Response to Interactive Comments

Troels Bøgeholm Mikkelsen, Aslak Grinsted and Peter D. Ditlevsen

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1 Remark from the authors

Dear editor and reviewers,

Once again we are grateful for your comments and suggestions, and for your valuable time spent reading the manuscript a second time.

We believe that we have now replied to all your comments and hope that this version of the manuscript is clearer and more precise than the previous version. Our replies to your comments and the changes made in the manuscript follow below.

Yours sincerely,
Troels Mikkelsen, Aslak Grinsted and Peter Ditlevsen

2 Report #2 by Anonymous Referee #3

General comments

> This manuscript has improved much from previous version and the context is clearer with the new abstract and introduction. The distinction that needs to be made between the minimal model (that authors state does not apply to Greenland ice sheet) and the estimates based on the results from Robinson is still not always clear, see comments below, particularly in the abstract (1m sle vs 30 GT/y) and in the conclusion "considering minimal model of the Greenland Ice sheet". It should be well separated and made clear when each model is applicable and what conclusions can be drawn – and what they mean.

Comment #1: Thank you for pointing this out.
Change #1: We have stated more explicitly in the abstract when the results relate to the simple model or the GrIS simulations by Robinson et al. (2012).

> The reviewer Fettweis points out that the bias authors are pointing at is not of concern when the ice sheet models are forced with climate model output, this should also be stated in the paper so that readers will not be mislead to think that there is a bias in all large scale

simulations of ice sheets. I have a few minor comments that could improve the text further.

Comment #2: Thank you for pointing this out.
Change #2: We now state in the conclusion "Temperature fluctuations may also be explicitly accounted for by forcing the ice sheet model with climate model output that reproduces the magnitude of observed interannual temperature variability."

Specific comments

Page 1

In the abstract it is not clear when authors are referring to the simple model (lines 4-7) and when to estimates based on simulations of Greenland Ice sheet (line 9) this should be clearly stated. For example by starting sentence in line 4 (We find) – by something like : By applying a simple circular symmetry model it is shown that steady state volume is biased toward a larger size if interannual temperature fluctuations are not taken into account, this can be approximately 1 m sea level equivalent for that setting. The text is confusing as it is now. The 1m sle is referring to the simple model, but the 30 GT is for Greenland, right?

Comment #3: Again, thanks for pointing out this imprecision. As per the comment above we have changed the wording in the abstract.

Lines 4 and 7, suggest to replace "temporal" with "interannual"

Change #3: Changed 'temporal' to 'interannual'

Line 18 not clear what "full regional climate model" is, do you mean "high resolution"

Comment #4: The word "full" here was meant to distinguish the regional model MAR used in Fettweis et al. (2013) from the simple models in the studies mentioned previously in that sentence. We already use the word "simple" to describe the simple models, and the word "full" is likely used in a non-standard way here.
Change #4: We have removed the word "full".

Page 2

Line 1 add "by" before solid

Change #5: Fixed.

Line 2, is there a reference for this statement, or is this your concern? Then state that

Change #6: We have added a reference and edited that sentence.

Line 4, suggest to replace "response from" with "response to"

Change #7: Fixed.

> Line 5 – the sentence starting in line 5 needs editing, is this a result that you are presenting here or is there a reference you can use to support this statement?

Change #8: We have changed the sentence to "Using a simple ice sheet model we will show how, as a consequence of this nonlinearity, the average mass balance will be different when forcing the model with a variable climate compared to a constant average climate."

> Line 10, also here is a statement that needs supporting reference or clarification

Comment #5: This statement is supported by the two references given in the previous sentence.
Change #9: We have edited the text so that this is clearer.

> Lines 17-19 this paragraph is misplaced, maybe it can be put into the previous paragraph, as it stands it is in no context with the rest of the section.

Comment #6: Thank you for pointing this out.
Change #10: We have moved the paragraph further down in the text where temperature variability is discussed further.

> Line 20 if something is well known you should add some reference for the reader who is interested to learn more about this well known fact.

Comment #7: Thank you for asking for clarification about this. "That the SMB of an ice sheet model is nonlinear with respect to temperature" is supported by the 3 references in the following sentences in that paragraph. However, that does of course not guarantee that the fact is "well known".
Change #11: We have changed "is well known" to "has previously been investigated in several studies".

> Line 24, take the plural s off models

Comment #8: Outputs of several models are compared in Fettweis et al. (2013, Fig 6.), so we believe that "models" should be plural. However we did write GCM and RCM as singular in the same sentence.
Change #12: Added plural s to GCM and RCM.

> Line 26, missing what the bias correction is applied to

Change #13: Added "to surface temperature".

> Line 32 replace "an" with "a" long-term

Change #14: Fixed.

> Line 33 suggest to replace "differs" with "differ" and "dependent" with "depending"

Change #15: Agree, fixed.

Page 3

> Line 3 add s to "influence"

Change #16: Fixed.

> Line 5 add "be" in front of applied OR replace "applied" with "apply"

Change #17: Fixed.

> Line 6, something is missing, relationship between T and what?

Comment #9: A keen eye was required for spotting this.
Change #18: Changed to "relationship between the magnitude of temperature fluctuations and ice sheet volume"

> Line 30, think it would be clearer to replace "mass balance" with "change in volume"

Change #19: Fixed.

Page 4

> Figure caption - the "runoff line" is not explained and in this context it is not clear what the line is

Comment #10: Agree, this could have been written more clearly.
Change #20: Changed to "The runoff line h_r specifies the simplified climatic conditions, as the specific balance is constant above h_r (see also Supplementing Information, Eq. (4)), and the balance gradient is constant below h_r (Oerlemans, 2003)'

Page 5

> Line 13, suggest to replace "for the model presented in Section 3" with Oer03 to clarify

Change #21: Fixed.

Page 9

> Line 11, suggest to edit "For this value it is seen" – change to something like Figure 4 shows, or it can be seen on

Change #22: Changed to "it can be seen on"

> Line 15 "Our results indicate..." This statement needs more clarification Figure 4. Shows that for 3°C warming the temperature bias is 0.12°C, how does that translate to the 1.6°C threshold for GrIS ice loss?

Comment #11: As mentioned previously in the paragraph, if assuming RCP45 scenario the bias correction of 0.12 °C (0.10 °C – 0.18 °C) should be added to any constant warming threshold; this will shift the threshold estimate to colder temperatures.
Change #23: We have changed the text to clarify this, adding "Applying the bias correction above [. . .]"

> Line 16 ",but it reduces the window available to avoid passing this threshold" is a strange sentence, what do you mean here? That there is less temperature change needed to reach the threshold?

Change #24: Changed to "but it places additional constraints on the maximum temperature increase admissible to avoid passing this threshold"

> Line 18 Figure 4 shows the ΔSMB with units mm SLE yr-1, which here is translated to Gt/yr, suggest to use only one unit, or explain the assumption made to transfer from one to the other.

Change #25: Agree, we have added the details of the conversion to the supplementing information.

> Line 31 to Page 10 line 6 - This paragraph is a strange way to start the conclusion section, suggest to move this to the discussion section

Change #26: That paragraph has been moved to the discussion. Additionally, the following paragraph has been moved down to improve the flow of the text.

Page 11

> Line 5 this sentence is misleading and in contrast to the replies to previous comments, you state there that the minimal model, Oer03, is there to show how equation 6 works and that it is not to model Greenland Ice sheet. Here, however, you state that you have considered a minimal model of the Greenland Ice sheet, suggest to edit this sentence

Comment #12: Agree, this is an oversight on our part. Thank you for pointing this out.
Change #27: Removed reference to the Greenland ice sheet from that sentence.

> Line 10-11, check the reference there should not be a parenthesis around the year within the parenthesis

Change #28: Fixed.

> The supplement for the paper is not very comprehensive and would benefit from a little more text to explain better its context. The text is very minimal and in bullet point style and does not provide the information needed to support the main text. The length of the main paper is not such that it would make it impossible to add the information in the supplement to the main text, making the paper more comprehensive and readable. The other option would be to provide more context in the supplement.

Comment #13: Thank you for this comment. We have added more text to the supplement and hope that it is now comprehensive.
Change #29: Added text to the supplement.

Page 7 (Supplement)

> note that data is plural so line 3 should be ".. data consist of monthly means and are . . . "

Change #30: Fixed.

Page 8 (Supplement)

> line 2, suggest to replace "yearly" with "annual"

Change #31: Fixed.

> line 3, parameters. . . are used

Change #32: Fixed.

Page 9 (Supplement)

> see comment above the figure caption does not clarify what is going on here, more text would help putting the context clearer. Same for page 10

Comment #14: Agree, as per the above comment we provide more context in the supplement.

3 Report #3 by Gerard Roe

> I've reviewed the revewers' comments and the authors' response. The authors have made comprehensive revisions in response to reviewer's suggestions. I think the analysis is worth publishing. I would recommend that the authors add a discussion of other sources of stochastic variability, so they can put their results in context. With 50% of Greenland ablation coming through calving, stochasticity in calving and in ice-stream dynamics are likely a source of stochastic forcing that will also have an impact on ice-sheet size. As a reader of the current manuscript, I'd want to know if the authors felt that it was just temperature variability I needed to incorporate or if there were potentially bigger problems out there.

Comment #15: Thank you for this comment, we are very happy to hear this. Regarding your point about other sources of stochastic variability, this is would undoubtedly lead to a very interesting discussion. Interesting work is being done in this field, such as that Mantellia et al. (2016) which we find interesting both in the context of ice sheets, as well as in the context of dynamical systems. As it stands now, in addition the mass balance response to to surface temperature variability, we discuss ablation-induced variability of the GrIS surface mass balance, the effects of ocean temperature on ice-discharge, and the case accumulation dominated mass balance, and feel that the addition of a further topic to the discussion would introduce too much material.

Notes:

> -Abstract: increases in future variability are small if at all (Simolo et al., 2011, Rhines and Huybers, 2013). Does this claim get repeated anywhere else? I did not see it, so maybe just leave out.

Change #33: We have removed that sentence.

> -Roe and ONeal is 2009, not 2005

Change #34: Fixed, thank you.

> Simolo CM, Brunetti MM and Nanni T (2011) Evolution of extreme temperatures in a warming climate. Geophys. Res. Lett., 38(16), L16701

> Rhines A and Huybers P (2013) Frequent summer temperature extremes reflect changes in the mean, not the variance. Proc. Natl. Acad. Sci., 110(7), E546–E546 (doi: 10.1073/pnas.1218748110)

References

X. Fettweis, B. Franco, M. Tedesco, J. H. van Angelen, J. T. M. Lenaerts, M. R. van den Broeke, and H. Gallée. Estimating Greenland ice sheet surface mass balance contribution to future sea level rise using the regional atmospheric climate model MAR. *The Cryosphere*, 7:469–489, 2013. doi: 10.5194/tc-7-469-2013.

Elisa Mantellia, Matteo Bernard Bertagnia, and Luca Ridolfia. Stochastic ice stream dynamics. *Proceedings of the National Academy of Sciences*, 113(47): E4594 – E4600, 2016. doi: 10.1073/pnas.1600362113.

Johannes Oerlemans. A quasi-analytical ice-sheet model for climate studies. *Nonlinear Processes in Geophysics*, 10(4/5):441–452, 2003. doi: 10.5194/npg-10-441-2003.

Alexander Robinson, Reinhard Calov, and Andrey Ganopolski. Multistability and critical thresholds of the Greenland ice sheet. *Nature Climate Change*, 2 (6):429–432, 2012. doi: 10.1038/nclimate1449.

Influence of temperature fluctuations on equilibrium ice sheet volume

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Abstract. Forecasting the future sea level relies on accurate modeling of the response of the Greenland and Antarctic ice sheets to changing temperatures. The surface mass balance of the Greenland ice sheet has a nonlinear response to warming. Cold and warm anomalies of equal size do not cancel out and it is therefore important to consider the effect of interannual fluctuations in temperature. We find that the steady state volume of an ice sheet is biased toward larger size if ~~temporal~~ interannual temperature fluctuations are not taken into account in numerical modeling of the ice sheet. We illustrate this in a simple ice sheet model and find that the equilibrium ice volume is approximately 1 meter sea level equivalent smaller when the simple model is forced with fluctuating temperatures as opposed to a stable climate. It is therefore important to consider the effect of ~~temporal~~ interannual temperature fluctuations when designing long experiments such as paleo spin-ups. We show how the magnitude of the potential bias can be quantified statistically. ~~We~~ For recent simulations of the Greenland ice sheet, we estimate the bias to be 30 Gt/y (24 Gt/y – 59 Gt/y, 95% credibility) for a warming of 3 °C above preindustrial values, or 13% (10% – 25%, 95% credibility) of the present day rate of ice loss. Models of the Greenland ice sheet show a 'collapse' threshold beyond which the ice sheet becomes unsustainable. The proximity of the threshold will be underestimated if temperature fluctuations are not taken into account. We estimate the bias to be 0.12 °C (0.10 °C – 0.18 °C, 95% credibility) for a recent estimate of the threshold. ~~Many scenarios of the future climate show an increased variability in temperature over much of the Earth.~~ In light of our findings it is important to gauge the extent to which this increased variability will influence the mass balance of the ice sheets.

1 Introduction

Ice sheet mass balance has a nonlinear dependence on temperature. This behavior is observed in simple ice sheet models (Weertman, 1961, 1964, 1976; Roe and Lindzen, 2001), in ~~full~~ regional climate modeling of Greenland surface mass balance (Fettweis et al., 2013), and the nonlinear effect of temperature on melt has been observed in Greenland river discharge (van As et al., 2017).

Ice sheets are characterized by a large interior plateau flanked by comparatively steeper margins. A warming will shift the equilibrium line altitude (ELA) to higher elevations, increasing the area exposed to melt. The area exposed to melt will increase nonlinearly with ELA because of the top-heavy hypsometry (van As et al., 2017). This mechanism explains the nonlinear dependence of mass balance on temperature for ice sheets where run-off is a significant fraction of the total mass balance. This mechanism is important for the mass balance of present-day Greenland, but less so for present-day Antarctica where mass loss is dominated by solid ice discharge (Church et al., 2013, p. 1170). ~~There is, however, some concern that Antarctic ice discharge and total mass balance may be highly nonlinear . The~~ However, observations show that the response of Antarctic melt to temperature is nonlinear (Abram et al., 2013), while the potential for a large nonlinear response of Antarctic mass balance is particularly evident in the simulations from Pollard et al. (2015).

The nonlinear relationship between mass balance and warming means that there is an asymmetry in the response ~~from~~ to cold versus warm anomalies. ~~The~~ Using a simple ice sheet model we will show how, as a consequence of this nonlinearity, the average mass balance will be different when forcing the model with a variable climate compared to a constant average climate. Simulations using constant climate will therefore be biased unless they make statistical corrections to allow for variance. Constant climate forcing is sometimes used to trace the long term equilibrium response of ice sheet models as a function of temperature (e.g. Robinson et al. (2012)).

Ice sheet modeling and evidence from paleoclimatic records indicate that ice sheets display a hysteresis response to climate forcing~~(Abe-Ouchi et al., 2013; Robinson et al., 2012). There is~~ , indicating a critical threshold in temperature, a tipping point, beyond which an ice sheet becomes unsustainable (Abe-Ouchi et al., 2013; Robinson et al., 2012). This is a generic saddle-node bifurcation point, estimated by Robinson et al. (2012) to be reached for the Greenland Ice Sheet (GrIS) at a global warming of $+1.6°$C ($0.8°$C – $3.2°$C) above the preindustrial value.

The stability of ice sheets is typically investigated by imposing a constant climate forcing and then letting the ice sheet model reach equilibrium (Huybrechts and de Wolde, 1999; Robinson et al., 2012; Solgaard and Langen, 2012). The hysteresis curve – and collapse thresholds – are then traced out by repeating these experiments for a range of temperatures and starting from ice free conditions. However, this approach disregards the effects of interannual temperature variability.

~~Previous studies of natural variability in the context of ice sheets include Fyke et al. (2014), who find that the variability of the GrIS surface mass balance will increase in a warmer climate due to increased ablation area, and **?** who find that large fluctuations in glacier extent can be driven by natural, fast fluctuations in climate.~~

That the SMB of an ice sheet model is nonlinear with respect to temperature ~~is well known.~~ has previously been investigated in several studies. In a simplified model of continental ice sheets, Roe and Lindzen (2001) show that the total annual ablation scales with the cube of temperature at the ice sheet margin. Ridley et al. (2010) specifically avoid using average monthly temperature and precipitation climatologies and instead use time series from individual months in order to include the effect of interannual variability in their study. Fettweis et al. (2013, see Fig. 6h) investigate the GrIS SMB simulated by regional climate models (~~RCM~~RCMs) as a function of mean surface temperature from general circulation models (~~GCM~~GCMs). Our contribution is a quantification of this effect, and an estimate of the necessary bias correction to surface temperature needed to account for temperature fluctuations in long term ice sheet simulations.

Previous studies of natural variability in the context of ice sheets include Fyke et al. (2014), who find that the variability of the GrIS surface mass balance will increase in a warmer climate due to increased ablation area, and Roe and O'Neal (2009) who find that large fluctuations in glacier extent can be driven by natural, fast fluctuations in climate. Sub-annual temperature variability in the context of positive degree-day (PDD) models is investigated in eg. Arnold and MacKay (1964); Reeh (1991); Hock (2003); Calov and Greve (2005); Seguinot (2013); Wake and Marshall (2015). PDD models connect surface melting and air temperature, and are used extensively due to their simplicity and wide availability of air temperature data (Hock, 2003). Seguinot (2013) compares Greenland SMB calculated from four different annual PDD formulations with a reference SMB calculated from a PDD scheme using a monthly air temperature and precipitation climatology and deviations from ~~an~~ a long-term interannual mean. At the scale of sub-annual climatology, there are large uncertainties as the estimates of the SMB ~~differs~~ differ significantly ~~dependent~~ depending on the simplifying assumptions used in the PDD formulation, highlighting the need to accurately model both spatial and temporal variability. These findings are built upon by Wake and Marshall (2015) who find that the standard deviation of monthly average temperature may be represented as a quadratic function of monthly average temperature. In the present study we are concerned with interannual variability and expect our results to apply independently of the chosen SMB model.

We investigate how climate variability ~~influence~~ influences the mass balance of ice sheets with a nonlinear response to climate forcing. We derive a simple statistical relationship which can be used to quantify the effect, and illustrate why it matters on a minimal ice sheet model. We then proceed to show how this may be applied to published results from a coupled ice sheet model. In Section 2 we derive an analytical relationship between the magnitude of temperature fluctuations and ice sheet volume, assuming a simple relationship between the mass balance, temperature and ice sheet volume. This relationship is shown to hold using a simple ice sheet model (that includes a surface mass balance model) in Section 3, and in Section 4 we estimate the consequences of temperature fluctuations on a recent long term ice sheet study (Robinson et al., 2012), assuming the effect of temperature fluctuation presented here is not already accounted for. The limitations of this approach, as well as further possible applications, are discussed in Section 5.

2 The Mass Balance of an Ice Sheet

2.1 A Minimal Ice Sheet Model

In order to investigate the influence of temperature fluctuations on the mass balance we consider a simple ice sheet model introduced by Oerlemans (2003) hereafter denoted *Oer03*. This model describes the essential dynamics of an ice sheet assumed to be axially symmetric and resting on a bed that slopes linearly downwards from the center. The ice is modeled as a perfectly plastic material, and the ice sheet is coupled to the surrounding climate by adjusting the height of the equilibrium line h_{Eq} (Oerlemans, 2008):

$$h_{Eq} = h_{E,0} + (T - \bar{T}) \cdot 1000/6.5. \tag{1}$$

Equation 1 represents an increase of the equilibrium line altitude of roughly 154 m $°\mathrm{C}^{-1}$. The influence of h_{Eq} on the specific balance B is illustrated in Fig. (1). It should be noted that the simple relationship described by Eq. (1) does not capture situations where the SMB may *increase* with increasing temperature, as discussed in Section 5. Further details of the Oer03 allowing the formulation in Eq. 2 below are described in the supplementing information.

The model is chosen for its simplicity, thus it is not accurately modeling a specific ice sheet; the two main reasons for choosing it for our analysis are: 1) The simplicity of Oer03 allows the analytical approach detailed below and 2) the Oer03

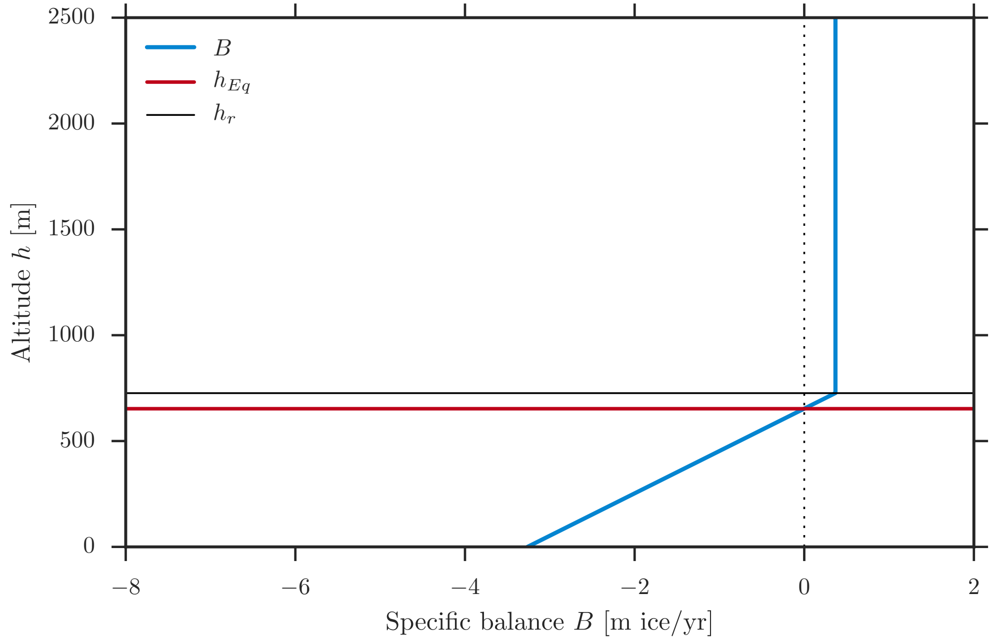

Figure 1. Specific balance B for $\overline{T} = 0$ from Eq. (1) using the parameters in Table 1 and Eqs. (3–4) of the supplement. h_{Eq} denotes the equilibrium line. The runoff line h_r specifies the simplified climatic conditions, as the specific balance is constant above ~~the runoff line~~ h_r (see also Supplementing Information, Eq. (4)), and the balance gradient is constant below h_r (Oerlemans, 2003).

model shows the same functional relationship between surface mass balance (SMB) and temperature as has been found for regional climate models (RCM) for a range of temperature scenarios (Fettweis et al., 2013). The change in volume or mass of the ice sheet depends on the balance between accumulation, ablation and ice sheet discharge which in turn depends on both the interplay between the fluctuating temperature and the state of the ice sheet itself.

Before proceeding with the simple model, we investigate the effect of interannual temperature fluctuations by considering the ice sheet as a simple dynamical system. Assume ~~the mass balance~~ change in volume of the ice sheet to depend only on the volume V itself and a single time-varying mean temperature over the ice sheet, T. Denoting the mass balance (change in ice sheet volume) as dV/dt,

$$dV/dt = f(T,V), \tag{2}$$

where $f(T,V)$ is some nonlinear function. The (stable) fixed point, $f(T,V) = 0$ corresponds to a balance between loss and gain in the ice volume. This is in general an implicit equation to determine the steady state volume $V_0(T)$ as a function of temperature, such that $f(T,V_0(T)) = 0$.

However, the fixed point is not identical to the statistically steady state volume with a temporally fluctuating temperature $T_t = T(t)$ with expectation value $\langle T_t \rangle = \overline{T}$. A numerical integration to equilibrium of an ice sheet model with and without interannual fluctuating temperature shows that in steady state the ice sheet volume V_t will fluctuate around $\langle V_t\rangle = \overline{V}$ where $\overline{V}$ is systematically smaller than the corresponding $V_0(T)$ (Fig. 2). In Fig. 2, $\overline{T}$ is shown on the horizontal axis in the right panel, and the corresponding $\overline{V}$ on the vertical axis (both panels).

Since the temperature T_t – and thus the ice sheet volume V_t – is a stochastic variable the following will characterize an equilibrium state:

$$\langle f(T_t,V_t)\rangle = 0. \tag{3}$$

To calculate $\overline{V}$ we perform a Taylor expansion of Eq. (3) around the – presently unknown – steady state $(\overline{T},\overline{V})$ and calculate the mean volume $\overline{V}$. We use the notation $f_T := \frac{\partial f}{\partial T}$, $f_{TV} := \frac{\partial^2 f}{\partial T \partial V}$, etc. Furthermore, $f^0 := f(\overline{T},\overline{V})$, $f^0_T := \frac{\partial f}{\partial T}(T,V)\Big|_{(\overline{T},\overline{V})}$ etc. We then get:

$$\begin{aligned}\langle f(T_t,V_t)\rangle \quad &= f^0 + \langle T_t-\overline{T}\rangle f^0_T + \langle V_t-\overline{V}\rangle f^0_V + \frac{1}{2}\langle (T_t-\overline{T})^2\rangle f^0_{TT}\\ &+\frac{1}{2}\langle (V_t-\overline{V})^2\rangle f^0_{VV} + \langle (T_t-\overline{T})(V_t-\overline{V})\rangle f^0_{TV} + \mathcal{O}(3),\end{aligned} \tag{4}$$

where $\mathcal{O}(3)$ represents higher order terms. We can simplify Eq. (4) considerably: First note that since $\overline{T}$ is the expectation value of T_t we have $\langle T_t-\overline{T}\rangle = \langle T_t\rangle - \overline{T} = \overline{T}-\overline{T} = 0$ and with the same argument $\langle V_t-\overline{V}\rangle = 0$. The quantity $\langle (T_t-\overline{T})^2\rangle$ is the variance of the fluctuating temperature – we will assume this is known in simulations and substitute $\langle (T_t-\overline{T})^2\rangle = \sigma_T^2$. Since the temperature variations are small with respect to the mean and have a symmetric distribution we may neglect higher order terms in Eq. (4) (Rodriguez and Tuckwell, 1996). We are left with:

$$\begin{aligned}\langle f(T_t,V_t)\rangle \quad &\approx f^0 + \frac{\sigma_T^2}{2} f^0_{TT}\\ &+\frac{1}{2}\langle (V_t-\overline{V})^2\rangle f^0_{VV} + \langle (T_t-\overline{T})(V_t-\overline{V})\rangle f^0_{TV}.\end{aligned} \tag{5}$$

We have evaluated the last two terms in Eq. (5) numerically for the ~~model presented in Section 3~~ Oer03 model and found that $\langle (V_t-\overline{V})^2\rangle$ and $\langle (T_t-\overline{T})(V_t-\overline{V})\rangle$ tend to zero as the ice sheet approaches equilibrium volume (Fig. 3, supplementing information) – neglecting the last two terms, Eq. (5) reduces to

$$\langle f(T_t,V_t)\rangle \approx f^0 + \frac{\sigma_T^2}{2} f^0_{TT}. \tag{6}$$

Equation (6) is the main observation in this work. We shall in the following estimate the implications of this result on realistic asynchronously coupled state-of-the-art ice sheet climate model simulations. As $\langle f(T_t,V_t)\rangle = 0$ at the steady state it can be seen from Eq. (6) that

$$\begin{aligned} 0 = \quad & f^0 + \frac{\sigma_T^2}{2} f^0_{TT} \Rightarrow\\ f^0 = \quad & -\frac{\sigma_T^2}{2} f^0_{TT} > 0\end{aligned} \tag{7}$$

since $f^0_{TT} < 0$ – this negative curvature of f^0 is the nonlinear effect causing the bias. $V_0(T)$ is the stable fixed point; $f(T,V_0(T)) = 0$, thus $f(T,V) > 0$ for $V < V_0$ and $f(T,V) < 0$ for $V > V_0$. This together with Eq. (7) implies that $\overline{V} < V_0$; that is, a positive temperature anomaly increases the mass loss more than what can be compensated by an equally large negative anomaly (van de Wal and Oerlemans, 1994).

3 Ice Sheet Simulations

3.1 Fluctuating Temperatures

To generate an ensemble of volume simulations we use time series T_t comparable to the observed temperatures over Greenland between year 1851 and 2011. For this we use the AR(1)-process (Hasselmann, 1976; Frankignoul and Hasselmann, 1977; von Storch and Zwiers, 2003; Mudelsee, 2010):

$$T_{t+1} = \overline{T} + a \times (T_t - \overline{T}) + \sigma_{AR} W_t, \tag{8}$$

where $W_t, t = 1, 2, \ldots$ are independent, random draws from a standard normal distribution. The exact form of the model used for generating temperature time series T_t is of less importance than the variance of the resulting T_t as only the variance enters into Eq. (6).

The parameters (a, σ_{AR}^2) were obtained by fitting Eq. (8) to the observed annual mean temperatures over Greenland between years 1851 and 2011 (supplementing information). We obtain $(a, \sigma_{AR}^2) = (0.67, 0.85)$ thus the AR(1)-process Eq. (8) has variance (Box et al., 2008) $\sigma_T^2 = \sigma_{AR}^2/(1-a^2) = 1.54\ ^\circ\mathrm{C}^2$.

As we quantify the effect of interannual stochastic variability we use annually averaged temperatures, consistent with the formulation of the Oer03 model (cf. Table 1 of the Supplementing Information). We find time step size of one year to be sufficient for integrating the Oer03-model (Fig. 1, supplementing information); thus T_{t+1} in Eq. (8) represents the temperature one year after T_t.

To find the steady state volume we run the Oer03-model forward long enough for the ice sheet to reach equilibrium, with and without fluctuating temperatures. The results of this procedure are shown in Fig. 2 (left) where it is clearly seen that the steady state volume is smaller for simulations with fluctuating temperatures than with constant temperature. We emphasize that the fluctuating temperature time series $\{T_t\}$ have as mean the constant temperature, $\langle T_t \rangle = \overline{T}$ so that the differences are due only to the annual temperature fluctuation.

In Fig. 2 (right) the effect of temperature fluctuations is shown in the (T, V)-plane: the markers "+" are steady states of numerical simulations with constant temperature, while the circles represent ensemble averages of simulations with fluctuating temperatures. It is evident that temperature fluctuations decrease the steady state ice volume. The yellow curve in Fig. 2 (right) was calculated using Eq. (6) and gives a good agreement with the results from ensemble simulations.

In order to illustrate the physics behind Eq. (6), consider values of the mass budget function f for different ice sheet volumes V, shown in Fig. 3. The insert shows, for a particular value of V, how the steady state is influenced by fluctuating temperatures: the average mass budget of a colder year and a warmer year is less than the mass budget of a year with a temperature corresponding to the average of "cold" and "warm"; to put it another way: the increased SMB of a single anomalously cold year cannot balance the increased melt from an equally anomalously warm year (van de Wal and Oerlemans, 1994). In particular let

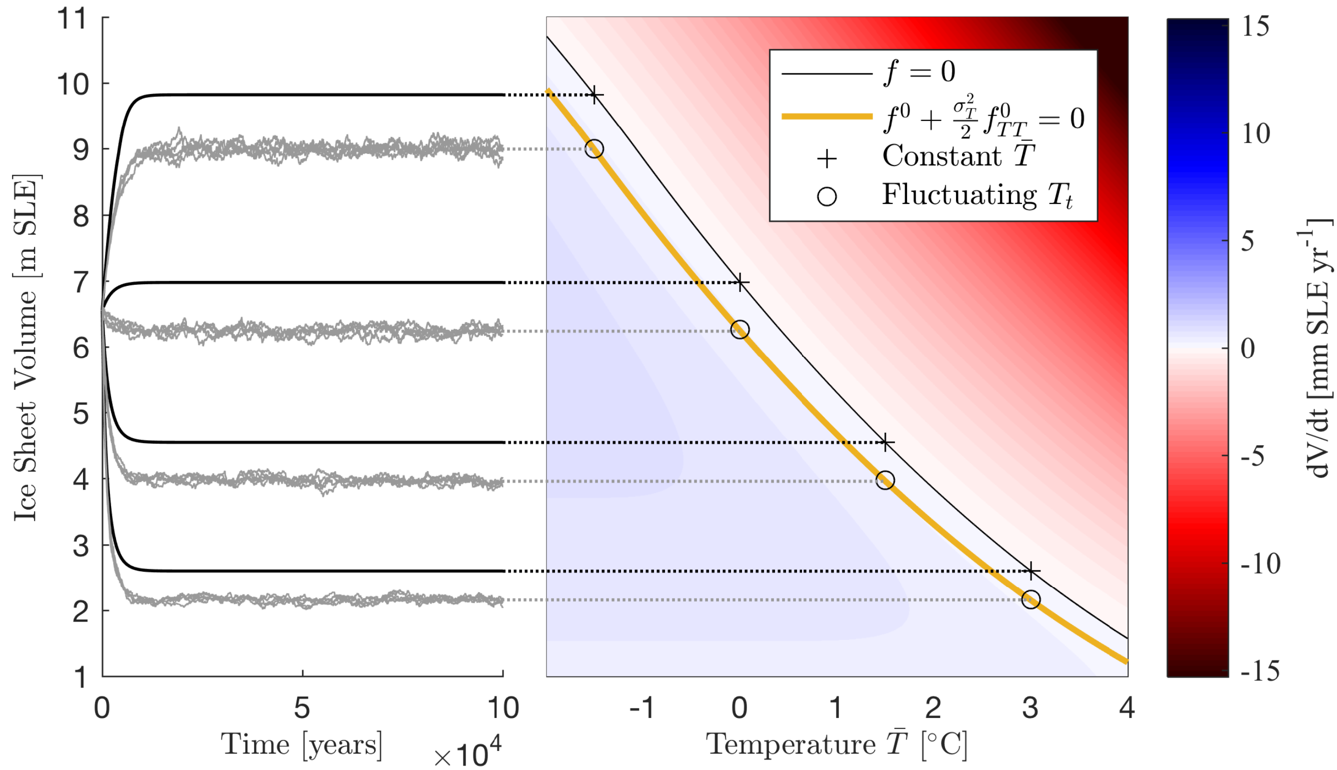

Figure 2. (Left) Simulations of the Oer03-model for $\overline{T} = -1.5, 0, 1.5$ and 3. The black curves denote a constant temperature and the grey curves fluctuating temperatures generated with Eq. (8). (Right) The mass balance Eq. (2) for the Oer03-model in the (T, V)-plane. The black contour is the steady state $f = dV/dt = 0$. The markers represent the average of the numerical simulation with constant $(+)$ and fluctuating $(\circ)$ temperature seen on the left. Finally the yellow contour shows the approximation derived in in Eq. (6).

$T_c = \overline{T} - \sigma$ and $T_h = \overline{T} + \sigma$:

$$\frac{f(T_c, V) + f(T_h, V)}{2} < f\left(\frac{T_c + T_h}{2}, V\right), \tag{9}$$

which is consistent with $f_{TT}^0 < 0$ as shown in Eq. (7).

4 Consequences for Long Term Ice Sheet Simulations

Here we investigate the effect of accounting for fluctuating temperatures when running long time scale climate simulations. These can be either transient runs, scenarios with specified changing CO_2-forcing or equilibrium runs with specified constant forcing. Specifically, we analyze the results of Robinson et al. (2012) where the long term stability of the GrIS is investigated. In that study, an ice sheet model is forced by the output of a regional climate model driven by the ERA40 climatology with a constant temperature anomaly applied, see Robinson et al. (2012) and Supplementary Information.

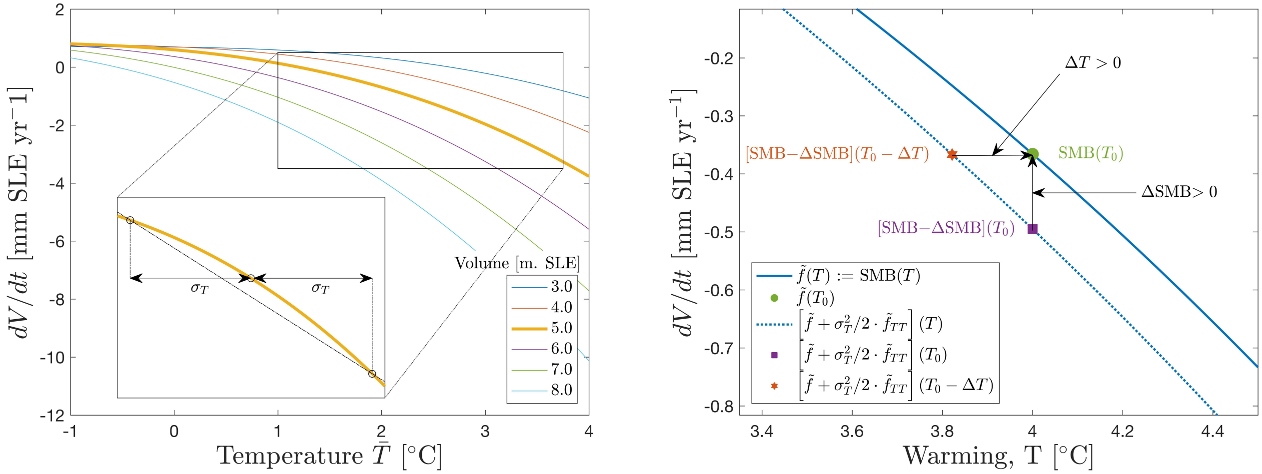

Figure 3. Left: Mass balance dV/dt of the ice sheet for different values of the total ice sheet ice volume V in the Oer03-model. Similar to Fig. 2 but here we show dV/dt as a function of $\overline{T}$ for different total volumes V. **Insert, left:** The curvature of $dV(\overline{T})/dt$ influences the steady state behavior – a cold year does not cancel out the effect of an equally warm year as shown in Eq. 9. The value of σ_T is used for illustration and is given as the square root of the temperature variance, $\sigma_T = \sqrt{1.54°\text{C}^2} = 1.24°\text{C}$. Note the similarity of the $dV(\overline{T})/dt$ found here to Fig. 6h in Fettweis et al. (2013). **Right:** Estimating the effect of fluctuating temperatures on GrIS projections. The full curve is obtained by fitting a third degree polynomial $\tilde{f}(T)$ to an SMB(T) from Robinson et al. (2012). The dotted line show the effect of temperature fluctuations obtained by applying Eq. (6). For a warming of 4°C the green circle shows the SMB. ΔSMB is obtained by applying Eq. (11) and represents the change in mass balance resulting from the temperature fluctuations. $-\Delta T$ is the temperature change required to negate this effect and is obtained implicitly from Eq. (12).

As parameters in ice sheet models are often tuned to reproduce an observed ice sheet history from a time series of forcing observations (eg., Muresan et al. (2016)), the ice sheet volume bias we describe may already be implicitly compensated. To estimate the size of the temperature fluctuation bias, we assume that this bias has *not* already been accounted for by parameter tuning.

Fettweis et al. (2013) compare the output of RCMs forced with multiple future climate scenarios and show that the effect of rising temperature on the GrIS SMB is well described by a third degree polynomial, consistent with the aforementioned findings of Roe and Lindzen (2001). The reader may note the qualitative similarities between Fig. 3 in the present article and Fig. 6h in Fettweis et al. (2013). We will follow Fettweis et al. (2013) and to the ensemble of simulations in Robinson et al. (2012) fit third degree polynomials to the SMB as a function of temperature at time t = 200 years (see also the supplementing information) and obtain third degree polynomials in T:

$$\left\{\tilde{f}_{ij}(T)\middle|\tilde{f}_{ij}(T) = A_{ij}T^3 + B_{ij}T^2 + C_{ij}T + D_{ij}\right\} \tag{10}$$

where the indices i and j run over two separate parameters in the model that take 9 – respectively 11 –values (Robinson et al., 2012) so in total we have 99 unique polynomial fits. These polynomials are then used as a simple description of the mass balance function as a function of temperature, $\mathrm{SMB}_{ij}(T) = \tilde{f}_{ij}(T)$. Differentiating twice we obtain $\tilde{f}_{TT}(T) = 6AT + 2B$ (suppressing indices i, j for clarity).

For all parameter pairs (i, j) we evaluate $\tilde{f}(T)$ and $\tilde{f}(T) + (\sigma_T^2/2)\tilde{f}_{TT}(T)$ – this is shown in Fig. 3 (right) as the full and dotted lines, respectively.

To illustrate this approach we pick a specific temperature T_0. $\tilde{f}(T_0)$ is thus the SMB for a constant temperature and $\tilde{f}(T_0) + (\sigma_T^2/2)\tilde{f}_{TT}(T_0)$ represents the effect of letting the temperatures fluctuate. This procedure gives us an expression for ΔSMB

$$\begin{aligned}\Delta\mathrm{SMB} = \quad & \tilde{f}(T_0) - \left[\tilde{f}(T_0) + \frac{\sigma_T^2}{2}\tilde{f}_{TT}(T_0)\right] \\ = \quad & -\frac{\sigma_T^2}{2}\tilde{f}_{TT}(T_0) \end{aligned} \tag{11}$$

where ΔSMB is positive in accordance with Eq. (7). Next we find the temperature difference ΔT such that

$$\tilde{f}(T_0 - \Delta T) + \frac{\sigma_T^2}{2}\tilde{f}_{TT}(T_0 - \Delta T) = \tilde{f}(T_0). \tag{12}$$

In this way ΔT is the *effective* temperature change resulting from considering fluctuating temperatures.

The results of applying the steps outlined above on the data from Robinson et al. (2012) are shown in Fig. 4. The red curves in Fig. 4 show the most likely ΔT and ΔSMB; the grey curves are estimates for the 9×11 individual parameter values and the blue shaded area represents the 95% credibility region.

The warmings quoted in Robinson et al. (2012) are relative to the preindustrial period whereas the reported warming from the preindustrial period to the present day is estimated to 1°C (Stocker et al., 2013, p. 78). Following the RCP45 scenario it is *more likely than not* that Earth will experience a further warming of 2.0°C (IPCC, 2013, p. 21) from today to the year 2100. Combining these numbers we arrive at a warming of 3.0°C in the year 2100 relative to the preindustrial period when considering the RCP45 scenario. For this value it ~~is seen in~~ can be seen on Fig. 4 (top) that an additional 0.12 °C (0.10 °C – 0.18 °C, 95% credibility) should be added to any constant warming term when considering simulations of the Greenland ice sheet, assuming the same temperature variance as in Section 3. We note that this bias correction is small compared to the spread in temperature projections. Nevertheless this is a known bias that should be accounted for. The threshold for GrIS ice loss has been estimated to be at $+1.6$°C (0.8°C – 3.2°C) (Robinson et al., 2012). ~~Our results indicate~~ Applying the bias correction above indicates that the threshold for GrIS may be 0.12 °C (0.1 °C – 0.18 °C) colder (Fig. 4, top). This is not a large adjustment considering other uncertainties, ~~but it reduces the window available to~~ but it places additional constraints on the maximum temperature increase admissible to avoid passing this threshold and the corresponding multi-millennial sea level commitment. Fig. 4 (bottom) shows the most likely ΔSMB resulting from temperature fluctuations at a 3°C warming to be 30 Gt/y (24 Gt/y – 59 Gt/y, 95% credibility) or – for context – 30 Gt/y (24 Gt/y – 59 Gt/y, 95% credibility) of the average GrIS SMB of -234 ± 20 Gt/y reported for the period 2003–2011 (Barletta et al., 2013).

Observe in Fig. (4) that ΔT goes to zero for low temperature anomalies and appears to reach a constant value for higher temperature anomalies. In the framework presented here this can be explained by considering the SMB(T)-curves shown in Fig. (3) (left). For low temperature anomalies the SMB(T) curve in Fig. 3 (left) is close to flat so the second derivative is small;

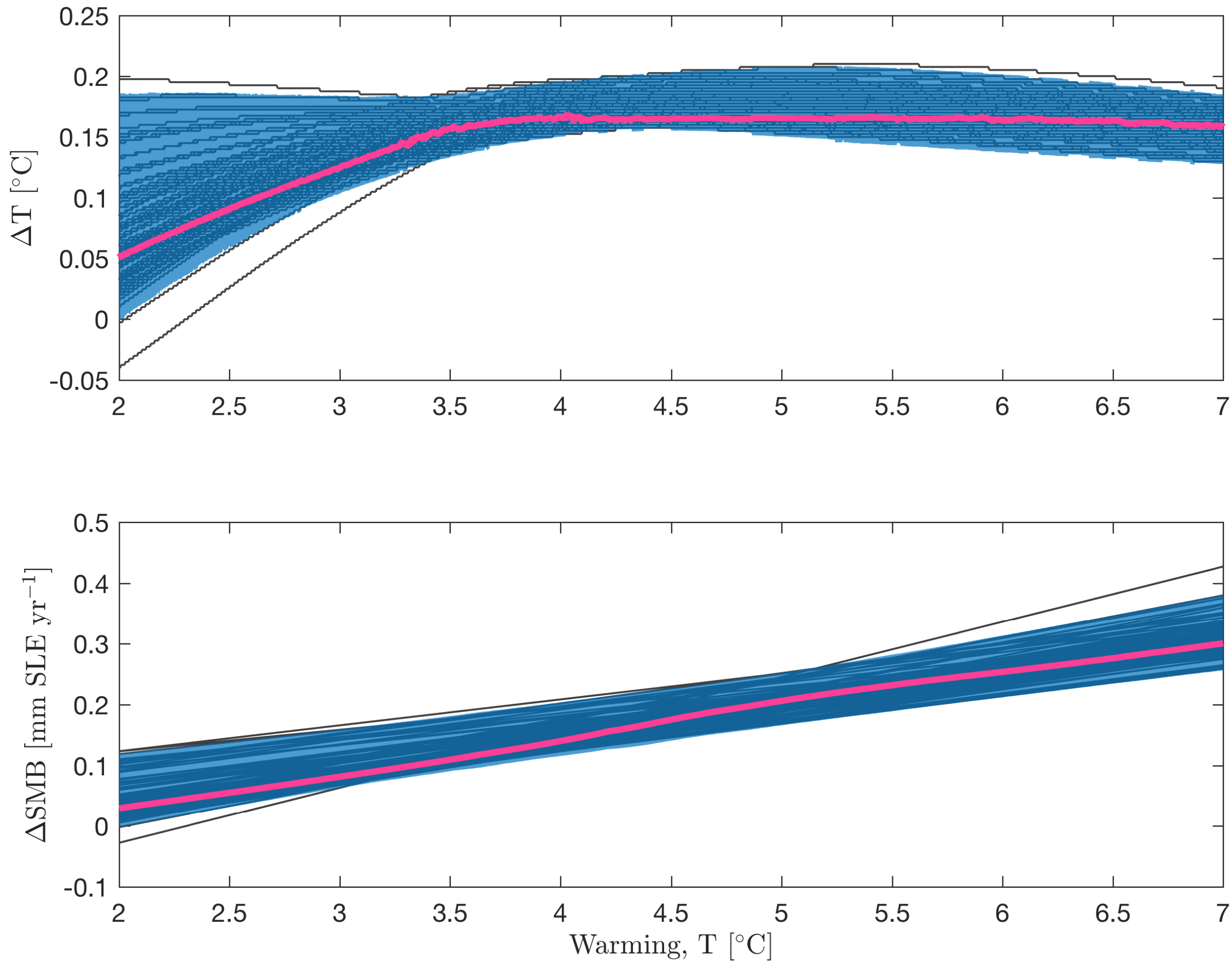

Figure 4. Maximum likelihood estimates of ΔT (effective temperature change) and ΔSMB (effective SMB change where positive values correspond to SMB loss, red curves) resulting from a given temperature increase. ΔT and ΔSMB defined as in Fig. 3, right. The grey curves are estimates from individual simulations and the blue shaded area denotes 95% credibility regions.

this gives a small contribution to ΔSMB from Eq. (11). On the other hand, as the SMB(T) curve in Fig. 3 (left) becomes progressively steeper, a correspondingly smaller ΔT in Eq. (12) is required to compensate for ΔSMB.

The results above highlight that interannual temperature variability cannot be neglected in long term studies involving ice sheet models. The straightforward approach would be to simply include the expected temperature variability in a number of simulations followed by calculating the ensemble average. Conversely, one could calculate the effect of temperature variability for a range of climate scenarios as a starting point for a following bias adjustment.

5 Conclusions

~~When calculating the $\tilde{f}$'s in Eq. (10) and Eq. (11) we assume a constant volume in the data from Robinson et al. (2012), but in reality the relative variations are as large as 9.5% when considering all the warming temperatures shown in Fig. 4 (Fig. 4, supplementing information). However, to draw the conclusion about the consequences of a 3°C warming it is adequate to consider warmings less than 4°C, and here the volume variation was less than 3% of the average (Fig. 5, supplementing information~~

From a theoretical argument and by considering a minimal model of an ice sheet we have shown that fluctuating temperatures forcing the ice sheet have an effect on the mass balance, and thus on the steady state volume of the ice sheet (Eq. 6 and Fig. 2). ~~Neglecting variations in volume does add uncertainty to our results, and it is not immediately clear to us how to quantify that uncertainty. Additionally, at time $t = 200$ years where we extracted the SMB data from the simulations in Robinson et al. (2012), the ice sheet model simulations had not yet reached steady state; thus, expanding the analysis using a data set from ice sheet simulations in steady state would be desirable.~~ The effect is explained by the curvature, or second derivative, of the mass balance as a function of temperature.

Temperature fluctuations can be accounted for in ice sheet modeling studies, either explicitly (eg. Ridley et al. (2010); Seguinot (2013)) or implicitly, as happens when tuning the ice sheet model to reproduce an observed ice sheet history with observed forcing as input (eg. Muresan et al. (2016)). Temperature fluctuations may also be explicitly accounted for by forcing the ice sheet model with climate model output that reproduces the magnitude of observed interannual temperature variability. Our results show the importance of considering temperature fluctuations in the mass balance schemes before bias correcting for other possible model deficiencies.

~~From a theoretical argument and by considering a minimal model of the Greenland ice sheet we have shown that fluctuating temperatures forcing the ice sheet have an effect on the mass balance, and thus on the steady state volume of the ice sheet (Eq. 6 and Fig. 2). The effect is explained by the curvature, or second derivative, of the mass balance as a function of temperature.~~

We find that the steady state ice sheet volume in Oer03 is $0.5 - 1$ $\mathrm{m_{SLE}}$ smaller when the minimal model is forced with fluctuating temperatures compared to constant temperature (Fig. 2). It is therefore necessary to consider the impact of temperature variability when designing long-term model experiments such as paleo spin-ups ~~(eg. Bindschadler et al. (2013); Golledge et al. (2015); Now~~ ~~)~~(Bindschadler et al., 2013; Golledge et al., 2015; Nowicki et al., 2016), especially when downsampling the paleo forcing series. Though differences between ice sheet models may be larger than the effect of temperature fluctuations estimated here, we expect the effect to be in the same direction and of similar magnitude for all models. Furthermore, models of sub-shelf melting, grounding line migration, and ice discharge have the potential to respond nonlinearly to changes in ocean temperatures (Favier et al., 2014; Joughin et al., 2014; Seroussi et al., 2014; Mengel and Levermann, 2014; Pollard et al., 2015; Fogwill et al., 2014), thus it is critical to take variability into account for quantitative assessments.

The response of a real ice sheet to temperature increase is naturally much more complex than what can be described in a simple study such as the present paper. In a model study, Born and Nisancioglu (2012) observe mass loss acceleration of the Northeastern GrIS as a response to warming. This part of the GrIS experiences comparatively little precipitation and thus increasing melt is not compensated by increasing accumulation. However, the opposite has been shown to be the case for Antarctica. Frieler et al. (2015) show that increasing temperatures will *increase* Antarctic SMB at continental scales due to increasing precipitation. This is a case of accumulation dominated mass balance where the curvature term in Eq. (6) has the opposite sign; thus an underestimated temperature fluctuation would lead to an underestimation of the growth of the ice sheet.

When calculating the $\tilde{f}$'s in Eq. (10) and Eq. (11) we assume a constant volume in the data from Robinson et al. (2012) , but in reality the relative variations are as large as 9.5% when considering all the warming temperatures shown in Fig. 4 (Fig. 4, supplementing information). However, to draw the conclusion about the consequences of a 3°C warming it is adequate to consider warmings less than 4°C, and here the volume variation was less than 3% of the average (Fig. 5, supplementing information). Neglecting variations in volume does add uncertainty to our results, and it is not immediately clear to us how to quantify that uncertainty. Additionally, at time $t = 200$ years where we extracted the SMB data from the simulations in Robinson et al. (2012), the ice sheet model simulations had not yet reached steady state; thus, expanding the analysis using a data set from ice sheet simulations in steady state would be desirable.

We have evaluated the consequences of the temperature fluctuation bias on long-term GrIS simulations and found that, if the full effects are taken into account with no further modifications, a significant *effective* temperature change would be required for an unbiased estimation of the equilibrium ice volume.

6 Code availability

The code for this study is available at https://bitbucket.org/bogeholm/ice-sheets-fluctuating-temp.

7 Data availability

Data used in this study was obtained from the authors of Robinson et al. (2012).

Author contributions. TBM, AG and PD designed the study. TBM performed the data analysis. TBM, AG and PD wrote the article.

Competing interests. The authors declare no conflict of interest.

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