Relevant changes made in the manuscript

- The introduction is restructured in 3 distinct subsections, and better settles the oldest-ice context, and the physical mechanisms that have to be accounted for in this research.

- The “heat model” section is reordered so that the heat equations, and boundary conditions are presented separately from the velocity model, and from the assumption under which the model is run.

- The discussion section is reordered to better distinguish between the comparison with litterature data, the model assessement, and the consequences for the old-ice research.

- More information is added concerning the shape parameter p, its validity and underlying assumptions (§2.3)

- Assumptions concerning the local origin of the basal water are reworded an put ogether with discussion on surface slope (§2.4.1)

- The §5.2 concerning the construction of the melt rate map is made more simple, since further mathematical explanations could lose the reader.

- Presentation of §5.4 concerning the emulator o the critical thickness is reworded.

Answers to referee #1

The paper presents a new method for estimating the geothermal heat flux in regions of slow flowing ice. The authors apply the method to the Dome C region. The study makes use of a combination of radar data to infer wet/dry conditions, the one-dimensional heat equation and inverse methods. The authors first construct a time-dependent, one-dimensional heat model including vertical advection of ice. The model is forced with past temperature and accumulation rates reconstructed from a deep ice core. The geothermal heat flux is initially assessed at ten spots, where the bed is known to change from wet to dry conditions from radar measurements. The geothermal heat flux is estimated by calculating a critical ice thickness necessary for basal melt, and then applying an inverse method to get the most likely geothermal heat flux. These values are interpolated to the entire region. The heat flux field is then used to calculate melt rates and the authors then arrive at a parameterisation for the melt rate that depend on geothermal heat flux, ice thickness and ice-flow parameter p. This parameterisation is used to calculate the melt rate over time in the region.

MAIN CONCERNS
Overall, the scientific method is sound and it is a nice combination of radar observations, simple ice-flow assumptions and inverse methods. The use of rational assumptions such as negligible variation of the geothermal heat flux on small spatial scales is a good example of how a complicated and under-measured parameter may be simplified. However, I found the structure of the manuscript rather confusing to a point where it detracts from the scientific content. I have listed some of my main points of concern below but overall the manuscript would greatly benefit from a critical revision by the authors with regards to structure, presentation and grammar.

We would like to thank the reviewer for the fruitful comments made on our work. We agree that the structure of certain sections needed to be modified to make the article more clear-cut. In particular, the introduction section now benefits from a better explained 'oldest ice' context, and section 2 (method) and 6 (discussion) were reordered. We also agree with almost all the minor comments and tried to correct the text accordingly.

The present version of the paper has been corrected by a native speaker for english wording.

When needed, the old sentences are colored in green with corresponding line, the new ones in red.

1. Introduction: It is never explicitly mentioned what “old-ice” is. I assume it refers to the on-going international effort of locating ice that is more than 1.2mio years old but the manuscript does not state this nor is the reader told why this is important. Instead the introduction jumps between general statements about geothermal heat and radar data processing, and specific descriptions of a dataset from the region. It is only at the end of p. 3 that the reader is told what the aim of the study is. I suggest splitting the introduction into three sections: i) A general introduction to why “old ice” is important etc. including the general effect of geothermal heat flux and ice thickness on basal melting, ii) an overview of past studies of radar data processing and what have been achieved so far with this technique, and

iii) an introduction to the study region and the specific dataset that this study uses. The authors are of course free to use a different structure but I strongly recommend rewriting the introduction in one way or another. Finally, a figure with a context map would be very helpful. For example, the introduction mentions studies from Thwaites Glacier, West Antarctica, but the Dome C region is in the central part of East Antarctica. A map could prevent confusion as to why the geothermal heat flux values differ significantly between the two sites.

We followed the above suggestions, in particular to explain how this work is involved in the more general frame of the oldest-ice research. We also added a configuration map. The introduction is now clearly separated in 3 subsections :
1.1) The oldest ice research
Why oldest ice, and how basal melting can be avoided
1.2) GF assessment methods unders ice sheets
Concerning Thwaites Glacier, we wanted to point out the method and its accuracy, not the particular value of the GF.
1.3) Exploitation of available RES dataset around Dome C
Here we explain how we want to use available RES data, and we set the goal of the paper : L. 107 : “this work aims at constraining sites known to be frozen today and that are very likely to have been frozen in the last 800 ka as well.”

2. Heat model: This section consists of several short subsections that not always follow each other in a logical order. For example, the one-dimensionality of the heat equation is presented first as a model assumption in section 2.1.1., then expanded on in section 2.2 and the reader is presented with the equations in 2.3 and 2.4, while the values of the parameters in the equations are mentioned in sections 2.5 and 2.6. It would greatly improve the readability of this section if the heat model is described first in its entirety, then the velocity model and then the assumptions about geothermal heat flux and water circulation.

We reordered the text as suggested for the “Heat model” section.

2.1 Geometry and coordinate system
2.2 Heat equation
    2.2.1 Ice thermal properties
    2.2.2 Basal boundary conditions
    2.2.3 Boundary condition at the surface
2.3 Velocity model
2.4 Proceeding assumptions
    2.4.1 Correspondance between wet and thawing areas
    2.4.2 GF spatial variability

3. Basal melt rate emulator: What is the advantage of the “emulator“ (I assume this is the same as a parameterisation)? The model is run for the whole domain over the period of 800kyr so why is the parameterisation needed? Can’t the model be applied directly to the different scenarios? Is it too computationally expensive?

We understand “emulator” as the process we use to empirically replace the whole heat model, to save computation time. We added this new sentence to make it more clear :

L. 344 : “As the computation for a given set of parameter lasts several minutes, computing the basal melt rate for each point of our domain would be far too expensive. Here, the result of the whole forward model is mimicked by an emulator that depends on the input parameters H, Φ g and p”

Giving an emulator will also be useful to help anyone who wants to compute realistic basal melt rate with a future refined bed for example, and starting from the map of GF.

4. Discussion: Again, I find the order of the sections confusing. The model assumptions and sensitivity tests are followed by a comparison to other studies and then a discussion of the geothermal heat pattern followed by a section titled “Interpretation” (interpretation of what?). I suggest having a separate section with comparison between this study and previous studies, then the discussion section that could start with the overall interpretation and then the discussion of sensitivity tests etc. in the context of the interpretation.

Following your suggestions, we reordered the section :
6.1 Consistency with published data and measurements
6.2 Model assessment
    6.2.1 Method validity
    6.2.2 Sensitivity to parameters
    6.2.3 Spatial variation of the GF field
6.3 Lessons drawn for the oldest-ice research
    6.3.1 Interpretation of the wet/dry pattern at the ice base
    6.3.2 Old-ice targets

MINOR COMMENTS
There are several small typos e.g. “southest”, “explicitely”, “conlude”, “flown” instead of “flowed”, “extend” instead of “extent”, “additionnal” that need to fixed. In addition, I have the following comments
Line 5-7: there is a word missing
The new sentences are the folowing :
“But, since basal conditions depend on heat transfer forced by climate but lagged by the thick ice, the basal ice may currently be frozen whereas in the past it was generally melting. For that reason, the risk of bias between present and past conditions has to be evaluated.”

Line 19: which ice core?
The EPICA Dome C ice core (EDC) is now explicitely mentionned.

Line 56: What are internal layers? Presumably radar layers but this need to specified and explained why they can be used.
L 56 : “basal melt rates have been estimated by fitting the vertical strains with dated internal layers”
Radar layers are used to constrain the vertical advection of ice, which is one of the component of the energy budget. The sentence is now :
L.80 : “basal melt rates for the region north of Dome C have been estimated by fitting the

vertical strain rates with dated radar layers, to constrain the vertical advection of ice, and its energy budget"

Line 60: How can the method of Carter et al. be used in this study without using the internal layers?
L60 : "Without using internal layers, which are not available everywhere, we will follow a similar approach to these two last studies"
Our formulation was ambiguous, we do not use the method of Carter et al. The sentence is now :
L.85 : "As dated layers are not available everywhere, we will follow a radar-based approach like that of Schroeder et al. (2014), but adapted to the specific pattern of radar echoes under Dome C"

Line 74: "amplitude difference" – is that the same as the difference in returned signal strength?
We can also talk of radar echo strenght but the calculation of reflectivity done in the paper of 2012 was based on the difference between the amplitude (strenght or power) of the radar wave reflected by the surface and the amplitude of the radar wave reflected from the ice/rock (or water) interface in dBm. We prefer to keep "amplitude difference" for consistency with the 2012 paper.

Line 105: the heat balanace is assumed to be only vertically dependent.
New sentence L134 :"Thus, the heat balance is assumed to be only vertically dependent."

Line 140: Need a few more details here. What are these tabulated parameters? What is the uncertainty in the method? How well do the results compare to those of a 3D model?
L139 : "At each timestep, explicit expressions for the thickness and the bedrock elevation are solved, that depend on the accumulation rate and six tabulated parameters."
The parameters account for the sensitivity of the ice thickness, surface, height, etc. to the climate forcing.We do not go into deeper details here, since the model is fully explained in Parrenin et al 2007, and since the influence of the accumulation and temperature reconstruction, used as forcing, does not deeply change the results. However, we added a few explanations :
L.143 : "... six tabulated parameters, that account for the sensitivity of physical quantities (in particular bedrock and surface height and ice thickness) to the climate forcing."

Line 143: Missing a word? Coordinate?
Yes, "coordinate was missing : L.148 : "The heat balance of ice only depends on the vertical coordinate"

Line 160: what is the physical meaning of the parameter p? Is higher values equivalent to more/less basal sliding? What is appropriate for a dome region.
We completed the paragraph with additionnal explanations concerning the construction of synthetic profiles :

L.217 : "Far from divides, and for an isotropic ice, this parameter depends on the non-linearity of the ice rheology and the vertical temperature gradient at the base (Lliboutry, 1979):

p=n-1+Q/RT2*dT/dz

where n is the exponent of the Glen’s flow law, Q = 60 kJ mol −1 is an activation energy, R = 8.314 J mol −1 K −1 the gas constant, and T b the basal temperature. Following Eq. (11), the values of p should range within 7 to 9 on the East Antarctic plateau. But in practice we will use p close to divides in a larger value range, as a parameter able to account for realistic vertical velocity profiles. For exemple, dome profiles are expected to correspond to low p values due to Raymond arches (Raymond, 1983), whereas basal sliding will make the profile more linear and increase the value of p.”

Line 174: The density of the firn layer from Dome C? or from somewhere else?
L.164 : “The density profile of the Dome C firn layer is taken from Parrenin et al. (2007)”

Line 180: This paragraph seems to contain some information that is irrelevant.
We wanted to be precise on our choice concerning the expression of the pressure melting point. But we understand these explanations may make the reader lose his train of thoughts, so now we just say what is our expression :
L.168 : “For thawing glacier ice, the melting temperature T m linearly depends on the ice pressure P and the partial pressure of air dissolved in the ice P 0 , which is expressed as (Ritz, 1992): ...”

Lines 191-193: These sentences are very confusing. What is too low for what?
L. 192 : “The melting temperature computed with B = 0.098 K.Pa −1 would be 0.8 K too low, whereas, it is found to be 270.96 K with Eq.(9)”
The sentence is reworded, just saying that the equation is close to the observation :
L173 : “The temperature profile can be extrapolated to the bedrock (similar to Dahl-Jensen et al. (2003) at North GRIP) to 271.04 K, where Eq. (5) finds 270.96 K.”

Line 209: Odd to use the value 1/6.04 instead of 1.656.
We prefer to keep it like this for consistency with literature (Lorius and Merlivat, 1975).

Line 261: Reference to Monte Carlo method missing (e.g. Tarantola, 2005 “Inverse Problem Theory and Methods for Model Parameter Estimation”). Also, from the description of the inversion it does not sound like a Monte Carlo approach but rather like a search of the parameter space. Is it a random parameter space exploration? And how is this done?
The parameter space is not explored in a uniform way, because each parameter couple (H, p) is given random values along a gaussian joint-distribution. So we think “Monte-Carlo is the appropriate word for what we did. The gaussian distribution for H and p are described in the rest of the section.

Line 274: This is the first mention of potential drill sites. Why C6 and H1?
Now a specific sentence introduces the denomination of the points in the introcution section and figure 1 is referred.

L.121 : “For convenience, the domain is referenced with letter-figure couples, corresponding to the grid of the Italian survey (Fig. 1). In particular, two promising old-ice candidates are located at C6 and H1.”

Line 285: Where is E4 and E6? Which figure is referred to here?
The previous sentence (L 121) now gives the needed information as well.

Line 311-317: There should be a reference to Fig. 7 somewhere in this paragraph. Generally, this paragraph is not very clear. E.g. in line 315: How do you assign values to some of the variables? Equations (15) and (16) do not help the reader nor do the several almost identical symbols for different GHF.
This paragraph is taken off, since it was unnecessarily complicated with respect to what we want to show. Now we simply present the past basal melt rate calculated with the central values of GF and p, which is the main message of the paper.

Line 323: Point N8 in Fig. 1.
Reference to figure 1 added L. 371

Line 361-373: This is another paragraph that is not clear. For example, what is meant by "much of the map is quite well assigned"? or "well described". Does this mean that the model agrees with the observations? Line 389: ". . . realistic transport of cold..." Cold snow?
We agree that this paragraph was not precise enough, and it was reworded to make it more clear-cut.
L.414 :
"Superimposed on the observed data, the model output shows that large-scale patterns of wet-
dry areas are respected, especially on steep bed slopes (candidate B, C, D, and to a lesser extent candidate A). On these bed reliefs, certain points however show a discrepancy between model and observation, but the gap to the critical thickness is often close to 0 m (D3, D5, D8, M3), meaning that a small change in GF forcing, or a better description of the ice thickness, would better assign these particular points. The 1 km-resolution of the Bedmap 2 bedrock dataset (Fretwell and coauthors, 2013) smoothed along-track subkilometric features detected by our RES survey.

The steeper the bed, the sharper the limit between melting and non-melting areas. In the central,
flatter, part of the domain, the position of this limit is more blurred. As we could not assess the
GF with our method there, it was interpolated. Despite this lack of constraints, several small-scale
features are well mimicked (dry areas at I9, G-H8, wet areas at G9 and L7). Other regions were not attributed in agreement with observations (G6-7-8, H-I8), meaning that the GF is overestimated,
probably up to 3mW m −2 , which is consistent with the uncertainties given by our method (inversion and interpolation)."

Line 391-385: How does this affect the conclusions?
L 391 : "Some of the given confidence intervals are quite low (E4, E6, L7 and N8), and this is a consequence of the tiny altitude difference measured on Fig. 1 between the highest wet points and thelowest dry ones for a given spot. Since the correlation between ice thickness and reflectivity was
weak, the confidence intervals at E4 and E6 are probably underestimated, and some local

effects
may not have been accounted for in this study (e.g. small relief and GHF variabilities)."

As the GF values found for these point are not strongly discordant with the closer points, no particular warning should be placed. But we had to mention these points because we know the observation constraint is probably not very good there (now this pargraph is at L.467)

Line 414-419: Is the accumulation rate influencing the results? That question is raised by not answered in this paragraph.
L. 487 : "The sensitivity of the GHF on the surface accumulation is less than a few tenths of mW m −2 so that, accounting for its spatial variations would not radically modify our results."

Line 447: Is it truly Occam's razor or just a lack of good quality data inhibiting model validiation?
L. 447 : "This application of the parcimony principle of Occam's
razor supports the validity of our 1D approach, and means that the main physical mechanisms have
been accounted for."
We agree with your remark. We think that our quite simple model works quite well on bed reliefs, where the results show patterns similar to the observations. On flatter areas, the quality of the method is more difficult to assess, given the quality of the bed DEM, and the compatibility with our assumptions (no upstream flow of basal water). So we reworded the paragraph thiws way :
L. 494 : "This means that the main physical mechanisms have been taken into account, at least where it was possible to evaluate the critical thickness on significant topographic features. Where the GF is interpolated, in flatter areas, the lack of constrains prevents us from really assessing the validity of the method, meaning that our method needs a sufficient hilly bedrock to be applied."

Line 482: What clue? Please clarify.
L. 537 : "Given that this hint of a low GF value is the result of only one observation,…"

Line 495: The amplitude analysis was not performed in this study but from the sentence it sounds like it was.
L. 550 : "a previous amplitude analysis"

FIGURES:
None of the map have an indication of scale. Presumably the axis are in metres but it is not stated anywhere. Figs. 1 and 9 are very busy and could be split up into several maps. Additionally, The combination of magenta/orange and red/blue in Fig. 9 makes it difficult to read.
The projection and coordinate system are specified in the caption of figure 1.
Colours of figure 9 were changed.
We could not split the figures 1 and 9, because the information they contain is much richer when superimposed the ones on the others. We hope their large size should be enough for the clarity.

Fig. 2 is a very nice schematic of the assumptions on this study.

Answers to referee #2

The authors do a nice job leveraging available data sets and modeling capabilities to provide constraints on the basal melt rate near Dome C. The methods applied seem reasonable and the results seem to advance how to choose a site that is most likely to have preserved more than million-year-old ice. While this is good work, the way the manuscript is written makes it hard to follow, and I think that it does not communicate results in a way that they can be readily used in the broader community effort in the search for an "oldest" ice site. I hope that the authors can both restructure and reword much of the paper in order to make this work more accessible. I do not know exactly how this should be done but will provide some suggestions and identify as many of the starting with reordering and reconfiguring the sections to make sure that each header offers something logical to understand the work, and that text in each section is complete. I recommend that all the authors carefully review the revised manuscript before publication.

We would like to thank the reviewer for the constructive comments made on our work. We agree that the structure of certain sections need to be modified to make the article more clear-cut. In particular, the introduction section now benefits from a better explained 'oldest ice' context, and section 2 (method) and 6 (discussion) were reordered. We also agree with almost all the minor comments and tried to correct the text accordingly.

The present version of the paper has been corrected by a native speaker for english wording.

When needed, the old sentences are colored in green with corresponding line, the new ones in red.

I strongly recommend that the authors change the terminology from "geothermal heat flux" to "heat flux" or "geothermal flux"; geothermal heat flux is redundant. (In the same way that 'thermal heat model' would be redundant.) I know the chosen term is often used but hope that stating what is meant more directly will also help the flow of the paper since the term is used throughout the paper.

That makes sense, and we changed the terminology for "geothermal flux".

Introduction to heat-flux estimates from Antarctica doesn't differentiate between east and west, which is confusing the way it is written.

We understand that the text was potentially confusing. Now we focus specifically on the method to assess the GF under ice sheets in general (§1.2 GF assessment methods unders ice sheets), and we specify East or West Antarctica when needed.

Section 2.1.2 header is misleading when referring to "water circulation", and I am not sure that it is only finding a new word that is needed – is there really a chance of a significant basal hydrological system here? If so, even if a basal spot has not water evident is there a way to constrain that water was not routed through this region?

The idea here is to check if wet and dry areas really correspond to thawing and frozen areas (title changed as follows) :

2.4.1 Correspondance between wet and thawing areas

The true basal hydrological system cannot be observed easily and we are limited to theory. If there is no basal water circulation, Zirizotti map is just enough, but this would be a too optimistic assumption : we need to investigate the pessimistic case where dry areas are in fact thawing. Two clues are giving us confidence that dry areas are frozen, as explained below : the continuous nature of the melting, but also the relatively steeper surface slope at the spots (previously discussed in §6.1.1, now in this section) :

"A continuous hydrological network upstream is only fed by local melting, so that, unless the network is disconnected, the thawed water cannot be driven out faster than the melt rate. As a consequence, some water would always remain at the base of the melting ice. Furthermore, water at the base enables basal sliding, and reduces the basal drag. For a given ice flux, the surface slope gives some indication on the relative importance of internal deformation and basal sliding in the ice motion. Local steeper surface may be associated with more basal drag and ice deformation, whereas local flatter surface may be associated with more sliding. Most of the spots where the model will be run are standing on slopes locally steeper than the regional slope"

I wonder if "emulator" is the right word. If you keep it, make sure it is really clear at the start of section where this is introduced what you are doing. Since you are trying to simply approximate the solution to physical equations maybe "approximation" or "parameterization" is a better word?

We added the following sentence to define the word properly and say why we need it. We understand "emulator" as the process we use to empirically replace the whole heat model, to save computation time. We do not intend to parameterize "true" physical relations between input parameters, as we do not specifically account for diffusion or advection for example, but aggregate all the effects in one empirical expression.

L.347 : "As computing a given set of parameter takes several minutes, computing the basal melt rate for each point of the Dome region would be far too expensive. Here, the result of the whole forward model is mimicked by an emulator that depends on the input parameters H, Φ g and p"

If you can find a location that has experienced no basal melting that is obviously ideal, but it seems like some melting at some time in the past million years may have occurred and what you really want to make sure is that very old ice is still at the bottom. If there was no melting how old could the deepest ice be? And, for continuity of the record is it critical that no melting has occurred? There needs to be more context on how your results inform the search for very-old ice and how good your results have to be before picking a site. Only at the end it is stated that this work may inform other modeling efforts – I might have missed it but this should be stated more clearly up front. It would really help to put this work in better context with the larger effort to find a drill site. Of course we won't know what is there until we drill, but I didn't know if your results have really helped to target a site or provided additional information that has to be weighed with all else.

Is the kriged product what would be used in follow-on 3-D modeling? Is this good enough? It seems like the validation you can do for point locations doesn't hold across this whole area in a kriged result, but maybe this interpolated map is the product you need to provide? That wasn't very clear.

If a bit of thawing occurred between 1.5M and 1M years ago, and no melting after, the basal ice age is clearly not "infinite", but 1.5M-year ice still exists somewhere, since it existed 1M years ago. The age resolution must be affected but this kind of question goes beyond the scope of this paper, and mechanical modelling is needed.

We now better set the wider frame of this paper (observations – heat model – mechanical model) in the introduction (1.1 The oldest ice research). As geothermal flux is largely unkonwn, any new information on it is useful in the decision process for the drill site. The 3D-mechanical model we use now tries to constrain the ice rheology ; as it is an inverse process, the informations given on the basal melting (average + confidence intervals) reduce the parameter space to explore. Now, we use these kriged maps as prior values for 3D modelling. As we are focusing on candidate A, where we do have some contrains on the geotharmal flux, the kriged products are good enough there, and they even are probably more realistic than the existing products in the region (almost uniform value).

L. 45 : "In a context of locating the oldest-ice, our objective was to constrain the value of the geothermal flux and of the basal melt rate around Dome C. Later, this information will be used as a boundary condition for 3D mechanical simulations. As 3D models require many input parameters and are computationnaly expensive, any additional information about the thermal regime of the ice that reduces parameters value range is welcome, and will help select a suitable drill site."

If there was more radar data could you do this a lot better? Where is the community at with respect to drilling for oldest ice – will more data be available that you can use in a follow-on study? As a general approach it seems that you already have a lot more data then you might have compared to most places. But, is more needed to really pick the best drill site?

From this method point of view, a better description of the bed topography and bed reflectivity in the central, flatter, part of our study area could bring some interesting constrains. Elsewhere we do not think we could do much better.
The present study gives us confidence in the absence of melting over the candidate A, but to constrain the effective age and age resolution we need additional radar information. Internal layers within the ice are now available over the candidate A (Young et al, TCD, in review and Parrenin et al, TCD, in review). By continuity from the EPICA core, these layers can be dated, so that the age-depth relation is described on th ¾ of the ice column. This information is mandatory to have a constrain on datation models (1D or 3D), and to know the quality of the stratigraphy.

L577 : "More specifically, over the candidate A, a recent steady sate model assimilates radar isochrone layers to invert the value of Φ g and p, and computes the basal age of the ice (Parrenin et al., 2017)."

Is there no chance for accretion? Or, have you already ruled out spots where that may have occurred?

Accretion is possible where the ice goes from wet to dry during interglacials. This could occurr on the flanks of the candidate A. At least on the candidate A, no accretion clue were visible in the last radar survey, so we did not investigate this particular point.

MINOR COMMENTS
Line 5: "climate forcing lagged by thick ice" – you mean lag in heat transfer through thick ice, right?
The sentence is completed as follows :
L. 5 : "But, as the basal conditions depend on heat tranfer forced by climate but lagged by the thick ice, the basal ice may be frozen today…"

"Temperate" isn't the wrong word, but maybe you want to keep the distinction between "frozen" and "melting"?
We agree with your words, that more focus on the melting process that detroy old-ice layers.
L.5 : "the basal ice may be frozen today whereas it was in average melting in the past."

Line 29: While the measurement at subglacial Lake Whillans is an important one, I think that it requires some assumptions about the thermal stratification of the lake in order to back out heat flux. I am not an expert in this area but more generally I would make sure that this section fits with your paper. How is this estimate from west Antarctica relevant to Dome C?
We just wanted to illustrate that measuring the heat flux is not an easy task in Antarctica, so that inverse methods and modelling are needed on the antarctic plateau. But as it was confusing, the text is now more straightforward on East Antarctica.
L. 52 : "The geothermal flux is usually derived from temperature gradients meassured in boreholes in the ground, but this cannot be done easily below ice sheets, because of the difficult access to the bedrock."

Line 37: Statement is about measurements but aren't the references to modeling efforts?
L37 : "Without any available temperature measurements, the value of the GHF has been estimated from geological considerations"
The sentence was confusing, now reworded as follows :
L.58 : "As deep boreholes are not numerous in East Antarctica, the value of the GF has first been estimated from geological considerations"

Line 49: should be "increase" instead of "increasing"
Correction made L. 72
I would suggest using "infer", instead of deduce (this occurred in multiple places)
Correction made

Lines 51-54: I didn't understand this sentence that talks about water routing models – not only was I unsure about how it related scientifically but I'm just not sure what point was being expressed. Perhaps a problem is also that it was a long sentence broken up

by citations.
We added a link between the first and the second part of the paragraph, to make clear why considering basal hydrology is needed :

L.75 : "But the presence of water is not a sufficient clue to infer the GF, since water either comes from local basal melting or was routed from elsewhere. Using a collection of water routing models, Schroeder et al. (2014) inferred the value of the GF needed to explain the observed pattern of radar echoes..."

Line 54: Who is "They"? I don't know what study is referred to
L54 : "They derived an average value of 114 ± 10 mW m −2 for the Thwaites Glacier catchment."
L. 77 : "Schroeder et al. (2014) inferred the value of the GF needed to
explain the observed pattern of radar echoes, and derived an average value of 114 ± 10 mW m −2 ..."

Line 57: "significative" should be "significant"
L. 57 : "The uncertainty on the GHF was estimated ±12 mW m −2 , which is a significative improvement"
L. 82 : "The uncertainty on the GF estimation was ±12 mW m −2 , which is a significant improvement"

Line 57: I am weary about making a point that uncertainties at a given level are a significant improvement – how uncertain are these uncertainty estimates
The current available (continental) estimations are affected by large uncertainties of +/- 20 mW/m2 (Fox Maule 2005 for example). From the "oldest-ice" point of view, these estimations cannot constrain efficiently the local basal melt rate. The key information for us would be a realistic upper boundary for the GF that would anyway allow the ice to be frozen through time, and the only way is to find places where there is no melting.

In that sense, any effort to reduce locally the uncertainties of continental estimations are welcome.

Line 58: I would restate to be something like ". . . but is still too large when trying to find a location that may preserve very old ice"
L. 82 : "The uncertainty on the GF estimation was ±12 mW m −2 , which is a significant improve-
ment compared to the previously available estimations, but is still too large when trying to find a
location that may preserve very old ice."

Line 60-62: Doesn't add much to the reader, either be specific about how what you are doing is or is not similar to previous work or I suggest taking it out
L58 :"Moreover, their study area does not completely cover the main old-ice candidates (east and south-west of Dome C). Without using internal layers, which are not available everywhere, we will follow a similar approach to these two last studies, but adapted to the specific pattern of radar echoes under Dome C"
Sentence reworded to say that we need a new local estimation.
L. 84 : "Their study area does not cover the main old-ice candidates located to the east and south-west of Dome C, so a new local estimation is needed. As dated layers are not

available everywhere, we will follow a reflectivity-based approach like Schroeder et al. (2014), but adapted to the specific pattern of radar echoes under Dome C"

Line 67: Suggest finding another word than "triggered"
I'm not sure it is clear here what you mean by "interesting" – state directly that the glaciology (not glaciologist) community is interested in sites where it has remained cold at the base without melting
This paragraph is taken out, but the main ideas are put in the first paragraph (§1.1 The oldest-ice research, L. 37).

Line 88: Again, I would state clearly what you are trying to do and somehow ". . .this paper primarily aims to assess the risk of past temperate conditions at places known to be cold today" doesn't quite get it across. Perhaps something more like "This work constrains sites where basal melting is less likely to have occurred over the past 800ka, even if they are frozen today, so that very old ice has the best chance of being preserved at that site"
L 88: "this paper primarily aims to assess the risk of past temperate conditions at places known to
be cold today"
We restated this sentence as follows :
L. 107 : "this work aims at constraining sites known to be frozen today and that are very likely to
have been frozen in the last 800 ka as well, increasing the probability that very old ice has been
preserved."

Line 90-94: A little bit of introduction isn't always helpful if the reader doesn't really have a good sense of what is coming. Consider expanding this to be more direct about what it means to do this in "forward mode" and "inverse model"
It may be more clear to talk about this as "running a forward model" and "solving an inverse problem"?
Be consistent with "heat model" or "thermal model" – used interchangeably in title, section headings, and text – but since they mean the same thing just pick one
Changed for "solve an inverse problem" and "run the model forward"
Title changed to "heat modelling"

The beginnig of the paragraph is now :
L. 112 : "We present a 1D heat model forced by reconstructed climatic conditions, and run it in two ways. First, we solve an inverse problem with this model to infer the value of the GF in the Dome C region,using radar echoes as observational constraints. The pattern of wet and dry areas allows to estimate a critical ice thickness that corresponds to a threshold between frozen and thawing ice. For a given GF and vertical advection, this thickness is unique, so that we may in turn infer a GF distribution from the pattern of basal echoes."

Line 96: I think you mean "relationships" instead of "relations"
Correction made, L.124

Line 104: "Characteristic" is more often used than "typical"
Correction made, L. 132

Line 117: “flown” should be “flowed”, or better yet “been transported by ice flow”
Correction made for “flowed”, L. 251

Line 121: “permanently” seems too strong
As “continuous” is characterizing the process, we took out “permanently”.
L. 256 : “A continuous hydrological network upstream is only fed by local melting,”

Section 2.1.3 doesn’t add much – why is this a section and why does this get mentioned in this order? It seems like you want to introduce the model equations first, then talk about caveats!
The new version of the “Model” section is reordered in this way :

2.1 Geometry and coordinate system
2.2 Heat equation
    2.2.1 Ice thermal properties
    2.2.2 Basal boundary conditions
    2.2.3 Boundary condition at the surface
2.3 Velocity model
2.4 Proceeding assumptions
    2.4.1 Correspondance between wet and thawing areas
    2.4.2 GF spatial variability

The last section 2.4.2 is needed as it is one simple but important assumption of the model.

Line 138: typo should be “which”
Again, “emulates” seems like the wrong word. Here, maybe “simulates” or “approximates”?
Corrextion made L. 141. As we answered in the “main concerns”, emulates seems to be the right word.

Line 145: Is D dimensionless? Is K a function of reduced depth? (don’t think so)
“Dimensionless” added, L. 154. New ordering of the paper provides information on K just after this section, L. 160. K depends on T, so indirectly on reduced depth.

Equation 2: These might be equivalent or just defined differently, but this equation is not the same as in Parrenin et al. (2007; equation 3) – check to make sure no typos between ζ and $(1-\zeta)$
No typo here, Parrenin et al 2007 is using reduced height, and we use reduced depths

Section 2.6 header – suggest stating as “Basal boundary conditions”
Correction made L. 165 §2.2.2

Line 187: Why is this an unusual choice?
L187 :”This is an unusual choice for such an important parameter, but we argue that Eq. (9) is consistent with the temperature profile of the EPICA Dome C ice core”
This formulation of the melting point temperature has not been published in a reviewed article yet, only in Ritz's PhD manuscript. It is needed since it seems to better match the measurements at Dome C. However, this choice is discussed later and has no crucial

impact on the results (L.396.), now we just say :
L. 171 : "This expression is compatible with the temperature profile at the EDC borehole"

Line 202: paleotemperatures aren't "known", they are still estimates
Changed for "estimated" L. 186

Line 209: Why is this represented as 1/6.04K – if report this way make sure to have typed as actual fraction since as it is now with inline slash it could be confusing
We keep 1/6.04 for consistency with literature (Lorius and Merlivat, 1975)
L193 : "In addition to the nominal value 1/6.04 K, we will perform sensitivity studies..."

Section 2.8 – Do you concentrate grid cells near the base, and is vertical spacing good enough? Also, are 1000 year timesteps good enough? Can you say anything about uncertainties related to your solution grid? The current statement mentions there was a tradeoff between accuracy and speed, how much was compromised?
The mesh is regular. The dependence of the result (basal temperature) is of the order of 0.1 K when changing the vertical spacing, and of the order of 0.01 K when changing the timestep, whereas the computation time linearly increases with the number of elements in the mesh.
L. 276 : "The dependence of the basal temperature on discretization is limited to a few tenths of K."

Everywhere you use "till" should be changed to "until"
Correction made

Section 3 heading is a bit confusing since it isn't yet clear what is being measured, and it is definitely not heat flux! I would change that to something more clearly related to your data analysis. In general, I suggest coming up with another term for "measurement spots".
L 235 : "3) Measurement spots"
We changed for a longer but more explicit title L285 : "3) Spots where GF will be inferred"

Suggest in the text to remind readers what p is. Same for any variable that was introduced awhile back in the text
"Shape parameter" added on L. 287

Line 269: Is there a short justification for the heat flux range of 40-70 mW/m^2?
Empirically, the GF values that were found in this range (in fact 45-65 + safety margin).
L. 319: "All the GF values were in the range 40-70 mW m –2"

Line 272: Suggest that "inferred" is better term than "derived"
Correction made

Line 286: "do not mismatch" – do not match? Or, mismatch?
We confirm that "do not mismatch" is what we intended to say. Since the confidence we have in these points is low, we could expect a mismatch between the surrounding GF field and the value at these points, but it is not the case. We reworded as folllows :
L335 : "The inferred values of the GF nevertheless matched the N-S gradient"

Line 299: I would put your estimated relationship between heat flux and ice thickness

in words and remind the reader that this is for the explored range of heat flux values, or does it hold for any heat flux (any melt rate)? Then, the relationship between heat flux and melt rate must correspond to a specific ice thickness, right?
I lost the details around equations 15-16 and suggest adding more text to explain what this means and how it was derived.
First we explain the goal of the emulator we need to build :
L. 347 : "As the computation for a given set of parameter lasts several minutes, computing the basal melt rate for each point of our domain would be far too expensive. Here, the result of the whole forward model is mimicked by an emulator that depends on the input parameters H, Φ g and p."

L. 355 : "Over the positive-melt-value domain, we used a least-square minimization method to compute the following relation:"

-Eq. 15-16 : This paragraph is taken off, since it was unnecessarily complicated with respect to what we want to show. Now we simply present the past basal melt rate calculated with the central values of GF and p, which is the main message of the paper.

Line 337: what do you mean by "at the opposite" here?
L 337 : "However, the average basal melt rate is changed at the opposite by 0.1 mm a −1"
When the GF is affected positively by a parameter change through the inverse problem, the melt rate is affected negatively during the forward problem, and vice-versa.
L380 : "However, the average basal melt rate is changed by 0.1 mm a −1 in the reverse direction"

Line 351: Again, not sure how to take this point about an "unusual choice" and why it was chosen that way to begin with
L351 : "Given that the expression of the pressure melting point is an unusual choice in glaciology (Eq. 9), the inverse method was reiterated with a more common value (B = 0.098 K.Pa −1) as a test"
As explained in section 2, we choose this dependence of temperature on pressure because it better matches with observation, instead of a widespread expression (for example in Cuffey and Paterson 2010).
L. 399 : "Given that the expression of the pressure melting point is an unusual choice in glaciology (Eq. 9), the inverse method was reiterated with a more common value corresponding to saturated air in the ice (T m = 273.16−0.098 P , Cuffey and Paterson, 2010)"

Line 354: Why is "order of magnitude" good enough?
L. 354 : "The order of magnitude of the results remains the same whatever the expression for the pressure melting point"
"Order of magnitude" was awkward, the sentence is reworded :
L. 401 : "The results in terms of basal melting are not significantly affected by the expression of the pressure melting point"

Line 357: The leading sentence doesn't seem to relate to the second sentence and I had to read back to see if I understood what was coming and has already been done. Suggest better lead-in to this section.
A new introduction to this section is proposed :

L.405 :“One way to assess the performance of our model is to compare observed basal state (wet or dry, Zirizzotti et al. (2012)) with the model simulation for present time. For p = 2, we compute the (Φ g , H c) relation, corresponding to the present basal state, by sampling H c between 2 700 m and 3 300 m. The two parameters are linked by the following empirical relation:”

Line 365: “build” should be “built”, and really do you mean “calculated”?
Correction made L. 414

Line 368: What small-scale structures are these E, G, H, I, L locations referring to?
A sentence now introduces specifically the coordinates we refer to (introduction section) :

L. 121 : “For the sake of convenience, the domain is referenced using pairs of letter and numeral, corresponding to the grid of the Italian survey (Fig. 1). In particular, two promising old-ice candidates are located at C6 and H1.”

Line 369: Should be “non-melting”
L 369 : “no-melting areas”
L423 : “non-melting areas”

Line 371: Need to rephrase part about “. . . is however often respected . . .”
“undoubtly” should be “undoubtedly”
Line 373: What do you mean by “local gap”?
This paragraph is now mainly restated :
L. 415 : “Superimposed with the observation data, the model output shows that large-scale patterns of wet-dry areas are respected, especially on steep bed slopes (candidate B, C, D, and to a lesser extent candidate A). On these bed reliefs, certain points however show a discrepancy between model and observation, but the gap to the critical thickness is often close to 0 m (D3, D5, D8, M3), meaning thata small change in GF forcing, or a better description of the ice thickness, would better assign theseparticular points. The 1 km-resolution of the Bedmap 2 bedrock dataset (Fretwell and coauthors, 2013) smoothed along-track subkilometric features detected by our RES survey.
The steeper the bed, the sharper the limit between melting and non-melting areas. In the central,
flatter, part of the domain, the position of this limit is more blurred. As we could not assess the
GF with our method there, it was interpolated. Despite this lack of constraints, several small-scale
features are well mimicked (dry areas at I9, G-H8, wet areas at G9 and L7). Other regions are
not assigned in compliance with observations (G6-7-8, H-I8), meaning that the GF is overestimated,
probably up to 3mW m –2 , which is consistent with the uncertainties given by our method (inversion and interpolation).”

Line 375: Should be “dependent”
Correction made L. 432

Section 6.1, maybe “Method validation”? There is no lead in, which may be fine but as

is now it isn't clear how come you need to have overarching section 6.1
The structure of the discussion section has now changed, and we changed for "Model assessment"

6.1 Consistency with published data and measurements
6.2 Model assessment
    6.2.1 Method validity
    6.2.2 Sensitivity to parameters
    6.2.3 Spatial variation of the GF field
6.3 Lessons drawn for the oldest-ice research
    6.3.1 Interpretation of the wet/dry pattern at the ice base
    6.3.2 Old-ice targets

Line 399: What do you mean "the surface slope is the source of motion"?
Talking about clues isn't very precise – is there a better word?
At first order there is motion because of the pressure gradients within the ice, and that depends on surface slope. These explanations are now put in section 2.4.1.
L. 260 : "For a given ice flux, the surface slope gives some indication on the relative importance of internal deformation and basal sliding in the ice motion"

Paragraph around lines 415-420: Not sure I understood the point of this paragraph, if this is just too hard to constrain state that directly. If it needs to be considered somehow and will significantly affect uncertainty estimate state that directly too. I can't tell if this is something that really could matters.
This paragraph (now L486) correspond to additional tests we made. We wanted to make sure of the influence of accumulation, since there is a NS gradient of both accumulation and GF. As we made the sensitivity test for a, it is worth saying a word about it, even if the sensitivity is low.

Line 422: Not sure what you mean with "litmus"
"Litmus test" means a "test of truth" (changed for "reliable comparison" L. 443).

Section 6.1.4 – by "structure" do you mean "spatial variation"
Correction made for "6.2.3 Spatial variations of the GF field" L. 492

FIGURES AND TABLES
Would be worth defining input parameters in Table 2. I don't quite understand what "total on m" represents.
The new caption is the following :

"Sensitivity of the GF (assessed from inverse runs, [mW m −2]) and basal melt rate (calculated with forward runs, [mm a −1]) to input parameters α, β and a. As the final value of m depends on both the inverse run to determine Φ g , and the forward run to compute the melt rate, the last column accounts for the sensitivity on the whole procedure (inverse+forward)."

As the final value of m depends on both the inverse run to determine the GF, and the forward run to compute the melt rate, the last column accounts for the sensitivity with the whole method (inverse+forward), which is less than the sensitivity with the forward mode only.

Figure 2: I might have missed it but check that critical ice thickness is defined in the text to this point, or add to caption
Caption completed :
“We define the critical thickness as the minimum thickness that allows basal melting at present.”

Figure 4: Not sure it was discussed in the text how the 10 spots were chosen?
The spots are chosen where a correlation could be found between reflectivity and ice thickness. Sentence modified in section 3 :
L. 293 : “Ten corresponding spots are selected (black rectangles in Fig. 1), where reflectivity and ice thickness are somehow correlated, and that are hereafter denoted by the indexes of their central point (Fig. 4).”

Figure 6: A lot of overlapping lines. How many discrete values of m are represented?
Now only 2 series of m-contours are represented, for p=1 and p=10.

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Geothermal flux and basal melt rate in the Dome C region inferred from radar reflectivity and heat modelling

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Abstract. Basal melt rate is the most important physical quantity to be evaluated when looking for an old-ice drilling site, and it depends to a great extent on the geothermal flux (GF), which is poorly known under the East Antarctic ice sheet. Given that wet bedrock has higher reflectivity than dry bedrock, the wetness of the ice-bed interface can be assessed using radar echoes from the bedrock. But, since basal conditions depend on heat transfer forced by climate but lagged by the thick ice, the basal ice may currently be frozen whereas in the past it was generally melting. For that reason, the risk of bias between present and past conditions has to be evaluated. The objective of this study is to assess which locations in the Dome C area could have been protected from basal melting at any time in the past, which requires evaluating GF. We used an inverse approach to retrieve GF from the radar reflectivity at the bed. A 1D heat model is run over the last $800\,\mathrm{ka}$ to constrain the value of GF by assessing a critical ice thickness, i.e. the minimum ice thickness that would allow the present local distribution. A regional map of the geothermal flux was then inferred over a $80\,\mathrm{km}\times130\,\mathrm{km}$ area, with a N-S gradient, and with values ranging from 48 to $60\,\mathrm{mW\,m^{-2}}$. The forward model was then emulated by a polynomial function, to compute a time-averaged value of the spatially variable basal melt rate over the region. Two main subregions appear to be free of basal melting because of the thin overlying ice, and a third one, north of Dome C, because of a low geothermal flux.

1 Introduction

1.1 The oldest ice research

Between 1.5 and 0.9 million years ago, the main periodicity of the global climate changed from $41\,\mathrm{ka}$ to about $100\,\mathrm{ka}$, as shown by temperature and sea level proxies retrieved in marine sediments (Mid-Pleistocene Transition, Lisiecki and Raymo, 2005). The causes of this major climate transition are still a matter of debate, and it would be easier to evaluate the different explanations proposed by several authors (e.g. Raymo et al., 2006; Bintanja and Van de Wal, 2008; Martínez-Garcia et al., 2011; Imbrie et al., 2011) if greenhouse gases and other environmental proxies of this climatic period were analysed. Hence, retrieving an ice archive as old as 1.5 million years is one of the greatest challenges facing the ice core scientific community today (Brook et al., 2006).

The Dome C region (East Antarctica, see map in Fig. 1) has been of great interest to paleoclimate scientists in recent decades, and two deep ice cores have already been retrieved (Lorius et al., 1979; Jouzel et al., 2007), including the oldest one ever dated (800 ka, EDC ice core). The region has also been identified as possibly hosting even older ice (Fischer et al., 2013; Van Liefferinge and Pattyn, 2013). Unfortunately, these studies lacked precise information on the geothermal flux (GF) at the base of the ice. To ensure that the prediction of the age of a future old-ice core site is reliable, the thermal conditions at the base of the ice must be well constrained, because basal melting strongly affects the possible presence of old ice by continuously removing the oldest layers (Rybak and Huybrechts, 2008). In a dome region, the heat budget of ice depends mainly on the geothermal flux, which warms the body of ice from below, and on the temperature at the surface. But the resulting vertical temperature gradient also depends on the ice thickness, and, because the ice acts as an insulator, the thicker the ice cover, the warmer its base (Pattyn, 2010). Vertical cold advection due to movement of the ice also affects basal thermal conditions and must consequently be taken into account. Its value is linked to accumulation rate. Surface temperatures, ice thickness and accumulation rates have been directly measured, or their past values reconstructed, so the parameter about which the least is known regarding its influence on basal melting is geothermal flux.

In the Dome C region, significant subglacial bedrock features make the ice thin enough to for there to be a chance has been protected from melting. In the glaciology community, these locations are widely thought to be good old-ice candidates (Fig. 1 - `A`,`B`,`C` and `D`). In a context of locating the oldest-ice, our objective was to constrain the value of the geothermal flux and of the basal melt rate around Dome C. Later, this information will be used as a boundary condition for 3D mechanical simulations. As 3D models require many input parameters and are computationnally expensive, any additional information about the thermal regime of the ice that reduces parameters value range is welcome, and will help select a suitable drill site.

1.2 Assessing GF under ice sheets

The geothermal flux is usually derived from temperature gradients measured in boreholes in the ground, but this is not easy below ice sheets, where it is difficult to access to the bedrock. However, geothermal flux can be estimated from temperature profiles in ice core boreholes (Dahl-Jensen et al., 1998; Engelhardt, 2004). More precise results are expected for cold basal conditions, since uncertainties affect the basal melt rate for temperate basal ice (Grinsted and Dahl-Jensen, 2002), in which case only a minimum value of GF can be estimated.

As deep boreholes are not rare in East Antarctica, the value of the GF could be estimated from geological considerations (Pollard et al., 2005; Llubes et al., 2006), where uniform values were attributed to large geologically homogeneous areas. But spatial variability of GF occur at a much smaller scale, and efforts to model this variability have been made in the last decade. Two approaches based on geophysical information have been proposed to provide comprehensive maps at continental scale: one using a seismic model of the crust and upper mantle (Shapiro and Ritzwoller, 2004), and one a crustal thickness model derived from observations of the magnetic field (Fox Maule et al., 2005; Purucker, 2013). These studies show that the GF on the East Antarctic plateau is about $60 \pm 25\,\mathrm{mW\,m^{-2}}$, which unfortunately is much too coarse an estimation to give any precise value at a specific location. Moreover, Terre Adélie and George V Land could form part of the Mawson craton, which also forms the southern central part of Australia, where the GF shows high spatial variability (Carson et al., 2014). Even if the exact extent of this craton in Antarctica is not well known, in East Antarctica the GF could include hot spots, with a typical lengthscale of $10\,\mathrm{km}$.

Radio echo sounding (RES) measurements may help infer the basal conditions at regional scale, since the presence of water at the ice-bed interface pis responsible for a remarkable increase in the amplitude of the reflected echoes. For this reason, RES made it possible to detect melting at the base of ice sheets (Fujita et al., 2012; Oswald and Gogineni, 2012; Zirizzotti et al., 2012), as well as to map subglacial lakes (Siegert et al., 2005). But the presence of water is not sufficient to infer the GF, since water can originate either from local basal melting or be routed from elsewhere. Using a collection of water routing models, Schroeder et al. (2014) inferred the value of the GF needed to explain the observed pattern of radar echoes, and derived an average value with an uncertainty of $\pm 10\,\mathrm{mW\,m^{-2}}$ for the Thwaites Glacier catchment (West Antarctica).

Finally, basal melt rates for the region north of Dome C have been estimated by fitting the vertical strains with dated radar layers, to constrain the vertical advection of ice, and its energy budget (Carter et al., 2009). The uncertainty on the GF estimation was $\pm 12\,\mathrm{mW\,m^{-2}}$, which is a significant improvement over the previous estimations, but is still too large to identify a location with very old ice. What is more, their study area does not cover the main old-ice candidates located to the east and south-west of Dome C, so a new local estimation is needed. As dated layers are not available everywhere, we used a reflectivity-based approach like that of Schroeder et al. (2014), but adapted to the specific pattern of radar echoes under Dome C, where the melting point is not reached everywhere.

1.3 Exploitation of available RES dataset around Dome C

Based on radar equations, Zirizzotti et al. (2012) proposed a method to recognize wet areas using the Italian RES dataset collected around Dome C (http://labtel2.rm.ingv.it/antarctica/). These authors used a linear model of electromagnetic wave adsorption in the ice column, based on analysis of the EDC ice core, to account for the differences in amplitude (in dB) between echoes at the surface of the ice and at the bottom. The thresholds used to ascribe an echo to a wet or dry basal contact were $\geq 7.7\,\mathrm{dB}$ and $< 1\,\mathrm{dB}$ respectively. Figure 1 shows the distribution of dry and wet areas under Dome C, which reveals interesting patterns: (i) the Concordia trench is characterised by the presence of wet points only, which is a consequence of the very thick ice; (ii) the tops of the two main bedrock reliefs (candidate $\mathtt{A}$ and candidates $\mathtt{B}-\mathtt{C}-\mathtt{D}$) appear to be dry; (iii) almost all the northern points are dry, despite the very thick ice at this location; (iv) in between, and in particular under the EPICA drilling site, dry and wet points are scattered, with no clear trend.

We would like to emphasise that these observations only refer to the present day bed conditions, whereas the historical conditions covering the glacial/interglacial periods need to be investigated to evaluate the quality of a future old-ice drilling site. The absence of basal water today means cold basal ice, but this may simply be a consequence of the very strong temperature signal of the Last Glacial Maximum (LGM) lagged by the thick ice. The present cold state may thus not be representative of all the glacial-interglacial periods, and thawing could have occurred in the past under different climatic conditions.

Hence, the aim of this work was to to constrain sites known to be frozen today and that are very likely to have also been frozen in the last 800 ka, increasing the probability that very old ice has been preserved. The dataset available from Zirizzotti et al. (2012) provides informations on whether melting occurrs, but not on its amount and its temporal and spatial variations. The only way to obtain this information is first to retrieve the spatial distribution of GF and then to use it to infer basal melting over time.

Here we present a 1D heat model forced by reconstructed climatic conditions, which we ran in two ways. First, we solved an inverse problem with the model to infer the value of the GF in the Dome C region, using radar echoes as observational constraints. The pattern of wet and dry areas made it possible to estimate a critical ice thickness that corresponding to a threshold between frozen and thawing ice. For a given GF and vertical advection, this thickness is unique, so that in turn the distribution of GF can be inferred from the pattern of basal echoes. Second, we ran the model forward to compute the average past basal melt rate under the GF inferred at the first step. We present an easy-to-use emulator of the past basal melt rate, which is a convenient thermal boundary condition for future modelling works, and key-criterion in the location of old-ice drilling sites. For the sake of convenience, the domain is referenced using pairs of letter and numeral, corresponding to the grid of the Italian survey (Fig. 1). In particular, two promising old-ice candidates are located at C6 and H1.

2 Heat model

In this section we present the heat model accounting for the relationships between GF, ice thickness, vertical advection, and the temperature of the ice. The main assumptions under which the model is run are presented at the end of the section. The areas for which these assumptions are valid are presented in section 3.

2.1 Geometry and coordinate system

In a dome region, horizontal velocities are very small (a few centimetres per year, Vittuari et al., 2004), meaning horizontal advection terms can be neglected. Similarly, the deformational heat is several orders of magnitude smaller than the vertical advection term. Finally, due to the small aspect ratio (thickness/characteristic horizontal length), the temperature field is mainly vertically stratified, so horizontal diffusion can also be neglected. Thus, the heat balance is assumed to be only vertically dependent.

Here we consider a one-dimensional vertical domain, oriented upwards along the z-axis. Instead of being set in the z-coordinates, the equations are expressed in reduced depth $\zeta = (s-z)/(s-b)$, s being the surface height, and b the bed height. In the ζ-coordinate system, the domain size remains the same whatever the ice thickness H, and no remeshing is needed in the resolution of the finite difference scheme. Therefore the ice thickness is a simple parameter evolving over time according to surface accumulation forcing. Changes in thickness are accounted for by the conceptual model of Parrenin et al. (2007), which, for Dome C, emulates the 3D large scale simulations of Ritz et al. (2001). At each timestep, for the thickness and the bedrock elevation, explicit expressions are solved, which depend on the accumulation rate and six tabulated parameters, which account for the sensitivity of physical quantities (in particular bedrock and surface height and ice thickness) to climate forcing. The typical difference in the ice thickness in the glacial and interglacial periods is $150\,\mathrm{m}$.

2.2 Heat equation

The heat balance of ice only depends on the vertical coordinate, and is written for the ice temperature T as follows in the ζ-coordinate system (Ritz et al., 1997):

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c D H^2} \frac{\partial}{\partial \zeta}\left(K \frac{\partial T}{\partial \zeta}\right) - u_\zeta \frac{\partial T}{\partial \zeta} \tag{1}$$

where K is the thermal conductivity of the material (firn or ice), c its heat capacity, ρ the density of ice ($917\,\mathrm{kg\,m^{-3}}$), and D the relative density of the material (<1 for the firn and 1 for the ice, dimensionless). The vertical velocity u_ζ accounts for the true ice velocity, but also for changes in the ice thickness, which in turn modifies the relationship between z and ζ.

2.2.1 Ice thermal properties

The specific heat capacity and heat conductivity depend on the absolute temperature T as follows (Cuffey and Paterson, 2010, p. 400):

$$c = 152.5 + 7.122 \cdot T \;[\mathrm{J\,K^{-1}\,kg^{-1}}] \tag{2}$$

$$K = 9.828 \cdot e^{-0.0057 \cdot T} \; [\mathrm{W\,m^{-1}\,K^{-1}}] \tag{3}$$

The presence of firn at the surface of the ice reduces conductivity in the upper part of the ice column, and this must be accounted for (Cuffey and Paterson, 2010, p. 401):

$$K = \frac{2K_i \times D}{3 - D} \tag{4}$$

where K_i is the conductivity of the ice. The density profile of the Dome C firn layer is taken from Parrenin et al. (2007).

2.2.2 Basal boundary conditions

At the ice/bed interface, the thermal conditions depend on whether or not the pressure melting point is reached. For thawing glacier ice, the melting temperature T_m linearly depends on the ice pressure P and the partial pressure of air dissolved in the ice P', which is expressed as (Ritz, 1992):

$$T_m = 273.16 - 0.074 \cdot P - 0.024 \cdot P' \tag{5}$$

In ice sheets, P' is of the order of $1\,\mathrm{MPa}$. This expression is compatible with the temperature profile at the EDC borehole, where the deepest measured temperature was $270.05\,\mathrm{K}$ at $3223\,\mathrm{m}$, $50\,\mathrm{m}$ above the bedrock. The temperature profile can be extrapolated to the bedrock (in the same way as Dahl-Jensen et al. (2003) at North GRIP) to $271.04\,\mathrm{K}$, where Eq. (5) gives $270.96\,\mathrm{K}$. As the ice in the dome region moves very slowly, we assume that the pressure only depends on ice thickness. Once the temperature field is computed, the melt rate m at the bottom can be explicitly known by

$$m = \frac{1}{\rho L}\left(\Phi_g - \frac{K}{H}\left.\frac{\partial T}{\partial \zeta}\right|_{\zeta=1}\right) \tag{6}$$

where Φ_g is the GF and L the latent heat of ice ($\mathrm{J\,kg^{-1}}$). In this equation, no friction heating is considered, since the basal velocities are very small, and the corresponding additional heat source is a second-order term (Krabbendam, 2016). For cold basal ice, Eq. (6) is used as a Neumann boundary condition, equating m to zero.

2.2.3 Boundary condition at the ice surface

The atmospheric temperature forcing is continuously transferred through the whole ice column, so the present thermal conditions at the bed are the result of the entire climate history. The atmospheric paleotemperatures have been estimated over the last 800 ka from the δD measurements made on the EDC ice core (Jouzel et al., 2007). Here we use the model and notations of Parrenin et al. (2007), linking the deviation of the deuterium content $\Delta\delta D$ of the ice to the surface temperature T_s w.r.t. a reference temperature T_s^0:

$$T_s = T_s^0 + \alpha\Delta\delta D \tag{7}$$

where α reflects the amplitude of past changes in T_s. The influence of the value chosen for α has to be examined, since uncertainties affect our knowledge of the isotopic thermometer at long timescales in Antarctica, and α can vary from -10% to +20% (Jouzel et al., 2003). In addition to the nominal value of $\frac{1}{6.04}$ K, we will perform sensitivity studies with two additional values of 0.13 and 0.20, chosen so that the maximum temperature difference over climatic periods with the nominal run is ± 2 K. The corresponding accumulation rates are also taken from Parrenin et al. (2007), who considered an exponential accumulation model, linking the accumulation rate a to a reference accumulation rate a^0:

$$a = a^0 \exp(\beta\Delta\delta D) \tag{8}$$

where β reflects the amplitude of past changes in a. For Dome C, the value of the β coefficient was evaluated at 0.0156 ± 0.0012 using an inverse method constrained by known age markers. We consequently checked the sensitivity of our model for three values of β (0.0138, 0.0156 and 0.0175). The two extreme values were chosen so that the maximum accumulation difference over climatic periods with the nominal run is $\pm 0.25\,\mathrm{cm\,a^{-1}}$, which is a more intuitive way to express the sensitivity.

Regarding the choice of the initial state, we considered that the duration needed for a step climatic signal to reach the bedrock is ~ 10 ka, plus ~ 100 ka to stabilise. As we do not know the true initial state of the temperature profile in the past, we decided to run the model over the whole reconstructed period (800 ka), so that the computation is independent of the initial state and the final vertical temperature profile is as realistic as possible.

2.3 Velocity model

In the heat balance, the vertical advection of ice acts by transporting cold towards the bed. Instead of solving the equations of motion, it will be accounted for by a 1D shape function ω (Parrenin et al., 2007; Ritz, 1987):

$$\omega(\zeta) = 1 - \frac{p+2}{p+1} \cdot \zeta + \frac{1}{p+1} \cdot \zeta^{p+2} \tag{9}$$

$$u_z = -(a - \frac{\partial H}{\partial t} - m) \cdot \omega(\zeta) - m \tag{10}$$

where u_z is the vertical velocity of ice in the z-coordinate system, a is the surface accumulation rate and m is the basal melt rate, which is considered to be positive when melting. The shape parameter p influences the temperature profile, since it controls the vertical advection of ice from the surface to the base. Far from divides, and in the case of isotropic ice, this parameter depends on the non-linearity of the ice rheology and on the vertical temperature gradient at the base (Lliboutry, 1979):

$$p = n - 1 + \frac{Q}{RT_b^2} \frac{\partial T}{\partial \zeta}\bigg|_{\zeta=1} \tag{11}$$

where n is the exponent of the Glen's flow law, $Q = 60\,\mathrm{kJ\,mol^{-1}}$ is an activation energy, $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$ the gas constant, and T_b is the basal temperature. Following Eq. (11), the values of p should theoretically range between 7 and 9 on the East Antarctic plateau. But in practice we will use p close to divides in a wider range of values, as a parameter able to account for realistic vertical velocity profiles. For exemple, dome profiles are expected to correspond to low p values due to Raymond arches (Raymond, 1983), whereas basal sliding makes the profile more linear and increase the value of p. Above a typical value of $p = 10$, the profile is very close to being linear (Fig. 3), and higher values of p do not significantly change the profile. In the case of a shape function in a dome region (Parrenin et al., 2007), the vertical velocity u_ζ in the ζ-coordinate system is then expressed as follows:

$$u_\zeta = \frac{1}{H}\left(\frac{\partial H}{\partial t}(1 - \zeta - \omega(\zeta)) + m(1 - \omega(\zeta)) + a\omega(\zeta)\right) \tag{12}$$

which is equivalent to

$$u_\zeta = \frac{1}{H}\left(-u_z + (1-\zeta)\frac{\partial H}{\partial t}\right). \tag{13}$$

2.4 Proceeding assumptions

In addition to the 1D geometric assumption allowed by the proximity of the dome, two further assumptions are made in this paper, and are now justified. First, as no routing model is used to make the basal water circulate at the ice-bed interface, this implicitly means that the presence of water at the bed corresponds to a thawing basal ice. Second, GF is assumed to be spatially uniform on short spatial scales.

2.4.1 Correspondence between wet and thawing areas

Since the wetness indicated by the basal echoes will be used as constraints in the inverse model, we first have to identify the origin of the observed basal water. Either the local ice is temperate and the water comes from locally thawed ice, or the local (cold) ice energy budget was upset by the latent heat of water flowing from a temperate location, along the gradient of hydraulic potential. In the Dome C area, the surface of the ice is very flat and the hydraulic potential almost follows the bedrock heights, which correspond to the local maxima of hydraulic potential. These topographic features prevent water from moving upwards (Fig. 2), so the water observed on the lee of a relief, close to the transition with dry areas, must be of local origin.

Could the dry areas in Fig. 1 correspond to thawing ice whose meltwater has flowed away? Meltwater can be driven out by different types of hydrological networks: connected cavities (Lliboutry, 1968; Kamb, 1987) or an efficient network of structured channels (Röthlisberger, 1972). Weertman (1972) showed that these channels cannot form upstream of the hydrological network, which is the case here, and would rather take the form of a film of water. Whatever the exact type of structure (film or cavities), basal melting is a permanent process that feeds the hydrological network. A continuous hydrological network upstream is only fed by local melting, so that, unless the network is disconnected, the thawed water cannot be driven out faster than the melt rate. As a consequence, some water always remains at the base of the melting ice. Furthermore, water at the base enables basal sliding, and reduces the basal drag. For a given ice flux, the surface slope gives some indication on the relative importance of internal deformation and basal sliding in the ice motion. Local steeper surface may be associated with more basal drag and ice deformation, whereas local flatter surface may be associated with more sliding. Most of the spots where the model will be run are located on slopes which are locally steeper than the regional slope (Table 1), meaning it is likely that sliding does not occur there, an additional clue that the base is cold. As a consequence, we suggest that the dry areas in Fig. 2 probably do correspond to cold ice.

2.4.2 GF spatial variability

Based on the little that we know about the geology, we can assume that the GF is uniform on a $\sim 10\,\mathrm{km}$-lengthscale. Under this assumption, the presence of basal water is only the consequence of thicker ice, and not of spatial variations in GF. As such, thickness and reflectivity have to be locally correlated, and the 1D model will be run for small areas where this correlation is reasonable (section 3).

2.5 Numerical method

Equations were solved with a finite difference implicit scheme on a 50-element regular mesh, and the temporal dependence of the model was solved at a 1000-year timestep. These values were selected as trade-offs between obtaining accurate results and computation speed. The dependence of the basal temperature on discretization is limited to a few tenths of K.

The physical coupling of flow and heat content of the ice is accounted for by non linear iterations, in which updated values of m, K and c are computed, until the temperatures no longer show a discrepancy larger than 10^{-5} K between two iterations. At each timestep, the heat equation is solved for a boundary condition corresponding to the thermal state of the basal ice (temperate or cold). The computed solution may be inconsistent with the imposed boundary condition, i.e. that the pressure melting point may be exceeded (cold → temperate), or the melt rate may become negative (temperate → cold). In these cases, the equation is solved again with the new consistent boundary condition.

3 Spots where GF will be inferred

We defined the critical ice thickness H_c as the minimum ice thickness at which present basal melting becomes possible for a given GF Φ_g and a given shape parameter p. To determine H_c, the model was run with increasing values of the GF, until the melting point was reached at present, so that we we were able to determine a unique tuple (H_c, p, Φ_g). As the unknown is the GF, an estimation of this critical ice thickness was needed, and this was done in two steps from the wet/dry map in Fig. 1.

First, we identified spots at which ice thickness and reflectivity are correlated, i.e. for which the top of bed reliefs are dry and their lees are wet (see Fig. 2). Melting starts to become physically possible between the two. Ten corresponding spots were selected (black rectangles in Fig. 1), where reflectivity and ice thickness are somehow correlated, and that are hereafter denoted by the indexes of their central point (Fig. 4). For two spots (E4 and E6), the correlation was weak, but we kept them as they are the only ones available in the central part of our study area.

Second, we evaluated the critical ice thickness for each spot. Along the radar line, moving upwards along a bed relief, we selected the wet point at which the ice is the thinnest, and the dry point at which the ice is the thickest. The same is done moving downwards, so that four points were selected along the radar line (black arrows on Fig. 2). The critical ice thickness H_c was estimated by the average of the ice thickness measured at these points. The uncertainty on H_c was taken as half the standard deviation of the four thicknesses, so that almost all the possible values $(H_c \pm 2\sigma)$ ranged within the extreme thicknesses measured at the spot considered. For the large spots C3 and C6, this operation was performed twice, on the two perpendicular radar lines going through them (radar lines C and 3, and C and 6 respectively), so that eight dry/wet points were selected.

4 GF inversion

The two main input parameters that influence the presence or absence of water on the bed are the ice thickness H and the shape parameter p, both of which are affected by uncertainties. The heat model was run for 200 (H, p) values spread along Gaussian distributions, so that a posterior distribution of Φ_g was put out (Tarantola, 2005).

The critical thickness H_c measured on the map was used directly as a prior for H. The shape parameter p cannot be less than -1, so the inversion was done on the derived parameter p':

$$p' = \ln(p+1) \tag{14}$$

The prior value for p' was taken as a Gaussian distribution of mean $\bar{p}' = 1.5$ and standard deviation $\sigma_{p'} = 0.3$ so that the corresponding values for p were mainly distributed between common values of 1 and 10. For a given present-day critical ice thickness H_c and a given p, the 1D heat model was run for increasing values of Φ_g (by steps of $0.25\,\mathrm{mW\,m^{-2}}$), and the first Φ_g value that allowed melting at the present time was selected as the local GF. The thicker the ice, the lower the GF needed to allow for melting. All the GF values were in the range between 40 and $70\,\mathrm{mW\,m^{-2}}$.

5 Results

5.1 Inverse mode: geothermal heat flux

The inferred values of the mean GF for the ten measurement spots ranged between 48 and 60 $\mathrm{mW\,m^{-2}}$ (Table 1), and the highest values of the GF were found south of Dome C. Given the model assumptions, the inferred value for the GF at two potential drilling sites (C6 and H1) were respectively 59.3 ± 2.2 and $53.9 \pm 3.3\,\mathrm{mW\,m^{-2}}$.

A kriging interpolation was used to evaluate the GF between the spots evaluated. Unfortunately, the spots are scarce (there are only ten points), and are unevenly distributed throughout the study area. As such, the computed experimental variogram was poorly described, and we had little confidence in the computed kriging standard deviation. Moreover, the possibility of local variability at a scale of a few tens of kilometres cannot be excluded, so that the validity of such an interpolation is limited. Nevertheless, we underline the importance of such a map to show the regional trend that can be expected around Dome C.

Figure 5 shows the interpolation between the ten spots, in which the GF field evolves smoothly along a N-S gradient, with a typical norm of $0.1\,\mathrm{mW\,m^{-2}\,km^{-1}}$. At the two sites where the correlation between reflectivity and ice thickness was weak (E4 and E6), the inferred values of the GF nevertheless matched the N-S gradient, even if the value at E4 appeared to be significantly lower than its neighbours. Given that the kriging standard deviation does not account for the uncertainty on the GF estimation at the spots, the GF at the EPICA drilling site was estimated at $54.5 \pm 3.5\,\mathrm{mW\,m^{-2}}$.

5.2 Forward mode: emulator of the basal melt rate averaged over the last 400 000 years

A time-averaged value of the basal melt rate has to be computed to assess the risk of the oldest ice layers being lost during the glacial/interglacial periods. To do so, the GF field inferred at the previous step was used as an input for the heat model. Running it transient enabled the computation of the basal melt rate at each time step, which was done for given values of H, Φ_g and p, whereas α and β were kept at $1/6.04$ and 0.0156 respectively. Because of the uncertainty on the initial state, it was not clear if our temperature profile for the first glacial cycles was accurate, so we averaged the computed melt rate over the last $400\,\mathrm{ka}$ only.

As computing a given set of parameter takes several minutes, computing the basal melt rate for each point of the Dome region would be far too expensive. Here, the result of the whole forward model is mimicked by an emulator that depends on the input parameters H, Φ_g and p. The empirical relationship linking the average melt rate to the model parameters appears to be very regular (Fig.6). The slope of the isomelt contour lines shows equivalence between GF and ice thickness ($1\,\mathrm{mW\,m^{-2}}$ roughly corresponds to $60\,\mathrm{m}$). The sensitivity of the melt rate on p ($1 \leq p \leq 10$) appears to be equivalent to a range of $2\,\mathrm{mW\,m^{-2}}$ of GF. An additional flux of $1\,\mathrm{mW\,m^{-2}}$ correspond to an increased melt rate of $0.09\,\mathrm{mm\,a^{-1}}$. To account for this relationship by a multivariable polynomial, it seems natural to make m depend linearly on Φ_g, and quadratically on H and p. Over the positive-melt-value domain, we used a least-square minimization method to compute the following relation, which is valid for the ranges of values in figure 6:

$$
\begin{aligned}
m = & -5.148 \times 10^{-7} H^2 + 4.688 \times 10^{-3} H + 89.519 \Phi_g \\
& + 3.08 \times 10^{-3} p^2 - 5.887 \times 10^{-2} p - 14.335.
\end{aligned} \tag{15}
$$

The performance of the emulator was assessed by comparing the melt rate output from the thermal model with the one computed with the emulator. The average absolute error was $0.014\,\mathrm{mm\,a^{-1}}$, and the corresponding standard deviation was $0.016\,\mathrm{mm\,a^{-1}}$, so that the errors due to the emulator were significantly lower than the corresponding uncertainties due to the GF and p. This polynomial function was thus considered to be sufficiently precise for our purpose. Here we emulated the values of m with H given by the Bedmap 2 dataset, but when a refined bedrock dataset is available, this emulator will be easy to use to compute updated estimations of the basal melt rate.

Figure 7 shows the basal melt rate corresponding to the GF field interpolated from the central values of Φ_g inferred at each measurement point (called $\hat{\Phi}_g^m$), and the central value of p' ($p = 3.5$). The basal melt rate increased to more than $0.6\,\mathrm{mm\,a^{-1}}$ where ice is very thick, and vanished over several spots. The candidates `A`,`B`,`C` and `D` appeared to be melt-free, which was expected, since they benefit from a relatively thinner ice. Less expected was the presence of a potentially melt-free area sixty kilometres north from Dome C (point N8 in Fig.1). The ice is quite thick there ($\sim$3300 m), but the computed GF is low enough to prevent ice from melting. Changes in the basal melt rate over time are also presented for two (H_c, Φ_g) temperate configurations, leading to the same average value (Fig. 8). The difference between the minimum and maximum value of the basal melt rate was $0.6\,\mathrm{mm\,a^{-1}}$ for $2770\,\mathrm{m}$-thick ice, and an additional $500\,\mathrm{m}$ of ice dampens this amplitude by half, and smooths the high frequencies. The difference between present melt rates and maximum past values suggests that, more generally, it is possible that present cold basal ice melted during the warmest periods.

5.3 Sensitivity tests

Next, we investigated the influence of climatic forcing, by running the model with different slopes of the isotopic thermometer. The results for the GF obtained using the inverse method shifted positively by $1.4\,\mathrm{mW\,m^{-2}}$ for $\alpha = 0.20$ and negatively for $\alpha = 0.13$. However, the average basal melt rate is changed by $0.1\,\mathrm{mm\,a^{-1}}$ in the reverse direction, so that the final difference in the inferred melt rate was only $0.04\,\mathrm{mm\,a^{-1}}$ (Table 2). Because parameter α affects both the derivation of the GF at the spots and the forward model melt rate, the two steps partly offset one another when producing the melt rate map.

For extreme values of the β coefficient (accumulation model), the GF results shifted positively by $0.3\,\mathrm{mW\,m^{-2}}$ for $\beta = 0.0138$ and negatively for $\beta = 0.175$. Like for α, the average basal melt rate shifted in the reverse direction by $0.01\,\mathrm{mm\,a^{-1}}$, so that the final difference in the inferred melt rate was only $0.02\,\mathrm{mm\,a^{-1}}$.

The accumulations reconstructed at the dome itself were spread equally across the whole region, whereas in pratice spatial variations surely affect the surface accumulation rate around Dome C (Frezzotti et al., 2005). The sensitivity of our model to a $\pm 10\,\%$ variation of the surface accumulation resulted in a $\pm 0.3\,\mathrm{mW\,m^{-2}}$ difference for the GF, and to an opposite $\pm 0.08\,\mathrm{mm\,a^{-1}}$ for basal melt rate. The final sensitivity of the basal melt rate was thus $\pm 0.05\,\mathrm{mm\,a^{-1}}$. A higher accumulation rate results in a lower basal melt rate, which is expected.

Given that the expression of the pressure melting point is an unusual choice in glaciology (Eq. 5), the inverse method was reiterated with a more common value corresponding to saturated air in the ice ($T_m = 273.16 - 0.098\,P$, Cuffey and Paterson, 2010) as a test. The inferred GF values were found shift by $-0.6\,\mathrm{mW\,m^{-2}}$ compared to our expression. The results in terms of basal melting were not significantly affected by the expression of the pressure melting point, nor was the regional pattern of GF.

5.4 Forward mode: emulator of the critical ice thickness

One way to assess the performance of our model is to compare observed basal state (wet or dry, Zirizzotti et al. (2012)) with the model simulation for present time. For $p = 2$, we computed the (Φ_g, H_c) relation, corresponding to the present basal state, by sampling H_c between $2\,700\,\mathrm{m}$ and $3\,300\,\mathrm{m}$. The two parameters are linked by the following empirical relation:

$$H_c = 1013272.4\Phi_g^2 - 170906\Phi_g + 9486.0 \quad (16)$$

This made it possible to compute the difference between critical ice thickness and the present ice thickness (Bedmap2), so that negative values correspond to melting areas, and positive values to melt-free areas. Our aim was to broadly mimic the wet/dry pattern in areas where no critical thickness had been measured (north of Dome C). To facilitate the comparison, we slightly tuned the imposed GF field, within the uncertainty range. The map in Fig. 9 was built with $\hat{\Phi}_g^m - 1\,\mathrm{mWm^{-2}}$. Superimposed on the observed data, the model output shows that large-scale patterns of wet-dry areas were respected, especially on steep bed slopes (candidate B, C, D, and to a lesser extent candidate A). However, at certain points on these bed reliefs, there was a discrepancy between model outputs and observed data, but the gap in the critical thickness was often close to $0\,\mathrm{m}$ (D3, D5, D8, M3), meaning that a small change in GF forcing, or a better description of the ice thickness, would assign these particular points more accurately. The $1\,\mathrm{km}$-resolution of the Bedmap 2 bedrock dataset (Fretwell and coauthors, 2013) smoothed the along-track subkilometric features detected by our RES survey.

The steeper the bed, the sharper the limit between melting and non-melting areas. In the central, flatter, part of the study area, the position of this limit was more blurred. As we were unable to assess the GF in that par of the study area using our method, it was interpolated. Despite the lack of constraints, several small-scale features are well mimicked (dry areas at I9, G-H8, wet areas at G9 and L7). Other regions were not attributed in agreement with observations (G6-7-8, H-I8), meaning that the GF is overestimated, probably up to $3\mathrm{mW\,m^{-2}}$, which is consistent with the uncertainties produced by our method (inversion and interpolation).

The heat model allows easy detection of the very strong climatic signal of the LGM when it reaches the bed, and the corresponding time lag Δt for basal temperature appears to be linearly dependent on the ice thickness in the range 2700-3500 m by the following relation

$$\Delta t = 10H - 16610. \tag{17}$$

The isocontours of time lag Δt are simply parallel to the thickness contours, but enable a temporal interpretation of the wet/dry patterns (Fig. 9). Around $10\,\mathrm{ka}$ are required for the signal to reach the tops of bed reliefs, but almost twice that for 3 400- m-thick ice. Hence, considering the duration of deglaciation, depending on the ice thickness the thermal state of the basal ice may correspond to very different climatic periods.

6 Discussion

6.1 Consistency with published data and measurements

Our GF values are comparable to those inferred by Fox Maule et al. (2005) and Purucker (2013), but, because the uncertainties of their method are quite large, they cannot be considered as a reliable comparison. For the northern part of our study area, the relatively low values of GF ($\sim 50\,\mathrm{mW\,m^{-2}}$ or less) are consistent with those estimated by Carter et al. (2009) at the same place, except on a bed relief (candidate D), where they found a positive heat flux anomaly and a high melt rate on its flanks ($\sim 2.5\,\mathrm{mm\,a^{-1}}$). The heat anomaly was not induced with our method, since the GF is interpolated at this location. This shows that our method is reliable at locations where we do have an estimation of the critical thickness (corresponding to potentially interesting dry, low-reflective cold spots), and elsewhere at least describes a GF regional background. The GF field is also compatible with the values inferred by (Siegert and Dowdeswell, 1996) to model the temperature above subglacial lakes around Dome C ($41-58\,\mathrm{mW\,m^{-2}}$). But their values were only minima compatible with ice at the pressure melting point, whereas the present work benefits from the dry areas detected by RES used as upper bounds constrains.

At the EPICA drilling site, we also computed an average melt rate of $0.32 \pm 0.25\,\mathrm{mm\,a^{-1}}$, and this value seems to be slightly lower than, but yet consistent with the value of $0.56 \pm 0.19\,\mathrm{mm\,a^{-1}}$ previously inferred by Parrenin et al. (2007), with the inverse model used for dating the ice core. The range of possible basal melt rates inferred with our method seems sufficiently realistic to contain its effective local value, at a location where no observed critical thickness is yet available. The basal melt rate inferred for the different old-ice candidates, for which we had observations of H_c, is thus reliable.

6.2 Model assessment

6.2.1 Method validity

Around Dome C, ice may flow over a hilly bedrock, and the velocity profile is not necessarily as smooth as a 1D synthetic shape function. However, the energy budget at the base of the ice actually depends on the total advection of cold from the top, not on small scale variations in the vertical velocity profile. Using a synthetic profile is just a convenient way to account for a realistic vertical advection of ice towards the bed, controlled by a single parameter, p.

Some of our confidence intervals are quite low (E4, E6, L7 and N8), one consequence of the tiny difference in altitude measured on Fig. 1 between the highest wet points and the lowest dry points at a given spot. Since the correlation between ice thickness and reflectivity was weak, the confidence intervals at E4 and E6 are probably underestimated, and some local effects (e.g. small relief and GF variabilities) may not have been accounted for in this study.

We cannot exclude assignment errors in the dry points, if a small water film is present but was not detected. If so, we would only be able to assess a lower boundary for the GF. However, the absence of detection of the meltwater likely means that the water film is very thin and the melt rate is very low. The average inferred GF would only be offset by a small amount, and the regional GF gradient would remain unchanged.

6.2.2 Sensitivity to parameters

The sensitivity of our results to climatic forcing shows that the confidence interval on the GF at the 10 spots could be larger than presented so far, possibly by $\sim 2\,\mathrm{mW\,m^{-2}}$. However, for the archiving process, the truly important parameter is the basal melt rate, which is much less sensitive. For example, a lower α corresponds to a lower inferred GF (inverse run), but the average melt rate is higher for the same GF (direct run). The latter compensates for two thirds of the effect of the former, so that the melt rates computed for the ten measurement spots are quite robust to our lack of knowledge on climate forcing.

As the present surface accumulation pattern shows a N-S gradient, we wonder if our GF gradient could be due to an artefact of our method, which does not account for the spatial variations in accumulation. The sensitivity of the GHF on the surface accumulation is less than a few tenths of $\mathrm{mW\,m^{-2}}$ so that, accounting for its spatial variations would not radically modify our results. Furthermore, we do not know if the pattern of accumulation over the glacial/interglacial periods remained stable, so assuming its stability would result in unnecessary additional uncertainties.

6.2.3 Spatial variation in the GF field

In our point-by-point method, there are no horizontal regularization constraints to make the GF realistic at a regional scale. Yet, the inferred heat flow at the ten spots reflects a certain spatial pattern, which is a good sign of plausibility. More fundamentally, at first order, we can explain a relatively complex pattern of wet/dry areas in a whole region with a single physical key (a smoothly-varying GF field) with no 2D water routing model, or horizontal heat transport depending on both ice flow and bedrock topography. This means that the main physical mechanisms have been taken into account, at least where it was possible to evaluate the critical thickness, on significant topographic features. Where the GF is interpolated, in flatter areas, the lack of constrains prevents us from really assessing the validity of the method, meaning that our method needs a sufficient hilly bedrock to be applied.

Of course, a higher-dimension ice flow and hydrological model, over a refined bedrock, would be necessary to go one step further to describe water variability in more detail, but more assumptions concerning the model parameters would also increase model uncertainties. The GF confidence intervals given for the surroundings of Dome C are now about five to ten times lower than those previously available. In the context of the oldest-ice project, we suggest that $\hat{\Phi}_g^m + 2\sigma_\Phi$ is a realistic upper boundary for the local GF value.

6.3 Lessons drawn for the oldest-ice research

6.3.1 Interpretation of the wet/dry pattern at the base of the ice

The map of basal melt rate (averaged over the last 400 000 years) suggests that the old-ice candidates are melt-free across the glacial/interglacial periods. Conversely, the northern part of Dome C is generally a thawing area, whereas Zirizzotti et al. (2012) observed no presence of water, despite the local thick ice. In this region, the climatic signal of the last deglaciation did not reach the bedrock later than in regions of thinner ice (Fig 9, contour lines). Where the basal ice was cold at the LGM, the pressure melting point has not yet been reached since the deglaciation signal has just reached the bed for the last $\sim 2\,000$ years. In places with thinner ice, the deglaciation signal is almost complete; if the basal ice is still cold today, one can infer it was also the case over the whole glacial/interglacial periods. Most of the always-cold areas could simply be detected by considering the present, cold, state of the basal ice, and how long since the LGM signal reached the bed. But our heat model is a more comprehensive approach, which provides additional information concerning the local value of the GF and the spatial extent of the cold spots.

6.3.2 Old-ice targets

As expected, the regions where the basal ice is assumed to have been cold throughout the glacial/interglacial periods are the candidate A and the candidates B, C and D. These three last places may have a lower GF than candidate A and could be better places to exclude the possibility of basal melting. However, they are separated from the topographic dome by the Concordia trench. The deep ice that could be drilled at these locations likely crossed the trench, and the stratigraphy may have been affected by this topographically-disturbed region. Furthermore, the ice velocity is probably higher than over candidate A because of the steeper slope, and this may enhance basal disturbance. Given the confidence intervals, difference in GF at candidate A is not significant enough to make candidates B, C and D first choices, but they remain spots of great interest. Three-dimensional modelling will now be performed to study the regional ice flow towards these sites in more detail, using the basal melt rate computed in this study as an important input parameter.

Our study also suggests that a certain spot in the northern part of Dome C have a low enough GF to prevent basal melt at long time scales. As the ice is significantly thicker than at other previously considered candidate sites, very old ice could be retrieved with a much better resolution than elsewhere. Given that this hint of a low GF value is the result of only one observation, we wish to emphasize the importance of carrying out additional survey (e.g. ground/airborne radar) to check the validity of this suggestion.

Finally, the evolution of the benthic δ^{18}O past sea level proxy (Lisiecki and Raymo, 2005), and its similarity to the one of the EDC reconstructed temperatures, show that the mean air temperature at Dome C was probably ~ 2 K higher before -800 ka than after. As a consequence, the mean basal melt rate was probably also $0.1\,\mathrm{mm\,a^{-1}}$ higher. Even if we conclude this study with favorable statements, it should be borne in mind that basal ice perhaps underwent some melting on the tops and flanks of bed reliefs during the Mid-Pleistocene.

7 Conclusions

The geothermal heat flux is a poorly constrained geophysical parameter in Antarctica, despite its crucial influence on ice flow properties and old-ice archiving. Around Dome C, the available continental estimations are currently of limited benefit, and a more precise local estimation is lacking. Here we present a simple inverse method based on a 1D heat model, constrained by a previous amplitude analysis of RES echoes recorded at the ice/bedrock interface trying to distinguish wet and dry areas on the bed. Assuming that the GF is locally uniform, the presence of basal water is only a function of ice thickness. The critical ice thickness, for which the pressure melting point is reached today, is inferred from wet/dry thresholds used in the analysis of RES amplitude data (Zirizzotti et al., 2012). The heat model accounting for the whole history of the ice (changes in temperature, in accumulation and in ice thickness), was inverted for this critical thickness, to retrieve the value of the GF and of the time-averaged basal melt rate for ten spots around Dome C.

Our method is valid in dome areas where horizontal advection and diffusion can be neglected, but its principle could also be adapted for other regions with a more complex physical model. However, it assumes that the origin of the observed basal water is local, and is thus better suited for flat regions, where there is no upwards water transport on the bed reliefs. In places where horizontal ice flow is significant, deformational heat should be taken into account in the energy balance of the ice.

Furthermore, we show that the ice thickness plays a dual role. Of course, on average, it limits the diffusion heat towards the surface and increases basal melting. But in a changing climate, it lags behind temperature forcing, so that the base of a thin layer of ice is more sensitive to climate change, while the base of a thick layer of ice has just begun to be concerned by deglaciation and may still be cold today. Hence, as the LGM was one of the coldest climatic conditions ever recorded, the lag effect of the ice thickness must be taken into account to correctly interpret basal conditions today.

The interpolated map of the GF shows a regional gradient, oriented north-south. Where no critical thicknesses have been measured, the GF values are consistent with the pattern of dry and wet points, particularly in the northern region, which appears to be dry today, and hosts one potential old-ice site. All the previously considered old-ice candidates are very likely cold-based, or to have undergone very little basal melting. The uncertainty at the old-ice targets on the local GF is dramatically reduced compared to previous estimations made at a continental scale. Our model reveal spatial variations in the basal melt rate in the Dome C region, which is a helpful and realistic boundary condition for future 3D ice-flow modelling. More specifically, over the candidate A site, a recent steady sate model assimilated radar isochrones to invert the value of Φ_g and p, and computed the basal age of the ice (Parrenin et al., 2017). It confirmed the melt-free areas, giving us confidence that ice as old as 1.5 million year can be retrieved near Dome C.

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Table 1. Physical parameters, observed (critical thickness $H_c \pm 1\sigma$ [m], ratio of the local surface slope to regional surface slope (ICESat)) and inverted (GHF $\Phi_g \pm 1\sigma_\Phi$ [$\mathrm{mW\,m^{-2}}$]). The surface slopes are computed in circles with a 3-km and 10-km radius centered on each spot, onto which the surface DEM (Bamber et al., 2009) is fitted by a biquadratic function. The ratio between the two slopes is presented below for each spot. A ratio > 1 corresponds to a slope locally steeper than its environment.

Spot	H_c	$\Phi_g^i \pm \sigma_\Phi^i$	Surf. slope ratio
B9	3157 ± 46	55.5 ± 0.9	1.03
C3	2926 ± 75	59.8 ± 1.5	1.20
C6	2957 ± 111	59.3 ± 2.2	1.09
E4	3181 ± 46	55.2 ± 0.9	0.97
E6	3124 ± 17	56.1 ± 0.5	2.01
H1	3249 ± 197	53.9 ± 3.3	1.80
L4	3319 ± 209	53.2 ± 2.9	1.37
L7	3408 ± 48	51.6 ± 0.8	1.46
N4	3662 ± 92	48.1 ± 1.2	1.27
N8	3698 ± 49	47.6 ± 0.7	1.42

Table 2. Sensitivity of the GF (assessed from inverse runs, [$\mathrm{mW\,m^{-2}}$]) and basal melt rate (calculated with forward runs, [$\mathrm{mm\,a^{-1}}$]) to input parameters α, β and a. As the final value of m depends on both the inverse run to determine Φ_g, and the forward run to compute the melt rate, the last column accounts for the sensitivity on the whole procedure (inverse+forward).

Parameter	Φ_g	m	Total on m
$\alpha = 0.13$	-1.4	$+0.1$	-0.04
$\alpha = 0.20$	$+1.4$	$+0.1$	$+0.04$
$\beta = 0.138$	$+0.3$	-0.01	$+0.02$
$\beta = 0.175$	-0.3	$+0.01$	-0.02
$a : -10\,\%$	-0.3	$+0.07$	$+0.05$
$a : +10\,\%$	$+0.3$	-0.07	-0.05

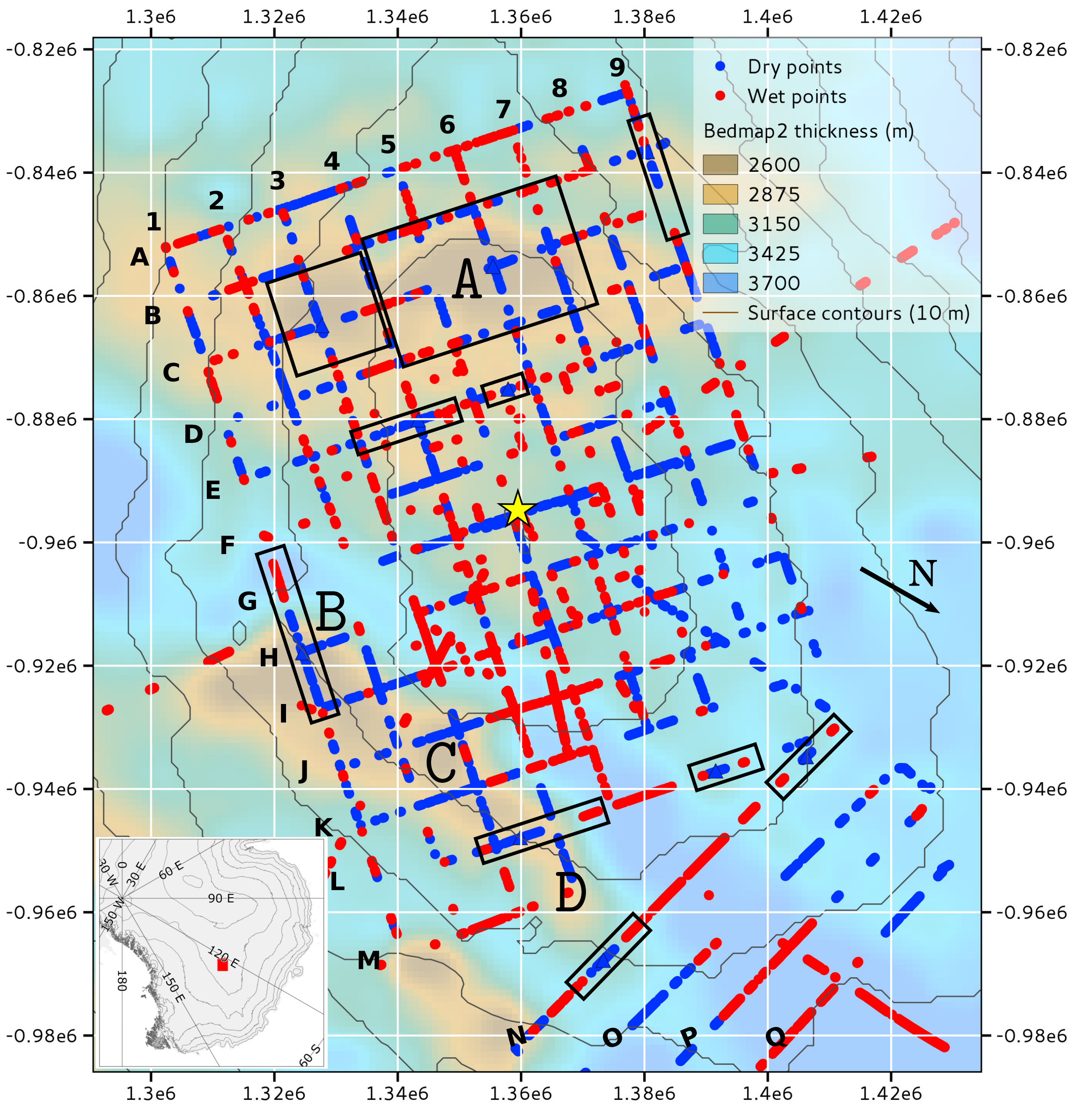

Figure 1. Wet (blue) and dry (dry), adapted from Zirizzotti et al. (2012). The yellow star shows the EPICA drill site, located at a distance of 1.4 km from the topographic dome. Side numbers and letters locate intersection points on the radar grid. Large letters identify for the main sites of interest (candidates A, B, C and D). Projection: WGS84/Antarctic Polar Stereographic - EPSG:3031 (metres). The north direction is bottom right. The situation map on the bottom left shows the position of our domain (red rectangle).

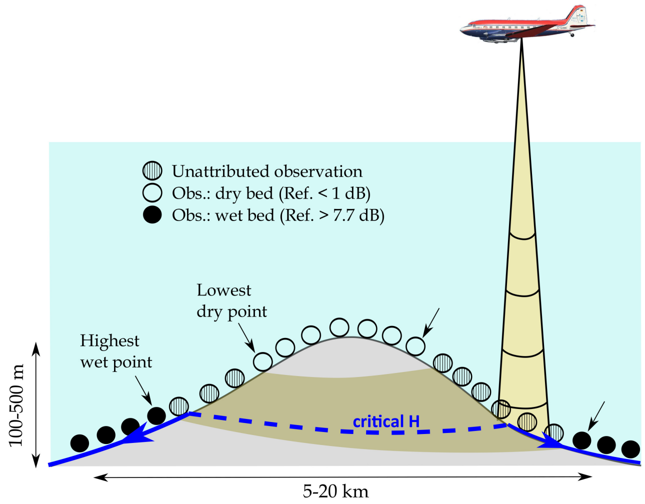

Figure 2. This figure illustrates how water and basal topography are linked with a region of flat surface for a uniform local GHF. The blue line shows the presence of water at the bed, and the direction of its flow. The brown area corresponds to a strip where the thermal conditions are unknown. We defined the critical thickness as the minimum thickness that allows basal melting today.

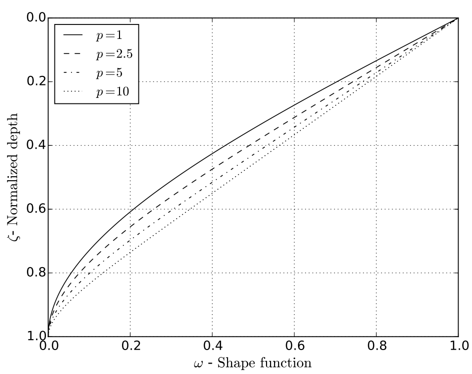

Figure 3. Shape function $\omega(\zeta)$ for several values of p (no basal melting).

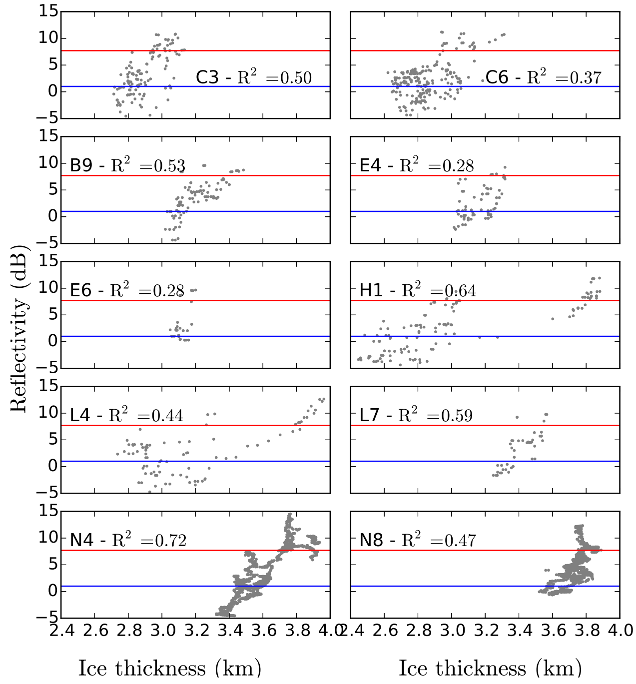

Figure 4. Basal reflectivity (dB) vs. ice thickness (km), at 10 favourable spots. Red and blue lines correspond to the thresholds of wet and dry points.

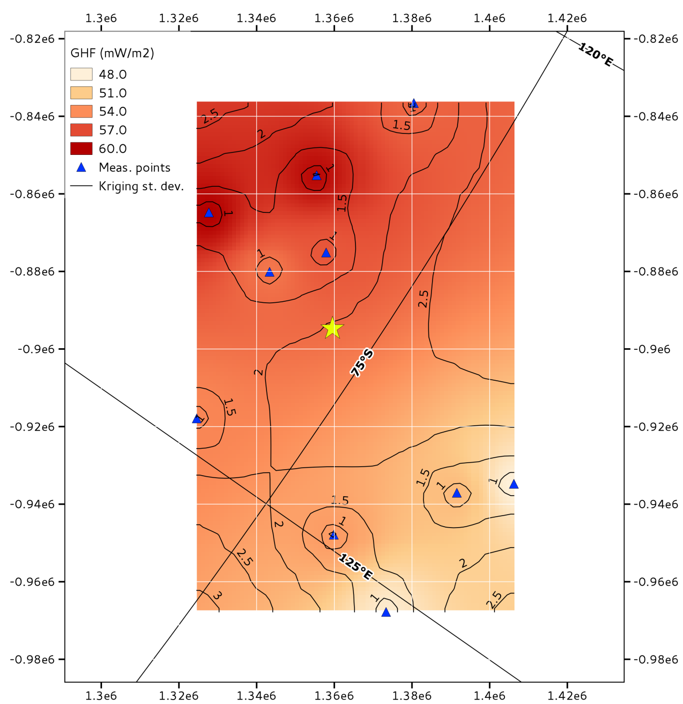

Figure 5. Geothermal heat flux, interpolated between the spots (blue triangles), and kriging standard deviation.

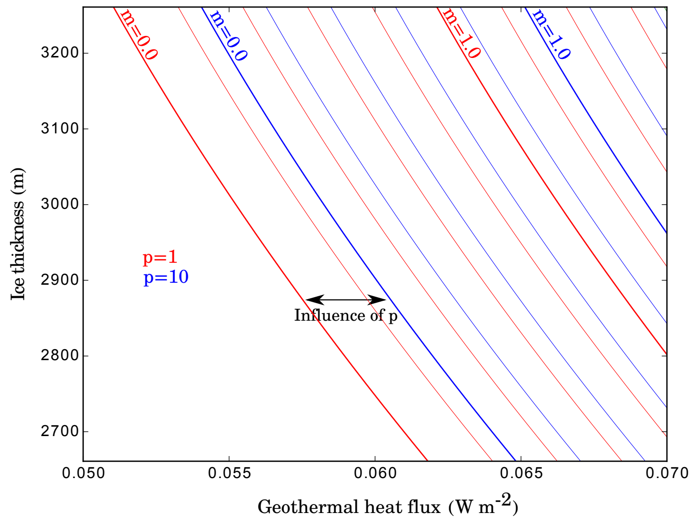

Figure 6. Basal melt rate in $\mathrm{mm\,a^{-1}}$, depending on the ice thickness, geothermal heat flux and parameter p.

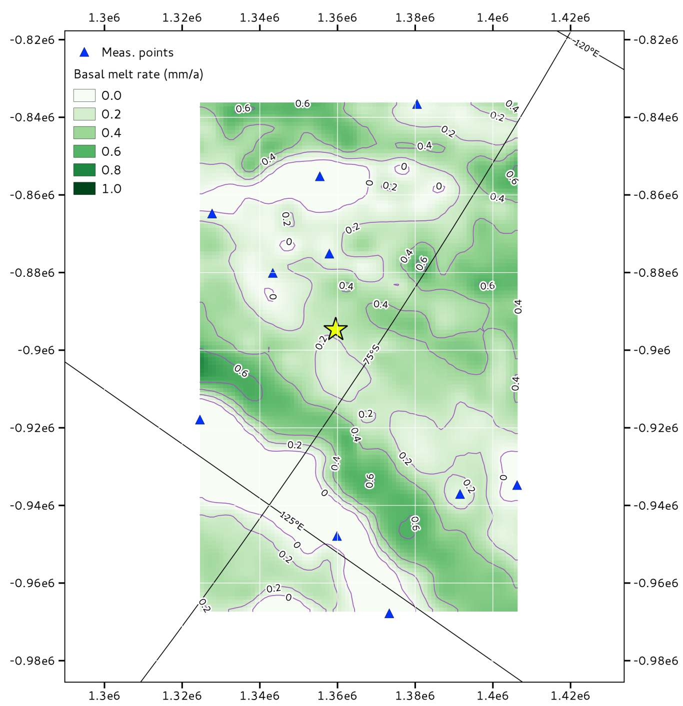

Figure 7. Averaged past basal melt rate in $\mathrm{mm\,a^{-1}}$, inferred from the emulating polynomial function with central values of Φ_g and p'. The ice thickness DEM is taken from the Bedmap 2 dataset (Fretwell and coauthors, 2013).

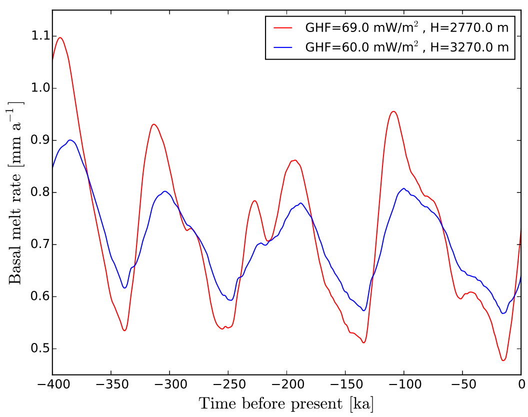

Figure 8. Changes in the basal melt rate, depending on the ice thickness and on the geothermal heat flux.

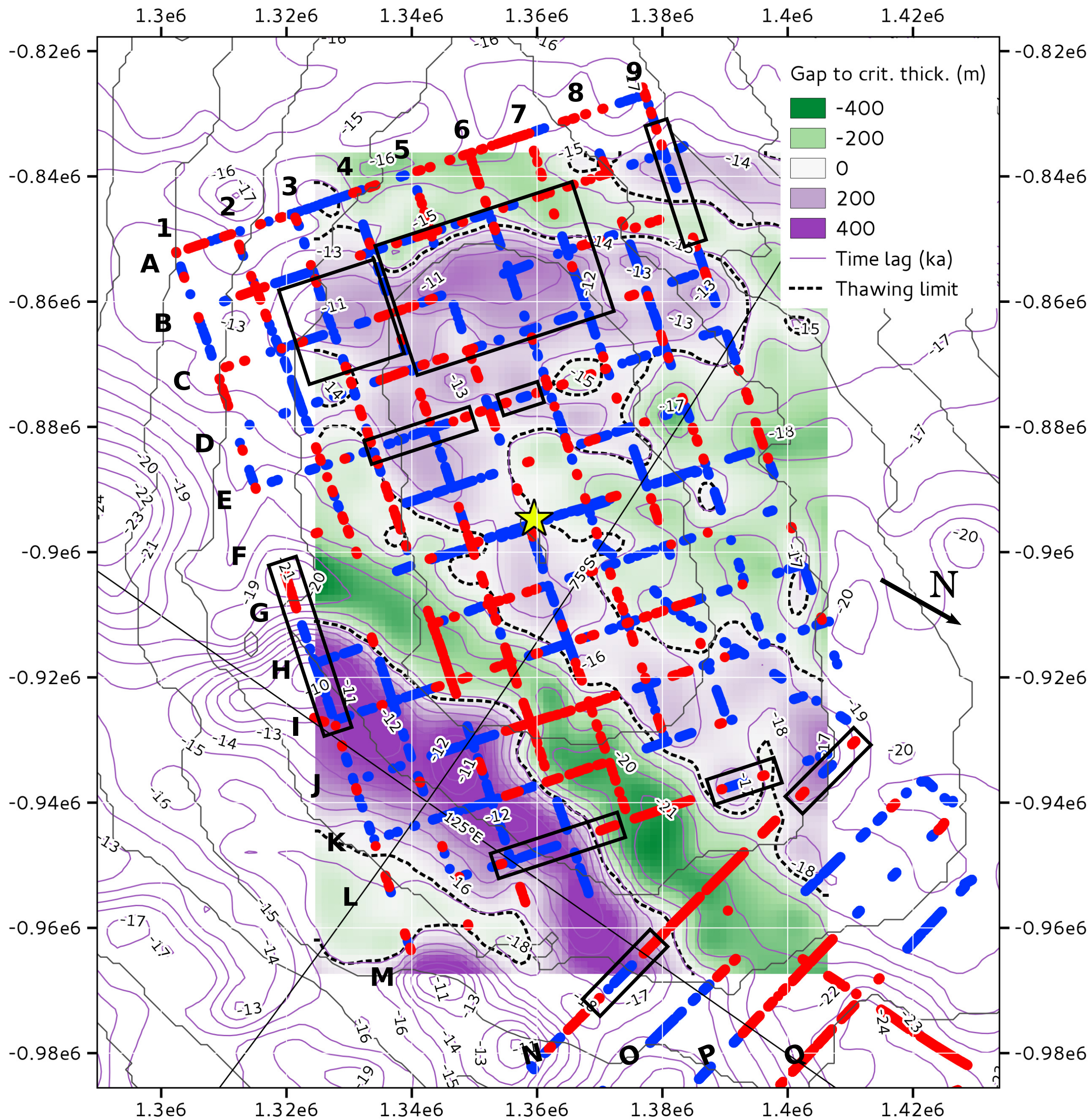

Figure 9. Difference between emulated critical ice thickness and observed ice thickness, for $p = 2$ and $\Phi_g = \hat{\Phi}_g^m - 1\,\mathrm{mWm}^{-2}$. The ice thickness DEM is taken from the Bedmap 2 dataset (Fretwell and coauthors, 2013). Isocontours show the time lag before the climatic signal to reach the bed.