We would like to thank both referees for their thorough assessment of our submitted paper. Thanks to their thoughtful comments, we made improvements to the paper. We added a figure and remade several others, expanded the description and discussion of our results, and heavily edited the manuscript again. The marked-up manuscript are attached in this response.

Response to referee 1: 'Ice shelf fracture parameterization in an ice sheet model' by Sainan Sun et al.

Received and published: 3 May 2017

The authors have implemented a new representation of continuum damage mechanics into the ice dynamics model BISICLES, providing a way to study feedbacks between flow dynamics and damage-induced softening of the ice. To compare results from their modified model to standard results in the absence of damage, they follow the design of the MISMIP+ experiment. We believe that a number of points need to be addresses before publication. We also suggest additional experiments and an expansion of the discussions section, which could strengthen the manuscript.

We would like to thank the referee for this detailed review.

In keeping with previously published papers, the authors adopt an advection scheme for damage. However, to the best of our knowledge, the authors suggest a new way to treat the source of damage in the advection equation, as detailed in section 2.2. The source is proportional to the local crevasse depth (surface + basal), where crevasse depths are calculated from a zero-stress criterion following (Nye 1957). We encourage the authors to expand on the differences/parallels with existing research on continuum damage mechanics, to put their work into context, and to better motivate this approach. How and why is it better than previous work such as [Krug et al. 2014, Borstad et al. 2012, Pralong and Funk 2005]?

We think the main aim of the study is not so much to propose a new damage model, but to look at the impacts of implementing one versus neglecting it. To that extent, we chose a simple relationship between damage and tensile stress. If we consider only surface crevasses and neglect the threshold stress (more on that later), our model would be rather more similar to Krug et al 2014: they have a non-zero damage source when the Cauchy stress is positive, so as $t \rightarrow \infty$ they will see $D \rightarrow 1$ to the depth given by Nye zero-stress model, at least as far as vertically integrated models are a good approximation to Stokes models. Basal crevasses could be included in a model along the lines of Krug 2014 by adding the water pressure to the Cauchy stress, as in the Nye zero-stress model for basal crevasses. Assuming that crevasses open at least as quickly as the viscous deformation, we decided to compute the strain rate and damage simultaneously rather than by choosing some rate (as in Krug 2014).

We added a note on the timescale: ``Specifying the damage-stress relationship in this way assumes that damage evolves on a similar or faster timescale to the ice velocity field. Many authors specify instead a damage evolution rate, which, in Krug et al 2014, and given typical stresses in an ice shelf, amounts to a timescale of around 1 year.'' and a note on the similarity with Krug 2014 ``Inasmuch as a shallow shelf model is a good approximation to the full Stokes model, our choice of the Nye zero stress model above is similar to the long-term behaviour of Krug et al 2014, at least for surface crevasses. In that model, damage grows where and only where the Cauchy stress is tensile, just as in the Nye model, giving the depth of surface crevasses. Basal crevassing could be included in such a model by adding the water pressure to the Cauchy stress, as in Keller and Hutter (2014).''

For example, contrary to these studies, the authors do not implement a stress threshold for the formation of damage, and assume that non-zero damage is present in any tensile stress environment. Is this realistic, and how does it affect the results? Presumable the qualitative nature of the results remains the same, but it might become important at a later stage, when e.g. calving criteria are considered?

Indeed, we do not impose a lower stress threshold for the formation of damge - this is in line with e.g the use of the same crevasse depth calculation in Nick 2010. Like them, we have assumed that the differences will be minor. At any rate, they will be limited to the upstream, slowing flowing parts of the ice where damage is low. We don't expect that such a threshold would make much difference to calving criteria, because we would only expect those criteria to be satisfied only in regions of large stress in either case. We added a note: "We also ignore any lower limit on the stress needed to open a crevasse, so that we will tend to produce small crevasses where there should be none. As we will see in the results, the major impact of the damage model is in the ice shelf and around the grounding line, where large tensile stresses readily exceed such limits".

With regards to the results, the text provides an adequate description and explanation of the findings, although we would like to add a few comments or suggestions:

The authors should specify how d_w (crevasse water depth) in Eq. 8 is determined. Is it set to a constant value, and how is this value chosen?

We added a note: A water depth $d_w \sim h/2$ is in fact required in Nick et al (2010) for any calving to take place at all, and could clearly have a substantial impact on our calculations too, but for this paper we consider only dry surface crevasses, with $d_w = 0$.

In Figure 8, a compelling argument is made that evolving damage could play an important role in simulating grounding line retreat/advance. However, the results are only discussed very briefly, which is disappointing. To strengthen their point, could the authors perform inversions for a spatially varying rate factor, using the surface velocity and geometry at different time steps in the IceD0 and IceD1 experiments? It would be interesting to see how the rate factor changes over time, as one incorporates the effects of damage into its value. This could inform present-day model initialization methods, as most models treat damage in the form of a spatially varying rate factor, which is kept constant in time.

We did originally consider a set of synthetic inversions following the IceD1 experiment, but we would expect to simply recover the field \$D\$ (or an approximation to it, with the difference being down to issues with the inversion method). Hence the experiment of figure 8 (now 9): in this case we do actually hold the damage constant in time as though it had been computed in an inversion at t=0. We then see a much lower rate of retreat than either the full damage model (IceD1), or the original model with uniformly softer ice (Ice1). We made a poor job of describing and discussing this result in the original, and have re-written the relevant parts of both the result and the discussion sections, and added a short paragraph to the conclusion. As the reviewer suggests, this does have serious implications for present-day model initialization methods.

In order to increase the impact of this work, we suggest highlighting how the results have altered our understanding of damage, and indeed, whether it should be treated as a vital part of future ice flow modelling studies. Perhaps the authors could discuss in more detail the future directions of research (incl. possible calving laws?) they like to pursue, and whether this model can become a prognostic tool for calving?

We think the paragraph added to the conclusion summaries at least the immediate impact - modellers may need to reconsider their initialization methods.

P2L5: “extremely sensitive to calving”: I believe this statement could be misinterpreted as “more likely to calve”. Therefore, please change the wording to “ice shelves in the Amundsen and Bellingshausen seas are thought to be more vulnerable in the event calving...” or similar.

Done.

P2L15: what do you mean by “magnify”? Do you mean that propagation and penetration of fractures causes calving?

Yes we mean that. We change the word to 'trigger'.

P5L25: point out that A’ is a constant, and not a spatially varying field

Done.

P6L1-3: The second question needs a better explanation. Perhaps write something along the lines of "If we adjust the rate factor such that the damaged model reaches a similar grounding line steady state compared to the undamaged model, how does the transient response between both setups differ, when subjected to an external forcing that leads to thinning of the ice shelf?

Done.

P6L12: reformulate this sentence as follows: "In order to start the MISMIP+ experiments from the required grounding line location at x=450m, we run a series of IceD simulations with different values of the rate factor A. For each value of A, a new steady state grounding line location is obtained, and we select the value A' for which the location is closest to the originally required grounding line at x=450m. We will refer to this steady state as IceD0."

Done.

P6L24: The reference to this table comes too late. Preferably refer the reader to this table before you start listing all the experiments, i.e. before line 5 on page 6.

Done.

P7L3: from here on, the authors use capital letter D to refer to damage. Should this not be small letter d, in line with the definition in Eq. 10 as the vertical integral of the damage?

We use small letter d to represent the vertical integral of the damage, but often it makes sense to refer to the vertical average, which we now denote $\bar{D}(= \mathrm{d}/\mathrm{h})$ to avoid the confusion caused by our earlier use of D.

P7L12: It is worth pointing out that a decrease in A leads to stiffer ice, making it intuitively easier to understand why this is the right thing to do.

Done.

P7L14: Reiterate that Figure 2 is for A' instead of A, and therefore the damage pattern looks different from Figure 1.

Done.

P7L14: Can you explain why the areas of high damage at the margins are not so well confined to narrow bands as in Figure 1?

The text should be "damage at the end of the IceD0 experiment". We modified now.

P7L18: From the small figure it is unclear that the damage starts to grow a few kilometres upstream of the GL. Perhaps provide a zoomed-in version as an inset in Figure 2?

Figure 2 (now 3) has been modified to include a zoomed in region around the GL as suggested, instead of showing the speed and effective viscosity, which appear in figure 3 (now 4).

Figures 1-5: There is a lot of white space in all these figures that could be used to better display the details of your results. You should also consider choosing a different colour to make the grounding line stand out better.

We have remade all the figures with less white space, and in some cases fewer panels (which can then be larger). We changed the colormap for the damage field to one that is light for $\bar{D} \approx 0$, progressing through red and blue to black for $\bar{D} \approx 1$. This allows us to use a black line for the grounding line, and we then use cyan (which stands out well against red, dark blue, and black, even when printed in black and white) for the 200 m thickness contour.

And a list of typos where the text can be improved...

We modified the typos according to the referee or rewrite the sentences, which are shown in detail in the marked-up manuscript bellow.

Response to referee 2 (J. Bassis): 'Ice shelf fracture parameterization in an ice sheet model' by Sainan Sun et al.

Received and published: 24 May 2017

1 J. Bassis (Referee)

1.1 Overview

This study incorporates a damage based parameterization of fracture in the BISICLES ice sheet model and uses this to assess the influence damage has on grounding line position. The BISICLES model is a sophisticated ice sheet model which includes mesh refinement. The goal of the present study is to examine the influence of damage on grounding line position using a MISMIP style setup. The authors introduce a damage formulation in which damage is determined based on the Nye crevasse depth formulation. This has the advantage that, unlike most damage evolution laws that are heuristically based on sparse laboratory or field measurements, damage evolution has a physical component based on some elementary physics. Moreover, because crevasse depth models are popular methods of simulating the advance and retreat of outlet glaciers, the formulation has the potential to provide a unifying theme linking the behavior of outlet glaciers and ice shelves. The distinction between these two regimes is that damage in ice shelves is dominated by advection whereas damage in glaciers tends to grow rapidly near the calving front. In general, I think that this study is interesting and merits publication. However, I have a few major points that the authors should consider addressing in addition to several more minor nit-picky comments. The first sequence of questions relates to the physical formulation of the model where the second relates to the overall structure of the manuscript and some difficulties I had working my way through it. Overall, however, the manuscript will be a valuable contribution to the field once these questions have been satisfactorily addressed.

I would like to thank Dr. Bassis for this thorough review and I will try to give a response.

1.2 Major issues:

1.2.1 Approach to damage:

The first comment that I have relates to the formulation of damage and advection of damage within the model. I like the general idea of the model and this feels unseemly to point out in a review proposed something similar several years ago (Bassis and Ma, 2015 Evolution of basal crevasses links ice shelf stability to ocean forcing, 10.1016/j.epsl.2014.11.003). There are, however, several key differences between the formulation proposed in that paper and in this one.

The work of Bassis and Ma, (2015) showed the instability effect of crevasses by strain rate weakening and the evolution of initially narrow crevasses. The penetration of crevasses depends on the the stress field and influenced by the basal melting or freezing in the crevasses. The work is definitely related to our study and we have cited it, noting in particular

that some of its physics is missing from our model.

In our model, we assumed that initial crevasse depths used to seed damage are determined by the Nye zero stress model, analogous to the model presented here. However, we used a perturbation analysis to examine how the crevasses evolve and in particular whether they deepen, widen or close. As we show in that paper, the evolution of the *ratio* of crevasse depth to ice thickness (a pseudo damage variable analogous to the one introduced in the present study) is controlled by three factors. The first factor is simply kinematic. If crevasses are passive tracers in the flow field then they will deform with the flow field and their depth (or height) will decrease in exactly the same proportion as the ice thickness. A consequence is that the ratio of crevasse penetration to ice thickness remains *constant*. It is unclear to me how the kinematic distortion is accounted for here. From Equation (11) and (10) it looks like crevasse depths are inherited from upstream without accounting for the distortion associated with ice flow. This could be problematic.

Eq. (11)

$$\boldsymbol{\nabla} \cdot (\mathbf{u}\, \boldsymbol{d_{tr}}) = (\boldsymbol{\nabla} \cdot \mathbf{u})\boldsymbol{d_{tr}} + (\boldsymbol{\nabla} \boldsymbol{d_{tr}}) \cdot \mathbf{u}$$

does include this purely kinematic factor (the second term above) because $(\boldsymbol{\nabla} \boldsymbol{d_{tr}}) \cdot \mathbf{u}$ is generally non-zero (with $\mathbf{u}$ being only the horizontal velocity). We added a note to the manuscript "The vertically averaged damage can be reduced through meteoric or marine ice accumulation, where ice thickens without an increase in $\boldsymbol{d_{tr}}$. It can also be reduced or increased through the stretching and compression represented by $(\boldsymbol{\nabla} \cdot \mathbf{u})\boldsymbol{d_{tr}}$, in such way that the ratio $\boldsymbol{d_{tr}/h}$ remains constant"

As we further show, the ambient stress field within the ice shelf will also result in crevasse growth or closure. In fact, crevasses are likely to widen, but penetrate a smaller portion of the ice thickness unless the tensile stress opening crevasses is larger than the stress for a freely spreading ice tongue. Again, this is based on a linear stability analysis and depends on the wavelength of the perturbation and is limited to the early stages of growth.

The crevasses in our model don't have a width, and it seems that our vertically integrated model does not include this particular effect. We have pointed that out in the manuscript.

Finally, we also show that the ratio of crevasse penetration to ice thickness will depend on the basal melt/refreezing regime of the ice shelf (for basal crevasses) or the surface mass balance (for surface crevasses). This again follows from kinematic considerations that depend on whether the melt/refreeze rates within crevasses is larger or smaller than the large-scale melt rate allowing the ocean to excavate crevasses or fill crevasses with marine ice. Again, it is unclear to me how the model proposed here accounts for these factors. To be clear we applied the formulation using observed ice shelf velocity and thickness fields as opposed to integrating it within an ice sheet model so our approach is not entirely transferable.

The right side of eq. (11) defines the effect of surface mass balance and melting. Snowing at the surface can heal the crevasse, as can melt at the base (since the crevassed layer is eroded before the layer above). The melt rates are the same with in and out of the crevasses. We would need to do some further development to improve on this.

I'm also somewhat confused by the model used. This might be because symbols are introduced without definitions making it harder to follow the logic. For example, I have not been able to find a definition of d_{tr}. Similarly, I'm not sure I understand the right hand side of Equation 11. This seems to account for surface/basal mass balance, but it is introduced without explanatory text to help the reader understand the physics and assumptions.

We have now defined the variable d_{tr}, "One more modification is needed to reflect the transport of damage by ice flow. At any one time and place we would have two fields, the $d_l(x, y, t)$ computed above, and a field of transported crevasse depths $d_{tr}(x, y, t)$ which would have originated at $(x', y', t' < t)$ and been carried downstream, stretched, compressed, and so on." We have expanded the description of eq. 11 to note that e.g. basal melt is assumed to erode the crevassed lower layer so that vertically integrated damage is reduced.

There is also another subtle issue with the damage model proposed. In Bassis and Ma (2015) we examined how individual crevasses would evolve using a perturbation analysis. The physical interpretation of damage here is more subtle. For example, suppose crevasses penetrate half of the ice thickness (or more generally X percent of the ice thickness) across a channel along the margin. Does that imply a channel cut into the ice shelf where the ice thickness is reduced by half? Does this also reduce the driving stress? Or are the crevasses assumed to be narrow so that they have little effect on the large-scale driving stress. In this case, the damage would then need to account for the fact that you have intact ice between crevasses, resulting in *lower* damage on a large-scale. Or perhaps crevasses are assumed to be filled with ice/melange? All of this is speculation and it would be helpful to have a cartoon or physical description of the process that readers can refer to.

In effect we are assuming that crevasses are filled with soft ice. We have added a diagram, (Fig. 1) and some more text: "Notice that damage affects only the deviatoric stress (as in Jouvet et al., 2011 and Krug et al., 2014) and does not affect the gravitational driving stress. We might expect such a modification if we had instead modified the full Cauchy stress (as in Pralong and Funk, 2005; Bassis and Ma, 2015 and Mobasha et al., 2016), but have assumed that damage has no impact with respect to isotropic compression or vertical shear, so that the usual hydrostatic vertical stress balance, and the usual vertical integral of the resulting horizontal pressure gradient holds. This is analogous to assuming that the crevasses are filled with an inviscid material having the same density as ice."

1.2.2 Organization

I also struggled to understand the main hypothesizes tested. In the introduction we are told that the authors perform numerical experiments to address how including damage influences the evolution of the ice sheet and how the geometry of the damage field affects the dynamic response to ocean forcing. Later, at the beginning of Section 3 we are told that the goal is to address three question, including “If similar grounding line steady states can be realized with or without the damage model”; “If the ‘hidden’ damage inherent in the difference between A and A’ is revealed in the response of the ice stream to thinning of the ice shelf” and; “If it is necessary to evolve the damage model in time or if one can get away with constructing a damage field at the start of a calculation and then merely hold it constant throughout the simulation.” I don’t object to any of the questions, but it would be helpful to have the main objectives of the study introduced together at the beginning. Perhaps the later three questions can be motivated as more specific versions of the initial questions? In fact, I’m not sure that all questions have been completely addressed–especially if the hidden damage is revealed by perturbation experiments. Perhaps I missed something. Nonetheless, these five motivational questions would ideally also be mentioned in the abstract along with the resolution to the questions posed.

Our aim is to construct a model that is amenable to large scale calculations, and to decide whether its impact on the ice flow justifies the further development of such a model, or whether even simpler prescriptions (e.g. a rule of thumb for reducing A in the ice shelf) might be just as good. We have modified the manuscript in several places, including the abstract, to make this more obvious.

In a similar vein, one of the questions that authors seek to address is whether there is an equivalence between the rheology of damaged ice and ice with an adjusted rate factor A. The answer to this question seems obvious, especially when comparing Equations 5 and 6. We see that so long as we define $A' = A(1-D)^3$ there is an exact correspondence. That this question can be addressed by a simple mathematical definition makes me suspicious that the authors are examining a more subtle question, but if so it would help to provide more signposts for readers to help bring us along.

We meant some simple rule of thumb e.g. $A' = A/8$ everywhere, rather than $A' = A(1-D)^3$, which would require knowledge of $D(x, y, z)$ (or in our case $d(x, y)$) We did not make a good job of explaining this and have made a number of modifications to the text. One of the outcomes is that we do seem able to emulate the damage model with a simple prescription, at least in terms of the ice flow.

1.3 Detailed comments:

The definition of ‘damage’ in Equations (6) and (7) doesn’t follow naturally to me. In the standard approach to continuum damage mechanics one introduces a mapping from the actual stress σ_{ij} to the effective stress $\tilde{\sigma}_{ij}$ of the form: $\tilde{\sigma}_{ij} = (1-D)\sigma_{ij}$.

Note here that the mapping applies to the Cauchy stress tensor and not merely the deviatoric stress, as implied by Equation (6). It is true that one can define an effective viscosity of the form of Equation (6), but presumably one also must apply a mapping to the pressure term?

Here we have followed others (e.g. Krug 2014, Jouvet 2011) in only modifying the part of the stress that maps onto strain-rate. It does seem that a modification to the pressure term would be necessary if we were considering the ice to be weaker under isotropic compression, but we have assumed that it is not (i.e. crevasses are either closed but not bonded, or, as the reviewer suggests earlier, filled with incompressible melange.

This leads me to my next question, typically the 'damage' is defined as a decrease in load bearing capacity associated with cross sectional area of microcracks within the ice. Hence, the damage takes on a value between zero and unity. Here damage is defined somewhat differently and damage is effectively unity everywhere there is a crevasse and zero elsewhere. Damage is thus binary instead of continuous. Upon depth integrating one obtains crevasse depth as the effective depth integrated damage variable. This new variable is no longer confined to the interval [0,1) and no longer behaves like a typical damage variable. However, one can define a new variable based on the ratio of crevasse penetration depth $(d_s + d_b)$ to ice thickness H, which then maps the problem make to a more traditional effective damage variable that is again constrained to the interval [0,1). This is what is done in Bassis and Ma (2015) and, as noted before, has several advantages in terms of the ability to account for kinematic distortion and passive advection. Nonetheless, the authors use the variable "D" to denote what they call damage, which is nebulously defined, but appears to be three-dimensional, dimensionless and is binary taking on values of either 0 or 1. The authors then denote depth integrated damage using the lower-case "d" and this variable mimics crevasse penetration depth, which has units of length. Most of the model description focuses on depth integrated damage "d". However, starting in Section 4, the authors talk exclusively about damage with a capital D (see, e.g., page 7). Similarly, we see damage D in Figures 1, 2, 4 and 5. This is acutely confusing because I thought that damage D was three-dimensional and binary and these figures all denote a single map view with a continuous variation in damage. I have a suspicion that the authors are really showing the ratio of crevasse penetration depth to ice thickness and these figures and much of the discussion is mislabeled, but I don't know for sure. Much of notation and discussion could be cleaned up to clear up reader confusion.

The reviewer is entirely correct here. We made this clear in the model description " We construct a vertically integrated damage model by treating the ice sheet as having upper and lower layers of ice entirely fractured by surface and basal crevasses respectively, and an undamaged central layer (Fig, 1). Therefore, the scalar damage variable, $\boldsymbol{D}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ employed in vertically varying models (Pralong and Funk, 2005; Jouvet et al, 2011; Keller and Hutter, 2014; Krug et al 2015; Bassis and Ma, 2015; Mobasha et al, 2016) takes on either the value 0 (in the central layer) or 1 (in the upper and lower layers). The principal damage variable in our model is $\boldsymbol{d}(\boldsymbol{x},\boldsymbol{y}) \in [\boldsymbol{0},\boldsymbol{h})$, the vertical integral of $\boldsymbol{D}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, and

our closest analogue to the usual D is its vertical average, $\overline{D}(x, y) \in [0, 1)$".

Page 2, Kachanov (1999) appears to be primarily based on metals and I am not aware of any observations presented therein that relate to ice. As such, I'm not sure that this is the best reference to support the hypotheses that micro-cracks are the ultimate source of crevasses. I think this is likely to be true, but one might thing about citing a more ice-centric study to support this.

We cite Rist et al., (1994) now, which examines the relationship between micro-cracking and ice strength. The sentence is modified to: "Macro-scale fractures are originate from micro-scale cracks, which appear when viscous strain is too high (Rist et al., 1994)."

Page 2, "calving rate increases with imbalance of forces". In the quasi-static approximation forces are always in balance. An imbalance of forces would result in acceleration and violate the Stokes flow hypothesis. Crucially, crevasses do not require an imbalance of forces to propagate.

We rewrote the sentence:

"Crevasse depth depends on the stress field (Nye, 1957; Benn et al., 2007; Weertman, 1973), but the stress field is affected in turn by the change in forcing that immediately follows a calving event."

Page 2, "The models discussed above have reasonably successfully reproduced the calving rate and fracture distribution on some individual glaciers." Is this really true? With the exception of Pralong and Funk, 2005, I don't think that damage mechanics have successfully predicted the calving rate on individual glaciers. This has always been in the hope of damage mechanics, but most of damage mechanics has so far been conceptually applied (e.g., Duddu and Waisman) or focused on the large-scale softening (e.g., Albrecht and Levermann, Borstad et al.).

The description is inappropriate here. We delete the sentence.

A complex bestiary of experiments. I had a very hard time keeping track of the bestiary of experiments and variants. The table is helpful and appreciated, but the authors can help readers a bit by reminding readers of key differences in the figure captions and providing a bit more explanation of what we are supposed to see in the figures. In general I prefer more descriptive figure captions that include a short figure title and then some more explanatory text to allow readers to quickly point readers to what the authors want to show.

Agreed. We have added to the figure captions.

Ice shelf fracture parameterization in an ice sheet model

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Abstract. Floating ice shelves exert a stabilizing force onto the inland ice sheet. However, this buttressing effect is diminished by the fracture process, which on large scales effectively softens the ice, accelerating its flow, increasing calving, and potentially leading to ice shelf breakup. ~~Here, we explore how the application of~~ We add a continuum damage model (CDM) to the ~~prognostic~~BISICLES ice sheet model ~~BISICLES can account for the effects of fracture processes on viscous ice dynamics. Damage~~which is ~~created by the local~~intended to model the localized opening of crevasses under stress ~~field~~, the transport of those crevasses through the ice sheet, and ~~advects downstream. This continuum damage model is coupled to~~ the ~~dynamical~~ coupling between crevasse depth and the ice flow ~~model by decreasing the effective viscosity proportional to the damage field. To evaluate the physical role of the fracture process on large-scale ice sheet dynamics and also discern the relative importance of the parameters used in the damage model, we~~ field, and carry out ~~a suite of~~idealized numerical experiments ~~based on the MISMIP+ (Marine Ice Sheet Model Intercomparison Project) marine ice sheet geometry. We find that behavior of the simulated marine ice sheet is sensitive to fracture processes on the ice~~examining the broad impact on large scale ice sheet and shelf dynamics. In ~~the~~each case ~~of a geometry that produces strong lateral stress, the stiffness of ice around the grounding line is essential to ice sheet evolution~~we see a complex pattern of damage evolve over time, with ~~softer~~ an eventual loss of buttressing approximately equivalent to halving the thickness of the ice shelf. We find that it is possible to achieve a similar ice flow pattern using a simple rule of thumb: introducing an enhancement factor ~10 everywhere in the model domain. However, spatially varying damage (or ~~more damaged ice leading~~equivalently, enhancement factor) fields set at the start of prognostic calculations to match velocity observations, as is widely done in ice sheet simulations, ought to ~~thinning and~~ evolve in time, or grounding line retreat~~.~~ can be slowed by an order of magnitude.

1 Introduction

The largest uncertainties in sea level rise prediction are the dynamic ice sheet contributions (Jevrejeva et al., 2016). Two recent ice sheet model studies came to ~~startling~~surprisingly different conclusions regarding the Antarctic contributions to 21st and 22nd century sea level rise: Ritz et al. (2015) concluded that Antarctic contributions of a meter or more are implausible, whereas DeConto and Pollard (2016) find that not only is this plausible, but should even be considered likely if the ~~former~~previously underappreciated hydrofracturing process is included and linked to climate dynamics. Observations show an increased rate of ice discharge from Greenland and Antarctica in recent decades (Shepherd et al., 2012). Calving is directly responsible for a mass loss comparable to that from ice shelf basal melting (Rignot et al., 2010; Depoorter et al., 2013; Liu et al., 2014). Furthermore, increased rates of calving and basal melt seem intertwined and act in concert to enhance mass loss from ice shelves that are in negative mass balance under the present climate (Liu et al., 2014; Åström et al., 2014). Mass loss from ice shelves does not contribute to sea level rise directly, but via the restraint ice shelves apply to the ice discharge from inland to ocean across the grounding line; ~~hence~~in other words mass loss from ice shelves is expected to weaken their buttressing effect. Fürst et al. (2016) show that ice shelves in the Amundsen and Bellingshausen seas are ~~extremely sensitive~~thought to be more vulnerable to calving~~,~~ events than other regions meaning that even a small amount of increased calving ~~will~~can trigger ~~essential~~ dynamical responses ~~to~~that raise sea level. Thus to better predict the future evolution of the ice sheets, processes related to calving should be better understood and described.

~~Calving events~~Macro-scale fractures originate from micro-scale cracks ~~(Kachanov, 1999),~~, which appear when viscous strain is too high~~.~~ (Rist et al., 1994). On microscopic scales fracturing is a discrete process which operates on time scales determined by the speed of sound in ice, far faster than the viscous processes typical of ice sheet flow. Combining these processes in a single model presents very difficult numerical challenges that have only been attempted for a few cases using discrete element models (Bassis and Jacobs, 2013; Åström et al., 2013; Åström et al., 2014). This discrete approach has a firm basis in physics with, for example, Glen's flow law emerging naturally. On macroscropic scales, the development of cracks may be seen as a ~~statistically~~ continuum and long-term process. The net effect is to soften ice, and hence potentially accelerate ice flow. Fractures form as the damage effect accumulates. Propagation and penetration of fractures ~~magnify~~trigger calving events. Previous continuum studies of calving fall into two main categories, but all ~~of them~~ are essentially empirical rather than being based on fundamental fracture process physics. ~~The~~ Models of the first ~~approach treats~~type treat calving as ~~a~~the result of macro-scale crevasses~~. Some of these methods are based on calculation of crevasse depth (Weertman, 1973). The crevasse~~, but do not consider any direct coupling between crevasses and the flow field. Crevasse depth depends on the stress field (Nye, 1957~~) and~~ ; Benn et al., 2007; Weertman, 1973), but the stress field is affected in turn only by the change in forcing that immediately follows a calving ~~rate increases with imbalance of forces (Benn et al., 2007). The model of Nick et al. (2010) takes basal crevasses into consideration as well as surface crevassing and combines an empirical criterion for calving. Models~~

~~of~~ event. Models in this category ~~successfully reconstruct the calving rate of some individual glaciers~~ (Nick et al., 2010; Nick et al., 2013; Cook et al., 2014), ~~however, they~~ compute crevasse depth based on instantaneous fields, and hence ~~could~~do not take into account ~~for~~ the stress history ~~on~~in the development of fractures. Models in the second category ~~are named~~use a Continuum Damage Mechanics (CDM~~),~~) approach, which ~~treat~~treats calving as a continuum process that develops from micro-scale cracks to macro-scale crevasses, and damage has an effect on the viscous ~~behavior~~behaviour of ice flow (Pralong and Funk, 2005; Jouvet et al, 2011; Duddu and Waisman, 2012; Borstad et al., 2012; Borstad et al., 2013; Albrecht and Levermann, 2014; Krug et al., 2014; Bassis and Ma, 2015).

~~The models discussed above have reasonably successfully reproduced the calving rate and fracture distribution on some individual glaciers.~~ Here we propose a continuum damage model (CDM) which ~~takes into account the accumulation~~considers conservation of damage during ~~the~~ ice flow as well as local sources of damage. The local source of damage comes from the stress field~~,~~ and water depth in crevasses following the physical mechanism of Nick et al (2010), while a conservation equation describes the evolution of the vertically integrated damage field due to advection, stretching and mass loss or gain from the glacier ~~surface~~'s upper and ~~bottom.~~lower surfaces. The development of damage has an impact on ice viscosity, and therefore influences the evolution of ice flow through Glen's flow law, ~~and provides a feedback to~~ so that the damage ~~field itself.~~ and flow fields are strongly coupled. Potentially important phenomena such as the detailed accretion of marine ice within basal crevasses, and the necking phenomena of Bassis and Ma (2015) are not included: our aim here is to construct a model that is amenable to large scale calculations and investigate the broad outline of its impact on ice flow.

We use the ~~state of art~~ adaptive mesh refinement model~~,~~ BISICLES (Cornford et al., 2013~~),~~) as our ice flow model, and add the CDM to it. The performance of the CDM is tested on the MISMIP+ (Marine Ice Sheet Model Intercomparison ~~Projects~~Project) experiments (Asay-Davis et al., 2016~~). The scheme is~~), which are based on an idealized marine ice sheet ~~geometry~~ that has strong lateral ~~stress gradients~~stresses (Gudmundsson et al., 2012). ~~Simulations under different ocean forced melt rates~~ A range of steady-state and time dependent simulations were carried out with ~~the coupled model are compared to the ice flow model~~ and without the CDM~~. Through the simulations, we~~ in an effort to answer ~~the following~~ four related questions~~:~~. How does damage influence the evolution of the ice sheet, ~~such as~~in particular the ~~speed and behavior~~behaviour of the grounding line, and the ice ~~flow~~ velocity field? ~~How~~Can we achieve the same steady state location of the grounding line in the model with or without the CDM component by making a rule of thumb adjustment to the (uniform in this case) rheology parameter A, which is typically unknown and so must be tuned to observations? If such an adjustment can result in similar steady states, how does the ~~geometry of the~~ transient response between the models then differ, when they are subjected to an external forcing that leads to thinning of the ice shelf? Is it necessary to evolve the damage model in time, or is it possible to simply construct a damage field ~~affect the dynamic response to ocean forcing?~~ at the start of a calculation and hold it constant throughout the simulation?

In the next Section, we describe the physics used in this study, including the CDM and ice flow model. Then we present in Section 3 a suite of experiments based on MISMIP+ to evaluate the model performance and the effect of damage on ice sheet evolution. We present the results in Section 4 and a discussion in Section 5.

~~2Model~~2 Model Description

2.1 Ice flow governing equations

We implemented the damage model in the marine ice sheet model BISICLES (Cornford et al., 2013). Ice flow velocity is computed by solving a vertically-integrated stress balance equation, in this case the Shallow-Shelf ~~approximation~~Approximation (SSA) of the Stokes equations (MacAyeal et al., 1996). This ice dynamics formulation performs well ~~in~~for ice shelves and fast-flowing ice stream simulations. Floating ice is assumed to be in hydrostatic equilibrium, so given a bed elevation b and ice thickness h, the surface elevation s is

$$s = max\left(h + b, \left(1 - \frac{\rho_i}{\rho_w}\right) h\right), \quad (1)$$

where $\rho_i = 910\ kg\ m^{-3}$ and $\rho_w = 1028\ kg\ m^{-3}$ are densities of ice and ocean water.

Ice thickness h and horizontal velocity $\boldsymbol{u}$ satisfy the mass conservation equation

$$\frac{\partial h}{\partial t} + \nabla \cdot [\boldsymbol{u}h] = a - M \qquad (2)$$

and the two~~-~~-dimensional stress-balance equation

$$\nabla \cdot [h\bar{\mu}(2\dot{\boldsymbol{\epsilon}} + 2tr(\dot{\boldsymbol{\epsilon}})I)] + \tau^b = \rho_i g h \nabla s \qquad (3)$$

together with lateral boundary conditions. In equation (2), a is the surface ice mass accumulation and M is the basal melt rate of the ice shelf. In equation (3), tr is the trace operator, $\dot{\boldsymbol{\epsilon}}$ is the horizontal strain rate tensor~~ and tr is the trace operator~~,

$$\dot{\boldsymbol{\epsilon}} = \frac{1}{2}[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T], \qquad (4)$$

I is the identity tensor and τ^b is the basal friction. Basal friction is computed according to Tsai et al. (2015), ~~defined as obeying~~tending to a power law far upstream ~~inland~~ from the grounding line, ~~transformed into~~and to a Coulomb friction law near the grounding line. $h\bar{\mu}$ is the vertically integrated effective viscosity, which would conventionally be given by Glen's law, but is modified here to include an additional damage parameter (see section 2.2).

BISICLES is constructed using the Chombo parallel AMR (Adaptive Mesh Refinement) framework, which allows us to use a non-uniform, evolving mesh during simulations. Here, we implement ~~a~~ three~~ level~~ levels of local refinement on top of ~~the~~a coarse mesh (level zero~~level) of~~) spanning the domain at 4 km resolution. The grid cell size is refined by a factor of 2 at each refinement level leading to a finest resolution of 0.5 km around the grounding line. The time step size satisfies the Courant-Friedrichs-Lewy condition everywhere, meaning, for the geometry here, about 16 time steps per year.

2.2 Damage model

~~In CDM, we consider the material as a continuous material, and so do not take into account the effect from individual crevasses but consider their integrated effect on ice flow. CDM has been proved to be effective in some case studies (e.g., Krug et al., 2014; Borstad et al., 2012; Pralong and Funk, 2005). The damage variable~~ ~~D~~ ~~is a scalar between 0 and 1, in which 0 means ice with no damage and 1 represents fully damaged ice, meaning a high spatial density of crevasses penetrating the whole column of the ice such that the ice provides no resistance to flow, and d is the vertical integral of~~ ~~D.~~

We construct a vertically integrated damage model by treating the ice sheet as having upper and lower layers of ice entirely fractured by surface and basal crevasses respectively, and an undamaged central layer (Fig, 1). Therefore, the scalar damage variable, $D(x,y,z) \in [0,1)$ employed in vertically varying models (Pralong and Funk, 2005; Jouvet et al, 2011; Keller and Hutter, 2014; Krug et al 2015; Bassis and Ma, 2015; Mobasha et al, 2016) takes on either the value 0 (in the central layer) or 1 (in the upper and lower layers). The principal damage variable in our model is $d(x,y) \in [0, h(x,y))$, the vertical integral of $D(x,y,z)$, and our closest analogue to the usual D is its vertical average, $\overline{D}(x,y) = d(x,y)/h \in [0,1)$.

Damage enters the stress balance equation through a modification to Glen's law. The usual relationship between deviatoric stress $\boldsymbol{\tau}$ and rate-of-strain $\dot{\boldsymbol{\epsilon}}$ is:

$$2\,A\,\tau^2\boldsymbol{\tau} = \dot{\boldsymbol{\epsilon}}, \qquad (5)$$

where the flow rate exponent $n = 3$, and A is a flow rate factor, which is typically temperature dependent but set to be constant here. We replace this with

$$2\,A\,\tau^2\boldsymbol{\tau} = (1 - D(\tau))^3\dot{\boldsymbol{\epsilon}}, \qquad (6)$$

which – given the shallow shelf approximation – results in an expression for the vertically integrated effective viscosity,

$$2\,h\bar{\mu} = [h - d(\tau_1)]A^{-\frac{1}{3}}\dot{\epsilon}^{-\frac{2}{3}}. \qquad (7)$$

A relationship between damage and the first principal stress, $d(\tau_1)$, must be ~~specified~~identified, at which point the stress balance equation can be solved numerically for d and $\boldsymbol{u}$ together. Specifying the damage-stress relationship in this way assumes that damage evolves on a similar or faster timescale to the ice velocity field. Many authors specify instead a damage evolution rate, which, in e.g. Krug et al (2014), and given typical stresses in an ice shelf, amounts to a timescale of around 1 year.

Notice that damage affects only the deviatoric stress (as in Jouvet et al, 2011 and Krug et al, 2014) and does not affect the gravitational driving stress. We might expect such a modification if we had instead modified the full Cauchy stress (as in Pralong and Funk, 2005, Bassis and Ma, 2015, and Mobasha et al 2016), but have assumed that damage has no impact with respect to isotropic compression or vertical shear, so that the usual hydrostatic vertical stress balance, and the usual vertical integral of the resulting horizontal pressure gradient holds. This is analogous to assuming that the crevasses are filled with an inviscid material having the same density as ice.

In the absence of advection, we could now prescribe $d(\tau_1)$ by equating it to the total depth of crevasses, computed following Nye (1957), Benn et al (2007), and Nick et al (2010). ~~The~~There, the depth of a mode I crevasse at the upper surface is:

$$d_s = \frac{\tau_1}{\rho_i g} h + \frac{\rho_w}{\rho_i} d_w, \qquad (8)$$

while at the lower surface it is

$$d_b = \frac{\rho_i}{\rho_w - \rho_i} \left(\frac{\tau_1}{\rho_i g} h - h_{ab} \right), \qquad (9)$$

where d_w is the water depth in the surface crevasse and h_{ab} is the thickness above floatation. The total local crevasse depth is then $d_l = \max(d_s, d_s + d_b, 0)$.

A water depth $d_w \sim h/2$ is in fact required in Nick et al (2010) for any calving to take place at all, and would clearly have a substantial impact on our calculations too, but for this paper we consider only dry crevasses, with $d_w = 0$. We also ignore any lower limit on the stress needed to open a crevasse, so that we will tend to produce small crevasses where there should be none. As we will see in the results, the major impact of the damage model is in the ice shelf and around the grounding line, where large tensile stresses readily exceed such limits.

Inasmuch as a shallow shelf model is a good approximation to the full Stokes model, our choice of the Nye zero stress model above is similar to the long-term behaviour of Krug et al (2014), at least for surface crevasses. In that model, damage grows to $D \rightarrow 1$ as $t \rightarrow \infty$ where and only where the Cauchy stress is tensile, just as in the Nye model, giving the depth of surface crevasses. Basal crevassing could be included in such a model by adding the water pressure to the Cauchy stress, as in Keller and Hutter (2014).

One more modification is needed to reflect the transport of damage by ice flow. At any one time and place (x, y, t) we would have two fields, the $d_l(x, y, t)$ computed above, and a field of transported crevasse depths $d_{tr}(x, y, t)$ which would have originated at $(x', y', t' < t)$ and been carried downstream, stretched, compressed, and so on. We assume that crevasse closure does not result in bonding of the crevasse surfaces, at least on the timescale of the closure itself, so that the final relationship $d(\tau_1)$ is given by

$$d(\tau_1) = \cancel{max(d_l(\tau_1)}\max(d_l(\tau_1), d_{tr}). \qquad (10)$$

This means that regions of the ice shelf under lower stress inherit damage from any higher stress region upstream. ~~The~~The transported total crevasse depth d_{tr} is found by solving a damage transport equation

$$\frac{\partial d_{tr}}{\partial t} + \nabla \cdot (\boldsymbol{u} d_{tr}) \cancel{= -[max(a,0) + max(-M,0)]} = -[\max(a,0) + \max(M,0)] \frac{d_{tr}}{h} \cancel{,}. \qquad (11)$$

The left hand side of Eqn. (11) expresses the conservation of the vertically integrated damage under ice flow, and includes both the motion of crevasses with the flow ($\boldsymbol{u} \cdot \nabla \mathrm{d}_{tr}$), and stretching and compression ($d_{tr} \nabla \cdot \mathbf{u}$), which, all else being equal, holds the ratio d_{tr}/h constant in a diverging or converging horizontal flow field. The right hand side assumes that accumulation at the upper surface (a) increases the thickness of undamaged ice, while basal melting (M) erodes the crevassed underside, so that both terms cause a reduction in vertically integrated damage for the cases considered here, where $a > 0$ and $M > 0$. Note that a and M are considered to be the same inside and outside of the crevasses. We specify Dirichlet conditions $d_{tr} = 0$ on all inflow boundaries, while initial conditions are determined ~~in the same way as ice thickness, that is,~~ by evolving the coupled system to ~~a~~ steady ~~state, but with~~states, starting from the ~~additional proviso~~initial guess $d_{tr}(t = 0) = d_l(t = 0)$. We also set $d_{tr}(t - \Delta t) \cancel{= d(t - \Delta t).} = d(t - \Delta t)$ at every time step, which in effect imposes a new initial condition to ensure that $d_{tr}(t - \Delta t) \geq d_l(t - \Delta t)$ everywhere. Note also that ~~although damaged ice does not strengthen over time, the vertically averaged damage can strengthen through meteoric or marine ice accumulation, since ice thickens without an increase in~~ $\cancel{d_{tr}}$~~.~~ ~~Eq.~~Eqn. (11) has no explicit healing term to represent the effect of overburden pressure, which will lead us to overstate the damage field. We assume it is relatively unimportant compared to crevasse opening in the largely tensile ice shelf flow fields most affected by damage in the results presented here.

3 Experimental design

~~We~~To answer the questions mentioned in Section 1, we carried out a set of numerical experiments based on the third Marine Ice Sheet Model Intercomparison Project (MISMIP+, Asay-Davis et al., 2016). MISMIP+ includes a number of experiments based on an idealized marine ice sheet geometry derived from Gudmundsson (2012) and Gudmundsson (2013). ~~In the experiments, ice~~Ice flows along an 800 km long and 80 km wide submarine bedrock trough, from an ice divide at one end to an ice shelf and calving front at the other. The geometry features an ice shelf with lateral stresses that buttress upstream ice to the extent that it is possible to obtain a stable equilibrium with its grounding lines ~~sited~~positioned on a retrograde bedrock slope, that is, a slope rising in the direction of ice flow ~~toward the ocean.~~. The suite of simulations based on different models and basal melt rates are summarized in Table 1 and described in detail below.

~~We had three questions in mind. First, although the CDM should weaken the ice shelf such that the steady state location of the grounding line would be altered for a given parameter~~ $\cancel{A}$ ~~in the Glen's flow law, is it possible to find an alternative value~~ $\cancel{A'}$ ~~which compensates for the damage? Note that~~ $\cancel{A}$ ~~is not measured directly but typically tuned so that simulated ice flow~~

~~matches observations. Second, if similar grounding line steady states can be realised with or without the damage model, is the 'hidden' damage inherent in the difference between A and A' revealed in the response of the ice stream to thinning of the ice shelf. Third, is it necessary to evolve the damage model in time, or is it possible to simply construct a damage field at the start of a calculation and hold it constant throughout the simulation?~~

We investigated equilibrium states by carrying out the MISMIP+ no ocean forcing experiment (Ice0) with and without the damage model. Ice0 requires that models are close to steady state with a grounding line crossing the centre of the trough (y = 40 km) at around x = 450 km, and that they demonstrate this by showing insignificant grounding line migration over 100 years in the absence of ice shelf basal melt. Without the damage model, we set $A = 2.0 \times 10^{-17}\, Pa^{-3}\, a^{-1}$, just as in Asay-Davis et al~~.~~ (2016), and found a steady state. We then switched on the damage model and ran for 1000 years to assess grounding line migration due to the weakened ice shelf. We will refer to this simulation as IceD (in general D indicates inclusion of the damage model). ~~Then,~~In order to ~~follow~~start the ~~MSIMIP~~MISMIP+ experiments ~~as intended~~from the required grounding line location at x = 450 km, we ~~carried out~~run a series of ~~experiments each running for 1000 years~~IceD simulations with ~~a~~ different values of ~~A~~ the rate factor A. For each value of A, a new steady state grounding line location is obtained, and we select the value A' for which the location is closest to ~~find a value A' that resulted in an ice sheet whose~~the originally required grounding line ~~was close to the original: we~~at x = 450 km. We will refer to ~~the test of that~~this steady state experiment as IceD0.

Once IceD0 had been completed, we carried out the remaining MISMIP+ experiments with the given values of A and A'. Ice1r and IceD1r see the models respond to a simple basal melt formula that concentrates ablation close to (but not at) the grounding line as it evolves over 100 years,

$$M = \frac{1}{5} tanh\left(\frac{H_c}{75.0}\right) max\big((100 - Z_d), 0\big) \qquad (12)$$

where H_c is the water column thickness and Z_d is ice shelf draft. Ice1ra and IceD1ra see the ice sheet change over the next 900 years when the basal melt rate is set to zero, while Ice1rr and IceD1rr continue for the next 900 years under the influence of basal melt (~~eq~~Eqn. 12). Ice2r, IceD2r, IceD2ra, IceD2ra, Ice2rr and Ice2Drr follow the same general pattern, but impose a different melt rate

$$M = \begin{cases} 100\ m\ a^{-1}, & x \geq 480\ km \\ 0, & x < 480\ km \end{cases} \qquad (13)$$

where x ~~represent~~represents the distance away from the ice divide. This melt rate is concentrated away from the grounding line and does not evolve with it, allowing a thick ice shelf to form in the wake of a retreating grounding line. This high basal melt rate at the ice front is designed to simulate the effect of mass loss far from the grounding line on ice flux, which is an analogue to calving events~~ in real world. Names of simulations based on different models and basal melt rates are summarized in Table 1~~.

Finally, we carried out versions of the Ice0, Ice1r, and Ice1ra experiments with the same A' and initial damage field as in IceD0, but without allowing the damage field to evolve over time. We will refer to these experiments as IceF0 and IceF1r, with the 'F' standing for fixed-in-time damage. This resembles realistic cases (e.g. Gong et al., 2014; Favier et al., 2015; Cornford et al., 2015; Sun et al., 2014) where an initial damage field is estimated to match observations of velocity in the ice shelf at the start of a simulation, and held constant thereafter.

4 Results

Switching the damage model on given the steady geometry of Ice0 with A produces widespread weakening of the ice shelf, resulting in 100 km of grounding line retreat. Fig. ~~1~~2 shows how the damage field evolves for 1000 years from the moment the damage mechanism begins. On grounded ice, the vertically averaged damage ~~D~~, $\bar{D} = d/h$ is generally low, which can be attributed to both the low viscous stress and the ice overburden acting against basal crevasse formation. In the ice shelf, ~~D~~ $\bar{D}$ is typically about $1/2$, varying from around $1/3$ close to the grounding line in the center of the channel, to nearly 1 where the grounding line crosses the channel walls. As time passes, damage is advected downstream so that strips of ice with ~~D~~$\bar{D}$ close to 1 extend all the way from the grounding line to the calving front. Meanwhile, the acceleration caused by this ice shelf weakening results in the grounded ice thinning and in turn the grounding line retreats, by around 70 km in the first ~~50~~100 years of the simulation and a further 30 km over the full 1000~~-~~-year simulation. ~~Decreasing~~ Increasing the stiffness of ice by reducing A to $A' = 1.5 \times 10^{-18}\, Pa^{-3}\, a^{-1}$ is sufficient to counter the damage and hold the grounding line steady at x = 450 km. Such an order of magnitude change in A corresponds to an approximately 20 K difference in ice temperature, or, more pertinently, the introduction of an enhancement factor ~ 10 in the model without damage. Fig. ~~Fig. 2 shows the effective viscosity, velocity, and damage at the start of the IceD0 experiment.~~3 shows the vertically averaged damage $\bar{D}$ at the end of the IceD0 experiment, computed with the damage model and the lower rate factor A' . At this point the model is close to steady state, having been allowed to evolve for 30,000 years: neither the grounded area nor the volume above flotation changes substantially over the course of 1000 years. Just as before, ice far upstream from the grounding line is not strongly damaged, whereas ~~D~~$\bar{D}$ is typically around $1/2$ in the ice shelf, and larger close to the domain boundaries. It is also apparent from Fig. ~~2~~3 that ~~D~~$\bar{D}$ begins to grow over a region a few kilometers upstream from the grounding line.

Although the pattern of damage and its impact on effective viscosity varies both laterally and between grounded and floating ice, the net effect is to produce a grounding line that is rather similar in shape and position to that of Ice0 (Fig. ~~3~~4). Having established that it is possible to 'emulate' the damage model, at least in steady state, by a simple (uniform) change to A, it is natural to ask if the same is true when the ice shelf is perturbed.

Fig. ~~4~~5 shows the evolution of ~~D~~ $\overline{D}$ and the grounding line location over time in the IceD1r~~,~~ and IceD1ra~~, and IceD1rr~~ simulations. IceD1r sees the grounding line in the centre of the channel retreat from around x = 450 km to x = 390 km over the course of 100 years while the basal melt rate (~~eq~~Eqn. 12) is applied. At the same time, the damage field evolves to maintain a pattern of low damage (~~D~~ $\overline{D}$ << 1) on the grounded ice and more damage (~~$D \sim 1/2$~~ $\overline{D} \sim 1/2$) in the ice shelf. Much of the ice shelf is thin (h< 200 m) so that a high value of ~~D~~ $\overline{D}$ implies only a small reduction in buttressing caused by the ice shelf. IceD1ra, which continues from IceD1r, specifies that basal melt rates return to zero, allowing the grounding line to advance toward the IceD0 steady state position by t~~=200~~ = 1000. Ice in newly grounded regions has almost no damage, since the advection of damage is significantly faster than grounding line advance. Between t~~=~~ = 100 and t~~=200~~ = 1000 years, the formerly heavily damaged region is completely lost downstream of the grounding line~~. A~~ and a tongue of less damaged ice (~~$D \sim 1/3$~~ $\overline{D} \sim 1/3$) ~~extruds~~extrudes from the grounding line and a pattern akin to IceD0 (Fig. ~~2~~3) is reached by t~~=1000. IceD1rr also follows on from IceD1r, but with basal melt rate continuously applied, driving grounding line retreat to around x = 250 km by t~~ = 1000.

Fig. ~~5~~6 plots the damage field and the grounding line during the evolution of ice in IceD2r~~,~~ and IceD2ra ~~and IceD2rr~~ experiments. The ice shelf is removed entirely beyond x = 480 km during the first 100 model years. The damage field in the remainder of the ice shelf appears much as it did in the initial state, albeit with a narrow strip of high (~~$D \sim$~~ $\overline{D} \sim$1) damage right at the calving front. The grounding line retreats by around 20 km in the center of the channel, while the damage field evolves so that ~~$D \sim 1/3$~~ $\overline{D} \sim 1/3$ immediately downstream, growing to ~~$D \sim 1/2$~~ $\overline{D} \sim 1/2$ before abruptly increasing at the calving front. ~~Experiment IceD2rr~~Maintaining the same forcing sees the same trend continue, with the grounding line retreating by a further 40 km~~.~~ in 1000 years (not shown here). Experiment IceD2ra, on the other hand, permits the calving front to advance to x = 640 km. It exhibits a travelling front of strong damage, separating thicker ice (h > 200 m) which has been carried downstream from the ice shelf of IceD2r from ice which had been permitted to accumulate on the open sea - had such accumulation not been permitted, this damage front would have been coincident with the calving front. Over time, the original pattern of IceD0 is recovered, with ~~$D \sim 1/3$~~ $\overline{D} \sim 1/3$ near the grounding line, ~~$D \sim 1/2$~~ $\overline{D} \sim 1/2$ in the lateral ~~center~~centre of the shelf for most of its length, and strips where ~~$D \sim$~~ $\overline{D} \sim$1 close to the domain boundaries.

We compare the grounded area change and ice volume change between different models and ocean forcing in Figs. ~~6~~7 and ~~7~~8. After tuning the parameters, our model with the CDM produces similar retreating/advancing trends to the published model results (Asay-Davis et al., 2016). For both BISICLES and BISICLES_D (BISICLES with CDM), some grounded regions become ~~floating~~afloat when we implement melt rates, and the grounded area decreases gradually until the melt rates cease. The floating area could be re-grounded if melt rates were no longer applied even on the retrograde bedrock slope. Ice volume change has the same trend as grounded area. The ice volume above floatation (i.e. that can affect sea level) decreases when forced by basal melt or calving, and increases to nearly the initial state when ocean forcing disappears. In both versions of

BISICLES, the ice sheet is more sensitive to basal melt near the grounding line than to extreme high basal melt representing calving at the front. However, the BISICLES_D produces less retreat under both ocean forcing scenarios.

The final experiment contrasts retreat rates between three models: the original model, the model with evolving damage, and a model where the damage is initially the same as in the evolving damage model, but remains constant in time. Fig. ~~In the experiment without basal melt, the model with fixed damage (IceF0) produces a stable ice sheet behavior similar to IceD0 and Ice0. In the experiment with high basal melt rate near the grounding line, ice behavior of the fixed damage (IceF1) ice sheet is stable and comparable to Ice1r and IceD1r (~~9 shows that all three models maintain a near steady state when not perturbed by ice shelf melting (Ice0, IceD0, IceF0), and indeed, the original and damage-evolving models (Ice1r, IceD1r) show a similar rate of grounding line retreat under ice shelf ablation. However, with the damage field fixed in time, experiment IceF1 shows a far slower rate of retreat – 500 km^2 over the course of a century, rather than the 3000 km^2 seen in the other two cases. To reiterate: ‘emulating’ the damage model by simply softening the ice uniformly gives far closer results to the full model than having an initially identical damage field but neglecting to evolve it over time.

~~Fig. 8). For the fixed damage model, the viscosity value in the region upstream of grounding line is higher, which greatly slows down the ice flux across the grounding line. The result indicates that the viscosity around the grounding line is essential to the stability of the ice sheet.~~

5 Discussion

The CDM produces a plausible damage distribution on this particular MISMIP+ geometry: ~~Damage~~damage is observed to be low on the grounded ice, while it increases dramatically when the ice crosses the grounding line and increases gradually downstream from there~~, (~~ similar to the crevasses distribution of Pine Island Glacier founded by Bindschadler et al., 2011~~).~~. The experiments ~~Ice0 and~~ IceD, which ~~show~~in comparison to Ice0, explicitly shows the result of adding damage to the ice shelf, produced rapid grounding line retreat over the full 1000 ~~years simulations~~year simulation (Fig. ~~1~~2). Thus, the buttressing force of the undamaged ice and its corresponding ice viscosity in the shelf is essential to the ~~behavior~~stability of the ice sheet when using the MISMIP+ geometry.

Although the damage distribution is highly localized, we showed that by tuning the viscous parameter A in the control experiment IceD0 we can match the grounding line in steady state achieved with Ice0. The required flow law parameter is an order of magnitude greater than the MISMIP+ value of A. Simulations IceD1r, IceD1rr, IceD1ra, IceD2r, IceD2ra, and IceD2rr all exhibit similar overall trends to the unmodified BISICLES model, despite extensive spatial variation in the damage field. However, the effective viscosity around the grounding line under the various experiments with and without the damage model is similar. This implies that for this geometry, the viscosity of ice around the grounding line essentially controls the grounding line position, while the state of the ice shelf far from grounding line has much less impact. The ice shelf is relatively thin (<200 m) except near the grounding line and thus contributes little buttressing to inland ice flux.

Including a CDM in a realistic simulation may, perhaps counterintuitively, result in lower sensitivity of the ice sheet to ice shelf ablation and sea level rise. Recall that, given an initial stable state with similar geometry and flow field, BISICLES_D saw lower rates of both retreat and advance in the Ice1 and Ice2 experiments than the BISICLES - especially the Ice2 experiment (Fig. ~~6~~7, Fig. ~~7~~8). We can attribute this to the lower value of A (more viscous ice) needed across the domain to compensate for the weaker ice shelf. Once that weaker ice shelf is removed, the remaining ice sheet tends to have a larger viscosity, certainly upstream from the grounding line, than it did in the undamaged case. Realistic simulations, ~~as~~at least if tuned to match observed geometry and velocity, might be expected to behave in the same way.

The relative effects on the grounding line position of experiments with ‘realistic’ basal melt patterns, where basal melt is highest close to (but not at) the grounding line (Ice1r), and those designed to mimic calving (Ice2r), where basal melt is very high near the ice shelf edge, show that calving has much less control on grounding line retreat. This does not mean that calving ~~process~~ is in general unimportant for the grounding line position, because in our special geometry, the side walls apply strong back stress and the removed ice is mostly downstream of the side wall, which will not always be the ~~general~~ case in reality, in particular for large ice shelves~~ in particular~~.

The evolution of the damage field in all of the experiments can be approximated, crudely at least, by a simple rule. There is little or no damage to grounded ice, while $D \sim 1/2$ at all times in the ice shelf, unless the ice is so thin as to contribute little buttressing. Thus, in the absence of a damage model, it might be possible to emulate its effects simply by setting $D \sim 1/2$ (or equivalently, multiplying A by ~ $1/8$, that is setting an enhancement factor E ~ 8) (Paterson, 2000). The primary cause of this simple pattern is the Nye crevasse formulae (~~eqns.6~~Eqns.8 and ~~7~~9): basal crevasses are deep – an order of magnitude deeper than surface crevasses – when and only when ice is close to or at flotation, while in a simple model of the ice shelf, with no lateral variation~~,~~ and no buttressing, it is straightforward to see that $d_b + d_s = \frac{h}{2}$~~,~~ by noting that the vertically integrated stress given by the calving front boundary condition is maintained throughout the shelf. As $d_b \gg d_s$ by about a factor of 10, depending on densities, modelled damage is much higher in the shelf than on grounded ice. This is consistent with observations that large-scale crevasse-like surface features are common on the ice shelves along the Amundsen and Bellingshausen Seas and on the smaller ice shelves between the Amery and Ross ice shelves of east Antarctica (Liu et al., 2015). Ground-penetrating radar show that many of these are, in fact, the surface expression of deep and wide transverse basal crevasses (Bassis and Jacobs, 2013; McGrath, et al., 2012) or longitudinal sub-glacial melt channels (Vaughan et al., 2012), that may penetrate 200 m into the base of the ice shelf, while the surface depressions are typically 30 m lower than the usual ice shelf surface (Liu et al., 2014).

~~Lack of healing from overburden pressure in the CDM will lead to an overstated damage field when applied to ice shelves with moderate stress environments. In that case, yield stress for healing should be considered for different conditions (Albrecht and Levermann, 2014).~~

~~In BISICLES_D, viscosity around the grounding line is reduced by the high damage value produced by the large principal stress (eqns. 6 and 7). From~~While the damage model might be approximated by the ansatz described above, it appears that the practise of estimating a spatially varying damage (or enhancement factor, or rate factor) field by solving an inverse problem to match observed velocities may be problematic, unless the damage evolves in time. An experiment along these lines (IceF1) was the only case where we saw substantially – nearly an order of magnitude – slower retreat rates in the MISMIP+ Ice1 experiment. In fact, the retreat rate was even slower than the typical Ice2r retreat rates, where melt is restricted to a downstream location and a thick ice shelf persists to buttresses the upstream ice. Since there is little ice shelf left in the IceF1 experiment, it is clear that the damage field close to the grounding line has a major role in the ice shelf dynamics, and neglecting to update it as the grounding line retreats leads to a lower flux q across the grounding line – a result that would be expected by considering the role of the rate factor in approximations to q (Schoof 2007, Tsai 2015).

A damage model with sufficient skill to represent all relevant processes would require further development. An obvious limitation is the choice of a vertically integrated model, when a vertically varying flow field is required to describe processes such a necking (Bassis and Ma 2015), or, even if a vertically integrated flow field is sufficient, a vertically integrated damage model may not be (Keller and Hutter, 2014). It is also clear that the Nye zero stress model cannot be the whole story: if nothing else it requires a phenomenological parameter (the crevasse water depth) to treat calving events, which may in effect be standing in for entirely different physics, such as brittle failure (Krug et al., 2015). It may be important to consider a threshold stress for damage initiations, and mechanisms of crevasse healing (Albrecht and Levermann, 2014).

~~Fig. 8, we see the retreat of grounding line in the fixed damage model (IceF1r) is much slower than in the model with evolving damage (IceD1r) and the model without damage (Ice1r), because the grounding line retreats inland where viscosity is much higher due to lower damage compared to IceD1r and smaller A compared to Ice1r. The comparison implies that the viscosity around the grounding line has essential control on ice flux.~~

6 Conclusions

We ~~developed~~added a continuum damage mechanics model component ~~of~~to the BISICLES ice sheet model. The model computes the evolution of a vertically integrated damage field by generating local damage ~~according to a modification to Glen's flow law~~, and transporting it downstream with the ice flow field. Although the modification to Glen's flow law is based upon the crevasse opening formulae of Nye (1957), Benn et al. (2007) and Nick et al. (2010) it can be adapted to any relationship between local stretching stress and damage.

The model was tested by carrying out the MISMIP+ (Asay-Davis et al 2016) experiments with and without the damage model. Simply introducing the damage calculation results in a much weaker ice shelf given the same flow law parameter (A), so that the grounding line retreats. However, realistic simulations tend to tune A to match observations, for example by solving an inverse problem to match velocities. We ~~were able to~~could produce similar steady states, defined primarily by grounding line position, for the two models by choosing a value of A around ten times ~~larger~~lower for the flow model with damage than for the model without. Once we had done so, the response of the ice stream to ablation of the ice shelf was ~~rather~~ similar in both cases, with the damage model resulting in slightly lower rates of retreat, especially when the ablation was limited to the downstream portion of the ice stream. We explain this lower rate of retreat by noting that damage is far greater in the ice shelf, which must be compensated for by stiffer ice upstream to have the same steady state: once the ice shelf is removed, we are left with the dynamics of stiffer ice~~.~~, albeit mildly stiffer ice because of the evolution of damage around the grounding line. Put another way, if ice shelves are generally weaker, their loss is of lower consequence.

Although tuning A in a simplistic way may be a plausible alternative to damage modelling, initializing an ice shelf rheology to match observed velocities and then holding that rheology constant in time is not. In this case, although the shelf still provides less buttressing, once the grounding line begins to retreat we are left with the dynamics of not just mildly stiffer, but far stiffer, ice, so that retreat rates might be underestimated by an order of magnitude.

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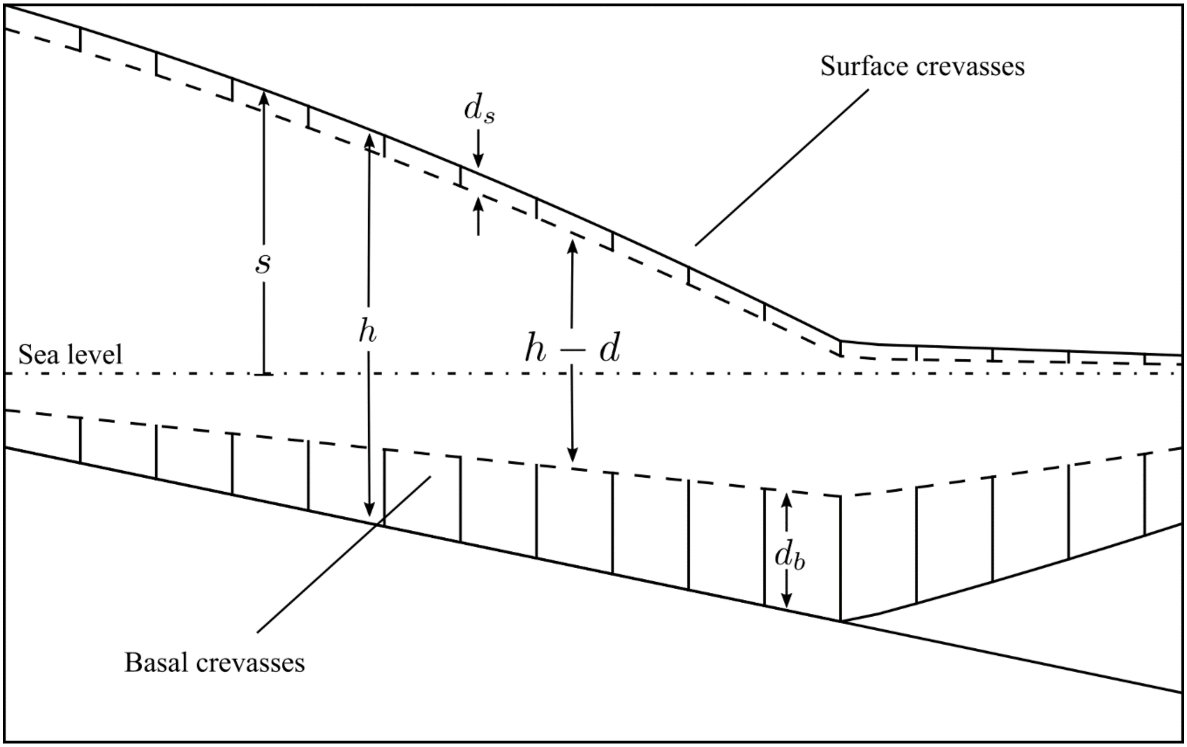
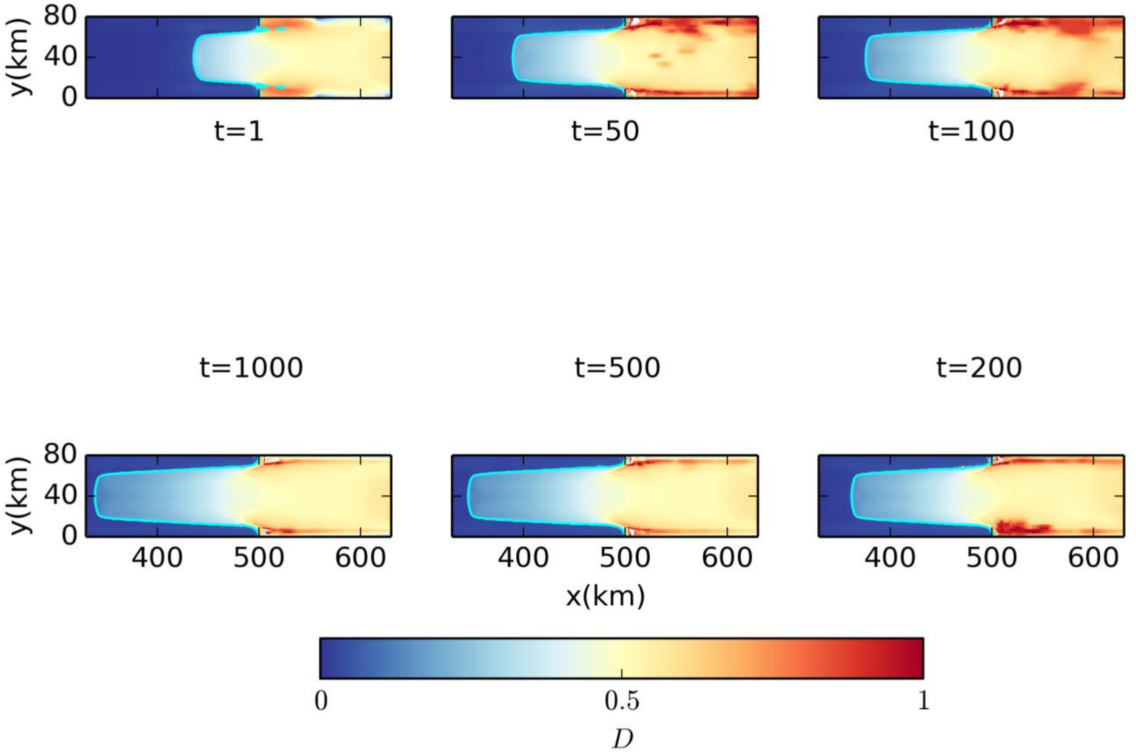


Figure 1: Sketch of the vertically integrated continuum damage model. The full thickness of the ice sheet (h) is divided into three layers. The upper and lower layers are assumed to be entirely riven by surface crevasses of depth d_s and basal crevasses of depth d_b, while the layer between them (with thickness $h-d=h-(d_s+d_b)$) is considered intact. The crevassed layers offer no resistance to horizontal longitudinal and lateral shearing, but behave in the same way as undamaged ice with regard to isotropic compression and vertical shear.



80
40
0
y(km)
t=1
t=50
t=100
t=1000
t=500
t=200
400
500
600
x(km)
0
0.5
1
D

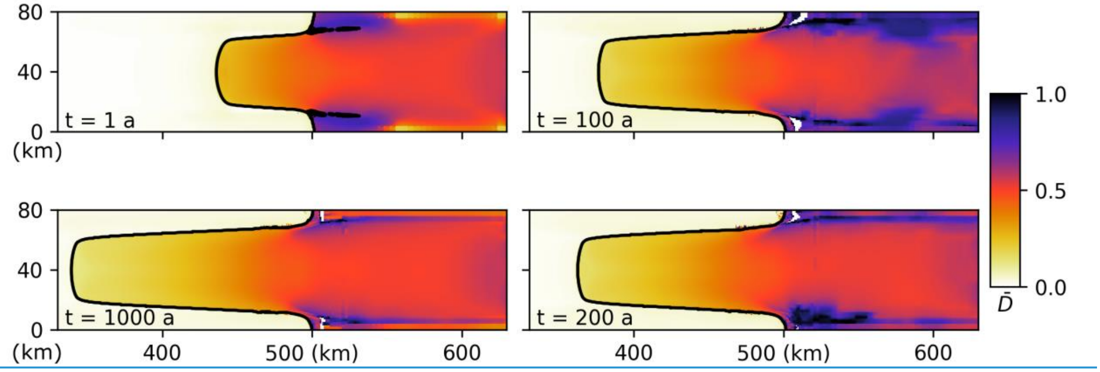
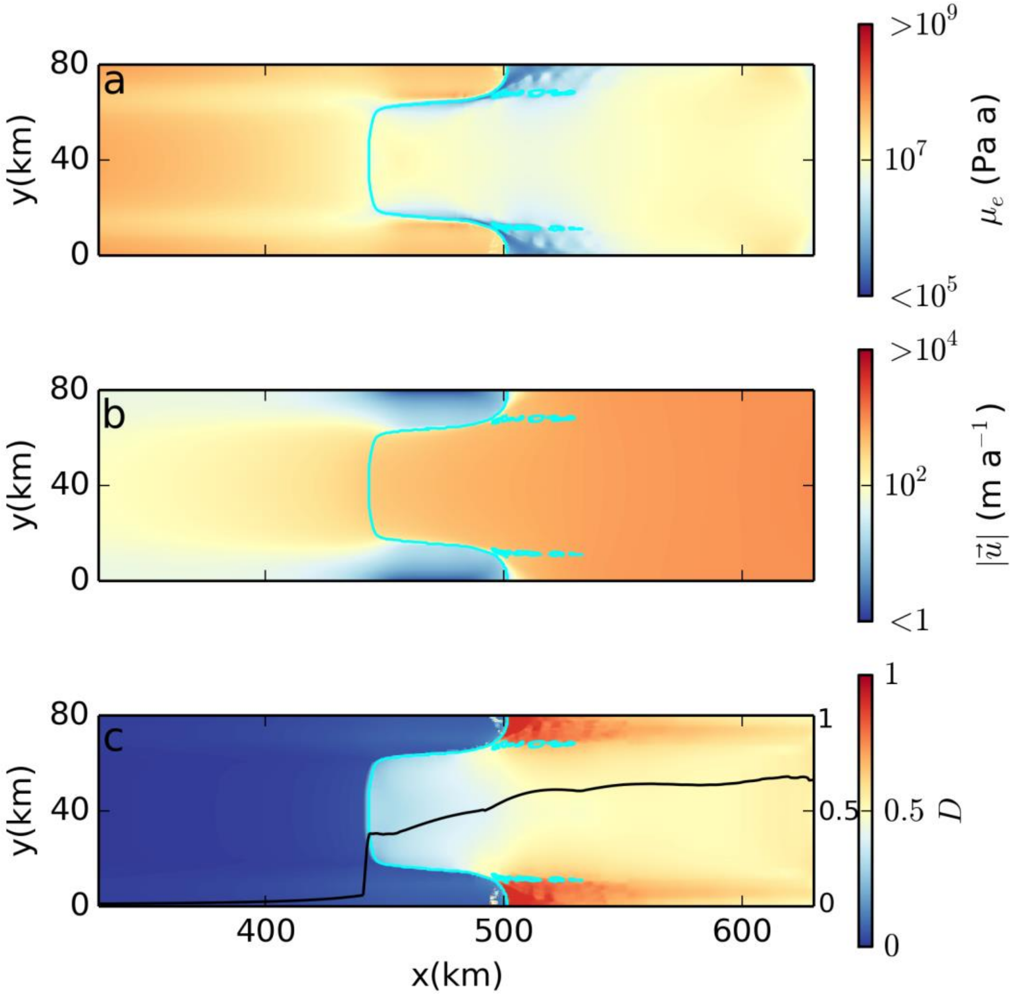


Figure ~~1~~2: Evolution of the damage field $\cancel{D}\bar{D}$ and the grounding line in IceD, starting from a steady state with $\cancel{D =}$ $\bar{D} = 0$. The immediate result of 'switching on' the damage model is the creation of a heavily damaged ice shelf (with $\cancel{D\sim}\bar{D}\sim 0.5$). Over time, this weaker ice shelf results in the grounding line (solid black contour) retreating over more than 100 km. At the same time spots of intense damage ($\cancel{D\sim}\bar{D}\sim 0.9$) generated where the grounding line crosses the channel walls around x = 500 km and y = {10, 70} km are transported downstream, to form strips that extend to the calving front.


a
b
c
y(km)
80
40
0
400
500
600
x(km)
$>10^9$
10^7
$<10^5$
μ_e (Pa a)
$>10^4$
10^2
<1
$|\vec{u}|$ (m a^{-1})
1
0.5
0
D

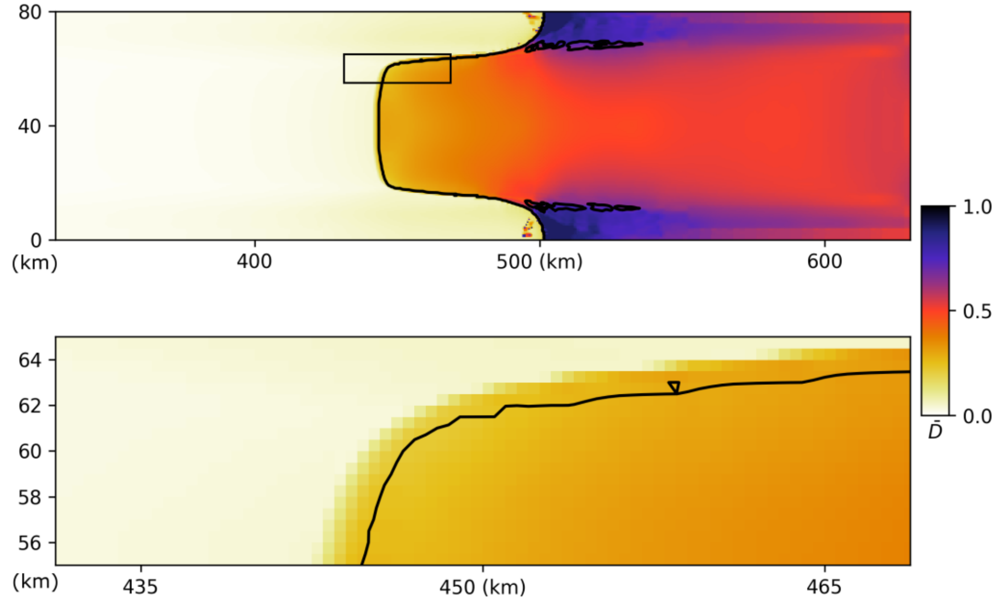
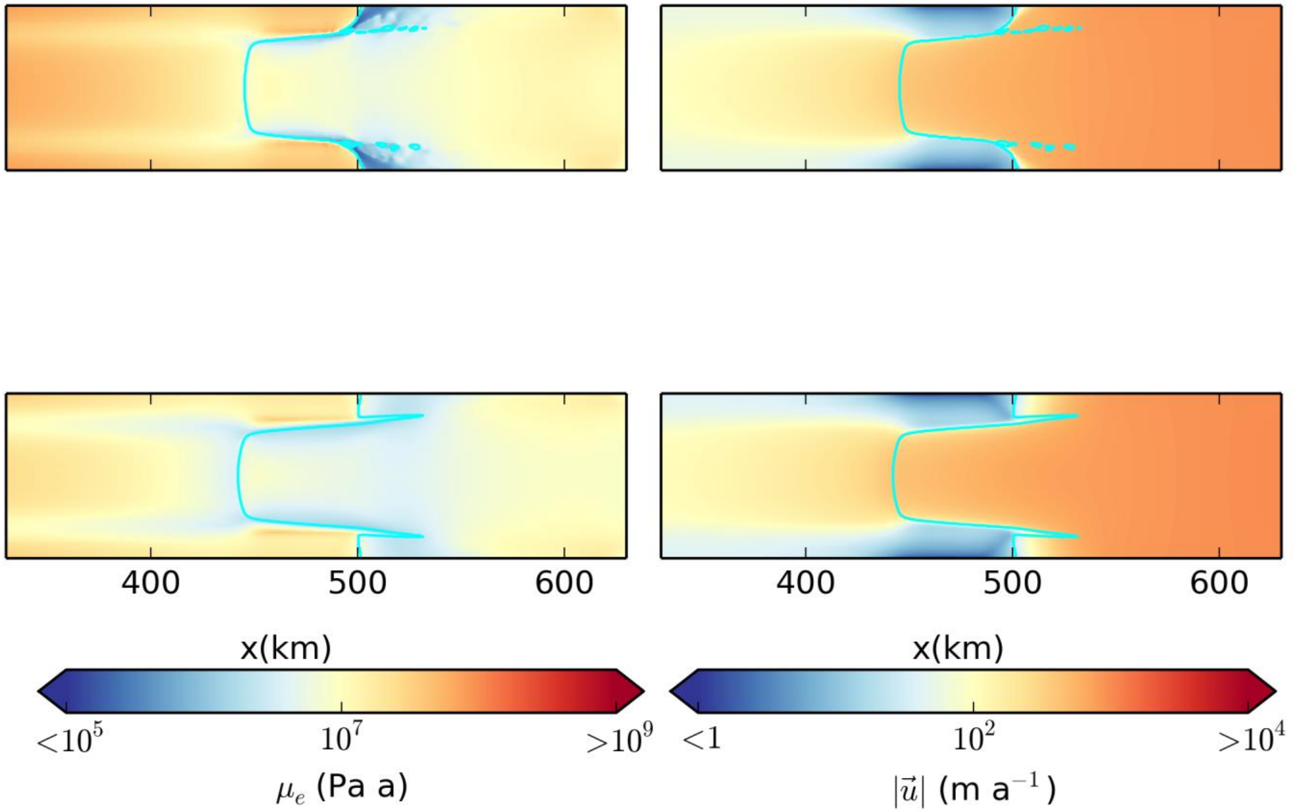


Figure ~~2: Initial variables~~3: Vertically averaged damage field $\bar{D}$ in experiment IceD0. ~~MuCoef (a), ice velocity (b), damage distribution (c), and~~ A steady state with its grounding line ~~location (shown in cyan on all panels). The~~ (solid black ~~curve in the bottom subfigure is the damage value in~~contour) crossing the centre of the channel ~~(The scale of *D* is to the right).~~around $x = 450$ km was found by decreasing the spatially uniform rate factor to $A' = 1.5 \times 10^{-18}\,\mathrm{Pa^{-3}\,a^{-1}}$. The resulting state is then close to the steady state without the damage model (i.e. with $\bar{D} = 0$) and $A = 2.0 \times 10^{-17}\,\mathrm{Pa^{-3}\,a^{-1}}$. The damage field grows from $\bar{D} \approx 0$ through $\bar{D} \sim 1/3$ around the grounding line to $\bar{D} \sim 1/2$ in the lateral center of the shelf and $\bar{D} \sim 1$ close to the domain boundaries. The lower panel plots a magnified portion of the upper panel, and shows damage increasing in the few kilometres upstream from the grounding line.



400
500
600
x(km)
$<10^5$
10^7
$>10^9$
μ_e (Pa a)
400
500
600
x(km)
<1
10^2
$>10^4$
$|\vec{u}|$ (m a^{-1})

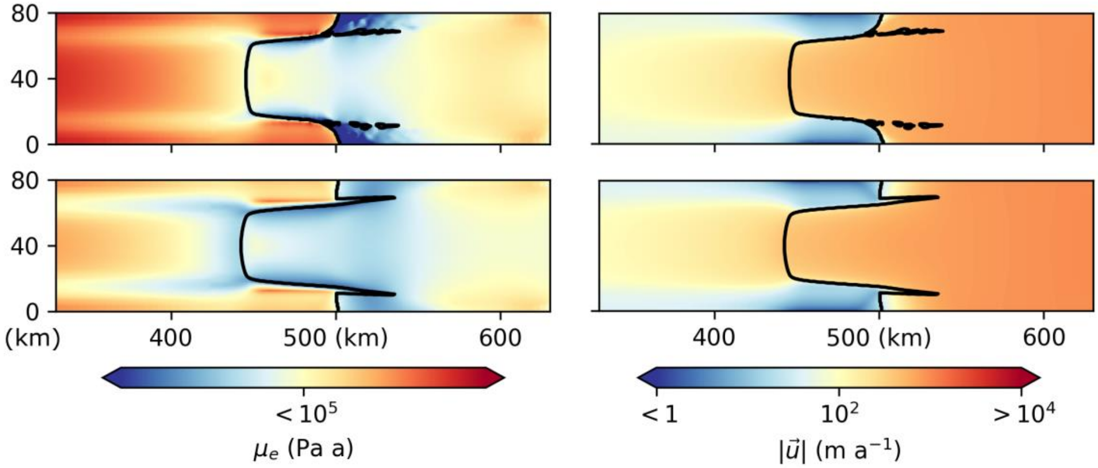
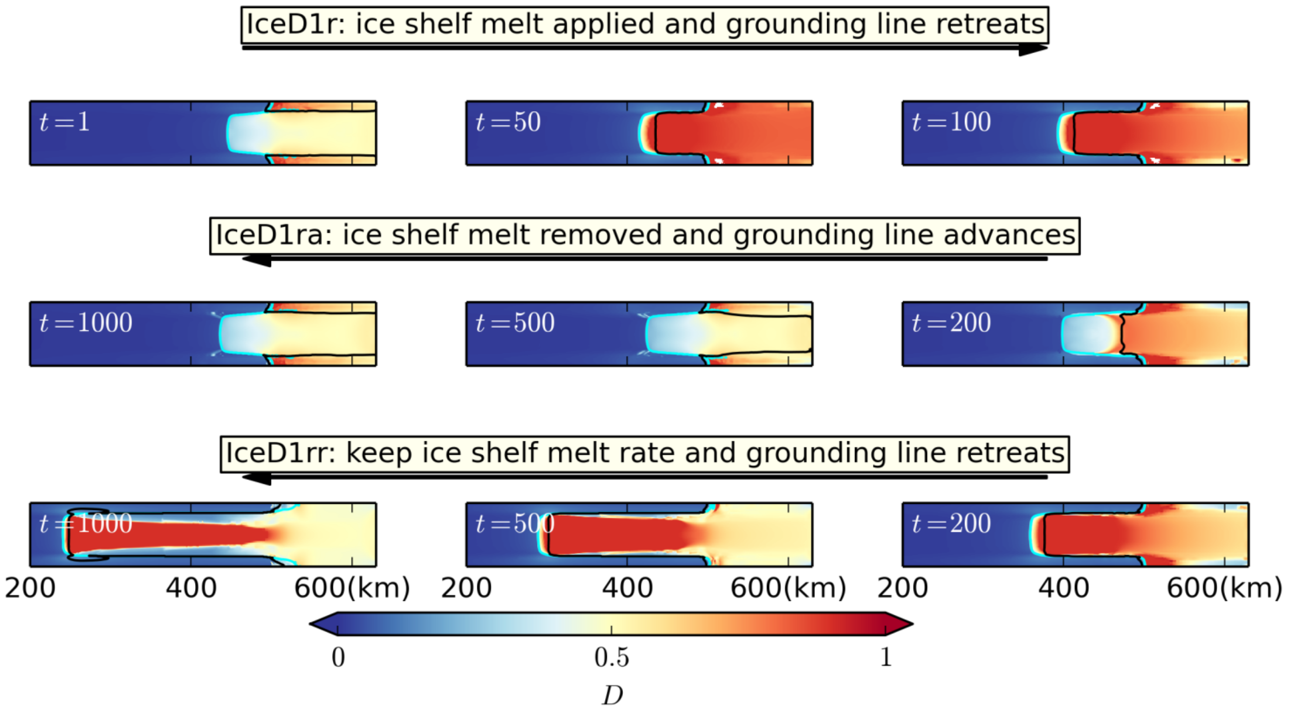


Figure ~~3~~4: Comparison of the ~~Ice0 (upper panel) and~~ final IceD0 (upper panel) and Ice0 (lower panel) patterns of ~~MuCoef and velocity at the end of the 1000 year simulations~~vertically averaged effective viscosity (μ_e) and speed ($|\vec{u}|$). The impact of damage on effective viscosity varies both laterally and between grounded and floating ice, but the shape and position of the grounding line (solid black contour) of IceD0 is similar to that of Ice0, as is the velocity field. These are approximately steady state patterns as the grounded area and volumes change very little after 1000 years (see also Figs ~~6~~7 and ~~7).~~8).


IceD1r: ice shelf melt applied and grounding line retreats
$t=1$
$t=50$
$t=100$
IceD1ra: ice shelf melt removed and grounding line advances
$t=1000$
$t=500$
$t=200$
IceD1rr: keep ice shelf melt rate and grounding line retreats
$t=1000$
$t=500$
$t=200$
200
400
600(km)
0
0.5
1
D

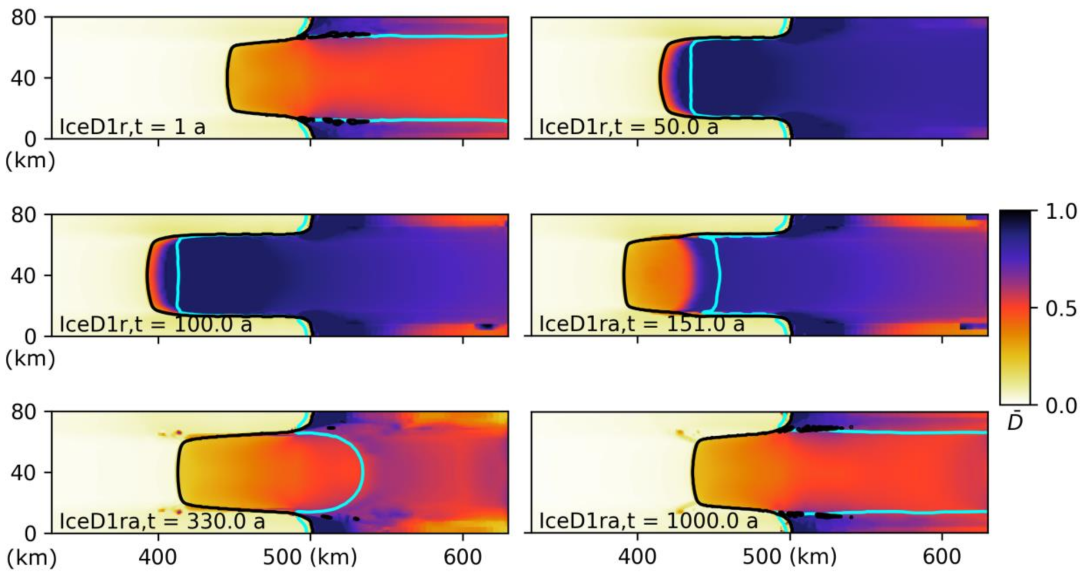
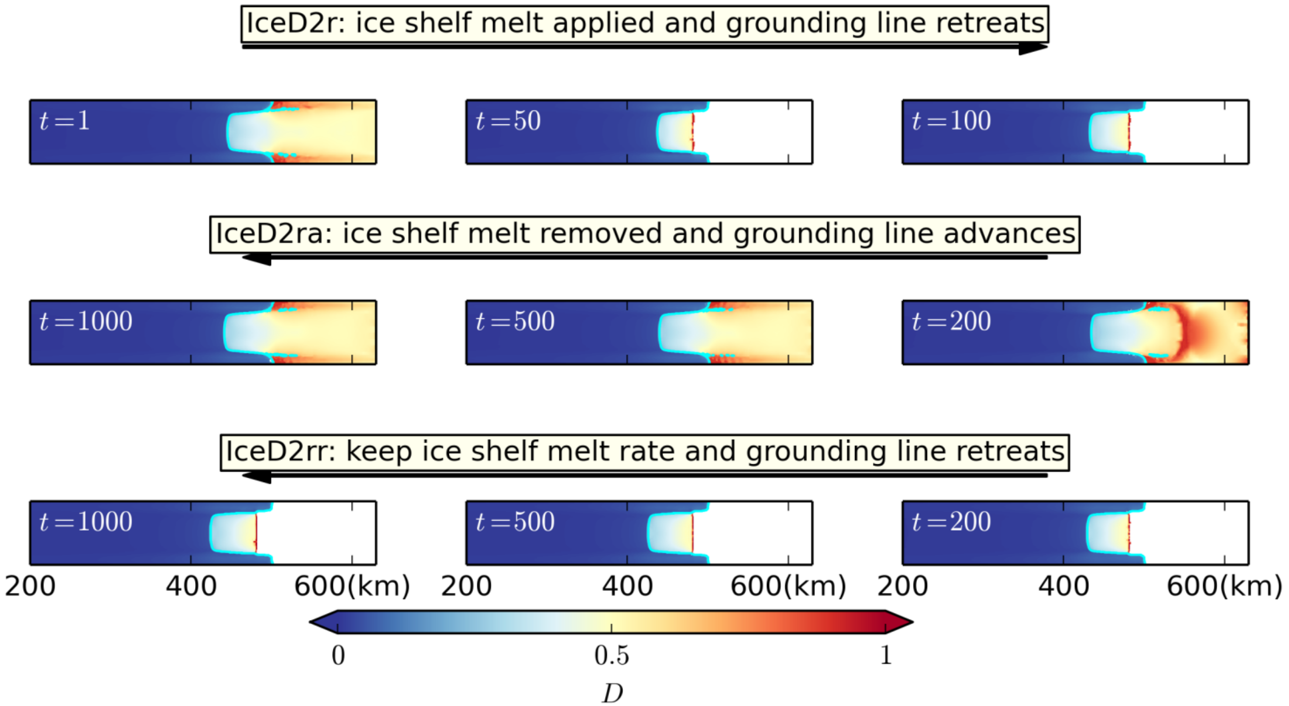
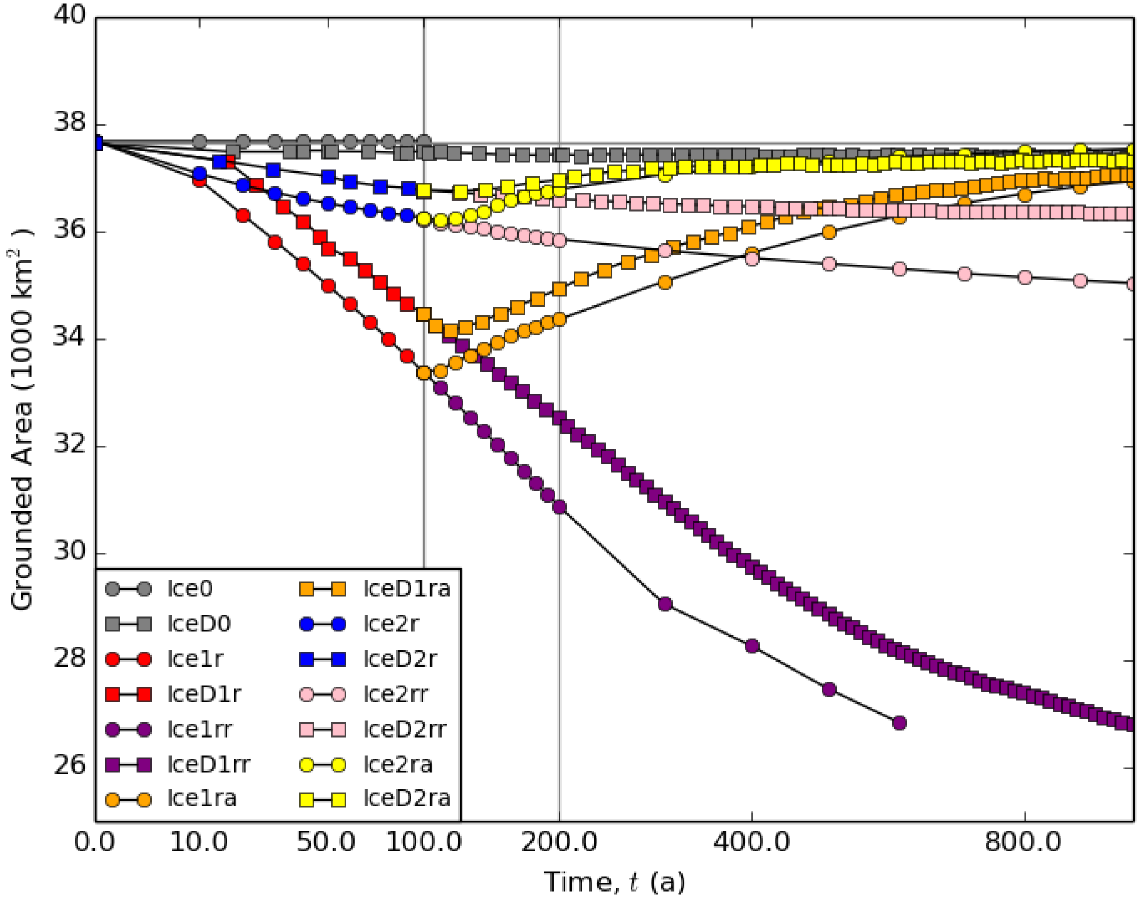

IceD1r,t = 1 a
IceD1r,t = 50.0 a
IceD1r,t = 100.0 a
IceD1ra,t = 151.0 a
IceD1ra,t = 330.0 a
IceD1ra,t = 1000.0 a
80
40
0
(km)
400
500 (km)
600
1.0
0.5
0.0
$\bar{D}$

Figure ~~4~~5: Comparison of the retreat and ~~recover speed~~recovery due to changes in ice shelf basal melt ~~forcing~~ (IceD1rr and IceD1ra) in MISMIP+. ~~We show h=200 m (black) contours. The~~ IceD1r sees the grounding line ~~is shown~~(solid black contour) in ~~cyan.~~



~~Figure 5: Compare~~ the centre of the channel retreat ~~and recover speed~~to x = 390 km over 100 years in response to strong ablation across the whole ice shelf. Parts of the ice shelf are thin ($h < 200$ m , outside cyan contour) so that a high value of D implies only a small reduction in buttressing from ~~calving (IceD2rr and IceD2ra) in MISMIP+.~~those regions. IceD1ra follows directly from IceD1r, setting the melt rate to zero so that the ice shelf thickness and the grounding line re-advances, recovering its original shape after around 1000 years.



40
38
36
34
32
30
28
26
Grounded Area (1000 km^2)
0.0
10.0
50.0
100.0
200.0
400.0
800.0
Time, t (a)
Ice0
IceD0
Ice1r
IceD1r
Ice1rr
IceD1rr
Ice1ra
IceD1ra
Ice2r
IceD2r
Ice2rr
IceD2rr
Ice2ra
IceD2ra

~~Figure 6~~

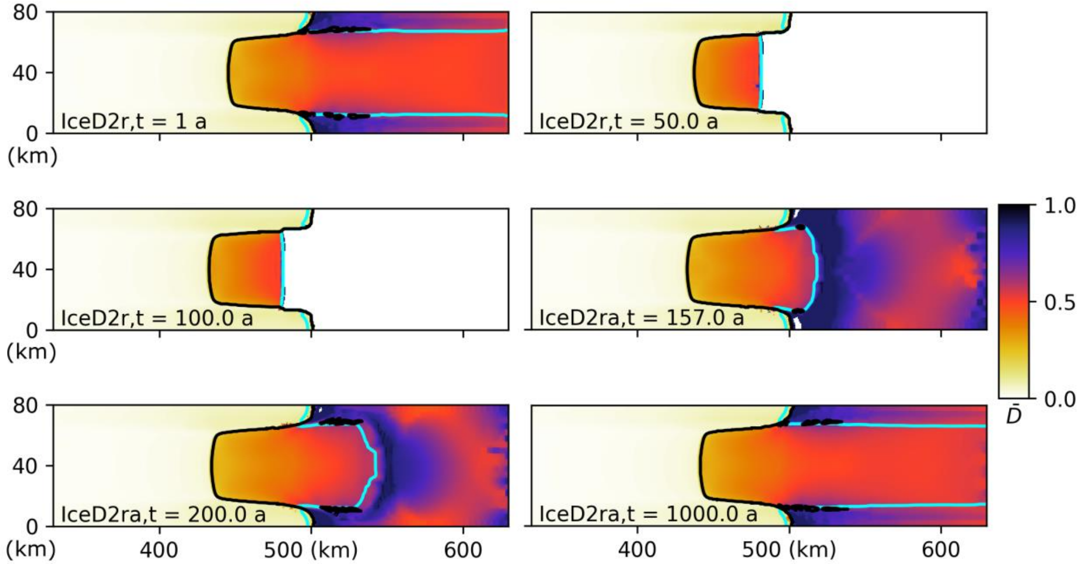


Figure 6: Comparison of the retreat and recovery due to changes in calving front position (experiments IceD2rr and IceD2ra in MISMIP+). Applying a melt rate of 100 m a^{-1} where $x > 480$ km results in a remnant ice shelf whose damage field resembles the initial state in their common area, but grows somewhat toward the front (IceD2r). Loss of buttressing leads to grounding line (solid black contour) retreat. Once the melt rate is set to zero (IceD2ra) the calving front advances, carrying a region of elevated damage with it across the domain edge. Thin, damaged ice downstream from the $h = 200$ m contour (cyan) forms from direct accumulation on the sea surface.

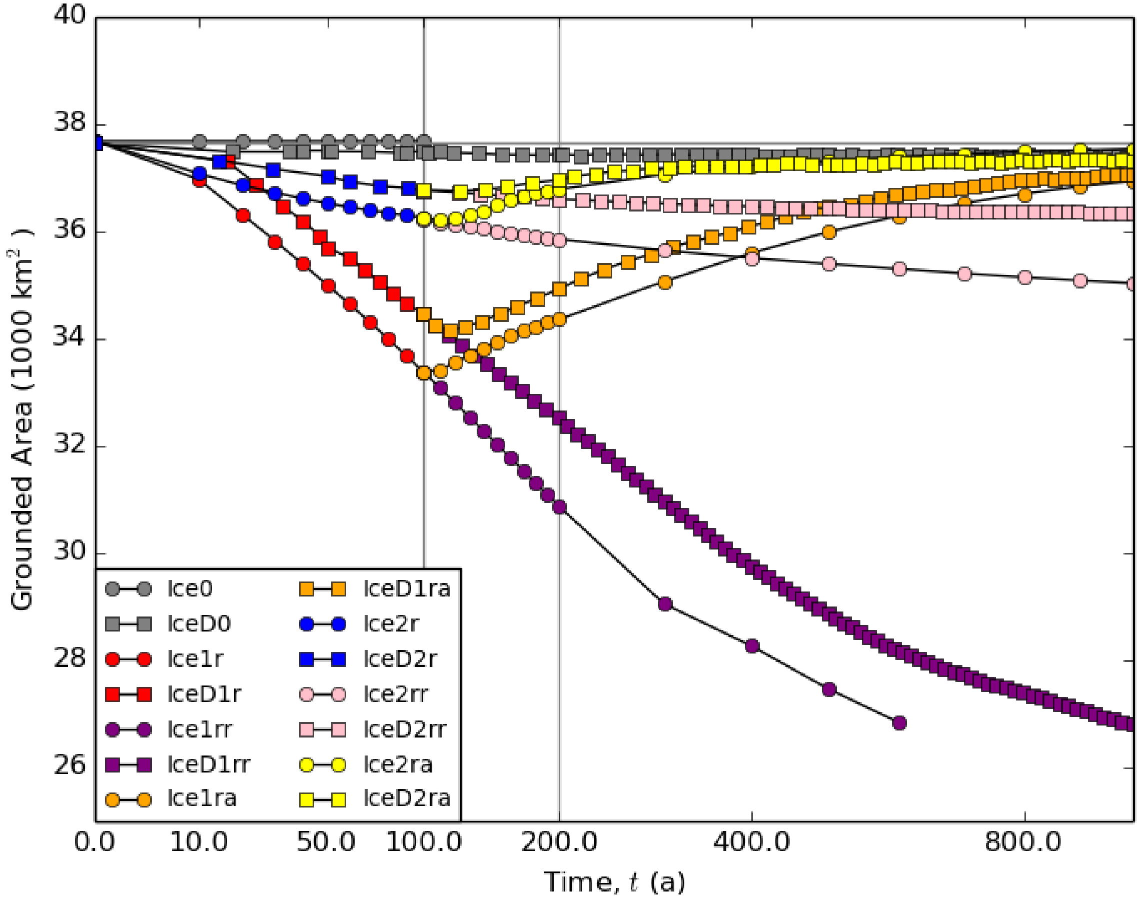
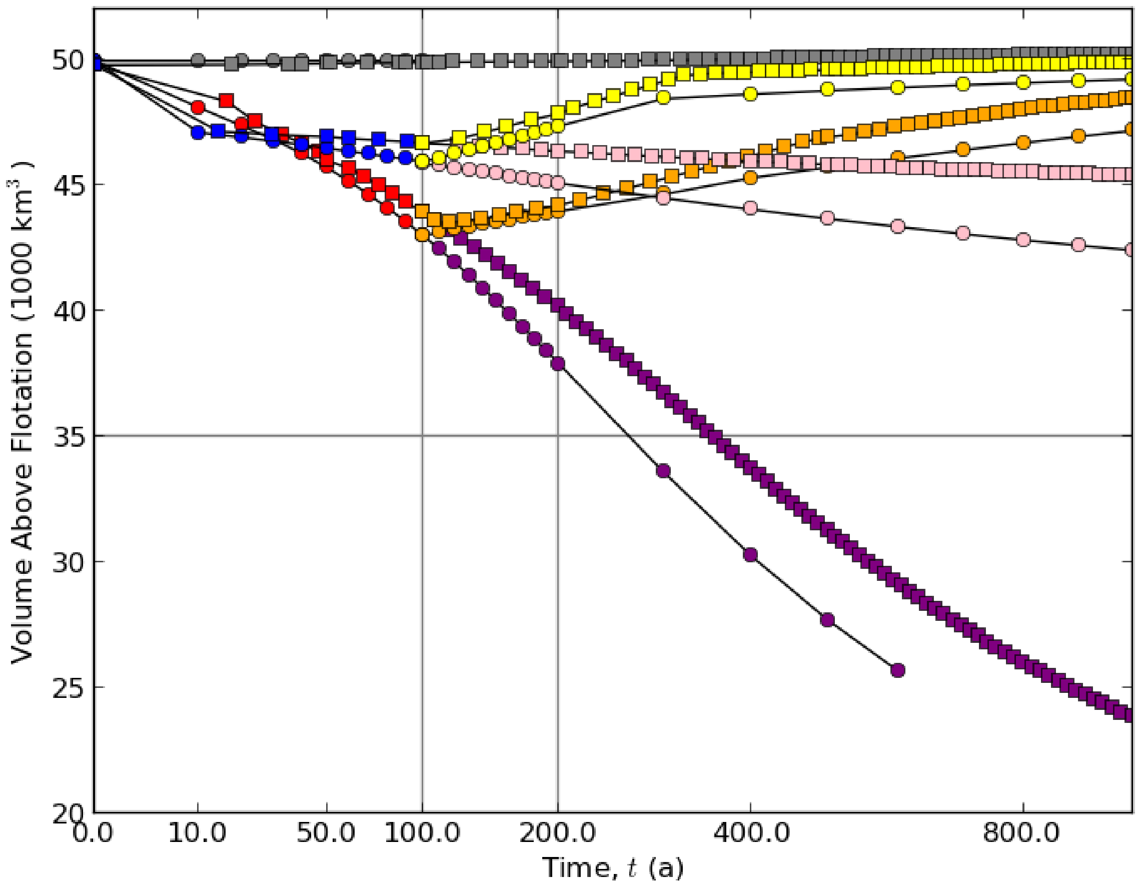
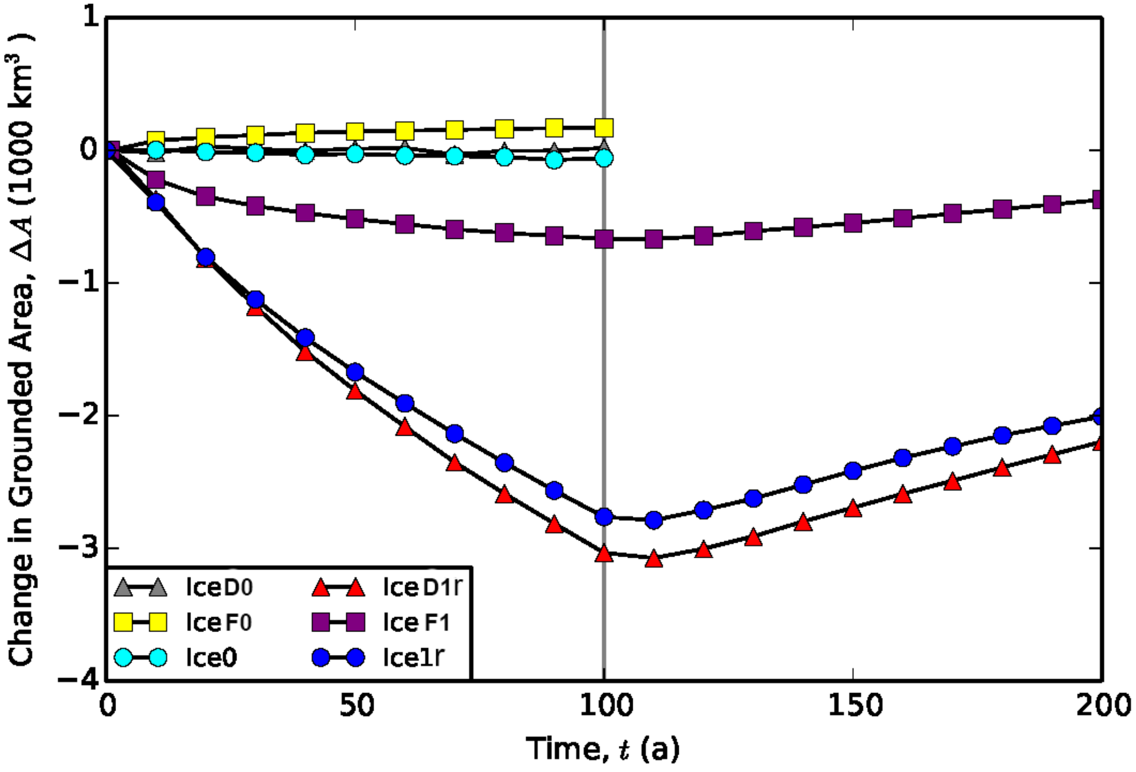


Figure 7: Grounded area against time for all of the simulations. Lines with circles show the results based on BISICLES without the damage model and those with squares show the results based on BISICLES with the damage model. Grey curves show the results of the control run Ice0, IceD0; red curves show the results of Ice1r, IceD1r; orange curves show the results of Ice1ra, IceD1ra; purple curves show the results of Ice1rr, IceD1rr; blue curves show the results of Ice2r, IceD2r; yellow curves show the results of Ice2ra; IceD2ra; pink curves show the results of Ice2rr, IceD2rr. BISICLES_D produces slightly less retreat than BISICLES.



~~**Figure 7: As that for Figure 6 but for volume above flotation.**~~


Change in Grounded Area, ΔA (1000 km³)
Time, t (a)
1
0
−1
−2
−3
−4
0
50
100
150
200
IceD0
IceD1r
IceF0
IceF1
Ice0
Ice1r

~~Figure 8:~~

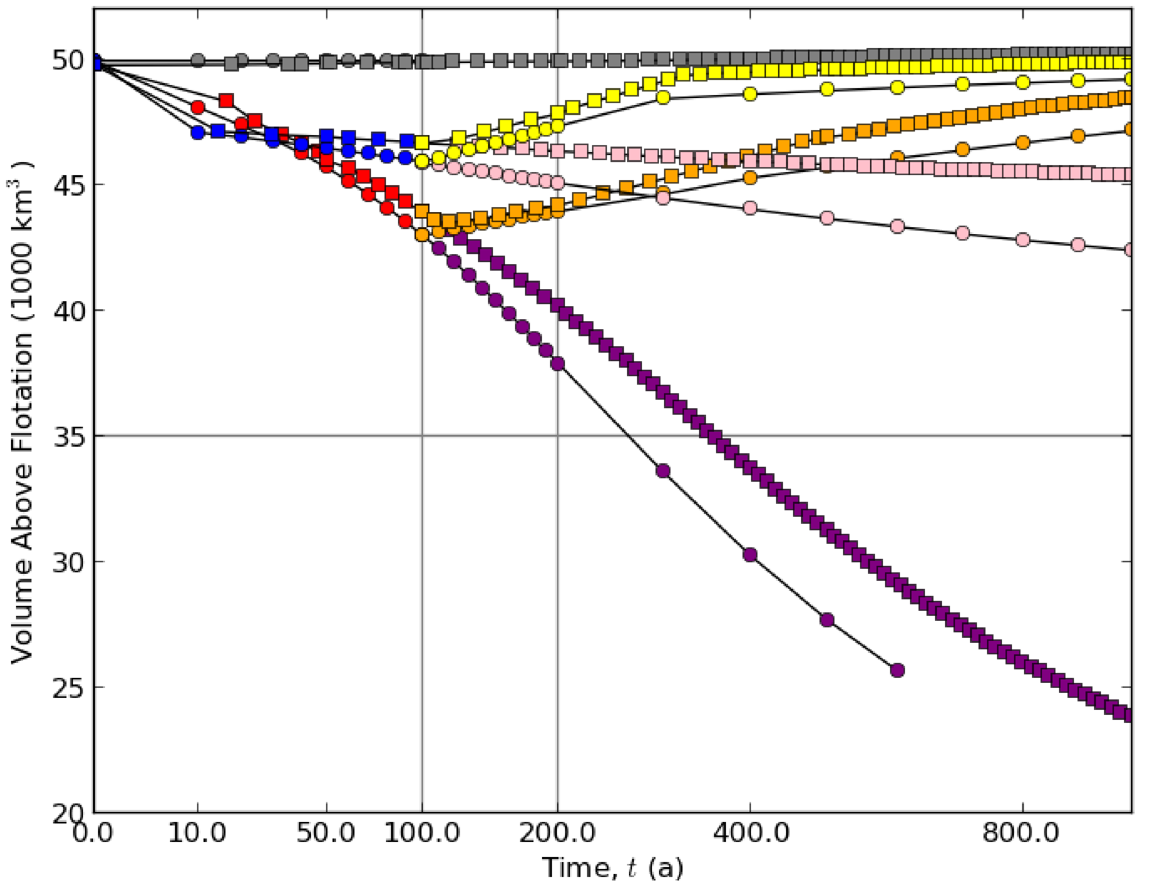


Figure 8: As for Fig. 7 but for volume above flotation (VAF). VAF follows a similar trend as the grounded area (Fig. 7: it decreases rapidly when forced by melt rates, and increases more slowly to nearly the initial state when ocean forcing disappears. In both models, the ice sheet is more sensitive to basal melt near the grounding line than to extreme high basal melt representing calving at the front. However, BISICLES_D produces slightly less retreat under both ocean forcing scenarios than BISICLES.

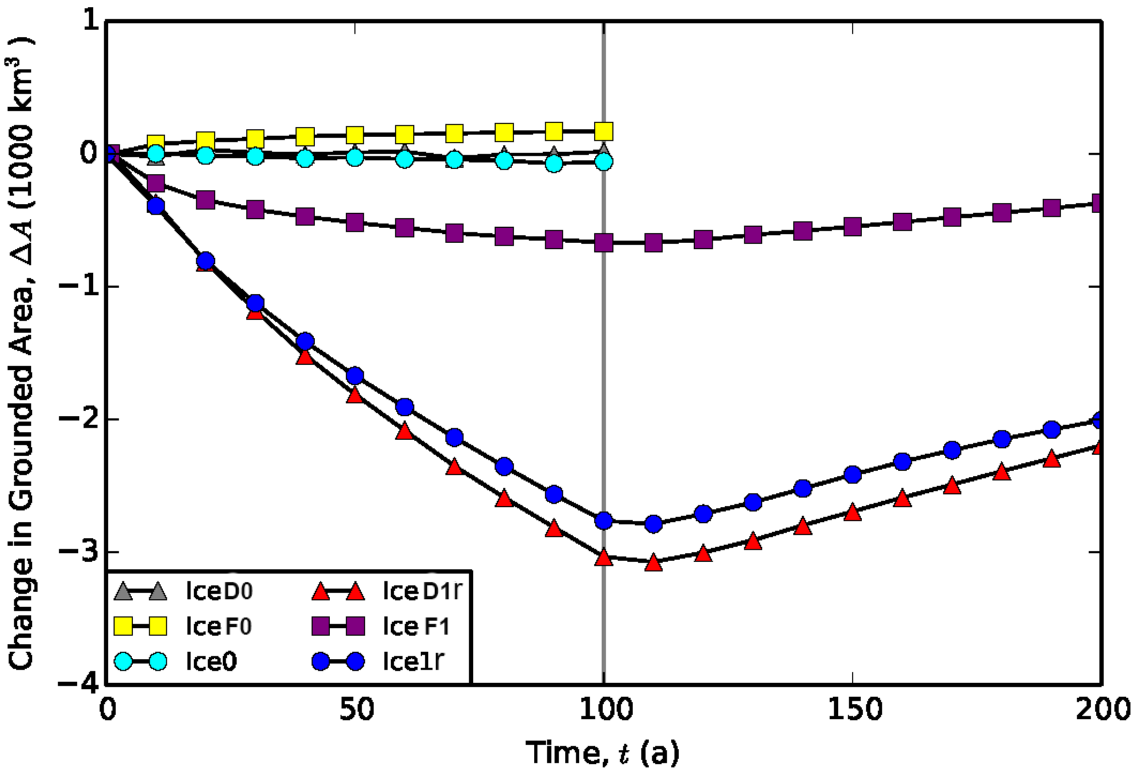


Figure 9: **Grounded area against time for the Ice0** (no forcing) **and Ice1** (basal melting) **experiments with no damage model (~~Ice0, Ice1r~~BISICLES) and the damage model** (BISICLES_D) **of Sect. 2.2 (IceD0, IceD1r) compared to a simulation where the damage field does not evolve with time but is taken from the initial state of IceD0 (IceF0, IceF1).** In the no forcing experiments, the model with fixed damage (IceF0) produces a near-steady state close to those of Ice0 and IceD0. In the forcing experiments, the retreat of the grounding line in the fixed damage model (IceF1r) is much slower than in the model with evolving damage (IceD1r) and the model without damage and larger A (Ice1r).

Experiment	Model	Viscous Parameter	Ocean forcing (1-100 year)	Ocean forcing (100-1000 year)
Ice0	BISICLES	A	0	0
Ice1ra	BISICLES	A	Melting	0
Ice1rr	BISICLES	A	Melting	Melting
Ice2ra	BISICLES	A	“Calving”	0
Ice2rr	BISICLES	A	“Calving”	“Calving”
IceD	BISICLES_D	A	0	0
IceD0	BISICLES_D	A'	0	0
IceD1ra	BISICLES_D	A'	Melting	0
IceD1rr	BISICLES_D	A'	Melting	Melting
IceD2ra	BISICLES_D	A'	“Calving”	0
IceD2rr	BISICLES_D	A'	“Calving”	“Calving”
IceF0	BISICLES-Fixeddamage	A'	0	0
IceF1	BISICLES-Fixeddamage	A'	Melting	0

Table 1: Summary of simulations carried out in the current study. The entries in the first column correspond to MISMIP+ experiment names (Asay-Davis et al., 2016).