Reply to Referee #2:

GENERAL COMMENTS

The manuscript describes a synthetic case study, in which a two-dimensional numerical model is used to estimate soil hydraulic parameters in idealized cross sections consisting of a permeable surface layer underlain by an impermeable bottom layer having undulating surface. Texts are generally well organized and written, and modelling approaches are technically sound. However, I am not convinced of the relevance and usefulness of the present manuscript in cryospheric science. It is an interesting numerical exercise, but the manuscript can be made much stronger and interesting to the readership of The Cryosphere. I will list my suggestions in specific comments below

Reply: We thank the reviewer for the constructive comments and suggestions. We revised the manuscript and thus refer to the revised manuscript.

SPECIFIC COMMENTS

1. Title. The study is motivated by GPR applications in the active layer, but the current version of the manuscript presents nothing specific to GPR or the active layer. For example, it does not deal with uncertainties and non-uniqueness in the relationship between dielectric permittivity, which is estimated by GPR, and volumetric water content. The model uses a two-layer structure consisting of permeable and impermeable soils separated by an undulating boundary, which is not specific to the active layer. The permeability of frozen soil is controlled by temperature, but the model does not account for coupled heat-energy transfer processes. Therefore, I think that the current title is somewhat misleading. It will be much better if the authors develop the paper to something that truly describes what is in the title.

Reply:

We don't agree that the manuscript presents nothing specific to GPR or the active layer. First of all, our study is completely based on the context of specific observations of soil depth and water content in 2D and the specific active layer with undulating thawing table, which is not common in warm regions. Given the well-established GPR observations, we decide to not use the deeper GPR information (dielectric permittivity or amplitude of electricmagnetic waves). As a preliminary study, we did not consider the effect of temperature on the permeability of frozen soil in the present model, but we gave an outlook of future development when being used for a seasonal case study.

Given the focus of the study, we changed the title to "Towards the estimation of hydraulic properties of active layers using ground-penetrating radar (GPR) and inverse hydrological modeling"

2. P3, L7. This form of the Richards equation is incorrect. The gravity term should have a unit vector, not a scalar "1". Also, please define the direction of z-axis.

Reply: We changed the manuscript accordingly.

3. P3, L22. What value was used for the Dirichlet upper boundary condition? How was it determined?

Reply: We agree. We did use a flux boundary condition. The description of boundary conditions is rephrased in P4 L30 and P5 L1-2 as "The infiltration is represented with a Neumann flux at the upper boundary and the groundwater table at the lower boundary is represented setting the Dirichlet potential to 0. The boundaries at the sides are impermeable."

4. P3, L25. GPR measures the travel time and amplitude of reflected radar waves, not the amount of soil water storage. The estimation of soil water storage from GPR data is not straight forward and has a large degree of uncertainty. To make this study relevant to The Cryosphere, it is highly desirable to incorporate uncertainties and non-uniqueness in GPR signal interpretation into numerical inversion. I believe that there is an established body of literature on this subject matter.

Reply: This study focuses mainly by GPR observations – soil water content and soil depth (integrated as soil water storage). As summarized in the introduction, the type of multi-channel GPR is an established tool to simultaneously measure soil water content and thawing depth. We choose to stand on this basis other than the step from original GPR signal interpretation and directly use the synthetic GPR observations incorporating real uncertainty in the inverse modeling. Thus, we could focus on the effect of undulating thawing table on the inverse modeling using the standard GPR observations.

5. P3, L27 - P4, L2. I do not understand this sentence. Please rephrase.

Reply: Changed word “later” to “lateral”.

6. P4, L2-3. It is assumed that the water table (i.e. matric potential = 0) is at the lower boundary. Does the lower boundary refer to the boundary between thawed and frozen soil? If so, does this “static condition” make sense hydrologically? For example, what is the condition at time step 7 in Figure 1? Should that be a more logical representative of the static condition after the complete drainage of the active layer?
Reply: As an initial condition, we assume the water table (i.e. matric potential = 0) is at the lower boundary, which refers to the bottom of the frozen layer. It is generalized as an averaged permafrost table. We acknowledge that this initial assumption might deviate from actual conditions, but it still can constrain the retention curve somehow and provide better starting points of the parameters for step 2.

7. P4, L5. It appears that a homogeneous soil is used in the model. It is well known that the near surface soil in natural environments is highly heterogeneous both vertically and horizontally. This severely limits the usefulness of the proposed approach to determining soil hydraulic property. I see this as a major weakness of this manuscript. It can be made much stronger by explicitly treating soil heterogeneity in numerical inversion.
Reply: We agree that the near surface soil in natural environments is highly heterogeneous. However, we can cope with it effectively when using the proposed method. Firstly, we could identify evident structural heterogeneity from GPR radargrams. Then we can exclude this case when using the proposed method. Secondly, given a certain error in the GPR observations, our inverse modeling yields effective hydraulic parameters, which certainly represents some small-scale heterogeneity. Thirdly, sandy active layers with weakly heterogeneous top soil are wide spread in alluvial fans in cryosphere regions, which would fit for the studied case.

8. Figure 1. It appears that a constant flux was applied to the upper model boundary, whereas the method section states that the upper boundary had a Dirichlet condition (P3, L22). What was the actual boundary condition?
Reply: See reply 3.

9. Table 1. The alpha values should be positive. The pore-size distribution coefficient (n) has a high value, and the residual water content is zero. I would say this is rather an unusual soil. Is this a good representative of typical soils in natural environments? Was this unusual soil purposely chosen for the synthetic case study? Why?
Reply: The product $\alpha \cdot h$ has to be positive, not α itself. In this study, we define h as negative for unsaturated soils, hence α also must be negative.
The parameters were derived from a sandy soil sample in lab experiments, and were selected to demonstrate the approach.

10. P5, L27-28. As the authors acknowledge, the water distribution over an irregular frost table is inherently three-dimensional. Two-dimensional models provide a useful tool for theoretical discussion, but its utility for practical application is limited. In addition, soil heterogeneity and a high degree of uncertainty in GPR data interpretation makes the present approach impractical to use in active-layer studies in natural environments. I suggest that the authors develop a full-length paper describing the development and application of a more realistic and useful inversion model using actual field examples of GPR data
Reply: We agree that a full-fledged demonstration including experimental verification of the effective parameters gained would be preferable. To the best of our understanding, this is not yet feasible. Still, the concept appears valuable to us and worth a brief communication.

Brief communication: Toward the estimation of hydraulic properties of active layers using ground-penetrating radar (GPR) and inverse hydrological modeling

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Abstract. Estimation of hydraulic properties of active layers is challenging due to the freeze-thaw effect in space and time. Provided an active layer with an undulating frost table, monitoring of spatial soil water dynamics in the thawed layer could provide significant information for commonly-used inverse estimation of soil hydraulic material properties. In this study, we assess the feasibility to estimate effective soil hydraulic properties using two-dimensional measurements of soil water storage and thawing depth, which can be efficiently derived from ground-penetrating radar (GPR) observations. The results of this study conceptually demonstrate that spatial and temporal observations of soil moisture in the active layer during a rain event are sufficient for inverse estimation of soil hydraulic parameters. The proposed method depends on the lateral water redistribution controlled by the undulating frost table. We suggest that this method could be used for seasonal-scale estimation of soil hydraulic properties.

1 Introduction

In permafrost regions, the active layer is a key stratum that controls the exchange of water and energy fluxes between the land surface and the atmosphere. However, the temporal development of the thawing front is hard to monitor at large scales because of its high spatial variability. The variability is not only a result of the local variation in surface features like microtopography and vegetation cover but is also related to subsurface soil properties which govern hydrological processes. Permafrost models such as empirical and process-based ones are normally used to predict the evolution of active layer and permafrost thickness (Riseborough et al., 2008). However, their predictions for thawing depth and permafrost degradation are still barely satisfactory, as this, e.g., lack of high spatial resolution of soil information like hydraulic material properties. Therefore, gaining accurate soil properties is crucial for understanding permafrost degradation and the associated permafrost hydrology. This is particularly important for permafrost studies in the regions with thick and unsaturated active layers like the Qinghai-Tibetan

Plateau (QTP). Soil properties controlling active layer dynamics are the thermal and hydraulic capacities and conductivities. Since thermal properties are less variable than the hydraulic ones, they are mostly derived from soil texture information and empirical models as well as literature data. In contrast, the soil hydraulic material properties are very sensitive to small variation in soil texture or even small-scale structural patterns. Thus, a site-specific determination of hydraulic material properties is essential for permafrost modeling.

Modeling water dynamics in permafrost soils is difficult due to the highly nonlinear hydraulics. Particularly, the soil hydraulic material properties are affected by considerable deformation and transport of soil material during freeze-thaw cycles (Ray et al., 1983; Boike et al., 1998). Additionally, lateral water redistribution is not negligible due to the high variability of the spatial thawing rate (e.g., Quinton et al., 2000; Wright et al., 2009). These features are adverse for the one-dimensional (1D) model, but this provides various possibilities for two-dimensional (2D) inverse modeling if spatial observations of the thawing front and the soil moisture are available. These spatial observations contain significant information as they monitor soil water dynamics. Lateral water redistribution is common in thawing active layers with undulating frost tables, and happens continuously during the thawing season. This rapid lateral water redistribution also occurs after strong rainfall events, which is often seen in the permafrost regions on the QTP (Pan et al., 2014), where precipitation is dominated by summer monsoon.

Inverse methods are commonly used to estimate hydraulic material properties from observed state variables at different scales. Progress in inverse modeling of soil hydraulic properties has been reviewed by Vrugt et al. (2008). Generally, results from the inverse method using in-situ 1D monitoring profile are limited to point scale, and this approach is usually hard to be used larger scales. Conversely, recent developments of soil hydraulic property inversion using satellite remote sensing yield some useful results at larger scales, e.g., from field-scale to catchment scale, which, however, are mostly based on the shallow measurement depth (< 5 cm) and an assumption of homogeneous soil column (Mohanty, 2013). In fact, root zone soils are heterogeneous whereas observations of the top soils are not representative. Accordingly, the model predictions using estimated hydraulic parameters solely based on near-surface soil moisture observations are not as good as the ones based the observations for the entire profile (Bandara et al., 2014). Furthermore, to yield robust results, all inverse parameter estimation methods rely on a time series of measurements that must contain significant information on soil water dynamics (Bandara et al., 2013).

Geophysical methods like ground-penetrating radar (GPR) of detecting soil layer boundaries developed rapidly (e.g., Neal, 2004; Annan, 2005; Jol and Bristow, 2003) and was extended for in real-time imaging of near-surface/layering soil water content (e.g., Huisman et al., 2003; Weihermüller, et al., 2007; Bradford, 2008; Pan et al., 2012b; Klenk et al., 2016). For permafrost soils, thawing depth of active layer and soil water content can be also simultaneously retrieved (e.g., Gerhards et al., 2008, Westermann et al., 2010), hence the total soil water storage can be derived accordingly as an integrated value of both. Quantitative observations of spatiotemporal variation in thawing depth and soil water content within plot-scale soil were successfully demonstrated on the QTP by Wollschläger et al. (2010) and Pan et al. (2014). As a primary modeling step, we go for a representation of the possibly heterogeneous soil as a set of uniform layers. The object then is to determine the effective hydraulic material properties, which we here propose to do based on GPR measurements. Whether such an effective representation is reasonable or not can be judged from the measured radargram.

Undulating frost tables are common in the permafrost regions resulting from the patterned surfaces such as vegetation cover, snow cover, and soil properties. Provided significant lateral water redistribution induced by an undulating frost table in active layers and spatiotemporal GPR observations, we investigate the applicability of the inverse modeling approach to efficiently estimate effective hydraulic material properties. In this study, we use synthetic data and discuss the effect of the amplitude of the undulating frost table on the estimating effective soil hydraulic material properties.

2 Scheme of hydraulic parameter estimation

2.1 The 2D hydrological model

Generally, two-dimensional Darcian water flow in a variably-saturated isotropic medium is described with Richards equation (Richards, 1931)

$$\partial_t \theta(h) - \nabla \cdot [K_w(\theta(h))\,[\nabla h - e_z]] = 0, \tag{1}$$

where θ is the volumetric soil water content, (m^3m^{-3}), K_w is the hydraulic conductivity, (ms^{-1}), and h is the matric head, (m), unit vector in z-direction e_z indicating the direction of gravity. The material functions involving soil hydraulic parameters in the Richards equation compose the hydraulic conductivity function K_w (h) and the soil water characteristic θ(h), usually in terms of the water saturation Θ (–). A widely employed model for these two relationships is the van Genuchten-Mualem model (van Genuchten, 1980; Mualem, 1976):

$$\Theta(h) = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = (1 + (\alpha h)^n)^{-m} \tag{2}$$

$$K(\theta) = K_s \Theta^{\tau}[1-(1-\Theta^{1/m})^m]^2, \tag{3}$$

where θ_s and θ_r denote saturated and residual water contents, respectively, α (m^{-1}), n (–) and m ($m = 1\text{-}1/n$) (–) are empirical parameters shaping the retention curve, K_s is the hydraulic conductivity at saturation condition, and τ is an empirical parameter shaping the hydraulic function, and is commonly set to 0.5 (Mualem, 1976). With this, there are five unknown parameters $p = \{\alpha, \mathrm{n}, \theta_\mathrm{r}, \theta_\mathrm{s}, K_\mathrm{s}\}$ to describe the hydraulic dynamics of the thawed active layer.

2.2 The parameter estimation

Given a time series of GPR observations of soil water content in the thawed active layer (Fig. 1c) and weather conditions, the parameters are estimated with the following three steps.

In step 1, the van Genuchten parameters $p_1 = \{\alpha, \mathrm{n}, \theta_\mathrm{r}, \theta_\mathrm{s}\}$ were estimated using the first GPR observation before the rainfall event, as inspired by the estimation of the initial state presented in Jaumann and Roth (2017). Since the subsequent water redistribution leads to a higher water storage in the active layer with a deeper thawing depth, the 2D water storage distribution is influenced by the thawing structure. Thus, a static hydraulic equilibrium at the initial stage is assumed that a water table at the lower boundary. Correspondingly, the van Genuchten parameters can be derived by fitting the relationship between observed soil water content (storage) and matric potential (depth).

In step 2, the effective soil hydraulic parameters $p_2 = \{\alpha, \mathrm{n}, \theta_\mathrm{r}, \theta_\mathrm{s}, K_\mathrm{s}\}$ were determined by minimizing the differences between observed water storages $l_\mathrm{obs}(x, t)$ and simulated water storages $l_\mathrm{mod}(x, p, t)$ at location x as an objective function

$$\chi^2(p) = \frac{1}{N}\frac{1}{M}\sum_t^N\sum_x^M [l_\mathrm{obs}(x,t) - l_\mathrm{mod}(x,p,t)]^2. \tag{4}$$

The Levenberg-Marquardt algorithm as implemented in Jaumann and Roth (2017) is used to minimize $\chi^2(p)$. M is the number of the grid cells in x-dimension ($M = 100$) and N is the number of observations ($N = 7$). Convergence typically requires less than 10 iterations. The optimization procedure also yields correlation coefficients for the resulting parameters. As the gradient-based optimization methods lead to local convergence, the Levenberg-Marquardt algorithm relies on good initial parameters. To address this, we used an ensemble of 50 inversion runs with different initial parameter K_s together with the initial estimated van Genuchten parameters from step 1.

In step 3, the resulting parameters $p_3 = \{\alpha, \mathrm{n}, \theta_\mathrm{r}, \theta_\mathrm{s}, K_\mathrm{s}\}$ are taken from the ensemble member with minimal χ^2.

The framework of the 2D inversion procedure is shown in Fig. 2. In step 1, the parameter estimation was solved using the function fminsearch from Matlab (Version: R2015a), which uses the Nelder-Mead-Simplex algorithm. In step 2, the Richards equation solver (muPhi, Ippisch et al., 2006) was used to simulate the spatiotemporal soil water dynamics, and the Levenberg-Marquardt algorithm was used to minimize the differences between the simulated state and observed state. In order to reduce the impact of the inversions with local minima, a mean value of the ensemble inversions with cost function values within 1-sigma of all (34 in 50) is used to assess the effect of the amplitude of the undulating frost table on the parameter estimation. In step 3, the final estimated parameters were determined from the 50 ensemble inversions by selecting the best one with the smallest cost function value. To assess the impact of the white Gaussian noise in GPR observations of soil water storage, the standard errors of the estimated parameters are calculated using 10 ensembles.

3 Case study

Here we used synthetic studies to demonstrate the proposed approach and assess the effects of undulating structure on the accuracy of the resulting material properties. The setup of the hydraulic model is shown with Fig. 1. The domain (12 m × 2 m) comprises a thawed layer and a frozen layer separated by an undulating frost table (Fig. 1b), and have the same sandy soil. The true soil hydraulic parameters of the active layer are listed in Table 1. As the Rechards solver requires a structured rectangular grid, the frozen layer has to be included into the domain. Due to numerical reasons, the according hydraulic conductivity has to be larger than 0. Hence we chose to scale the parameters of the active layer with a Miller scaling factor of 10 ensuring a constant hydraulic conductivity which is 1% of the one of the thawed active layer. The infiltration is represented with a

Neumann flux at the upper boundary and the groundwater table at the lower boundary is represented setting the Dirichlet potential to 0. The boundaries at the sides are impermeable.

The soil water dynamics were simulated with a time step of 100 minutes over a short period of 5.9 days. The time series of the forcing at the upper boundary by rainfall is shown in Fig. 1a. Seven snapshots of soil water storage observations were created using the forward simulations and adding a white Gaussian noise to the evaluated soil water storage associated with an extent of 1m and a mean soil water content of 15%. We assume that the uncertainty of the resulting depth is 0.05 m (Pan et al., 2012a), and the uncertainty of water storage is deduced as 0.15×0.05 = 0.0075 m.

Since the non-uniform change of soil water storage is essential to the inversion, one controlling factor is the undulating structure of frost table. To investigate its influence, three active layers (S1, S2, and S3 in Fig. 3) with different undulating amplitudes (0.25 m, 0.5 m, and 0.75 m, respectively) were investigated. Additionally, the parameter estimations were repeated 10 times using the GPR observations with the same uncertainty but different realizations of the random errors, to investigate the influence of the errors in GPR observations on the approach. The 1500 2D inversions were run on a cluster using parallel computation requiring specifically 192 processors for 10 days.

4 Results and discussion

Figure 4 shows the results of the parameter estimation in step 2. On the left panel, the estimated water retention curves of three structures (S1, S2, and S3) can be compared with the synthetic one after step 1. The middle and right panels visualize the ensemble estimates of water retention curve and hydraulic conductivity curve, respectively, together with the synthetic ones after step 2. In each plot, the curves represent the best 34 estimates, according to their χ^2, accounting for 68% (1 σ) of the 50 ensemble inversions. The darker the curve, the better is the estimation. Comparing the best χ^2 for step 1 and the mean values of the 34 best χ^2 for step 2, we found that $\chi^2_{best,S1} > \chi^2_{best,S2} > \chi^2_{best,S3}$. Therefore, the larger undulating amplitude, the better are the estimated hydraulic material properties. We attribute this to the increasing intensity of lateral water redistribution. Generally, the proposed approach works well for the studied cases.

The final estimates of parameters $p_3 = \{\alpha,\ \mathrm{n},\ \theta_\mathrm{r},\ \theta_\mathrm{s},\ K_\mathrm{s}\}$ were derived from the best one, according to χ^2, among the 50 ensemble inversions through step 3. The effects of the white Gaussian noise and the thawing structure on the inversion of the five van Genuchten-Mualem parameters are shown in Fig. 5. The histograms show the estimated parameters from step 3 together with the ones from step 1. Through the 10 ensembles, we can find that the impact of the white Gaussian noise on the parameter estimation is much more significant in step 1 than in step 2. Given an assumption of equilibrium state, the curve fitting is sensitive to the noise due to the narrow cover range of the matric head over the 2D transect. Based on this preliminary estimation, the sensitivity of the parameter estimation in step 2 is much less. In addition, the larger the amplitude of the undulating structure, the smaller are the standard errors. Overall, the results from step 3 show the robustness of the method to the errors of GPR observations, and the mean standard errors for S1, S2 and S3 are 0.40, 0.45, 0.0, 0.02, $8\cdot10^{-4}$ for the five parameters, respectively.

Apart from the white Gaussian noise, the autocorrelation of the parameters is also investigated. The resulting correlation coefficients of the parameters of the best ensemble member for S3 are given in Table 2. The hydraulic material functions are not unique in the sense that different parameter sets produce similar hydraulic material functions within a small intervals of hydraulic head or hydraulic conductivity. As the proposed method merely evaluates the integrated soil water storage, it is less sensitive on specific characteristics of the material functions governed by the individual parameters rather than on the material functions as a whole. Hence, the method yields the inherent parameter correlations of the parameterization model for the material properties. Therefore, the resulting parameters are expected to vary for different ensemble members, the resulting material functions, however, stay within a narrow interval (Fig. 4) indicating high sensitivity of the method on the material properties. Generally, the proposed method works well for the studied case, and the structure of frost table determines the significance of lateral water redistribution. In addition, precipitation features like intensity and duration can strengthen or weaken the effect of the structure on the lateral water redistribution. In essence, parameter estimation of soil water dynamics depends on the coverage of possible hydraulic states by the measurement data, e.g., regulated by precipitation features (e.g., Steenpass et al., 2011; Scharnagl et al., 2011).

For practical application, there are some necessary conditions for this approach to work. First of all, a continuous and undulating frost table is required that leads to lateral water redistribution. Applying this approach for a three-dimensional frost table could be handled in analogy. Secondly, for the studied case, prerequisite conditions like large rainfall intensity and good soil permeability are necessary which are only available at specific regions, e.g., the northeastern QTP. Finally, to capture the lateral water redistribution, selecting the time-slice observations is essential. As a rule of thumb, time-evenly distributed observations are best during the infiltration process. These requirements limit the range of possible applications. However, these limitations can be alleviated when applying this approach at seasonal-scale. Yet, this requires including evapotranspiration and a variable frost table in the model. Since the process of lateral water redistribution does not only rely on precipitation but also melt water by thawing. The latter one is relatively slow, but the amount can be considerable in wet active layers. Here, the timing of GPR observations will not be as sensitive as in the rain-based application.

5 Summary

Permafrost tables are often undulating and therefore lead to lateral redistribution of water within the active layer. We propose a method for the inverse estimation of soil hydraulic material properties that exploits this situation. It uses the observed data of thawing depth and soil water storage (which can both be derived from GPR measurements simultaneously) and a 2D simulation of the soil hydrology. We demonstrate this method using synthetic data. Based on a single rain event, seven snapshots of soil water storage were sufficient to accurately estimate the true material properties. The reasonable accuracy of the estimated parameters indicates the good feasibility of the proposed method at desired conditions. The applicability of this method depends on the site of lateral water redistribution, which is mainly controlled by the undulating frost table in the studied case.

As a conceptual study, we assume perfect model and observations with a white Gaussian noise. We comment that GPR works best in coarse-textured soils and may not work at all silty or clay soils. This coincides with the applicability of the hydraulic method, however. Despite its limitations, this approach provides one step forward to accurately estimate hydraulic parameters at the field scale. Its major advantages include non-destructive observations, a fast gain of field-scale soil hydraulic material properties in application.

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Table 1 Hydraulic parameters of a sand at the unfrozen condition and their allowed ranges for parameter estimation.

symbol	description (unit)	value	allowed range
α	inverse of air entry suction (m^{-1})	-2.0	-10...-0.1
n	measure of the pore-size distribution (-)	4.0	1.3...8.0
θ_r	residual water content (m^3 m^{-3})	0.0	0.001...0.02
θ_s	saturated water content (m^3 m^{-3})	0.3	0.25...0.4
$\log_{10}(K_s)$	K_s: saturated hydraulic conductivity (m s^{-1})	-4	-6...-3

Table 2 Example of the resulting correlation coefficients for the parameters for one realization of S3 ensemble inversions.

	α	n	θ_r	θ_s	$\log_{10}(K_s)$
α	1	-0.91	-0.91	0.55	0.79
n	-0.91	1	0.88	-0.75	-0.91
θ_r	-0.91	0.88	1	-0.57	-0.72
θ_s	0.55	-0.75	-0.57	1	0.93
$\log_{10}(K_s)$	0.79	-0.91	-0.72	0.93	1

5

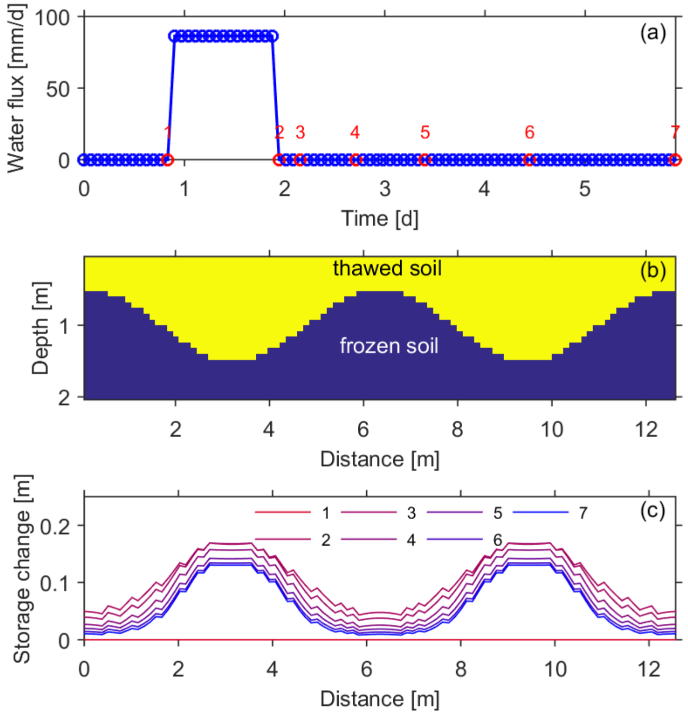

Figure 1. An example of the forward modeling. (a) Water flux through the upper boundary. Red circles show the times of seven snapshots of soil water storage observations. (b) The structure of a thawing active layer. (c) A time series of soil water storage observations in the thawed layer, corresponding to the time markers in (a).

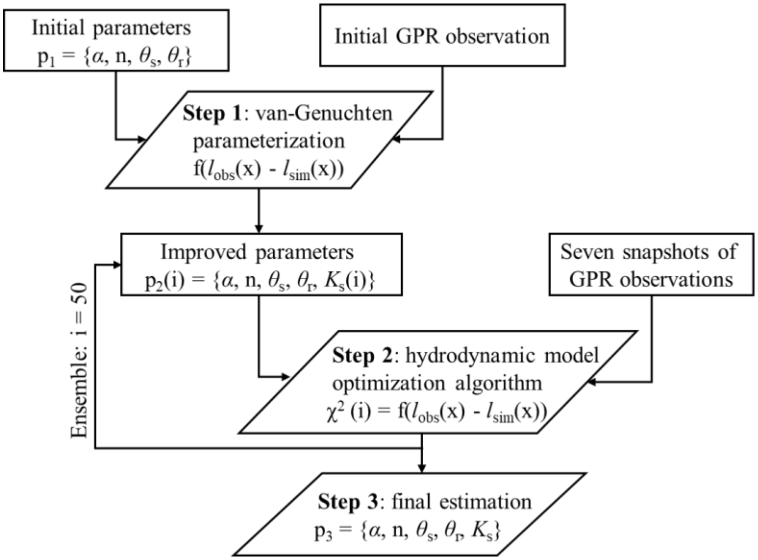

Figure 2. The framework of the 2D inversion procedure.

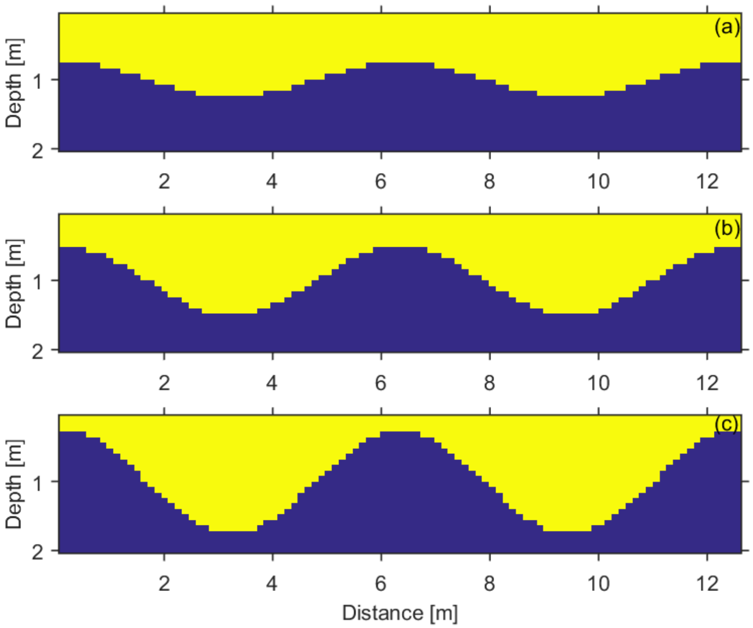

Figure 3. The active layer with different thawing structures. The amplitudes of the undulating frost table are (a) S1: 0.25 m; (b) S2: 0.5 m, and (c) S3: 0.75 m, respectively.

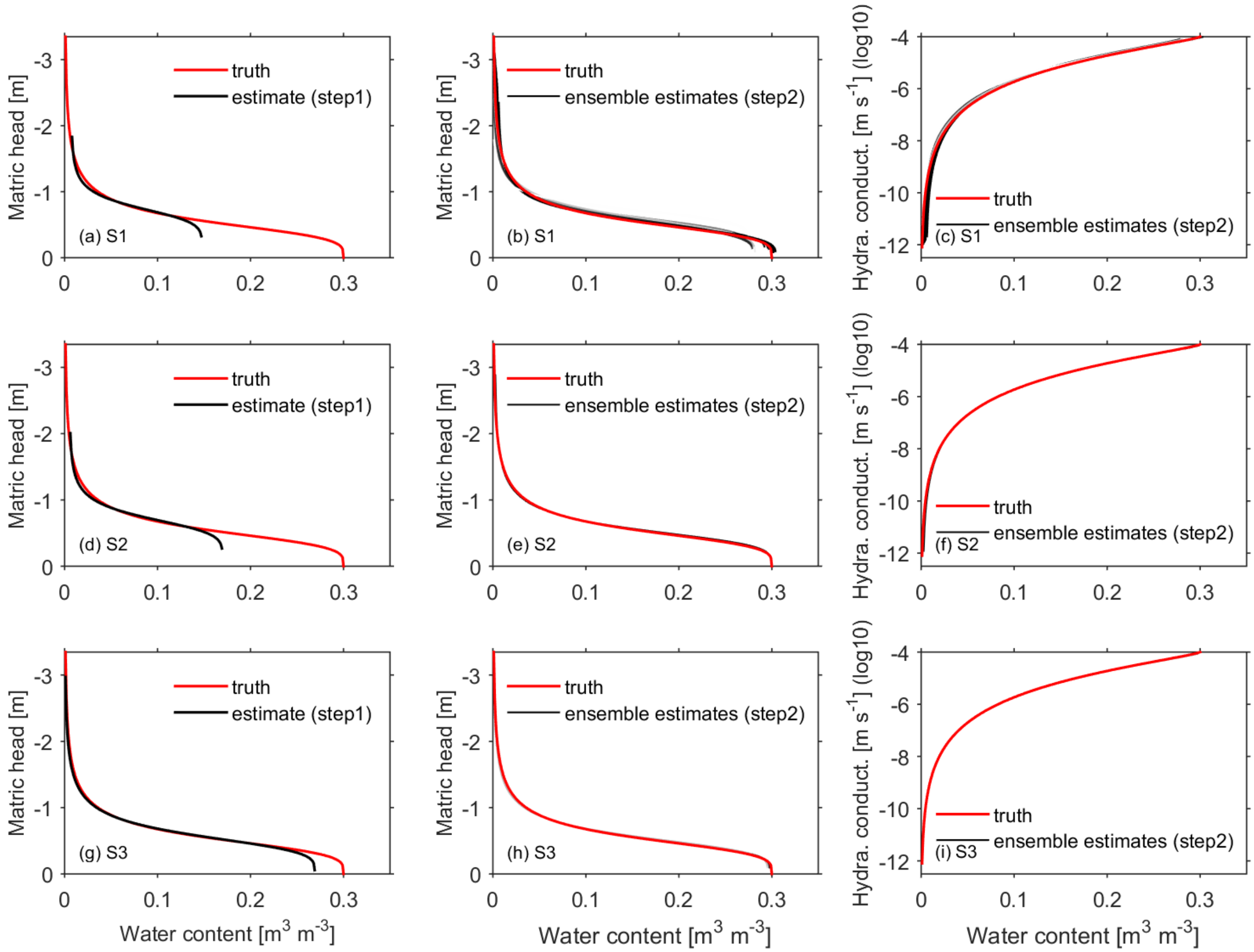

Figure 4. A comparison of estimated water retention curves (step1 and step2) and hydraulic conductivity curve with the synthetic ones for three structures (S1, S2, and S3). Left panel: initial estimates of water retention curve. Middle panel: final estimates of water retention curve with the best 34 ensemble inversions. Right panel: final estimates of hydraulic conductivity curve with the best 34 ensemble inversions. 5

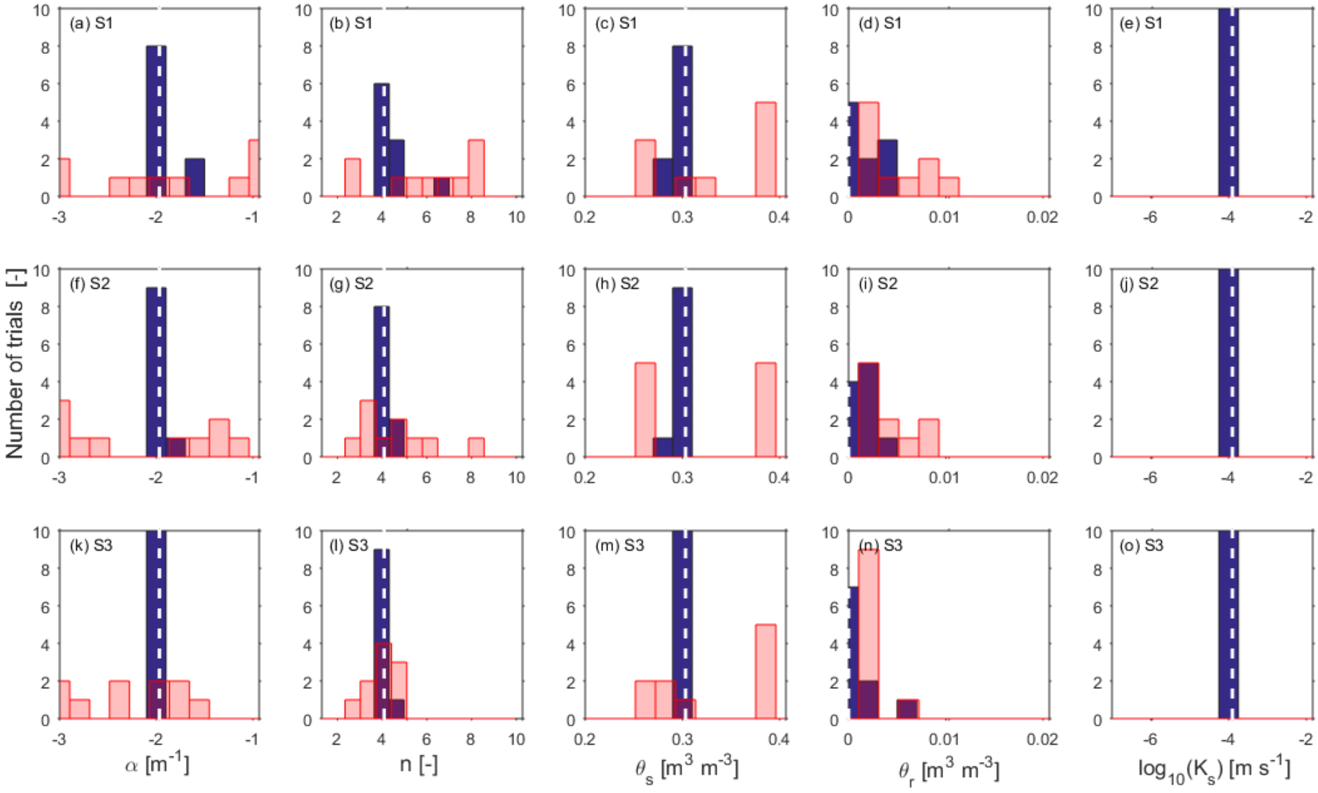

Figure 5. Effects of thawing structure on the inversion of the five van Genuchten-Mualem parameters. Given 10 repeated GPR observations with different random errors, histograms of the initial (step 1, light red bars) in and final (step 3, blue bars) estimated parameter values are shown. Only blue bars are shown in the right most panel because
5 **there are no initial estimates in step 1 for K_s. Dashed lines show the true parameter locations.**