

## ***Interactive comment on “Reply to “Basal buoyancy and fast-moving glaciers: in defense of analytic force balance” by C. J. van der Veen (2016)” by Terence J. Hughes***

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The exchange between C.J. van der Veen and me boils down to his belief there is only one way to skin a cat for the force balance in glaciology, integrating the Navier-Stokes equations, the standard analytical approach, and I believe there is another way, a simpler geometrical approach. As an aside, my geometrical approach is also "analytical" in the broad definition in that it provides an "analysis" leading to a quantitative solution.

My "misplaced antagonism" about van der Veen's "equation formatting" was based on a version of his 2016 paper provided by my colleague James Fastook that had those formatting defects. The defects were missing in the version that actually appeared in The Cryosphere, as I subsequently acknowledged when I saw that version. So let's put

C1

this "objection" to rest.

What remains is important. Van der Veen's Figure 3 misrepresents my geometrical force balance. There is no other word to describe it. I can only conclude he doesn't understand the geometrical force balance (which is my fault; I didn't present it clearly enough), or he was just careless by not making his lines AF and BE parallel. We all make careless mistakes.

Van der Veen made a careless mistake in his 31 January 2017 response to my reply: his Equation (1) contains a longitudinal force gradient along flow direction  $x$ , not a longitudinal stress gradient, yet his plot of that equation in his Figure 2 labels it "gradients in longitudinal stress". So which is it? And how does he sort out a "gradient in longitudinal stress" from his Equation (1) anyway? Differentiating his force gradient by parts gives a longitudinal stress gradient times ice thickness plus a longitudinal stress times an ice thickness gradient.

His Equation (2) has another careless mistake. It presents the Navier-Stokes equation written for direction  $x$  of ice flow, yet the last term does not contain  $x$ , only the  $yz$  stress and the  $z$  direction. It should be the  $zx$  stress in the  $z$  direction.

Once again, Van der Veen insists "Flow of glaciers is driven by gradients in the gravitational lithostatic stress that can only be estimated over a certain horizontal distance." His conclusion is based on his Equation (1) which presents stresses after integrating the Navier-Stokes stress-gradient equations. Force gradients in his Equation (1) are stresses. Weertman (1957) obtained the force balance at the calving front of a flat ice shelf by integrating these equations, but Robin (1958) obtained the very same solution geometrically with no gradients in gravitational stresses over no horizontal distances. His force balance is obtained at  $x = 0$ . Van der Veen will have to explain, very carefully so everyone can understand, why it is impossible to get the same solution using the two different approaches, with and without stress gradients.

Here it is worth emphasizing that Van der Veen accepts my geometric force balance for

C2

linear shelf flow and linear sheet flow. He only objects to my solution for linear stream flow, which readily connects sheet flow to shelf flow by progressively uncoupling ice from the bed by progressively drowning the bed to produce a floating fraction of overlying ice, my variable  $\phi$ . My  $\phi$  produces the typical concave profile of ice streams in a very direct way, where he struggles with convoluted "bridging stresses" over cavities without specifying a water pressure (if any) in his cavities. I do specify the water pressure in my floating fraction.

Van der Veen also needs to explain, using his "force budget" approach to solving the Navier-Stokes equations, how it produces water pressure in his subglacial cavities that pushes water far above sea level in West Antarctic ice streams, as Kamb and Engelhardt demonstrated by measuring water heights in many boreholes. My geometrical approach produces this water height directly from the floating fraction of ice along an ice stream calculated from known ice thicknesses and surface slopes. You can't beat that.

I agree with Van der Veen on one point. "It is always a good idea not to put words in someone else's mouth." My interpretation of his Figure 4a, as it is drawn, does not allow horizontal spreading, as each horizontal arrow at each depth cancels the opposite arrow. That's all I said. Does he deny that? We both know how to draw a geometrical figure that allows horizontal spreading of a flat ice shelf (or ice divide), see Weertman (1957) and Robin (1958), and our books, mine in 1998 and 2012 and his in 1999 and 2013.

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