuss the processing of a representative profile, Line 4~~
610 ~~(Figure 8), which~~ was collected on the morning of March 11, 2015 in a flat meadow just south of
611 Wyoming State Highway 130. This line is 98 meters long and shows an abundance of
612 diffractions along the snow/ground interface (Figure 8). After interpolating to equal spacing,
613 ~~the trace spacing was 0.024 m.~~

614 ~~Migration velocities~~ on this line range from 0.237 to 0.277 m/ns with an average
615 uncertainty of +/- 0.01 m/ns. The corresponding snow velocities are 0.207 and 0.268 m/ns.
616 Here, the exceptionally high velocities are confined to a region between x = 65 and x = 85 meter
617 where a number of diffractions from obviously large objects are present (Figure 7). If we
618 exclude velocity picks from this region, we get a maximum migration velocity of 0.266 m/ns and
619 a maximum snow velocity of 0.251 m/ns. Estimated snow depths range from 0.7 to 2.1 meters
620 with an average uncertainty of +/- 0.1 m. Estimated snow densities range from 228 to 532
621 kg/m³ with an average uncertainty of +/- 50 kg/m³. Estimated SWE ranges from 0.25 to 0.71
622 meters with an average uncertainty of +/- 0.09 meters.

Deleted: 0.32 +/- 0.07 g/cc, SWE = 0.41 +/- 0.07

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Deleted: 9a). Picking velocities along this line required significantly more discretion than was required on Line 19. Whereas on Line 19 we were confident in choosing velocities with well-defined varimax peaks, on this line we rejected some velocity observations between x = 0 and x = 10 that appeared to produce well focused diffractions that would have resulted in snow-density estimates greater than 1 g/cc.

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647 Snowpits 3 and 4 are located at 50 and 97 meters along the profile and showed average
648 snow densities of 379 ± 50 and $360 \pm 48 \text{ kg/m}^3$; SWE values of 0.54 ± 0.13 and 0.64 ± 0.13
649 meters; snowdepth was 1.44 ± 0.05 and $1.8 \pm 0.05 \text{ m}$ respectively. The GPR-derived
650 depth, density and SWE estimates at 50 and 97 meters were 1.50 ± 0.08 and $1.91 \pm 0.12 \text{ m}$;
651 389 ± 92 and $394 \pm 97 \text{ kg/m}^3$; and 0.53 ± 0.09 and $0.70 \pm 0.13 \text{ m}$. GPR-derived estimates
652 for the whole profile are shown in Figure 9. We also measured 21 snow depths at 5 meter
653 intervals (Figure 9b) along this profile. The RMS error between observed and estimated depths
654 is 0.13 meters.

655 During data acquisition on Line 7, the air temperature was 5°C , raising the possibility of
656 liquid water in the snow. The average frequency of the snow reflection for every trace in the
657 image is 435 MHz with a standard deviation of 27 MHz and the average frequency of the soil
658 reflection across the whole line is 464 MHz with a standard deviation of 38 MHz. Again, the
659 frequency content of the soil reflection appears to be higher than the reference frequency.

660 Within the uncertainty bounds there is no resolvable frequency change, and we conclude that
661 our dry snow assumption is valid.

662

663 4. Discussion

664 The primary purpose of this study is to develop an efficient processing flow for
665 measuring GPR velocity and thus snow density SWE from common-offset data that requires a
666 minimum amount of human interpretation. Common-offset GPR data are fast and easy to
667 obtain, and velocity estimates can be made when diffractions are present. However, the
668 common methods of visually inspecting migrated images or fitting curves to diffraction

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Deleted: estimated snow-depths are generally low (Figures 10b and 11) and, on average, within 8% of
Deleted: probed depths. The correlation coefficient between predicted and observed
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Deleted: , however given these uncertainties there may be up to a 36 MHz shift, which would result in a volumetric water content of less than 0.03 (Figure 3).

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696 hyperbolas ~~are~~ time-consuming and subject to human error. The migration velocity analysis
697 described in this paper provides an efficient means for extracting velocity information from
698 large GPR data sets. Here we discuss the ~~accuracy and efficiency of the method as well as the~~
699 level of automation.

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700 ~~To validate the method, we compared estimated snow densities, depths, and SWE to~~
701 observations made in four snow pits and to 86 probed snow depth measurements. The results
702 are summarized in Table 2 and in Figure 9. If we exclude the two obvious outliers (Figure 10a),
703 the RMS error for our depth predictions for the remaining 88 depth observations is 12% of the
704 mean snowdepth observation. The RMS error for snow density and SWE relative to the mean
705 observed values are 15% and 18%. Averaging the velocities across the entire line (Figure 10 red
706 crosses) reduce the difference between predicted and observed depth values to an RMS error
707 of 9%, suggesting that lateral variations in snow velocity are minimal. Averaging the velocities
708 across the entire line reduces the RMS errors for density and SWE to 8% and 10%, respectively.

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709 The greatest potential for systematic error in this analysis is the presence diffracting
710 objects whose dimensions exceed the radius of the first Fresnel zone. The field data offer the
711 opportunity to evaluate the influence of diffractor size on velocity estimates. Line 1, for
712 example, shows four prominent diffractions between 50 and 70 meters. The Varimax norm has
713 a maximum value at 0.256 m/ns, which is the velocity that focuses the two leftmost diffractions
714 (Figure 6c). The diffractions on the right are clearly not focused because they are caused by an
715 object (most likely a log) with a radius greater than the first Fresnel zone. Because the leftmost
716 two have a higher amplitude than the others, they have the largest influence on the varimax
717 value. Thus, although there are clearly events in the field data that have the potential to give

724 erroneous results, our results suggest that reliable velocity estimates can be achieved so long as
725 the majority of the diffracted energy is related to objects that can be considered point
726 diffractors.

727 One of our main goals was to produce a processing flow that allows for the rapid
728 processing of common offset GPR data with minimal user interaction. The two most time
729 computationally expensive parts of the processes are the migrations and the varimax
730 calculations. As an example, on a 2016 MacBook Pro with a 2GHz processor, for the ~100-
731 meter-long Line 4, performing 51 migrations takes approximately 5 minutes, the varimax
732 calculation takes about half as long, and the PWD filtering takes a few seconds. The most time-
733 consuming part of the process is picking the arrival times of snow surface and ground surface
734 reflections.

735 Although the processing flow is relatively efficient, it does require some user
736 interaction. The PWD method of separating continuous reflectors from diffractions treats the
737 GPR image as the superposition of locally planar waves. Estimating the slope of these waves
738 from the image requires the solution of a regularized inverse problem and the smoothness of
739 the slope-field depends on the choice of regularization parameter. This is the most subjective
740 step of the process, as it may require several attempts to find the optimal smoothness
741 constraints to adequately suppress reflections in the GPR image. However, for our data the
742 majority of the diffractions are located along the ground surface and the internal structure of
743 the snowpack shows dips that closely parallel the ground reflection. A good first guess, and
744 often a good final guess, for the dip field can be computed by picking the arrival times of the

Deleted: We found that areas of rapidly changing slope can result in noise from incompletely suppressed reflection events and poorly preserved diffractions. Noise from inadequate filtering may cause the Varimax Norm value to be high even when the diffractions are not optimally focused and low when they are. Visually checking the migrated images before committing to a velocity pick can help mitigate this issue.

753 ground reflection. Because the ground reflection has to be interpreted to measure snow depth,
754 this strategy can significantly reduce the processing time for each data set.

755 The data presented in this paper contained an abundance of diffractions located near
756 the soil/ground interface allowing an average velocity for the entire snowpack to be obtained.

757 These events are likely due to small-scale variations in surface topography, rocks, and/or
758 vegetation along the ground surface, which may not be present in all environments. However,
759 we note that mountain watersheds free of vegetation, small undulations in surface topography,
760 and surface rocks are probably rare. Thus, the method may be useful in many regions where
761 seasonal snowpacks exist.

762

763 5. Conclusions

764 We applied the migration focusing analysis presented in Fomel (2007) to the problem of
765 estimating SWE in seasonal snow. The method was most accurate for the case when the GPR
766 was in contact with the snow, providing GPR-derived SWE estimates within 6 % of the manual
767 observation. When the GPR was mounted on a snowmobile, the results were within 12-21% of
768 the manual observations.

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Deleted: ... In particular, Line 7 required a substantial amount of user intervention to avoid picking obviously incorrect velocities. The performance of the MVA analysis along this line may have been due to several complicating factors: 1. When mounted on the snow-mobile, the GPR antenna is fixed at the rear and can wobble up and down at the front by up to ~5 cm. The change in orientation of the antenna with respect to subsurface targets as well as the change in distance between the snow surface and the GPR antenna may be additional noise sources and cause diffractions to migrate incorrectly. This situation is likely to be of concern when the snow-surface is uneven, or when the snowmobile is accelerating. Indeed, the greatest variability along this line occurred during the first few meters when the snowmobile was accelerating. 2. On this day the air-temperatures were above freezing and, although our frequency analysis suggests that we can make the dry snow assumption, it is likely that some water was present in the snowpack the presence of water in the snowpack would result in decreased velocities and increase the apparent dry snow density. - ... [11]

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Deleted: small bushes near the base of the snow-pack

Deleted: Areas likely to contain point diffractors suitable for this type of analysis can

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Deleted: ... The processing flow that we presented in this paper proved to be an efficient way to measure radar velocities within seasonal snow. While not fully automated, the method requires less processing time than visually scanning each migrated image and could make GPR a more attractive tool for estimating SWE at the watershed ... [12]

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815 **Acknowledgements**

816 This work was funded by the U. S. National Science Foundation (NSF) EPSCoR Program,

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817 NSF award EPS-1208909. We would like to thank Matt Provart for assisting with data collection

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818 and Mehrez Elwaseif for assistance with REFLEX software. Data used in this paper are available

819 at <https://data.uwyo.edu>.

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869 **Figure Captions**

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870 **Figure 1 a)** Synthetic hyperbolas for 4 rectangular diffractors with lateral dimensions of 0.1, 0.2,

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871 0.3 and 0.4 meters (from left to right) and a round diffractor with radius = 0.4 meters (far right.)

872 **b)** Varimax curve for windows depicted in a, V curve colors match the windows in a. V curves

873 for all four rectangular diffractors show peaks at 0.24 m/ns, while the round diffractor is peaked

874 at 0.268 m/ns. **c)** Varimax curve for the entire image showing a peak at the correct migration

875 velocity of $v = 0.24$ m/ns.

876

877 **Figure 2** Justification for uncertainty estimates. A synthetic hyperbola that is obviously

878 undermigrated (a), migrated at indistinguishable velocities (b-d), and obviously overmigrated

879 (e). f) The corresponding varimax curve for a-e showing a peak at the true migration velocity

880 (0.24 m/ns), the shaded area under the curve corresponds to velocities in b-d and represent

881 varimax values that are 95% of the maximum. Panels (g-l) show the same for a section of field

882 data extracted from Line 1.

883

884 **Figure 3** Raypaths and travel-times for point diffractors. **a)** 0.5 meters of air overlying a 230

885 m/ns snowpack with point diffractors buried at 0.5 meter intervals. **b)** two-way travel-times for

886 each of the diffractors showing the characteristic hyperbolic shape.

887

888 **Figure 4** Dix velocities for point diffractors as a function of depth for different hyperbola

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889 widths. The true interval velocity is 0.230 m/ns (red line) and the Dix velocities are shown as

892 black lines. The red dashed line is at 0.234 m/ns, which is 2 percent greater than the true
893 velocity.

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895 **Figure 5** Synthetic Data set and velocity picking. **a)** synthetic data before filtering. **b)** after PWD
896 filtering. **c)** Varimax norm for sliding window 8 meters wide. **d)** Velocities from synthetic data
897 set as a function of diffractor depth. Solid blue line shows measured migration velocities,
898 dashed blue lines show uncertainty bounds. Solid red line show velocities computed with the
899 Dix equation, dashed red lines show uncertainty bounds. Solid black line shows the true velocity
900 (0.24 m/ns). Light gray region indicates where velocities are within 2% of the true velocity and
901 dark gray region shows where velocities are with 5% of the true velocity.

902

903 **Figure 6** **a)** raw GPR data for Line 1 **b)** GPR data after PWD filtering **c)** diffractions migrated at
904 the mean velocity (0.245 m/ns) for the entire line **d)** Normalized varimax curves for sliding
905 window 10 meters wide. Blue curve shows the peak value for every curve, red line is smoothed
906 with a box car averaging filter 10 meters wide.

907

908 **Figure 7** Line 1 Results. **a)** density, **b)** snow depth (black line) and SWE (blue line) estimates
909 from the GPR data, snow pit data are shown in red. **Grayed out region corresponds to areas**
910 **where velocity picks are unreliable.**

911

912 **Figure 8** **a)** raw GPR data for Line 4, red lines indicate interpreted ground and snow reflection.
913 **b)** GPR data after PWD filtering **c)** diffractions migrated at the mean velocity (0.256 m/ns) for

Deleted: **Figure 3** Snow wetness for typical GPR velocities in and snow and peak frequency shifts for a reference frequency of 500 MHz. For the data presented in this paper, typical uncertainties in the frequency measurements are 10 to 40 MHz and the velocities range from 0.22 m/ns to 0.25 m/ns. For the typical range of velocity and frequency shift estimates reported in this paper, snow wetness values less than 0.03 cannot be resolved. [... \[13\]](#)

Deleted: **b)** the unmigrated data

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Deleted: plotted against velocity showing a peak at 0.246 m/ns.

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Deleted: Synthetic data set from a model with the lateral velocity trend and no air layer. **b)** The recovered velocities (black line) show the same trend as the true model (red) but systematically underestimate the true values by 2.1% at x = 0 meters and 1.6 % at x = 10 meters. [... \[15\]](#)

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Deleted: **a)** the radar velocity within the snow along the profile.

Deleted: **c)** snow densities estimated from the GPR data (blue line) and the density measured in the snow-pit at 68 meters (red).

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Deleted: 0.254 m/ns showing the focused diffraction events. [... \[16\]](#)

965 the entire line **d)** Normalized varimax curves for sliding window 10 meters wide. Blue curve
966 shows the peak value for every curve, red line is smoothed with a box car averaging filter 10
967 meters wide.

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969 **Figure 9** Line 1 Results. **a)** density, **b)** snow depth (black line) and SWE (blue line) estimates
970 from the GPR data. snow pit data are shown in red. Grayed out region corresponds to areas
971 where velocity picks are unreliable.

Deleted: as well as the probe depths (red) and the
Deleted: at 50 and 97 meters (red). **c)** snow densities
estimated from the GPR data (blue line) and the densities
measured

Deleted: pits at 50 and 97 meters (

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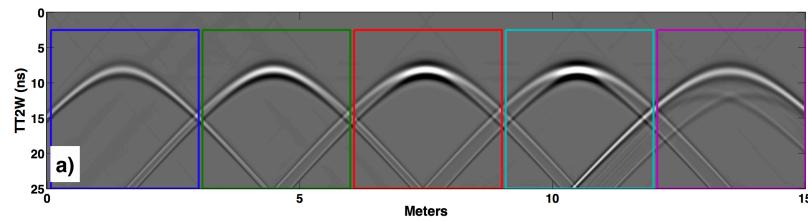
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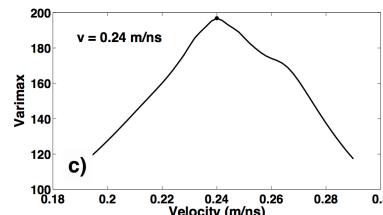
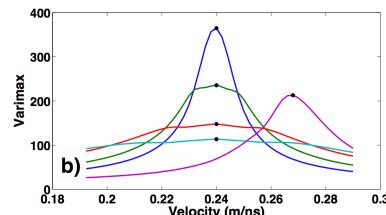
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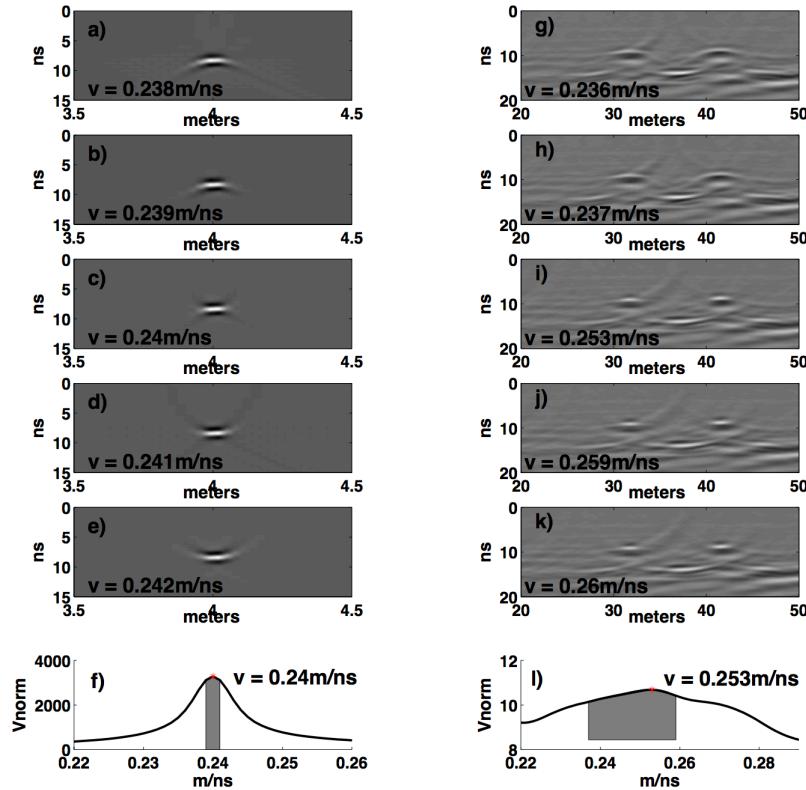


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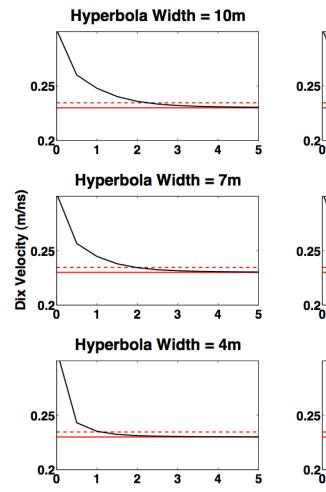
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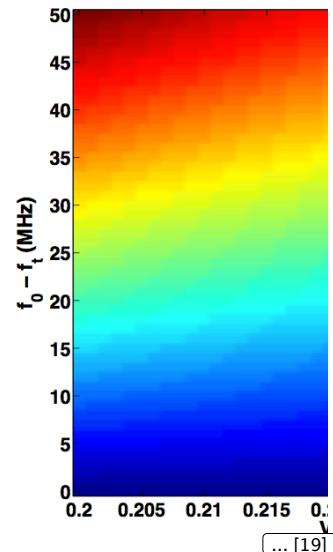
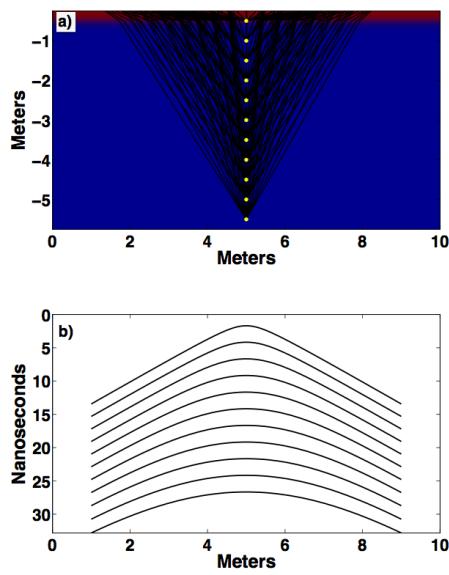
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1021 **Figure 3**

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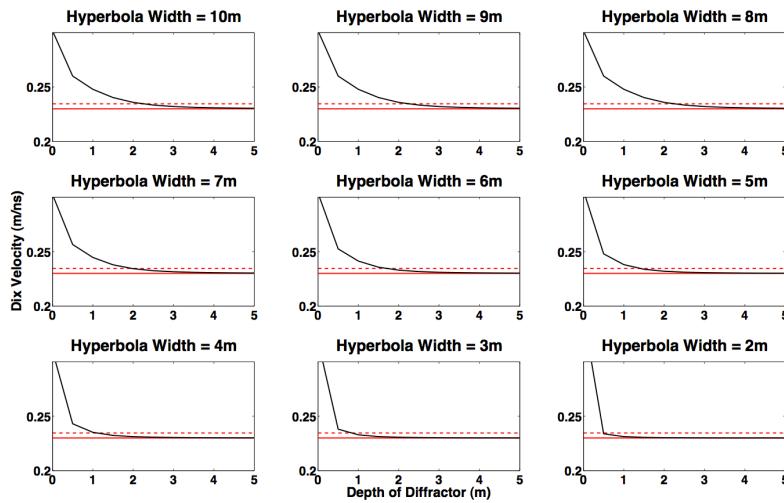
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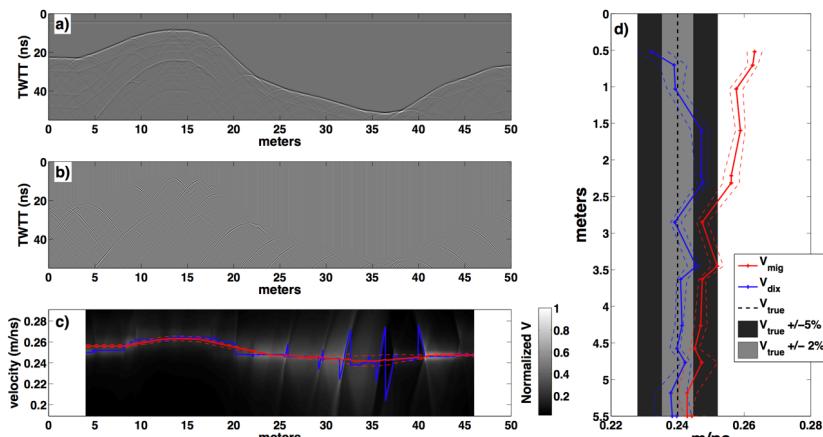
Figure 4



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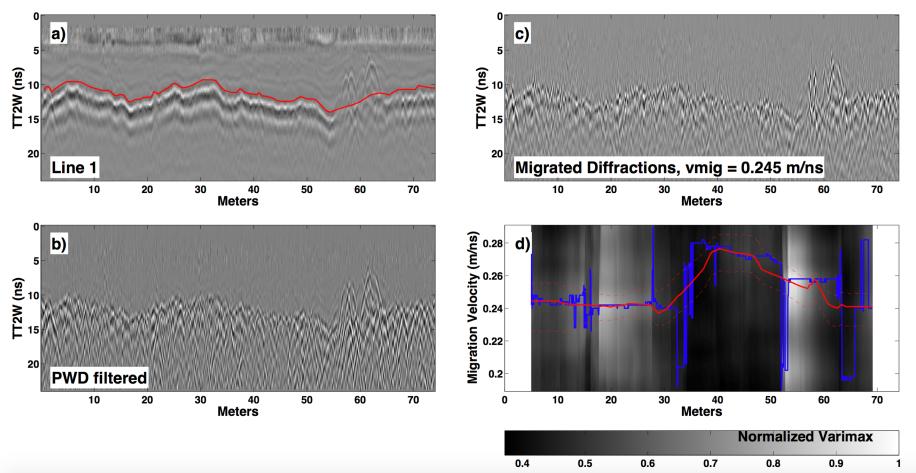
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1043 **Figure 6**

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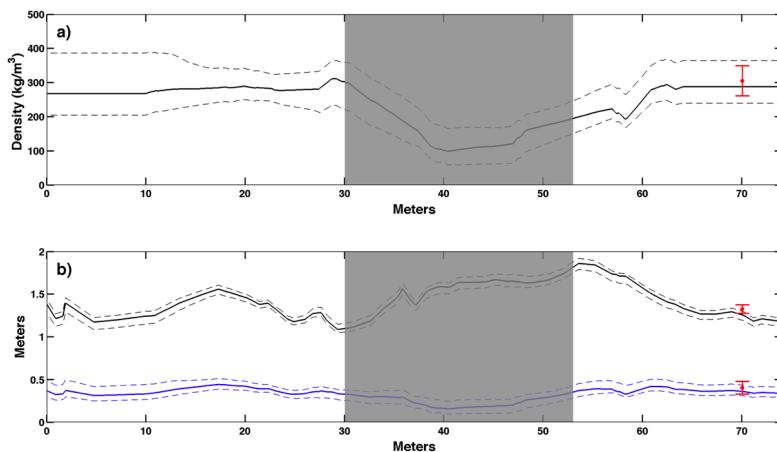
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Figure 7



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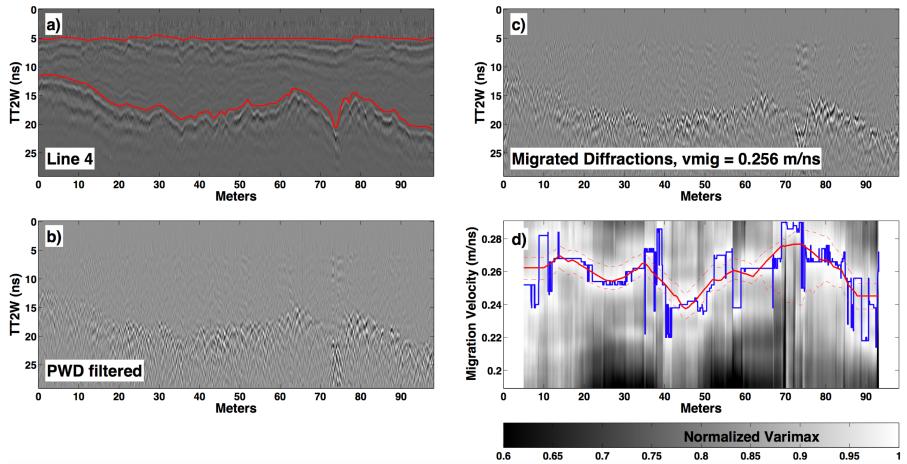
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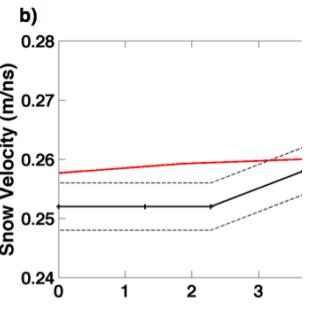
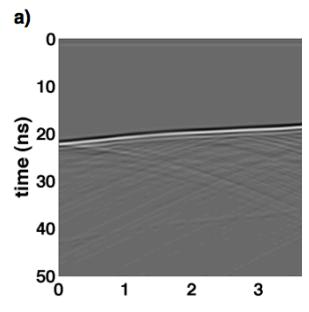
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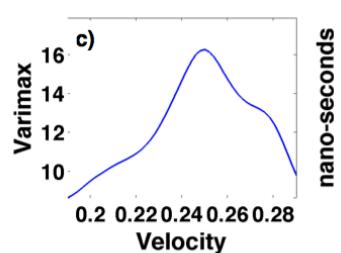
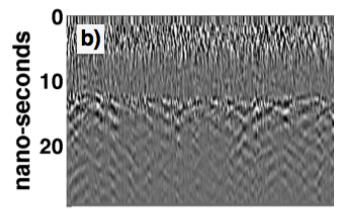
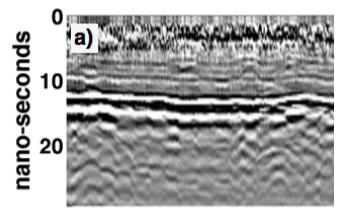
Figure 8



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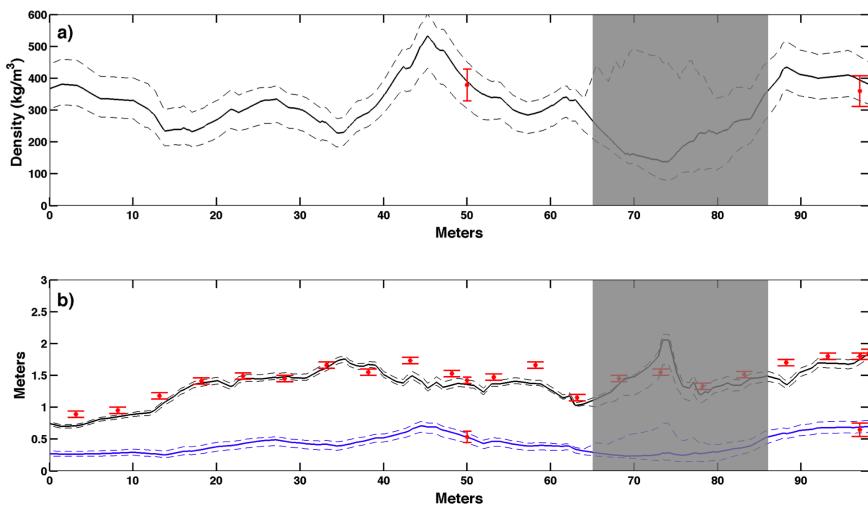
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1069 **Figure 9**



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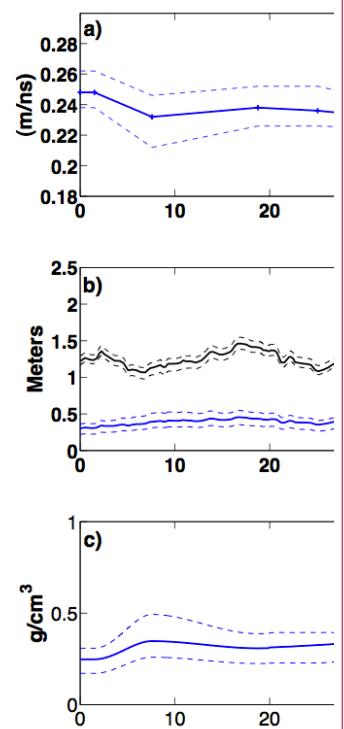
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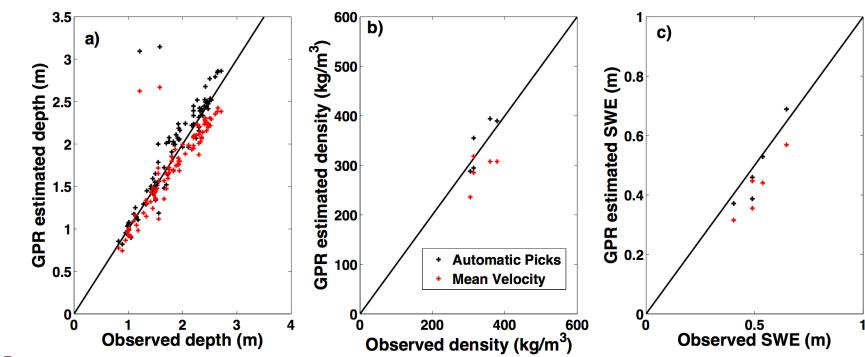
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1104 [Tables](#)

1106 [Table 1. Snowpit summary](#)

| <u>Pit Name</u> | <u>Date</u> | <u>Depth (m)</u> | <u>Rho (kg/m³)</u> | <u>SWE (m)</u> | <u>GPR profiles</u> |
|-----------------|------------------|----------------------|-------------------------------|----------------------|----------------------|
| Pit 1 | <u>25-Feb-15</u> | <u>1.33 +/- 0.05</u> | <u>305 +/- 44</u> | <u>0.40 +/- 0.14</u> | <u>Line 1</u> |
| Pit 2 | <u>26-Feb-15</u> | <u>1.56 +/- 0.05</u> | <u>314 +/- 44</u> | <u>0.49 +/- 0.14</u> | <u>Lines 2 and 3</u> |
| Pit 3 | <u>11-Mar-15</u> | <u>1.44 +/- 0.05</u> | <u>379 +/- 50</u> | <u>0.55 +/- 0.13</u> | <u>Line 4</u> |
| Pit 4 | <u>11-Mar-15</u> | <u>1.80 +/- 0.05</u> | <u>360 +/- 48</u> | <u>0.65 +/- 0.13</u> | <u>Line 4</u> |

Table 2. Summary of GPR field data and comparison to manual measurements

| <u>GPR Profile</u> | <u>Collection Date</u> | <u>Acquisition Mode</u> | <u>Pits/Probe</u> | <u>GPR Predictions at pit</u> | | | <u>Error Compared to Pit/Probe</u> | | |
|--------------------|------------------------|-------------------------|--------------------------|--------------------------------|-------------------------------|--------------------------------|------------------------------------|------------|--------------|
| | | | | <u>Depth Pred (m)</u> | <u>Rho (kg/m³)</u> | <u>SWE (m)</u> | <u>Depth</u> | <u>Rho</u> | <u>SWE</u> |
| Line 1 | 25-Feb-15 | Ski | Pit 1 | 1.29 +/- 0.06 | 288 +/- 50 | 0.37 +/- 0.07 | 2.6% | 5.5% | 8.0% |
| †Line 2 | 25-Feb-15 | Ski | Pit 2 | 1.59 +/- 0.04 | 294 +/- 40 | 0.46 +/- 0.03 | 0.1% | 6.0% | 6.0% |
| †Line 3 | 25-Feb-15 | Snowmobile | Pit 2 | 1.10 +/- 0.05 | 354 +/- 65 | 0.39 +/- 0.06 | *30.0% | 13% | *21% |
| Line 4 | 11-Mar-15 | Snowmobile | Pit 3 Pit 4 Probes | 1.50 +/- 0.08 1.91 +/- 0.12 | 389 +/- 92 394 +/- 97 | 0.53 +/- 0.09 0.69 +/- 0.13 | 6.0% 6.0% | 3% 10% | 2.0% 6.6% |
| †Line 5 | 17-Mar-15 | Snowmobile | Probes | | | | **RMSE = 0.13 m (9%) | | |
| †Line 6 | 17-Mar-15 | Snowmobile | Probes | | | | **RMSE = 0.38 m (18%) | | |
| | | | | | | | **RMSE = 0.19 m (11%) | | |

*Line 3 was located 1.5 meters off of Pit 2, disagreement between depth and SWE measurements at this site reflect lateral variations in snowdepth.

**RMSE percentages are calculated relative to the mean observed depth along each profile

†Lines 2, 3, 5, and 6 are described in the supplementary materials.

Supplementary Materials

In addition to the data presented in the main text, we include snow density profiles for Pits 1-4 (Figure S1) and show data and results for GPR lines 2, 3, 5 and 6:

Line 2 is a skier pulled data set collected on February 25, 2015 in sub-freezing conditions. Pit 2 was located at $x = 100$ meters. After interpolating the data to equal spacing, the trace spacing was 0.027 meters. The data and velocity picks are depicted in Fig S2 and the resulting snow depth, density and SWE estimates are shown in Fig S3.

Line 3 is a snowmobile driven data set collected on February 25, 2015 in sub-freezing conditions. Pit 2 was located at $x = 54$ meters. After interpolating the data to equal spacing, the trace spacing was 0.027 meters. The data and velocity picks are depicted in Fig S4 and the resulting snow depth, density and SWE estimates are shown in Fig S5. Notably, Pit 2 was located ~ 1.5 meters off of the GPR line, which we suggest explains the discrepancy in the depth and SWE predictions at the pit site.

Line 5 is a snowmobile driven data set collected on March 17, 2015 in above-freezing conditions. Air temperature reached $10^\circ C$ on this day and we infer that the dry snow assumption was not valid. After interpolating the data to equal spacing, the trace spacing was 0.0245 meters. The data and velocity picks are depicted in Fig S6 and the resulting snow depth, density and SWE estimates are shown in Fig S7. We probed snowdepth along this line at 2 meter intervals.

Line 5 is a snowmobile driven data set collected on March 17, 2015 in above-freezing conditions. Air temperature reached $10^\circ C$ on this day and we infer that the dry snow assumption was not valid. After interpolating the data to equal spacing, the trace spacing was 0.0148 meters. The data and velocity picks are depicted in Fig S8 and the resulting snow depth, density and SWE estimates are shown in Fig S9. We probed snowdepth along this line at 2 meter intervals.

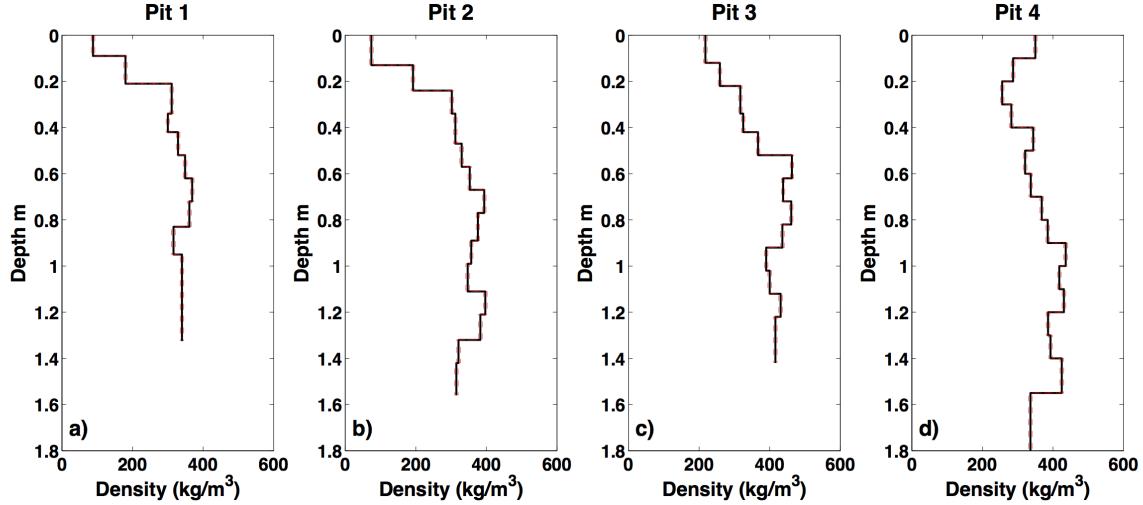


Figure S1. Snow density profiles. Black lines are measured density values, red lines indicate uncertainty estimate.

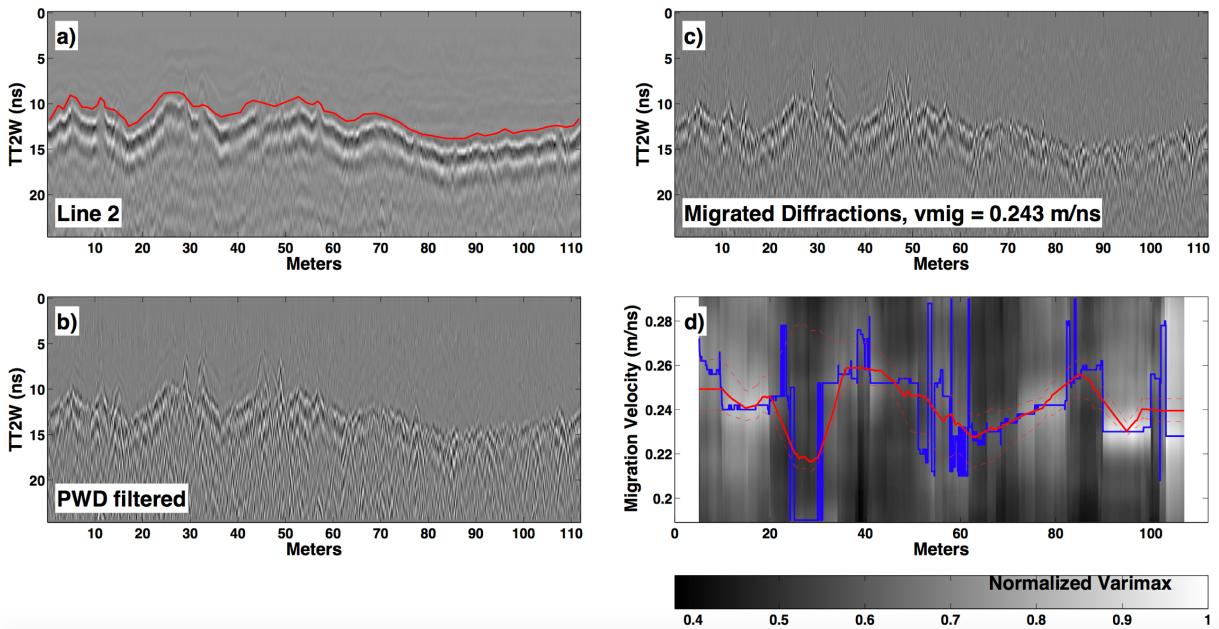


Figure S2. a) raw GPR data for Line, red line indicates interpreted ground reflection 2 b) GPR data after PWD filtering c) diffractions migrated at the mean velocity (0.243 m/ns) for the entire line d) Normalized varimax curves for sliding window 10 meters wide. Blue curve shows the peak value for every curve, red line is smoothed with a box car averaging filter 10 meters wide.

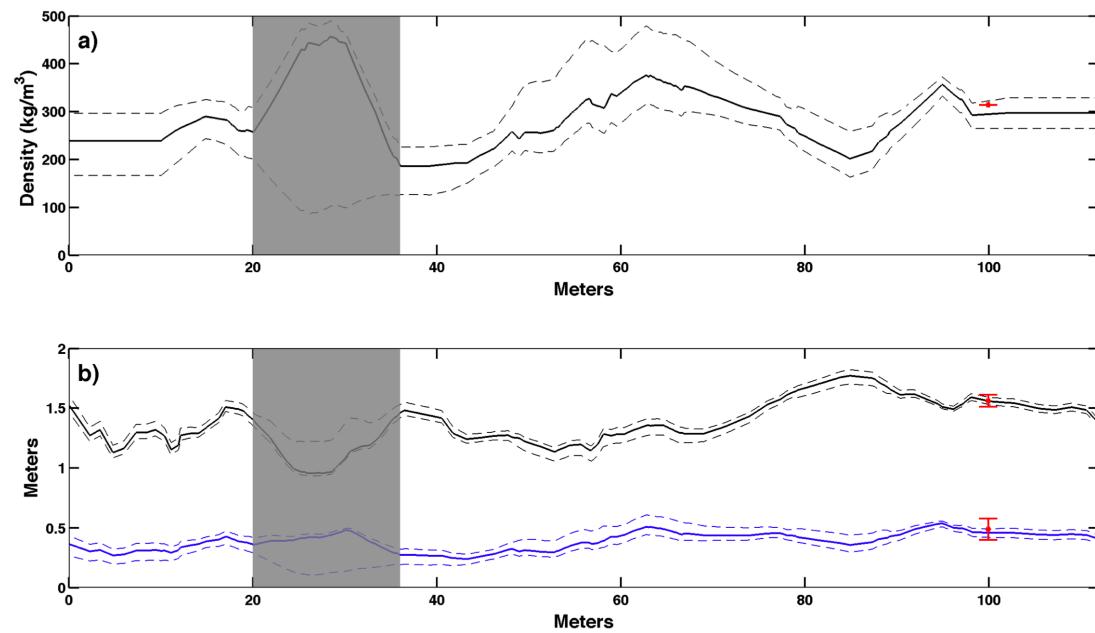


Figure S3. Line 2 Results. **a)** density, **b)** snow depth (black line) and SWE (blue line) estimates from the GPR data, snow pit data are shown in red. Grayed out region corresponds to areas where velocity picks are unreliable.

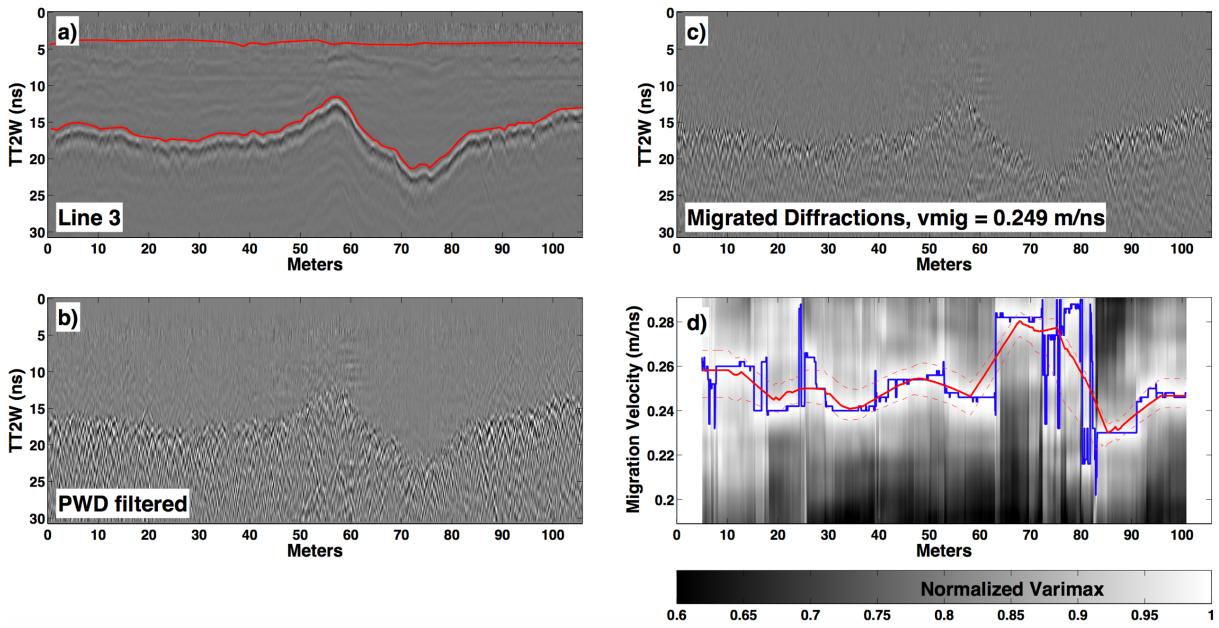


Figure S4. a) raw GPR data for Line 3, red lines indicates interpreted ground and snow reflections b) GPR data after PWD filtering c) diffractions migrated at the mean velocity (0.247 m/ns) for the entire line d) Normalized varimax curves for sliding window 10 meters wide. Blue curve shows the peak value for every curve, red line is smoothed with a box car averaging filter 10 meters wide.

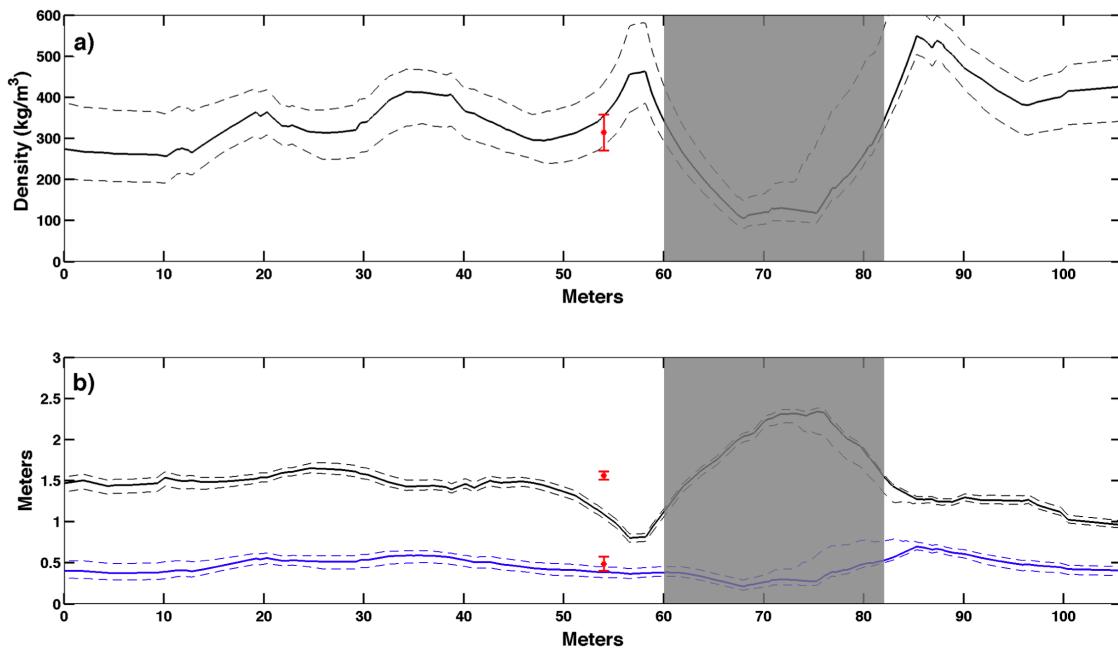


Figure S5. Line 3 Results. a) density, b) snow depth (black line) and SWE (blue line) estimates from the GPR data, snow pit data are shown in red. Grayed out region corresponds to areas where velocity picks are unreliable.

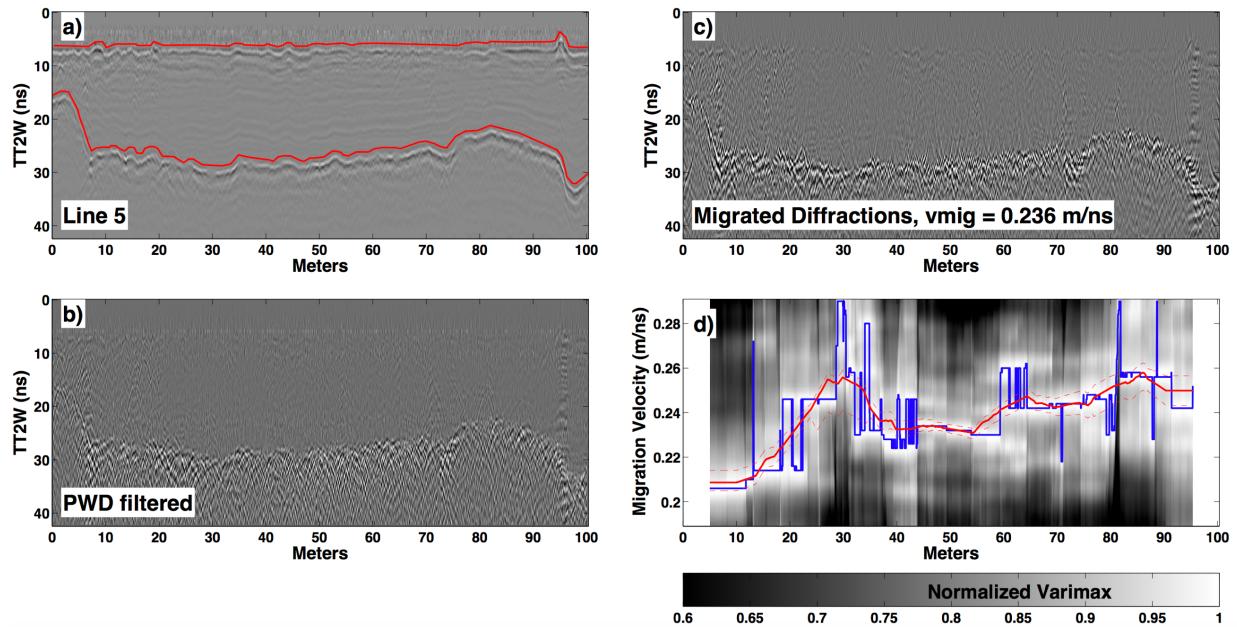


Figure S6. **a)** raw GPR data for Line 5, red lines indicates interpreted ground and snow reflections **b)** GPR data after PWD filtering **c)** diffractions migrated at the mean velocity (0.233 m/ns) for the entire line **d)** Normalized varimax curves for sliding window 10 meters wide. Blue curve shows the peak value for every curve, red line is smoothed with a box car averaging filter 10 meters wide.

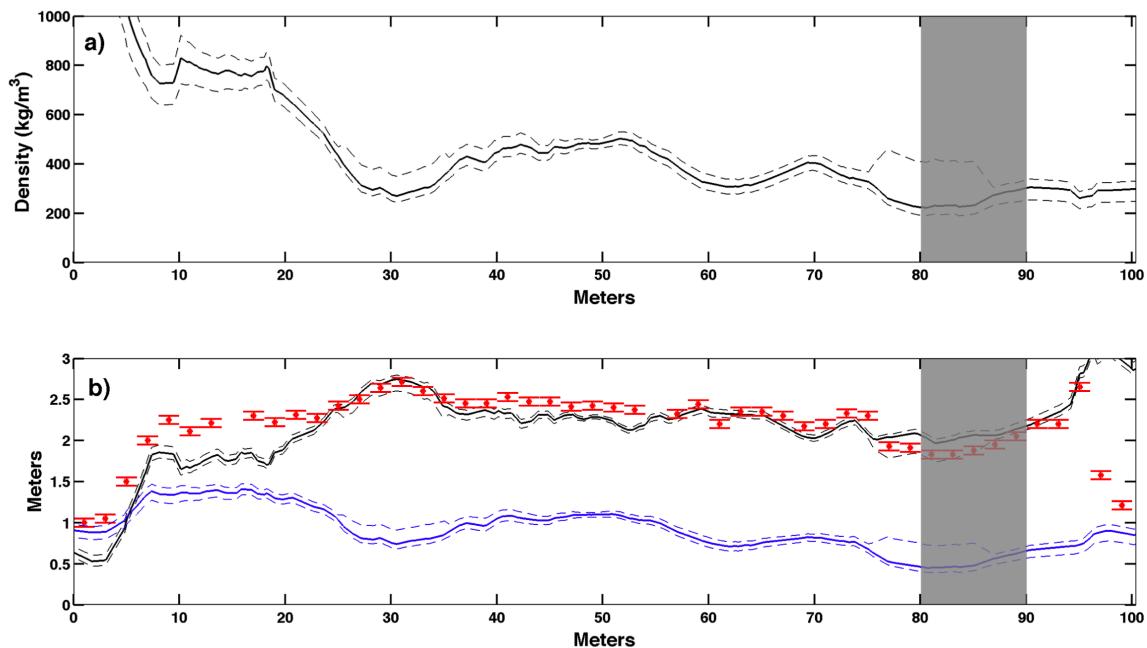


Figure S7. Line 5 Results. **a)** density, **b)** snow depth (black line) and SWE (blue line) estimates from the GPR data, snow pit data are shown in red. Grayed out region corresponds to areas where velocity picks are unreliable.

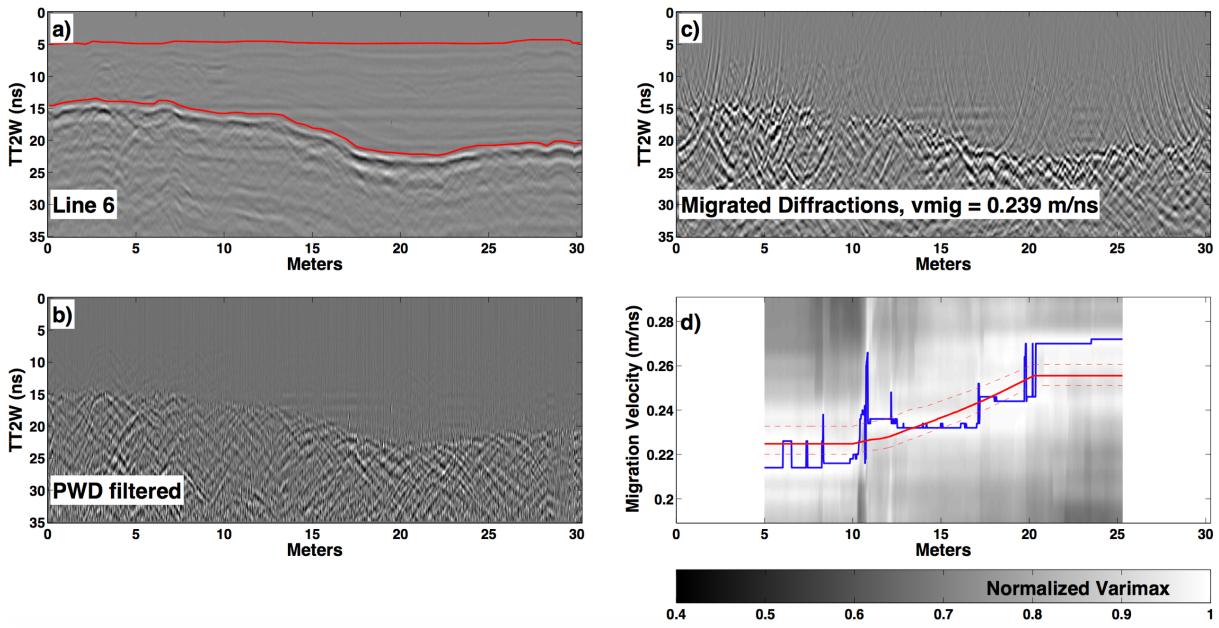


Figure S8. a) raw GPR data for Line 6, red lines indicates interpreted ground and snow reflections b) GPR data after PWD filtering c) diffractions migrated at the mean velocity (0.245 m/ns) for the entire line d) Normalized varimax curves for sliding window 10 meters wide. Blue curve shows the peak value for every curve, red line is smoothed with a box car averaging filter 10 meters wide.

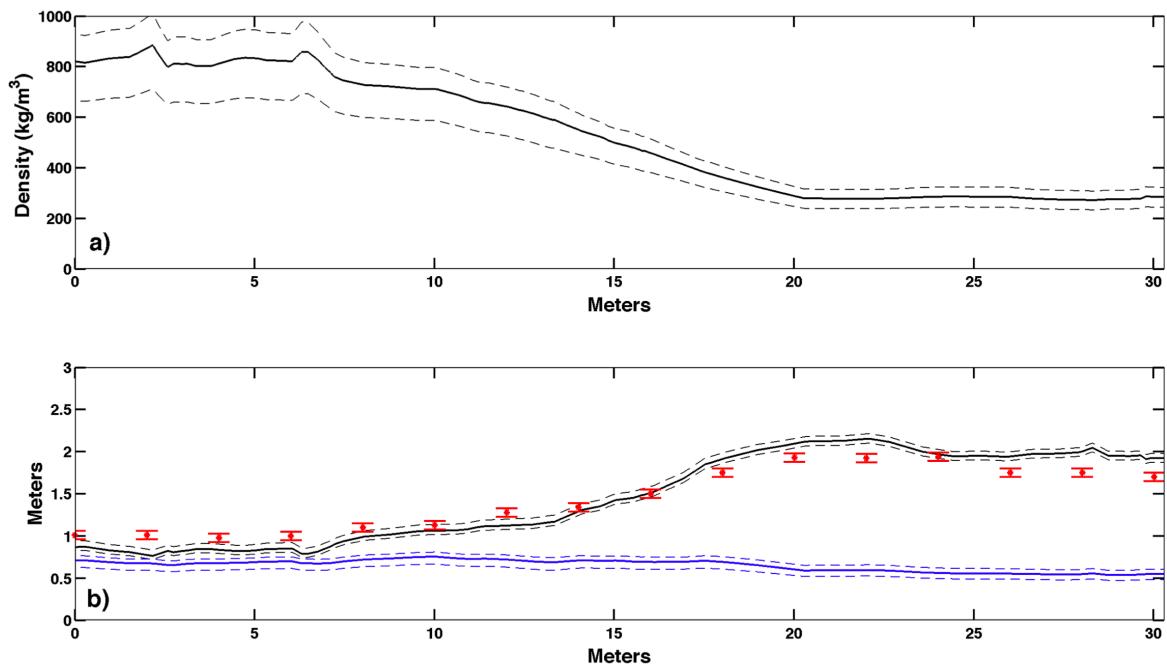


Figure S9. Line 6 Results. a) density, b) snow depth (black line) and SWE (blue line) estimates from the GPR data, snow pit data are shown in red.