

Dear Dr. Fettweis,

Thank you for handling the review process of our manuscript. We are very happy that both reviewers appreciate the novelty of our study and consider it interesting. We present here a detailed response to each of the reviewers' comments *in italic and blue*.

We highly appreciate that both reviewers have taken so much time and effort in reviewing our study. The manuscript has improved significantly thanks to their corrections and suggestions. However, we must admit that we are slightly frustrated by some of the very confident, strong and strict demands made by reviewer #1, because they seem to be based in places on a misunderstanding of the theory (e.g. concerning equations (3) and (4)) and in other places simply on a different appreciation for theory and a different understanding of the purpose of this study than we have. In particular, we could not meet the demand that we make the computation of the melt time the central aspect of our study. There exist complex numerical models which are much better suited to address this question and our approach is not comprehensive in the number and detail of the processes that are represented – on the contrary, it is purposefully simple in order to emphasize what can be explained by the melt-elevation feedback and what physical characteristics this feedback has.

That being said, we really appreciate that both reviewers have provided such a thorough review of the manuscript, including the correction of typos and other issues, and we took on very seriously that he or she finds that the discussion was insufficient and have sought to improve this in the revised manuscript - detailed answers to all points raised by the reviewer are given below.

We are very sorry that we were not able to comply with all of reviewer #1's requests. Generally we try to do this, but it was not possible in this case. We hope that you and the reviewers will never-the-less agree with us that while our paper does not solve all matters of ice sheet melting, it is still useful and interesting for the readers of The Cryosphere.

Best wishes,

Anders and Ricarda

Dear Dr. Fettweis,

Thank you for handling the review process of our manuscript. We are very happy that both reviewers appreciate the novelty of our study and consider it interesting. We think that the manuscript has improved significantly due to their corrections and suggestions. We present here a detailed response to each of the reviewers' comments *in italic and blue*.

Reviewer #1:

General Comments

This paper shows an interesting way of deriving the melt time of the Greenland Ice Sheet for different warming levels using a very simple approach based on three observable quantities: the equilibrium-line altitude (ELA), the atmospheric lapse rate and the melting sensitivity of the ice surface to temperature. The most interesting result is that the derived decay time quantitatively reproduces the range given by existing process-based numerical simulations. This study is relevant in the current context of Greenland Ice Sheet mass loss. However, the approach suffers from several drawbacks that we detail in the Specific Comments below, especially the non-applicability of the decay time equation if dynamic discharge is taken into account, the lack of experiments to confirm the results given by the proposed equation, the lack of connection between different sections of the paper and the poor discussion. Therefore, we advise the authors to revise the paper either for providing a more substantial analysis of their work or for summarizing their results into a brief communication.

Response:

We would like to thank the reviewer for taking the time to work so intensely with our manuscript and we are very happy that the reviewer considers our work interesting. We agree with the reviewer that the comparison of the simple equation that we derive with the complex process-based models is interesting, but we do not consider this the “most interesting result” of the paper. As reflected in our choice of the title we seek to provide the simplest possible representation of the melt-elevation feedback and derive some (hopefully) curious characteristics of the resulting mini-theory.

Some of the specific comments of the reviewer (below) aim at changing this spirit of the study. We fully appreciate that there are different views on what is important and relevant science. The reviewer has one take on this and we have our own. As much as we appreciate the reviewer's comments, we would also very much appreciate if we could keep the general nature of our paper as it was intended: An equation that captures one specific and important aspect of the large-scale melting of an ice sheet.

Our study is purposefully simple. It purposefully does not include any ice dynamic effects and it is purposefully not a modelling study. It is a simple theory paper. There is no doubt that our study is not a comprehensive analysis of ice-sheet mass-loss and we make this very clear throughout the paper. We believe that there is special merit in extracting specific processes from complex physical phenomena and this is one small contribution in this direction. We hope

and believe that it will be of interest to other researchers in the field and that it can be used as a conceptual approach to further understanding the melt-elevation feedback.

Having said that we very much appreciate the reviewer's suggestions on the manuscript and we will, of course, address every aspect that the reviewer raises. We think that the manuscript's clarity and legibility have improved significantly thanks to the many helpful suggestions.

We will give a detailed response to all of the reviewer's comments together with a revised manuscript that also includes the second reviewer's comments.

Specific Comments

1. The decay time equation proposed here does **not take into account ice dynamics**, as the authors state in section 5. However, a number of studies have shown that, even if the contribution of the dynamic part in Greenland ice loss seems to be less important than surface mass balance (SMB) changes, it is still quite substantial. One of the most recent modeling studies about this topic (Furst et al., 2015, TC) shows that 40% of the recent loss (2000-2010) is due to an increase in ice dynamic discharge (60% is due to SMB decrease). In terms of projections, using a 3D higher-order model with climate anomalies coming from 10 AOGCMs forced by the four RCPs climate scenarios, Furst et al. (2015) conclude that the sea-level rise of 1.4 to 16.6 cm by 2100 is predominantly caused by SMB decrease. They suggest the dynamic discharge contribution is limited by margin thinning and retreat as well as a competition between surface melting that removes ice before it reaches the calving front. Another modeling study based on four outlet glaciers that drain 22% of the Greenland Ice Sheet (Nick et al., 2013, Nature) shows that the dynamic contribution would be about 4-8.5 cm sea-level rise by 2100 versus 2.5-9.8 cm for SMB. Finally, radar (ERS-2) and laser (ICESat) altimetry observations show that mass changes in Greenland were dominated by SMB changes between 1995 and 2001, and then both SMB and dynamics equally contributed to the negative mass balance from 2001 to 2009 (Hurkmans et al., 2014, TC). Therefore, we think that not taking into account the dynamic part is a very strong assumption and we question the pertinence of the results presented here. At least, a scaling taking into account dynamics could be proposed in the decay time equation as well as a stronger discussion related to those three studies.

Response:

There is no doubt that ice dynamics is important for both ice sheets on Antarctica and Greenland. However, as explained above, this paper is not about being comprehensive. For example, the very important articles that derived the Shallow Ice Approximation made the very strong assumption of zero ice velocity at the ground which is not the case in most regions in Antarctica that are crucial for the ice loss of the continent. The Shallow Ice Approximation was nevertheless a very important contribution to glaciological theory. While we cannot claim that our approach is even remotely as relevant as that, we are convinced that extracting only one specific feedback from a complex problem is helpful for the understanding, and may it only be theoretical. It is for example curious to view the melt-elevation feedback (as we now call it in

response to both reviewers' requests) as a linear response function with an increasing, not decreasing long-term tail. It is also curious to see the "critical slowing down" that can be observed in many non-linear systems near their threshold. These two phenomena would, for example, be diluted and become less clear by adding more processes to the equation.

We understand that the reviewer thinks that our discussion of previous work was not sufficient (although we cite more than 60 articles while in a brief communication only 20 references would be allowed). In order to give a more comprehensive discussion we have added additional references (including the ones mentioned by the reviewer) and their discussion.

2. Even if we assume that ice loss only comes from SMB changes (which is the case of this study), the study **lacks some proofs that the decay time equation is robust against process-based studies**. Only Figure 2 clearly shows that the results agree well with two process-based numerical simulations, even if it does not show the time to lose 10% of ice for Ridley et al. (2010) under 1_C warming above threshold. Figure3 shows the same quantity but for 50% of ice loss with only one numerical simulation (Robinson et al., 2012). What about Ridley et al. (2010) in Figure 3? In order to validate the simple equation proposed here, we think that the decay time for other values (20%, 30%, 40%, 100%) should be shown along with results from process-based simulations.

Response:

This comment reflects the disagreement that we have with the reviewer with respect to the purpose of our study. We would like to emphasize that this is not a modelling study, but a simple piece of theory. We were searching for an analysis of the melt-elevation feedback in the same spirit as they have been carried out by a number of authors in other contexts (e.g. Gnanadesikan, Science, 1999; Stommel, 1961, Levermann et al. PNAS 2009 and a number of publications by J. Oerlemans in Nature and Science etc.). Since we did not find this kind of equation we derived it, analyzed some interesting mathematical properties of the solution (see above) and compared it to some available model results. In fact, there are not too many model simulations that increase the temperature on Greenland by a constant value and report the result for long enough to lose 10% of the ice sheet (we only know of Ridley et al. 2010 and Robinson et al. 2012).

The Figure 3 of Ridley et al. (2010) that the reviewer refers to, can be compared with our results in the sense that it obviously shows that there is more complex physics at play in the numerical model than there is in our simple model. That is the reason why Ridley et al. find a multi-stability of the ice sheet while we only find a bi-stability. Ridley et al. show that this multi-stability arises predominantly from horizontal differences in topography and surface-mass balance which is why some parts of the ice sheet are more and some are less sensitive to a surface temperature increase. This is not surprising and it is not at odds with our equation. It is just not captured by a simple conceptual model without horizontal resolution.

3. It is not straightforward to understand **how sections 2 and 3 really fit** into the paper since the authors do not use the equations (1) to (10) related to the Vialov profile and the critical SMB for deriving the decay time equation (17), except equation (6) that relates surface melt rate and elevation. It is nice to see how the critical SMB and surface elevation below which a meltdown is inevitable are calculated but they are not really used in computing the main results of the paper (since the decay time only depends on the warming level, lapse rate, ELA and melting sensitivity). As far as we understood, one of the main purposes of sections 2 and 3 is to show where Figure 1 (which is quite nice) comes from.

Response:

We believe that this issue arises because the reviewer views the purpose of the paper as providing a means to estimate the decay time of the Greenland Ice Sheet. While this is an interesting application, it is not the purpose of the manuscript. Instead we aim to provide a simple conceptual theory for the decay of an ice sheet due to the melt-elevation feedback beyond the critical temperature threshold. We believe that to this end it is interesting for the reader to see a derivation of this threshold and a computation of the threshold position as a starting point for the temporal equation. We have added sentences to explain the purpose of these sections to the introduction and each of the sections to clarify.

4. The **discussion** clearly misses a robust analysis of the results. For example, some drawbacks related to the use of the decay time equation are presented at the end of section 5 but we think that they should really go into the discussion and be more detailed.

Response:

We have shifted parts of the discussion in section 5 to the discussion section and expanded it. We hope that it is now clearer that we are presenting a conceptual model and not a method to comprehensively estimate the melt time of the Greenland Ice Sheet.

5. The whole paper talks about the surface-elevation feedback but in reality this is the **SMB-elevation feedback** (IPCC, 2013; Edwards et al., 2014, TC; Goelzer et al., 2013, J. Glaciology). Furthermore, the paper does not talk about the feedback of ice sheets in general but of the Greenland Ice Sheet in particular. Finally, the results of the paper focus less on the SMB-elevation feedback than on the melt time. Therefore, we suggest a different title: 'A simple equation for the melt time of the Greenland Ice Sheet'.

Response:

We changed the name of the feedback to "melt-elevation feedback" in response to both reviewers' comments. As we explained above, we disagree with the reviewer on the purpose of the study. Since we consider it to be a conceptual paper that is relevant for all ice sheets in general we would like to keep the title with the change in the feedback's name. We think this is

reflected in the fact that all but one of the six sections of the paper are general in nature and only one section deals specifically with Greenland as an example.

6. In **section 1 (Introduction)**, the first paragraph is very long and could be separated into two different paragraphs, one with the general Greenland ice loss context and the other one with the temperature threshold and the SMB-elevation feedback. In any case, the link with the last paragraph of section 1 is not really done. We would add a clear explanation about the SMB-elevation feedback and the importance of determining the melt time for Greenland.

Response:

Done.

Technical Corrections

P2, L9: 'has been loosing' instead of 'is been loosing.

Response: Done.

P2, L12: 'the' instead of 'The'.

Response: Done.

P2, L13: Rephrase 'the lower the ice surface reaches into the atmosphere' since this is not clear.

Response: Changed to: "the more ice is lost, the lower the ice surface and the warmer the surface air temperature which fosters further melting and ice loss."

P2, L14-15: The sentence 'The rate of ice loss is highly relevant for coastal protection worldwide' does not really fit here. It could go to the beginning or the end of the abstract.

Response: The sentence was moved to the beginning of the abstract.

P2, L16: Delete 'as it should be'.

Response: Done

P2, L16: Is the bit 'In order to contribute a little to the conceptual understanding' really needed? We would remove it.

Response: We have rephrased this part to make our intentions clearer:

"The computation of this rate so far relies on process-based numerical models which are the appropriate tools to capture the complexity of the problem. By contrast, we aim here at gaining conceptual understanding by deriving a purposefully simple equation for the self-enforcing feedback and use it to estimate the melt time for different levels of warming using three observable characteristics of the ice sheet itself and its surroundings."

P2, L18: We would cite the three observable 'characteristics', which we think are better defined as 'parameters'.

Response: We have defined the characteristics of the ice sheet that we use in the main text. We cannot see why the word “parameters” is better defined than “characteristics”.

P2, L20: ‘critically depends’ instead of ‘depends critically’.

Response: We have checked with a native speaker that holds a master degree in English literature and were assured that this is not a grammatical error, but a matter of choice. In accordance with our advisor’s opinion, we prefer our original version of the wording.

P2, L21: Use of ‘critical’ and ‘critically’ in the same sentence. Maybe replace ‘critically’ by ‘strongly’.

Response: Done.

P2, L21: ‘the’ instead of ‘The’.

Response: Done.

P2, L24: ‘meltdown’ instead of ‘melt down’.

Response: Done.

P3, L27: The first sentence is not totally accurate. Maybe: ‘Global sea level rise has been raising in the past decades mainly due to ocean thermal expansion and melting ice (Church et al., 2013).’ The last reference is more accurate than ‘IPCC (2013)’.

Response: Yes, thank you - we changed the sentence to “In past decades global mean sea level has been rising mainly by expansion of ocean waters and melting of ice on land (Church et al. 2013).”

P3, L28: ‘past two decades’ instead of ‘two past decades’.

Response: Done.

P3, L29-31: We think that some older references could be deleted and some newer studies could be added, e.g. Kjeldsen et al. (2015, Nature) who study the Greenland ice loss since 1900 using aerial imagery, Khan et al. (2015, Reports on Progress in Physics) who provide a review of Greenland Ice Sheet mass balance, Shepherd et al. (2012) who provide results from the Ice sheet Mass Balance Inter-comparison Exercise (IMBIE).

Response: We appreciate the reviewer’s suggestions and have happily added them to our reference list. We prefer not to delete any older references since these earlier studies (between 2011 and 2016) are still highly relevant as is the Shepherd et al. 2012 study suggested by the reviewer.

P3, L36: ‘Greve, 2000’ instead of ‘Greve, n.d.’.

Response: Done.

P3, L38: ‘critically depends’ instead of ‘depends critically’.

Response: See our comment above (P2, L20).

P3, L43: The authors need to agree whether they use ‘meltdown’ or ‘melt-down’ throughout the article (see also L24).

Response: Done.

P3, L45: We did not find that Howat et al. (2014) mention a sea level rise contribution from Greenland of 7 m. Maybe Gregory et al. (2004, Nature Brief Communications) is a more suitable reference. Please also check your references for Howat et al. (2014) because you list both the TC and TCD articles: is it really necessary?

Response: No, this was a mistake. Thanks for the hint.

P3, L50: 'surface mass balance (SMB)-elevation feedback' instead of 'surfaceelevation feedback'. Please check this for the whole paper (e.g. title of section 2).

Response: Since "surface mass balance-elevation feedback" seems very long, we would like to follow the second reviewer and denote it as "melt-elevation feedback". We hope this is alright with the editor and both reviewers. We have changed this through-out the manuscript.

P3, L51: 'one dimension' instead of 'zero dimension'.

Response: Done.

P3, L51: 'section 2' instead of 'section 1'.

Response: Done.

P3, L53: 'section 3' instead of 'section 2'.

Response: Done.

P3, L51: 'section 4' instead of 'section 3'.

Response: Done.

P3, L55: 'feed' instead of 'enter'.

Response: We would like to keep "enter" because the verb "feed" would suggest that the model is really a means to transform parameters into melt rates and that is (as we stated) not the purpose of the model.

P5, L61: '(e.g.' instead of 'e.g. ('.

Response: Done.

P5, L71: The authors already mention the Vialov profile above (L65), so there is no need to recall it.

Response: We would like to keep it here. It is just a half-sentence and it allows the reader to skip the introduction and still understand the following chapters.

P5, L73: Please define all quantities, i.e. h , x and L just after equation (1).

Response: Done.

P5, L73: h_m is more the surface elevation at the ice divide rather than the maximum surface elevation (Greve and Blatter, 2009).

Response: As can be seen from the equation it is also the maximum elevation of the ice sheet.

P5, L74-75: Rephrase 'we do not aim for a realistic representation of the ice flow', which is not 'politically correct'.

Response: Done.

P5, L77: 'constant and equal to surface accumulation' instead of 'homogeneous at a value, a' '.

Response: Done.

P5, L78: Define L after equation (1) instead of here.

Response: Done.

P5, L79: 'icebergs' instead of 'ice bergs'.

Response: Done.

P5, L83-85: What is the purpose of writing down equations (2) and (3)? Mean surface elevation is not used at all in the study. If the authors demonstrate their usefulness, what is the derivative in equation (3)?

Response: We consider it interesting to the reader to relate the maximum surface elevation to the mean surface elevation, both of which characterize the ice-sheet geometry in the Vialov profile. Since this is not a long derivation we would like to keep it. There is no derivative in equation (3), so we are confused as to what the reviewer refers to?

P6, L90: Precise which quantities you normalize.

Response: They are given in the same sentence and refer to the surface elevation, surface mass balance and the ice softness.

P6, L93: 'equilibrium-line altitude (ELA)' instead of 'equilibrium line'.

Response: Done.

P6, L108: We did not really understand how you 'rescaled' the SMB by A in equation (7). Don't we miss A in this equation, i.e. $hmA - \eta h - a_0 = 0$?

Response: We have added a sentence with an additional equation to explain the rescaling.

P8, L121: Is it really necessary to have an entire section only for 13 lines? Wouldn't it be more useful to merge it with section 2?

Response: We merged it with section 2.

P8, L122: 'ice sheet' instead of 'ice-sheet'.

Response: Done.

P8, L124: Rephrase. Maybe: 'conditions, i.e. being a solution of the governing equation (7) and a minimum of the function...'

Response: Done.

P8, L131: Equation (10) could be written more easily if starting by '(1-m)' instead of '-(m-1)'.

Response: Thanks for the advice. Since m is generally larger than 1 we prefer this form so that it can be seen immediately that a_{0c} and that the exponent is positive, i.e. the a_{0c} is an increasing and not decreasing function of Γ and γ .

P9, L136-137: The first sentence is not really necessary since it was done in the previous section.

Response: We reformulated in order to meet this comment by the reviewer as well as the earlier request to explain the role of the former sections 2 and 3 (which are now merged into section 2).

P9, first paragraph: Since you extensively compare your analysis to Ridley et al. (2010) and Robinson et al. (2012), maybe it would be useful to give us more insights about their methodology in the introduction (e.g. which models they use) and to try to provide an explanation for the differences between their models and the simple equation.

Response: Done.

P9, L145: Why did you choose a threshold of 1.6_C? Is it only based on Ridley et al. (2010)?

Response: Robinson et al (2012) provide a number for their threshold which is 1.6°C for the parameter setting that we used. Ridley et al. (2010) carried out fewer simulations and a value of 1.6°C is consistent with their results, but no precise threshold value can be derived from their publication. Since this part of the paper is merely an illustration of the possible application of the theory, we consider it best to use the Robinson value of 1.6°C and state this clearly and transparently. We have added a half-sentence to clarify.

P9, L145: What do you mean by ‘both models’?

Response: The Robinson et al. and the Ridley et al model. We hope this has become clearer by the addition of the half-sentence.

P9, L148: Define $_T$: is it $T-T_c$ with T being the temperature above the threshold?

Response: Yes. We have added the definition.

P9, L158: You previously defined the melt rate ($_a0$) as negatively related to melting sensitivity and warming level (see L147)? And now it is positive. What is right?

Response: Thank you very much for spotting this typo again. The negative value is correct and it has been corrected here.

P10, L170: ‘with time, which’ instead of ‘with time which’.

Response: Done.

P11, L188: ‘choose’ instead of ‘chose’.

Response: Done.

P11: The second and third paragraphs should be re-organized as they are a bit confusing: Figure 2 is explained only in the third paragraph but is already mentioned in the second paragraph.

Response: We would like to thank the reviewer for this advice. We have shifted the second paragraph down in the section in order to keep the flow of the text, i.e. first explain what is done

in order to obtain the simple model results including their uncertainty range and then add the complex model results. We hope the text is now clearer.

P11, L200: 'translate' instead of 'translates'.

Response: Done.

P11, L204: 'strongly depends' instead of 'depends strongly'.

Response: Done.

P11, L204: 'threshold' instead of 'thresholds'.

Response: Done.

P12, L211: Figure 4 is (almost) not discussed in the paper.

Response: We have expanded the discussion of Figure 4.

P12, L212-213: Rephrase, maybe: 'Since results obtained with equation (17) do not account for any dynamical discharge or even ice motion, they strongly deviate ...'.

Response: We would like to start the paragraph with a clear statement introducing the discussion. To this end we find the sentence "The simple equation provided here is clearly limited in its applicability." very clear and would like to keep it. The content is the same as suggested by the reviewer.

P12, L215: It is not really apparent in Figure 3 that results deviate more strongly with a higher ice loss. Rephrase or rescale the figures.

*Response: We agree with the reviewer that this formulation was misleading. We rephrased the sentence to say that it can be clearly seen that the **functional form** is not captured by our simple equation. We believe that this is indeed the case.*

P12, L221: Dynamic discharge is not as limited as suggested by different studies (see first specific comment).

Response: We have reformulated to say "Some studies suggest".

P13: Rewrite discussion taking into account all specific comments.

Response: We have changed the discussion in accordance with the changes made.

P13, L234: Rephrase 'For these curves in this figure'.

Response: Done.

P13, L238: Precise that you mean sea-level contribution from the Greenland Ice Sheet.

Response: Done.

P13, L241: 'dominant' instead of 'dominate'.

Response: Done.

P13, L245: 'multi-millennial' instead of 'mult-millennial'.

Response: Done.

P14, Tab. 1: Write down _T somewhere in the table.

Response: We do not understand the request.

P16, Fig. 2: 'median (...), likely (...) and very likely (...)' instead of 'median (...) and the likely (...) and very likely (...)'.
and very likely (...)'.

Response: Done.

P17, Fig. 3: To be consistent with Fig. 2, it would be better to get the time in years (and not kyears).

Response: We have changed the numbers in Fig. 3 to kyrs in order to be consistent.

P21, L338: Complete reference Greve (journal, year, volume, etc.).

Response: Done.

P22, L347: Do we need to have the Howat TCD article since the TC article is listed in L342?

Response: No, this was a mistake. Thanks for the hint.

Reviewer #2:

The authors derive a simple relationship for the elevation-melt feedback based on an analytical 1D flowline profile model of an ice sheet. The relationship allows calculation of the critical mean height that leads to melting of the ice sheet, given a prescribed rate of accumulation that changes with height and temperature anomaly. It also allows the decay time to be estimated, which compares well with more complex models for the Greenland ice sheet domain – serving as a validation of the approach. The method is described well and looks promising for distilling this feedback into a simple relationship.

I would therefore only suggest some minor changes before publication. The decay time equation is interesting and could certainly be useful for risk management planning. However, it seems that its validity is questionable for higher volume losses, ie, 50% or more. The authors discuss this briefly, but then the tables give decay times for both 50% and 100% volume loss as if it has equal weight to the 10% loss time estimates. I would recommend differentiating these results somehow, and emphasizing that this approach is more useful for diagnosing the earlier stages of decline.

In addition, I am missing the transition from height reduction to volume loss. The fraction α is introduced to represent the volume loss, but as I understand it, this is applied interchangeably with the mean height reduction. A justification of why $\text{mean}(H) = V$ is needed.

Finally, I realize the authors are interested in promoting this as a tool for risk assessment, but I think the manuscript would benefit more from discussion of the theoretical implications of the approach. What does the form of your equation mean in terms of the process(es) represented? Why does the rate of fastest melting saturate (Fig. 4) – what in the equations limits the time scale of melt? When could this be violated?

Response: We very much appreciate the reviewer's positive assessment and are grateful for the constructive comments. We are sorry that we missed to provide the translation from height to volume. In fact we just assumed a constant area by which the percentage change in ice thickness translates directly to the percentage change in ice volume. We have now added this to the paper in section 3 (end of first paragraph) together with another statement that the quantitative interpretation of the results is subject to a number of limitations and that the results are supposed to be taken conceptual in nature.

In fact we do not plan to use this study for any risk analysis and do not think that it should be used to this end. We have added a sentence concerning this issue to the abstract and are sorry if we gave the impression that the method can be used for risk assessments. Given the clear limitations, including the exclusion of dynamic effects and the strong assumption that the ice height directly translates into ice volume through a constant area, forbid this kind of quantitative interpretation. We hope this has now become clearer.

With respect to the distinction of 10%, 50% and 100% volume loss, we do not fully know how to handle this issue. At the moment we say that 10% is the most useful regime in which the method gives relatively good representations of the complex models. This has two reasons: for one it is enough volume change so that the melt-elevation feedback is indeed significant and secondly the time that elapsed during the melt was not too long for ice dynamic effects (at least in the complex models that we used for comparison) to become dominant. We say this in the

text. We would like to keep the table with all percentages because we believe that it is interesting to see how long a complete melting without ice dynamics and without horizontal distinction would take. But this is not crucial. If the reviewer or editor prefer to take out the part of the table, we will definitely do this. For the time being we have added a sentence to the caption of the table stating that the 10% values have some quantitative merit while the other numbers cannot be taken as valid estimates of real world ice loss time scales.

Minor comments

Title: As with the first reviewer, “Surface-elevation” feedback seems incorrect. Either “Temperature-elevation” or more likely “SMB-elevation” feedback makes more sense to me.

Response: We are happy to change the name of the feedback. In order not to make it too long, we would like to use the term melt- elevation feedback, if that is alright.

Page 2, line 9: is been losing => has been losing

Response: Done.

Page 2, line 16: Suggest deleting “as it should be” and “a little”.

Response: Done.

Page 3, line 46: The sentence starting with “The framework” seems to belong to a new paragraph. Furthermore, so far, no framework has been introduced so it seems out of place without a bit of introduction.

Response: The sentence was shifted to the end of the section so that the framework is mentioned prior to this statement.

Page 3, line 47: imclude => include

Response: Done.

Page 3, line 51: dimension => dimensions

Response: Done.

Page 6, line 98: Consider rephrasing “observed”. This is open to debate.

Response: We rephrased to “commonly chosen to be”.

Page 7, line 112: melt-down => melt. Also note that the lower branch (ie, the melted state) is also a stable state, as shown in Fig. 1. Consider rephrasing slightly.

Response: We agree with the reviewer that physically there is a third solution with $H=0$. Since this solution is not captured mathematically by the theory we believe that the current formulation is correct, isn't it?

Page 8, line 132: It seems that this point “ a_0 to decline linearly” should be mentioned earlier. This is in fact a pretty critical assumption to the whole approach, no?

Response: We fully agree and have mentioned this now on page 7 where Fig. 1 is first mentioned. In fact we could just change the x-axis of the figures to be a_0 , but we thought it

would be more illustrative to show the dependence on temperature change. In this sense there is no “loss of generality”, but we agree that it has to be made very clear and we hope that this has now become clearer.

Page 8, line 133: off-set => offset

Response: Done.

Page 12, line 225: can thus be used when => can thus be used if

Response: Done.

Page 13, line 241: dominate => dominant

Response: Done.

Page 13, line 243: the 015 Paris => the 2015 Paris

Response: Done.

Page 15, line 254: melt-down => melt

Response: Done.

Figure 4: Grid lines would help to be able to compare the panels. It would also be easier if they were presented in a vertical column, to emphasize the shift in time scale for higher temperatures.

Response: We agree and have changed the figure accordingly.

1 | **A simple equation for the surface-melt-elevation feedback of**
2 | **ice sheets**

3 | A. Levermann^{1,2,3*} & R. Winkelmann^{1,3}

4 | ¹*Potsdam Institute for Climate Impact Research, Potsdam, Germany.*

5 | ²*LDEO, Columbia University, NY, USA.*

6 | ³*Institute of Physics, Potsdam University, -Potsdam, Germany.*

7 | *Correspondence to: anders.levermann@pik-potsdam.de

8 **Abstract:**

9 In recent decades, the Greenland Ice Sheet ~~has~~ been losing mass and thereby contributed to
10 global sea-level rise. ~~The rate of ice loss is highly relevant for coastal protection worldwide.~~ The
11 ice loss is likely to increase under future warming. Beyond a critical temperature threshold, a
12 meltdown of the Greenland Ice Sheet is induced by the self-enforcing feedback between its
13 lowering surface elevation and its increasing surface mass loss: ~~t~~The more ice is lost, the lower
14 the ice surface ~~reaches into the atmosphere~~ and the warmer the ~~surrounding surface~~ air
15 ~~temperature becomes~~ which fosters further melting and ~~further~~ ice loss. ~~The rate of ice loss is~~
16 ~~highly relevant for coastal protection worldwide.~~ The computation of this rate so far relies on
17 complex numerical models ~~as it should be~~ which are the appropriate tools to capture the
18 complexity of the problem. ~~In order to contribute a little to the~~ By contrast we aim here at
19 gaining conceptual understanding ~~we by~~ deriving ~~here~~ a purposefully simple equation for the
20 self-enforcing feedback ~~which is then used~~ and use it to estimate the melt time for different levels
21 of warming using three observable characteristics of the ice sheet itself and its surroundings. The
22 analysis is purely conceptual in nature and is missing important processes like ice dynamics to be
23 useful for applications to sea-level rise on centennial time scales, but ~~When when~~ the volume
24 loss is dominated by the feedback, the resulting logarithmic equation unifies existing numerical
25 simulations and shows that the melt time depends ~~critically~~ strongly on the level of warming
26 with a critical slowing-down near the threshold: ~~t~~The median time to lose 10% of the present-day
27 ice volume varies between about 3500 years for a temperature level of 0.5°C above the threshold
28 and 500 years for 5°C. Unless future observations show a significantly higher melting sensitivity
29 than currently observed, a complete melt-down is unlikely within the next 2000 years without
30 significant ice-dynamical contributions.

1. Introduction

~~Anthropogenic climate warming~~In past decades global mean sea level has been rising mainly by ~~expanding expansion of~~ ocean waters and melting ~~of ice is raising global sea level on land~~ (IPCCChurch et al., 2013). Over the ~~past~~ two ~~past~~ decades, the Greenland Ice Sheet has lost mass at an accelerating pace (Bamber et al., 2000; Box et al., 2012; van den Broeke et al., 2009; Fettweis et al., 2013; Mernild et al., 2011; Nick et al., 2009; Rignot et al., 2008, 2011; Shepherd and Wingham, 2007; Thomas et al., 2011). The ice loss is likely to increase under unabated greenhouse-gas emissions (Clark et al., 2016; Fettweis et al., 2013; Goelzer et al., 2012; Graversen et al., 2011; Harper et al., 2012; Huybrechts et al., 2011; Levermann et al., 2013; Nowicki et al., 2013; Price et al., 2011).

Numerical simulations suggest that a decline of the Greenland Ice Sheet is inevitable once its surface temperature permanently exceeds a certain threshold (Charbit et al., 2008; Greve, 2000; Huybrechts and Wolde, 1999; Huybrechts et al., 2011; Ridley et al., 2005, 2010; Robinson et al., 2012; Solgaard and Langen, 2012). If and when this temperature threshold is passed, depends critically on past and future greenhouse-gas emissions (Fettweis et al., 2013; Goelzer et al., 2013; Gregory et al., 2004a; Rae et al., 2012). Even if emissions were reduced to zero, temperatures would not drop significantly for thousands of years because of the long life-time of anthropogenic CO₂ in the atmosphere and reduced oceanic heat uptake if oceanic convection is extenuated (Allen et al., 2009; Solomon et al., 2009; Zickfeld et al., 2013). This implies a possible commitment of a melt-down of the Greenland Ice Sheet in the near future which would eventually raise global sea-level by more than 7 meters (Gregory et al., 2004a) ~~(Howat et al., 2014b)~~. Whether this occurs on a multi-centennial or rather a multi-millennial time scale is of relevance for coastal planning. ~~The framework that we provide here can also be used to include new physical processes that might be discovered in the future, e.g. potential changes in surface albedo through melting (Box et al., 2012) or aerosol induced surface melt or the lack thereof (Polashenski et al., 2015).~~

~~Here~~In this article we first recap the Vialov profile and add a simple representation of the ~~surface elevation~~melt-elevation feedback towards a governing equation for a steady-state ice-sheet in ~~zero-one~~ dimensions ~~(Section 1)~~, then we derive the critical warming threshold for the existence of an ice sheet in this simple model (Section 2). In Section 3 we derive a simple time-

61 evolution equation for the decay of the ice sheet after surface temperatures have exceeded the
62 threshold. Finally we use observational estimates of the three ~~characteristics parameters~~ that
63 enter the model to estimate the decay time of the ice sheet under melting above the threshold
64 ([Section 4](#)). Here solid ice discharge is neglected as well as any other ice sheet dynamics
65 (Andresen et al., 2012; Howat and Eddy, 2012; Moon et al., 2012; Nick et al., 2009; Price et al.,
66 2011; Straneo et al., 2011; Walsh et al., 2012). The framework that we introduce here can be
67 used to include new physical processes that might be discovered in the future, e.g. potential
68 changes in surface albedo through melting (Box et al., 2012) or aerosol-induced surface melt or
69 the lack thereof (Polashenski et al., 2015).

2. Governing equation for shallow-ice steady states under ~~surface-elevation~~melt-elevation feedback

A nonlinear threshold behaviour is generally associated with a fundamental self-enforcing feedback and thereby an associated system memory—e.g. (Levermann et al., 2012). For the Greenland Ice Sheet, such a feedback is given by the interaction between surface elevation and surface melting (Weertman, 1961). For illustration, we include this feedback in a well-established highly idealized ice-profile of an ice-sheet in one dimension, the so-called Vialov-profile (Vialov, 1958). We introduce the ~~surface-elevation~~melt-elevation feedback in the simplest possible way by assuming that the surface melt rate depends linearly on the surface temperature and that the temperature decreases linearly with the height of the ice surface following a constant atmospheric lapse rate.

2.1 Governing equation

We consider a highly simplified flowline model for an isothermal ice sheet grounded on a flat and rigid bed. The solution of the shallow-ice approximation in one dimension for the ice-sheet elevation under these simplifying assumptions is ~~called~~ the Vialov-profile:

$$\tilde{h}(x) = h_m \left(1 - (x/L)^{(n+1)/n}\right)^{n/(2n+2)} \quad (1)$$

where h_m is the maximum surface elevation and n is Glen's flow law exponent (Glen, 1955). ~~x denotes the horizontal position and L the horizontal limit of the ice sheet.~~ The inherent assumption of isothermal ice is a strong simplification, ~~but we do not~~which needs to be kept in mind when interpreting the results. The aim of this derivation is purposefully not for a realistic comprehensive representation of the ice flow but ~~will to~~ derive a measure for the average height of the ice sheet and its dependence on changes in the surface mass balance. The surface mass balance is considered to be ~~homogeneous at a value~~spatially and temporally constant at a value, a , which will later be considered dependent on the surface elevation and thereby temporally variable. The overall horizontal extension of the ice sheet is set to L , and it is thereby assumed that any ice flow across this point is calved off into ice-bergs. This situation represents a confined ice-bearing bedrock topography as in most of Greenland's interior (Howat et al., 2014).

The mean surface elevation can then be computed to be

$$\bar{h} = L^{-1} \int_0^L dx h(x) = \omega \cdot h_m \quad (2)$$

It is proportional to the maximum surface elevation h_m with a proportionality factor

$$\omega \equiv \int_0^1 d\xi \left(1 - \xi^{(n+1)/n}\right)^{n/(2n+2)} \quad (3)$$

which only depends on the flow law exponent.

The maximum surface elevation is determined by the surface mass balance \tilde{a} and the ice softness

\tilde{A}

$$h_m = 2^{(n-1)/(2n+2)} \cdot L^{1/2} \cdot \left(\frac{(n+2)\tilde{a}}{(\rho g)^n \tilde{A}} \right)^{1/(2n+2)} \quad (4)$$

with ρ being the ice density and g the gravity constant. We normalize all three quantities by

defining $h \equiv \omega \cdot h_m / \bar{h}_0$, $a \equiv \tilde{a} / a_0$ and $A \equiv \tilde{A} / A_0$ where a_0 is the accumulation rate on the

ground, i.e., in the absence of an ice-sheet, and $A_0 = a_0 / \left((\rho g)^n (\varepsilon \cdot L)^{(n+1)} \right)$ with $\varepsilon = \bar{h}_0 / L$ being

the typical height-to-width ratio. \bar{h}_0 is the equilibrium-line altitude of the considered ice sheet in

the initial equilibrium situation. Values for a_0 , \bar{h}_0 and L are later chosen to resemble the

conditions of the Greenland Ice Sheet.

The non-dimensional surface elevation, h , of the ice sheet can then be expressed as

$$h = \left(\frac{a}{A} \right)^{1/m} \quad (5)$$

For the Vialov profile, $m=2(n+1)$ where ~~n denotes~~ the Glen flow-law exponent ~~observed-is~~ commonly chosen to be around $n=3$ which yields $m=8$.

We introduce the ~~surface-elevation~~melt-elevation feedback in its simplest form through a dependency of the surface melt rate on the surface elevation:

$$a = a_0 + \gamma \Gamma \cdot h \quad (6)$$

with the atmospheric lapse rate $\Gamma > 0$. γ denotes the melting sensitivity of the ice surface, i.e. the

increase in surface melt-rate per degree of warming, which is regularly measured and comprises

a large number of physical processes (e.g. (Box, 2013)). For simplicity we rescale the surface

mass balance by the constant ice softness parameter, A , ~~which is considered to be constant to~~

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123 | obtain $h = (a_0 + \gamma \Gamma \cdot h)^{1/m}$. The steady state solution for the surface elevation of the ice-sheet is
124 | thus governed by the following equation

$$125 \quad h^m - \gamma \Gamma \cdot h - a_0 = 0 \quad (7)$$

126 | which has two positive solutions for h as long as the surface mass balance on the ground is
127 | negative, i.e., $a_0 < 0$. Note that the surface mass balance can be positive even if $a_0 < 0$. If the
128 | ice-sheet is in an unstable configuration, a slight perturbation will either cause it to converge into
129 | the stable state with a positive surface mass balance or to melt ~~down~~ completely.

130 | Our simple approach qualitatively captures the basic hysteresis behavior of the Greenland Ice
131 | Sheet caused by the ~~surface-elevation~~melt-elevation feedback (Fig. 1, in which we have assumed
132 | the surface mass balance to depend linearly on temperature): For a given surface temperature, a
133 | stable state of the ice sheet (red line) annihilates an external perturbation in surface elevation by
134 | changes in surface mass balance (grey arrows). The unstable solution branch defines the basin of
135 | attraction for the stable state. A surface elevation that is lower than the unstable solution branch
136 | cannot be sustained. In that case the melting reduces the surface elevation to practically zero
137 | even without further external perturbation (grey arrows). Beyond a certain surface temperature
138 | threshold (vertical dotted line) no ice sheet can be sustained.

2.2. Critical surface mass balance in steady state

As illustrated in Fig. 1, there is a critical temperature above which the ice-sheet is not sustainable. Let us denote the corresponding surface elevation by h_c . The critical point (T_c, h_c) has to fulfill two conditions. ~~First, i.e. it has to be~~ a solution of the governing Equation 7 and ~~second it has to be a~~ minimum of the function

$$F(h) = h^m - \gamma \Gamma \cdot h - a_0 \quad (8)$$

which we can determine by setting the derivative of F to zero.

Consequently,

$$h_c = \left(\frac{\Gamma \cdot \gamma}{m} \right)^{1/(m-1)} \quad (9).$$

Inserting this into the governing equation yields the critical surface mass balance at the ground

$$a_{0c} = -(m-1) \cdot \left(\frac{\Gamma \cdot \gamma}{m} \right)^{m/(m-1)} \quad (10).$$

For illustrative purposes we have assumed a_0 to decline linearly with the surrounding temperature and plotted the solution of Equation 7 against that temperature with an arbitrary off-set in Fig. 1.

3. A simple temporal equation for the ~~surface-elevation~~melt-elevation feedback

~~Based on the governing equation, we can~~Once the derive the critical surface-mass balance and surface elevation threshold (as derived in the previous section 2) is transgressed, below which a meltdown of the ice-sheet is inevitable in our conceptual model. Let us define the time τ_α as the time it takes to melt a fraction α of the initial ice volume and the threshold temperature T_c as the temperature above the pre-industrial level at which the surface mass balance becomes negative. Robinson et al. (2012) find a range of 0.8 – 3.2°C for the threshold warming beyond which no ice sheet can be sustained on Greenland. Their best estimate for the threshold is 1.6°C above pre-industrial level. The study uses a regional climate model of intermediate complexity (Robinson et al., 2009) coupled to the SICOPOLIS ice sheet model (Greve, 1997). Using a different model combination, Ridley et al. (2010) find that in their model the ice sheet cannot be sustained for a warming of 2°C. They combine the HadCM3 Atmosphere-Ocean-General Circulation Model (Gordon et al., 2000) with an atmospheric resolution of 2.5° x 3.75° (Pope et al., 2000) to an ice sheet model of 20 km horizontal resolution (Huybrechts and Wolde, 1999).

Some studies assume that the threshold is associated with a mean negative surface mass balance (Gregory et al., 2004b; Ridley et al., 2005; Toniazzo et al., 2004). In Fig. 2 we use 1.6°C as a threshold value for both models because this value is given by Robinson et al. (2012) and consistent with Ridley et al. (2010) and thus a simple and transparent choice. This number can be easily adjusted if new estimates are obtained. For the translation from percentage ice thickness change to percentage ice volume change a constant horizontal ice surface area was assumed which renders the analysis conceptual in nature. Thus the quantitative interpretation of the melt times are subject to this additional simplification.

For a fixed anomalous melt rate $\Delta a_0 = -\gamma \cdot \Delta T$ in response to an anomalous temperature increase $\Delta T = T - T_c$ above this threshold temperature T_c , the decay time without any feedbacks would be

$$\tau_0 = -\frac{h_0}{\Delta a_0} = \frac{h_0}{\gamma \cdot \Delta T} \quad (11)$$

178 Since the surface temperature increases with decreasing elevation, this zero-order estimate for
 179 the decay time is higher than the actual value. As a first-order correction to the situation of fixed
 180 melting, let us assume that the anomalous surface mass balance behaves as

$$181 \quad \Delta a = \Delta a_0 + \frac{1}{\tau_\gamma} \cdot (h - h_0) \quad (12)$$

182 where $\tau_\gamma = 1/(\gamma \cdot \Gamma)$.

183 From the relation $dh/dt = \Delta a$, we then obtain

$$184 \quad \frac{d\Delta h}{dt} = -\Delta a_0 + \frac{\Delta h}{\tau_\gamma}, \quad (13)$$

185 if $\Delta h \equiv h_0 - h$ is defined as the reduction in height. For a time-dependent melting induced by
 186 surface warming $\Delta a_0 = -\gamma \cdot \Delta T$ the general solution of Equation 13 is

$$187 \quad \Delta h(t) = \gamma \cdot \int_0^t dt' \Delta T(t') \cdot e^{(t-t')/\tau_\gamma} \quad (14)$$

188 This equation corresponds to a linear response theory with the melting $-\gamma \cdot \Delta T$ as forcing and an
 189 exponential response function

$$190 \quad R(t') = e^{t'/\tau_\gamma} \quad (15)$$

191 Linear response theory states that the convolution of Equation 14 yields the linear response of
 192 the system (Good et al., 2011; Winkelmann and Levermann, 2013). Note that generally linear
 193 response theory is used as an approximation of a non-linear system to relatively weak forcing. In
 194 these circumstances the response function has to decline with time because it represents the
 195 history of the system's response to past perturbation. For example, if the response function was a
 196 declining exponential $(t') = e^{-t'}$ ~~that, this~~ would mean that the effect of forcing that occurred
 197 in the past, i.e. prior to the time t that is considered, becomes exponentially less relevant for the
 198 current system response. Here, however, the response function is *increasing* with time, which
 199 means that the past deviation from the steady state is amplified as expected near an unstable
 200 fixed point, which is exactly what an unstable situation should do. The exponent $1/\tau_\gamma$ can be
 201 considered the Lyapunov exponent of the system.

202 Given the boundary condition $\Delta h(t=0)=0$, for a constant temperature increase ΔT , Equation 14
 203 becomes

$$\Delta h(t) = h_0 \cdot \left(\frac{\tau_Y}{\tau_0} - \frac{\tau_Y}{\tau_0} \cdot e^{t/\tau_Y} \right) - \frac{h_0}{\tau_0} - \frac{h_0}{\tau_Y} \quad (16).$$

The decay time for a relative volume reduction of α is then given by:

$$\tau_\alpha = \frac{1}{\gamma\Gamma} \cdot \log \left(1 + \alpha \cdot \frac{\Gamma \cdot h_0}{\Delta T} \right) \quad (17),$$

where \log denotes the natural logarithm. Equation 17 is denoted the *decay-time equation* hereafter.

3. Estimating the Melt Time of the Greenland Ice Sheet from Observables

In this simplified approach, the collapse time is thus a function of three observable quantities: the equilibrium-line altitude, h_0 , the atmospheric lapse rate, Γ , and the melting sensitivity to temperature, γ . The average equilibrium-line altitude of the Greenland Ice Sheet is at about 1150 meters (Box & Steffen 2001)). The observed range for the atmospheric lapse rate is estimated to be between 5 ± 2 °C/km (Fausto et al. 2009; Gardner & Sharp 2009), and current estimates for the melting sensitivity scatter around 4.4 ± 2 cm/year/°C (Box 2013). In order to obtain an estimate of the decay time and the uncertainty around this estimate we use Equation 17 and choose the lapse rate and melting sensitivity uniformly randomly from these observed intervals (Tab. 1, Figs. 2 – 4).

~~Existing numerical simulations for a decay of the Greenland Ice Sheet (Ridley et al. 2010; Robinson et al. 2012) differ in their trajectories for the total ice volume, but exhibit a characteristic functional form when the relative ice volume is expressed as a function of the temperature anomaly above the critical temperature threshold (Fig. 2). This characteristic relation is captured by our first order equation for the decay time, embedding the results from process based models into a simple analytical framework. This approach provides a good approximation if, on the one hand, the volume loss is significantly large for the surface elevation feedback to become relevant and, on the other hand, the melting is dominating the ice loss compared to the dynamic ice discharge.~~

Following the decay-time Equation 17, the observational constraints for the atmospheric lapse rate, Γ , and the melting sensitivity, γ , translates into an uncertainty range for the melt time of the Greenland Ice Sheet, assuming uniform probability distributions for both Γ and γ within the above intervals. Fig. 2 shows the histograms of the time until 10% of its present-day ice volume (corresponding to 0.7 m global sea-level rise) are melted for different warming scenarios. The melt time strongly depends ~~strongly~~ on the level of warming beyond the temperature threshold: The median estimate varies from more than 2000 years for a warming of +1°C to less than 500 years for a warming of +5°C.

~~Since the melt time is a monotonically decreasing function of both the lapse rate and the melting sensitivity, the upper and lower limits of the estimates can be directly computed from the~~

~~observed uncertainty interval of these quantities. However, the functional form of Equation 17 introduces a specific structure into the histogram of the melt time which is highly skewed towards the low end (Tab. 1 and Fig. 4).~~chosen

Existing numerical simulations of a decay of the Greenland Ice Sheet (Ridley et al. 2010; Robinson et al. 2012) differ in their trajectories for the total ice volume, but exhibit a characteristic functional form when the relative ice volume is expressed as a function of the temperature anomaly above the critical temperature threshold (Fig. 2). This characteristic relation is captured by our first-order equation for the decay time, embedding the results from process-based models into a simple analytical framework. This approach provides a good approximation if, on the one hand, the volume loss is significantly large for the melt-elevation feedback to become relevant and, on the other hand, the melting is dominating the ice loss compared to the dynamic ice discharge.

~~The simple Equation provided here is clearly limited in its applicability. Since the simple equation provided here# does not account for any dynamic discharge or even ice motion, the results from Equation 17 strongly deviate from numerical simulations when the ice has time to adjust dynamically to the volume loss. This can be seen for a stronger ice loss of 50% of the initial volume where the functional dependence between the decay time and the temperature anomaly clearly follows a different functional form than predicted by Equation 17 (Fig. 3). Also the role of the ice material properties is comprised into one parameter, the melting sensitivity of the ice to a temperature increase at the surface. This sensitivity will in general vary not only with time but also spatially and due to the melting itself. Similarly the feedback role of the surrounding climate is represented by only one parameter, the atmospheric lapse rate which will again vary spatially but also with time as the ice surface declines.~~

~~(Graversen et al., 2010; Price et al., 2011)The dynamic discharge from Greenland is strongly limited by the ice sheet's bottom topography, for which estimates yield an upper bound of approximately 5-13 cm during the next century (Graversen et al., 2010; Price et al., 2011). Over a period during which the ice loss is dominated by the feedback and the ice dynamic effect is limited, our approach provides a quantitative estimate of the melt time based on observable~~

quantities. Equation 17 can thus be used when new observations suggest an altered melting sensitivity or changes in the atmospheric response to Greenland ice loss.

Since the melt time is a monotonically decreasing function of both the lapse rate and the melting sensitivity, the upper and lower limits of the estimates can be directly computed from the observed uncertainty interval of these quantities. However, the functional form of Equation 17 introduces a specific structure into the histogram of the melt time which is highly skewed towards the low end (Tab. 1 and Fig. 4). For increasing warming levels the histogram is shifting towards lower decay times. At the same time the histogram narrows and higher decay times become less frequent within the chosen parameter range (see description above).

4. Discussion and conclusion

Our estimate for the decay time captures the characteristic slow-down near the critical threshold as can be seen from the divergence of the decay time, τ_a , in the limit of vanishing warming above the threshold (Equation 17). The simple equation of the decay time quantitatively reproduces the range given by simulations with process-based models. the relative speed-up of ice loss due to the melt-elevation feedback~~The feedback becomes more dominant near the threshold compared to larger temperature increase for which the external climatic forcing is more relevant~~ (Fig. 5). ~~For these curves in this figure we used~~ is estimated, using the central values of the ~~parameters~~parameter ranges, i.e. equilibrium-line altitude $h_0=1150\text{m}$, atmospheric lapse rate $\Gamma=5^\circ\text{C/km}$ and melting sensitivity $\gamma=4.4\text{ cm/year/}^\circ\text{C}$. The feedback becomes more dominant near the threshold compared to larger temperature increase for which the external climatic forcing is more relevant.

The simple equation provided here is clearly limited in its applicability. The role of the ice material properties is comprised into one parameter, the melting sensitivity of the ice to a temperature increase at the surface. This sensitivity will in general vary not only with time but also spatially and due to the melting itself. Similarly, the feedback role of the surrounding climate is represented by only one parameter, the atmospheric lapse rate which will again vary spatially but also with time as the ice surface declines.

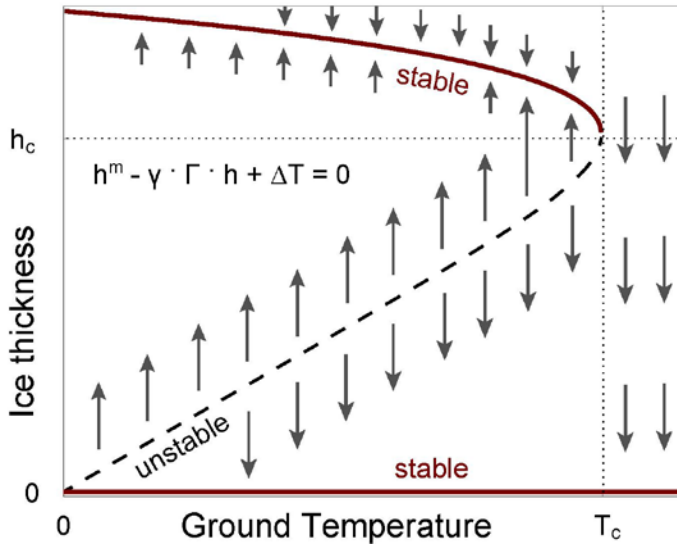
Ice dynamics are deliberately excluded in our simple conceptual approach in order to separate and characterize the melt-elevation feedback. In reality, ice dynamics of course play an important role in the ice-sheet mass balance: Radar (ERS-2) and laser (ICESat) altimetry observations show that mass changes in Greenland were dominated by changes in the surface mass balance (SMB) between 1995 and 2001, and both SMB and dynamics contributed equally to mass loss from the Greenland Ice Sheet between 2001 to 2009 (Hurkmans et al., 2014). (Fürst et al., 2015) estimate that 40% of the recent loss (2000-2010) is due to an increase in ice dynamic discharge, 60% due to changes in the surface mass balance. Their results suggest that the future volume loss from the Greenland Ice Sheet might be predominantly caused by surface melting and dynamic discharge is limited by margin thinning and retreat.

Some studies suggest (Graversen et al., 2010; Price et al., 2011) that the dynamic discharge from Greenland is strongly limited by the ice sheet's bottom topography, for which estimates yield an upper bound of approximately 5-13 cm during the next century. Over a period during which the ice loss is dominated by the feedback and the ice-dynamic effect is limited, our approach provides a quantitative estimate of the melt time based on observable quantities. Equation 17 can thus be used if new observations suggest an altered melting sensitivity or changes in the atmospheric response to Greenland ice loss.

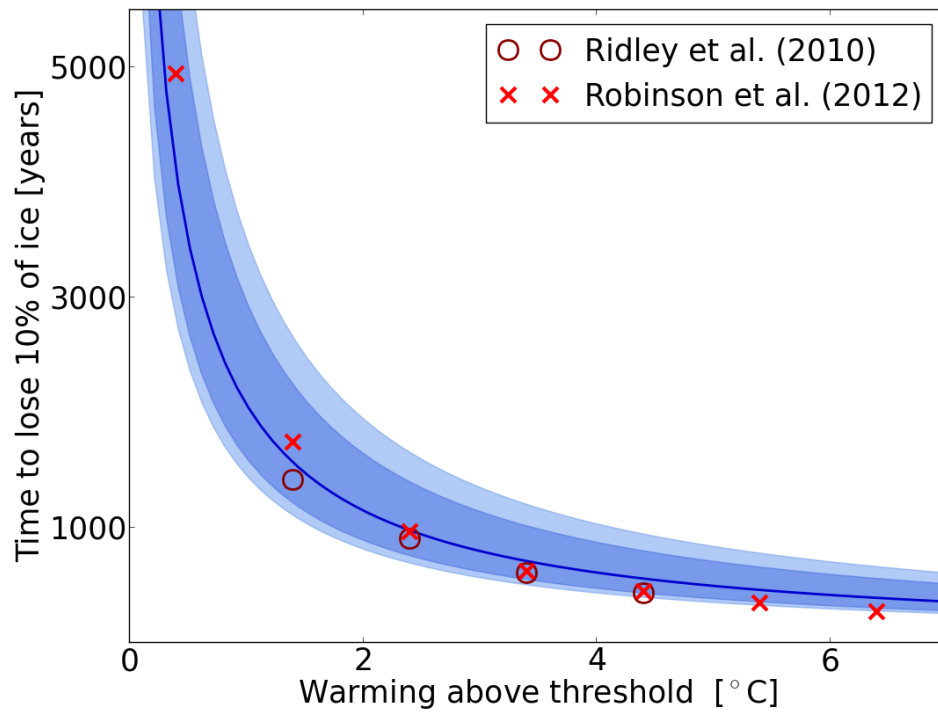
For a temperature increase of 5°C, which could be reached within this century (IPCC, 2013), the median rate of sea-level contribution is about 1.4 mm per year which is about four times that of its current contribution of about 0.4 mm per year (Rignot et al., 2011; Shepherd et al., 2012). Even for extremely high temperatures however, the Greenland Ice Sheet cannot melt infinitely fast – our results show that a complete disintegration within the next two millennia is highly unlikely unless ice dynamics effects become dominant~~te~~ or the melting sensitivity is significantly higher than currently observed. For a global mean temperature increase below two degrees, as agreed upon during the ~~1015–2015~~ Paris UNFCCC climate summit, the threshold temperature would only be exceeded mildly and the decay time of the Greenland ice sheet would be multi-millennial.

Volume loss		0.5°C	1°C	2°C	3°C	4°C	5°C
10%	Lower	2140 yr	1320 yr	760 yr	530 yr	410 yr	330 yr
	Median	3430 yr	2040 yr	1140 yr	790 yr	610 yr	500 yr
	Upper	7290 yr	4120 yr	2210 yr	1520 yr	1150 yr	930 yr
50%	Lower	4920 yr	3600 yr	2460 yr	1900 yr	1550 yr	1320 yr
	Median	8740 yr	6170 yr	4040 yr	3040 yr	2450 yr	2090 yr
	Upper	20740 yr	13920 yr	8640 yr	6310 yr	4980 yr	4120 yr
100%	Lower	6340 yr	4920 yr	3600 yr	2910 yr	2460 yr	2140 yr
	Median	11610 yr	8730 yr	6160 yr	4840 yr	4020 yr	3500 yr
	Upper	28710 yr	20740 yr	13920 yr	10630 yr	8640 yr	7290 yr

Table 1: Decay time. Time period after which different percentages of volume loss have occurred at different warming levels. Provided are the median values of the distributions from Figures 2 and 3 together with the lower and upper limit that are derived respectively from the upper and lower limits of the uncertainty range of the observed melting sensitivity and atmospheric lapse rate. The simple decay time equation (Equation 17) does not take any ice dynamic effects into account and its translation to ice volume assumes a constant horizontal ice-sheet area. Thus the values provided here best fit the complex model simulations only when these assumptions are reasonably well justified which is most likely not the case for high ice loss such as 50% or 100% of the original ice volume.



331
 332 **Figure 1: Ice-sheet hysteresis.** If the ice-sheet is in an unstable configuration (dashed black
 333 branch), a slight perturbation will either cause it to converge into the stable state (upper red
 334 branch) or to melt ~~down~~ completely. For a given temperature, the dotted line gives the critical
 335 surface elevation (Section 3). If the surface elevation is lower than h_c , a complete meltdown of
 336 the ice sheet is inevitable. Once the temperature threshold, T_c , is crossed, the time for a collapse
 337 of a certain fraction of the ice-sheet can be estimated via Equation 17.



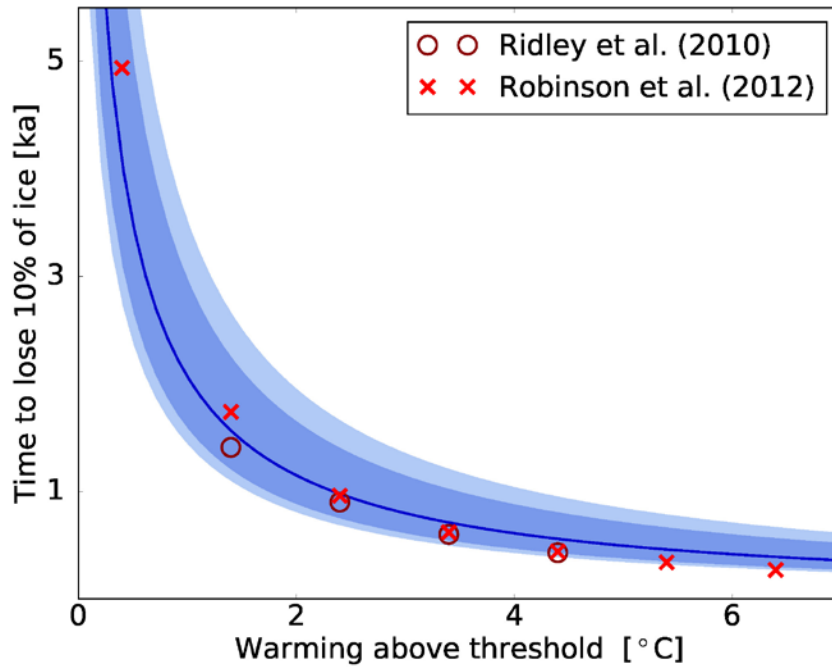
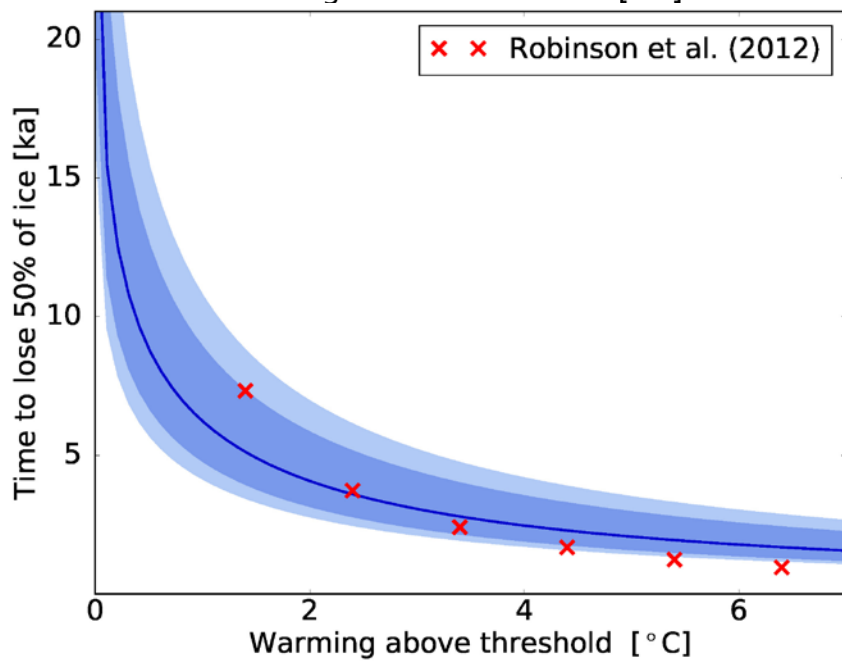
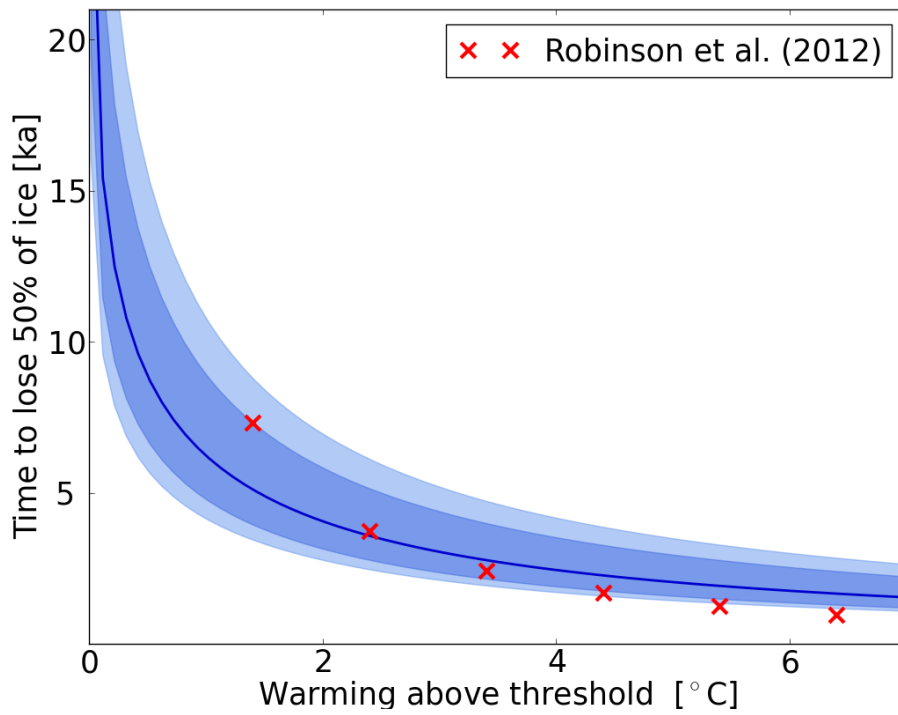
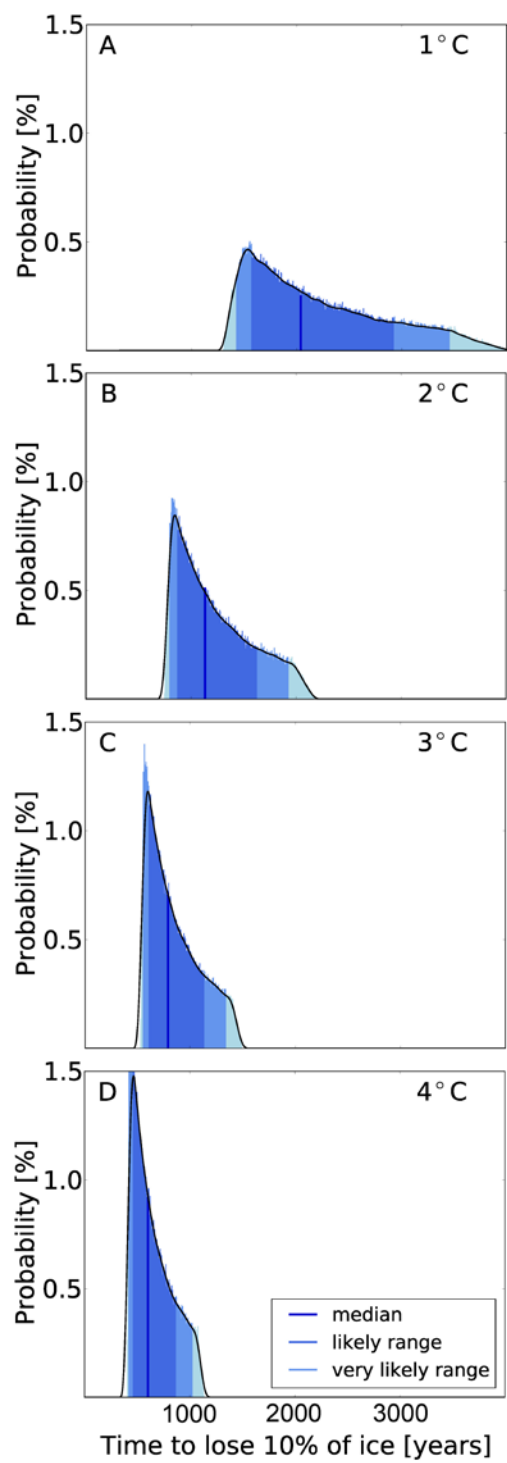
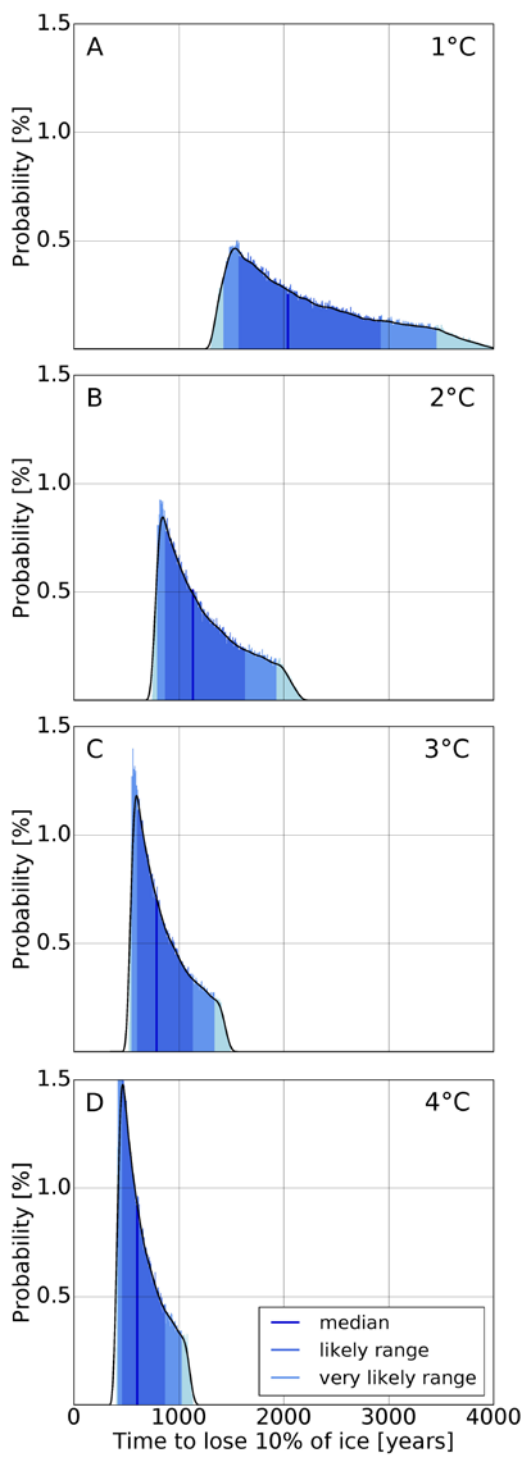


Figure 2. Decay-time of the Greenland Ice Sheet. The decay time depends critically on the level of warming above the temperature threshold. Shown are the median (black line), ~~and the~~ likely (18% to 83% quantiles, dark blue shading) and very likely (5% to 95% quantiles, light blue shading) ranges for the time to melt 10% of the present-day ice volume, estimated via Equation 17. The red circles and crosses indicate the results from process-based model simulations by Ridley et al. (2010) and Robinson et al. (2012), respectively.

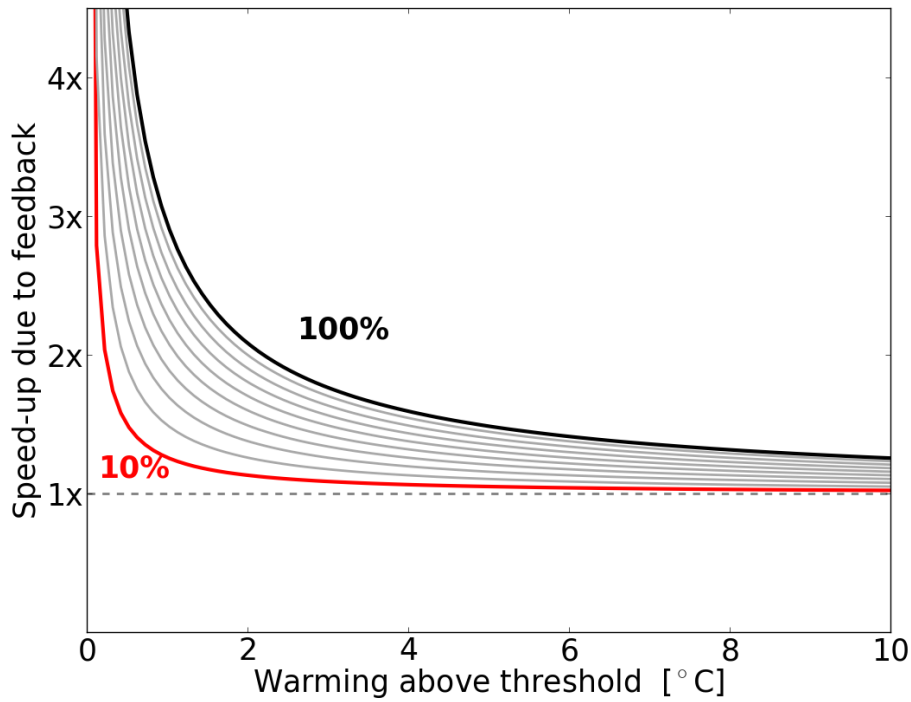


348 **Figure 3: Time until 50% of the Greenland Ice Sheet are melted.** Shown are the median (black
349 line) and the likely (18% to 83% percentiles, dark blue shading) and very likely (5% to 95%
350 percentiles, light blue shading) ranges for the time to melt 50% of the present-day ice volume,
351 estimated via the equation for the decay time τ_a . The red crosses indicate the results from
352 process-based model simulations by Robinson et al. (2012).



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Figure 4. Likelihood for 10%-decay of Greenland Ice Sheet. Shown are the probabilities for the ice-sheet to lose 10% of its initial ice volume in a certain time period for surface warming of +1°C (A), +2°C (B), +3°C (C) and +4°C (D) above the threshold. The median is indicated by the black line, and the likely and very likely ranges are shaded in dark and light blue, respectively.



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 370 **Figure 5: Role of ~~surface-elevation-melt-elevation~~ feedback in melting of Greenland ice sheet**
 371 **declines with increasing temperature.** Shown is the ratio of melting time with ~~surface-~~
 372 ~~elevation-melt-elevation~~ feedback over melting time without the feedback τ_a/τ_0 . Each line
 373 represents the ratio for a loss of different percent of the initial ice volume. The red line shows the
 374 ratio of the decay time with feedback over the decay time without feedback for a 10% ice loss
 375 (corresponding to Figures 2 and 4). The influence of the feedback becomes less dominant with
 376 stronger warming above the critical threshold (x-axis). Near the threshold the melting time
 377 without feedback diverges stronger ($1/\Delta T$) than the melt time with feedback which declines
 378 logarithmically.

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