RESPONSES TO REFEREE 1

General Comments:

This paper presents a topological and statistical analysis of novel three dimensional images of sea ice microstructure. In this paper, a directed graph is mapped to the microstructure of the sea ice and a throat size assigned to each node from the semi-minor axis of a best fit ellipse around a brine pocket viewed from a horizontal slice. Edges are assigned when moving though the ice if a brine pocket continues immediately below the previous elevation. Edges themselves can represent a splitting or joining of a brine channel depending on the overlap of the pockets being compared. Using this network model, statistical analysis is carried out relating morphological characteristics to depth and temperature. The results of the analysis are consistent with observed characteristics of sea ice microstructure for both columnar and frazil ice. The analysis presented here is then to be used as a basis for model development.

This paper presents an extensive analysis of rare and difficult to obtain microstructural data. Sea ice microstructure moderates a broad range of physical processes in both the Arctic and Antarctic. As a result, this statistical analysis should be of broad interest to polar science community. Overall, the analysis is well thought out, and well executed. I recommend this paper for publication with the following specific issues being addressed and or considered. I might also suggest the authors carefully read through the manuscript, there are sporadic minor grammatical errors.

We thank Referee #1 for the overall comments and thorough reveiw of the manuscript.

Specific Comments:

Page 1, Line 1: "The brine network in sea ice is a complex labyrinth whose. . ." I understand what you are trying to get at here but the description is not completely accurate. A labyrinth would imply that there is no order to the channel development, this is not the case. It might be better to say something like, "The brine pore space in sea ice can form complex connected structures whose geometry is critical in the governance of important physical transport processes between the ocean, sea ice and surface."

We have made the recommended change to the first sentence of the abstract in the revised manuscript.

Page 2, Line 32: "since viewed in two-dimensional slices" I think it would be prudent to add the fact that these are horizontal slices for clarity.

We have added the word "horizontal" in front of slices in the revised manuscript.

Page 3, Line 4: "This definition captures both the location and the size of the brine phase at any point. . ." Im not sure I understand this. The brine phase refers to the whole of the brine pore space. I think you might mean that it captures the location and size of a brine pocket at any point. However, this is a 2d slice so you may want to find some other type of phrasing.

We thank Referee $^{\#}1$ for identifying this confusing terminology. We edited the last sentence of this paragraph and the sentence now reads as follows:

This definition captures both the location and quantity of the brine at any point in the sea ice.

Page 3, Near Line 25: The probability of remaining notation is a bit awkward and phrasing confusing. Saying you are counting the number of connections made me think of the total number of connections that can trace back to that pocket. How about: For example, to calculate the probability that brine pockets of a given size remain we simply divide the number of pockets for a fixed throat size r that connect once from z_i to z_i 1 by the total number of pockets of that size. Markov chain: I agree that the network model is representing a Markov chain, and that seems to make sense, it should be that way physically I think. But is there a physical justification that this is true? I believe it would strengthen the paper. We have edited the definition of calculating the probability of remaining in the revised manuscript as suggested by Referee #1. In regards to the Markov chain, we agree that it seems to make sense physically due to the downward growth mechanism of sea ice. However, a complete justification is beyond the scope of this paper.

Section 3 Results: You might want to add a reference with definitions of the some of the quantities listed in Figure 2.

We have added the sentence below to the start of Section 3, referencing definitions for all quantities listed in Figure 2.

We first used standard morphological metrics as defined in previous work to describe the brine network shape and size (Lieb-Lappen et al., 2017)

Page 4, Line 7: "but instead produces a slush that has x-ray attenuating properties between ice and brine". Is the slush in the pore space? The wording makes it sounds as if the whole thing is slush, that of course would not make sense at -7 C. It becomes clear later, but saying that the slush is in the pore space immediately would make things clearer. Could it also be that at the bottom of the core you had more brine leakage at the time of extraction? In this case, what was in the brine space may have been less saline and thus slushy at the in-situ temperatures.

We have added a clarification that the slush is in the pore space. During extraction, we did not notice any brine leakage, but we also can not definitively state that there was none. It is possible that the brine space may have been less saline and thus slushy at the in-situ temperatures but we have no direct observations to support this statement.

Page 4, Line 11: It would be helpful to describe what is meant by "as best as possible".

We have edited the identified sentence to be more accurate, and the sentence reads as follows in the revised manuscript:

We used segmentation thresholds that split the difference with a threshold halfway between the peak of the brine phase and the peak of the ice phase, recognizing that there was indeed error in segmentation for these warmer samples.

Page 4, Line 27: "Salinity Values Measured in the Field" Were these bulk salinity measurements from adjacent cores? That should be stated if so.

We thank Referee #1 for catching this error. Bulk salinity was estimated from ion chromatography measured chloride concentrations. The identified paragraph has been edited and now starts as follows:

We compared the μ CT-measured brine volume fraction to expected values derived from the Frankenstein and Garner relationship relating temperature, salinity, and brine volume fraction (Frankenstein and Garner, 1967; Cox and Weeks, 1983). For this analysis, we used the core temperatures measured in the field and salinity values estimated from ion chromatography measured chloride concentrations presented in Lieb-Lappen and Obbard (2015).

Page 5, Line 13: "This is an important observation since we did not record the vertical orientation of the samples during cutting" Cant you tell by the direction of splitting, or are the samples too small to see that structure?

The samples were indeed too small to see that structure. We know the orientation of the vertical z-axis. However, unfortunately we did not have the direction of the vertical z-axis.

Page 5, Line 21: Change maximize to maximum.

We have made the recommended change in the revised manuscript.

Page 5, Line 22: The description of Figures 8 and 9 should be rethought and made more clear. It is not clear to me what the unsorted figures represent. I assume node index is just a way to label each node and to me it seems arbitrary. Ordering them by size makes sense but what gives the unsorted part of the figure any relevance? I think a description of how each node is labeled in the unsorted figure is needed to understand what it is meant to represent. Is it done by physical distance from the node with the

largest throat size? It was not clear to me. This may all be fixed by clearly defining "node index" which I did not see in the figure.

We have edited the language to clearly define the sorting in Figures 8 and 9 in the revised manuscript. The following has been added to the captions for Figures 8 and 9:

The left panels show the throat sizes at each depth in the sample with nodes sorted by location, not by size. The right panel sorts the nodes by throat size in descending order.

Additionally, the third paragraph of section 4.2 now starts as follows:

To gain insight into the behaviour of a channel, we visualized the number of branches and distribution of throat sizes by plotting the throat size r_i of each node p_i for the largest brine channel. Fig. 8 and Fig. 9 shows the throat sizes as a function of depth in the sample for three different representative sample depths: top, middle, and bottom of the Butter Point and Iceberg Site cores, respectively. For each channel shown, there is a plot of $\{r_i\}$ at each depth sorted by physical location in a two-dimensional grid (working line by line), not by size. A second corresponding plot shows node sizes sorted by descending $\{r_i\}$ for a given depth in the channel. The first plots illustrate the connectivity of given branches, while the second plots provide a visualization of the distribution of r_i .

Page 10, Line 22: "The probability distributions shown represent a sampling of the various possibilities. . ." Consider rephrasing, maybe change possibilities to microstructural behaviors? For your future model development, you might want to consider the effect salinity might have on the statistics you consider. It is encouraging to see that the other two previous cores you use do follow the most recent though. However, in the Arctic summer snow melt can get into the pore space decreasing salinity and reducing permeability. Just a thought.

We have made the recommended change to the identified sentence. We thank Referee #1 for the observation in regards to future model development and will indeed strive to include salinity in future development.

Figure 1: A figure showing how a split or join is assigned would be nice but is not completely necessary.

We thank Referee #1 for this suggestion but we feel as though the written description of a split and a join was sufficient and did not require an additional figure.

Figure 2: Rename object volume to brine volume fraction?

We have made the suggested recommendation in the revised manuscript.

Figure 14: Just a comment, the up and down motion might be able to be captured if you had someway to include horizontal edges in your network model.

We thank Referee #1 for this observation. We are intrigued by a definition for a horizontal edge, but it is not possible with the model as currently defined.

Figure 15: Connecting the dots with thin lines may make the figure easier to read, not sure, it is ok as is though.

With so many values having zero probability or no data, we made the stylistic choice to not include thin lines between the dots.

Technical Corrections:

Page 1, Line 25: produces should be produce. "Since different growth rates in natural sea ice produce. . ."

We thank Referee $^{\#1}$ for catching this error and have made the recommended revision.

Page 4, Line 4: Remove the word "sufficiently"

We have removed the word "sufficiently" in the revised manuscript.

Page 4, Line 6: change "this ambient cooling" to "the ambient temperature".

We have made the recommended revision in the revised manuscript.

Page 5, Line 21: Change maximize to maximum.

We have made the recommended change in the revised manuscript.

Figure 10: X-axis title is cut off a bit.

We have fixed the X-axis in Figure 10 in the revised manuscript.

Figure 11: In the caption "black squares" should be "blue squares".

We thank Referee $^{\#1}$ for catching this oversight and have edited the caption to read "filled squares".