Journal: *The Cryosphere*
Manuscript: tc-2017-54: Implementing an empirical scalar tertiary anisotropic rheology (ESTAR) into large-scale ice sheet models
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Reviewer: E.D. Waddington
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1 Overview

I was invited by the editor to review this manuscript late in the process. After reading the 30 March version of the paper, and the reports by the two anonymous reviewer, I wrote the review in Section 3 below. Then, later, after seeing the response from the authors to the initial two reviewers, I wrote the additional comments following immediately as Section 2. I hope the points in both reviews will be considered.

I think the revised manuscript does a good job of demonstrating that the ESTAR flow relation of Budd et al. (2013) is an effective and low-cost constitutive relation for representing the flow of ice sheets and ice shelves. Because its prefactor varies depending on the local stress state, ESTAR appears to capture features of the flow with demonstrable advantages over the Glen flow law, which is limited by its constant prefactor.

Many years ago, in a Glaciology group discussion about anisotropy and enhancement factors here at UW, Charlie Raymond noted that a scalar enhancement factor in Glen's Flow Law should be a fine to incorporate anisotropic-fabric effects in a flow model, as long as it accurately reproduced the strain rate that was the most important or dominant strain-rate component at each position and at each time. Whatever the other strain-rate components did was less important. This manuscript shows that the ESTAR flow relation appears to be a rigorous way to do exactly that.

2 Comments on revised submission (July, 2017)

Page numbers and line numbers refer to the author-annotated version of the revised manuscript, which was attached to the authors' responses to the reviewers.

2.1 Scientific points

With some justification, the authors have pointed out to the reviewers that the message of their manuscript is *not* about whether the Budd et al. (2013) ESTAR flow law is anisotropic; the point of their manuscript is to test how well ESTAR works in a variety of situations in comparison with the Glen flow law.

- While I am sympathetic to their point, life also doesn't always work out the way we hope it will, and I fear that the semantics of anisotropy may continue to hound any discussion of the paper among readers (as it did among all the reviewers, including me), to the detriment of focus on their primary message. So if the authors want the paper to be widely read, and cited for the right reasons, I think it might be helpful to include a short section in the **Introduction** to address this concern head-on, as outlined in the following item. This could come at the expense of removing some to the other introductory material about anisotropy, which I felt didn't always make the point that is needed here.

- In their response to reviewers, the authors have written a long description and review of anisotropy and its treatment, which perhaps merits publication in its own right, but after reading it, I was left feeling that they have still not clearly pin-pointed the issue of contention, which is a lack of agreement among glaciologists as to what it means to be an *anisotropic flow law.* I admit to having felt ambivalent over the past few years myself, with particular reference to CAFFE and ESTAR. However, after re-reading Placidi et al. (2010) and Budd at al. (2013), and after reviewing this manuscript, I no longer feel ambivalent.

  Perhaps if these authors can clarify exactly what they mean by an *anisotropic flow relation* in this paper with a few sentences or paragraphs, it will convince readers to focus on their results, and their paper will be more widely cited.

  I expect that glaciologists agree that ice develops a non-uniform preferred crystal orientation fabric as a result of deformation, and that because an ice crystal has essentially just one slip system, this preferred crystal orientation fabric affects the deformation rate.

  (1) First definition: prefactor must be tensorial
  Many glaciologists would express the view that in order to be *anisotropic*, a constitutive relation between the deviatoric-stress tensor and the strain-rate tensor should be able to produce different effective viscosities associated with different stress components. For example, with an anisotropic ice fabric such as a strong vertical single pole, that ice parcel should be much stiffer to

vertical compression than to shear along horizontal planes. With this definition of anisotropy, a constitutive relation should have a tensorial prefactor. (Experimental results with multiple stress components in Budd et al. (2013) suggested that perhaps a tensor prefactor isn't needed after all, but having a tensor prefactor is still one working definition of an anisotropic flow relation.)

(2) Second definition: response changes when sample is rotated
Placidi et al. (2010) (Section 4.1) defined an isotropic constitutive relation as one in which the strain-rate response does not change if the sample were to be rotated in a steady stress field. They went on to show that with this definition, the CAFFE constitutive relation is anisotropic, even though it has a scalar prefactor.

They also pointed out that although many glaciologists assume that a tensorial prefactor is a necessary requirement for anisotropy (i.e. (1) above), continuum mechanics experts view definition (1) as an overly restrictive requirement for anisotropy. In general, Placidi et al. stated that (a) anisotropy, and (b) lack of collinearity between strain rate and deviatoric stress are independent properties of flow laws.

They also remarked (end of section 4.2) that their scalar prefactor in CAFFE (collinearity) is a shortcoming, because it precludes associating different effective viscosities for different components of the deviatoric stress, even though ice crystals have only one slip system.

(3) Third definition: Relation replicates strain rates for material with anisotropic crystal fabric
Other researchers, including the authors of this manuscript, may take the view that a constitutive relation that can realistically describe the behavior of ice with a non-uniform preferred crystal orientation (i.e. anisotropic ice) is an anisotropic flow relation, regardless of its mathematical form.

ESTAR appears to do a good job of modeling the flow of ice in tertiary creep, and has a scalar prefactor, which makes it easy to apply (that's the point of this manuscript). But, by the widely used (but restrictive) glaciologicial definition (1) above, ESTAR is not anisotropic, because it has a scalar prefactor.

By the less-restrictive Placidi et al. definition (2), the ESTAR constitutive relation is also isotropic, but *not* because of the scalar prefactor. The ESTAR constitutive relation is isotropic because if an ice sample (which by assumption has developed a tertiary fabric in equilibrium with the current stress field) is then rotated, the ice fabric in the sample cannot rotate with the sample, because by the ESTAR assumptions, the fabric is locked to the stress field, which does not rotate in this thought experiment. There is no change in the strain-rate response according to ESTAR, and so the flow law is not anisotropic.

Since the authors are apparently assuming a definition similar to (3), they should state this clearly (and early). However, I personally would prefer to see it clearly stated that ESTAR is a *mathematically isotropic* flow relation that works well for ice with fully developed *tertiary anisotropic* fabric.

With this perspective, I am also concerned about the places in the text that compare "*the ESTAR relation*" with "*the "isotropic Glen relation*", because they are both isotropic relations. I think the text would be more effective to focus on the differences between "*ESTAR with its stress-dependent prefactor*", and "*the Glen relation with its constant prefactor*". That's where the important new science that makes this paper valuable lies.

Perhaps the authors are moving in this direction with their text on page 6, line 30: " *... and could be regarded as providing a variable enhancement factor for the Glen relation that incorporates the effect of anisotropy.*"

- I am puzzled by Equation (21), which represents the incoming boundary condition for flow. Equation (21) does not go to zero at the lateral boundaries ($x = 0$ and $x = L$), yet the boundary condition along the lateral margins is zero flow, i.e. $v_y(0, y) = v_y(L, y) = 0$. Since $x_{mid} = L/2$, these two conditions are incompatible in the corners where they meet. Admittedly exp(-5.96)=0.003 is small but it is not zero. Does this create some of the fine structure there, for example in Figure 4a?

- It is a pity that the ISMIP experiments used only sinusoidal bed variations, because, as the authors point out, they are unable to examine the ESTAR response in what we might call *the far field*, away form the influence of an isolated bump. Perhaps in future work, experiments with isolated bumps in a longer model domain would shed more light on the effectiveness of the ESTAR formulation?

2.2 Editorial points and clarity

- Page 9, line 10:
  What is a *binormal to a flow-line*?
  Also see comment about *flow line* on page 20.

  Be consistent with terminology. Do you want a hyphen in flow line (as here) or not (as on page 20, line 26)?

- Page 11, line 11:
  Consistent with what theory?

- Page 21, Line 25:
  Should be Fig. 3f.

- **Can times be slow?**
  Page 13, Line 21:
  Rates can be slower, but the corresponding times are *longer*.
  "... no more than 3% *longer* than ..."

- **Misplaced *only***

  Page 4, line 21:
  " ... are appropriate only for highly localized ..."

  Page 5, line 11:
  " ... occur only gradually ..."

  Page 6, line 29:
  " ... Eq.2 differs from the Glen relation only ..." Page 20, line 1:
  " ... varied only on scales ..."

- Page 18, Line 1:
  "... between *extensile* and ...
  Better to avoid *extensive* which can also mean *over a broad area.*
  Page 20, line 13: Same.

- Page 20, line 26:
  "*... the ice surface is a flow line, ...*"
  Most glaciologists would describe the surface as a *particle path*, or equivalently in a steady state, as a *streamline*. A *flowline* would be a line on the surface that was always tangential to the ice flow at the surface, or it would be the projection of a particle path or streamline onto the free surface (assuming the velocity azimuth did not rotate with depth).

- **Hyphens**

  Page 13, line 22:
  Generally, adverb-noun pairs and adjective-noun pairs are not hyphenated when the adverb or adjective ends in a y.
  "... as computationally efficient as ..."

  Page 18, line 4:
  "... leading to near-surface spikes ..."

- **Figure 1 axes**
  The tic marks on the horizontal axis are given in units of strain, but the axis label says the units are log strain. There is a big difference. The tic labels and the axis label need to be consistent.
  The vertical axis has no magnitudes indicated. So what is the point of attributing units of inverse seconds to strain rate $\dot{\epsilon}$? See note in earlier review about the factor of 8/3.

- **Use of *e* notation on graphs**
  Many of the figures (4, 5, 6, 7, 9, 10) use e notation on graph axes. In computer code, e notation is the way to indicate powers of ten, but in text and publications I think explicitly writing the powers of ten is the more appropriate way to report large and small numbers (i.e. 5.2×10^{-6}, not 5.2e-6).

3 Review of initial submission (March 30, 2017)

The manuscript demonstrates that the ESTAR constitutive relation from Budd et al. (2013) for ice-sheet flow can be used in a large ice-sheet model, with a negligible increase in computation time relative to computation time using the standard Glen relation with a constant enhancement factor, while producing more-realistic flow fields in the presence of anisotropy in crystal fabric orientation distributions. This is a worthy contribution meriting publication in *The Cryosphere*. However, the points raised by the two earlier reviews should be addressed first.

3.1 Scientific points

- Both anonymous reviewers questioned whether ESTAR is really an anisotropic flow law. I agree with their conclusion that it is not. ESTAR is a clever way to adjust the scalar Glen enhancement factor E at each point to judiciously reflect the local stress state and its impact on the local fluidity, but locally it is still an isotropic flow law. As reviewer #1 stated it,
  "*for a given stress state, the mechanical response (the strain-rate state) does not change by rotation of the sample ...*"
  A *changing* response when a sample is rotated is the essence of an anisotropic constitutive relation.

  It would be fair to say that ESTAR offers an improved flow law for ice that is anisotropic, but it is incorrect to say that ESTAR is an anisotropic flow law.

- Page 1, Line 17 -
  " *Data from laboratory ice deformation experiments can be used to define flow relations suitable for implementation in numerical ice sheet models.*"
  This may be correct, but at such dramatically different rates of strain, isn't there a strong possibility that the dominant micro-scale processes are different, and if that's the case, why should the lab experiments offer very much insight? Some more discussion might be in order.

- Both previous reviewers expressed concern that ESTAR is based on the assumption that tertiary creep has been reached everywhere. This is equivalent to the expectation (Page 4, line 24) that there is
  "*an anisotropic flow relation for polycrystalline ice in which the nature of the crystal fabric and the magnitude of strain rate enhancement, E, are both determined by the stress regime.*"
  I think we all understand that fabric actually evolves in response to strain, so in order for these two different views to be compatible, the time required for ice to undergo ≈10% strain must be significantly less than the time required for it to move into a regime with a significantly different stress.

  Perhaps the paper by Thorsteinsson et al. (2003) would offer some ideas and discussion points about where this might be justified and where not.

  Thorsteinsson, T., E.D. Waddington and R.C. Fletcher. 2003. Spatial and temporal scales of anisotropic effects in ice-sheet flow. *Annals of Glaciology*, 37, 40-48.

- Figure 1 capton -
  "Note that the ratio ... is approximately 8/3, ..."
  How is a reader supposed to note this when there are no numbers on the vertical axis? The figure is just a cartoon.

3.2 Editorial points and clarity

- Some purists would say that *rheology* denotes a field of study, like *geology*, and a better term in the title would be *constitutive relation.*

- Page 2, line 13 -
  Cuffey and Paterson is a big book. It helps readers when you include a page number.
  Page 13, line 17 - Same comment for Paterson (1994).

- Page 3, line 9
  Section 4 is not mentioned in the outline, although all other sections are mentioned.

- Page 4, line 7 -
  Text is generally easier to read if long strings of adjectives can be avoided, or if not avoided, then correctly hyphenated to make the groupings clear.
  Suggestion:
  "a discrete vector-based description of fabric based on c-axes ..."

- Misplaced "only"s.
  Page 4, line 8 - ...appropriate *only* for highly ...
  Page 4, line 26 - ...occur *only* gradually ...
  Page 5, Line 15 - ...Eq. 3 differs from the Glen flow relation *only* in the form ...

- Page 5, Equation (5) and below -
  The idea of a simple shear τ' requires an explanation of the plane on which is measured. However, the definition of that plane follows only after τ' has been used in λ_s. This backwards order can leave a reader puzzled and unnecessarily confused for half a page or so.

- Page 8, line 12 -
  Readers would probably like at least a brief description of Taylor Hood and P1×P1 finite elements, and why they are the elements of choice. Otherwise, why mention those types of elements specifically?

- Personally, I think that authors do themselves no favours when they introduce unnecessary acronyms into their papers. The acronyms can become stumbling blocks to readers who may not read the paper front to back in a single sitting, and may abandon it, rather than bothering to dig through the paper to find the definitions of unusual acronyms. In this case, eliminating the acronyms HO (higher order) and FS (full Stokes) and just writing those terms out in full for the perhaps 2 dozen times they are used would make the paper more readable, and would add perhaps 100 words, which I (and probably *TC* editors) would view as a small price to pay for improved clarity.