

Dear Editor, dear Reviewers,

The authors wish to thank you again for valuable remarks and patience. We fully understand your request to provide a clear physical justification for our modeling approach. In order to address this issue we adopted a more general (transmissivity) formulation for our aquifer model that can be directly derived (by multiplication with the constant hydraulic conductivity K) from the corresponding evolution equation for aquifer thickness of De Fleurian et al. 2016 or by multiplying with constant aquifer thickness b from our formulation for the hydraulic conductivity. Using this generalized model, we adopted the same set of assumptions ($K=\text{const}$) as in De Fleurian et al. 2016 instead of ($b=\text{const}$) used in our previous revision (we also thank the reviewers for pointing out a problem with units). By doing this, we ensure that we use the exact same parameterization as the original works – as we intended – and simplify the comparison of the results. The generalized model retains all the features (single layer for effective/ineffective system, combination of confined and unconfined Darcy flows) of our model presented in earlier revisions. We have re-run all our experiments, because we had to adjust the parameters, but the basic model is still the same as before. This new round of tuning was also necessary, because we added – as Referee #1 suggested – a term representing cavity opening into the model.

Referee #1: Doug Brinkerhoff

Physical Validity

The authors have addressed many of my original complaints, and I am convinced that their model is implemented and solved correctly. However I am still not convinced that this model is physically justified. The authors give the classic derivations for the equations that govern changes in channel area by dissipative heating and by creep closure. These are not at all controversial. What is controversial is implicitly stating that $K = A_c$, or that bulk hydraulic conductivity is equal to the area of a hypothetical channel. These things don't even have the same units. A_c is cross sectional area with units m^2 . K is hydraulic conductivity, with units m s^{-1} . They clearly cannot be equal, yet looking at equation A10 versus A11, that is apparently the exact assumption that is being made. It also implies that the first term on the right hand side of Eq. A11 has incorrect units (m s^{-1} , while the other two terms have units m s^{-2}). Even if a constant were included to deal with the unit problem, I still need to understand why hydraulic conductivity should scale linearly with channel cross-sectional area, when it's typically thought that transmissivity is a non-linear function of channel cross-sectional area.

After carefully checking our equation again, we found that we have made an error in the conversion to conductivity. We fixed this error and decided to generalize the formulation by evolving the transmissivity instead of the hydraulic conductivity. This re-formulation allows us to confirm the reviewer's claim of non-linear dependence between the transmissivity and the channel cross-section that is somewhat awkward to handle numerically. We came to the

conclusion that it is simpler if we use the original equation of de Fleurian et al. 2016 and evolve the transmissivity under assumption ($K=\text{const}$) resulting in a linear dependence.

Inclusion of sliding

Fowler (1987) does not necessarily justify the neglect of cavity opening, particularly near the margins. The specific line in Fowler (1987) stating that cavitation is unlikely for ice sheets also assumes low water pressure, which is likely not the case for NEGIS. Similarly, it is mostly unknown whether bedrock undulations are actually longer wavelength in the sub-stream environment than elsewhere. The low slope component of Fowler's conclusion (namely that slopes are around 10^{-3}) are also not valid here, especially near the margins, where surface slopes are closer to 2.5×10^{-2} , not so different from a mountain glacier. Finally, there are many observations from boreholes in Greenland that seem to be consistent with a linked cavity system. Ignoring it seems unjustified to me, regardless of the lack of parameter knowledge.

We realize that it really is a good idea to include sliding/a cavity opening term and did so in our evolution equation. The advantage is, that now we do not need to use the minimum for the conductivity/transmissivity to have an initial opening and preventing the system from going completely watertight, but have a proper term to control this.

Parameter Choices

I cannot accept the conclusion that because the system being modelled is imaginary, that including parameters from two other physically justifiable models at the same time somehow makes this model capable of capturing the behavior of both.

Whereas there is a clear physical justification of the ineffective model, the representation of channels using the Darcy model relies on a wide array of assumptions (especially those concerning the channel geometry and size distribution) that are not so easy to consider realistic. Our idea of creating an effective model is to use a somewhat modified set of assumptions to represent large scale (bulk) behavior for both types of drainage. Similar approaches are widely used for applications in subsurface modeling (see, e.g., double porosity models for fractured aquifers). Certainly the way the porosity is treated is different in a channelized system than in an inefficient system, but the proposed approach assumes that an active effective system clearly dominates the flow thus the choice of parameters in our model is clearly biased towards the effective type of flow. For ineffective flow situations dominated by 'background' conductivity, the model uses limiting values T_{min} with the corresponding classical Darcy behavior – just as the separate ineffective layer models do in two-layer approaches.

Technical corrections

P19L14 What does 'empirical nature' mean? Doesn't look like the correction made it into the next version.

We wanted to express our idea to use the available data to build the simplest possible model and we have updated the sentence like this:

Since extensive observations of the subglacial system are rare, its large scale approach to fit a simple parametrization of the flow to the available data can be an advantage.

Referee #2

1. In the opening comments, the authors better describe the aim of the model:

"Our goal is not to fit an aquifer model to the really existing subglacial aquifers by finding correct geometrical and physical parameters. This approach would likely work for flows through till or similar types of drainage but just as likely fail for channel networks. Instead we chose Darcy model as an 'idealized' representation of all drainage processes (including channels) and tuned it to give the best fit to the available data."

A few sentences down, they remark that to better explain this point, they have added a corresponding sentence in the introduction. However, the sentence I think they're referring to isn't sufficient. I think it would be best for the authors to add a new paragraph, essentially laying out clearly their in motivation as in the above quote. This would really avoid confusion for readers.

[This is a good point and we modified the introduction to attempt to better explain our motivation and modeling assumptions.](#)

2. I'm not certain I fully understand the author's reply to my point 5 (regarding instability with channels). Further the authors have chosen to add a sentence P19L15 in the conclusion stating that no instability is a positive aspect of their model. I think this requires them to add a corresponding section in the discussion about the instability, since the conclusions should cover the discussion (unless I missed it there). This would be a good opportunity to try to be more lucid about what's going on. Can this be related to physics, or is this an outcome of the model parameterization?

[The reason for why we do not have this instability is, that in the parametrization that we adopted from De Fleurian et al. 2016 the melt term, as well as the creep term depend linearly on the transmissivity/layer thickness. This means, that there is no potential for instability, because the melt term does not grow way faster than the creep term, when it is slightly perturbed. The instability usually occurs when a turbulent flow parametrization, such as Darcy-Weisbach is used \(see e.g. Schoof 2010\).](#)

[We added a small paragraph explaining this in the paper.](#)

3. For my point number 6, the authors direct me to the response of point number 9 of the other reviewer. I'm not clear what their response exactly means, except that perhaps this goes back to the point that their model isn't meant to be physical. They perhaps need to do a better job guiding us to what parts of the model are seen as physical, and what parts are parameterizations tuned for the model to fit other models/reality.

First review point 6: The argument that the amount of water released from an unconfined aquifer is larger than a confined aquifer is counterintuitive (P4L5). If the head drops 1m in the unconfined case, then wouldn't the water released be much greater than in the confined case, due to the volume of water in the latter being limited by the porosity?

We think we have a (likely somewhat syntactic rather than in substance) misunderstanding regarding the role of the storage coefficient in both types of model. In the unconfined case, the water can be stored in the empty pore space with little effort (i.e. small changes in head result in large changes in the volume of stored water). In the confined case, all pore space is filled, some additional water can be stored by compressing the solid matrix usually possible due to trapped air; this type of storage increase requires a lot of effort (i.e. small changes in head cannot produce large changes in volume of stored water). This is the meaning behind $S_s b \ll S_y$ expression in our previous response. Also see Bear, Cheng (Springer, 2010, p. 214) for more detail.

4. I'm not certain that keeping the SSA is in the best interest for the manuscript. It seems like a poor choice, especially since there doesn't seem to be a computational limitation for using a higher order model. I wouldn't derail the paper over this, but I wouldn't think that rerunning the model would be that hard. This effects the author's misfit quantification, because the SSA model clearly fails in the slow moving regions, while improves when hydrology is introduced. If they use Blatter-Pattyn, the misfit would be lower in the no-hydrology case for slow moving regions, and then the fit may decrease over this region when hydrology is introduced. Overall however, since the fit leaves room for improvement, I suspect most suspect readers will only go as far as interpreting the results visually.

We agree that this is a valid point and we have repeated our experiments using higher order equations (Blatter-Pattyn). Due to the additional deformation part, the overall velocity increases (more visible in the no-hydrology case, because of the color scale) and the modelled velocity is closer to the observed data. The result is still the same: using hydrology improves the RMS by about 50 m/a. We updated the text in the manuscript to reflect these changes.

A confined–unconfined aquifer model for subglacial hydrology and its application to the North East Greenland Ice Stream

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Abstract. Subglacial hydrology plays an important role in the ice sheet dynamics as it determines the sliding velocity of ice sheets. It also drives freshwater into the ocean, leading to undercutting of calving fronts by plumes. Modeling subglacial water has been a challenge for decades, and only recently new approaches have been developed such as representing subglacial channels and thin water sheets by separate layers of variable hydraulic conductivity. We extend this concept by modeling a confined and unconfined aquifer system (CUAS) in a single layer. The advantage of this formulation is that it prevents unphysical values of pressure at reasonable computational cost. We also performed sensitivity tests to investigate the effect of different model parameters. The strongest influence of model parameters was detected in terms governing the opening and closure of channels. Furthermore, we applied the model to the North East Greenland Ice Stream, where an efficient system independent of seasonal input was identified about 500 km downstream from the ice divide. Using the effective pressure from the hydrology model in the Ice Sheet System Model (ISSM) showed considerable improvements of modeled velocities in the coastal region.

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1 Introduction

Subglacial water has been identified as a key component in glacial processes, it is fundamental in driving large ice flow variations over short time periods. Recent studies show considerable progress in modeling these subglacial networks and coupling them to ice models. Water pressure strongly influences basal sliding and can therefore be considered a fundamental control on ice velocity and ice-sheet dynamics (Lliboutry, 1968; Röthlisberger, 1972; Gimbert et al., 2016).

Generally, two fundamentally different types of drainage are identified: discrete channel / conduit systems and distributed water sheets or thin films. Distributed flow mechanisms are, for example, linked cavities (Lliboutry, 1968), flows through sediment/till (Hubbard et al., 1995), or thin water sheets (Weertman, 1957); those are considered to be an inefficient and slow system to transport water. Channels (Röthlisberger, 1969; Shreve, 1972; Nye, 1976) are seen as discrete single features or

arborescent networks; they usually develop over the summer season when a lot of melt water is available. It is assumed that these channelized or efficient drainage systems able to drain large amounts of water in short time spans are predominant in alpine glaciers and on the margins of Greenland, where substantial amounts of surface melt water are capable of reaching the bed (van den Broeke et al., 2017). In the interior of Greenland and also in most parts of Antarctica, the water supply is limited to melt due to the geothermal and frictional heating within the ice (Aschwanden et al., 2016) – a circumstance favoring distributed systems.

Seasonal variations of ice velocity have been observed and attributed to the evolution of the drainage system switching between an efficient and inefficient state in summer and winter (Bartholomew et al., 2010). For this reason, a new generation of subglacial drainage models has been developed recently that is capable of coupling the two regimes of drainage and reproducing the transition between them (Schoof, 2010; Hewitt et al., 2012; Hewitt, 2013; Werder et al., 2013; De Fleurian et al., 2014; Hoffman and Price, 2014). While these models demonstrate immense progress for modeling spontaneously evolving channel networks, it is still a challenge to apply them on a continental scale. A comprehensive overview of the various operational and newly emerging glaciological hydrology models is given in Flowers (2015).

Distributed or sheet structures can naturally be well represented using a continuum approach, while channels usually require a secondary framework, where each feature is described explicitly. Water transport in channels is a complex mechanism that depends on the balance of melt and ice creep (Nye, 1976; Rothlisberger, 1969), channel geometry, and network topology. Additionally, the network evolves over time which further complicates modeling of this process. When simulating channel networks, particular care must be also taken to prevent the emergence of instabilities due to runaway merging of channels (see the discussion in Schoof et al. (2012)). This leads to increased modeling complexity and high computational costs. An exception to this is the work of De Fleurian et al. (2014), where both systems are represented by Darcy flow through separate porous media layers. The layer representing the channels has its parameters (namely hydraulic conductivity and storage) adjusted to exhibit the behavior of an effective system.

We take this idea even further and only use a single layer of Darcy flow with locally adjusted transmissivity of the layer at locations where channels form, ~~while, at the same time, accounting for a decrease in storage.~~ This means that we approximate the channel flow as a fast diffusion process similarly to work in De Fleurian et al. (2014); however, a single Darcy flow layer with spatially varying parameters (effective hydraulic ~~conductivity~~transmissivity) accounts for both drainage mechanisms. Similar approaches are known to have been applied to modeling of fracture networks in rock Van Sice (2002). This reduced complexity model does not capture channels individually but represents their effect by changing specific local properties. Since our model aims to simultaneously represent the main properties of both drainage mechanisms (efficient and inefficient), special care must be exercised when choosing the model parameters and relating them to the physical properties of a specific scenario. In particular, the geometrical and physical parameters used in this model are not directly comparable to observed quantities, but instead describe an idealized representation that gives the best fit to the available data. While this strategy may not help to advance the precise understanding of channel formation processes, it captures the overall behavior, is computationally efficient, and allows to examine the complex interactions on larger spatial and temporal scales. ~~Currently we focus on channelised~~

~~hydrology, whereas in future we will consider to expand the model towards cavity formation and closure. This is primarily due to the fact that cavity formation is not considered a dominant effect underneath the large ice sheets (Fowler, 1987).~~

In addition, we introduce a new Confined–Unconfined Aquifer Scheme (CUAS) that differentiates between confined and unconfined flow in the aquifer (Ehlig and Halepaska, 1976). While the assumption of always saturated – and therefore confined – aquifers may be true for glaciers with large water supply, it does not hold in areas with lower water input. Especially in locations far from the coast, the water supplies are often insufficient to completely fill the aquifer. Ignoring this leads to significant errors in the computed hydraulic potential and unphysical, i.a. negative, water pressure. This problem has been analyzed in detail by Schoof et al. (2012), but here we study the effect in the context of equivalent aquifer models using unconfined flow as a possible solution.

Large scale ice flow models often compute the basal velocity using a Weertman-type sliding law, where the inverse of the effective pressure (difference between ice overburden pressure and water pressure) determines the velocity at the base. Low effective pressure leads to high basal velocity. Without subglacial hydrology models, the ice models simply take the ice overburden pressure as effective pressure completely neglecting water pressure. This is a major reason why these models struggle to represent fast flowing areas such as ice streams. The effective pressure computed by our model can be easily coupled to an ice sheet model and improve results for fast flowing areas.

~~The following~~ Our work is structured as follows. In the next section, we present the one-layer model of subglacial aquifer ~~and briefly describe its discretization~~. In Sect. 3 the model is applied to artificial scenarios, and the sensitivity to model parameters and stability are investigated. In addition, results for seasonal forcing are presented there, and we show how the model evolves over time. Section 4 demonstrates the first application of the proposed methodology to the North East Greenland Ice Stream (NEGIS), which is the only interior ice stream in Greenland. It penetrates far into the Greenland mainland with its onset close to the ice divide, so sliding apparently plays a major role in its dynamics. A short conclusions and outlook section wraps up the present study.

2 Methods

2.1 Confined–Unconfined Aquifer Scheme

The vertically integrated continuity equation in combination with Darcy’s law leads to the general *groundwater flow equation* (see e.g. Kolditz et al. (2015)):

$$S \frac{\partial h}{\partial t} = \nabla \cdot (T \nabla h) + Q \quad (1)$$

with h the hydraulic head (water pressure in terms of water surface elevation above an arbitrary datum also known as the piezometric head), S the storage coefficient (change in the volume of stored water per unit change of the hydraulic head over a unit area), ~~h the hydraulic head (water pressure in terms of water surface elevation above an arbitrary datum; piezometric head)~~, T transmissivity of the ~~porous medium~~ aquifer, and Q the source term. For a confined aquifer, $T = Kb$, where K is the

hydraulic conductivity, and b is the aquifer thickness. $S = S_s b$ with specific storage S_s given by

$$S_s = \rho_w \omega g \left(\beta_w + \frac{\alpha}{\omega} \right) \quad (2)$$

with material parameters for the porous medium (porosity ω , compressibility α) and water (density ρ_w , compressibility β_w).

In order to consider the general form covering both cases (confined and unconfined), we follow Ehlig and Halepaska (1976) and write the general form for the confined–unconfined problem:

$$S_e(h) \frac{\partial h}{\partial t} = \nabla \cdot (T_e(h) \nabla h) + Q. \quad (3)$$

Now the transmissivity and the storage coefficient depend on the head and are defined as

$$T_e(h) = \begin{cases} T, & h \geq b & \text{confined} \\ K\Psi, & 0 \leq h < b & \text{unconfined} \end{cases} \quad (4)$$

where $\Psi = h - z_b$ is the local height of the head over bedrock z_b and effective storage coefficient S_e is given by

$$S_e(h) = S_s b + S'(h) \quad (5)$$

with

$$S'(h) = \begin{cases} 0, & b \leq \Psi & \text{confined,} \\ (S_y/d)(b - \Psi), & b - d \leq \Psi < b & \text{transition,} \\ S_y, & 0 \leq \Psi < b - d & \text{unconfined.} \end{cases} \quad (6)$$

This means that as soon as the head sinks below the aquifer height, the system becomes unconfined, and therefore only the saturated section contributes to the transmissivity calculation. This also prevents the head from falling below the bedrock as detailed in Section 3.2. Additionally, the mechanism for water storage changes from elastic relaxation of the aquifer (confined) to dewatering under the forces of gravity (unconfined). The amount of water released from dewatering is described by the specific yield S_y . Since this amount is usually orders of magnitudes larger than the release from confined aquifer ($S_y \gg S_s b$), it is useful to introduce a gradual transition as in Eq. (6) controlled by a user defined transition parameter d .

Note that the transmissivity is not homogeneous making Eq. (3) nonlinear. This fits with our approach to describe the effective system (channels) by locally increasing the ~~hydraulic conductivity~~transmissivity. The benefit of this approach is discussed in Sect. 3.2.

Water pressure P_w and effective pressure N are related to hydraulic head as

$$P_w = \Psi \rho_w g \quad (7)$$

and

$$N = P_i - P_w \quad (8)$$

with g acceleration due to gravity, $P_i = \rho_i g H$ the cryostatic ice overburden pressure exerted by ice with thickness H and density ρ_i .

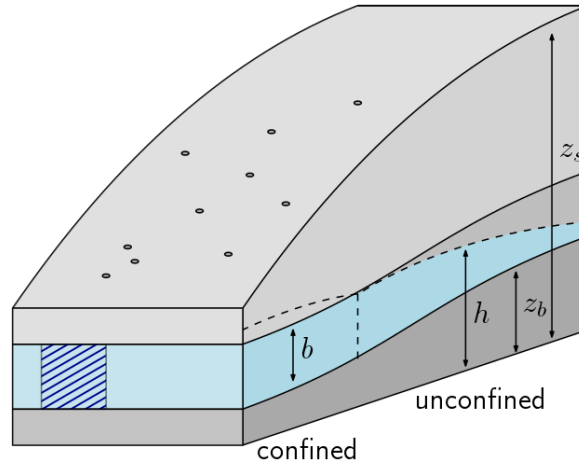


Figure 1. Schematics of the confined–unconfined aquifer scheme and artificial geometry for experiments. The hatched zone represents an area where the system is efficient. Dots on top indicate moulins.

2.2 Opening and closure

Opening and closure of channels is governed by melt at the walls due to the dissipation of heat and the pressure difference between the inside and outside of the channel leading to creep deformation. We follow de Fleurian et al. (2016) in using the classical channel equations from Nye (1976) and Röthlisberger (1972) to scale our transmissivity in order to reproduce this behavior.

- 5 In contrast to de Fleurian et al. (2016), we do not evolve the aquifer thickness but the conductivity K , which leads to However, the transmissivity T is evolved directly in our formulation instead of the aquifer thickness b in de Fleurian et al. (2016), even though both models are fully equivalent in the way they represent the melt rate.

$$\frac{\partial K}{\partial t} \frac{\partial T}{\partial t} = \underline{v}a_{\text{melt}} + \underline{a}_{\text{cavity}} - \underline{v}a_{\text{creep}}, \quad (9)$$

in which

$$10 \quad \underline{v}a_{\text{melt}} = \frac{rg\rho_w \min(b, \Psi)K}{\rho_i L} \frac{g\rho_w KT}{\rho_i L} (\nabla h)^2, \quad (10)$$

and

$$\underline{a}_{\text{cavity}} = \beta |\mathbf{v}_b| K \quad (11)$$

and

$$\underline{v}a_{\text{creep}} = 2An^{-n} |N|^{n-1} N \underline{KT} \quad (12)$$

with L the latent heat, r roughness factor, β a factor governing opening via sliding over bedrock protrusions, v_b basal velocity of the ice, A the creep rate factor depending on temperature, and n the creep exponent, which we choose as $n = 3$. Depending on the sign of N , creep closure as well as creep opening can occur. Negative effective pressure over prolonged time is usually considered unphysical, and the correct solution to this would be to allow the ice to separate from the bed (see e.g. Schoof et al. (2012) for a possible solution). However, in the context of our equivalent layer model, Eq. (12) is still applicable because this is how a channel would behave for $N < 0$. In Sect. 3.1, we test the sensitivity of K and N to the magnitudes of r , K , β and A .

3 Experiments with artificial geometries

Testing out equivalent layer model and finding parameters for it is not straightforward, because there are no directly comparable physical properties. Moreover, observations and measurements of subglacial processes are in general difficult and sparse. We address this by testing the model with some of the benchmark experiments of the Subglacial Hydrology Model Inter-comparison Project (shmip.bitbucket.io) (de Fleurian Mauro A. Werder et al., 2018, in prep.).

The proposed artificial geometry mimics a land-terminating ice sheet margin measured 100 km in the x -direction and 20 km in the y -direction. The bedrock is flat ($z_b(x, y) = 0$ m) with the terminus located at $x = 0$, while the surface z_s is defined by a square root function $z_s(x, y) = 6 \left((x + 5e3)^{1/2} - (5e3)^{1/2} \right) + 1$. Here, we use the SHMIP/B2 setup, which includes 10 moulins with constant in time supply. Boundary conditions are set to zero influx at the interior boundaries ($y = 0$, $y = 20$, $x = 100$) and zero effective pressure at the terminus. All experiments start with initial conditions that imply zero effective pressure and are run for 50 years to ensure that they reach a steady state.

3.1 Parameter estimation and sensitivity

SHMIP is primarily intended as a qualitative comparison between different subglacial hydrology models, where results from the GlaDS model (Werder et al., 2013) serve as a “common ground”. Here, we use it as a basis for an initial tuning and a study of the sensitivity of our model with regard to parameters. The upcoming results from the SHMIP are also the reason why we do not show a comparison to other models in this study but refer to the manuscript in preparation instead.

In Table 1, we show the physical constants used in all setups and runs. The values in the lower half are properties of the porous medium and are only estimated. Since they are utilized in the context of the equivalent layer model this is not an issue. Table 2 contains the model parameters in the upper part and the variables computed by the model in the lower part.

We divide the sensitivity analysis into a general block investigating the sensitivity to the amount of water input into moulins, the layer thickness b , the confined / unconfined transition parameter d , grid resolution dx (Fig. 3) and a block that examines the parameters directly affecting channel evolution such as channel roughness factor r , creep rate factor A , and conductivity K , and the bounds for the allowed conductivity K_{\min} and K_{\max} , transmissivity T_{\min} and T_{\max} (Fig. 4). In Table 3, we list values that lead to the best agreement with the SHMIP benchmark experiments and thus are used in the following as the baseline for our sensitivity tests.

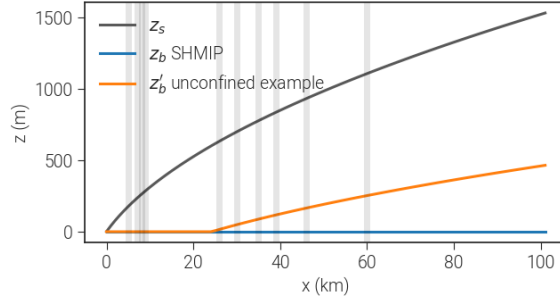


Figure 2. Experiments with artificial geometries. Vertical lines denote moulin positions for SHMIP/B2. The orange line shows the modified bedrock used to illustrate the impact of the confined/unconfined scheme as discussed in Sect. 3.2

Table 1. Physical constants used in the model. We distinguish between well known (upper half) and estimated / uncertain (lower half) parameters.

Name	Definition	Value	Units
L	latent heat of fusion	334	kJ kg^{-1}
ρ_w	density of water	1000	kg m^{-3}
ρ_i	density of ice	910	kg m^{-3}
n	flow law exponent	3	-
g	gravitational acceleration	9.81	ms^{-2}
β_w	compressibility of water ^a	5.04×10^{-10}	Pa^{-1}
α	compressibility of porous medium ^a	10^{-8}	Pa^{-1}
ω	porosity ^a	0.4	-
S_s	specific storage (Eq. (5))	$\approx 1 \times 10^{-4} \approx 1 \times 10^{-3}$	m^{-1}
S_y	specific yield	0.4	

^a Values from De Fleurian et al. (2014)

In Figs. 3a and b, the model's reaction to different amounts of water input through the moulins is shown. With deactivated conductivity evolution ($K = \text{const.}$, transmissivity evolution ($T = \text{const.}$), dashed lines), larger water inputs lead to higher water pressure, hence lower effective pressure N . In this case, a moulin input of $18 \text{ m}^3 \text{ s}^{-1}$ leads to negative values of N . With activated evolution of K , the conductivity T , the transmissivity adapts to the water input: as more water enters the system through moulins, the conductivity-transmissivity rises. Vertical gray bars show the location of moulins along the x -axis, and the most significant increase in K - T occurs directly downstream of a moulin. This happens because the water is transported in this direction leading to increased melt. At the glacier snout ($x = 0$), the ice thickness is at its lowest so almost no creep closure takes place; hence, the conductivity reaches its allowed maximum of 0.5 ms^{-1} transmissivity grows large for all tested param-

Table 2. Model parameters (upper) and variables computed in the model (lower)

Name	Definition	Units
$K_{\min} T_{\min}$	min. conductivity <u>transmissivity</u>	ms⁻¹ <u>m²s⁻¹</u>
$K_{\max} T_{\max}$	max. conductivity <u>transmissivity</u>	ms⁻¹ <u>m²s⁻¹</u>
b	aquifer thickness	m
d	confined / unconfined transition (Eq. (6))	m
Q	water supply	ms ⁻¹
r <u>roughness factor</u> A	creep rate factor	Pa ⁻³ s ⁻¹
h <u>K</u>	hydraulic head <u>transmissivity</u>	m <u>ms⁻¹</u>
K <u>v_b</u>	hydraulic conductivity <u>basal ice velocity</u>	ms ⁻¹
<u>β</u>	<u>cavity opening parameter</u>	
<u>h</u>	<u>hydraulic head</u>	<u>m</u>
S	storage	-
S_e	effective storage	-
T	transmissivity	m ² s ⁻¹
v_{melt} <u>a_{melt}</u>	opening by melt	ms⁻² <u>m²s⁻²</u>
v_{creep} <u>a_{cavity}</u>	opening <u>by sliding over bedrock</u>	<u>m²s⁻²</u>
<u>a_{creep}</u>	<u>opening</u> /closure by creep	m⁻² <u>m²s⁻²</u>
P_w	Water pressure	Pa
P_i	Ice pressure	Pa
N	effective pressure	Pa

Table 3. Selected baseline parameters for all experiments unless otherwise noted. These parameters best match the SHMIP targets.

Name	Value	Units
$K_{\min} T_{\min}$ <u>$K_{\min} T_{\min}$</u>	0.003 <u>1</u> $\times 10^{-7}$	ms⁻¹ <u>m²s⁻¹</u>
$K_{\max} T_{\max}$ <u>$K_{\max} T_{\max}$</u>	0.5 <u>100</u>	ms⁻¹ <u>m²s⁻¹</u>
b	10 <u>0.1</u>	m
d	0	m
dx	1000	m
r <u>1</u> A	5×10^{-25}	Pa ⁻³ s ⁻¹
<u>K</u>	<u>10</u>	<u>ms⁻¹</u>
<u>β</u>	<u>5×10^{-4}</u>	
$Q_{\text{per moulin}}$	9	m ³ s ⁻¹

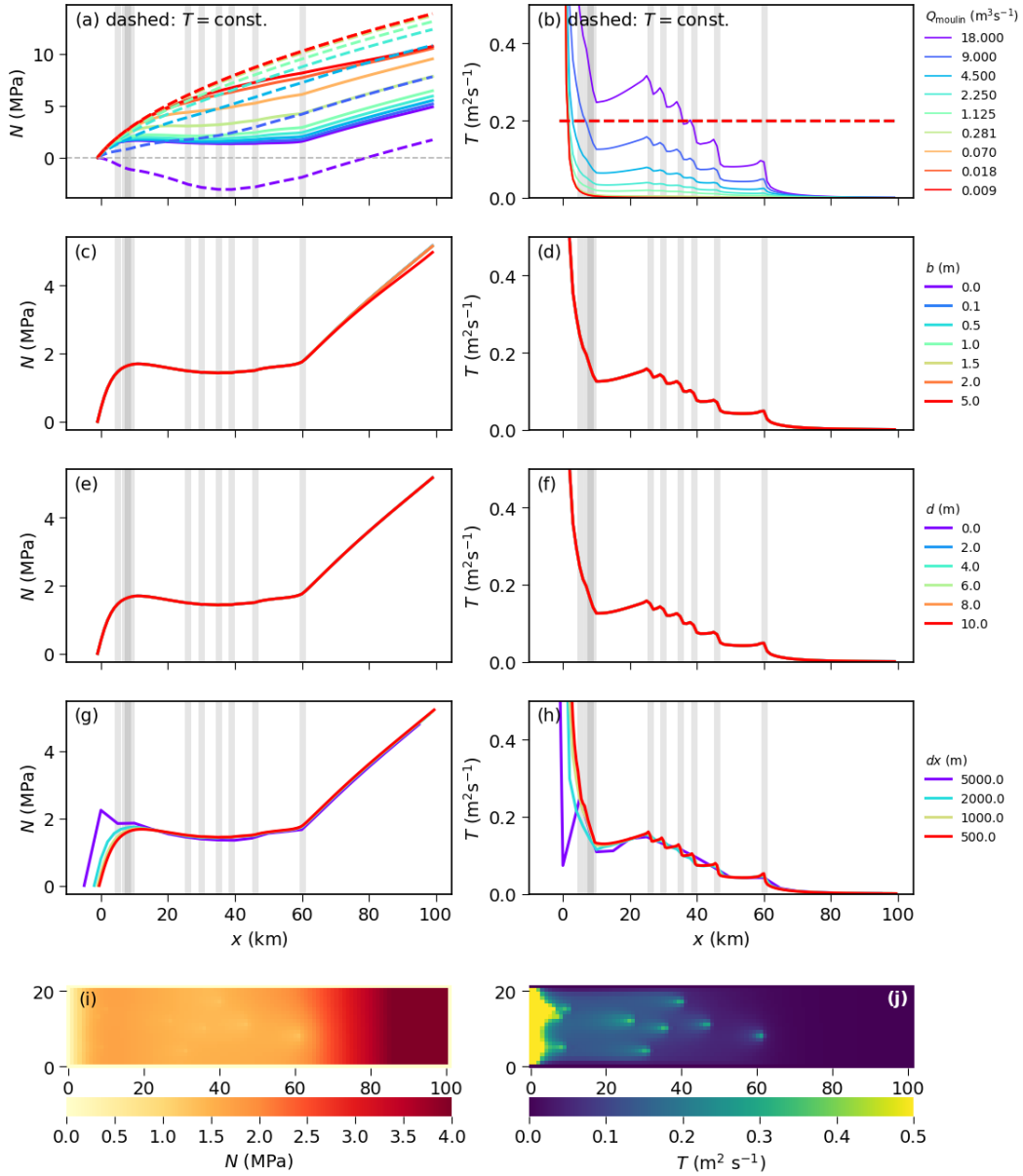


Figure 3. Results from the general sensitivity experiments showing the dependence of N (left) and $K-T$ (right) on: (a)–(b) Water supply from moulins Q_{moulin} (results for deactivated conductivity-transmissivity evolution are shown using dashed lines), (c)–(d) aquifer layer thickness b , (e)–(f) confined/unconfined transition parameter d , (g)–(h) grid resolution dx . Shown values are averaged along the y-axis to represent cross-sections-cross-sections at flow lines. Conductivity-Transmissivity plots are cut off at 0.06ms^{-1} – $0.5\text{m}^2\text{s}^{-1}$ to improve visibility of the relevant range. (i) and (j) show the two-dimensional distributions (map view) of the results using the best-fit baseline parameters.

eter combinations. Significant development of effective drainage is visible for inputs above $2.25\text{m}^3\text{s}^{-1}$ – $0.07\text{m}^3\text{s}^{-1}$ (yellow line). The resulting effective pressure decreases with rising water input as the system becomes more efficient at removing water. Up to ca. 35km distance from the snout this results in ~~almost identical~~ similar values of N for all forcings above $2.25\text{m}^3\text{s}^{-1}$ – $0.28\text{m}^3\text{s}^{-1}$. The system adapts so that it can remove all of the additional water efficiently. In Figs. 3i and j, the two-dimensional distributions of N and K – T are shown for the baseline parameters; ~~in addition, we detail the temporal development of K in Fig. ??.~~

~~Development of conductivity over time~~

“Channels” (indicated by regions of high ~~conductivity~~ transmissivity) form downstream from moulins and continue straight towards the ocean. The effective pressure drops around water inputs and along the “channels”.

10 ~~The~~ We observe no sensitivity of our result to the layer thickness b ~~directly influences the transmissivity according to Eq.~~ ~~A thicker layer can transport more water out of the system and therefore leads to lower water pressure and higher effective pressure (Figs. 3c and d). At the same time, a thinner layer leads to increased water pressure and higher pressure gradients which results in higher v_{melt} and can induce negative effective pressure and creep opening~~ Because we use transmissivity, b does not influence the flow of water directly, but is important to decide when the system becomes unconfined, as well as determining the Storage (see Eq. 5). However, in this experiment the system has sufficient water input so that all cells are confined in the steady state and also the storage has not influence on the long time solution (The storage determines how fast a pressure change travels through the system, but is irrelevant for the steady state).

15 ~~The large availability of water also explains why the~~ confined–unconfined transition parameter d does not show noticeable effects on the results (Figs. 3e and f) because the experiment has sufficient water input so that all cells are confined in the steady state– the system is always confined.

20 Grid resolution dx has low influence on the pressure distribution and a minor effect on the ~~conductivity–transmissivity~~ downstream (Figs. 3g and h). However, coarse resolutions are unable to resolve the steps that appear at the moulins.

In Figs. 4a and b, we show the results for different values of K_{min} . ~~K_{min} ensures that the system is never completely watertight and determines how much water can pass through independently of the system’s current state. Higher values lead to more water transport, lower water pressure, and higher T_{min} . These act as a numerical limit to avoid infinite growths for ill-posed conditions and do generally not show influence on the results. If T_{min} is chosen very large ($0.1\text{m}^2\text{s}^{-1}$ or larger), this dominates the balance between opening and closure and leads to high water flux, increasing the effective pressure. In this experiment, values higher than $0.01\text{m}^2\text{s}^{-1}$ show no reactions of K to moulins upstream of 10 km.~~

25 ~~K_{max} – T_{max} (Fig. 4e and f) and c) has no visible impact on the resulting pressure distribution. The differences in K are only large at the snout, where the maximum is always reached because the ice thickness is too low to counteract the melt term. No significant influence on the upstream area has been detected.~~

The creep rate factor A determines the “softness” of the ice and therefore effects the creep term in Eq. (9). Larger values of A imply warmer ice; hence, more creep closure (see Figs. 4e and f). Note, that this also effects creep opening if $N < 0$.

35 ~~The roughness factor r is meant as a measure for small bumps and imperfections, where a rougher “channel” (larger r) would experience more melt because of the larger contact area and more turbulent mixing.~~ conductivity K describes the flux of

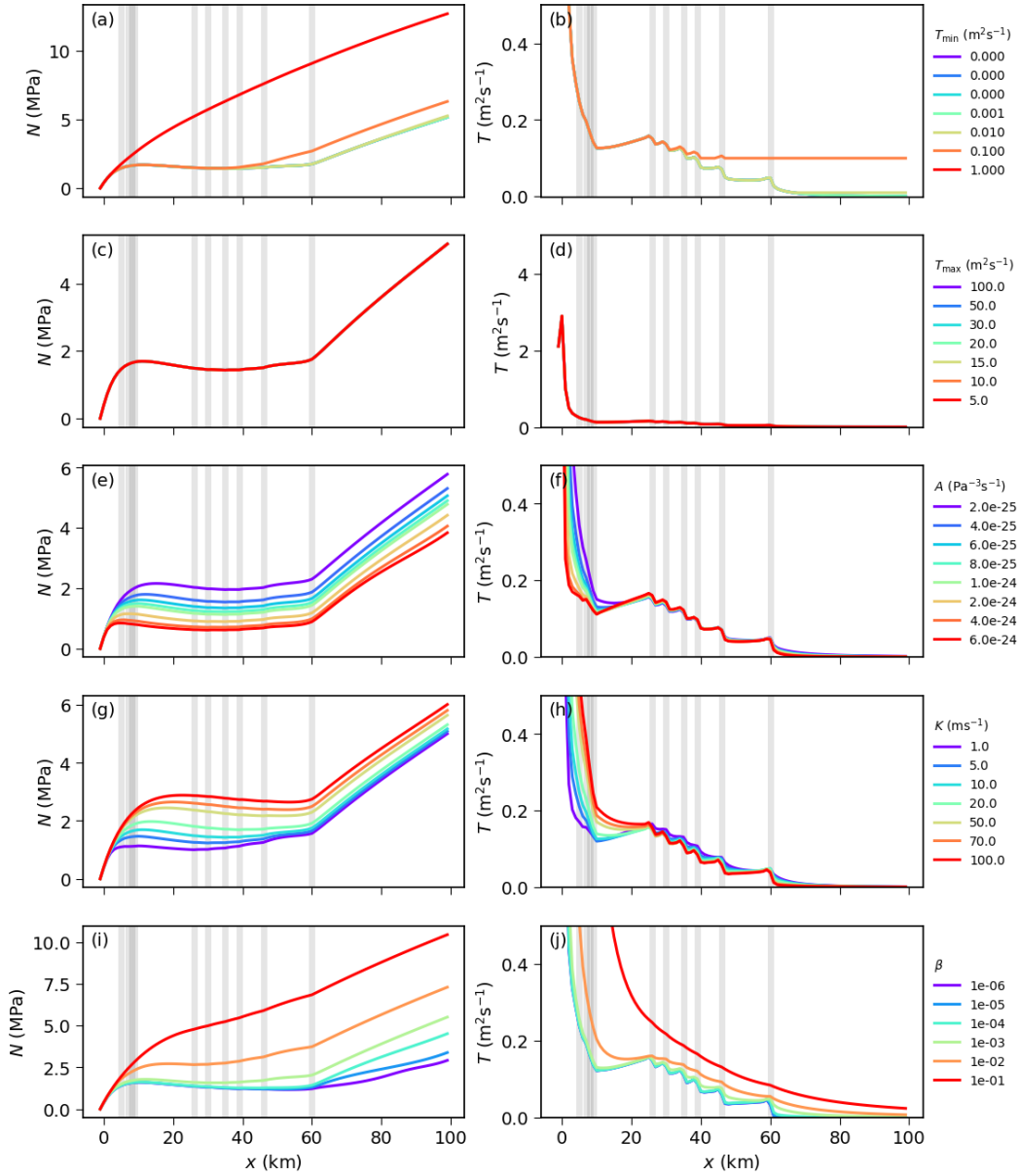


Figure 4. Results from parameters directly related to opening and closure: Limits on the K_{\min} T_{\min} (panels a and b) and K_{\max} T_{\max} (panels c and d), creep rate factor A (panels e and f) and roughness factor r , conductivity K (panels g and h) and cavity opening parameter β (panels i and j). Shown values are averaged along the y-axis to represent cross-sections at flow lines. Conductivity-Transmissivity plots are cut off at 0.06 ms^{-1} $0.5 \text{ m}^2 \text{ s}^{-1}$ to improve visibility of the relevant range.

water through the system and therefore determines the melt term (see Eq. 10). Larger values of r K lead to higher conductivity transmissivity and more water transport resulting in lower P_w and higher N .

In order to explore the dependence on the cavity opening term, we assume the basal ice velocity $v_b = 1 \times 10^{-6}$ (as in SHMIP) and vary the β term. β parametrizes the bedrock geometry and incorporates the height and distance of protrusion. As expected, larger values of β lead to more opening and, therefore, a higher effective pressure. With values as high as 1×10^{-1} , the cavity opening completely dominates the transmissivity evolution, and the effect of moulins is not visible anymore.

5 3.2 The benefit from treating unconfined aquifer

As described above, the confined–unconfined aquifer approach is advantageous for obtaining physically meaningful pressure distributions. In the example illustrated in Fig. 5, we use a slightly modified geometry, where the bedrock rises towards the upstream boundary forming a slab $z'_b(x, y) = \max(3((x + 5e3)^{1/2} - (5e3)^{1/2}) - 300, 0)$. The supply is constant in time and space, and we choose a low value of $7.93e-11$ m/s (≈ 2.5 mm/a) to compare our improved scheme to the simple confined only case. Fig. 5 shows a comparison of the steady state solutions: For the confined-only case, the hydraulic head drops below the bedrock at the upstream region. This results in negative water pressure for these regions. Addressing this by simply limiting the water pressure to zero would result in inconsistencies between the pressure field and the water supply. Our new scheme limits the transmissivity when the head approaches the bedrock and by this means ensures $p_w \geq 0$ in a physically consistent way. Additionally, the confined-only solution completely depends on boundary conditions and supply terms, basal topography has no influence in this case (apart from governing dK/dt). The possibility of the aquifer to become unconfined captures the expected behaviour much better: At high water levels, water pressure distribution dominates water transport, while at low levels the bed topography becomes relevant.

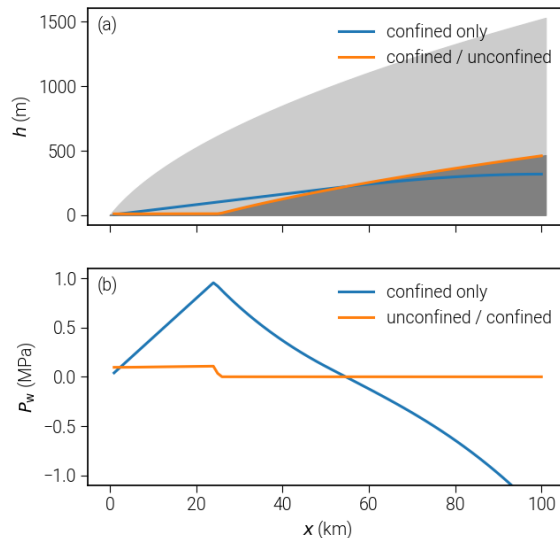


Figure 5. Advantages of using the confined/unconfined aquifer scheme (CUAS): Values of head and water pressure for geometries with non-flat bedrock. (a) Computed head for the confined and combined scheme with ice geometry in the background. In the confined only case, the head goes below bedrock. (b) Resulting water pressure, only for the combined scheme the pressure is always non-negative.

3.3 Seasonal channel evolution and properties

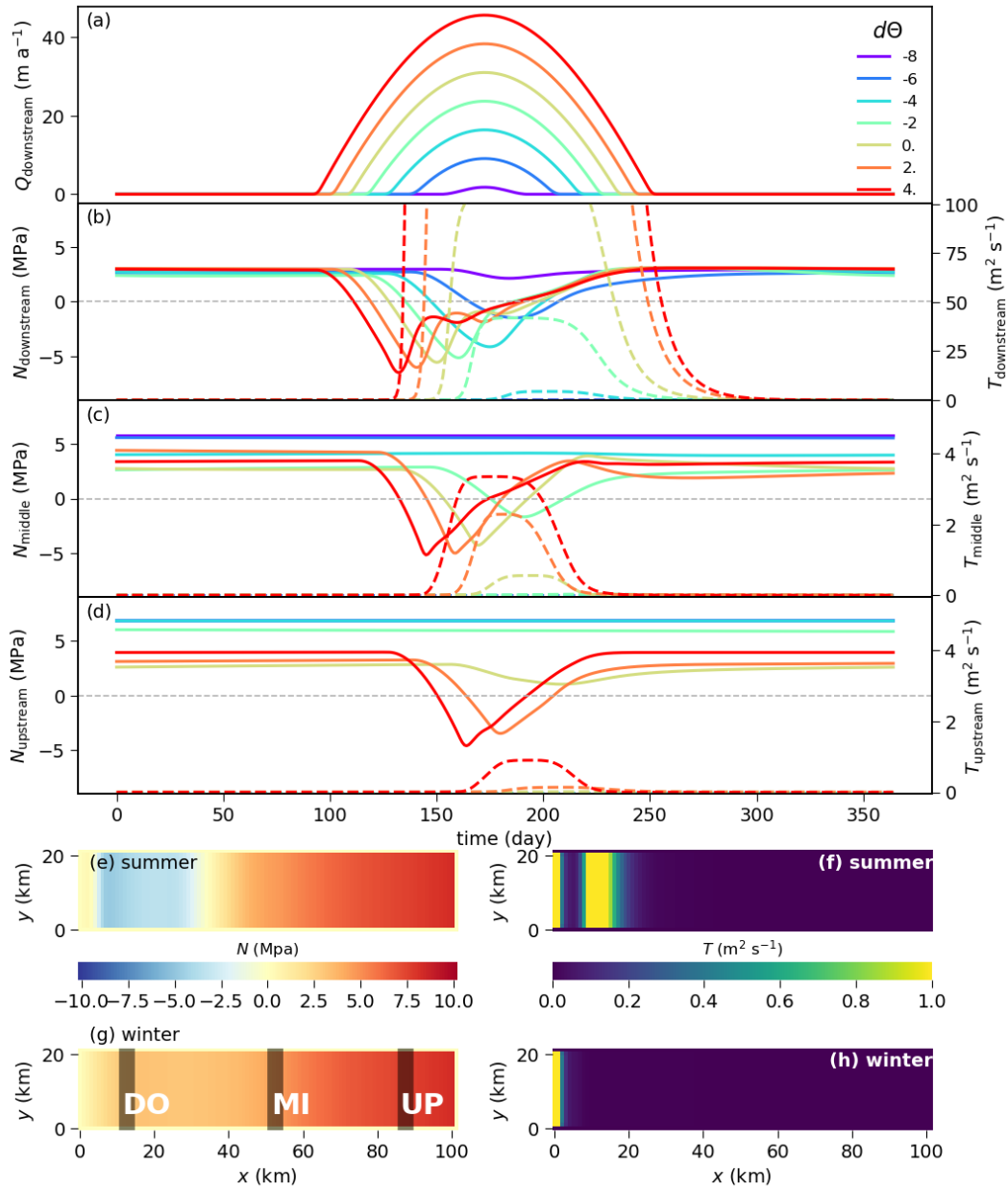


Figure 6. Results for one season of the SHMIP/D experiment. In panels (b)–(d), the left axis (effective pressure) corresponds to the solid lines, while the right axis (conductivity-transmissivity) specifies the values for the dashed lines. The values at the given positions (upstream, middle, downstream) are averaged over the corresponding areas indicated in panel (g). Panels (e)–(h) show two-dimensional distribution maps of $d\Theta = 0$ runs $d\Theta = -4$ run.

In order to understand our model’s ability to simulate the seasonal evolution of subglacial systems, we selected the setup SHMIP/D and ran it with different values of key model parameters. This experiment does not include any moulins but prescribes a non-uniform spatial distribution of supply instead that also varies seasonally. A simple degree day model with varying temperature parameter $d\Theta$ provides water input rising from the downstream end (lowest elevated) of the glacier towards the higher elevated areas over summer:

$$\Theta(t) = -16 \cos(2\pi/\text{yr } t) - 5 + d\Theta \quad (13)$$

$$Q_{\text{dist}}(z_s, t) = \max(0, (z_s \text{LR} + \Theta(t)) \text{DDF}) + Q_{\text{basal}}. \quad (14)$$

Here, $\text{yr} = 31536000 \text{ s}$ denotes the number of seconds per year, $\text{LR} = -0.0075 \text{ Km}^{-1}$ the lapse rate, $\text{DDF} = 0.01/86400 \text{ mK}^{-1} \text{ s}^{-1}$ is the degree day factor, and $Q_{\text{basal}} = 7.93 \times 10^{-11} \text{ ms}^{-1}$ is additional basal melt. The resulting seasonal evolution of the supply is shown in Fig.6a. The model is run for 10 years so that a periodic evolution of the hydraulic forcing is generated. Here, we present the result for one parameter set only since the model is not very sensitive in this setup. ~~For the prescribed water supplies, the conductivity only reaches its lower limit K_{min} for very short periods, and the upper limit K_{max} is never reached at all (apart from the terminus, where it does not have any significant influence on the upstream behavior of the system as we have shown in the previous experiment).~~

We chose three different locations to present N and K - T during the season: downstream of the glacier close to the snout, in the center, and at a far upstream location (Figs. 6b–d; the locations are marked in panel g). Shown time series are spatially averaged over these locations with solid lines representing the effective pressure and dashed lines the ~~conductivity.~~

~~transmissivity.~~ Water input increases during the summer months, while the corresponding effective pressure drops. With a time lag the ~~conductivity-transmissivity~~ rises in response. Supply develops from downstream towards the upstream end of the glacier over the season so the decline in N at the downstream location (Fig. 6b) is instantaneous when the supply rises, while, at the further inland locations (Figs. 6c and d), N reacts later during the year. At the middle location, the drop in N is ~~more intense almost reaching —5 MPa for the only visible for~~ temperature parameters of ~~2–2~~ and higher. ~~Temperature parameters below 4 show no response at this location. The same applies to the change in conductivity: only~~ ~~The rise in transmissivity occurs for~~ the three highest temperatures ~~lead to a significant rise of K .~~ Finally, at the upstream position, only for $d\Theta = 4$ and $d\Theta = 2$ the effective pressure drops below zero, while for $d\Theta = 0$ the drop is smaller in magnitude and more prolonged. The ~~conductivity-transmissivity~~ rise is only significant for $d\Theta = 4$ at this location. While the onset and minima of the decline in N strongly depend on the amount and timing of the water input for all values of $d\Theta$, the maximum of K - T and also the time when N returns to winter conditions is similar. For the downstream position, the maximum ~~conductivity-transmissivity~~ is reached for day 210 (~~not visible in the figure~~), and N reaches its background value approximately 25 days later. At the center and upstream positions, this behavior is less pronounced but generally similar.

The observed behavior is expected and indicates that our model is able to represent the seasonal evolution of the subglacial water system. Increasing water supply over the year leads to rising water pressure and dropping effective pressure. When the ~~conductivity-transmissivity~~ rises in response, the effective pressure goes up again despite the supply not yet falling again because the more efficient system is able to transport the water away. For the cases, where no visible change in K - T occurs

such as $d\Theta = -6$ (blue line in Fig. 6b), the effective pressure ~~directly~~ follows the supply at the terminus with a small delay, while at the center position ($d\Theta = -2$, cyan line, Fig. 6c), the minimum is offset by the time needed for the supply to reach that location. The maximum in ~~conductivity K~~ transmissivity T is reached later because, once the system becomes efficient, increased water transport stimulates melting that opens the system even more. This self-reinforcing process is only stopped when enough water is removed and the reduced water flux reduces the melt again. We assume that this leads to similar locations of the ~~conductivity~~ transmissivity maxima for different $d\Theta$ and the resulting similar reemerging of winter conditions in N .

In this experiment, N becomes negative during the seasonal evolution, which is not physically meaningful. We attribute such behavior to a lack of adjustment of water supply to the state of the system. In reality, the supply from runoff or supraglacial drainage would cease as soon as the pressure in the subglacial water system becomes too high; here we simply continue to pump water into the subglacial system without any feedback. This then leads to negative values of N . It is also consistent with the finding that N becomes negative earlier in the season in cases of higher supply. This deficiency will be addressed in future work.

4 Subglacial hydrology of NEGIS, Greenland

The role of subglacial hydrology in the genesis of ice streams in general is not well understood yet. NEGIS is a very distinct feature of the ice sheet dynamics in Greenland; thus, the question about the role of subglacial water in the genesis of NEGIS is critical. The characteristic increase in horizontal velocities becomes apparent about 100 km downstream from the ice divide (Vallelonga et al., 2014). Further downstream, the ice stream splits into three different branches: the 79° North Glacier (79NG), Zacharias Isbrae (ZI), and Storstrømmen. Thus far, large scale ice models have only been able to capture the distinct flow pattern of NEGIS when using data assimilation techniques such as inverting for the basal friction coefficient (see e.g. horizontal velocity fields in Goelzer et al., 2017). It is assumed that most of the surface velocity can be attributed to basal sliding amplified by basal water instead of ice deformation (Joughin et al., 2001). This means that the addition of a subglacial hydrology might have the potential to improve the results considerably. While many glaciers in Greenland have regularly draining supraglacial lakes and run-off driving a seasonality of the flow velocities, little is known about the effect at NEGIS (Hill et al., 2017). Because of this lack of data, to avoid an increased complexity, and to focus on the question if basal melt alone can account for the development of an efficient system, we do not include any seasonal forcing into our experiment.

Our setup includes the major parts of this system. The ~~pressure-adjusted~~ pressure-adjusted basal temperature Θ_{pmp} obtained from PISM (Aschwanden et al., 2016) is utilized to define the modeling region. We assume that for freezing conditions at the base ($T_{\text{pmp}} < 0.1\text{K}$) basal water transport is inhibited and take this as the outline of our model domain. Fig. 7 shows the selected area and PISM basal melt rates used as forcing.

For the ice geometry, we use the bed model of Morlighem et al. (2014) interpolated on a 1.2 km grid. Boundary conditions at lateral margins are set to no flux, whereas the termini at grounding lines are defined as Dirichlet boundaries with a prescribed head that implies an effective pressure of zero. This means that the water pressure at the terminus is equal to the hydrostatic water pressure of the ocean assuming floating condition for the ice at the grounding line. Parameters used for this experiment

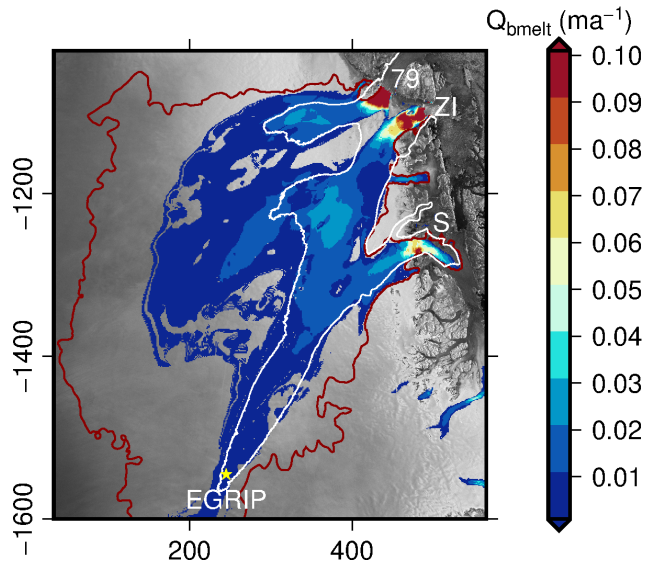


Figure 7. Boundary conditions and forcing for NEGIS experiment. Shown is the basal melt rate from PISM and contour line for $\Theta_{\text{pmp}} = -0.1\text{K}$ (red) used as model boundary. [The white line indicates the \$50\text{ m a}^{-1}\$ velocity contour.](#)

are the same as ~~those for the previous experiments in our sensitivity study~~ (Table 3) ~~but with K_{max} reduced to 0.3ms^{-1} to speed up the computation. The experiment.~~ [The simulation](#) is run for 50 a to reach steady state. Despite ~~the large number of cells a high resolution~~ (444×481), computing time for this setup is still reasonable (~~48~~ [3.5](#) hours on a single core of Intel Xeon Broadwell E5-2697).

- 5 The resulting distributions of effective pressure and ~~conductivity-transmissivity~~ are shown in Figs. 8a and b, respectively. As expected, effective pressure is highest at the ice divide and decreases towards the glacier termini. ~~Conductivity-Transmissivity~~ is low for the majority of the study area with the exception of ~~areas in~~ the vicinity of grounding lines and two distinct areas that touch in between 79NG and ZI. The northern area (marked I in Fig. 8b) is located at the northern branch of 79NG and has no direct connection to the snout. The second area (marked II in Fig. 8b) emerges in the transition zone between the southern
 10 branch of 79NG and Zacharias Isbrae and covers an area approximately twice as large as area I with higher values of ~~K~~ . ~~It is connected to~~ [It reaches down](#) to the snout of ZI ~~with narrow band of high conductivity cells.~~

Comparing the effective pressure distribution to the observed velocity (Rignot and Mouginot, 2012) – we chose the 50 m a^{-1} contour line as indicator of fast flow – we observe a high degree of overlap between the fast flowing regions and those with low effective pressure (below 1MPa) over most of the downstream domain of our study area. Storstrømmen shows ~~slightly~~
 15 higher effective pressure [downstream](#) than 79NG and ZI, which is in accordance with lower observed horizontal velocities for that glacier (Joughin et al., 2010). [At the location where the small sidearm branches north, we observe extremely low effective pressure and high transmissivity; however, we attribute this problem to an anomalously high basal water supply in our forcing data.](#) [At the](#) onset of the NEGIS, the effective pressure is high, and no relationship to the flow velocity can be observed.

To further examine the possible influence of our hydrology model to basal sliding, we investigate the impact on the sliding law. We chose to compare our computed N_{CUAS} to the reduced ice overburden pressure N , defined in Huybrechts (1990) as $N_{HUY} = P_1 + \rho_{sw}g(z_b - z_{sl})$ for $z_b < z_{sl}$ and $N_{HUY} = P_1$ otherwise.

~~We show the~~ The quotient of H_{HUY} ~~and to~~ N_{CUAS} ~~is shown~~ in Fig. 8c. ~~This demonstrates to demonstrate~~ where the application of our hydrology model would increase basal velocities.

In order to demonstrate the effect of the modeled subglacial hydrology system on the NEGIS ice flow, we setup a simple, one-way coupling to an ice flow model. Here, we use the Ice Sheet System Model (ISSM, Larour et al., 2012), an open source finite element flow model appropriate for continental scale and outlet glacier applications (Bondzio et al., 2017; Morlighem et al., 2016) (e.g. Bondzio et al., 2017; Morlighem et al., 2016). The modeling domain covers the grounded part of the whole NEGIS drainage basin. The ice flow is ~~approximated with the Shallow Ice Approximation (SSA, MacAyeal, 1989; Morland, 1987) within a 2D plan-view modeled by the higher order approximation (HO, Blatter, 1995; Pattyn, 2003) in a 3D model, which is appropriate for fast flowing ice but not for the slow flowing parts in our model area. Since we aim do demonstrate that our addition of the hydrology model improves results in fast flowing areas where the SSA is valid, this is not a huge concern. As we use the SSA we accounts for transversal and longitudinal stress gradients. In the HO-model we~~ do not perform a thermo-mechanical coupling, but prescribe a α -depth-averaged hardness factor in Glens flow law ~~instead~~. Model calculations are performed on an unstructured finite element grid with a ~~high resolution of 1 km resolution of 1 km~~ in fast flow regions and ~~coarse resolution of 20 km of 20 km~~ in the interior. The basal drag τ_b is written in a Coulomb-like friction law:

$$\tau_b = -k^2 N \mathbf{v}_b, \quad (15)$$

where \mathbf{v}_b is the basal velocity vector tangential to the glacier base, N the effective pressure, and k^2 is a positive constant. We run two different scenarios, where (1) the effective pressure is parametrized as the reduced ice overburden pressure, $N = N_{HUY}$, and (2) the effective pressure distribution is taken from the hydrological model at steady state, $N = N_{CUAS}$. The value of k^2 is tuned in order to have ice velocities of approximately 1500 m a^{-1} at the grounding line at the 79NG. For both scenarios, the value of k^2 is ~~0.1 sm^{-1}~~ 0.067 sm^{-1} . The results for both scenarios are shown in Fig. 9a and c, respectively. Additionally, we show the observed velocities (Fig. 9d, Rignot and Mouginot, 2012) and the PISM surface velocities (Fig. 9b, Aschwanen et al., 2016). Note that the latter is a PISM model output on a regular grid interpolated to the unstructured ISSM grid.

Velocities computed with the reduced ice overburden pressure are generally too ~~slow~~ ~~low~~ and do not resemble the structure of the fast flowing branches at all. The result from PISM shows distinct branches for the different glaciers, which display a relatively sharp separation from the surrounding area. Note, that PISM also uses a basal hydrology model as described in Bueller and van Pelt (2015). Velocities are slightly lower than observed velocities, especially for Zacharias Isbrae and in the area, where ZI and 79NG are closest. In the upper part towards the ice divide, the ice stream structure is not visible in the velocities. The ISSM model using effective pressure computed by CUAS produces high velocities towards the ocean that closely resemble N . The ~~observed sharp~~ transition between the ice streams and the surrounding ice is poorly reproduced. While the stream structure is way too diffused, the ~~different branches can be discerned and the~~ velocity magnitude for the glaciers appears reasonable. The inland part is similar to observed velocities but – as in the PISM simulation – the upper part where

NEGIS is initiated is not present. The onset of NEGIS is thought to be controlled by high local anomalies in the geothermal flux (Fahnestock et al., 2001), which PISM currently does not account for. Higher geothermal flux would lead to more basal melt, hence, water supply in the hydrology model. However, the consequences for the modeled effective pressure would require further experiments which are not in the scope of this paper.

- 5 In Tab. 4, we show the root mean square error (l_2 -norm), Pearson correlation coefficient r^2 and Δv (l_1 -norm) between the modeled and observed velocities.

Table 4. Comparison of modeling results for horizontal ice velocity to observed values (Rignot and Mouginot, 2012). Herein RMS denotes the root mean square error or l_2 -norm, r^2 is the Pearson correlation coefficient, and ΔV is the l_1 -norm.

	RMS (ma ⁻¹)	r^2	Δv (ma ⁻¹)
ISSM with reduced ice overburden pressure	176.83 <u>152.30</u>	0.69 <u>0.77</u>	90.13 <u>78.63</u>
PISM (Aschwanden et al., 2016)	132.05	0.84	65.42
ISSM with N computed from CUAS	126.86 <u>101.95</u>	0.80 <u>0.88</u>	53.28 <u>44.61</u>

We find it impressive that even without extensive tuning, we can considerably improve the velocity field in ISSM by our simple one-way coupling to the hydrology model. However, the results in this section are to be understood not as a thorough study of the NEGIS but as a first application of the model to a real geometry. A complete study requires extended observations in order to determine the optimal model parameters. However, we are confident that our results represent the general aspects of the hydrological system at NEGIS. Based on our sensitivity and seasonal experiments (Sect. 3.1 and Sect. 3.3) we expect the ~~high-conductivity-areas~~high-transmissivity-areas to be a stable feature, which would extend or retract depending on the chosen values of the melt and creep parametrizations but not change their location. Available supply plays a more important role here, and we assume that different basal melt distributions – or the addition of surface melt – might considerably change the position and the extent of the efficient system and, therefore, the effective pressure distribution as can be seen in Sect. 3.3.

The onset of NEGIS is not well reproduced in the PISM simulation as well as in our ISSM result. Since the ice is slow in the PISM results in that area, basal melt rates are low, and, since we use these as input in our hydrology model, it is expected that our model computes low water pressure here. In our opinion ~~this another point~~, this represents another point in favor of having a real two-way coupling between the ice model and the basal hydrology model in order to obtain good results. These results could then in turn be used to guide further optimization of the modeling parameters in our hydrology model in the future.

5 Conclusions

We present the first equivalent aquifer layer model for subglacial hydrology that includes the treatment of unconfined water flow. It uses only a single water-conductive layer with adaptive ~~conductivity~~transmissivity. Since extensive observations of the subglacial system are rare, ~~its relative simplicity and empirical nature~~ our approach to fit a simple parametrization of the effective Darcy model to the available data can be an advantage.

We find strong model sensitivity to ~~the lower limit of conductivity K_{\min} , grid spacing dx , and the parametrization of melt v_{melt} and creep v_{creep} , a_{melt} , creep closure a_{creep} , and the cavity opening parameter~~, while the sensitivity to the ~~upper limit of conductivity K_{\max}~~ limits of transmissivity and the confined–unconfined transition parameter d is low. Our model robustly reproduces the seasonal cycle with the development and decline of the effective system over the year. ~~Another positive aspect of our modeling approach was the complete absence of instabilities similar to those arising due to runaway channel enlargement—even at high flow rates. We attribute this fact that our combined confined/unconfined aquifer model quickly transports excess water away from confined aquifer parts, where high water pressure could lead to steep increases in effective conductivity (via negativ v_{creep} term).~~

In our NEGIS experiments, we find the presence of a partial efficient system for winter conditions. The distribution of effective pressure broadly agrees with observed velocities, while the upstream part is not represented correctly. When coupled to ISSM, our hydrology model notably improves computed velocities.

A number of aspects of the proposed model can be further developed; those include improved parametrizations of several physical mechanisms (e.g. adding feedback between pressure and water supplies), changing the hydraulic ~~conductivity~~ transmissivity coefficient to a tensor-valued on to better represent the anisotropy of channel networks, and, last but not least, transition to a mixed formulation of the Darcy equation discretized on an unstructured mesh in order to preserve mass conservation and to improve resolution in the areas of interest.

Appendix A: Parametrization of evolution of ~~conductivity~~ transmissivity

We use the same parametrization as de Fleurian et al. (2016) detailed here using the notation in Cuffey and Paterson (2010).

Opening and closure

The conduit expands when there is more melt than ice inflow due to creep, thus the mass change per unit length is given as:

$$\rho_i \frac{\partial A_c}{\partial t} = \dot{M}_{\text{melt}} - \dot{M}_{\text{creep}} \quad (\text{A1})$$

(Cuffey and Paterson, 2010, Eq. 6.42), in units of mass change per unit length ($\text{kg m}^{-1} \text{s}^{-1}$).

This is equivalent to

$$\rho_i \frac{\partial b}{\partial t} = \dot{m}_{\text{melt}} - \dot{m}_{\text{creep}}, \quad (\text{A2})$$

which describes the mass change per unit area ($\text{kg m}^{-2} \text{s}^{-1}$) or specific mass balance.

Creep term

Nye (1976), found for the closure on channels due to creep that

$$\frac{1}{R_c} \frac{\partial R_c}{\partial t} = A \left[\frac{N}{n} \right]^n, \quad (\text{A3})$$

with R_c denoting the channel radius and A_c the channel area ($= \pi R_c^2$) (notation as in (Cuffey and Paterson, 2010, Eq. 6.15)).

Multiplication by $2\pi\rho_i R_c^2 = 2\rho_i A_c$ on both sides, leads to

$$2\pi\rho_i R_c \frac{\partial R_c}{\partial t} = 2\rho_i A_c A \left[\frac{N}{n} \right]^n \quad (\text{A4})$$

Rewriting the left side to area, using the chain rule ($\frac{\partial A_c}{\partial t} = 2\pi \frac{\partial R_c}{\partial t}$) yields

$$5 \quad \rho_i \frac{\partial A_c}{\partial t} = 2\rho_i A_c A \left[\frac{N}{n} \right]^n \quad (\text{A5})$$

thus,

$$\dot{M}_{\text{creep}} = 2\rho_i A_c A \left[\frac{N}{n} \right]^n, \quad (\text{A6})$$

or again as a change per unit area

$$\dot{m}_{\text{creep}} = 2\rho_i b A \left[\frac{N}{n} \right]^n. \quad (\text{A7})$$

10 Melt term

Heat produced over ds in unit time is $Q_w G$ and pressure melting point effects are $\rho_w Q_w c_w B \frac{dP_i}{ds}$, which leads to

$$\dot{M}_{\text{melt}} L_f = \underbrace{Q_w G}_{\text{turb. heat produced}} \underbrace{\text{heat produced}}_{\text{heat produced}} - \underbrace{\rho_w Q_w c_w B \frac{dP_i}{ds}}_{\text{PMP effect}} \quad (\text{A8})$$

(Cuffey and Paterson, 2010, Eq. 6.16), where \dot{M}_{melt} represents the melt rate (mass per unit length of wall in unit time) and the magnitude of gradient of the hydraulic potential is given by

$$15 \quad G = |\nabla \phi_h|, \quad \text{where} \quad \phi_h = \rho_w g h. \quad (\text{A9})$$

Neglecting the PMP effects we get

$$\dot{M}_{\text{melt}} = \frac{Q_w G}{L_f}. \quad (\text{A10})$$

using $Q_w = qb$

As before, we can write that as a change per unit area instead:

$$20 \quad \dot{m}_{\text{melt}} = \frac{Q'_w G}{L_f}, \quad (\text{A11})$$

where Q' is now the flux per unit length(?). Using $Q'_w = qb$ (confined case, unconfined would be $Q_w = q(h - z_b)$ $Q'_w = q(h - z_b)$) and $q = K \nabla(h)$ (ommiting the minus, because we need the magnitude here) this is

$$\dot{m}_{\text{melt}} = \frac{K \nabla(h) b \nabla(\rho_w g h)}{L_f} \quad (\text{A12})$$

which can be rewritten to

$$\dot{m}_{\text{melt}} = \frac{\rho_w g K b (\nabla h)^2}{L_f}. \quad (\text{A13})$$

Opening and closure Evolution equation The conduit expands when there is more melt than ice inflow due to creep

$$\rho_i \frac{\partial A_c}{\partial t} = \dot{M} - \underbrace{2\rho_i A_c A \left[\frac{N}{n} \right]^n}_{\text{creep term from A5}}$$

5 (Cuffey and Paterson, 2010, Eq. 6.42). Inserting \dot{M}

Inserting \dot{m}_{creep} from Eq. A13 A7 and \dot{m}_{melt} from Eq. A13 into Eq. A2 and dividing by ρ_i results in

$$\frac{\partial A_c}{\partial t} \frac{\partial b}{\partial t} = \frac{\rho_w g K b (\nabla h)^2}{L_f \rho_i} - 2b A_c A \left[\frac{N}{n} \right]^n, \quad (\text{A14})$$

which is equation (6) in de Fleurian et al. (2016).

Change of area to conductivity Formulation in transmissivity

10 This is a purely geometrical argument, since we are interested in the transmissivity $T = Kb$ ($T = Kh$ for unconfined) and therefore, a change in By multiplying Eq. A14 with the constant hydraulic conductivity coefficient K is equivalent to a change in the aquifer thickness b .

Hence, we add a roughness term r to be able to investigate the sensitivity of our model to the melt term and write we obtain our evolution equation for the transmissivity:

$$15 \frac{\partial K}{\partial t} \frac{\partial T}{\partial t} = \frac{r g \rho_w b K (\nabla h)^2}{L_f \rho_i} \frac{g \rho_w K T (\nabla h)^2}{L_f \rho_i} - 2 K A T \left[\frac{N}{n} \right]^n. \quad (\text{A15})$$

and considering unconfined case as well, it is

$$\frac{\partial K}{\partial t} = \frac{r g \rho_w \min(b, h - z_b) K (\nabla h)^2}{L_f \rho_i} - 2 K A \left[\frac{N}{n} \right]^n.$$

Our reasoning behind scaling K and not evolving T instead of b are unintended side effects of changes in twofold: first, our combination of confined/unconfined aquifer flows would be conceptually confusing when formulated in terms of b on storage and evolution and may cause unintended side effects on the storage term; second, the transmissivity formulation is more general, since it can also model situations when K is varying without any re-formulation.

To account for cavity opening by the ice sliding over bedrock protrusions, we add another term to the behaviour when the aquifer becomes unconfined. A thicker aquifer in areas which represent channels would imply larger storage and also less transmissivity when the area eventually becomes unconfined (because the unconfined case triggers at a higher head already, limiting transmissivity). However, we do not expect the storage to increase, because channels usually do not store a lot of water and pressure changes travel faster than in the inefficient system. Also, the effective system should still have higher transmissivity when the water pressure drops (as long as it does not empty completely). evolution equation (9).

Appendix B: Discretization

We discretize the transient flow equation (Eq. (3)) on an equidistant rectangular grid using ~~an explicit forward in time central in space (FTCS) finite difference approximation~~ a Crank-Nicolson scheme. For sake of completeness, we give the equations for a non-equidistant grid here.

- 5 For the spatial discretization, we use a second-order central difference scheme (e.g., Ferziger and Perić, 2002) leading to the spatial discretization operator for the head \mathcal{L}_h :

$$\mathcal{L}_h = T_{i+\frac{1}{2},j} \frac{h_{i+1,j} - h_{i,j}}{(\Delta_f x)_i (\Delta_c x)_i} - T_{i-\frac{1}{2},j} \frac{h_{i,j} - h_{i-1,j}}{(\Delta_b x)_i (\Delta_c x)_i} + T_{i,j+\frac{1}{2}} \frac{h_{i,j+1} - h_{i,j}}{(\Delta_f y)_j (\Delta_c y)_j} - T_{i,j-\frac{1}{2}} \frac{h_{i,j} - h_{i,j-1}}{(\Delta_b y)_j (\Delta_c y)_j} + Q$$

$$\mathcal{L}_h = T_{i+\frac{1}{2},j} \frac{h_{i+1,j} - h_{i,j}}{(\Delta_f x)_i (\Delta_c x)_i} - T_{i-\frac{1}{2},j} \frac{h_{i,j} - h_{i-1,j}}{(\Delta_b x)_i (\Delta_c x)_i} + T_{i,j+\frac{1}{2}} \frac{h_{i,j+1} - h_{i,j}}{(\Delta_f y)_j (\Delta_c y)_j} - T_{i,j-\frac{1}{2}} \frac{h_{i,j} - h_{i,j-1}}{(\Delta_b y)_j (\Delta_c y)_j} + Q \quad (\text{B1})$$

- 10 where half-grid values of T denote harmonic rather than arithmetic averages computed using Eq. (4), where

$$(\Delta_c x)_k = (x_{k+1} - x_{k-1})/2, \quad (\text{B2})$$

$$(\Delta_f x)_k = x_{k+1} - x_k, \quad \text{and} \quad (\text{B3})$$

$$(\Delta_b x)_k = x_k - x_{k-1} \quad (\text{B4})$$

denote central, forward, and backward differences, respectively. Re-writing this more compactly in compass notation

15 $\mathcal{L}_h = d_S h_S + d_W h_W + d_P h_P + d_E h_E + d_N h_N + Q \quad (\text{B5})$

with

$$d_W = \frac{T_{i-\frac{1}{2},j}}{(\Delta x)_i^2}, \quad d_E = \frac{T_{i+\frac{1}{2},j}}{(\Delta x)_i^2}, \quad d_S = \frac{T_{i,j-\frac{1}{2}}}{(\Delta x)_j^2}, \quad d_N = \frac{T_{i,j+\frac{1}{2}}}{(\Delta x)_j^2},$$

and $d_P = -(d_W + d_E + d_S + d_N). \quad (\text{B6})$

- We use the ~~explicit Euler method for the time discretization, where the next time step $m+1$ is computed from the previous time step m ($\Delta t = t_{m+1} - t_m$) (using a somewhat sloppy notation)~~

$$h^{m+1} = h^m + \frac{\Delta t}{S_e} (d_S^m h_S^m + d_W^m h_W^m + d_P^m h_P^m + d_E^m h_E^m + d_N^m h_N^m - Q^m).$$

~~Conductivity is updated in each time step by Eq. ⋮~~

Crank-Nicolson semi-implicit method for computing our hydraulic head

$$\frac{\Delta h}{\Delta t} = \Theta \mathcal{L}_h(h^{n+1}) + (1 - \Theta) * \mathcal{L}_h(h^n) \quad (\text{B7})$$

(with $\Theta = 0.5$ for Crank-Nicolson) and then update the transmissivity with an explicit Euler step:

$$\underline{KT}^{m+1} = \underline{KT}^m + \Delta t \left(\underline{va}_{\text{melt}}^m + \underline{a}_{\text{cavity}}^m - \underline{va}_{\text{creep}}^m \right), \quad (\text{B8})$$

5 where we use a combined forward- backward-difference scheme for the discretization of $(\nabla h)^2$ in Eq. (10):

$$(\nabla h)^2 \approx \frac{1}{2} \left[\left(\frac{h_{i,j} - h_{i-1,j}}{(\Delta_b x)_i} \right)^2 + \left(\frac{h_{i+1,j} - h_{i,j}}{(\Delta_f x)_i} \right)^2 + \left(\frac{h_{i,j} - h_{i,j-1}}{(\Delta_b y)_j} \right)^2 + \left(\frac{h_{i,j+1} - h_{i,j}}{(\Delta_f y)_j} \right)^2 \right]. \quad (\text{B9})$$

Compared to central differences, this stencil is more robust at nodes with large heads caused by moulins.

The time step is chosen sufficiently small so that the discretization error is dominated by the spatial discretization (~~the Courant-Friedrichs-Lewy condition is always satisfied~~). Additionally, we check that the time step is small enough for the
10 unconfined component of the scheme to become active by restarting the time step with a decreased Δt if at any point $h < z_b$.

All variables are co-located on the same grid, but the ~~conductivity K~~ transmissivity T is evaluated at the midpoints between two grid cells using the harmonic mean due to its better representation of ~~conductivity~~ transmissivity jumps (e.g. at no-flow boundaries).

A disadvantage of this discrete formulation is that it is not mass-conservative (see, e.g. Celia et al. (1990)). The solution to
15 this is to use a mixed formulation for Darcy flow in which also the Darcy velocity is solved for. However, in our application, the resulting error is very small, and we plan to implement the mixed formulation approach in future work.

Competing interests. The authors declare that they have no conflict of interest.

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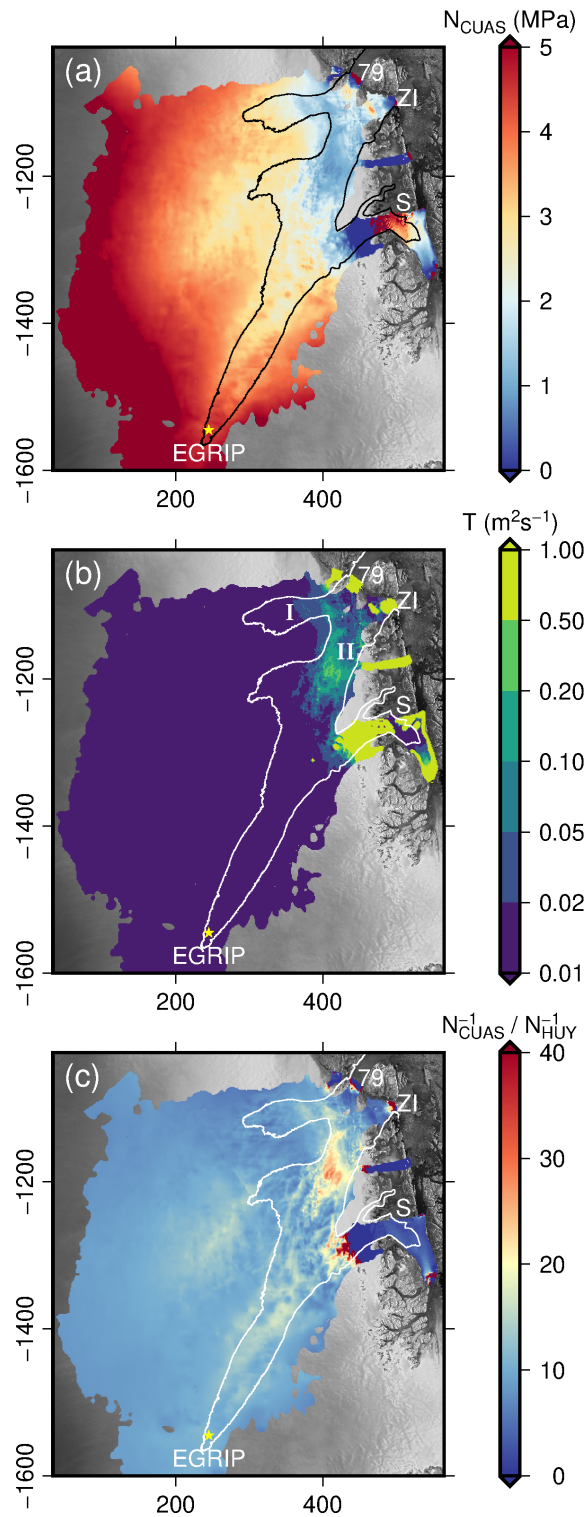


Figure 8. Results for NEGIS region with forcing due to basal melt (PISM) representing winter conditions. White lines indicate the 50 m a^{-1} velocity contour. Panel (a) shows effective pressure N_{CUAS} , (b) ~~conductivity-K-transmissivity~~ T (logarithmic scale), and (c) shows the quotient of the ice overburden pressure above flotation and the effective pressure computed by CUAS.

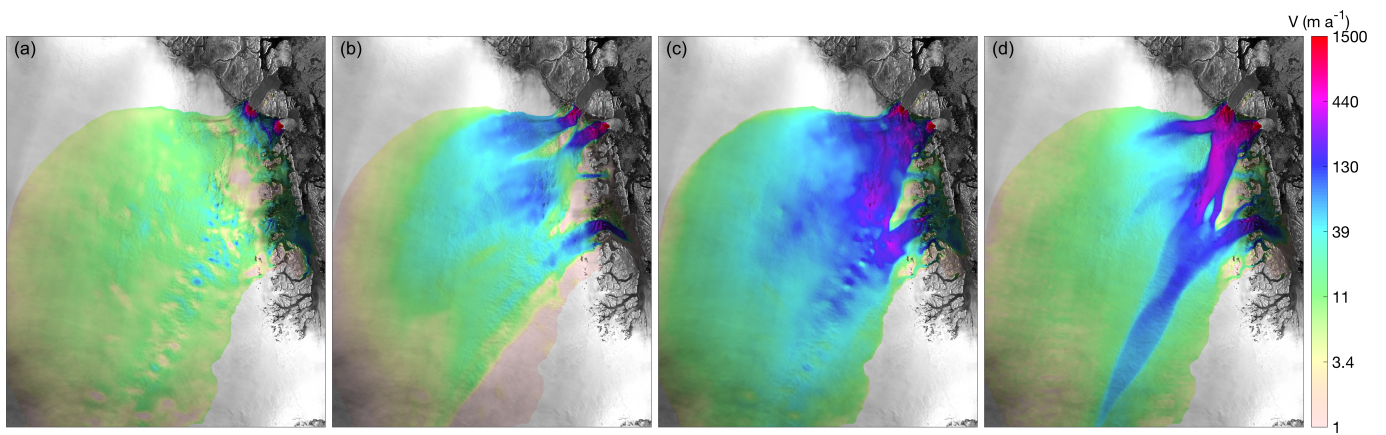


Figure 9. Horizontal surface velocity: ISSM with reduced ice overburden pressure N_{HUY} (a), PISM result from Aschwanden et al. (2016), interpolated to unstructured ISSM grid (b), ISSM with effective pressure from our hydrology model N_{CUAS} (c), and observed velocities (Rignot and Mouginot, 2012) (d).