## Dear Sebastian,

I have now had a chance to look at the revised version of the manuscript. Thank you for including the additional descriptive material on the physical rationale behind the model. This gives a clearer picture of how the model works and I think it does provide an avenue for us to take the manuscript forwards. However, I still do not think we can publish the manuscript in current form. The main reason is that I think you still need to include additional explanatory material on the reasons why you are using the equations that you are using. I have gone into further details of these below.

The important question raised by both referees is why you are using channel opening and closing equations to evolve the conductivity (now transmissivity). In the current version you are prescribing a conductivity and evolving the transmissivity. To do this you are appealing to the de Fleurian et al. (2016) study, where the same equations are used to evolve the thickness of an Equivalent Porous Layer. However, I still think that the model description in the manuscript will cause confusion to many readers, unless you spell out the assumptions behind your approach more fully.

In their study, de Fleurian et al. (2016) do not go into much detail in how they translate evolution equations for channel cross-sectional area into an equivalent porous layer thickness. However, their formulation can be recovered quite straightforwardly under the following assumptions:

- 1) Channels have area  $S_c$  and are spaced apart by  $L_c$ .
- 2) EPL thickness is identified as the average water thickness, given by the average crosssectional area of channels per unit length (along a line perpendicular to channels), which is  $d_{EPL} = S_c / L_{c.}$
- 3)  $S_{\rm c}$  evolves according to the channel opening and closing equations.
- 4) *L<sub>c</sub>* remains constant.

For Darcy flow with a particular conductivity and pressure gradient, the water flux is proportional to the cross-sectional area, so channels of cross sectional area  $S_c$ , spaced at distance  $L_{c,}$ , carry the same flux as the equivalent porous layer of thickness  $d_{EPL}$ . The assumption of Darcy flow through channels is somewhat unconventional in the glaciological literature, where the Darcy-Weisbach equation is often used. Nevertheless, I think that the explanation above is enough to consider the de Fleurian et al. (2016) interpretation as a physically-motivated model, particularly if channels are assumed to contain sediment, debris or other obstacles, so that Darcy flow is an appropriate assumption.

Now. In the current manuscript, you are evolving the transmissivity (T) using the same equations, but you are simply replacing Equivalent Porous Layer thickness with transmissivity. I think this is the source of the difficulty we have all had in understanding the manuscript (in which I include both referees and myself). Unless this is clarified I think most readers will also be confused. You are appealing to the de Fleurian et al. (2016) model (in which  $d_{EPL}$  can be conceptualised, as above, as the thickness of a water film that would be produced if all the water in the channels were distributed uniformly). But, it is clear from the description that you are assuming that this film acts as though it is filled with material that has a conductivity K, and that this takes the same value as is used for the groundwater flow through the aquifer. In effect then, you are using the channel evolution equations to bring about new aquifer that can transmit flow (channel opening terms), or to remove aquifer (channel closing terms). This is only affecting the transmissivity (T). For all other purposes, such as storage, the thickness of the aquifer (b) is being kept fixed. Perhaps the best route forwards is to explain more fully some of the steps that lead to your system of equations. I give two examples of how this could be done below, but I really mean these to illustrate the level of detail needed, not to be prescriptive about the interpretation of the model.

One option would be to include an equation where Transmissivity (T) is written as the sum of two terms, as appropriate for flow in parallel through the aquifer ( $T_a$ ) and the channel system ( $T_c$ ).

$$T = T_a + T_c$$

Unconfined case:  $0 < \psi < b$ , with  $\psi = h - z_b$ ,

$$T_a = K_a \ \psi,$$
$$T_c = 0,$$

where  $K_a$  is the conductivity of the aquifer.

Confined case:  $\psi > b$ ,

$$T_a = K_a \ b,$$
$$T_c = K_c \ d_{EPL}.$$

where  $K_c$  is the conductivity of the equivalent porous layer (or equivalently of the Darcy flow through channels of cross-sectional area  $S_c$ , spaced apart by distance  $L_c$ ).

As above, the Equivalent Porous Layer  $(d_{EPL})$  for fixed channel spacing  $(L_c)$  is

$$d_{EPL} = S_c / L_c$$

Channel cross-sectional area  $(S_c)$  evolves (as already described in the appendix),

$$d(S_c)/dt = Melt + Cavity_opening - Creep_closure$$

The system that you are solving appears to assume that conductivities are the same in the aquifer and in the equivalent porous layer, so that  $K_c = K_a$ . You also seem to be neglecting flow in the aquifer for the confined case, so that  $T = T_c$ , in the confined case, rather than  $T = T_a + T_c$ .

If you agree with this interpretation of your equations then I think you need to include these arguments, and these additional steps, to the model derivation in the manuscript (using your own preferred notation for the quantities referred to above), otherwise the manuscript presents no logic as to why you are solving the system that you are solving. If you do this, then the assumptions need to be justified. The assumption  $K_c = K_a$  is perhaps appropriate if channels are filled with sediment, but you will then need to explain that any effect of the sediment on creep closure has been neglected. The assumption  $T = T_c$  in the confined case could perhaps be justified if  $d_{EPL} >> b$ , and you could test this from analysis of your existing results.

One problem with the above interpretation is that your model only seems to include storage in the aquifer, not in the equivalent porous layer, but if  $d_{EPL} >> b$  this does not seem appropriate. An alternative interpretation, is that the equivalent porous layer is thinner than the aquifer, so that  $d_{EPL} << b$ , allowing you to neglect storage in channels. If channels are much more conductive than the aquifer, so that  $K_c >> K_a$ , then it is possible that  $T_c >> T_a$ , so that T is approximately proportional to  $d_{EPL}$  in the confined case, despite the equivalent porous layer being thinner than the aquifer. This solves the storage problem, but does not explain why you are using one value of K throughout. In that case, you should be using  $K_c$  in the melting term, and the cavity opening term, so, under this interpretation, you are either underestimating these terms, or overestimating flux through the

aquifer. The cavity opening term could be dealt with simply by changing the cavitation step height, so that the parameter  $\beta$  is unchanged from your simulations, but that still leaves either the melt term underestimated, or the flux through the aquifer overestimated.

If you do not agree with either of the above interpretations, then you will need to supply a similarly detailed picture of how you consider that the model can be derived from some physical picture of the system under consideration. The description should make it clear what assumptions have been made and what the consequences of those assumptions might be. Unless I feel that this description has been provided I will reject the manuscript. Simply appealing to similarities with de Fleurian (2016) study, as you do in the present version of the manuscript, is not enough. It does not provide enough guidance for the readers to assess whether the model can be expected to behave realistically or not.

To be more specific, the main difficulties are with the derivation in the appendix.

P18. Equation A2. As described above, you need to provide a physical interpretation here that makes sense. This interpretation says that aquifer thickness grows when channels grow in area, but melting does not bring a new layer of aquifer into existence. I think you need to separate out aquifer thickness b and Equivalent Porous Layer thickness  $d_{EPL}$  and be much clearer about which concept is in use at each stage. Please think carefully about this and present a coherent explanation for the model equations. You cannot replace one quantity (channel area  $A_c$ ) with another (aquifer thickness b) unless you provide some physical reasoning why you are doing this.

P19. Equation A7. Same problem. Please provide an explanation that has some physical reasoning behind it. You need to be clearer about what is aquifer thickness (*b*) and what is equivalent porous layer thickness ( $d_{EPL}$ ). It is the latter that is controlled by the opening equations. Creep closure of channels is not usually considered to destroy aquifer.

P19. Line 17. Same problem. Please provide an explanation that has some physical reasoning behind it.

P20. Line 5. Same problem. Please provide an explanation that has some physical reasoning behind it. This only applies if aquifer thickness b is changing, but I think it is the thickness of the Equivalent Porous Layer that is changing.

Please include the cavity opening term and give the reason why it takes the form that it does. Cavity opening creates channel area at rate  $v_b h_{step}$ , where is  $v_b$  is sliding speed and  $h_{step}$  is step height. This provides a source of channel area, not a source of aquifer thickness. However, if  $T_c = K_c d_{EPL}$  and  $d_{EPL} = S_c / L_c$  a cavity opening term of similar form can be recovered. Please go through the steps needed to relate cavity opening to channel area and transmissivity and include this chain of reasoning in the manuscript.

To be clear. I will reject the paper if these questions are not clarified. I don't think this needs to happen, because I think there are conditions (as outlined above) for which the system of equations that you are solving, or perhaps a slight modification of them using two conductivities ( $K_c$  and  $K_a$ ), can be justified. You need to do a much better job at explaining the physical motivation behind the model in the manuscript.

I have also included some more minor technical corrections below.

Yours sincerely,

**Robert Arthern** 

## Minor technical corrections

Please go through carefully and check which equations should be using the hydraulic head (h) and which should be using the relative value (psi).

In particular,

- i) Equation 4. Shouldn't  $\psi$  be used to determine whether the system is confined or not.
- ii) Equation 7. Shouldn't pressure be  $P = \rho_w g h$ , not  $\rho_w g \psi$ .

Appendix A.

P19. Please correct description of chain rule. There is a missing value of  $R_c$ .

P19. Probably better to leave the sign on the gradient (G) and the flux (q) rather than taking magnitudes. If you do this, please go through carefully and make sure melt term is defined to have the correct sign.

P19. flux per unit length(?).