Supplementing Information for “Influence of temperature fluctuations on equilibrium ice sheet volume”

Troels Bøgeholm Mikkelsen[1], Aslak Grinsted[1], and Peter Ditlevsen[1]

[1]Centre for Ice and Climate, Niels Bohr Institute, Juliane Maries Vej 30, DK-2100 Copenhagen Ø

Correspondence to: Troels Mikkelsen (bogeholm@nbi.ku.dk)

Contents

The *Oer03* model

The *Oer03* model is introduced in Oerlemans (2003) – some details are briefly summarized here. The model is “highly parameterized” and coupled to the surrounding climate by the altitude of the runoff line. Effectively the model consists of three steps: 1) describing the shape of the ice sheet, 2) analytically integrating the mass balance over the ice sheet and 3), numerically integrating the resulting expression for dR/dt where R is the radius of the ice sheet; the volume V is then uniquely determined from R.

Above the runoff line the accumulation is constant, below the balance gradient is constant; this is illustrated in Fig. (1) in the main article. The ice sheet is axially symmetric and rests on a sloping bed; furthermore ice is assumed to be a perfectly plastic material (Oerlemans, 2003).

The parameters we use are shown in Table 1. We have kept most parameters fixed as compared to Oerlemans (2003) but changed a total of 7 values to crudely approximate Greenland – note that we do not claim to be able to make accurate predictions of the GrIS even with this parametrisation. The temperature $\overline{T} = 5.8°\text{C}$ has been chosen so that no temperature anomaly (i.e. setting $T = 0$ in Eq. 2) gives a equilibrium volume of about 7m SLE, corresponding roughly to the GrIS (Church et al., 2013).

Name	Unit	Value	Notes
A_0	m ice yr^{-1}	1.0 †	Characteristic specific balance.
β	m ice yr^{-1}m^{-1}	0.005 †	Specific balance gradient.
c	m$^{1/2}$	2×10^6 †	Bed slope effect parameter.
C_R	m	5×10^5 †	e-folding radius for "desert effect" from large ice sheets; see Eq. 3
d_0	m	$= h_{E,0}$ *)	Undisturbed bed height at center of ice sheet.
h_{Eq}	m	See Eq. 2 *)	Height of equilibrium line.
$h_{E,0}$	m	1545 *)	Equilibrium line height at $T = 0$. Approximate 1990 - 2010 average (NOAA (2015), Fig 3.2a)
f	yr^{-1}	0.5 $^+$	Bulk flow parameter related to ice discharge.
μ_0	m$^{1/2}$	8 †	Bed slope effect parameter.
$\mu = \mu_0 + cs^2$			Equation 4 in Oerlemans (2003)
ρ_i	kg m^{-3}	900 $^+$	Density of ice.
ρ_w	kg m^{-3}	1025 $^+$	Density of sea water.
ρ_m	kg m^{-3}	3500 $^+$	Density of bedrock.
r_c	m	8×10^5 *)	Continental radius. Approximate width of Greenland.
r_{gr}	m	8×10^5 *)	Initial value – dynamical value in the model.
s	m/m	$d_0/r_c \approx 0.002$ *)	Bed slope.
$\bar{T}$	°C	5.8*)	Temperature offset.

Table 1. †: Suggested in Oerlemans (2003). $^+$: suggested in private communication with Hans Oerlemans. *): chosen by the present authors.

Steps 1 through 11 below describe the *Oer03* model setting used – these steps describe calculations performed at every time step that give an expression for

$$\frac{dR}{dt} = f(T,R); \tag{1}$$

dR/dt is then integrated using the Euler scheme with a time step of 1 year. We find that using a smaller time step size than this only produce negligible differences – see Figure 1 for an example.

1. We couple the ice sheet to the ambient temperature by introducing the following relationship between temperature and height of the equilibrium line (Oerlemans, 2008):

   $$h_{Eq} = h_{E,0} + (T - \bar{T}) \cdot 1000/6.5. \tag{2}$$

   As in the main article, Eq. 2 represents an increase of the equilibrium line altitude of approximately 154 m $°\mathrm{C}^{-1}$.

2. Equation 3 reflects that the accumulation rate will likely decrease for a large ice sheet (Eq. 20 in Oerlemans (2003)):

   $$A \leftarrow A_0 \mathrm{e}^{-R/R_C}. \tag{3}$$

3. Height of the runoff line (Eq. 15 in Oerlemans (2003)):

   $$h_R \leftarrow h_{Eq} + A/\beta. \tag{4}$$

4. Height of the bedrock where the ice sheet ends:

   $$h_E \leftarrow d_0 - sR. \tag{5}$$

5. Location where the runoff line and the ice sheet surface meet (Eq. 17 in Oerlemans (2003)):

   $$r_R \leftarrow R - (h_R - h_E)^2/\mu. \tag{6}$$

6. Check if the ice sheet extends into the sea, i.e. if $R > r_c$. If so, use Eq. (7) in Oerlemans (2003) to define the radial coordinate of the grounding line r_{gr}:

   – **if** $R > rc$:

   $$r_{gr} = R - h_E^2/\mu. \tag{7}$$

7. If the radial coordinate of the runoff line is larger than of the grounding line, set runoff coordinate to grounding coordinate:

   – **if** $r_R > r_{gr}$:

$$r_R \leftarrow r_{gr}. \tag{8}$$

8. If the height of the runoff line is smaller than the height of ice sheet termination, set radial coordinate of the runoff line to radius of the ice sheet:

   – **if** $h_R < h_E$

$$r_R \leftarrow R. \tag{9}$$

9. If $R < r_c$ the ice sheet is continental. Equations 10 and 11 are included for numerical reasons.

   – **if** $R \le r_c$

     – **if** $r_R < 0$

$$r_R \leftarrow 0 \tag{10}$$

     – **if** $R < 1$

$$R \leftarrow 1 \tag{11}$$

     – Calculate total $dV/dt = B_{tot}$:

$$\begin{aligned} B_{tot} \quad & \leftarrow \pi A R^2 && (12)\\ & -\pi\beta\left(h_R - h_E\right)\left(R^2 - r_R^2\right) && (13)\\ & +\frac{4\pi\beta\mu^{1/2}}{5}\left(R - r_R\right)^{5/2} && (14)\\ & -\frac{4\pi\beta\mu^{1/2}}{3}R\left(R - r_R\right)^{3/2}. && (15)\\ & && (16) \end{aligned}$$

10. If $R > r_c$ the the ice sheet extends into the sea:

   – **if** $R > r_c$

$$\begin{aligned} B_{tot} \quad & \leftarrow \pi A r_{gr}^2 && (17)\\ & -\pi\beta\left(h_R - h_E\right)\left(r_{gr}^2 - r_R^2\right) && (18)\\ & +\frac{4\pi\beta\mu^{1/2}}{5}\left(\left(R - r_R\right)^{5/2} - \left(R - r_{gr}\right)^{5/2}\right) && (19)\\ & -\frac{4\pi\beta\mu^{1/2}}{3}\left(R\left(R - r_R\right)^{3/2} - R\left(R - r_{gr}\right)^{3/2}\right) && (20)\\ & -2\pi r_{gr}\left(\frac{\rho_w}{\rho_i}\right) f\left(s r_{gr} - d_0\right)^2. && (21) \end{aligned}$$

   Here the last term corresponds to Eq. 19 in Oerlemans (2003) and is related to the flux across the grounding line.

11. Relationship between $\frac{dR}{dt}$ and B_{tot}, corresponding to Eq. 13 in Oerlemans (2003):

– **if** $R \leq r_c$

$$Q \leftarrow \pi \left(1 + \frac{\rho_i}{\rho_m - \rho_i}\right)\left(\frac{4}{3}\mu^{1/2}R^{3/2} - sR^2\right), \tag{22}$$

$$\frac{dR}{dt} \leftarrow B_{tot}/Q. \tag{23}$$

– **if** $R > r_c$

$$Q \leftarrow \pi \left(1 + \frac{\rho_i}{\rho_m - \rho_i}\right)\left(\frac{4}{3}\mu^{1/2}R^{3/2} - sR^2\right) \tag{24}$$

$$-2\frac{\rho_w}{\rho_m - \rho_i}\left(\pi sR^2 - d_0 R\right), \tag{25}$$

$$\frac{dR}{dt} \leftarrow B_{tot}/Q. \tag{26}$$

Integrating steps 1-11 yield a time series of the ice sheet radius. To convert to volume we use the following relations (Eqs. 9, 11 and 12 in Oerlemans (2003)); the volume of the continental part of the ice sheet:

$$V_{cont} = \frac{8\pi\mu^{1/2}}{15}R^{5/2} - \frac{1}{3}\pi sR^3. \tag{27}$$

In the case of the ice extending to the sea, the volume of the sea water replaced by ice:

$$V_{sea} = \pi\left(\frac{2}{3}s\left(R^3 - r_c^3\right) - d_0\left(R^2 - r_c^2\right).\right) \tag{28}$$

V_{sea} is set to zero if the ice does not extend to the sea and thus $R < r_c$. The total volume is given by:

$$V_{tot} = V_{cont}\left(1 + \frac{\rho_i}{\rho_m - \rho_i}\right) - \frac{\rho_w}{\rho_m - \rho_i}V_{sea}. \tag{29}$$

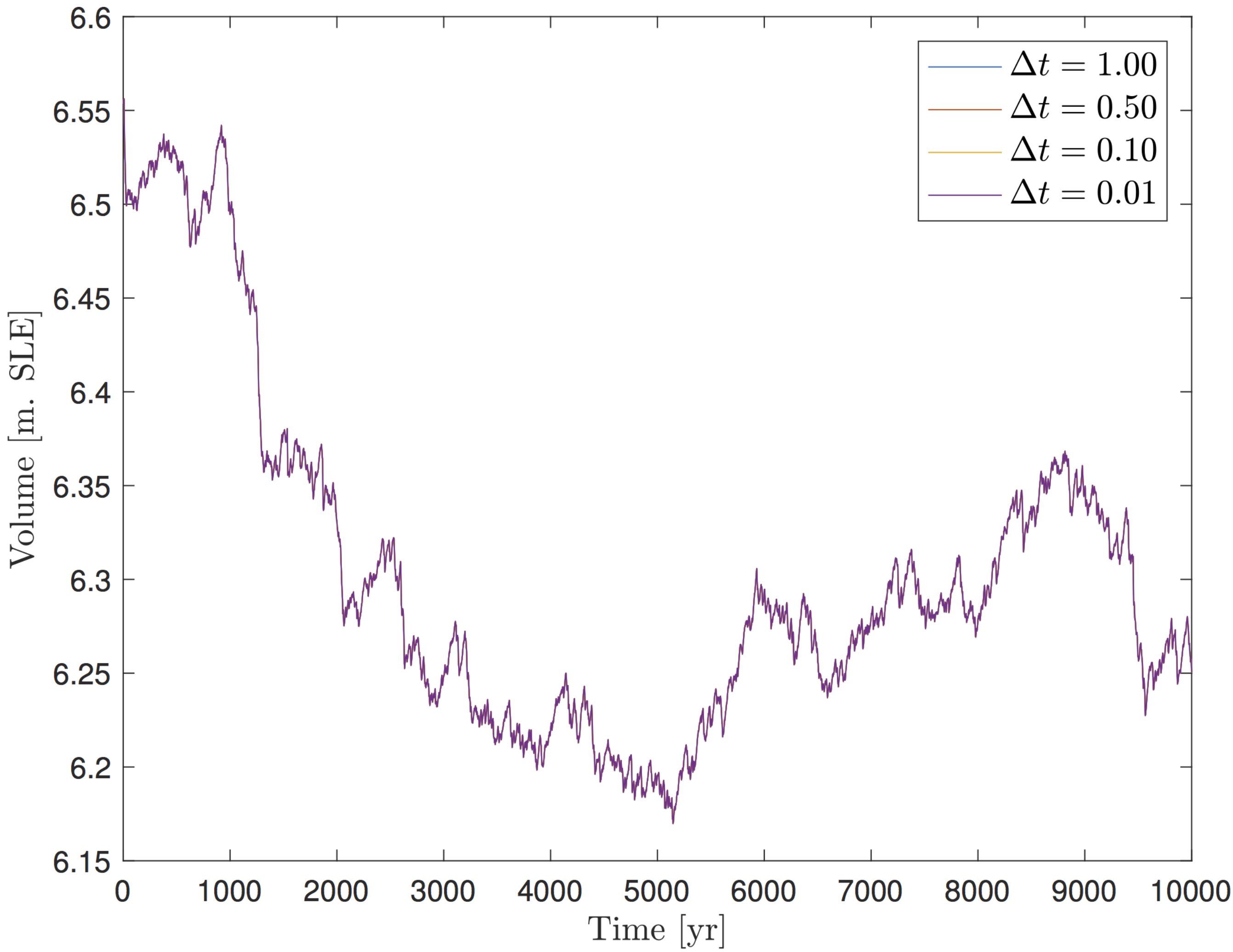

Figure 1. Varying the integration stepsize Δt from 1 year to 0.01 years for a simulation with $\overline{T} = 0$, such that the (random) fluctuating temperature T_t is the same for each whole year. A visual inspection confirms qualitatively that the graphs for varying Δt coincide and we do not further analyze the consequences of varying Δt.

Observed Fluctuations in Greenland temperature

Surface temperature anomalies were obtained from (KNMI). We use the "Twentieth Century Reanalysis V2c" from the years 1851 to 2011 in a box spanning 68°N to 80°N and 25°W to 60°W. The raw data consists of monthly means and is shown in Figure 2 as the blue curve.

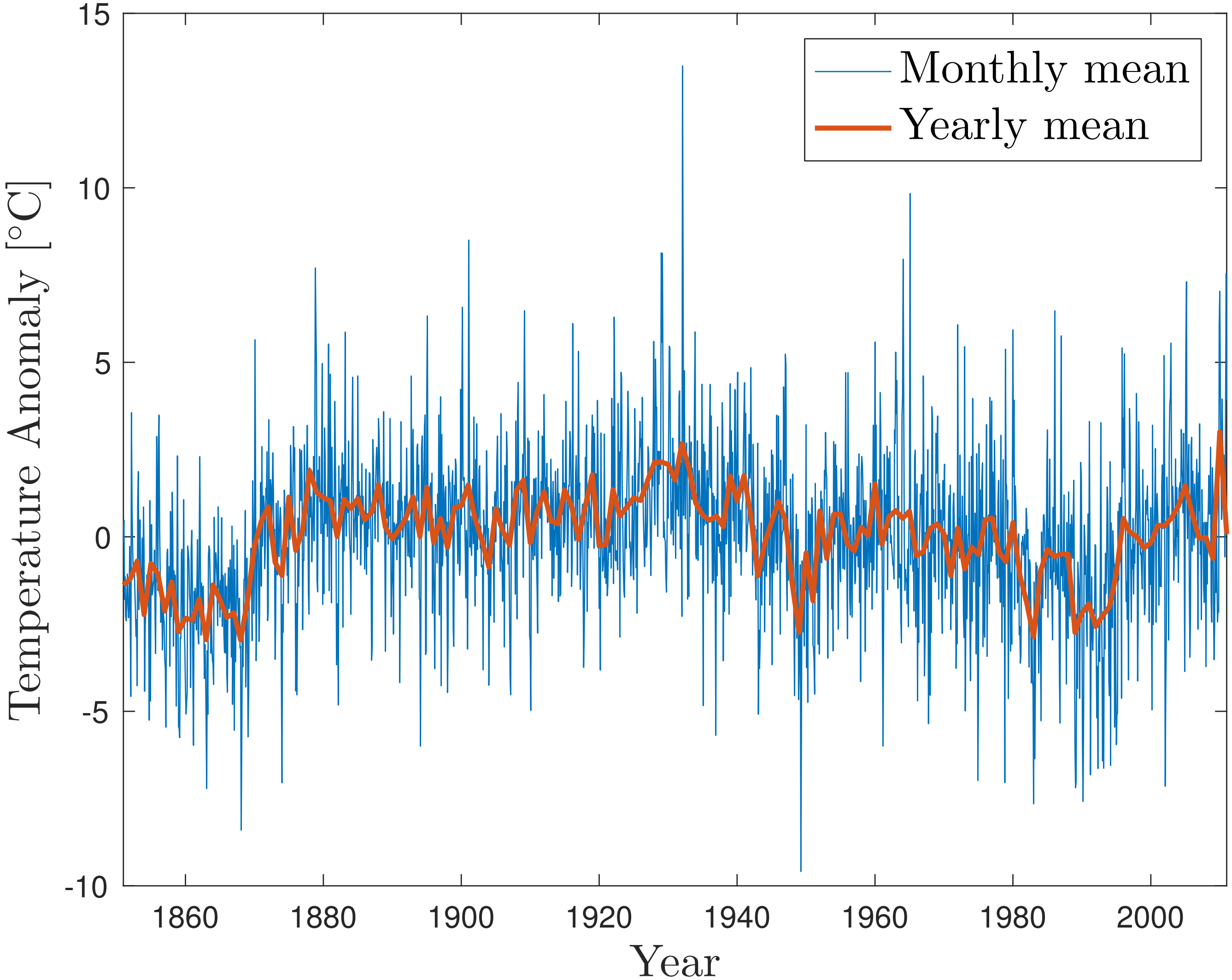

Figure 2. Reanalysis data showing monthly mean surface temperature anomaly (blue curve) over the area 68°N – 80°N, 25°W – 60°W covering a large part of Greenland. The red curve is the annual mean surface temperature anomaly.

We treat the temperature data as follows:

1. We calculate the yearly mean (the red curve in Figure 2),

2. To the yearly means we fit an autoregressive model of order 1 or an AR(1)-model,

3. The parameters from this model is used to generate artificial temperature time series $\{T_t\}$ that fluctuate in a way similar to the observed temperatures over Greenland.

An AR(1) model describing describing $\{T_t\}$ has the form

$$T_{t+1} = c + aT_t + \sigma_{AR} W_t. \tag{30}$$

where (c, a, σ_{AR}) are parameters to be determined and W_t is white noise with unit variance and zero mean. The parameters (a, σ_{AR}) are found using `MATLAB`'s `estimate()`. We find

$$(a, \sigma_{AR}^2) \qquad = (0.67, 0.85). \tag{31}$$

Evaluating Neglected Terms in Eq. 6)

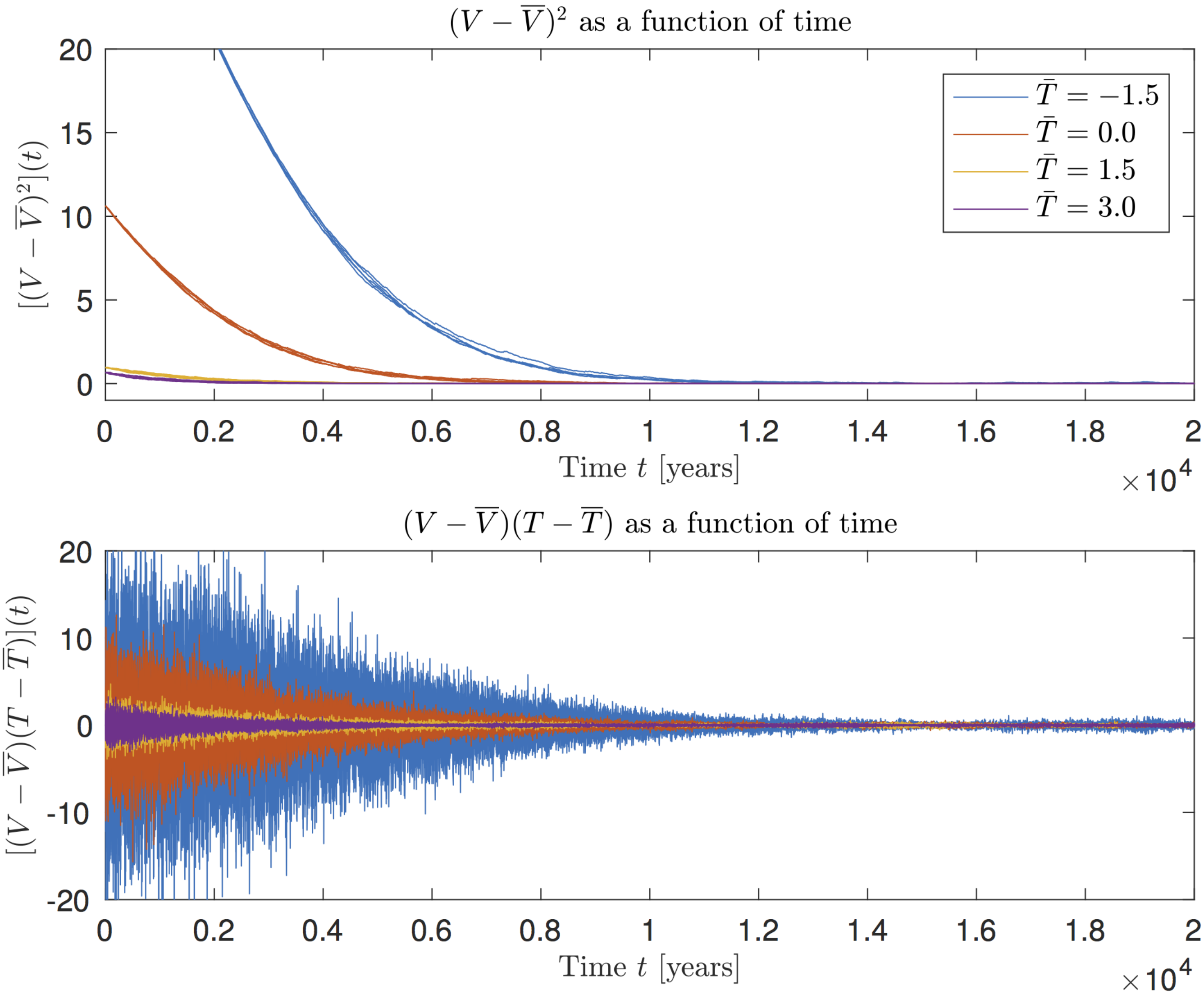

Figure 3. Evaluation of part of the terms dropped from Eq. 5 in the main article, for simulations with same parameters as in Figure 1. It is clear that $\langle (V_t - \overline{V})^2 \rangle$ and $\langle (T_t - \overline{T})(V_t - \overline{V}) \rangle$ tend to zero.

Analysis of Robinson et al. (2012)'s Data

We aim to estimate what effect a fluctuating temperature would have on the results quoted in Robinson et al. (2012) concerning the stability of the Greenland Ice Sheet (GrIS).

Methodology

The Surface Mass Balance (SMB) as a function of warming ($\bar{T}$) is extracted as follows:

- In Robinson et al. (2012) the warming is ramped for the first 100 years for numerical reasons. We wait until $t = 200$ years to extract SMB(T),
- Robinson et al. (2012) employ 9×11 values of two separate parameters deemed "equally likely" in their simulations,
- For each of these 99 simulations a 3rd degree polynomial is fitted to SMB(T) following Fettweis et al. (2013). We denote these fits $g_{ij}(T)$,
- We proceed as outlined in Section 4 (main article).
- Finally we calculate 95% credible intervals for each value of T. This is done by fitting a densities to the obtained ΔT and ΔSMB and calculating the interval containing 95% of the observations.

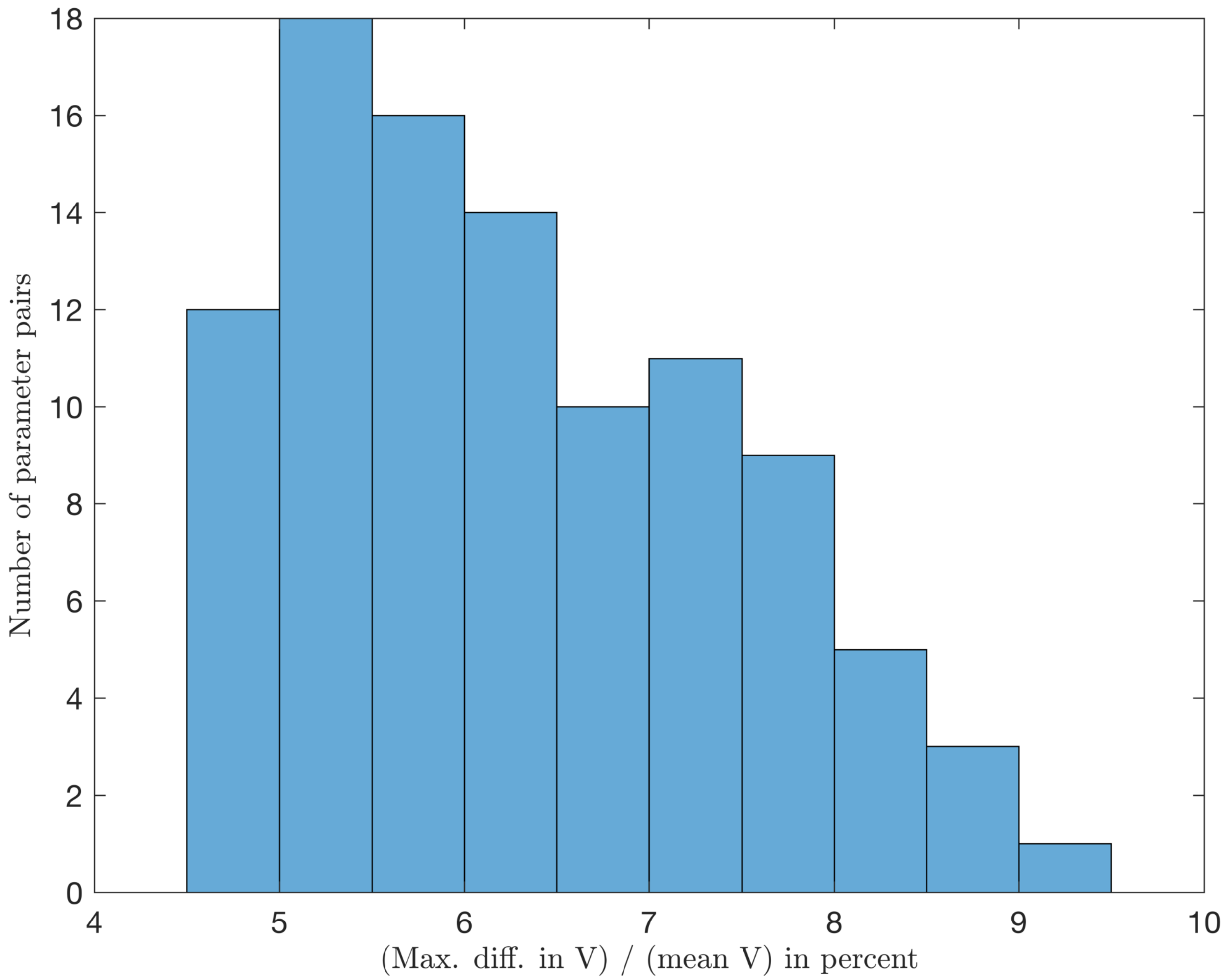

Figure 4. Histogram of the maximum difference in volume for different temperature anomalies divided by the mean volume $t = 200$ years in the data from Robinson et al. (2012), calculated for each parameter combination; in total there are 9×11 combinations of two separate parameters .

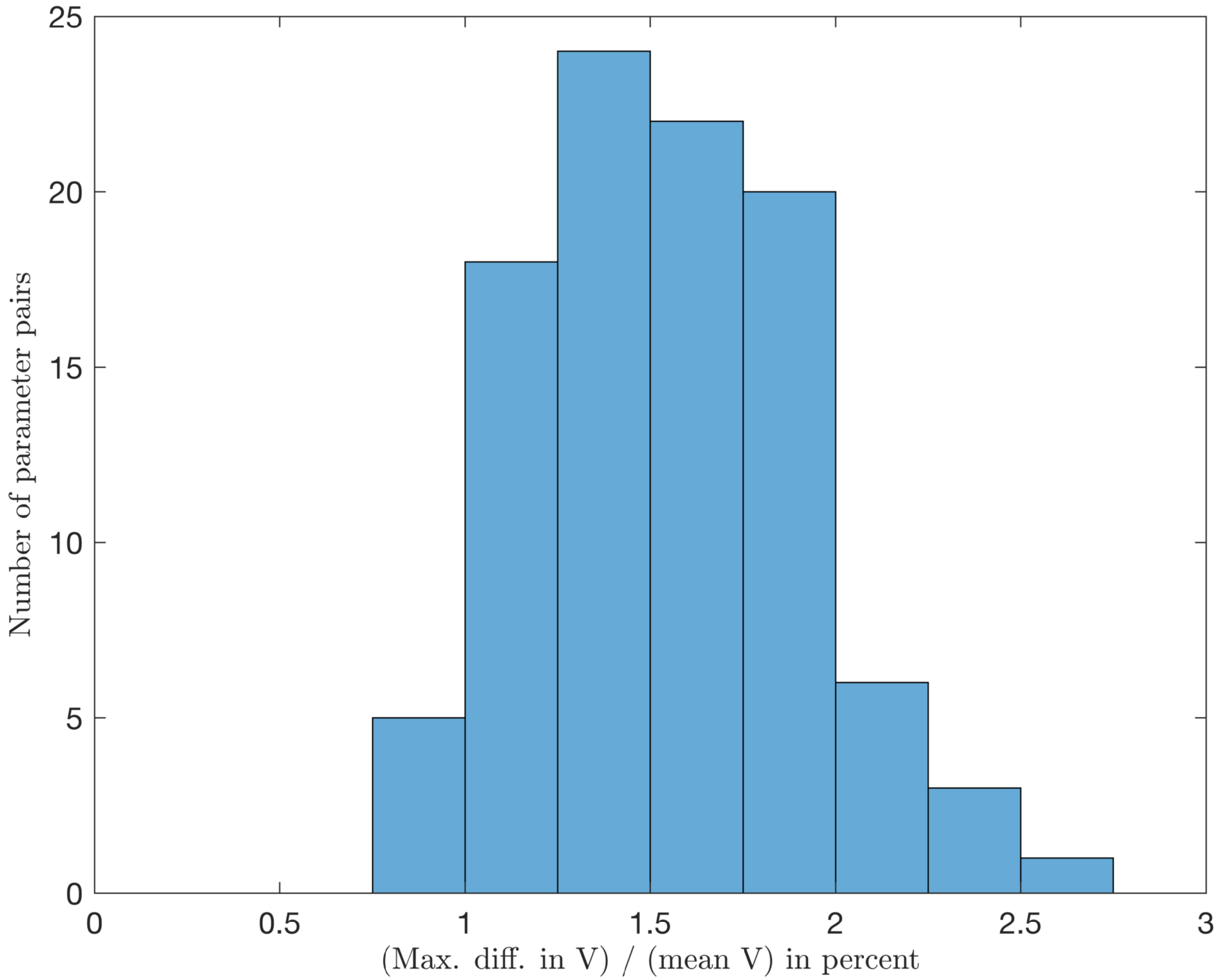

Figure 5. Same as Figure 4 but for a maximum warming of 4°C.

References

Church, J., Clark, P., Cazenave, A., Gregory, J., Jevrejeva, S., Levermann, A., Merrifield, M., Milne, G., Nerem, R., Nunn, P., Payne, A., Pfeffer, W., Stammer, D., and Unnikrishnan, A.: Sea Level Change, book section 13, pp. 1137–1216, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, doi:10.1017/CBO9781107415324.026, www.climatechange2013.org, 2013.

Fettweis, X., Franco, B., Tedesco, M., van Angelen, J. H., Lenaerts, J. T. M., van den Broeke, M. R., and Gallée, H.: Estimating Greenland ice sheet surface mass balance contribution to future sea level rise using the regional atmospheric climate model MAR, The Cryosphere, 7, 469–489, doi:10.5194/tc-7-469-2013, 2013.

(KNMI), R. N. M. I.: KNMI Climate Explorer, http://climexp.knmi.nl/, 2015.

NOAA: Greenland Ice Sheet, http://arctic.noaa.gov/Report-Card/Report-Card-2015/ArtMID/5037/ArticleID/219/Greenland-Ice-Sheet, 2015.

Oerlemans, J.: A quasi-analytical ice-sheet model for climate studies, Nonlinear Processes in Geophysics, 10, 441–452, doi:10.5194/npg-10-441-2003, 2003.

Oerlemans, J.: Minimal Glacier Models, Igitur, Utrecht Publishing & Archiving Services, Universiteitsbibliotheek Utrecht, second print in march 2011 (with some revisions) edn., 2008.

Robinson, A., Calov, R., and Ganopolski, A.: Multistability and critical thresholds of the Greenland ice sheet, Nature Climate Change, 2, 429–432, doi:10.1038/nclimate1449, 2012.