Response to Reviewer Comments
Date: 20 April 2018
By R. Reese, R. Winkelmann, G. H. Gudmundsson

Journal: TC
Title: Grounding-line flux formula applied as a flux condition in numerical simulations fails for buttressed Antarctic ice streams
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First of all, we would like to thank the editor Olivier Gagliardini, the anonymous reviewer and Christian Schoof for their helpful and excellent comments and their efforts to create the detailed reviews!
In our response and in the revision of the manuscript we addressed the main issues raised by the first reviewer:

1. We detailed the description of our inverse methodology (see, e.g., comments on 'flux formula' and first comment on the 'buttressing factor') and created figures to underpin that our findings are independent of the details of the inversion as requested (Figs. 2, 1).
2. We discuss that the exact grounding line location does not affect our results (see, e.g., second comment on 'buttressing factor', lines 4f on page 19 of the marked-up manuscript, and Figs. 2, 1 here).

We are further very happy about the evaluation of our manuscript by Christian Schoof and would like to thank him for his discussion about the underlying reasons for the formula to fail at Antarctic grounding lines. We would be very happy to collaborate with him along the lines of his suggestions to approach this task in an further study.
We provide detailed answers to all comments below. The reviewer's comments are given in black and the authors responses in blue. The changes made to the main document can be found at the end (created with latexdiff). Page and line numbers given below relate to this document.

Anonymous referee 1

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Interactive comment on The Cryosphere Discuss., https://doi.org/10.5194/tc-2017-289, 2018

The comment was uploaded in the form of a supplement:
https://www.the-cryosphere-discuss.net/tc-2017-289/tc-2017-289-RC1-supplement.pdf

Summary

As reported in several studies, resolution is key for an appropriate representation of grounding line migration in ice-flow models. Yet for continental-scale forward simulation, a trade-off has to be made between resolution and computational costs. On longer time-scales, the mesh has furthermore to be adapted and updated according to the migration of the grounding line. Avoiding these mesh issues, boundary layer theory allows to infer ice-flux values across the grounding line (Schoof, 2007). This flux relation (Eq. 1) has since been implemented as an internal boundary condition in several ice-sheet models using coarse resolution ($\sim$ 5km). Inter-comparison studies confirmed the utility of this parameterisation. Yet the effect of ice-shelf buttressing, which can be included in the parameterisation, did not receive much attention. The authors of this study try to shed light on the applicability of the buttressing correction term (buttressing factor) in the flux parameterisation at the grounding line. For this purpose, a state-of-the-art ice-flow model is applied over Antarctica. Ice velocity observations are matched by a bi-variate inversion for the basal slipperiness and the rate factor. For this setup, it is reported that the computed buttressing factor shows high spatial variability along the grounding line of the two largest ice-shelves in Antarctica. Moreover, negative values are widespread, which leads to unphysical flux values in the parameterisation. The authors therefore strongly question the applicability of this correction term. The study is soundly structured, to the point and well written. Yet I have major questions and concerns on important details of the methodology and on theit interpretation. As the simultaneous inversion of two parameters is not well posed, their quantification is underdetermined. Thus, essential parameters for the flux parameterisation are not well constrained. Further issues concerning the buttressing factor θ come from uncertainties in the exact location of the grounding line, which might have severe consequences for the interpretation. Therefore, I recommend that this manuscript undergoes a major revision. I advise the editor to only consider publication of this article, if the authors were able to adequately address the comments below.

We would like to thank the anonymous reviewer for his/her effort to review our manuscript and appreciate his/her comments for improving our study. His/Her main points are raised on the inversion method to initialize our model and the positioning of the grounding line in the study. We give in-depth responses to both issues that the reviewer detailed in the questions below (see comments on ‘buttressing factor’ and ‘flux formula’).
However, we would like to point out that θ becomes negative independently of the initialization

method of our study and of the exact grounding line position in the setup: In Institute Ice Stream, this fact is already visible in the velocity data set. Ice velocities decrease approaching the grounding line (see Fig. 1 below, also added to Fig. 1 of the main manuscript). Negative, longitudinal strain rates are the underlying reason for the buttressing value to become negative. Similar strain rates are found within a region that extends 20km upstream and 100km downstream of the grounding line. Negative strain rates in longitudinal direction relate to compressive stresses and $\theta \leq 0$, as described on page 10 in lines 15ff of the marked-up manuscript. Hence, $\theta \leq 0$ is independent of both main issues raised by the reviewer: (1) the inversion methodology to determine the basal slipperiness and ice rate factor and (2) the exact position of the grounding line in the model experiments. Unrelated to both, the formula will hence fail in this case.

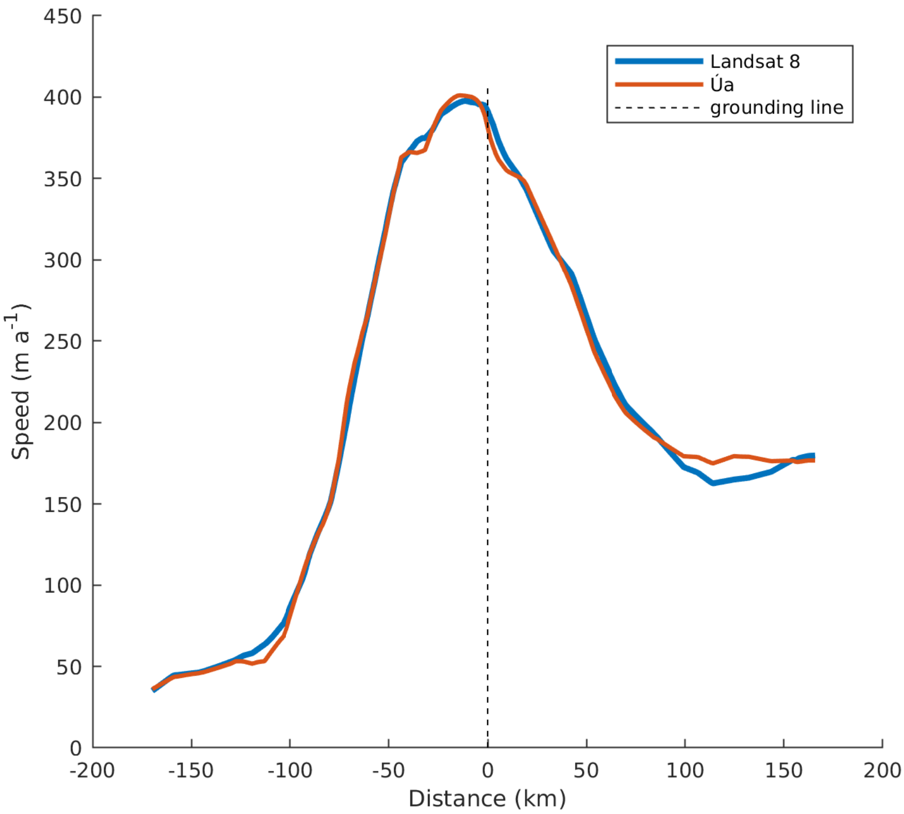

Figure 1: Speed along the centerline of Institute Ice Stream. Observed and modelled velocities decrease towards the grounding line.

Main comments

Buttressing factor

- As you seek to infer flux values at the grounding line, special attention has to be given to its positioning in the model. Errors in the positioning can have large impacts on the flux formula. I therefore highly appreciate the effort to correct the ice density by firn information from a regional climate model. Yet in regions of high inflow from outlet glaciers, the used flotation criterion is possible not very accurate. Certainly in light of the fact that the Bedmap2 thickness values near the grounding line are subject to large uncertainties. For any mismatch of the grounding line position or in ice thickness, the inversion has to compensate by accordingly adjusting the basal slipperiness (C) and the rate factor (A) such that observed velocities are reproduced. I cannot foresee what effects this might have on the stress regime and ultimately on the buttressing parameter (θ).

We agree with the reviewer that the position of the grounding line needs particular care, especially in respect with the uncertainties in the Bedmap2 data set. That is the reason why we adjusted the Bedmap2 bedrock data within a range of 50m so that the grounding line position aligns with the observed position from (Bindschadler et al., 2011). This is described on page 7 in lines 7f in the marked-up manuscript. Thus, the modelled grounding line position, which is determined from the physically-based flotation criterion, matches closely observed grounding lines. However, the results of our study are independent of the exact position of the grounding line. As Fig. 1 below as well as Fig. 3 in the main manuscript show, velocities decrease as the ice approaches the grounding line, which gives rise to the negative θ values along the grounding line of Institute Ice Stream. This pattern is not restricted to the grounding line exclusively, but extends also upstream and downstream of the grounding line and will hence not depend on the exact location of the grounding line in that region. This is already discussed on page 16, lines 22ff as well as page 10, line 32ff of the manuscript.

- Considering uncertainties in the grounding line position, you could rely on Antarctic-wide grounding line observations (e.g. Rignot et al. (2011a)), rather than using a flotation criterion. Bedmap2 also comprises a flotation mask. A sensitivity assessment to the grounding line position might indeed be valuable.

  As stated above, we make use of the observed grounding line position from Bindschadler et al. (2011) in order to adjust the bed topography around the grounding line. The adjustment of the bed topography within the range of Bedmap2 uncertainties was done only for the Antarctic-wide setup presented in the main manuscript, for the alternative mesh, described in the Appendix B and shown in Figs. B1 and B2, this procedure was not applied. While its grounding line position hence deviates from the observed positions, the results are qualitatively and quantitatively very similar and we conclude that our findings are robust with respect to the grounding line position. We added this on page 19, line 4f of the marked-up manuscript.

- You nicely explain how you retrieve the grounding line position. For the calculation of the buttressing factor θ and the flux parameterisation (Eq. 1), you interpolate the relevant stress and thickness values within the corresponding mesh element. Any grounding-line mesh element will by definition include nodal information from the grounded side. Therefore, your buttressing calculation is biased by a stress regime that is influenced by basal friction. From how I understand the buttressing factor, it should represent the stress regime on the ice shelf-side only because it is this side which exerts buttressing upstream. Unfortunately, this is highly uncharted research territory and there are certainly no best practices. From my experience, some spatial smoothing of the shelf-side deviatoric stress regime is beneficial.

We do not fully understand this comment and hope that we can address it appropriately: we calculate the buttressing number from the stress fields at the grounding line position diagnosed from the floatation criterion. Since stresses within the ice are continuous across the grounding line using an interpolation method is valid here. A closer look into the stress fields (see Fig. 2) reveals that it is rather uniform in the vicinity of the grounding line and hence the stresses are independent of the exact position of the grounding line and of the interpolation method.

- Using your three buttressing definitions, you compute the 'analytical' parameter θ all along the grounding line. Yet these values remain uncontested or unvalidated. You concentrate on the fact that negative values are widespread and that the flux parameterisation would yield 'unphysical' results. Buttressing can however be estimated in another way for comparison. The idea is that you should remove the ice shelf entirely and recompute the associated unbuttressed velocity field. Then, you can directly compute the ratio between ice fluxes in the buttressed and unbuttressed case. These values would be very informative to quantify the buttressing effect along the grounding line. Moreover, the variability could be compared to the θ values. With some luck the two values show a significant correlation, which you might want to exploit for an improved adjusted quantification of buttressing in the flux formula.

While we agree that the effect of buttressing could also be quantified by relating modelled fluxes in the presence and absence of an ice shelf, it is unclear to us how such an experiment relates to the topic of our manuscript, which is to test the accuracy of the flux formula under realistic conditions. In particular, it is unclear how this would produce θ values of relevance for calculating analytical ice fluxes. As there appears to be some misunderstanding on the behalf of the reviewer as to how θ is defined in the original work, it is done by replacing condition (9) with $2\bar{A}^{-1/n}h|\frac{\partial u}{\partial x}|^{1/n-1}\frac{\partial u}{\partial x} = \theta \cdot \left(\frac{1}{2}\rho_i\left(1-\frac{\rho_i}{\rho_w}\right)gh^2\right)$ at $x = x_{gl}$ in order to account for modification in stress at the grounding line from the 1HD unbuttressed case (see description in Section 4.2 of (Schoof, 2007)).
The reviewer appears to suggest that if θ was somehow defined or calculated in a differ-

ent way, the analytical fluxes would show a better agreement with calculated ones (but provides no evidence to support this). As we explain in the manuscript several slightly different definitions of θ have been used in the literature. While our first definition of θ (i.e. θ_1) is in our opinion the correct definition, we found after having conducted a thorough review of the literature that two further definitions have also been used. We therefore decided to conduct our analysis using all three definitions of θ. In all cases our conclusions are the same: The flux formula produces either grossly incorrect or quite simply physically unrealistic fluxes for all major Antarctic ice streams.

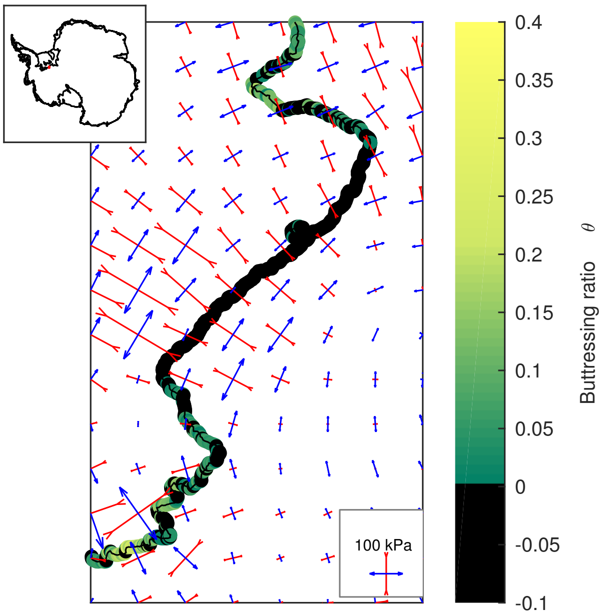

Figure 2: Close-up of the grounding line of Institute Ice Stream from Fig. 3a in the main manuscript (location indicated in the inset).

Flux formula

- Similar to above, any uncertainties in the location and ice thickness of the grounding line will affect the inferred parameters (C, A). Moreover, it was shown by (Arthern and Gudmundsson, 2010) that a similar bi-variate inversion for C and A is highly underdetermined. This means that multiple combinations of C and A (on grounded ice) are possible. As both parameters enter the flux parameterisation, with different weighting, these two issues cannot be ignored. I even fear that the usage of the flux parameterisation in such a situation is almost a vain exercise. An option to prove me wrong would be the following: (1) Infer A and C using velocity observations. (2) Remove all ice shelves, either by reducing the computational domain or by setting the ice-shelf thickness to a very small value. Then compute the corresponding velocity field for this unbuttressed case (3) Compare modelled ice flux with the 'analytical' flux parameterisation. I fear that no consistency will be found in the unbuttressed flux values between the 'modelled' and 'analytical' values. This is certainly a very useful and informative exercise.

We have some difficulties understanding the purpose of the proposed exercise. In the unbuttressed case the analytical and the numerical fluxes will (of course) agree. This has been tested many times in the past both with our ice-flow model Úa, and with other similar models. There seems to be little use in repeating this exercise here. However we would like to point out that simply removing ice shelves will not provide a flowline-type unbuttressed situation as the reviewer seems to suggest, as even in that situation the flow can be convergent/divergent at the ice margin, in which case the buttressing parameter θ will not be equal to unity. We would also like to point out that the example given in Arthern and Gudmundsson, 2010, referred to a datum flow where there are no spatial variations in ice or bed properties. The more general situation (i.e. where A and C vary spatially) has yet to be studied, but we expect the results of such a study to be somewhat similar to the conclusions by Gudmundsson and Raymond, 2008, who studied a similar type of an inverse problem involving spatial variations in both basal slipperiness and basal topography. There it was found that the problem was, giving the right surface data, not necessarily under-determined. We did test for a number of different degrees of regularization as well as different values of m, which yield different A and C fields. We find our results to be robust, with details given in Appendix B. Again, we would like to stress that the reason for $\theta \leq 0$ in the region of Institute Ice Stream is already visible in the observed flow fields. This is true for observed fields from (Gardner et al., 2017) as well as Rignot et al. (2011b). The simplest explanation for the differences between the numerically and analytically calculated fluxes is that not all the assumptions made to derive the formula are satisfied. An in-depth discussion on these assumptions is given by Reviewer 2.

Unbuttressed situation

- I understand why you strongly focus on highly buttressed ice shelves. In this way, your evaluation of the buttressing parameter θ is deliberately biased to the buttressed cases. I would therefore suggest to add an unconfined ice-shelf setup. An example could be the Thwaites Glacier area, though there might be some complication from a pinning point not present in the Bedmap2 geometry. The pinning point might however not matter to much, as the western portion of the floating tongue is certainly not much buttressed. In such a clearly unconfined setup, I would expect θ values consistently close to 1.0 or even above.

In our study, we focus on the current Antarctic Ice Sheet. Since most of its major ice streams and glaciers are highly buttressed, we believe that focusing on buttressed ice streams is of particular importance. Figs. 1 from (Fürst et al., 2016) and (Reese et al.,

2017) show that all Antarctic ice shelves buttress upstream ice flow. In Table 1 of the manuscript we compare the modelled to formula-predicted fluxes and find that also for the (probably) less buttressed grounding line of Thwaites glacier, fluxes disagree by more than 50%. This is true for all major ice shelves. We added a Figure for West Ice Shelf (see Fig. 3, as the reviewer points out in Thwaites a pinning point is missing and we hence do not fully trust the results for Thwaites). The western part of West Ice Shelf is largely unconfined, and θ values are generally close to 1, but since the grounding line is not perfectly straight, they vary locally and in some areas, θ values are very low. Fluxes here do not show perfect agreement, which might be pinned down to the fact that one of the assumptions in the derivation of the formula fails as discussed by the second reviewer Christian Schoof. We tested our calculations and the model also for a flowline setup where we find very good agreement between the formula and the modelled fluxes.

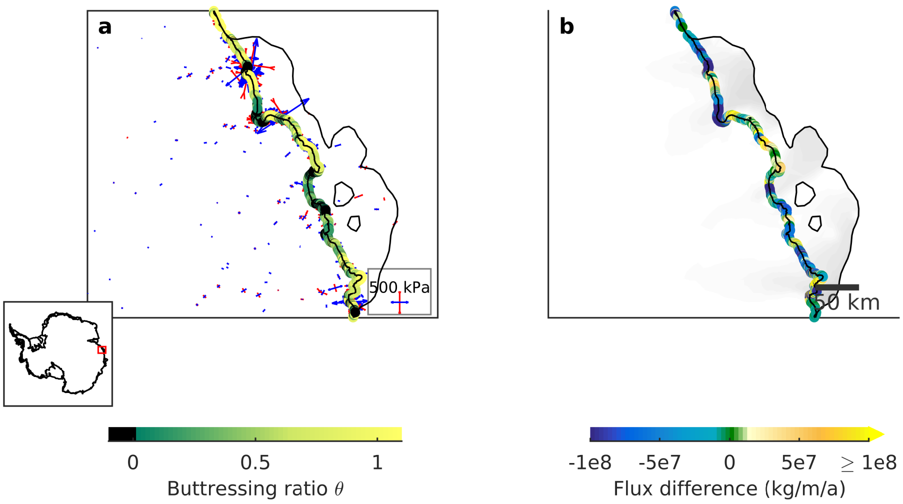

Figure 3: Same as Figure 3 in the paper, here for West Ice Shelf.

Specific comments

P7 L21-23 I suppose that you compute the 'modelled' ice flux over the grounding line by using the velocity component perpendicular to the grounding line. Is that right?
Yes.

P8 L12-13 I do not see the velocity decrease along the central flowline of Institute Ice Stream in Fig. 1. A velocity profile, as an inset to Fig. 1 or 3, would help.
Please find the velocity profile in Fig. 1, also as an inset to Fig. 1 of the main manuscript.

Eq. 14 I wonder how you compute the velocity mismatch in the cost function. Do you do this on the model mesh nodes or directly at the location of each velocity observation. This matters, because you have refined your grid near the grounding line. So the nodal difference computation would introduce a strong bias in the cost towards the grounding line area.

This might even be desirable in your case.
We are not sure that we understand this comment correctly. The misfit function is given as an integral over velocity mismatches. The integral is numerically evaluated at nodes of the mesh. But by integrating over the entire domain, the areas of the respective elements are taken into account and the misfit function is not biased towards the highly-resolved regions of the mesh.

Fig. 1 The vectors in this plot are hard to discern. I would prefer a 2D magnitude plot of the velocity fields with superposed streamlines. As mentioned above, an along-flow profile of velocity magnitudes would help to see the velocity decrease upstream of the grounding line.
Done.

Fig. 3b, 5, B2 Use a different colour map for the flux difference because the colours are very similar to the buttressing parameter θ.
Done.

Fig. B3-5 Legend entries are too tiny. Please take a larger font size.
Done.

Fig. S3 A comparison of the inferred viscosity field on Ronne Ice Shelf by Larour et al. (2005), I miss well imprinted weak zones along the lateral ice-shelf margins. Do you have any explanation for that?
We did not invert for the viscosity but for the rate factore A, and there are clearly defined bands of weak ice (i.e. high A values) in the vicinity of most margins.

Referee 2: Christian Schoof

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This paper presents a systematic comparison of remotely-sensed ice fluxes through Antarctic grounding lines with the fluxes predicted by a suite of "flux formulae" based on a particular boundary layer model for ice flow at the grounding line. The basal friction parameter - a key variable in the flux formula - estimated using an inversion of the same velocity data set. In general, the procedure adopted shows terrible agreement, demonstrating that said flux formulae do not work at all well when applied to present-day Antarctica. I believe the result is robust, and I am generally happy for the paper to be published more or less as is.

We would like to thank you for your efforts and your in-depth, detailed discussion on the underlying reasons for the formula to fail along Antarctic grounding lines. We are happy that you agree with our main conclusion that the applicability of the formula to Antarctic grounding lines is limited. As you mention, $\theta \leq 0$ is more of a symptom and the underlying reason for the formula to fail is that one of the assumptions made to derive the formula is not satisfied for the Antarctic grounding lines. These assumptions are (a) that the boundary layer is in quasi-steady state, (b) that the flow is transversally homogeneous and (c) the flow is unidirectional and perpendicular to the grounding line. We added a remark in the main text to clarify that we do not focus on the underlying reasons here (page 16, lines 10f and page 17, line 13) and we addressed all minor changes (see also specific comments below).
We want to thank you for pointing out various tests to understand which of those conditions fails and we would be very happy to collaborate with you in future along those lines. In this manuscript, we aim to make the point that the amended formula does not work for Antarctic ice streams, independently from the underlying reasons for it to fail, and we are happy that you recommend the paper to be published in its current state.

There are a few items that one could go after a bit more. My view of the review process has become pretty cynical (is my role really to hold up decent work just because it could be improved? Where does that process end?). In short, I am under no illusion that the points I raise will, or even should, feature in a revised paper, and so I have no desire to force the authors to address them. Rather, my argument is that writing off the methodology behind the offending flux formulae may be somewhat premature, and this could be investigated further. In that vein, here goes:

1. The discussion regarding the reason for the discrepancy between observed fluxes and fluxes predicted by the flux formulae is a bit weak. The discrepancy is primarily put down to large buttressing factors. I actually think that is inaccurate, in the following sense: if I really had

a locally uniaxial flow, and buttressing due to a larger-scale reduction in extensional stress at the grounding line, I'd expect an amended flux formula (with buttressing factor θ_1 as defined in the paper here) to work pretty well, and I wouldn't expect $\theta_1 = 1/4$ to cause major issues. In fact, it had been my impression that Gudmundsson et al. (2012) had found reasonably good agreement, at least where the grounding line cuts across the channel in the geometry used in that paper, rather than at its sides (where the flow is presumably heavily affected by shearing parallel to the grounding line).

2. The flux formula being discussed was dervied under a number of conditions (for this you really have to look in detail at Schoof 2007 in JFM, the JGR version won't help), which can really be boiled down as follows: there is a boundary layer near the grounding line over which extensional stress decays to values compatible with a shallow ice approximation further inland. In order for the flux formula being tested to work, the following conditions must be met a) the boundary layer must be in a pseudo-steady state. This is justified by a separation of time scales: the boundary layer should equilibrate much faster than the ice sheet as a whole, so in the absence of rapid changes in forcing (or in bed condition due to internal feedbacks! - see Robel et al 2016 in The Cryosphere) b) the flow at the boundary layer scale must not depend on the transevrse position (that is, if I move parallel to the grounding line by a distance comparable with the boundary layer length scale, the flow field should still look the same) c) the flow must be unidirectional and perpendicular to the grounding line. The condition that depth-averaged extensional stress ('R') at the grounding line is equal to $1/2rho(1 - rho/rho_w)gH$ is really not essential at all, which is where the simple correction factor theta in Schoof (2007) came from. It is true that making theta very small should change things - in fact, the extensional stress can become comparable to those that are experienced in a shallow ice flow and the need for a boundary layer almost goes away, as discussed in the appendix to Schoof (2007, JFM), Kowal et al (2013, JFM) as well as the supplementary material to Schoof et al (2017, The Cryosphere). A superficial reading of Reese et al would suggest that the value of theta is the crux of the problem, and I think that obscueres a few things (in the sense that some of the theta values observed, notably the negative ones, are likely to be the symptom rather than the cause; bear with me).

3. Out of the conditions listed above, c) is actually the easiest to surmount (I am currently working on an extension to the unidirectional flow model that would take shearing parallel to the grounding line into account, this changes the relationship between flux, thickness and extensional stress to take account of grounding-line-parallel shear stress, but nothing too exciting happening here). My impression is that the conditions that are most likely to be violated by the real data considered in this paper are a) and b). I'll touch on both below, in a way that may be at least partially testable.

4. Pseudo-steady state (point a above). This is likely to be violated relatively frequently, at

least *in the data*. By that, I mean that bedmap ice thicknesses and ice velocities are taken at face value here, and the inversion is done purely as a snapshot inversion to find C, presumably at the cost of generating quite large ice flux divergences, if one were to compute them. If so, then it is likely that a "progonostic" forward computation of the model with time stepping would lead to significant transients that result from an incompatibility between velocity field and bed geometry, and these transients may not be 'real'. See the work by Morlighem et al on inverting for bed topography to suppress the effect, and also the work by Goldberg and Heimbach on the use of data assimiliation techniques to avoid the pitfalls of using snapshot inversions for bed properties.

From the MISMIP model intercomparison in one horizontal dimension (Pattyn et al 2012, The Cryosphere), we know that violating the pseduo-steady state assumption (in that case, due to step changes in the ice viscosity parameter) can lead to singificant but sort-lived departures from the 'flux formula', with the ice flux at the grounding line potentially settling back onto the flux formula over a time scale that is equal to the advective time scale for the boundary layer (boundary layer length / scale for velocity in the boundary layer). I'm not saying that this is likely to happen here, but it's worth illustrating - if you run your model forward over the time scale I identify, do you end up somewhere closer to the flux formula?

A corollary of this is actually the negative theta values computed. If I supposed that the flow were unidirectional and laterally homogeneous as per points c) and b) above, then the only way I could have a negative theta value would be if $du/dx < 0$ (u and x being measured along the flowline, naturally. If that is the case, then the assumption of the boundary layer model of a fixed flux through the boundary layer, corresponding to a pseudo-steady state, must fail, since such a flux would require $d(hu)/dx = hdu/dx + uhd/dx = 0$ and we can assume that $u, h > 0$ and, as we ought to be thinning towards the grounding line, $dh/dx < 0$. If also $du/dx < 0$, both terms are negative and there is no way that we can have $hdu/dx + uhd/dx = 0$.

That leads to two possibliities: either the pseudo-steady state condition is violated (so a) does not apply, which could occur due to short-time-scale changes in forcing) or the assumption of laterally homogeneous flow (point b) above) must be false. I expect we are looking at the latter rather than the former: the flow of the ice streams in question slows as the grounding line is approached. Mass is not lost here, but rather, I expect that the flow simply spreads laterally: as the centreline slows, a wider region flows at ice stream speeds. I confess I haven't bothered to check this in the data set, but it would certainly be worth doing that.

5. Lateral homogeneity. This is probably the big one. The original boundary layer theory did not deal with this at all, and I fully expect that ice streams violate this assumption pretty much every time. There is plenty of evidence for the force balance of ice streams to be singificantly affected by lateral shear stresse, and I expect this holds true near the grounding line. This is intrinsically a loss of lateral homogeneity: we have gradients in the velocity component normal

to the grounding line with respect to the coordinate that measures distance parallel to the grounding line, and an extra term appears in the force balance that does not feature in the original Schoof (2007, JFM) boundary layer theory. In that sense, I would not expect the resutls of the latter paper to hold, though I know how one would update the boundary layer theory presented there to account for it. In fact, it may be worth pointing out that various papers have already tried to do that, though only in a form where lateral drag is parameterized. The most complete version of this can be found in Schoof et al (2017, The Cryosphere), with parts of the problem also addresed by Pegler (2016 and earlier papers).

Is this testable? The answer is yes. Both Schoof (2007) papers give estimates for the size of the boundary layer as a function of the various model parameters, such as C, m etc. All you would have to do is check whether any mdoel parameters (such as extensional stess at the grounding line, or C, or ice thickness at the grounding line) vary singificantly within that boundary layer length scale. It is not acutally enough to do that *along* the grounding line, you should really also check that C in particular does not vary by an O(1) fractional amount when going a single boundary layer length scale inland. If there is singificant variability, you immediately have good reason to say that the flux formula won't hold.

6. I'm not particularly bothered by wanting to "defend" simulation models that use a flux formula. I can ceratinly see their appeal in simulating long time scales, and for testing qualititative ice sheet behaviour without getting lost in computational detail. But I haven't built a career on such a model. That said, I would be quite interested in a comparison of long-time-scale prognostic modelling using a flux formula model - admitting that it is going to give locally terribly wrong fluxes at a given point in time - and a fully resolved model like Ua. If I care about grounded ice volume changes over long time scales, how different are the predicitons? And are they *qualitiatively* different, in the sense that irreversible retreat occurs not just at different values of forcing parameters, but depends in a fundamentally different way on the forcing that is imposed?

Minor comments:

- "Within the context of the shallow ice-stream computational models — a commonly-used flow approximation for describing the flow of ice streams and ice shelves (e.g., MacAyeal, 1989; Morland, 1987) — it has, for example, been suggested that for many applications a horizontal resolution of around one ice thickness or less is suitable (Cornford et al., 2016; Gladstone et al., 2012; Pattyn et al., 2012)." This is a somewhat bizarre thing to say; the whole point about a shallow ice theory is that it does not know about how long a horizontal distance equal to one ice thickness is: the limit of a small aspect ratio is already implicit in constructing a shallow ice theory. The fact that a grid or node spacing comparable to something like 1-3 km (or whatever) is regarded as adequate should not be equated with a mesh element size of one ice thickness. More relevant is what fraction of the linear domain

size the typical distance between grid points or nodes should be - probably 1 in 1000 is really implied here.
Many thanks for pointing this out. We changed the wording to '..it has, for example, been suggested that for many applications a horizontal resolution of around one kilometer or less is suitable...'.

- equation 14 "$I(f) = ...$" what does the argument "f" signify, given that f does not appear on the right-hand side of equation (14)? Should this be $I(v)$?
Thanks for bringing this typo to our attention. The argument should indicate the data misfit $\boldsymbol{v}_{modeled} - \boldsymbol{v}_{observed}$, which we omit now for simplicity.

- page 9 "can *not* be used": this should be "*cannot* be used"
Done.

- figure 4: The Ua flux looks terrible - as in, very non-smooth. I'd expect H1-type convergence of ice velocities under grid refinement - is the jaggedness mostly a result of a misalignment between grounding line and mesh, or of forcing the grounding line to lie along a mesh (and therefore having sharp corners at every node?) Probably worth explaining.
This is a discretization issue, depending on the diagnostic location of the grounding line. We diagnose it as a curve of line-segments crossing the (triangular) elements with floating and grounded nodes, with an example given in Fig. B1, Panel 2. This creates a 'wiggly' grounding line. To obtain fluxes, we calculate the grounding line normal for each such segment which causes an element-by-element variation of the normal, and in fluxes. Especially in areas where the flow direction is more or less aligned with the grounding line, this leads to positive-to-negative fluctuations in fluxes. We added this discussion to our manuscript on page 13, lines 1 and 2. If we apply a moving average smoothing to the data (in Fig. 4 displayed for a span of 5 neighbours) Úa fluxes look much better, but we felt better to show the element-wise results.

- page 13 bottom "However, in the presence of ice-shelf buttressing no such simple conclusions can be drawn (e.g Goldberg et al., 2009; Gudmundsson et al. ,2012; Gudmundsson, 2013; Pegler, 2016)." Without wishing to advertise my own work too much, Schoof et al 2017 in the Cryosphere also gives a qualitatively different example (with calving, which I believe differs from the other references given here) where the usual stability argument is reversed.
We added this study to our manuscript.

- multiple instances, for instance figure caption B1. "Exemplary" is not usually used synonymously with "an example of". A brief internet search gives me the following meanings 1. serving as a desirable model; representing the best of its kind. "an award for exemplary community service" synonyms: perfect, ideal, model, faultless, flawless, impeccable, irreproachable; More excellent, outstanding, admirable, commendable, laudable, above/beyond reproach; textbook, consummate, archetypal "her exemplary behavior" antonyms: deplorable 2. (of a punishment) serving as a warning or deterrent. "exemplary sentencing may discourage the ultraviolent minority" synonyms: deterrent, cautionary, warning, admonitory; raremonitory "exemplary jail sentences" You might want to consider whether this is what you mean

Thanks for bringing this to our attention. We changed the wording accordingly.

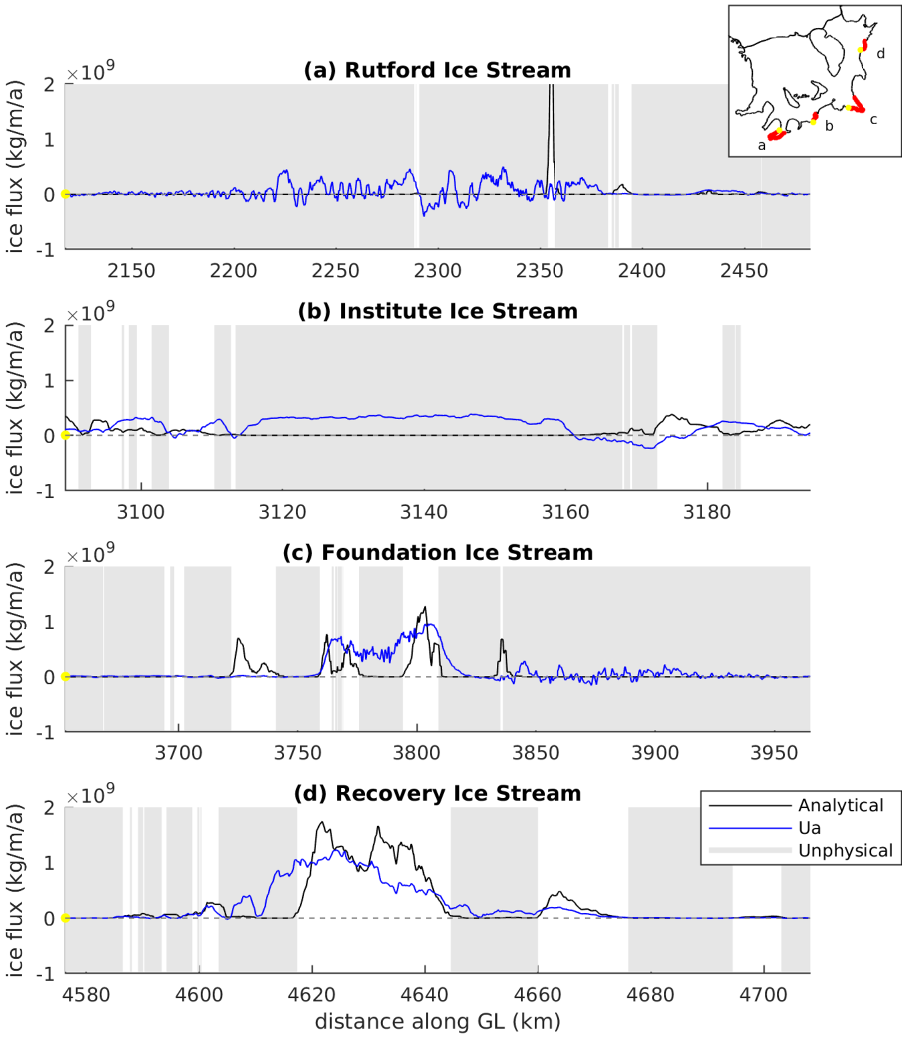

Figure 4: Comparison of fluxes calculated with Úa (blue) and analytical fluxes (black) along the grounding lines of four major ice streams draining into Filchner-Ronne Ice Shelf. All values are smoothed applying a moving average filter with span 5. Locations where the flux formula provides unphysical results are marked in grey. Plotted grounding line segments are located as displayed in the inset with western margins indicated by a yellow dot.

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Grounding-line flux formula applied as a flux condition in numerical simulations fails for buttressed Antarctic ice streams

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Abstract. Currently, several large-scale ice-flow models impose a condition on ice-flux across grounding lines using an analytically motivated parameterization of grounding-line flux. It has been suggested that employing this analytical expression alleviates the need for highly resolved computational domains around grounding lines of marine ice sheets. While the analytical flux formula is expected to be accurate in an unbuttressed flow-line setting, its validity has hitherto not been assessed for complex and realistic geometries such as those of the Antarctic Ice Sheet. Here the accuracy of this analytical flux formula is tested against an optimized ice flow model that uses a highly-resolved computational mesh around the Antarctic grounding lines. We find that when applied to the Antarctic Ice Sheet the analytical expression provides inaccurate estimates of ice fluxes for almost all grounding lines. Furthermore, in many instances direct application of the analytical formula gives rise to unphysical complexed-valued ice fluxes. We conclude that grounding lines of the Antarctic Ice Sheet are, in general, too highly buttressed for the analytical parameterization to be of practical value for the calculation of grounding-line fluxes.

1 Introduction

Estimating the future impact of the Antarctic Ice Sheet (AIS) on global sea levels invariably involves calculating changes in ice fluxes across grounding lines, as well as determining the migration of the grounding lines themselves. Accurately describing grounding-line dynamics can therefore be considered an essential prerequisite for any numerical ice-flow simulation of marine ice sheets such as the AIS. Accordingly, over the last decades, considerable efforts have focused on ensuring that large-scale ice-flow models are capable of capturing correctly the dynamical behavior of grounding lines (e.g. Goldberg et al., 2009; Gladstone et al., 2010; Seroussi et al., 2014; Feldmann et al., 2014; Gagliardini et al., 2016; Pattyn et al., 2017). As part of these efforts, several model inter-comparison experiments have been conducted to assess different approaches within the ice-sheet modeling community regarding the numerical modeling of marine-type ice sheets (Pattyn et al., 2012; Drouet et al., 2013; Pattyn et al., 2013; Asay-Davis et al., 2016). Although still a subject of active research, one of the outcomes of these inter-comparison experiments has been to stress the need for a sufficiently fine resolution of the computational domain around grounding lines. Within the context of the shallow ice-stream computational models — a commonly-used flow approximation for describing the flow of ice streams and ice shelves (e.g., Morland, 1987; MacAyeal, 1989) — it has, for example, been suggested that for many applications a horizontal resolution of around one ~~ice thickness~~ kilometer or less is suitable (Gladstone et al., 2012; Pattyn et al., 2012; Cornford et al., 2016). However, for large-scale ice flow models using uniform grids employing such a high resolution globally for large ice sheets such as the AIS can be computationally prohibitively expensive. As a way of resolving this issue, and to allow for an accurate description of grounding-line dynamics without resorting to high spatial resolution, in a number of numerical modelling studies a 'flux condition' is imposed at the grounding line whereby the grounding-line flux is prescribed using an analytical expression (e.g., Docquier et al., 2011; Thoma et al., 2014; DeConto and Pollard, 2016; Pattyn, 2017)~~or grounding line retreat is decided depending on such an expression~~. In other instances, the grounding-line migration rate is prescribed directly (e.g. without buttressing parameterisation, Ritz et al., 2015).

The analytical flux expression most often used is based on a theoretical study by ~~Schoof (2007a)~~ Schoof (2007a, b) and was derived under the assumption that the ice shelf provides *no buttressing* to the ice at the grounding line. The absence of buttressing implies that the (vertically integrated) horizontal stresses at the grounding line are not affected by the presence of the ice shelf, and were the ice shelf to be removed and replaced by ocean water, the state of stress (in a vertically integrated sense) would remain unaffected (e.g., Schoof, 2007a; MacAyeal and Barcilon, 1988). However, in general, and this is certainly the case for the AIS today (e.g. De Rydt et al., 2015; Fürst et al., 2016; Reese et al., 2017), ice shelves do provide some buttressing. To account for this, numerical models use a modified analytical expression of ice flux based on Schoof (2007a) involving an additional buttressing parameter (θ) describing the modification in axial stress due to the mechanical impact of the ice shelf on the stress state at the grounding line. The buttressing parameter (θ) needs to be calculated by the numerical ice flow model, and then inserted into the analytical flux expression. The resulting flux is then used by the corresponding numerical model as a flux condition along all grounding lines.

Previous numerical model inter-comparison experiments (Pattyn et al., 2012) have shown that in the unbuttressed case there is, in general, a good agreement between the analytically and numerically calculated ice fluxes for steady-state conditions. For one particular synthetic model setup, Gudmundsson (2013) also found, in places, a good agreement between analytically and numerically calculated ice fluxes for buttressed ice. The question now arises as to how accurately the analytical expression predicts grounding-line ice fluxes for *realistic* geometries such as that of the present-day AIS. More specifically, if one were to apply sufficiently high resolution around all Antarctic grounding lines, would fluxes calculated directly by such a high-resolution numerical model agree with the predictions of the analytical flux formula? Answering this question is the subject of this study.

Here we assess the accuracy and the general applicability of the analytical flux formula for calculating ice fluxes across grounding lines of present-day Antarctica. We do this by comparing predicted analytical fluxes with independently numerically calculated ice fluxes using the community ice-flow model Úa (Gudmundsson, 2013). The ice flow model is applied continent-wide, using high spatial resolution around all grounding lines of few hundreds of meters.

The paper is structured as follows: First, we describe our numerical ice flow model Úa, and the model initialization procedure in Sect. 3. We then give a brief overview over the flux formula derived by Schoof (2007a), and discuss several different approaches to quantifying ice-shelf buttressing. ~~We then describe our numerical ice flow model Úa, and the model initialization procedure in Sect. 3.~~ The following Sect. 4 on the comparison between numerically calculated grounding-line ice fluxes and those by the flux formula forms the main part of the paper. This is followed by a discussion of the results and final conclusions, Sect. 5 and Sect. 6.

2 ~~Ice shelf buttressing and grounding-line ice flux~~Model description

~~In Schoof (2007a) , an expression for the grounding-line flux (q) of marine ice sheets is derived. While the analysis is primarily focused on a flow-line configuration where ice-shelf buttressing plays no role, Schoof (2007a) also estimates how the flux might be affected by a reduction θ in axial stress at the grounding line due to ice-shelf buttressing. The resulting analytical flux expression is~~

$$q(x) = \theta^{\frac{nm}{m+1}} \rho_i h^{\frac{1+m(n+3)}{m+1}} \frac{1}{4^n} A(\rho_i g)^{n+1} (1-\rho_i/\rho_w)^n C^{1/m} \frac{m}{m+1}$$

~~where q is the ice flux across the grounding line, h the ice thickness, ρ_i the ice density, ρ_w the density of ocean water and g the gravitational acceleration (please note that in the related Eq. 17 of Gudmundsson (2013) for the flux q there is a typo in the exponent of the basal slipperiness C). For grounded ice, the tangential component of the basal traction (τ_b) is related to the basal velocity (v_b) through the Weertman-type sliding law~~

$$\tau_b = C^{-1/m} |v_b|^{1/m-1} v_b,$$

~~where C is the basal slipperiness, and m the stress exponent, while deviatoric stresses and strain rates $\dot{\epsilon}_{ij}$ in ice flow are linked via Glen's flow law~~

$$_{ij} = A\tau^{n-1}\tau_{ij},$$

~~with $\tau = \sqrt{\tau_{ij}\tau_{ij}/2}$ the second invariant of the deviatoric stress tensor, exponent n (often set to 3) and rate factor A. Here τ_{ij} denote the components of the deviatoric stress tensor and $\dot{\epsilon}_{ij}$ the components of the strain rate tensor.~~

~~As mentioned above θ is a scalar quantity that describes the deviation in deviatoric axial stress at the grounding line from the unbuttressed situation. For an unbuttressed grounding line in one horizontal dimension (i.e. no variations in any quantities transverse to the flow direction) and assuming that the x-axis of the coordinate system is aligned with the flow, we have $\tau_{xx} = \tau_f$ (see Appendix A) where~~

$$\tau_f = \frac{\rho_i g}{4} 1 - \frac{\rho_i}{\rho_w} h.$$

~~In the buttressed case, τ_{xx} is however no longer necessarily equal to τ_f, and θ is defined as~~

$$\theta^{1HD} = \frac{\tau_{xx}^{1HD}}{\tau_f}.$$

~~We have here used the superscript $1HD$ to indicate that this definition of θ is only unambiguous in the one horizontal dimensional situation (1HD). In the more general two horizontal dimensional situation (2HD), where the flow direction is~~

~~not necessarily aligned with the (horizontal) normal to the grounding line, several different definitions of θ are possible, and in the literature at least three different definitions of θ have been suggested. In the following we denote these by θ_1, θ_2, and θ_3, with~~

$$\theta_1 = \frac{\boldsymbol{n}_1 \cdot \boldsymbol{R}\boldsymbol{n}_1}{2\tau_f},$$

~~where $\boldsymbol{n}_1$ is a normal to the grounding line pointing horizontally outwards from the grounded ice into the ice shelf, and~~

$$\theta_2 = \frac{\boldsymbol{n}_1 \cdot \boldsymbol{\tau}\boldsymbol{n}_1}{\tau_f},$$

~~and~~

$$\theta_3 = \frac{\boldsymbol{n}_2 \cdot \boldsymbol{\tau}\boldsymbol{n}_2}{\tau_f},$$

~~where $\boldsymbol{n}_2$ is the direction of ice flow at the grounding line and~~

$$= \tau_{xx}\tau_{xy}\tau_{xy}\tau_{yy},$$

~~is the (horizontal) deviatoric stress tensor, and~~

$$= 2\tau_{xx} + \tau_{yy}\tau_{xy}\tau_{xy}\tau_{xx} + 2\tau_{yy},$$

~~the tensor of resistive stresses. In the 1HD unbuttressed case where $\boldsymbol{n}_1 = \boldsymbol{n}_2$, $\tau_{xx} = \rho_i g h(1 - \rho_i/\rho_w)/4$, and $\tau_{yy} = \tau_{xy} = 0$, all these three definitions of θ result in $\theta_1 = \theta_2 = \theta_3 = 1$. The first definition (i.e. θ_1) has, for example been used by Gudmundsson (2013) to diagnose buttressing at the grounding line of an idealized setup, the second definition by Pollard and DeConto (2012) , Thoma et al. (2014) a Pattyn (2017) as a flux condition, and the third one by Fürst et al. (2016) to diagnose 'flow-buttressing' within Antarctic ice shelves. Note however that for instance Pollard and DeConto (2012, see section 2.3) , Thoma et al. (2014, see section 4.4) , Fürst et al. (2016, see Supplementary Eq. 2) and Pattyn (2017, see Eq. 20) appear to use a different expression for τ_f, with $\tau_f = \rho_i g h(1 - \rho_i/\rho_w)/2$, in which case $\theta = 1/2$ in the unbuttressed case and θ in Eq. (6) must be replaced by 2θ.~~

~~The definition of θ_1 is motivated by the form of the boundary condition at the calving front in the shallow ice-stream approximation (see Appendix A). For $\theta_1 = 1$ the normal traction at the grounding line equals that of a calving front. In the general 2HD situation, this same interpretation does not hold for the definitions of θ_2 and θ_3. If $\theta_1 > 1$ the ice shelf can be considered to be 'pulling' the ice at the grounding line, while $\theta_1 < 1$ implies that the ice shelf is reducing the normal traction at the grounding line. Note that for all these three different definitions, it is possible for θ to become negative. If, however, a negative θ value is inserted into Eq. (6), the resulting value for the flux q is a negative or even a complex number for most combinations of n and m — a clear indication that the analytical flux formula fails in such situations. Only the specific combinations of n and m such that $nm/(m+1) = 2k$ for $k \in \mathbb{N}$ (for instance the combination $n = 3$ and $m = 2$) 'fix' the flux back to a positive real number, however they introduce a non-substantiated dependency between the flow law and the sliding~~

~~law. Furthermore, for these combinations and $\theta < 0$, enhanced buttressing - inconsistently - yields an increase in ice flux. Physically, $\theta_1 < 0$ corresponds to a situation where the traction vector at the grounding line points in upstream direction. One possible situation giving rise to $\theta_1 < 0$ would be $\tau_{xx} < 0$ while $\tau_{yy} = 0$, with x being the flow direction and the grounding line aligned with the y axis. In this case, the ice at the grounding line experiences compression in along-flow direction and, hence, longitudinal strain rates are negative and ice velocities become smaller as the grounding line is approached from upstream direction. Another situation giving rise to $\theta_1 < 0$ is that of equal transversal compression and vertical extension of the ice column at the grounding line, i.e.$\tau_{yy} = -\tau_{zz} < 0$ while $\tau_{xx} = 0$.~~

2.1 ~~Model description~~

We diagnose the fluxes at the grounding line with the finite-element ice-flow model Úa (Gudmundsson, 2013). The flow model Úa has been used to calculate the ice-flow for various geometries involving ice-shelf buttressing (e.g. De Rydt and Gudmundsson, 2016; Royston and Gudmundsson, 2016; Gudmundsson et al., 2017), and results obtained by the model submitted to a number of model inter-comparison experiments (MISMIP, Pattyn et al., 2012) and (MISMIP3d, Pattyn et al., 2013). The model employs an unstructured grid and hence allows for resolving the grounding line zone locally with high resolution. The model further allows for nodal-based or element-based, and simultaneous inversion of the ice rate factor (A) and the basal slipperiness (C) using either Bayesian or Tikhonov type regularization.

Here we use Úa to solve the *shallow ice-stream equations* (e.g., Morland, 1987; MacAyeal, 1989) in a diagnostic mode using a Weertman-type sliding law (see Eq. 7) and Glen's flow law (see Eq. 8). In the glaciological literature the shallow ice-stream equations are also referred to as the Shallow-Shelf/Shelfy-Stream Approximation and often abbreviated as SSA. In 2HD the SSA momentum equations are

$$\nabla_{xy} \cdot (h\boldsymbol{R}) - \boldsymbol{\tau}_{bh} = \rho_i g h \nabla_{xy} s + \frac{1}{2} g h^2 \nabla_{xy} \rho_i \,, \tag{1}$$

where

$$\nabla_{xy} = (\partial_x, \partial_y) \tag{2}$$

and $\boldsymbol{R}$ is the tensor of resistive stresses given by Eq. (15), h is the ice thickness, s the ice surface elevation, ρ_i the ~~vertically averaged~~ vertically-averaged ice density, and $\boldsymbol{\tau}_{bh}$ is the horizontal part of the bed-tangential basal traction $\boldsymbol{\tau}_b$. Where the ice is floating $\boldsymbol{\tau}_{bh} = 0$. In the SSA the flotation criterion has the form $h < h_f$ with

$$h_f = (S - B)\rho_w / \rho_i, \tag{3}$$

where S is the ocean surface, B the bedrock, and ρ_w is the ocean density. The flotation criterion in Úa is evaluated at each integration point of the elements of the finite element mesh and the basal drag term evaluated accordingly through a standard finite-element procedure involving element-wise integration.

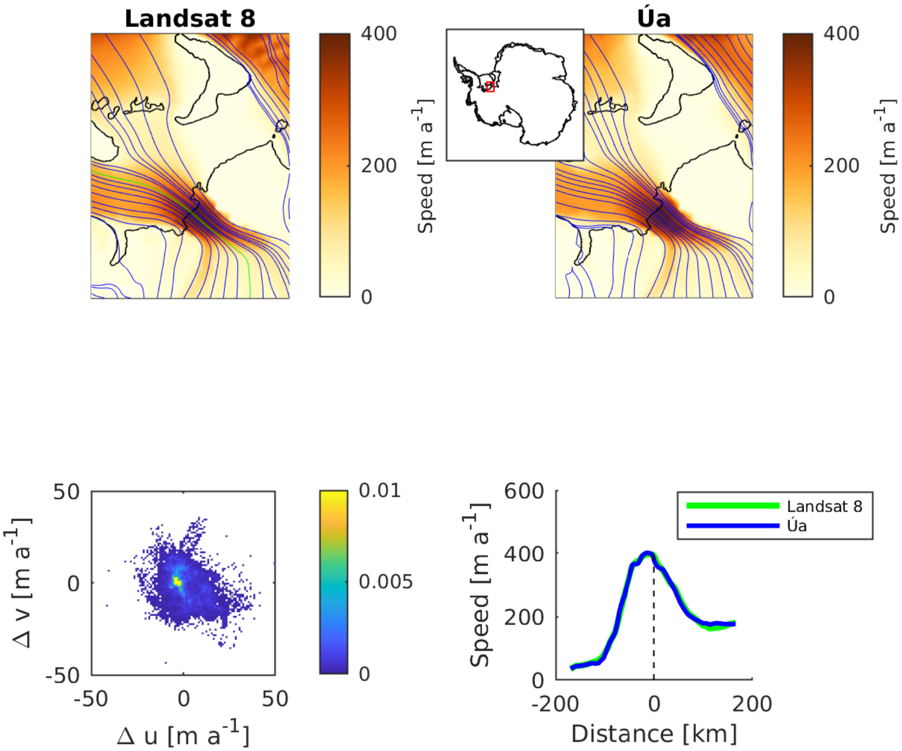

Figure 1. Observed (upper left panel) and modelled (~~middle~~ upper right panel) ice ~~velocities~~ speed in the region of Institute Ice Stream. The inset displays the location of the plotted area in Antarctica. Grounding lines are shown as black lines and ~~colorscales indicate the speed~~streamlines are displayed in blue. The ~~right~~ lower left panel shows a normalised bivariate histogram of the velocity residuals which are the differences between modeled and observed velocities within this area, that is, $\Delta u = u_{\text{modeled}} - u_{\text{observed}}$ and $\Delta v = v_{\text{modeled}} - v_{\text{observed}}$, and u and v are the horizontal components of the surface velocity vector, respectively. The lower right panel shows an ice-speed profile along the central line of Institute Ice Stream that is indicated in green in the upper left panel.

2.1 Methodology

Using the ice flow model Úa, we calculate ice velocities for the entire Antarctic Ice Sheet, including all ice shelves. The SSA equations are solved throughout the computational domain. Stress boundary conditions (i.e. Neumann boundary conditions) are applied at the margins of the computational domain. Since the modeling domain covers the whole of the AIS, no inflow or outflow boundary conditions (i.e. Dirichlet boundary conditions) need to be applied at any sections of the boundary.

Two different computational meshes were generated and the sensitivity of the results evaluated using linear (3-node), quadratic (6-node), and cubic (10-node) triangular elements. All results presented here were obtained using a very high resolution mesh generated with the finite-element mesh generator *Gmsh* (Geuzaine and Remacle, 2009) with $1,360,894$ triangular linear elements and $689,042$ nodes. Within $5\ \mathrm{km}$ distance to the grounding line, the mesh was refined such that element sizes decrease towards the grounding line to a maximum size of $250\ \mathrm{m}$ directly at the grounding line. Overall, the elements have a maximal size of $179,307$ m in the interior of the continent and minimal size of 56 m along the grounding line. Mean element size is 1596 m and median 480 m. A regional example of the mesh is given in Fig. S.1. The robustness of the results was also tested based on the mesh used in Reese et al. (2017), as discussed in Appendix B.

Ice thickness and bed geometry input is based on the Bedmap2 estimates (Fretwell et al., 2013). Vertically averaged ice densities were calculated using firn thickness fields from RACMO2 (Lenaerts et al., 2012) and assuming a constant ice density of $910\,\mathrm{kg\,m^{-3}}$ and a firn density of $500\,\mathrm{kg\,m^{-3}}$. Resulting densities range from $770\,\mathrm{kg\,m^{-3}}$ to $910\,\mathrm{kg\,m^{-3}}$ and the horizontal gradients in vertically averaged densities are hence small, see Fig. S.2. In a few places the bathymetry around grounding lines was vertically modified to improve its alignment with Bindschadler et al. (2011), with vertical adjustments of maximally 50 m being allowed.

For the entire Antarctic setup we inverted for basal slipperiness C (see Eq. 7) and ice softness fields A (see Eq. 8) to match observed 2015/2016 velocities derived from Landsat 8 imagery (Gardner et al., 2017). The stress exponent of Glen's flow law was set to $n = 3$ and we repeated the inversion for a whole sequence of sliding law exponents $m = 1,2,3,4,5,7,9,11$. We inverted for A and C over the computational nodes using Tikhonov type regularization. The inversion procedure minimizes the function

$$J(u,p) = I(u) + R(p)$$

with respect to p, where p stands for model parameters to be determined (i.e. A and C, here C was set to 0 within ice shelves), u are modeled surface velocities, I the data misfit function, and R the regularization term. The misfit function I has the form

$$I(f) = \frac{1}{2\mathcal{A}} \int (\boldsymbol{v}_{modeled} - \boldsymbol{v}_{observed})^2 / e^2 \, dA \tag{4}$$

where $\mathcal{A} = \int dA$ is the total area, $\boldsymbol{v}_{modeled}$ and $\boldsymbol{v}_{observed}$ modeled and observed velocities, respectively, and e data errors. The regularization function R has the form

$$\begin{aligned} R &= \frac{1}{2\mathcal{A}} \int \left(\gamma_s^2 \left(\nabla \left(\log_{10}(p) - \log_{10}(\hat{p}) \right) \right)^2 + \gamma_a^2 \left(\log_{10}(p) - \log_{10}(\hat{p}) \right)^2 \right) dA \\ &= \frac{1}{2\mathcal{A}} \int \left(\gamma_s^2 \left(\nabla \log_{10}(p/\hat{p}) \right)^2 + \gamma_a^2 \left(\log_{10}(p/\hat{p}) \right)^2 \right) dA \end{aligned} \tag{5}$$

where γ_a and γ_s are regularization parameters, and $\hat{p}$ the *a priori* values for model parameters. Inversions were done for a wide ranges of γ_s and γ_a and optimal values determined from an L-curve analysis. In the results shown here, we use $\gamma_a = 1$ and $\gamma_s = 10,000\,\mathrm{m}$. However our results are insensitive to the particular values chosen.

For $\gamma_s = 10,000\,\mathrm{m}$, $\gamma_a = 1$ and the sliding exponent $m = 3$, the corresponding basal slipperiness C and the ice rate factor A distributions are shown in Figs. S.3 and S.4. The average difference between modeled and observed ice speed is 29 meters per year with a median of 13 meters per year and a root mean square error of 103 meters per year. The measured and modeled velocity fields for the region of Institute Ice Stream are displayed in the ~~left and middle~~ upper panels of Fig. 1. They agree well in this area, as the residual histogram for this region shows in the ~~right~~ lower left panel, but also for the entire continent, see Fig. S.5. As a consequence of our inverse methodology, modeled ice velocities are in close accordance with measurements.

From the modeled stresses obtained with our ice-flow model we calculate the buttressing parameter θ as defined in Sect. 3. We do this for each of the three different definitions for θ (see Eqs. 11, 12, and 13). We then calculate the *analytical* fluxes predicted by the flux formula, i.e. Eq. (6). Note that we refer to these fluxes as 'analytical' fluxes although their calculation involves the use of our numerical ice-flow model for estimating the buttressing number θ.

We also calculate *modeled* grounding-line fluxes from modeled ice velocities. Since our modeled velocities are in a good agreement with observed velocities, these modeled grounding-flux estimates will be in an equally good agreement with fluxes estimated directly from observed velocities. The analytical and the modeled flux estimates are then compared and analysed.

When calculating grounding-line fluxes we interpolate nodal quantities of the computational mesh onto the (calculated) grounding line. The grounding line does not, as such, enter the numerical calculations done by our numerical ice flow model. As described in Sect. 2.1, it is the flotation mask — evaluated at the integration points — that determines the impact of the basal drag term. However, in a post-processing step we determine the positions of the grounding lines from the flotation mask. Our approximation of the grounding line is a piecewise linear curve, with each linear segment representing the grounding line within a given computational element (see Figs. S.1 and B1). We then interpolate nodal values onto the central point of each such linear segment. This same procedure is employed when calculating both analytical and modeled fluxes.

3 Ice shelf buttressing and grounding-line ice flux

In Schoof (2007a) , an expression for the grounding-line flux (q) of marine ice sheets is derived. While the analysis is primarily focused on a flow-line configuration where ice-shelf buttressing plays no role, Schoof (2007a) also estimates how the flux might be affected by a reduction θ in axial stress at the grounding line due to ice-shelf buttressing. The resulting analytical flux expression is

$$q(x) = \theta^{\frac{nm}{m+1}} \rho_i h^{\frac{1+m(n+3)}{m+1}} \left(\frac{1}{4^n} A(\rho_i g)^{n+1} (1-\rho_i/\rho_w)^n C^{1/m} \right)^{\frac{m}{m+1}} \tag{6}$$

where q is the ice flux across the grounding line, h the ice thickness, ρ_i the ice density, ρ_w the density of ocean water and g the gravitational acceleration (please note that in the related Eq. 17 of Gudmundsson (2013) for the flux q there is a typo in the exponent of the basal slipperiness C). For grounded ice, the tangential component of the basal traction (τ_b) is related to the basal velocity (v_b) through the Weertman-type sliding law

$$\tau_b = C^{-1/m} |v_b|^{1/m-1} v_b, \tag{7}$$

where C is the basal slipperiness, and m the stress exponent, while deviatoric stresses and strain rates $\dot{\epsilon}_{ij}$ in ice flow are linked via Glen's flow law

$$\dot{\epsilon}_{ij} = A \tau^{n-1} \tau_{ij}, \tag{8}$$

with $\tau = \sqrt{\tau_{ij}\tau_{ij}/2}$ the second invariant of the deviatoric stress tensor, exponent n (often set to 3) and rate factor A. Here τ_{ij} denote the components of the deviatoric stress tensor and $\dot{\epsilon}_{ij}$ the components of the strain rate tensor.

As mentioned above θ is a scalar quantity that describes the deviation in deviatoric axial stress at the grounding line from the unbuttressed situation. For an unbuttressed grounding line in one horizontal dimension (i.e. no variations in any quantities transverse to the flow direction) and assuming that the x-axis of the coordinate system is aligned with the flow, we have $\tau_{xx} = \tau_f$ (see Appendix A) where

$$\tau_f = \frac{\rho_i g}{4}\left(1 - \frac{\rho_i}{\rho_w}\right) h. \tag{9}$$

In the buttressed case, τ_{xx} is however no longer necessarily equal to τ_f, and θ is defined as

$$\theta^{1HD} = \frac{\tau_{xx}^{1HD}}{\tau_f}. \tag{10}$$

We have here used the superscript $1HD$ to indicate that this definition of θ is only unambiguous in the one horizontal dimensional situation (1HD). In the more general two horizontal dimensional situation (2HD), where the flow direction is not necessarily aligned with the (horizontal) normal to the grounding line, several different definitions of θ are possible, and in the literature at least three different definitions of θ have been suggested. In the following we denote these by θ_1, θ_2, and θ_3, with

$$\theta_1 = \frac{\boldsymbol{n}_1 \cdot \boldsymbol{R}\boldsymbol{n}_1}{2\tau_f}, \tag{11}$$

where $\boldsymbol{n}_1$ is a normal to the grounding line pointing horizontally outwards from the grounded ice into the ice shelf, and

$$\theta_2 = \frac{\boldsymbol{n}_1 \cdot \boldsymbol{\tau}\boldsymbol{n}_1}{\tau_f}, \tag{12}$$

and

$$\theta_3 = \frac{\boldsymbol{n}_2 \cdot \boldsymbol{\tau}\boldsymbol{n}_2}{\tau_f}, \tag{13}$$

where $\boldsymbol{n}_2$ is the direction of ice flow at the grounding line and

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_{xx} & \tau_{xy} \\ \tau_{xy} & \tau_{yy} \end{pmatrix}, \tag{14}$$

is the (horizontal) deviatoric stress tensor, and

$$\boldsymbol{R} = \begin{pmatrix} 2\tau_{xx} + \tau_{yy} & \tau_{xy} \\ \tau_{xy} & \tau_{xx} + 2\tau_{yy} \end{pmatrix}, \tag{15}$$

the tensor of resistive stresses. In the 1HD unbuttressed case where $\boldsymbol{n}_1 = \boldsymbol{n}_2$, $\tau_{xx} = \rho_i g h(1 - \rho_i/\rho_w)/4$, and $\tau_{yy} = \tau_{xy} = 0$, all these three definitions of θ result in $\theta_1 = \theta_2 = \theta_3 = 1$. The first definition (i.e. θ_1) has, for example been used by Gudmundsson (2013) to diagnose buttressing at the grounding line of an idealized setup, the second definition by Pollard and DeConto (2012) , Thoma et al. (2014) a Pattyn (2017) as a flux condition, and the third one by Fürst et al. (2016) to diagnose 'flow-buttressing' within Antarctic ice shelves. Note however that for instance Pollard and DeConto (2012, see section 2.3) , Thoma et al. (2014, see section 4.4) , Fürst et al. (2016, see Supplementary Eq. 2) and Pattyn (2017, see Eq. 20) appear to use a different expression for τ_f, with $\tau_f = \rho_i g h (1 - \rho_i/\rho_w)/2$, in which case $\theta = 1/2$ in the unbuttressed case and θ in Eq. (6) must be replaced by 2θ.

The definition of θ_1 is motivated by the form of the boundary condition at the calving front in the shallow ice-stream approximation (see Appendix A). For $\theta_1 = 1$ the normal traction at the grounding line equals that of a calving front. In the general 2HD situation, this same interpretation does not hold for the definitions of θ_2 and θ_3. If $\theta_1 > 1$ the ice shelf can be considered to be 'pulling' the ice at the grounding line, while $\theta_1 < 1$ implies that the ice shelf causes a reduction in normal traction at the grounding line, i.e. the ice shelf 'holds the ice back'. Note that for all these three different definitions, it is possible for θ to become negative. If, however, a negative θ value is inserted into Eq. (6), the resulting value for the flux q is a negative or even a complex number for most combinations of n and m — a clear indication that the analytical flux formula fails in such situations. Only the specific combinations of n and m such that $nm/(m+1) = 2k$ for $k \in \mathbb{N}$ (for instance the combination $n = 3$ and $m = 2$) 'fix' the flux back to a positive real number, however they introduce a non-substantiated dependency between the flow law and the sliding law. Furthermore, for these combinations and $\theta < 0$, enhanced buttressing - inconsistently - yields an increase in ice flux. Physically, $\theta_1 < 0$ corresponds to a situation where the traction vector at the grounding line points in upstream direction. One possible situation giving rise to $\theta_1 < 0$ would be $\tau_{xx} < 0$ while $\tau_{yy} = 0$, with x being the flow direction and the grounding line aligned with the y axis. In this case, the ice at the grounding line experiences compression in along-flow direction and, hence, longitudinal strain rates are negative and ice velocities become smaller as the grounding line is approached from upstream direction. Another situation giving rise to $\theta_1 < 0$ is that of equal transversal compression and vertical extension of the ice column at the grounding line, i.e. $\tau_{yy} = -\tau_{zz} < 0$ while $\tau_{xx} = 0$.

4 Results

From the numerically modeled stress field we calculate the buttressing parameter θ_1 (given by Eq. 11) for all grounding lines of the Antarctic setup described in Sect. 2.1. While we here focus on the buttressing parameter θ_1, our findings are independent of the exact definition of θ, the choice of the sliding law exponents m, the mesh and the details of the inverse methodology applied (see Appendix B).

We find that the grounding lines of Filchner-Ronne and Ross ice shelves are, in general, highly buttressed with buttressing values significantly different from unity (see Fig. 2). Typically, $\theta_1 \leq 0.4$, and in many cases $\theta_1 < 0$. Among the ice streams of these two biggest ice shelves of the AIS, the dormant Kamb Ice Stream is the relatively least buttressed one, with $\theta_1 \approx 0.4$. Over all other ice streams θ values are even smaller. Negative θ values are also found over grounding-line segments located between active ice streams, for example along the grounding line running between the Rutford and Institute Ice Streams.

An example of an ice stream where $\theta_1 < 0$ over most of its grounding line is the Institute Ice Stream (see Figs. 3 and 4). Inspection of the velocity field in the vicinity of the grounding line of that ice stream reveals that ice flow velocities decrease

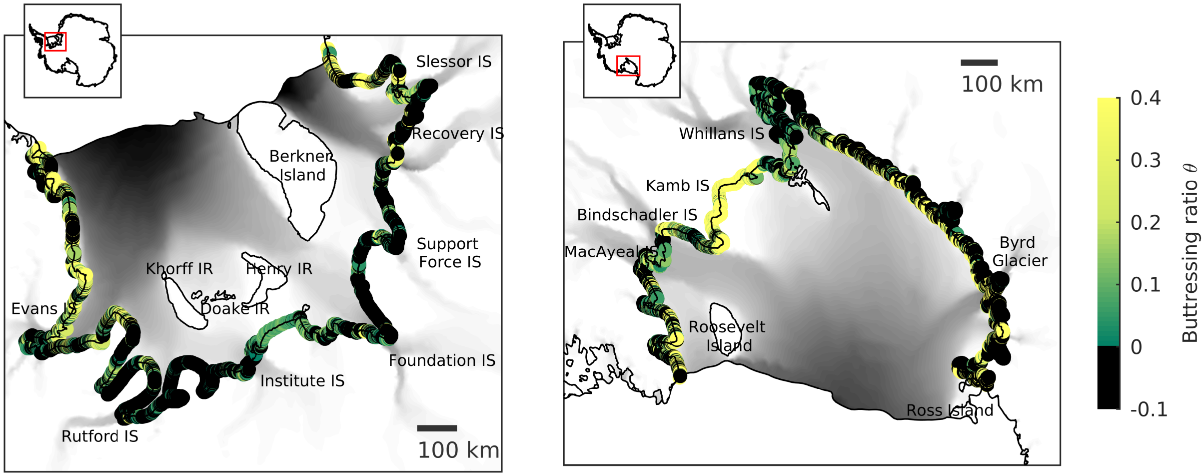

Figure 2. Buttressing ratio θ_1 along the grounding lines of Filchner-Ronne Ice Shelf (left panel) and Ross Ice Shelf (right panel). Insets indicate the ice shelves' locations in Antarctica, correspondingly. Regions where the grounding line is 'over-buttressed', that is, $\theta \leq 0$, are displayed in black. Modelled speed is plotted in gray ranging up to $1,500\ \mathrm{ma}^{-1}$. Grounding line and ice front locations are indicated in black. IS denotes ice streams, IR denotes ice rises or rumples.

with distance as the grounding line is approached from up-stream direction (see also Fig. 1, lower right panel). Consequently, both along-flow strain rates and along-flow deviatoric stresses are negative (compressive). This general feature of ice flow around the grounding line of Institute Ice Stream implies that its grounding line is 'over-buttressed' with the traction vector at the grounding line pointing in inland direction. Hence, independently of our numerical simulation of the stress field, it is clear that for this ice stream θ must be negative.

As discussed in Sect. 3 the analytical flux formula (Eq. 6) is clearly not applicable in situations where θ becomes negative~~(see also Sec. 3)~~. As θ is found to become negative over large sections of the grounding lines of many the ice streams of the two largest Antarctic ice shelves, i.e. Ross and Filchner-Ronne ice shelves, it follows that the formula ~~can not~~*cannot* be used to calculate grounding-line ice fluxes over significant parts of the AIS.

We furthermore compare analytical and numerically modeled grounding-line ice fluxes in all regions where $\theta_1 \geq 0$, i.e. where the application of the analytical flux formula (Eq. 6) results in real-valued ice fluxes. In particular we compare both the flux values point-wise along all grounding lines (Fig. 5) and the total cumulative fluxes over grounding-lines of ice streams and ice shelves (Table 1). When comparing cumulative analytical fluxes, we are forced to assume values for those sections of grounding lines for which θ is negative (and q complex). There we assume $q = 0$, which is equivalent to setting $\theta = 0$.

In general, we find significant differences between analytically calculated and numerically modeled flux values. Analytical fluxes are much lower than modeled in many locations of Filchner-Ronne Ice Shelf, especially along the grounding lines of the Rutford, Institute and Moeller ice streams (Fig. 5). However, cumulative analytical fluxes over all grounding lines of the Filchner-Ronne Ice Shelf are about 30% larger than modeled for θ_1, and this difference is considerably larger for θ_2 and

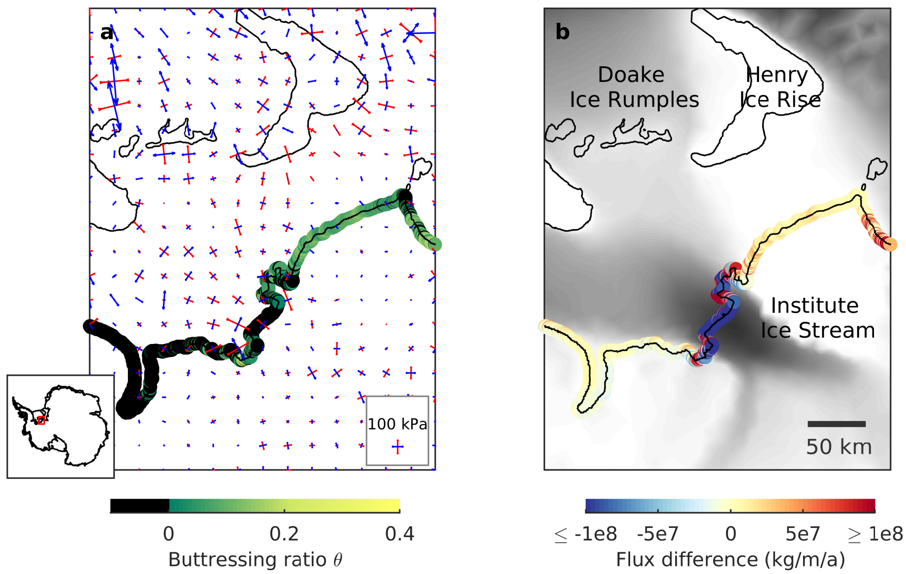

Figure 3. Buttressing ratio and differences in grounding line flux for Institute Ice Stream draining into the Filchner-Ronne Ice Shelf (location shown in inset). (a) Buttressing values θ_1 are displayed along the grounding line and principle deviatoric stresses are shown, with compression in red and extension in blue. The length of the vectors indicate the magnitude of each principle stress. (b) Differences between analytical and modeled fluxes and observed ice velocities ranging up to $500\,\mathrm{m\,a^{-1}}$ (Gardner et al., 2017). Analytical fluxes are set to 0 where $\theta_1 < 0$. Grounding line positions are indicated in black.

θ_3 (Table 1). Similar disagreement between analytical and modeled fluxes is found for the Siple Coast Ice Streams such as Bindschadler and MacAyeal Ice Streams, and for Byrd Glacier (Panel b of Fig. 5). For Ross Ice Shelf the overall difference is only 5%, but given the fact that θ_1 is negative over significant sections of its grounding line (where we set the analytical flux values to zero), this agreement appears somewhat fortuitous.

For other ice shelves, cumulative fluxes are generally underestimated by the flux formula. Analytical fluxes for Pine Island Glacier and Thwaites Glacier, for example, deviate by -33% and -52% from the modeled fluxes, respectively. For George VI ice shelf, cumulative analytical fluxes are several times smaller than modeled ones (Table 1).

The analytical flux formula tends to strongly overestimate fluxes over grounding lines where ice flow is approximately tangential to the grounding line. This failure of the flux formula to correctly predict fluxes in such circumstances is not surprising as the underlying assumptions of the formula are clearly not met in such situations. Nevertheless, this demonstrates the inherent conceptual difficulties in applying the formula to the Antarctic Ice Sheet.

Moreover, the analytical formula produces much higher spatial variability in fluxes than the numerically modeled ones. This can be clearly seen in Fig. 4 where analytical and modeled ice fluxes are plotted along the grounding lines of Rutford, Institute, Foundation and Recovery ice streams. Here, gray background indicates sections of the respective grounding lines where the flux formula yields unphysical results. Variability in fluxes calculated with Úa occurs when ice flow is nearly aligned with the

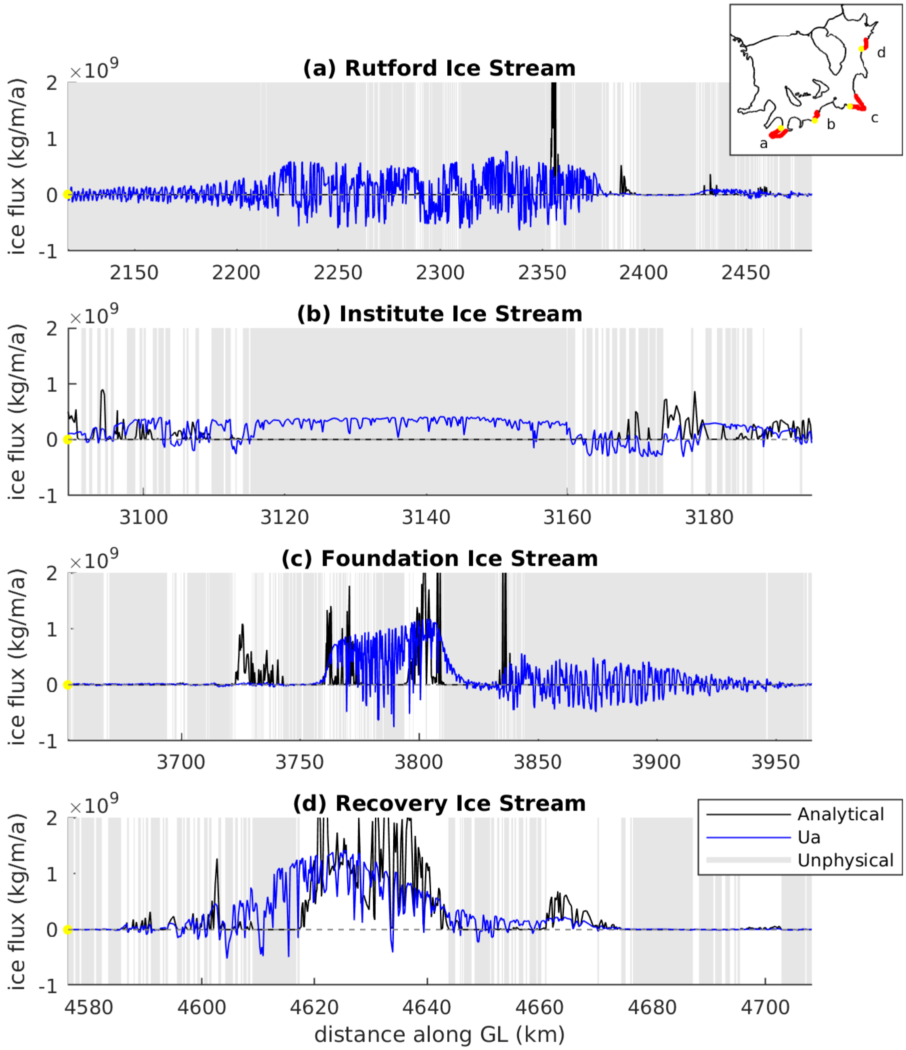

Figure 4. Comparison of fluxes calculated with Úa (blue) and analytical fluxes (black) along the grounding lines of four major ice streams draining into Filchner-Ronne Ice Shelf. Locations where the flux formula provides unphysical results are marked in grey. Plotted grounding line segments are located as displayed in the inset with western margins indicated by a yellow dot.

grounding line. We calculate fluxes within each triangular element using the normal of the piecewise-linear grounding line curve which may vary in-between individual line segments.

We test the sensitivity of our analytical flux calculations to different degrees of regularization (γ_s and γ_a) and different values of the sliding law stress exponent (m), for which our findings are summarized in Figs. B3 and B5. Numerically modeled fluxes are, as expected, mostly independent of the value of the sliding law stress exponent m. This can be considered to be a consequence of the inversion procedure, which ensures that modeled velocity fields agree closely with measured data,

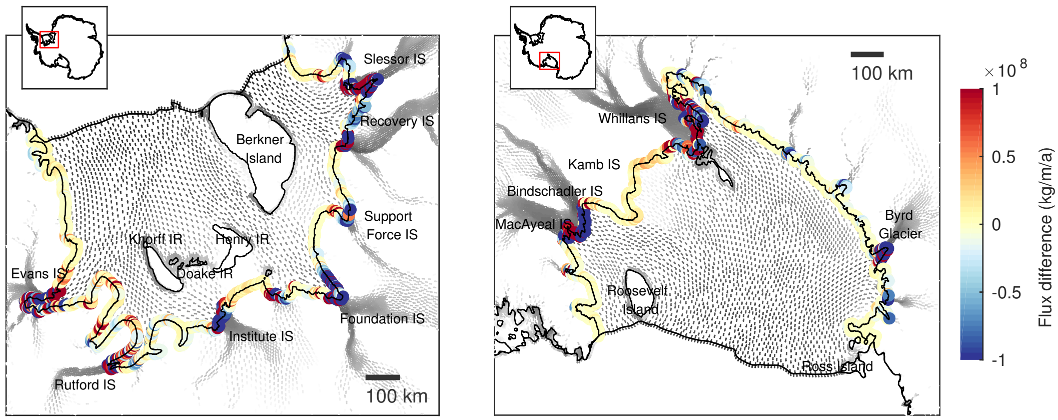

Figure 5. Difference between the analytical and the modeled fluxes along the grounding lines of Filchner-Ronne Ice Shelf (left panel) and Ross Ice Shelf (right panel). Analytical fluxes are calculated based on θ_1 defined in Eq. 11. In locations where the formula yields unphysical results, fluxes are set to zero. Grey arrows show the modeled ice flow. IS denotes ice streams, IR denotes ice rises or rumples. Grounding line and ice front locations are indicated in black.

independently of the value of m. On the other hand, analytically calculated flux values are highly sensitive to the value of m (see Fig. B3). For example, cumulative analytical fluxes for Filchner-Ronne Ice Shelf increase by about a factor of five as m is changed from 1 to 7, while numerically modeled fluxes change by less than 10 %. Numerically modeled fluxes are also insensitive to the exact degree of regularization applied, whereas analytically calculated flux values change significantly (Fig. B5). The dependency of the analytically calculated fluxes on the amount of regularization used in the numerical model is due to the impact regularization has on modeled stresses and, therefore, on the value of θ.

We also compare analytical fluxes as calculated using the three different definitions (11), (12) and (13) for θ. While overall spatial variability of θ is similar for these three definitions, with all definitions giving rise to extended areas of negative θ values, the cumulative flux for the alternative definitions θ_2 and θ_3 are generally higher than for θ_1 (see also Fig. B4).

Table 1. Ice flux integrated along the grounding lines of Antarctic ice shelves. $Q_{\text{Úa}}$ denotes the modeled ice flux with Úa, Q_1 was derived from the analytical flux formula based on θ_1, Q_2 based on θ_2 and Q_3 based on θ_3, respectively. Last column shows the deviation of the analytical flux Q_1 from the modeled $Q_{\text{Úa}}$.

Ice shelf	$Q_{\text{Úa}}$ $[\text{Gt a}^{-1}]$	Q_1 $[\text{Gt a}^{-1}]$	Q_2 $[\text{Gt a}^{-1}]$	Q_3 $[\text{Gt a}^{-1}]$	$(Q_1 - Q_{\text{Úa}})/Q_{\text{Úa}}$ [%]
Filchner-Ronne	216	282	694	755	30
Pine Island	123	82	148	190	-33
Ross	120	126	280	155	5
Thwaites	117	57	82	133	-52
Getz	91	27	52	60	-70
Totten	65	44	158	243	-32
George VI	64	9	21	21	-85
Amery	55	16	135	56	-70
Moscow-University	43	16	44	120	-63
West	40	27	33	49	-32
Shackleton	37	20	62	61	-48
Crosson	34	17	38	38	-51
Larsen C, D	25	9	19	38	-64
Brunt/Stancomb-Wills	22	18	24	40	-16
Fimbul	21	7	15	15	-67
Stange	16	3	13	15	-81
Riiser-Larsen	12	9	20	25	-29
Dotson	11	2	13	19	-84

5 Discussion

The analytical grounding-line flux formula Eq. (6) was derived for a flow-line configuration (Schoof, 2007a), and there is no reason to doubt its validity in that particular case. When applied to a flow-line configuration, many current ice-flow models employing the shallow ice-stream approximation (SSA) with Weertman-type sliding law, have demonstrated an excellent agreement between modeled and analytical grounding-line fluxes (Pattyn et al., 2012). Ice fluxes and grounding line positions calculated with the ice flow model Úa also agree closely with those predicted by Eq. (6) where such an agreement is to be expected. The inclusion of the buttressing parameter θ was used by Schoof (2007a) to illustrate the potential impacts of ice-shelf buttressing on ice flux, provided its effects were sufficiently small as not to invalidate too strongly the basic assumption of a flow-line setting. However, we find that most of the grounding lines of the AIS are highly buttressed with θ significantly different from unity. ~~Therefore the assumptions of the flux formula are in most cases not met.~~ It seems likely that at least part of the reason why the analytical flux formula fails relates to the high degree of buttressing that we find to be characteristic for most Antarctic ice streams.

When applied to the current geometry and the current flow field of the AIS, the flux formula predicts either unphysical or highly inaccurate flux values when compared to modeled ones. While we have done the comparison with numerically modeled fluxes, comparison with observed fluxes — ~~as~~ calculated from measured surface velocities, observed grounding-line positions, and measured ice thicknesses — would not alter our conclusions as, due to our inversion procedure, observed and modeled surface velocities are in good agreement.

The strongest indication that the analytical flux formula fails when applied to the Antarctic Ice Sheet is arguably the fact that it predicts non-real valued fluxes over significant parts of Antarctic grounding lines. This happens whenever θ becomes negative~~except~~. Although for specific combinations of n and m ~~which also yield unphysical solutions as discussed in~~ (such as $n = 3$ and $m = 2$) the resulting exponents in the flux formula are even numbers — in which case the analytical fluxes are always real positive numbers — the flux values are still unphysical (see Sect. 3). As we point out above, even a cursory inspection of the velocity field of the AIS suffices to show that θ is negative for a number of grounding lines (e.g. the Institute Ice Stream grounding line). Hence, the occurrence of negative θ values is not simply a feature of our particular numerical approach, but a general aspect of the current ice-flow regime of the AIS.

As analytical ice fluxes are strongly dependent on ice thickness (h) at the grounding line, they depend somewhat on the specifications of the numerical model: the exact location of the grounding line is influenced by the mesh resolution used by the model. The resulting error is an example of a discretization error that becomes smaller as the mesh is refined. Other numerical models using a different computational mesh may locate the grounding line differently and hence calculate flux values different to some extent. We tested the dependency of our modeled ice fluxes to grid resolution by using several different meshes — an example of two such meshes is given in Fig. B1 — and found none of our main conclusions to be affected by differences in mesh resolution.

As measured by the buttressing parameter θ_1, almost all grounding lines of the AIS can be considered to be strongly buttressed with, in most cases, $\theta < 0.4$. Hence, theoretical concepts based on the assumption of none, or insignificant, ice-shelf buttressing may not apply to present-day Antarctica. One such theoretical prediction of considerable relevance for the possible future of the AIS relates to the stability of its grounding lines. In the absence of ice-shelf buttressing, grounding-line stability is predicted to be related to local bed slope (Weertman, 1974; Thomas and Bentley, 1978; Schoof, 2007a, b, 2011). However, in the presence of ice-shelf buttressing no such simple conclusions can be drawn ~~(e.g Goldberg et al., 2009; Gudmundsson et al., 2012; Gudmu~~ Possibly, rather than being dominated by local bed slope, the stability regime of the Antarctic Ice Sheet is to a leading-order dependent on the properties of the ice shelves downstream of its grounding lines (e.g. geometry and structural integrity), as also supported by, e.g., Pegler et al. (2013); Haseloff and Sergienko (2018) . Further work is needed to address the question of the stability of Antarctica's grounding lines.

6 Conclusions

In our study, we compare grounding-line ice fluxes obtained by an ice-sheet model with fluxes predicted by an analytical flux formula based on ~~Schoof (2007a)~~ Schoof (2007a, b) . The formula includes a parameter (θ) to account for ice-shelf buttressing, and the resulting flux is sometimes applied as a grounding-line flux condition in numerical simulations. We find that the formula results in unphysical and grossly inaccurate grounding-line fluxes for most of the AIS. We furthermore find that almost all Antarctic grounding lines are highly buttressed, suggesting that the ~~assumption~~ underlying assumptions of the analytical flux formula ~~of weakly buttressed grounding lines is~~ are not met for the current configuration of the Antarctic Ice Sheet.

Appendix A: Vertically integrated stress boundary condition at a free calving front

A derivation of the boundary condition at the calving front for the momentum equations in 2HD can be found for example in Cuffey and Paterson (2010) and van der Veen (1999). At the calving face:

$$\int_b^s \sigma \cdot n_{\mathrm{cf}}\, dz = -\int_b^S p_w \cdot n_{\mathrm{cf}}\, dz$$

where $n = (n_x, n_y, 0)$ is the normal of the calving front pointing outwards, s the ice surface, S the sea-level and p_w is hydrostatic pressure in the ocean $p_w = \rho_w g(S-z)$. The balance in x-direction reads:

$$\int_b^s (\sigma_{xx} n_x + \sigma_{xy} n_y)\, dz = \int_b^S -\rho_w g(S-z) n_x\, dz = -\frac{\rho_i^2 g}{2\rho_w} h^2 n_x \tag{A1}$$

We can rewrite $\sigma_{xx} = 2\tau_{xx} + \tau_{yy} + \sigma_{zz}$ (since $\sigma_{xx} = \tau_{xx} + p$, $\sigma_{zz} = \tau_{zz} + p$ and $\tau_{xx} + \tau_{yy} = -\tau_{zz}$). Under the assumptions of the hydrostatic approximation, $\sigma_{zz} = -\rho_i g(s-z)$. The vertically integrated horizontal stress balance equals

$$\int_b^s (\sigma_{xx} n_x + \sigma_{xy} n_y)\, dz = 2h\tau_{xx} n_x + h\tau_{yy} n_x + h\tau_{xy} n_y - \frac{\rho_i g}{2} h^2 n_x, \tag{A2}$$

since $\tau_{xx}, \tau_{yy}, n_x$ and n_y do not vary vertically. Inserting this in Eq. A1 yields:

$$(2\tau_{xx} + \tau_{yy}) n_x + \tau_{xy} n_y = \frac{\rho_i g}{2}\left(1 - \frac{\rho_i}{\rho_w}\right) h n_x. \tag{A3}$$

Similarly for the y-direction. This can be abbreviated as

$$R \cdot n = \frac{\rho_i g}{2}\left(1 - \frac{\rho_i}{\rho_w}\right) h n. \tag{A4}$$

Following Gudmundsson (2013) we obtain the normal buttressing value which compares the RHS and LHS of the equation above in direction of the normal n at the grounding line:

$$\theta = \frac{n \cdot Rn}{\frac{\rho_i g}{2}\left(1 - \frac{\rho_i}{\rho_w}\right) h} = \frac{n \cdot Rn}{2\tau_f}. \tag{A5}$$

In the case of a laterally uniform unconfined ice shelf with $\tau_{yy} = 0$ and $\tau_{xy} = 0$, this reduces to τ_{xx}/τ_f.

A different approach to define θ would be based on this vertically integrated stress boundary condition in 1HD with $\theta^{1HD} = \tau_{xx}/\tau_f$. In 1HD the normal at the grounding line is equal to the flow direction. In 2HD, this is not necessarily true. Thus, to generalize the longitudinal direction in the 1HD buttressing ratio, a choice needs to be made. The longitudinal direction can either be generalized as the normal at the grounding line (θ_2) or as the flow direction (θ_3).

Appendix B: Consistent results using different model parameters

We test the robustness of our findings with respect to the mesh, the sliding law stress exponent m, the definition of the buttressing parameter θ and the regularization parameter γ_s. In a second Antarctic setup, based on a different, continent-wide mesh with quadratic base functions (instead of linear elements, see Fig. B1), we find a similar pattern of $\theta_1 \leq 0$ which yields similar flux differences as exemplified in Fig. B2 for the Filchner-Ronne Ice Shelf. In this case, inversion was done for element-based basal slipperiness and ice softness (instead of inverting on a nodal basis) using a Bayesian methodology (instead of Tikhonov regularization) and the MEASURES velocity data set (Rignot et al. (2011) instead of Landsat 8 (Gardner et al., 2017)). This setup is further described in Reese et al. (2017). In this setup, Bedmap2 bathymetry is not adjusted around the grounding line. This indicates that the exact location of the grounding line does not affect our findings.

For the Antarctic-wide setup described in Sect. 2.1, we test for the choice of the stress exponent m in the sliding law. Different choices $m = 1, 3, 7$ yield good agreement in modeled fluxes but large disagreement in-between analytical fluxes, see Fig. B3. Comparing shelf-wide integrated fluxes for major Antarctic ice shelves shows that also the definitions θ_2 and θ_3 of the buttressing parameter yield large deviations from the modeled fluxes, see Fig. B4. Similarly, we find that the choice of the regularization parameter γ_s does not influence the results significantly, see Fig. B5. Our findings are hence independent of the details of numerical modeling choices.

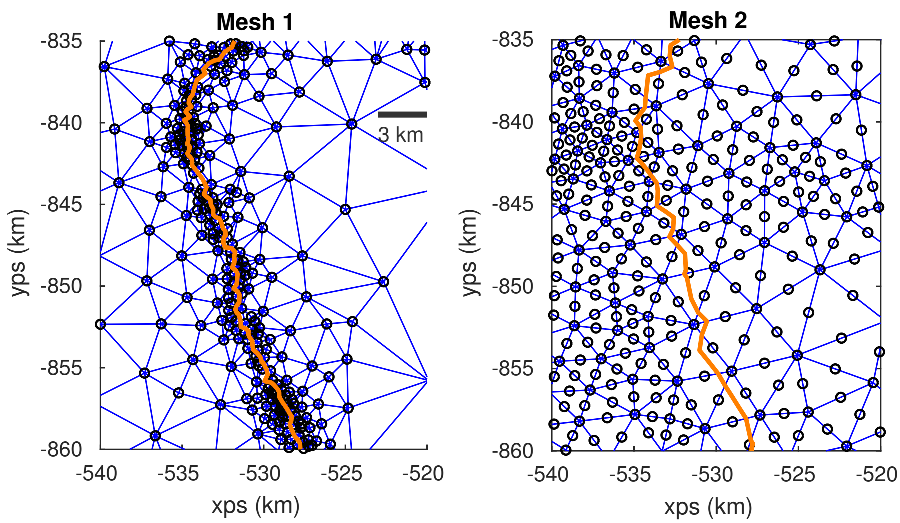

Figure B1. ~~Exemplary zoom~~ Zoom into the Bindschadler grounding line for two different meshes. Left panel: elements and nodes of the mesh presented in the main text. The mesh was refined especially around the grounding line and linear 3-node elements were employed. Right panel: alternative mesh with 6-node elements with quadratic base functions. The grounding line position is indicated in both meshes in orange.

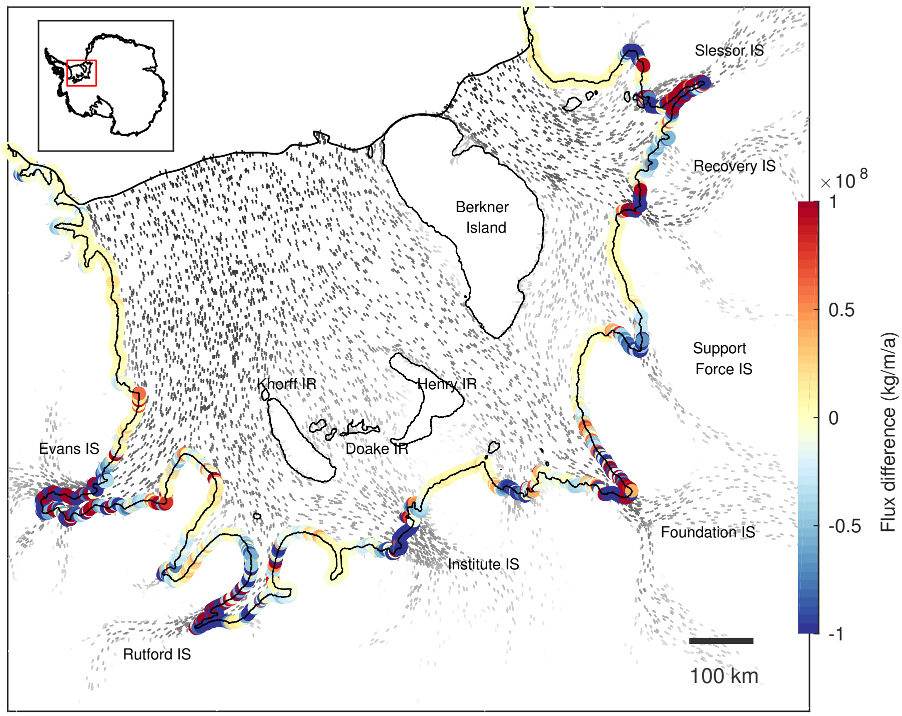

Figure B2. Difference between formula-derived and modeled fluxes along the grounding lines of Filchner-Ronne Ice Shelf. In contrast to Fig. 5 a different mesh was employed (exemplified in the right panel in Fig. B1), the data assimilation was conducted using Bayesian inversion and based on the MEASURES velocity data set (Rignot et al., 2011). The analysis was done using quadratic elements. This Antarctic-wide setup is described in more detail in Reese et al. (2017).

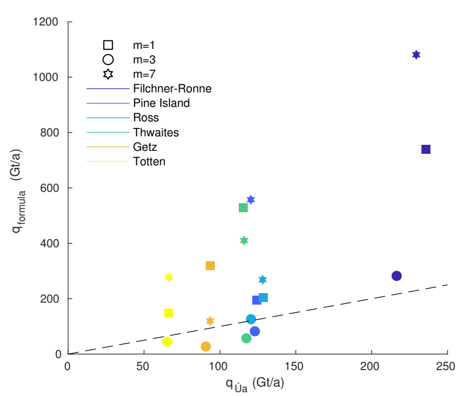

Figure B3. Comparison of fluxes calculated with Úa (~~x-axis~~x-axis) and with the analytical flux formula (~~y-axis~~y-axis), integrated along the grounding lines of ~~exemplary~~ the ice shelves indicated in the legend. Symbols indicate the different sliding law exponents $m = 1,3,7$ employed. All other parameters agree with the reference run (indicated by a circle). The ~~dotted~~ dashed line shows where fluxes calculated with Úa and predicted by the formula would agree.

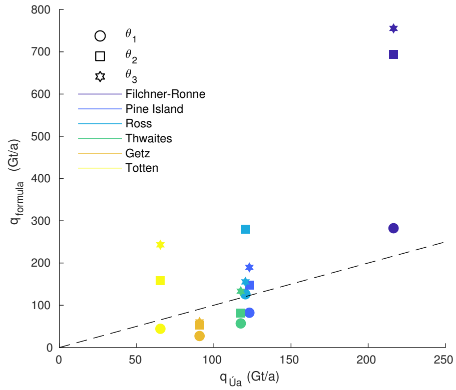

Figure B4. Comparison of fluxes calculated with Úa (~~x-axis~~x-axis) and with the extended flux formula (~~y-axis~~y-axis), integrated along the grounding lines of ~~exemplary~~ the ice shelves indicated in the legend. Symbols indicate the different definitions of θ as described in Sect. 3. All other parameters agree with the reference run (indicated by a circle). The ~~dotted~~ dashed line shows where fluxes calculated with Úa and predicted by the formula would agree.

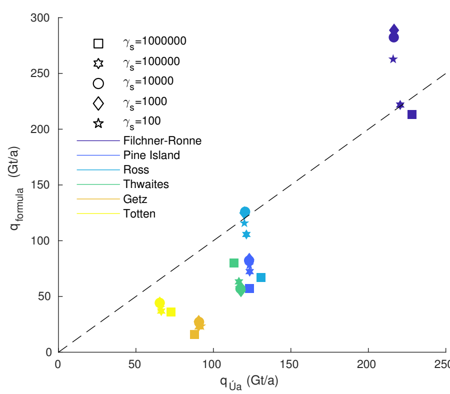

Figure B5. Comparison of fluxes calculated with Úa (~~x-axis~~x-axis) and with the extended flux formula (~~y-axis~~y-axis), integrated along the grounding lines of ~~exemplary~~ the ice shelves indicated in the legend. Symbols indicate the different regularization parameters γ_s used. All other parameters agree with the reference run (indicated by a circle). The ~~dotted~~ dashed line shows where fluxes calculated with Úa and predicted by the formula would agree.

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