A simple equation for the melt-elevation feedback of ice sheets

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8   **Abstract**:

9   In recent decades, the Greenland Ice Sheet has been losing mass and thereby contributed to

10   global sea-level rise. The rate of ice loss is highly relevant for coastal protection worldwide. The

11   ice loss is likely to increase under future warming. Beyond a critical temperature threshold, a

12   meltdown of the Greenland Ice Sheet is induced by the self-enforcing feedback between its

13   lowering surface elevation and its increasing surface mass loss: the more ice is lost, the lower the

14   ice surface and the warmer the surface air temperature which fosters further melting and ice loss.

15   The computation of this rate so far relies on complex numerical models which are the

16   appropriate tools to capture the complexity of the problem. By contrast we aim here at gaining

17   conceptual understanding by deriving a purposefully simple equation for the self-enforcing

18   feedback which is then used to estimate the melt time for different levels of warming using three

19   observable characteristics of the ice sheet itself and its surroundings. The analysis is purely

20   conceptual in nature and is missing important processes like ice dynamics to be useful for

21   applications to sea-level rise on centennial time scales, butwhen the volume loss is dominated by

22   the feedback, the resulting logarithmic equation unifies existing numerical simulations and

23   shows that the melt time depends strongly on the level of warming with a critical slowing-down

24   near the threshold: the median time to lose 10% of the present-day ice volume varies between

25   about 3500 years for a temperature level of 0.5°C above the threshold and 500 years for 5°C.

26   Unless future observations show a significantly higher melting sensitivity than currently

27   observed, a complete meltdown is unlikely within the next 2000 years without significant ice-

28   dynamical contributions.

1. Introduction

In past decades global mean sea level has been rising mainly by expansion of ocean waters and melting of ice on land (Church et al., 2013). Over the past two decades, the Greenland Ice Sheet has lost mass at an accelerating pace (Bamber et al., 2000; Box et al., 2012; van den Broeke et al., 2009; Fettweis et al., 2013; Mernild et al., 2011; Nick et al., 2009; Rignot et al., 2008, 2011; Shepherd and Wingham, 2007; Thomas et al., 2011). The ice loss is likely to increase under unabated greenhouse-gas emissions (Clark et al., 2016; Fettweis et al., 2013; Goelzer et al., 2012; Graversen et al., 2011; Harper et al., 2012; Huybrechts et al., 2011; Levermann et al., 2013; Nowicki et al., 2013; Price et al., 2011).

Numerical simulations suggest that a decline of the Greenland Ice Sheet is inevitable once its surface temperature permanently exceeds a certain threshold (Charbit et al., 2008; Greve, 2000; Huybrechts and Wolde, 1999; Huybrechts et al., 2011; Ridley et al., 2005, 2010; Robinson et al., 2012; Solgaard and Langen, 2012). If and when this temperature threshold is passed, depends critically on past and future greenhouse-gas emissions (Fettweis et al., 2013; Goelzer et al., 2013; Gregory et al., 2004a; Rae et al., 2012). Even if emissions were reduced to zero, temperatures would not drop significantly for thousands of years because of the long life-time of anthropogenic CO_2 in the atmosphere and reduced oceanic heat uptake if oceanic convection is extenuated (Allen et al., 2009; Solomon et al., 2009; Zickfeld et al., 2013). This implies a possible commitment of a melt-down of the Greenland Ice Sheet in the near future which would eventually raise global sea-level by more than 7 meters (Gregory et al., 2004a). Whether this occurs on a multi-centennial or rather a multi-millennial time scale is of relevance for coastal planning.

In this article we first recap the Vialov profile and add a simple representation of the melt-elevation feedback towards a governing equation for a steady-state ice-sheet in one dimensions, then we derive the critical warming threshold for the existence of an ice sheet in this simple model (Section 2). In Section 3 we derive a simple time-evolution equation for the decay of the ice sheet after surface temperatures have exceeded the threshold. Finally we use observational estimates of the three characteristics that enter the model to estimate the decay time of the ice sheet under melting above the threshold (Section 4). Here solid ice discharge is neglected as well as any other ice sheet dynamics (Andresen et al., 2012; Howat and Eddy, 2012; Moon et al.,

59   2012; Nick et al., 2009; Price et al., 2011; Straneo et al., 2011; Walsh et al., 2012). The

60   framework that we introduce here can be used to include new physical processes that might be

61   discovered in the future, e.g. potential changes in surface albedo through melting (Box et al.,

62   2012) or aerosol-induced surface melt or the lack thereof (Polashenski et al., 2015).

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2. Governing equation for shallow-ice steady states under melt-elevation feedback

A nonlinear threshold behaviour is generally associated with a fundamental self-enforcing feedback and thereby an associated system memory (Levermann et al., 2012). For the Greenland Ice Sheet, such a feedback is given by the interaction between surface elevation and surface melting (Weertman, 1961). For illustration, we include this feedback in a well-established highly idealized ice-profile of an ice-sheet in one dimension, the so-called Vialov-profile (Vialov, 1958). We introduce the melt-elevation feedback in the simplest possible way by assuming that the surface melt rate depends linearly on the surface temperature and that the temperature decreases linearly with the height of the ice surface following a constant atmospheric lapse rate.

2.1 Governing equation

We consider a highly simplified flowline model for an isothermal ice sheet grounded on a flat and rigid bed. The solution of the shallow-ice approximation in one dimension for the ice-sheet elevation under these simplifying assumptions is the Vialov-profile:

$$\tilde{h}(x) = h_m \left(1 - (x/L)^{(n+1)/n}\right)^{n/(2n+2)} \tag{1}$$

where h_m is the maximum surface elevation and n is Glen's flow law exponent (Glen, 1955). x denotes the horizontal position and L the horizontal limit of the ice sheet. The inherent assumption of isothermal ice is a strong simplification, which needs to be kept in mind when interpreting the results. The aim of this derivation is purposefully not a comprehensive representation of the ice flow but to derive a measure for the average height of the ice sheet and its dependence on changes in the surface mass balance. The surface mass balance is considered to be spatially and temporally constant at a value, a, which will later be considered dependent on the surface elevation and thereby temporally variable. The overall horizontal extension of the ice sheet is set to L, and it is thereby assumed that any ice flow across this point is calved off into icebergs. This situation represents a confined ice-bearing bedrock topography as in most of Greenland's interior (Howat et al., 2014).

The mean surface elevation can then be computed to be

$$\bar{h} = L^{-1} \int_0^L dx\, h(x) = \omega \cdot h_m \tag{2}$$

It is proportional to the maximum surface elevation h_m with a proportionality factor

92      $\omega \equiv \int_0^1 d\xi \left(1 - \xi^{(n+1)/n}\right)^{n/(2n+2)}$ (3)

93   which only depends on the flow law exponent.

94   The maximum surface elevation is determined by the surface mass balance $\tilde{a}$ and the ice softness

95      $\tilde{A}$

96      $h_m = 2^{(n-1)/(2n+2)} \cdot L^{1/2} \cdot \left(\frac{(n+2)\tilde{a}}{(\rho g)^n \tilde{A}}\right)^{1/(2n+2)}$ (4)

97   with ρ being the ice density and g the gravity constant. We normalize all three quantities by

98   defining $h \equiv \omega \cdot h_m / \overline{h_0}$, $a \equiv \tilde{a} / a_0$ and $A \equiv \tilde{A} / A_0$ where a_0 is the accumulation rate on the

99   ground, i.e., in the absence of an ice-sheet, and $A_0 = a_0 / \left((\rho g)^n (\varepsilon \cdot L)^{(n+1)}\right)$ with $\varepsilon = \overline{h_0} / L$ being

100   the typical height-to-width ratio. $\overline{h_0}$ is the equilibrium-line altitude of the considered ice sheet in

101   the initial equilibrium situation. Values for a_0, $\overline{h_0}$ and L are later chosen to resemble the

102   conditions of the Greenland Ice Sheet.

103   The non-dimensional surface elevation, h, of the ice sheet can then be expressed as

104      $h = \left(\frac{a}{A}\right)^{1/m}$ (5)

105   For the Vialov profile, m=2(n+1) where the Glen flow-law exponent is commonly chosen to be

106   around n=3 which yields m=8.

107   We introduce the melt-elevation feedback in its simplest form through a dependency of the

108   surface melt rate on the surface elevation:

109      $a = a_0 + \gamma \Gamma \cdot h$ (6)

110   with the atmospheric lapse rate $\Gamma > 0$. γ denotes the melting sensitivity of the ice surface, i.e. the

111   increase in surface melt-rate per degree of warming, which is regularly measured and comprises

112   a large number of physical processes (e.g. (Box, 2013)). For simplicity we rescale the surface

113   mass balance by the constant ice softness parameter, A to obtain $h = (a_0 + \gamma \Gamma \cdot h)^{1/m}$. The steady

114   state solution for the surface elevation of the ice-sheet is thus governed by the following equation

115      $h^m - \gamma \Gamma \cdot h - a_0 = 0$ (7)

116 which has two positive solutions for h as long as the surface mass balance on the ground is

117 negative, i.e., $a_0 < 0$. Note that the surface mass balance can be positive even if $a_0 < 0$. If the

118 ice-sheet is in an unstable configuration, a slight perturbation will either cause it to converge into

119 the stable state with a positive surface mass balance or to melt completely.

120 Our simple approach qualitatively captures the basic hysteresis behavior of the Greenland Ice

121 Sheet caused by the melt-elevation feedback (Fig. 1, in which we have assumed the surface mass

122 balance to depend linearly on temperature): For a given surface temperature, a stable state of the

123 ice sheet (red line) annihilates an external perturbation in surface elevation by changes in surface

124 mass balance (grey arrows). The unstable solution branch defines the basin of attraction for the

125 stable state. A surface elevation that is lower than the unstable solution branch cannot be

126 sustained. In that case the melting reduces the surface elevation to practically zero even without

127 further external perturbation (grey arrows). Beyond a certain surface temperature threshold

128 (vertical dotted line) no ice sheet can be sustained.

129 **2.2.   Critical surface mass balance in steady state**

130 As illustrated in Fig. 1, there is a critical temperature above which the ice sheet is not

131 sustainable. Let us denote the corresponding surface elevation by h_c. The critical point $\left(T_c, h_c\right)$

132 has to fulfill two conditions, i.e. being a solution of the governing Equation 7 and minimum of

133 the function

134 $$F(h) = h^m - \gamma\, \Gamma \cdot h - a_0 \tag{8}$$

135 which we can determine by setting the derivative of F to zero.

136 Consequently,

137 $$h_c = \left(\frac{\Gamma \cdot \gamma}{m}\right)^{1/(m-1)} \tag{9}.$$

138 Inserting this into the governing equation yields the critical surface mass balance at the ground

139 $$a_{0c} = -(m-1) \cdot \left(\frac{\Gamma \cdot \gamma}{m}\right)^{m/(m-1)} \tag{10}.$$

140 For illustrative purposes we have assumed a_0 to decline linearly with the surrounding temperature

141 and plotted the solution of Equation 7 against that temperature with an arbitrary offset in Fig. 1.

3. A simple temporal equation for the melt-elevation feedback

Once the critical surface-mass balance and surface elevation threshold (as derived in the previous section 2) is transgressed, a meltdown of the ice-sheet is inevitable in our conceptual model. Let us define the time τ_α as the time it takes to melt a fraction α of the initial ice volume and the threshold temperature T_c as the temperature above the pre-industrial level at which the surface mass balance becomes negative. Robinson et al. (2012) find a range of 0.8 – 3.2°C for the threshold warming beyond which no ice sheet can be sustained on Greenland. Their best estimate for the threshold is 1.6°C above pre-industrial level. The study uses a regional climate model of intermediate complexity (Robinson et al., 2009) coupled to the SICOPOLIS ice sheet model (Greve, 1997). Using a different model combination, Ridley et al. (2010) find that in their model the ice sheet cannot be sustained for a warming of 2°C. They combine the HadCM3 Atmosphere-Ocean-General Circulation Model (Gordon et al., 2000) with an atmospheric resolution of 2.5° x 3.75° (Pope et al., 2000) to an ice sheet model of 20 km horizontal resolution (Huybrechts and Wolde, 1999).

Some studies assume that the threshold is associated with a mean negative surface mass balance (Gregory et al., 2004b; Ridley et al., 2005; Toniazzo et al., 2004). In Fig. 2 we use 1.6°C as a threshold value for both models because this value is given by Robinson et al. (2012) and consistent with Ridley et al. (2010) and thus a simple and transparent choice. This number can be easily adjusted if new estimates are obtained. For the translation from percentage ice thickness change to percentage ice volume change a constant horizontal ice surface area was assumed which renders the analysis conceptual in nature. Thus the quantitative interpretation of the melt times are subject to this additional simplification.

For a fixed anomalous melt rate $\Delta a_0 = -\gamma \cdot \Delta T$ in response to an anomalous temperature increase $\Delta T = T - T_c$ above this threshold temperature, T_c, the decay time without any feedbacks would be

$$\tau_0 = -\frac{h_0}{\Delta a_0} = \frac{h_0}{\gamma \cdot \Delta T} \tag{11}$$

168 Since the surface temperature increases with decreasing elevation, this zero-order estimate for
169 the decay time is higher than the actual value. As a first-order correction to the situation of fixed
170 melting, let us assume that the anomalous surface mass balance behaves as

171 $$\Delta a = \Delta a_0 + \frac{1}{\tau_\gamma} \cdot (h - h_0) \tag{12}$$

172 where $\tau_\gamma = 1/(\gamma \cdot \Gamma)$.

173 From the relation $dh/dt = \Delta a$, we then obtain

174 $$\frac{d\Delta h}{dt} = -\Delta a_0 + \frac{\Delta h}{\tau_\gamma}, \tag{13}$$

175 if $\Delta h \equiv h_0 - h$ is defined as the reduction in height. For a time-dependent melting induced by
176 surface warming $\Delta a_0 = -\gamma \cdot \Delta T$ the general solution of Equation 13 is

177 $$\Delta h(t) = \gamma \cdot \int_0^t dt' \, \Delta T(t') \cdot e^{(t-t')/\tau_\gamma} \tag{14}$$

178 This equation corresponds to a linear response theory with the melting $-\gamma \cdot \Delta T$ as forcing and an
179 exponential response function

180 $$R(t') = e^{t'/\tau_\gamma} \tag{15}$$

181 Linear response theory states that the convolution of Equation 14 yields the linear response of
182 the system (Good et al., 2011; Winkelmann and Levermann, 2013). Note that generally linear
183 response theory is used as an approximation of a non-linear system to relatively weak forcing. In
184 these circumstances the response function has to decline with time because it represents the
185 history of the system's response to past perturbation. For example, if the response function was a
186 declining exponential $(t') = e^{-t'}$, this would mean that the effect of forcing that occurred in the
187 past, i.e. prior to the time t that is considered, becomes exponentially less relevant for the current
188 system response. Here, however, the response function is *in*creasing with time, which means that
189 the past deviation from the steady state is amplified as expected near an unstable fixed point. The
190 exponent $1/\tau_\gamma$ can be considered the Lyaponov exponent of the system.

191 Given the boundary condition $\Delta h(t=0)=0$, for a constant temperature increase ΔT, Equation 14
192 becomes

193 $$\Delta h(t) = h_0 \cdot \left(\frac{\tau_\gamma}{\tau_0} - \frac{\tau_\gamma}{\tau_0} \cdot e^{t/\tau_\gamma}\right) - \frac{h_0}{\tau_0} - \frac{h_0}{\tau_\gamma} \tag{16}.$$

194 The decay time for a relative volume reduction of α is then given by:

195 $$\tau_\alpha = \frac{1}{\gamma\,\Gamma} \cdot log\left(1 + \alpha \cdot \frac{\Gamma \cdot h_0}{\Delta T}\right) \tag{17},$$

196 where *log* denotes the natural logarithm. Equation 17 is denoted the *decay-time equation*

197 hereafter.

3. Estimating the Melt Time of the Greenland Ice Sheet from Observables

In this simplified approach, the collapse time is thus a function of three observable quantities: the equilibrium-line altitude, h_0, the atmospheric lapse rate, Γ, and the melting sensitivity to temperature, γ. The average equilibrium-line altitude of the Greenland Ice Sheet is at about 1150 meters (Box & Steffen 2001)). The observed range for the atmospheric lapse rate is estimated to be between 5 ± 2 °C/km (Fausto et al. 2009; Gardner & Sharp 2009), and current estimates for the melting sensitivity scatter around 4.4 ± 2 cm/year/°C (Box 2013). In order to obtain an estimate of the decay time and the uncertainty around this estimate we use Equation 17 and choose the lapse rate and melting sensitivity uniformly randomly from these observed intervals (Tab. 1, Figs. 2 – 4).

Following the decay-time Equation 17, the observational constraints for the atmospheric lapse rate, Γ, and the melting sensitivity, γ, translate into an uncertainty range for the melt time of the Greenland Ice Sheet, assuming uniform probability distributions for both Γ and γ within the above intervals. Fig. 2 shows the histograms of the time until 10% of its present-day ice volume (corresponding to 0.7 m global sea-level rise) are melted for different warming scenarios. The melt time strongly depends on the level of warming beyond the temperature threshold: The median estimate varies from more than 2000 years for a warming of +1°C to less than 500 years for a warming of +5°C.

Existing numerical simulations of a decay of the Greenland Ice Sheet (Ridley et al. 2010; Robinson et al. 2012) differ in their trajectories for the total ice volume, but exhibit a characteristic functional form when the relative ice volume is expressed as a function of the temperature anomaly above the critical temperature threshold (Fig. 2). This characteristic relation is captured by our first-order equation for the decay time, embedding the results from process-based models into a simple analytical framework. This approach provides a good approximation if, on the one hand, the volume loss is significantly large for the melt-elevation feedback to become relevant and, on the other hand, the melting is dominating the ice loss compared to the dynamic ice discharge.

Since the simple equation provided heredoes not account for any dynamic discharge or even ice motion, the results from Equation 17 strongly deviate from numerical simulations when the ice

227 has time to adjust dynamically to the volume loss. This can be seen for a stronger ice loss of 50%
228 of the initial volume where the functional dependence between the decay time and the
229 temperature anomaly clearly follows a different functional form than predicted by Equation 17
230 (Fig. 3).

231 Since the melt time is a monotonically decreasing function of both the lapse rate and the melting
232 sensitivity, the upper and lower limits of the estimates can be directly computed from the
233 observed uncertainty interval of these quantities. However, the functional form of Equation 17
234 introduces a specific structure into the histogram of the melt time which is highly skewed
235 towards the low end (Tab. 1 and Fig. 4). For increasing warming levels the histogram is shifting
236 towards lower decay times. At the same time the histogram narrows and higher decay times
237 become less frequent within the chosen parameter range (see description above).

238

4. Discussion and conclusion

Our estimate for the decay time captures the characteristic slow-down near the critical threshold as can be seen from the divergence of the decay time, τ_α, in the limit of vanishing warming above the threshold (Equation 17). The simple equation of the decay time quantitatively reproduces the range given by simulations with process-based models. the relative speed-up of ice loss due to the melt-elevation feedback (Fig. 5) is estimated, using the central values of the parameter ranges, i.e. equilibrium-line altitude h_0=1150m, atmospheric lapse rate Γ=5 °C/km and melting sensitivity γ=4.4 cm/year/°C. The feedback becomes more dominant near the threshold compared to larger temperature increase for which the external climatic forcing is more relevant.

The simple equation provided here is clearly limited in its applicability. The role of the ice material properties is comprised into one parameter, the melting sensitivity of the ice to a temperature increase at the surface. This sensitivity will in general vary not only with time but also spatially and due to the melting itself. Similarly, the feedback role of the surrounding climate is represented by only one parameter, the atmospheric lapse rate which will again vary spatially but also with time as the ice surface declines.

Ice dynamics are deliberately excluded in our simple conceptual approach in order to separate and characterize the melt-elevation feedback. In reality, ice dynamics of course play an important role in the ice-sheet mass balance: Radar (ERS-2) and laser (ICESat) altimetry observations show that mass changes in Greenland were dominated by changes in the surface mass balance (SMB) between 1995 and 2001, and both SMB and dynamics contributed equally to mass loss from the Greenland Ice Sheet between 2001 to 2009 (Hurkmans et al., 2014). (Fürst et al., 2015) estimate that 40% of the recent loss (2000-2010) is due to an increase in ice dynamic discharge, 60% due to changes in the surface mass balance. Their results suggest that the future volume loss from the Greenland Ice Sheet might be predominantly caused by surface melting and dynamic discharge is limited by margin thinning and retreat.

Some studies suggest (Graversen et al., 2010; Price et al., 2011) that the dynamic discharge from Greenland is strongly limited by the ice sheet's bottom topography, for which estimates yield an upper bound of approximately 5-13 cm during the next century. Over a period during which the ice loss is dominated by the feedback and the ice-dynamic effect is limited, our approach

268 provides a quantitative estimate of the melt time based on observable quantities. Equation 17 can
269 thus be used if new observations suggest an altered melting sensitivity or changes in the
270 atmospheric response to Greenland ice loss.

271 For a temperature increase of 5°C, which could be reached within this century (IPCC, 2013), the
272 median rate of sea-level contribution is about 1.4 mm per year which is about four times that of
273 its current contribution of about 0.4 mm per year (Rignot et al., 2011; Shepherd et al., 2012).
274 Even for extremely high temperatures however, the Greenland Ice Sheet cannot melt infinitely
275 fast – our results show that a complete disintegration within the next two millennia is highly
276 unlikely unless ice dynamics effects become dominant or the melting sensitivity is significantly
277 higher than currently observed. For a global mean temperature increase below two degrees, as
278 agreed upon during the 2015 Paris UNFCCC climate summit, the threshold temperature would
279 only be exceeded mildly and the decay time of the Greenland ice sheet would be multi-
280 millennial.

Volume loss		0.5ºC	1ºC	2ºC	3ºC	4ºC	5ºC
10%	Lower	2140 yr	1320 yr	760 yr	530 yr	410 yr	330 yr
	Median	3430 yr	2040 yr	1140 yr	790 yr	610 yr	500 yr
	Upper	7290 yr	4120 yr	2210 yr	1520 yr	1150 yr	930 yr
50%	Lower	4920 yr	3600 yr	2460 yr	1900 yr	1550 yr	1320 yr
	Median	8740 yr	6170 yr	4040 yr	3040 yr	2450 yr	2090 yr
	Upper	20740 yr	13920 yr	8640 yr	6310 yr	4980 yr	4120 yr
100%	Lower	6340 yr	4920 yr	3600 yr	2910 yr	2460 yr	2140 yr
	Median	11610 yr	8730 yr	6160 yr	4840 yr	4020 yr	3500 yr
	Upper	28710 yr	20740 yr	13920 yr	10630 yr	8640 yr	7290 yr

281 **Table 1**: **Decay time.** Time period after which different percentages of volume loss have
282 occurred at different warming levels. Provided are the median values of the distributions from
283 Figures 2 and 3 together with the lower and upper limit that are derived respectively from the
284 upper and lower limits of the uncertainty range of the observed melting sensitivity and
285 atmospheric lapse rate. The simple decay time equation (Equation 17) does not take any ice
286 dynamic effects into account and its translation to ice volume assumes a constant horizontal ice-
287 sheet area. Thus the values provided here best fit the complex model simulations only when
288 these assumptions are reasonably well justified which is most likely not the case for high ice loss
289 such as 50% or 100% of the original ice volume.

The equation shown within the figure:

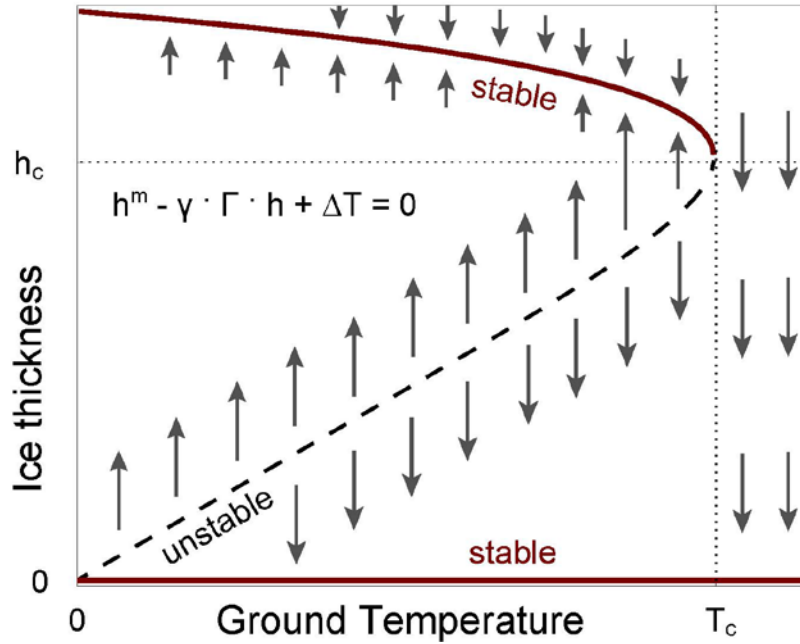$$h^m - \gamma \cdot \Gamma \cdot h + \Delta T = 0$$

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Figure 1: Ice-sheet hysteresis. If the ice-sheet is in an unstable configuration (dashed black branch), a slight perturbation will either cause it to converge into the stable state (upper red branch) or to melt completely. For a given temperature, the dotted line gives the critical surface elevation (Section 3). If the surface elevation is lower than h_c, a complete meltdown of the ice sheet is inevitable. Once the temperature threshold, T_c, is crossed, the time for a collapse of a certain fraction of the ice-sheet can be estimated via Equation 17.

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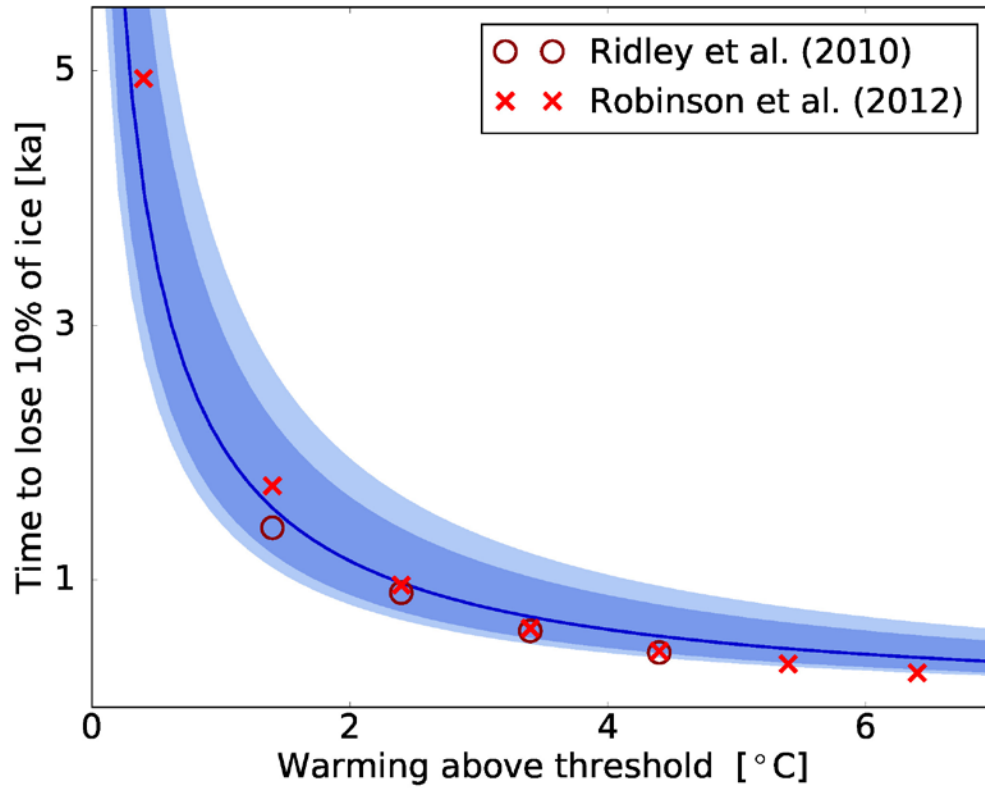
Figure 2. **Decay-time of the Greenland Ice Sheet.** *The decay time depends critically on the level*
of warming above the temperature threshold. Shown are the median (black line), likely (18% to
83% quantiles, dark blue shading) and very likely (5% to 95% quantiles, light blue shading)
ranges for the time to melt 10% of the present-day ice volume, estimated via Equation 17. The
red circles and crosses indicate the results from process-based model simulations by Ridley et al.
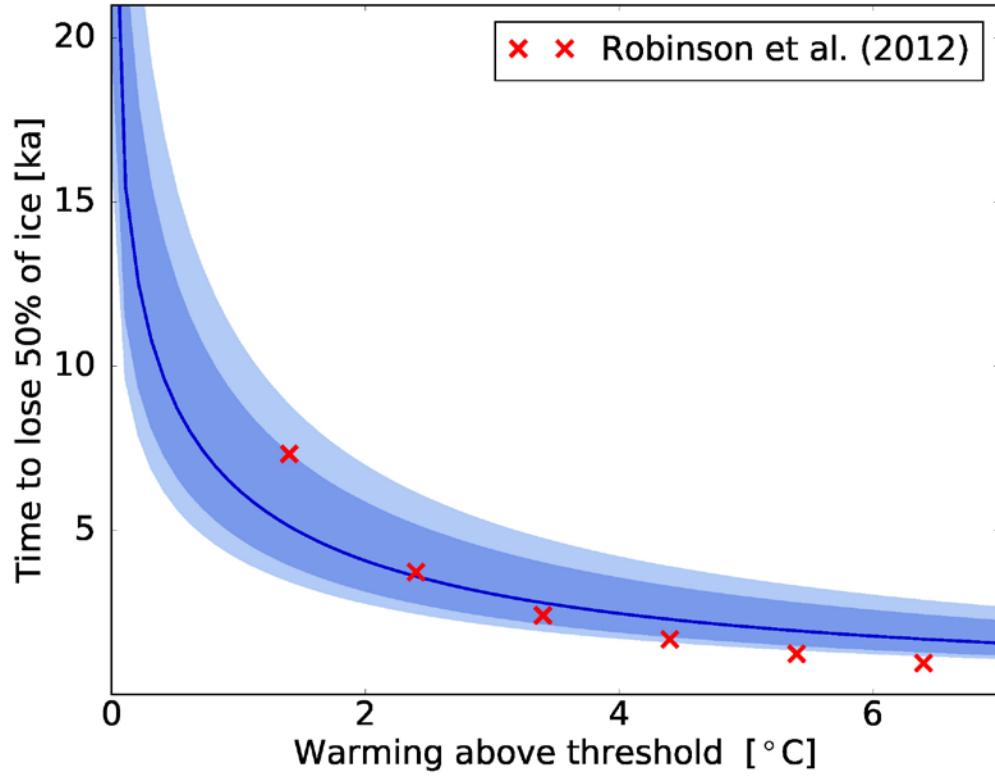*(2010) and Robinson et al. (2012), respectively.*

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Figure 3: Time until 50% of the Greenland Ice Sheet are melted. *Shown are the median (black line) and the likely (18% to 83% percentiles, dark blue shading) and very likely (5% to 95% percentiles, light blue shading) ranges for the time to melt 50% of the present-day ice volume, estimated via the equation for the decay time τ_α. The red crosses indicate the results from process-based model simulations by Robinson et al. (2012).*
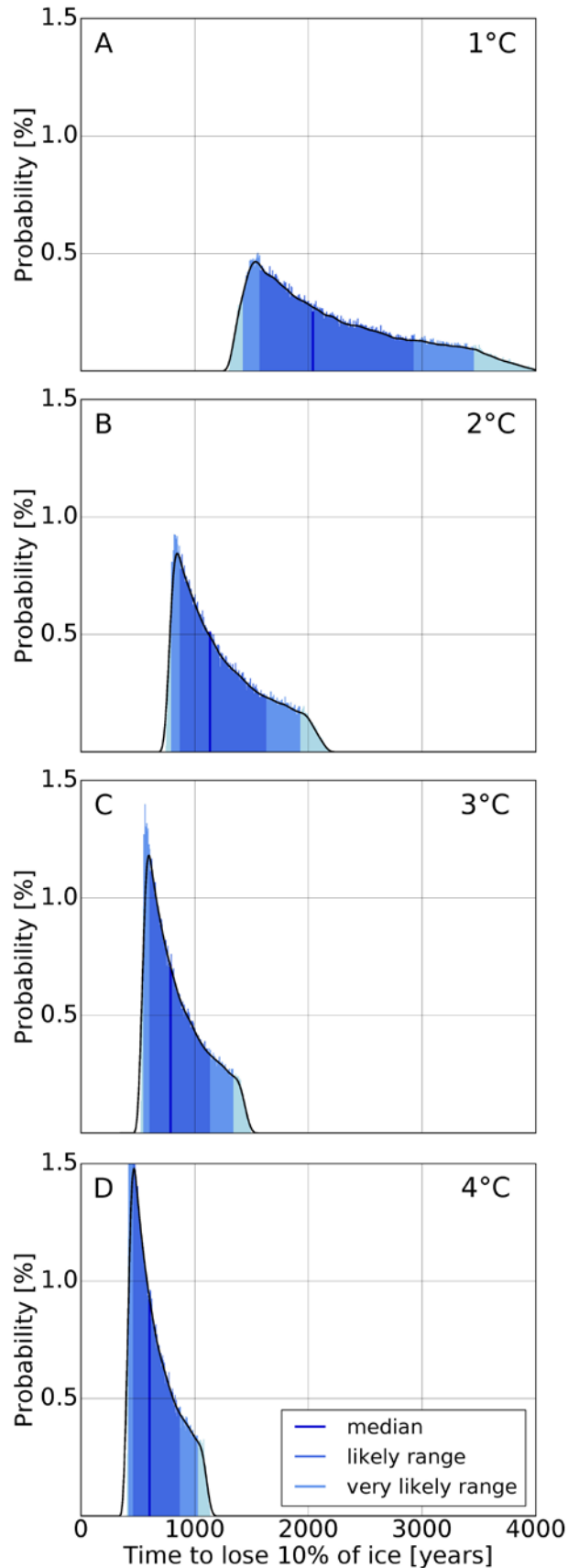
Figure 4. Likelihood for 10%-decay of Greenland Ice Sheet. Shown are the probabilities for the ice-sheet to lose 10% of its initial ice volume in a certain time period for surface warming of +1°C (A), +2°C (B), +3°C (C) and +4°C (D) above the threshold. The median is indicated by the black line, and the likely and very likely ranges are shaded in dark and light blue, respectively.
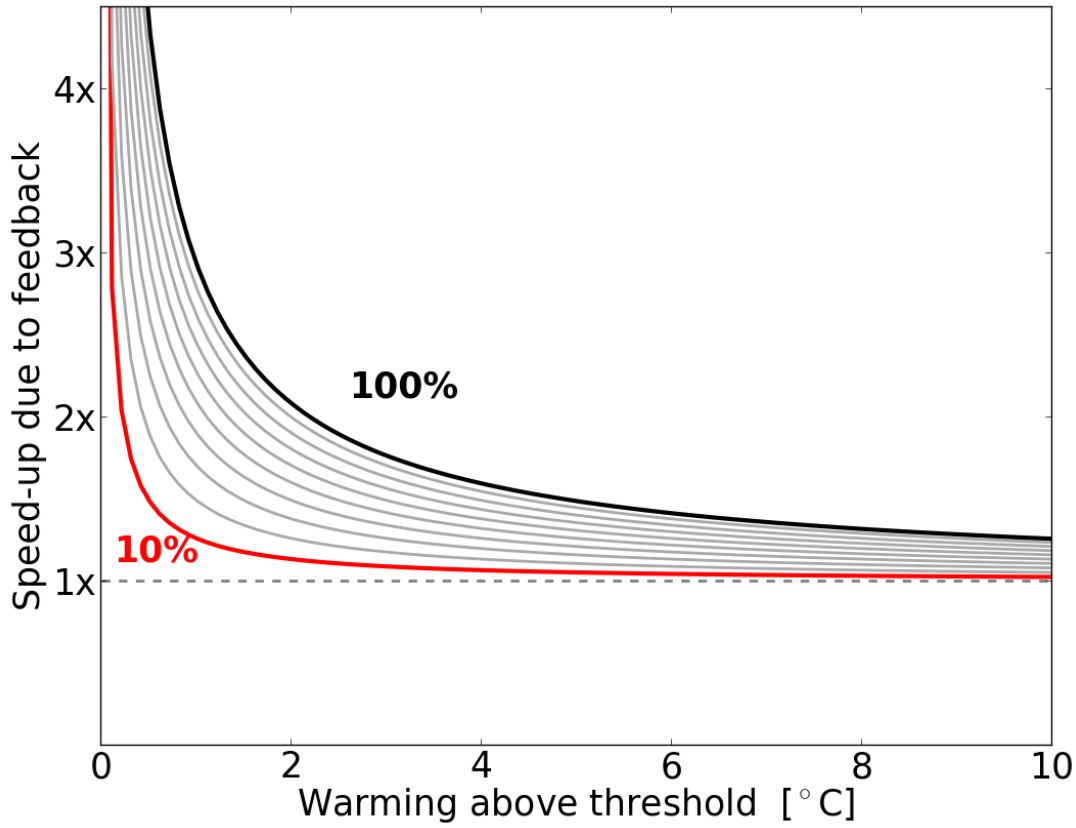
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Figure 5: Role of melt-elevation feedback in melting of Greenland ice sheet declines with increasing temperature. Shown is the ratio of melting time with melt-elevation feedback over melting time without the feedback τ_α/τ_0. Each line represents the ratio for a loss of different percent of the initial ice volume. The red line shows the ratio of the decay time with feedback over the decay time without feedback for a 10% ice loss (corresponding to Figures 2 and 4). The influence of the feedback becomes less dominant with stronger warming above the critical threshold (x-axis). Near the threshold the melting time without feedback diverges stronger ($1/\Delta T$) than the melt time with feedback which declines logarithmically.

329 **References**

330 Allen, M. R., Frame, D. J., Huntingford, C., Jones, C. D., Lowe, J. A., Meinshausen, M. and

331 Meinshausen, N.: Warming caused by cumulative carbon emissions towards the trillionth tonne,

332 Nature, 458(7242), 1163–1166, doi:10.1038/nature08019, 2009.

333 Andresen, C. S., Straneo, F., Ribergaard, M. H., Bjørk, A. A., Andersen, T. J., Kuijpers, A.,

334 Nørgaard-Pedersen, N., Kjær, K. H., Schjøth, F., Weckström, K. and Ahlstrøm, A. P.: Rapid

335 response of Helheim Glacier in Greenland to climate variability over the past century, Nat.

336 Geosci., 5(1), 37–41, doi:10.1038/ngeo1349, 2012.

337 Bamber, J. L., Hardy, R. J. and Joughin, I.: `An analysis of balance velocities over the Green

338 land ice sheet and comparison with synthetic aperture radar interferometry', J. Glaciol., 46(152),

339 67–74, doi:10.3189/172756500781833412, 2000.

340 Box, J. E.: Greenland ice sheet mass balance reconstruction. Part II: Surface mass balance (1840-

341 2010), J. Clim., 26(18), 6974–6989, doi:10.1175/JCLI-D-12-00518.1, 2013.

342 Box, J. E., Fettweis, X., Stroeve, J. C., Tedesco, M., Hall, D. K. and Steffen, K.: Greenland ice

343 sheet albedo feedback: Thermodynamics and atmospheric drivers, Cryosphere, 6(4), 821–839,

344 doi:10.5194/tc-6-821-2012, 2012.

345 van den Broeke, M., Bamber, J., Ettema, J., Rignot, E., Schrama, E., van de Berg, W. J., van

346 Meijgaard, E., Velicogna, I. and Wouters, B.: Partitioning recent Greenland mass loss., Science,

347 326(5955), 984–986, doi:10.1126/science.1178176, 2009.

348 Charbit, S., Paillard, D. and Ramstein, G.: Amount of CO_2 emissions irreversibly leading to the

349 total melting of Greenland, Geophys. Res. Lett., 35(12), 1–5, doi:10.1029/2008GL033472, 2008.

350 Clark, P. U., Shakun, J. D., Marcott, S. A., Mix, A. C., Eby, M., Kulp, S., Levermann, A., Milne,

351 G. A., Pfister, P. L., Santer, B. D., Schrag, D. P., Solomon, S., Stocker, T. F., Strauss, B. H.,

352 Weaver, A. J., Winkelmann, R., Archer, D., Bard, E., Goldner, A., Lambeck, K., Pierrehumbert,

353 R. T. and Plattner, G.-K.: Consequences of 21st Century Policy for Multi-Millennial Climate and

354 Sea-Level Change, Nat. Clim. Chang., (February), doi:10.1038/NCLIMATE2923, 2016.

355 Fettweis, X., Franco, B., Tedesco, M., van Angelen, J. H., Lenaerts, J. T. M., van den Broeke, M.

356 R. and Gallée, H.: Estimating Greenland ice sheet surface mass balance contribution to future sea

357 level rise using the regional atmospheric climate model MAR, Cryosph., 7, 469–489,

358   doi:10.5194/tcd-6-3101-2012, 2013.

359   Fürst, J. J., Goelzer, H. and Huybrechts, P.: Ice-dynamic projections of the Greenland ice sheet

360   in response to atmospheric and oceanic warming, Cryosph., 9(3), 1039–1062, doi:10.5194/tcd-8-

361   3851-2014, 2015.

362   Glen, J. W.: The Creep of Polycrystalline Ice, R. Soc. London Proc. Ser. A, 228, 519–538, 1955.

363   Goelzer, H., Huybrechts, P., Raper, S. C. B., Loutre, M.-F., Goosse, H. and Fichefet, T.:

364   Millennial total sea-level commitments projected with the Earth system model of intermediate

365   complexity LOVECLIM, Environ. Res. Lett., 7(4), 045401, doi:10.1088/1748-9326/7/4/045401,

366   2012.

367   Goelzer, H., Huybrechts, P., Fürst, J. J., Nick, F. M., Andersen, M. L., Edwards, T. L., Fettweis,

368   X., Payne, a. J. and Shannon, S.: Sensitivity of Greenland ice sheet projections to model

369   formulations, J. Glaciol., 59(216), 733–749, doi:10.3189/2013JoG12J182, 2013.

370   Good, P., Gregory, J. M. and Lowe, J. A.: A step-response simple climate model to reconstruct

371   and interpret AOGCM projections, Geophys. Res. Lett., 38(1), 1–5, doi:10.1029/2010GL045208,

372   2011.

373   Gordon, C., Cooper, C., Senior, C. A., Banks, H., Gregory, J. M., Johns, T. C., Mitchell, J. F. B.

374   and Wood, R. A.: The simulation of SST, sea ice extents and ocean heat transports in a version

375   of the Hadley Centre coupled model without flux adjustments, Clim. Dyn., 16, 147–168,

376   doi:10.1007/s003820050010, 2000.

377   Graversen, R. G., Drijfhout, S., Hazeleger, W., Wal, R., Bintanja, R. and Helsen, M.:

378   Greenland's contribution to global sea-level rise by the end of the 21st century, Clim. Dyn.,

379   2010.

380   Graversen, R. G., Drijfhout, S., Hazeleger, W., van de Wal, R., Bintanja, R. and Helsen, M.:

381   Greenland's contribution to global sea-level rise by the end of the 21st century, Clim. Dyn.,

382   37(7-8), 1427–1442, doi:10.1007/s00382-010-0918-8, 2011.

383   Gregory, J. M., Huybrechts, P. and Raper, S. C. B.: Threatened loss of the Greenland ice-sheet,

384   Nature, 428, 616, 2004a.

385   Gregory, J. M., Huybrechts, P. and Raper, S. C. B.: Threatened loss of the Greenland ice-sheet,

386   Nature, 428(April), 2513–2513, doi:10.1038/nature02512, 2004b.

387   Greve, R.: Application of a polythermal three-dimensional ice sheet model to the Greenland Ice

388   Sheet: Response to steady-state and transient climate scenarios, J. Clim., 10(5), 901–918,

389   doi:10.1175/1520-0442(1997)010<0901:AOAPTD>2.0.CO;2, 1997.

390   Greve, R.: On the response of the Greenland ice sheet to greenhouse climate change,

391   Clim.Change, 46(3), 289–303 [online] Available from:

392   http://www.scopus.com/inward/record.url?eid=2-s2.0-

393   0033843980&partnerID=40&md5=c8f0b28433d6b41a7c9a47e4b44080bc, 2000.

394   Harper, J., Humphrey, N., Pfeffer, W. T., Brown, J. and Fettweis, X.: Greenland ice-sheet

395   contribution to sea-level rise buffered by meltwater storage in firn., Nature, 491(7423), 240–3,

396   doi:10.1038/nature11566, 2012.

397   Howat, I., Negrete, A. and Smith, B.: The Greenland Ice Mapping Project (GIMP) land

398   classification and surface elevation data sets, Cryosph., 8(4), 1509–1518, doi:10.5194/tc-8-1509-

399   2014, 2014.

400   Howat, I. M. and Eddy, A.: Multi-decadal retreat of Greenland ' s marine-terminating glaciers, J.

401   Glaciol., 57(203), 389–396, 2012.

402   Hurkmans, R. T. W. L., Bamber, J. L., Davis, C. H., Joughin, I. R., Khvorostovsky, K. S., Smith,

403   B. S. and Schoen, N.: Time-evolving mass loss of the Greenland Ice Sheet from satellite

404   altimetry, Cryosph., 8(5), 1725–1740, doi:10.5194/tc-8-1725-2014, 2014.

405   Huybrechts, P. and Wolde, J. De: The Dynamic Response of the Greenland and Antarctic Ice

406   Sheets to Multiple-Century Climatic warming, J. Clim., 12 (8), 2169–2188, 1999.

407   Huybrechts, P., Goelzer, H., Janssens, I., Driesschaert, E., Fichefet, T., Goosse, H. and Loutre,

408   M. F.: Response of the Greenland and Antarctic Ice Sheets to Multi-Millennial Greenhouse

409   Warming in the Earth System Model of Intermediate Complexity LOVECLIM, Surv. Geophys.,

410   32(4-5), 397–416, doi:10.1007/s10712-011-9131-5, 2011.

411   IPCC: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to

412   the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, edited by T. F.

413   Stocker, D. Qin, G.-K. Plattner, M. Tignor, S. K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex,

414   and P. M. Midgley, Cambridge University Press, Cambridge, United Kingdom and New York,

415 NY, USA., 2013.

416 Levermann, A., Bamber, J. L., Drijfhout, S., Ganopolski, A., Haeberli, W., Harris, N. R. P.,

417 Huss, M., Krüger, K., Lenton, T. M., Lindsay, R. W., Notz, D., Wadhams, P. and Weber, S.:

418 Potential climatic transitions with profound impact on Europe, Clim. Change, 110(3-4), 845–

419 878, doi:10.1007/s10584-011-0126-5, 2012.

420 Levermann, A., Clark, P. U., Marzeion, B., Milne, G. A., Pollard, D., Radic, V. and Robinson,

421 A.: The multimillennial sea-level commitment of global warming., Proc. Natl. Acad. Sci. U. S.

422 A., 110(34), 13745–50, doi:10.1073/pnas.1219414110, 2013.

423 Mernild, S. H., Mote, T. L. and Liston, G. E.: Greenland ice sheet surface melt extent and trends:

424 1960-2010, J. Glaciol., 57(204), 621–628, doi:10.3189/002214311797409712, 2011.

425 Moon, T., Joughin, I., Smith, B. and Howat, I.: 21st-Century Evolution of Greenland Outlet

426 Glacier Velocities, Science (80-.)., 336(May), 576–578, doi:10.1126/science.1219985, 2012.

427 Nick, F. M., Vieli, a., Howat, I. M. and Joughin, I.: Large-scale changes in Greenland outlet

428 glacier dynamics triggered at the terminus., Nat. Geosci., 2(2), 110–114, doi:10.1038/ngeo394,

429 2009.

430 Nowicki, S., Bindschadler, R. A., Abe-Ouchi, A., Aschwanden, A., Bueler, E., Choi, H.,

431 Fastook, J., Granzow, G., Greve, R., Gutowski, G., Herzfeld, U., Jackson, C., Johnson, J.,

432 Khroulev, C., Larour, E., Levermann, A., Lipscomb, W. H., Martin, M. A., Morlighem, M.,

433 Parizek, B. R., Pollard, D., Price, S. F., Ren, D., Rignot, E., Saito, F., Sato, T., Seddik, H.,

434 Seroussi, H., Takahashi, K., Walker, R. and Wang, W. L.: Insights into spatial sensitivities of ice

435 mass response to environmental change from the SeaRISE ice sheet modeling project II:

436 Greenland, J. Geophys. Res. Earth Surf., 118(2), 1025–1044, doi:10.1002/jgrf.20076, 2013.

437 Polashenski, C. M., Dibb, J. E., Flanner, M. G., Chen, J. Y., Courville, Z. R., Lai, A. M.,

438 Schauer, J. J., Shafer, M. M. and Bergin, M.: Neither dust nor black carbon causing apparent

439 albedo decline in Greenland's dry snow zone: Implications for MODIS C5 surface reflectance,

440 Geophys. Res. Lett., 42(21), 2015GL065912, doi:10.1002/2015GL065912, 2015.

441 Pope, V. D., Gallani, M. L., Rowntree, P. R. and Stratton, R. a.: The impact of new physical

442 parametrizations in the Hadley Centre climate model: HadAM3, Clim. Dyn., 16(2-3), 123–146,

443 doi:10.1007/s003820050009, 2000.

444 Price, S. F., Payne, A. J., Howat, I. M. and Smith, B. E.: Committed sea-level rise for the next

445 century from Greenland ice sheet dynamics during the past decade., Proc. Natl. Acad. Sci. U. S.

446 A., 108(22), 8978–83, doi:10.1073/pnas.1017313108, 2011.

447 Rae, J. G. L., Aoalgeirsdóttir, G., Edwards, T. L., Fettweis, X., Gregory, J. M., Hewitt, H. T.,

448 Lowe, J. a., Lucas-Picher, P., Mottram, R. H., Payne, a. J., Ridley, J. K., Shannon, S. R., Van De

449 Berg, W. J., Van De Wal, R. S. W. and Van Den Broeke, M. R.: Greenland ice sheet surface

450 mass balance: Evaluating simulations and making projections with regional climate models,

451 Cryosphere, 6(6), 1275–1294, doi:10.5194/tc-6-1275-2012, 2012.

452 Ridley, J., Huybrechts, P., Gregory, J. M. and Lowe, J. A.: Elimination of the Greenland Ice

453 Sheet in a High CO2 Climate, J. Clim., 18, 3409–3427, 2005.

454 Ridley, J., Gregory, J. M., Huybrechts, P. and Lowe, J. A.: Thresholds for irreversible decline of

455 the Greenland ice sheet, Clim. Dyn., 35, 1049–1057, doi:10.1007/s00382-009-0646-0, 2010.

456 Rignot, E., Box, J. E., Burgess, E. and Hanna, E.: Mass balance of the Greenland ice sheet from

457 1958 to 2007, Geophys. Res. Lett., 35(20), 1–5, doi:10.1029/2008GL035417, 2008.

458 Rignot, E., Velicogna, I., van den Broeke, M. R., Monaghan, A. and Lenaerts, J.: Acceleration of

459 the contribution of the Greenland and Antarctic ice sheets to sea level rise, Geophys. Res. Lett.,

460 38, 2011.

461 Robinson, a., Calov, R. and Ganopolski, a.: An efficient regional energy-moisture balance

462 model for simulation of the Greenland ice sheet response to climate change, Cryosph. Discuss.,

463 3(3), 729–764, doi:10.5194/tcd-3-729-2009, 2009.

464 Robinson, A., Calov, R. and Ganopolski, A.: Multistability and critical thresholds of the

465 Greenland ice sheet, Nat. Clim. Chang., 2(6), 429–432, doi:10.1038/nclimate1449, 2012.

466 Shepherd, A. and Wingham, D.: Recent Sea-Level Contributions of the Antarctic and Greenland

467 Ice Sheets, Science (80-.)., 315(5818), 1529–1532, doi:10.1126/science.1136776, 2007.

468 Shepherd, A., Ivins, E. R., A, G., Barletta, V. R., Bentley, M. J., Bettadpur, S., Briggs, K. H.,

469 Bromwich, D. H., Forsberg, R., Galin, N., Horwath, M., Jacobs, S., Joughin, I., King, M. A.,

470 Lenaerts, J. T. M., Li, J., Ligtenberg, S. R. M., Luckman, A., Luthcke, S. B., McMillan, M.,

471 Meister, R., Milne, G., Mouginot, J., Muir, A., Nicolas, J. P., Paden, J., Payne, A. J., Pritchard,

472 H., Rignot, E., Rott, H., Sorensen, L. S., Scambos, T. A., Scheuchl, B., Schrama, E. J. O., Smith,

473 B., Sundal, A. V., van Angelen, J. H., van de Berg, W. J., van den Broeke, M. R., Vaughan, D.

474 G., Velicogna, I., Wahr, J., Whitehouse, P. L., Wingham, D. J., Yi, D., Young, D. and Zwally, H.

475 J.: A Reconciled Estimate of Ice-Sheet Mass Balance, Science (80-.)., 338(6111), 1183–1189,

476 doi:10.1126/science.1228102, 2012.

477 Solgaard, A. M. and Langen, P. L.: Multistability of the Greenland ice sheet and the effects of an

478 adaptive mass balance formulation, Clim. Dyn., 39(7-8), 1599–1612, doi:10.1007/s00382-012-

479 1305-4, 2012.

480 Solomon, S., Plattner, G.-K., Knutti, R. and Friedlingstein, P.: Irreversible climate change due to

481 carbon dioxide emissions, Proc. Natl. Acad. Sci., 106, 1704–1709,

482 doi:10.1073/pnas.0812721106, 2009.

483 Straneo, F., Curry, R. G., Sutherland, D. a., Hamilton, G. S., Cenedese, C., Våge, K. and Stearns,

484 L. a.: Impact of fjord dynamics and glacial runoff on the circulation near Helheim Glacier, Nat.

485 Geosci., 4(3), 322–327, doi:10.1038/ngeo1109, 2011.

486 Thomas, R., Frederick, E., Li, J., Krabill, W., Manizade, S., Paden, J., Sonntag, J., Swift, R. and

487 Yungel, J.: Accelerating ice loss from the fastest Greenland and Antarctic glaciers, Geophys.

488 Res. Lett., 38(10), 1–6, doi:10.1029/2011GL047304, 2011.

489 Toniazzo, T., Gregory, J. M. and Huybrechts, P.: Climatic impact of a Greenland deglaciation

490 and its possible irreversibility, J. Clim., 17(1), 21–33, doi:10.1175/1520-

491 0442(2004)017<0021:CIOAGD>2.0.CO;2, 2004.

492 Vialov, S. .: Regularities of glacial shields movement and the theory of plastic viscours flow, Int.

493 Assoc. Sci. Hydrol. Publ., 47(2), 266–275, 1958.

494 Walsh, K. M., Howat, I. M., Ahn, Y. and Enderlin, E. M.: Changes in the marine-terminating

495 glaciers of central east Greenland, 2000-2010, Cryosphere, 6(1), 211–220, doi:10.5194/tc-6-211-

496 2012, 2012.

497 Weertman, J.: Stability of ice-age ice sheets, J. Geophys. Res., 66(11), 3783,

498 doi:10.1029/JZ066i011p03783, 1961.

499 Winkelmann, R. and Levermann, A.: Linear response functions to project contributions to future

500 sea level, Clim. Dyn., 40(11-12), 2579–2588, doi:10.1007/s00382-012-1471-4, 2013.

501 Zickfeld, K., Eby, M., Weaver, A. J., Alexander, K., Crespin, E., Edwards, N. R., Eliseev, A. V.,

502 Feulner, G., Fichefet, T., Forest, C. E., Friedlingstein, P., Goosse, H., Holden, P. B., Joos, F.,

503 Kawamiya, M., Kicklighter, D., Kienert, H., Matsumoto, K., Mokhov, I. I., Monier, E., Olsen, S.

504 M., Pedersen, J. O. P., Perrette, M., Philippon-Berthier, G., Ridgwell, A., Schlosser, A., Von

505 Deimling, T. S., Shaffer, G., Sokolov, A., Spahni, R., Steinacher, M., Tachiiri, K., Tokos, K. S.,

506 Yoshimori, M., Zeng, N. and Zhao, F.: Long-Term climate change commitment and

507 reversibility: An EMIC intercomparison, J. Clim., 26(16), 5782–5809, doi:10.1175/JCLI-D-12-

508 00584.1, 2013.

509