

Reply to reviewers comments

Summary of changes.

- 1) I have changed the organisation of the paper:
 - a. I now first describe the heat production by frictional heating, and the problem of the growth and maintenance of a temperate layer (new Section 2 and 3). This takes care of the awkward 'intermezzo', highlighted by the Anonymous Reviewer.
 - b. Then I deal first with the creep component, which is likely to be more important than pressure melting, as commented upon by D Cohen.
- 2) The section of creep (new section 4) has been completely rewritten, to take care of the comments of M Montagnat, and some comments of the Anonymous Reviewer.
- 3) I've added a short section on the critical obstacle size, that now stresses the importance of creep over that of pressure melting (new Section 8 + new Figure 6), which is indeed important as indicated by D Cohen.
- 4) I've also rewritten the Introduction, which now focusses now more on the general issue of temperate ice (e.g. comments by the Anonymous reviewer), and less of a 'criticism' of Weertman sliding (e.g. comments by D Cohen), which was not meant to be the case.

Note 1; My replies on the comments below are in red

Note 2: in the tracked changes below, the changes in the Reference list have NOT been tracked.

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Detailed reply to comments by D Cohen.

It was not my intention at all to deconstruct or criticise Weertman's original model ; all I wanted to do is to point out it is not applicable to temperate ice with the original assumptions, which, unfortunately, are at times still perpetuated in probably inappropriate conditions. I have reworded and changed the text to be more respectful to Weertman's original model, which of course was pioneering, and has become classic.

I agree that, for larger-scale obstacles, creep is more important than pressure melting against the obstacles. I've changed the order of the manuscript to reflect this, and added a section on Critical Obstacle size to discuss this, and how this differs from the original Weertman model.

Page 1 last line: Probably "near steady-state situations" should be replaced with something like "near-instantaneous situations". **MK: done.**

Page 2 Line 10. I think the author meant to refer to Cohen et al 2005 and not Cohen et al 2006. **MK: Reference to Cohen et al. 2005: corrected.**

Page 2 Line 24. I would insert the word "either" before "pose problems" **MK: Corrected**

Page 3 Line 20. I think the mechanisms of sliding are clear: ice at the melting temperature contains water and water between ice and bedrock forms a thin lubricating layer with near zero shear resistance that allows ice in contact with the bedrock to have non-zero velocities. The question is how to quantify sliding and what glaciological parameters control it. **MK:** Reworded. The actual sliding (over a flat area) is fairly clear, the basal motion over a rough surface is not.

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Page 4 Line 20. Equation 4. There is an typo/error. It should be: $V_{pm} = Q_{obs}/(H_{ice} * \rho_{ice})$. **MK:** Oops, indeed, corrected.

Page 4 Line 24. Strictly speaking in regelation ice does not flow around the obstacle. That's the viscous part of motion. In regelation ice melts on one side, the water flows to the other side and refreezes there. May be change wording. **MK:** Reworded.

Page 5 Line 2. The vertical stress could even be higher than the effective pressure since, due to melting, there is a component of ice flow towards the bed that creates a vertical downward force on the debris. This force could be significant and further increase basal friction".

MK: That is strictly true, but that concerns the very localised contact stress of the debris onto the bed, according to the 'Hallet model;'. On this contact the friction coefficient would also be much higher, probably in the order of $\mu = 0.5$ or so. Since I use the friction coefficient averaged over a large area, I have also used the vertical stress over a large area, rather than looking at the very localised stresses. See longer explanation in the reply to Anonymous Reviewer.

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Page 6 Line 11 (ii). Strictly speaking this is not true. There will be differences in temperature in temperate ice due to differences in stresses (if only with depth). These temperature differences will cause thermal gradients and heat fluxes (arguably small). These gradients will only serve to melt ice or freeze water. See Lliboutry 1993."

MK: This is strictly true, but here I'm referring here to a bulk thermal gradient, capable of transporting significant heat from the base to the CTB. This bulk thermal gradient is zero (or rather it is slightly negative, due to the increase in cryostatic pressure). I've reworded this, to make it clear.

Page 7 Line 10. The use of the words 'cold patch' is confusing. The ice is at the melting temperature so it's not cold. I think the term cold patch should be restricted to cold ice not ice at the melting temperature that is colder because under a higher pressure. See also Figure 1. **MK:** Cold patch: good point: changed to 'cool/warm spot', also on Figure.

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Detailed reply to comments by M Montagnat.

M Montagnat makes some very astute points, and as a result I've rewritten the section on creep. I have discussed more the role of different deformation mechanisms, including enhanced dislocation creep, dynamic recrystallisation and grain boundary sliding. I've stressed now that grain boundary melting is not a 'new' mechanism, but has in fact been observed before. The behaviour of temperate ice is NOT very similar to other geological materials and metals at high temperatures. No other such material (with the apparent exception of plutonium!) shows the near-unique behaviour of H₂O, namely that the liquid phase is denser than the solid phase. This fundamentally different phase-transition behaviour is likely to lead to a fundamentally different deformation behaviour. In other words, the whole concept of pressure melting, namely that higher pressure leads to a lowering of the melting temperature only works for H₂O. Both the dramatic and sudden increase in strain rate close to the melting temperature, and the (albeit limited) evidence for near-Newtonian ($n \sim 1$) behaviour in temperate ice do suggest that a) a power-law does not describe the behaviour (both in terms of the breakdown of the Arrhenius relation, as well as evidence for $n \sim 1$ behaviour – this is now described better; b) there is a switch in rate-controlling deformation mechanism(s), rather than 'merely' a variation of dislocation creep.

- p 9, 2d paragraph: I don't think that results on temperate ice show that power-law creep does not adequately describe ice creep above -0.2°C. With increase in temperature, the activation energy can change, and therefore, we do not expect a linear relation between strain rate and temperature. To get rid of a power-law creep relation, one needs to plot minimum strain-rate as a function of stress... and show that this is not linear. Which is not provided in Morgan 1991. Results by De La Chapelle et al tend to show that power-law creep remains even when a liquid phase exist at GB, which is mostly what happens when ice is temperate (Wilson et al. 1996)?

MK: I've now described better that a) the Arrhenius relationship does not work **and** b) that the stress dependency changes. Note the La Chapelle experiments are not truly in temperate ice, but in an cold ice/brine mixture.

- p9, last paragraph: to my point of view, there are not enough information to be able to suggest a mechanism such as the one suggested here (GBPS), that has never been observed in ice, and that appears more than unlikely regarding knowledge about ice deformation behavior, with or without liquid layers... A discussion would nevertheless be required concerning the possibility of grain boundary sliding at such high temperatures. **MK:** grain boundary melting has been described before – this is now discussed better. The possibility of grain boundary sliding is now discussed, as well as the enhancement of dislocation creep.

- p10, 1st paragraph: As far as I know, pressure solution requires different phases to be present, in order for some to be dissolved under local pressure, and migrate in some fractures together with the fluid phase, and re-precipitate further away (see J-P. Gratier, D. K. Dysthe and F. Renard. The role of pressure solution creep in the ductility of the Earth's upper crust.

Advances in Geophysics, vol. 54, 2013). I do not see at all how this can occur in the temperate ice layer at the bottom of glaciers and ice sheets...

MK: The analogy lies in the fact that solid changes to a liquid and vice versa, and that this leads to strain. The analogy with pressure melting (and the difference) is now described better.

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- p11, 1st paragraph: Once again, there are not enough proof or information to assess so directly the occurrence of some grain boundary pressure melting... Power-law creep can also be fast, if accommodation mechanisms are efficient (dynamic recrystallization, GBS, liquid intergranular phase)... so it can not be ruled out so easily.

MK: completely right, which is why I've rewritten the section and discussed all these deformation mechanisms. Grain boundary sliding is, however, unlikely, due to very large grain sizes, as now discussed.

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Detailed reply to comments by Anonymous Reviewer

This paper, however, has serious deficiencies:

15 1) Most importantly, it contains little new analysis or data that help shed light on sliding physics. For example, the calculation of p. 4 yields conclusions that could have been reached without the calculation (see below, comments on p. 4, 1-21) and is used inappropriately to assert that the Weertman model is "illogical" (p. 4, 23-26).

MK: I beg to differ. It contains an important element of consideration concerning frictional heat production, which is widely ignored; it reviews what little is known about temperate ice, and questions, with good reason, the use of Glen's flow law for such ice; it proposes a different way of heat transfer for pressure melting, and hence suggests that ice sheet modelling need to take into account 3 rather than 2 thermo-mechanical modes of sliding. As the reviewer does not state that this has been done before, these analyses (conceptual and qualitative as they are) are new.

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and is used inappropriately to assert that the Weertman model is "illogical" (p. 4, 23-26).

MK: Fair point, I've rephrased this.

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2) Misconceptions/errors indicate a muddled understanding of relevant physics related to sliding (e.g., p. 7, 5-7; p. 7, 8-11; p. 8, 26; p. 10, 26-27; p. 11, 19) and ice rheology (e.g., p. 9, 8-15).

MK: some poor phrasing on my part, but I reject the term 'muddled understanding of relevant physics':

- p. 7, 5-7; p. 7, 8-11. No, this is entirely logical. The question is whether water flow is sufficient – and that needs observations to confirm – see below.
- p. 8, 26. Merely poor phrasing on my part. Have change to: "albeit enhanced by stress concentration around the obstacle".
- p. 10, 26-27; Merely poor phrasing on my part, see below. :
- p. 11, 19: Can't think of what is muddled here, unless the reviewer refers to the negligible (negligible!!!) contribution of cold-based sliding – see further below. The remainder is mainstream, accepted glaciology.

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- p. 9, 8-15 I have changed this section, as per comments to M Montagnat

The attempt to assess the extent to which temperate ice obeys a power-law flow rule by considering the dependence of strain rate on temperature, rather than on stress, is a particularly major error.

- 5 **MK:** Not it is not. The reviewer needs to realise that the Arrhenius relation is part and parcel of a power law in most if not all materials, and certainly ice (Glen 1955). Nevertheless, I've probably phrased this too strongly too early on, but the reviewer also misses the point that later the stress dependency is dealt with, with good evidence in some circumstances for $n \sim 1$. I've now restated that the Arrhenius temperature relationship does not work anymore, but also provided more examples of experiments/observations that show that $n < 3$, i.e. a departure from the 'normally assumed' $n=3$. The main point is that
- 10 temperate ice behaves fundamentally different from cold ice. With the sudden strain rate increase at -0.2°C AND evidence for $n \sim 1$ behaviour, I feel this is justified. It is unreasonable to change the Activation energy by some 'fudge factor' if there's a fundamental and significant change in rheological behaviour near the melting temperature. Thus, temperate ice does not follow a standard power-law creep behaviour, and it would be irresponsible to claim that it does if there's evidence to the contrary. This should now be clearer in the rewritten section on 'Creep component'.

- 15 3) References are used inappropriately to support conclusions (p. 9, 8-15; p. 9, 8-23).

MK: p. 9, 8-15 I have strengthened this, and provided more references that show that the stress dependency in many cases is close to 1. I do not see how this is inappropriate. See new section on 'Creep component' at the end of this reply.

- p. 9, 8-23: I do not understand what the reviewer refers to, but if so probably covered in the new section on 'Creep component' at the end of this reply..
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4) Inadequate justification is provided for some of the paper's assumptions (p. 5, 2; p.7, 5-7; p. 8, 1-4; p. 8, 5)

MK: p. 5, 2. "The normal vertical stress σ_{nv} can be taken as the effective pressure" Explained in detail below

- 25 **MK:** p.7,5-7; "If sufficient water is flowing through the system, heat advection by flowing water will be much more efficient than heat conduction through rock or ice": As such this is true, but the question is whether water flow is sufficient – fair point. I have rephrased this

- MK** p. 8, 1-4; " Increase of basal water pressure P_w , resulting in a drop in effective pressure P_e , lowering the friction on flat surfaces. Frictional heating and drag on the flats will drop, as long as P_w remains high. Because there is less drag on the flat surfaces, the normal stress σ_{ss} onto the stoss side of obstacles, however, increases (also temporarily), enhancing stoss-side melting and basal melting". These are simple physic/mechanical principles, frankly. But have changed the last part into: "... enhancing both stoss-side melting as well as creep".
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MK: p. 8, 5. Explained in detail below

5) The introduction would benefit from substantial revision. The attempt to motivate the subject of this paper (i.e., sliding mechanics) with bullets 1-5 is not successful (see my comments below on p. 2-3).

5 **MK:** Fair point, I've rewritten the Introduction.

Specific comments keyed to page and line numbers:

10 p. 1, 14-15. "Thermal equilibrium" is vague. There is "thermal equilibrium" in Weertman's model. The intended meaning needs to be clarified.

MK: Oh dear. Weertman's model relies on a thermal gradient between the stoss and the lee-side, so there is no thermal equilibrium. Muddled thermodynamics, I'm afraid. Thermal equilibrium has a very clear meaning, i.e. having a uniform temperature distribution = absence of a thermal gradient. (I do of course state later on there is thermal gradient near the stoss-side only, but this is the abstract, so 'near-thermal equilibrium' is a good enough). No reason to change.

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MAIN TEXT

p. 1, 18. Power Law Creep should not be capitalized. **MK.** Fair point, rewritten as: 'power-law creep'.

20 p. 2, 13-15. "The essence of Weertman's sliding model is that basal ice movement past an obstacle is controlled either by stoss-side pressure melting around the obstacle or by ductile flow enhanced by stress concentrations near the obstacle, whichever is the fastest." This wording is a bit misleading. It suggests that either pressure-melting or ductile flow occurs, when regardless of bump size, there will always be components of both, albeit with one more important than the other (except for the transition bump size for which they equally important).

25 **MK:** Rewritten as: "The essence of the classic Weertman (1957) sliding model is that basal ice movement past an obstacle occurs by stoss-side pressure melting around the obstacle and by ductile flow enhanced by stress concentrations near the obstacle, with ductile flow being more important for larger obstacles."

p. 2, 25-29. It is not clear why fast ductile flow and soft ice somehow contradict Glen-type power law flow (i.e., the nonlinear dependence of strain rate on stress).

30 **MK:** this is now dealt with in detail in the new section on Creep.

p. 2-3, 30-4. Although these are valid observations, the author needs to be more explicit about how they contradict pressure melting or a power-law ice rheology.

MK: the introduction has been rewritten

p. 3 12-18. These comments on basal thermal regime and basal hydrology in Greenland come as a surprise because there is no allusion to Greenland earlier in the introduction. Such an allusion is necessary because temperate ice of thicknesses much larger than bump size and water access to the glacier bed are, of course, normal for temperature glaciers. If sliding OF THE GREENLAND ICE SHEET, is how the author wants to motivate this paper, then that should be made clearer at the beginning of the introduction.

MK: the introduction has been rewritten, and the motivation to deal with the NEGIS is now clear

p. 3. Omit “worked” before “example” here and elsewhere.

10 **MK:** OK.

p. 4. 13. “was seen” indicates that this was actually observed. Express this differently here and where used elsewhere.

MK: changed to ‘was regarded’

15 p. 4 “is then”, earlier “was seen”. Here and elsewhere the author tends to change tense in midstream. **MK:** reworded

p. 4. 1-21. The reason for this calculation is unclear. The points made after it– that cavity formation can impede heat transport and that regelation speed decreases linearly with bump length–could be made without presenting the calculation. Even if there were better motivation for presenting this calculation, the source of numerical values, such as those for ice thickness and bed shear stress, is unclear (not in the text or appendix but presumably from somewhere in Greenland) and the choice of bump size and spacing is seemingly arbitrary.

20 **MK:** The main point is to show that the thermal gradient set up through the obstacle is very small, so that other heat transport mechanisms are likely to be more efficient and hence more important. I’ve taken the original Weertman parameters, for a fair comparison, as is clear from the text.

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p. 4, 23-26. The author concludes at the end of the calculation: “This implies that ice flowing around an obstacle that is, say, four times longer than another obstacle (Fig. 1b), would be four times slower, even though this obstacle is more streamlined (having a longer aspect ratio). This result is illogical, contradicts most observed geomorphology (Stokes and Clark, 1999; Bradwell et al., 2008), and is a major weakness of the Weertman model.” This decrease in speed with elongation is a major weakness of the Weertman model ONLY if one neglects viscous flow in the Weertman model, as the author does here. And how can the author consider only pressure melting and refreezing–most relevant to bumps less than 0.5 m in wavelength–and assume that the calculation has relevance to the much larger landforms considered by geographers like Stokes and Clark? The Weertman model is indeed flawed, but this calculation adds nothing new to the subject, and the verbiage toward the end of the paragraph is misleading.

MK: Fair point. Rephrased as: “This result contradicts most observed geomorphology (Stokes and Clark, 1999; Bradwell et al., 2008) and supports the notion that pressure melting is not dominant for large obstacles”.

p. 5, 2. Explain why this approach is valid. In Hallet’s abrasion model (1979; 1981) for example, debris-bed friction is independent of normal stress (and effective normal stress) and instead depends on the rate of ice convergence with the bed, so the assumption made here requires justification. Even for a flat bed, ice will converge with it due to basal melting, and that process exerts a downward drag on clasts, increasing friction between them and the bed. I think equation 5 can, in fact, be justified, but the necessary justification is not provided here.

MK: What I’ve taken are the bulk friction coefficients from experiments (lab and subglacial) and which represent friction coefficients averaged over an area (eg. Budd et al, 1979; Zoet et al. 2013, Cohen et al. 2005) and then applied standard Coulomb friction, rather than the theoretical approach of Hallet, which depends on the contact friction coefficient at the (sparse) debris/bed contact points. Hallet’s model is somewhat different, as it looks at abrasion by clast-bed wear rather than friction (pure ice over bedrock gives friction but negligible abrasion of the bedrock). Therefore, Hallet focussed on the clast-bed contact forces, and these are indeed in his model dependent on ice convergence to the bed rather than the normal stress. Note that the friction coefficient used in the Hallet model is the rock-rock friction coefficient (in the order of $\mu = 0.5-0.6$) at the debris-bed contact spot (which is likely to be quite small), rather than the bulk ice / bed friction coefficient that results from the experiments

Note further that Budd et al. (1979) experiments were for clean ice, so a standard Coulomb friction law is applicable. Only Budd et al. (1979) varied the normal stress, so the other experimental work cannot be used to validate/invalidate the notion that normal stress has no effect. Budd et al. (1979) reported a c. 6 x increase in wear at constant sliding velocity associated with a doubling of the normal stress, suggesting that normal stress *does* play a role. Possibly, the disparity between Hallet’s theory and the empirical observations arises because if debris is present, some of it will be in contact with the bed regardless of the convergence rate (if only due to negative buoyancy forces) combined with the effect that in fast moving ice, the sliding rate is orders of magnitude greater than the convergence rate. A further possibility is that the contact force of a clast is a combination of the Hallet model (independent of normal stress) and the Boulton (1974) model (proportional to normal stress).

I have added a caveat, to better justify the standard Coulomb friction model.

p. 6. 11. A temperate ice layer, in fact, has a thermal gradient—one that reflects the decrease in melting temperature with pressure, as pictured in Figure 1 of this paper. Rewrite.

MK: rewritten as: “a temperate ice layer has no bulk thermal gradient (it has arguably a very small negative gradient, but this is ignored here), so no heat can be conducted through it; it forms a near-ideal thermal barrier (e.g. Aschwanden and Blatter, 2005)”. Key here is that the normal increase in temperature cannot be sustained by conduction alone, the slope of

the gradient (compared to the geothermal gradient below and the effective gradient (sustained by advection) above) is miniscule and opposite. This should now be obvious. (For information: the temperature gradient is c. 0.0007 °C/m or 0.7 °C/km)

5 p. 6. This page-long digression (“Intermezzo”) distracts from the theme of the paper and will leave readers wondering what this paper is supposed to be about.

MK: Good point, now covered by the re-organisation of the manuscript.

10 p. 6 34. This is a melt rate rather than a flux, which begs the question why melt rate was not expressed in Equation 6 with these units, as it normally is, either by defining the heat of fusion volumetrically or including density.

MK: 1) No, it is not the melt rate *sensu stricto*, although the melt rate puts an upper bound on to the flux. It is possible that some/much of the meltwater produced by frictional heating at the base is NOT percolated through the temperate layer towards the CTB, but flows towards the terminus and (eventually) escapes through a subglacial drainage system. So there’s a difference between meltwater production (melt rate) and water flux through the temperate layer

15 2) the unit point is valid, and I have now shown the equation (now equation 2) in SI units.

20 p. 7. 5-7. **(A)** The assertion here that heat flow through advection by flowing water is “more efficient” than heat flow due to thermal gradients in rock may be correct but is not demonstrated here or later in this paragraph. **(B)** Also, if some of the frictional and geothermal heat is advected by water, why doesn’t equation 6 reflect that? It should have a heat sink term in it associated with advection by flowing water.

MK: Equation (6) (now equation 2) describes the melting rate of over a very large area, whereas the advection here merely transports (potentially) heat from one spot to another (from bump to bump) within the larger overall system. Similar small thermal gradient will also for instance occur at the high-friction clast/bed contacts, which may locally and possibly short-lived have high temperatures above the melting point on the very small (mm²) contact areas....

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30 p. 7. 8-11. The physics here is muddled. The ice temperature will be pinned everywhere along the bump surface at values set by the distribution of ice pressure (unless the temperature of the water in the film that divides ice from rock is not in equilibrium with the ice temperature, which seems unlikely). Although the water flow will cause some extra melting, if lee-side cavities do not form, the thermal gradient will be set by the pressure deviation from hydrostatic on both the stoss and lee sides of the bump. For a reason I don’t understand, the author is assuming pressure is only important on the upstream side (see Figure 1c also).

MK: The key here is a) we’re in a net-melting environment, with excess water and hence high P_w ; b) cavities may occur; c) therefore no net-negative deviatoric stress can be set up on the lee-side and the pressure there will be controlled by P_w ,

rather than the deviatoric stress. In that case (deviatoric) stress (not pressure!) is indeed only important at the stoss-side.

Reworded to:

5 “In the conceptual model here (Fig. 5c), the temperate layer is thicker than the height of the obstacle. Water is continually produced by frictional heating, there is a net-melting environment with an excess of water, and water pressure on the ice-bed contact will be high. Depending on the basal melt rate and the amount of water flowing along the ice-rock interface, heat advection by flowing water may well be more efficient than heat conduction through rock or ice. In that case no significant thermal gradients can build up and the entire basal system (temperate ice, water, and top rock) is kept at thermal equilibrium at T_m ”

10 Also the word “cold patch”, as used in the classic paper by Robin (1976) and by subsequent textbook writers (e.g. Hooke), describes ice below the pressure melting temperature (PMT). To use it in this context, where all ice is at the PMT, is thus confusing.

15 **MK:** Not at all: “The only exception (to PMT) is the stoss-side of a bedrock obstacle, where the melting temperature is continually depressed as a result of the concentrated deviatoric stress acting onto it” this is the same as the use of Robin (1976), and is perfectly clear and not confusing at all. The other reviewer suggested to think of a different term, as cold patches can be taken to mean (larger scale) frozen patches in a polythermal situation

20 p. 8, 1-4. Debris-bed friction can be affected by water flux to the bed only if water can gain entry to zones where there may be small cavities beneath debris particles. This requires moving water from the channel at the point of entry to the bed out through the thin film that divides debris-bearing ice from bedrock. This propagation of pressure will be diffusive and slow, so it is unclear how much a sudden increase in warm meltwater will really decrease effective pressure and thereby reduce frictional drag.

25 **MK:** This is the increase in P_w that occurs locally near to the point to where supraglacial lake drainage events occur. It has been observed that this can lead to minor but measurable local uplift (Das et al. 2008; Hoffman et al. 2011), suggesting that, despite what the reviewer asserts, P_w increases do occur and quite rapidly so, although they maybe localised and short-lived.

30 p. 8, 5. The thermal gradient towards stoss surfaces depends not only on the temperature of the incoming warm water but on the distance between it and the stoss surfaces of bumps. Because the distances between channels carrying water and stoss surfaces is poorly known and could be far larger than the distance across a bedrock bump, the importance of this effect for stoss-side melting is uncertain, contrary to the certitude of the statement made here. (B) Perhaps the author is assuming that all of the warm water in a Das-like event moves in the thin film that divides ice from rock. If so, that is a dubious assumption.

MK: I do not assume this (all of the warm water in a Das-like event moves in the thin film), but the reviewer assumes that all water in a ‘Das-like’ event will flow through channels, which is even more unlikely: it is clear from the scale of these events, and the measured vertical uplift that the subglacial channel system cannot carry all the water away, and thus some will develop a film. Nevertheless, I agree that the effect and importance are poorly known, and have rephrased to ‘soften’ the certitude.

p. 8, 26. “Weertman (1957) assumed that the creep component of ice flowing around a hard obstacle worked with a rheology” according to ‘Glen’s Flow law’, albeit enhanced by stress concentration on the stoss side.” Again here, the author makes the error of assuming deviatoric stresses are concentrated only on the stoss sides of bumps in Weertman’s theory. Rather, deviatoric stresses are symmetric across bumps in the theory, with lee-side deviatoric stresses equal to but opposite in sign of those on stoss surfaces.

MK: rephrased as: ” albeit enhanced by stress concentration around the obstacles”

p. 8, 27. “strain-rate” rather than “strain”. **MK:** changed

p. 8, 28. Power Law should not be capitalized, here and elsewhere. **MK:** OK changed.

p. 9, 3. “comparisons” “suggests” Correct subject-verb correspondence. **MK:**, corrected.

p. 9, 8-15. Here the author asserts that if the log of strain rate plots as a straight line against the reciprocal of temperature, then power law creep is indicated. He needs to be aware that power-law creep is defined on the basis of the relationship between stress and strain rate, rather than between stress and temperature. Note that Morgan (1991) (the source of the data reproduced here) never commented on whether his data conformed to power law creep rules because all of his tests were done at a single stress (0.1 MPa).

MK: Reworded as: “Thus, at constant stress, ice above c. -0.2°C shows a sudden weakening of a factor 5 to 10, evidently not described by the Arrhenius relation of equation (7), potentially implying that power-law creep is not dominant close to the melting temperature (see also Barnes et al. 1971; Colbeck and Evans, 1973; Morgan, 1991). “

The reviewer needs to be aware that the Arrhenius relation is part & parcel of the power law (certainly for ice), so that if the Arrhenius relation breaks down suddenly it is only correct to start questioning power law. Clearly something is happening...

I then have provided more evidence from the literature that the stress dependency is NOT 3, but in many cases is close to $n = 1$. Thus, power-law creep ‘breaks down’, but I’ve phrased this differently (although any metallurgist would agree..).

p. 9, 8-23. This point of the paragraph—that temperate ice obeys a near Newtonian flow rule—could conceivably be correct, given the relative lack of work on warm ice, but is not convincingly argued here. Other authors cited, such as Byers et al

(2012) and Chandler et al (2008), did indeed suggest values of n near 1.0 but were careful to attribute those low values to low deviatoric stresses, for which there is some micromechanical justification for low n . For ice near glacier beds, however, and particularly near bumps, deviatoric stresses will tend to be high, and thus justification for low values of n is weak.

MK: I have rewritten this, and added more evidence from the literature. The reviewer has a point with the low-stress / high stress, and this is now described. However, there is ample evidence that the stress dependency in both experiments and in nature is NOT the 'standard' $n=3$ that is usually assumed. Note that Chandler's work IS on a real glacier. The main point is that glacial modellers should not assume $n = 3$ in temperate ice, if there's no evidence. See revised section on the "Creep Component in Temperate ice"

p. 9, 31. Why does the abbreviation, GBPS, not coincide to the first letters of "grain boundary pressure melting"? And why choose a new term here when there is ample discussion of this sort of deformation in the literature, much of which is not cited here? Overall, I am left with the impression that the author does not have sufficient familiarity with the ice rheology literature.

MK: I have rewritten this, with more emphasis on dislocation creep & recrystallisation enhanced by the presence of melt, discussed the potential role of Grain Boundary Sliding, and used the more traditional term 'grain boundary melting' / 'internal pressure melting' (mainly on the instigation of reviewer M Montagnat). There is again much evidence for partial melting in deforming temperate ice, and I've strengthened this. As it is unclear as to which mechanism is dominant (I admit I was pushing too hard for a single mechanism in the original MS), I use the term 'melt-assisted creep' as a bucket term. The problem is that, to my knowledge, no-one has studied the stress-strain behaviour AND the microstructures, which can tell us about the deformation mechanisms. See revised section on the "Creep Component in Temperate ice"

p. 10, 26-27. "In summary, ice flow around a bedrock obstacle in temperate ice is constrained either by stoss-side pressure melting or by enhanced creep." Again this is misleading. Any obstacle is accommodated by both mechanisms, although one can dominate the other depending on bump size.

MK: not so much misleading, as poorly phrased. "Ice flow around a bedrock obstacle in temperate ice is accommodated by stoss-side pressure melting or by enhanced creep, with creep being more important for larger obstacles. In temperate ice, the creep component is...[]. And the pressure melting is [...]"..

p. 10, 29-32 & p. 11, 1-4. None of these bullets, or the following assertion, has been demonstrated in this paper.

MK: Note: "it is proposed here that stoss side pressure melting is constrained by". Given the preceding matter, these is a very reasonable propositions to make, that are testable. It is clear from the English that I propose a hypothesis here, rather than demonstrate or assert.

p. 10, 5-9. The conclusion here that temperate ice does not obey a power-law rheology has not been demonstrated in this paper.

MK: It is now convincingly shown that temperate ice cannot be responsibly modelled according to a standard power-law rheology with the standard $n=3$. What it should be modelled as maybe unclear, but part of the point of this paper is to point out where and why temperate ice is important, but that there are large gaps in the knowledge of that material. It appears to me that the reviewer thinks that temperate ice behaves the same as cold ice, and can be modelled in the same way. This is clearly not true. Power law cannot be regarded as operating in temperate ice.

p. 11, 19. The idea that no sliding occurs at subfreezing temps is not strictly correct. See Shreve 1984 J Glaciol.; Cuffey et al 1999, GRL.

MK: that maybe true, especially near cold/warm boundaries. However, both these papers admit that compared to normal, let alone fast warm-based sliding velocities, the sliding is 'negligible', and therefore as an overall approximation, the scale of which is clear here, it is correct.

Section 8.2. This is an interesting story but no aspect of it has been demonstrated in this paper.

MK: The first part is mainstream glaciology, the parts of 3) and 4) follow logically from the paper. A dismissive comment, that is not very useful for an author to act upon. However, I will rephrase the introductory line as: "In such a model, three thermomechanical basal regimes may thus occur, with a potential fourth operating seasonally". Note that reviewer Cohen noted that the figure accompanying this section was very useful.

p. 12, 8-9. "The corollary of the processes described herein is that if a thick temperate layer is present, basal motion over a hard bed with bedrock humps provides less drag than previously thought." This is misleading. The fact that traditional sliding theories, including Weertman's, under-predict rates of glacier sliding and over-predict drag has been discussed for many decades, and is well described in the leading reference book by Cuffey and Paterson (2010), for example. Under-emphasized in this paper is the role of cavity formation in reducing basal drag (e.g., Schoof, 2005).

MK: I assume that the reviewer refers to the phrase 'previously thought'; if so, fair point. Rephrased as: "The corollary of the processes described herein is that if a thick temperate layer is present, basal motion over a hard bed with bedrock humps provides less drag than ice modelled with cold ice properties".

p. 12, 11. "weaker bulk rheology"? I can see how ice can be "weak", but I don't understand how the relationship between stress and strain rate (rheology) can be "weak".

MK: Rheology is the general study of plastic/viscous deformation, so has a rather wider meaning than merely the stress / strain rate relation – it includes such important things as temperature for instance. In some disciplines, this is then (possibly

erroneously) used in term of 'rheological behaviour' and hence the 'rheology of material X'. Rephrased as: "Instead they suggest a different, weaker bulk rheological behaviour"

5 Section 8.3. The problem of fast flow on a hard bed is indeed important and may certainly involve soft ice and the mechanical and thermal effects of water flow under glaciers.

MK: exactly, that's part of the point of the paper

The problem is that this paper does not provide new analyses (or data) that convincingly bear on the issue, so these comments on ice streaming come off as speculative and poorly motivated.

10 **MK:** I stated right in the introduction that this paper is conceptual in character. It does provide new concepts that do bear on the issue, because, to my knowledge, no-one has invoked a thick temperate layer as an explanation for the NEGIS, nor an explanation for how palaeo-ice streams with poor topographic steering operated during the Pleistocene. As I understand, the fast flow of the NEGIS is explained either by very high geothermal heat flow that is geologically unreasonable, and for which the presented geophysical evidence (gravity, aeromag) is unconvincing, or by the presence of soft sediment, which is
15 at odds with the rough nature of the bed, as well as with the observations that hard gneiss in deglaciated areas is normally free of till where significant ice sliding has occurred during Pleistocene (see also my Reply to O Eisen). Thus, there is currently *no* satisfactory explanation for the observed fast ice flow in the NEGIS. Yes, the existence of a thick temperate layer is speculative, but so are all previous explanations put forward. Science cannot move forward if hypotheses cannot be proposed: as long as they are testable. This hypothesis is eminently testable (by drilling or doing radar echo sounding with
20 an appropriate frequency): whether it is correct, time will tell. I feel it is justified to suggest the possibility of the presence of a thick layer of temperate ice at the base of the NEGIS. Speculative: yes, but in a responsible way of posing a testable hypothesis; poorly motivated: no, quite the opposite. See also comments by the other reviewers...

Sliding of temperate basal ice on a rough, hard bed: creep mechanisms, pressure melting, and implications for ice streaming

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5

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10 Abstract

Basal ice motion is crucial to ice dynamics of ice sheets. The classic Weertman sliding model for basal sliding over bedrock obstacles proposes that sliding velocity is controlled by pressure melting and/or ductile flow, whichever is the fastest; it further assumes that stoss-side melting is limited by heat flow through the obstacle and ductile flow is controlled by power-law creep. These last two assumptions, it is argued here, are not applicable if a substantial basal layer of temperate ($T \sim T_{melt}$) ice is present. In that case, frictional melting results in excess basal meltwater and efficient water flow, leading to near-thermal equilibrium. High temperature ice creep experiments have shown a sharp weakening of a factor 5-10 close to T_{melt} , suggesting normal power-law creep does not operate due to a switch to melt-assisted creep with a component of grain boundary melting. Stoss-side melting is controlled by melt water production, heat advection by flowing meltwater to the next obstacle, and heat conduction through ice/rock over half the obstacle height. No heat flow through the obstacle is required. Ice streaming over a rough, hard bed, as possibly in the Northeast Greenland Ice Stream, may be explained by enhanced basal motion in a thick temperate ice layer.

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1 Introduction

The manner in which ice deforms within an ice sheet and moves or slides over its base are critical to accurately model the dynamic past, present and future behaviour of such ice bodies (e.g., Marshall, 2005). Internal deformation of cold ice is fairly well understood, and predictions made on the basis of physical laws (e.g., Glen's flow law) are broadly confirmed by observations (e.g., Dahl-Jensen and Gundestrup, 1987; Paterson, 1994; Ryser et al., 2014, but see Paterson (1991) for

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problems with dusty ice, and Hooke (1981) for a general critique). This is not the case for basal sliding, for which many parameters are poorly constrained. Instead, many models of modern ice sheets use an empirical drag factor or slip coefficient, derived from observed ice velocity and estimated shear stresses (e.g., MacAyeal et al., 1995; Gudmundsson and Raymond, 2008; Ryser et al., 2014). Using an empirical slip coefficient is reasonable to describe and understand present-day near-instantaneous ice sheet behaviour, but cannot reliably predict or reconstruct ice velocities if parameters such as ice thickness, driving forces and meltwater production change significantly. ▾

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This problem is particularly acute for ice streams with poor topographic steering. For such ice streams it is commonly assumed that the necessary low drag at their base can be explained by the presence of soft sediment or deformable till (e.g., Alley et al., 1987; Hindmarsh, 1997; Winsorow et al., 2010), which has indeed been shown to occur below some ice streams in West Antarctica (e.g. Alley et al. 1987; King et al. 2009) and also documented in the geomorphological record (e.g. Margold et al. 2015). However, there is increasing geomorphological evidence for palaeo-ice streaming on rough, hard bedrock-dominated beds without clear topographic steering. Hard, rough beds are widespread on the beds of the former Pleistocene ice sheets and also likely beneath large parts of the present-day Greenland and Antarctic ice sheets (e.g., Kleman et al., 2008; Eyles, 2012; Rippin, 2013; Krabbendam and Bradwell, 2014; Krabbendam et al., 2016). Evidence for paleo-ice streaming has been reported from the former Pleistocene Laurentide and British ice sheets and deglaciated parts of West Greenland (Smith, 1948; Stokes and Clark, 2003; Roberts and Long, 2005; Bradwell et al., 2008; Eyles, 2012; Bradwell, 2013; Eyles and Putkinen, 2014; Krabbendam et al., 2016). In these areas, the deforming-bed models cannot apply because little or no soft-sediment is present. These palaeo-ice stream zones are surrounded by areas also subjected to less intense, but still warm-based ice erosion suggesting intermediate ice velocities (e.g., Bradwell, 2013), consistent with ice velocity analysis and borehole observations from the Greenland Ice Sheet that show significant warm-based sliding (10-100 m yr⁻¹) outside ice-streams (Lüthi et al., 2002; Ryser et al., 2014, Joughin et al., 2010). Thus, fast ice flow appears to be possible on hard, rough beds and cannot be explained by a simple cold/warm thermal boundary (cf. Payne and Dongelmans, 1997). In Greenland, the massive Northeast Greenland Ice Stream remains difficult to explain, as current explanations invoke geologically unreasonably high geothermal heat flows (e.g., Fahrenstock et al., 2001) and a deformable bed with an unknown till source (Christianson et al. 2014).

A solution may be presented by the presence of a basal layer of temperate ice (ice at the melting temperature T_m), below cold ice that makes up the remainder of the ice sheet. Drilling in Greenland Ice Sheet adjacent to the Jakobshavn Isbrae has documented a basal layer of temperate ice below cold ice of some 30 m thickness (Lüthi et al., 2002), equal or greater in height than typical bedrock obstacles (roches moutonnées, whalebacks) in most crystalline gneiss terrains (Krabbendam and Bradwell, 2014). Temperate ice has also been modelled to occur beneath other parts of the Greenland Ice Sheet (e.g., Dahl-Jensen, 1989; Calov and Hutter, 1996; Greve, 1997). Two pertinent questions follow from these observations:

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1) How does such a temperate layer develop and how is it maintained, given that it is overlain by cold ice? In-situ measurements at a glacier base and experiments have recently shown that warm-based basal sliding occurs under significant friction, caused by basal-debris / bedrock contacts (Iverson et al., 2003; Cohen et al., 2005; Zoet et al., 2013), generating significant frictional heat at the base, which is important for the development of a temperate ice layer.

2) How does basal sliding operate in temperate ice, and how does this differ from classic sliding models? The essence of the classic Weertman (1957) sliding model is that basal ice movement past an obstacle occurs by pressure melting around the obstacle and by ductile flow enhanced by stress concentrations near the obstacle, with ductile flow being more important for larger obstacles. The rheology of enhanced ductile flow is normally described by 'Glen's flow law', whereas pressure melting is regarded to be limited by heat flow through the obstacle. Sparse experimental evidence, however, suggests that temperate ice is considerably weaker than cold ice, and that creep may not be modelled reliably according to the standard Glen's flow law (e.g. Colbeck and Evans, 1973; Duval, 1977; Morgan, 1991). Furthermore, in a basal temperate ice layer it maybe that no thermal gradient across an obstacle can be maintained, and that pressure melting at the stoss side has thermal different controls than in the classic model.

Numerous studies have improved upon the Weertman sliding model, focussing on more realistic geometries of the bedrock obstacles, more sophisticated analyses of the stresses near the obstacles, and models that allow for lee-side cavitation, which was not allowed in the original model (e.g., Kamb and La Chapelle, 1964; Kamb, 1970; Nye, 1970; Lliboutry, 1993; Schoof, 2005). Nevertheless, the mechanical behaviour of temperate ice moving over a rough, hard bed remains poorly constrained. This paper deals with four issues. Firstly, the problem of how a temperate layer can grow and be maintained below cold ice is discussed. Secondly, it will be shown that the two critical assumptions of classic Weertman sliding (enhanced ductile flow controlled by Glen's flow law and pressure melting controlled by heat flow through an obstacle) cannot be applied to temperate ice, and alternative controlling mechanisms are proposed. Thirdly, the implications of the different behaviour of temperate ice below cold ice for thermo-mechanical modelling of ice sheets are discussed. Finally, it is suggested that the development of a temperate basal ice layer may help to explain the occurrence of ice streaming on rough, hard beds, such as seen on deglaciated terrains and possibly also below the Northeast Greenland Ice Stream.

The paper takes a conceptual approach, focussing on the primary thermodynamic and rheological controls, so to achieve an improved conceptual model of basal ice motion around bedrock obstacles, rather than the exact quantification of geometries and stress distributions around such obstacles.

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Deleted: Hard, rough beds are widespread on the beds of the former Pleistocene ice sheets and also likely beneath the present-day Greenland and Antarctic ice sheets (e.g., Kleman et al., 2008; Eyles, 2012; Rippin, 2014; Krabbendam and Bradwell, 2014; Krabbendam et al., 2015). Warm based basal motion over such beds involves frictional sliding over 'flat areas' without roughness (where ice motion is parallel to the bed) as well as motion past bedrock obstacles, which are typically on the 1-100 m scale. Friction on flat areas is largely controlled by basal debris concentration (e.g., Hallet, 1979; Schweizer and Iken, 1992), with debris/bed contacts responsible for most friction, as demonstrated in subglacial observations and experiments (Iverson et al., 2003; Cohen et al., 2006; Zoet et al., 2013). The basis of our understanding of motion past bedrock obstacles on a rough bed goes back to Weertman (1957). The essence of Weertman's sliding model is that basal ice movement past an obstacle is controlled either by stoss-side pressure melting around

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2 Basal meltwater production by frictional sliding

Consider sliding over a flat area without obstacles, over which debris-laden ice slides under friction. Friction produces drag and heat. Frictional heat production Q_{fr} and geothermal heat flow Q_{geo} together control the heat budget at the base of ice sheets. Frictional heat production is controlled by the friction coefficient μ , the normal vertical stress σ_{nv} and the sliding velocity V_{sl} according to:

$$(1) \quad Q_{fr} = \mu \sigma_{nv} V_{sl} \approx \mu(P_i - P_w) V_{sl} \text{ in [W m}^{-2}\text{].}$$

The normal vertical stress σ_v can be taken as the effective pressure, that is ice pressure P_i minus water pressure P_w . (This is the standard friction model, using empirically derived bulk friction coefficients averaged over an areas containing many debris particles (eg Budd et al. 1979; Cohen et al. 2005; Zoet et al. 2014), rather than the theoretical models that consider individual clasts (e.g. Boulton, 1974; Hallet, 1979)). A temperate layer of ice has no significant thermal gradient and cannot conduct heat (see Section 3). Any heat produced at the base therefore causes basal melting, with the resultant melting rate M_{rm} (see Appendix) given by:

$$(2) \quad M_{rm} = \frac{(Q_{geo} + Q_{fr})}{H_{ice}} \text{ in [kg s}^{-1} \text{ m}^{-2}\text{]}$$

where H_{ice} is the heat of fusion of ice. The friction coefficient μ , controlled largely by debris concentration, and water pressure P_w are likely to vary significantly in space and time. Iverson et al. (2003) measured $\mu = 0.05$ *in situ* below a glacier; Budd et al. (1979) measured $\mu = 0.01 - 0.4$ for experimental sliding over rocks of different micro-roughness and Zoet et al. (2013) reported $\mu = 0.01 - 0.05$ for experimental sliding at the pressure melting point, but much higher values ($\mu = 0.2 - 0.6$) at colder temperatures and also, intriguingly, for warm-based sliding of ice over sandstone, which suggest that different bedrock lithologies can result in very different friction coefficients and hence basal melting rates.

The potential contribution of frictional heat due to basal sliding as a function of μ and P_w is graphically illustrated in Fig. 1. It is clear that under a range of circumstances, frictional heating can be equal or greater than the geothermal heat flow (see also Paterson, 1994; Calov and Hutter, 1994). Even at $\mu = 0.05$ (close to the friction coefficient of Teflon) heat production can be significant. At moderate sliding velocities ($<10 \text{ m yr}^{-1}$), heat production at a warm base can be twice the heat production at a cold base, whilst at high sliding velocities typical of ice streams ($>50 \text{ m yr}^{-1}$), frictional heating swamps geothermal heat flow. Note that the likely significant spatial variations in friction coefficient and bed roughness at the base of an ice sheet imply that it is not reliable to derive geothermal heat flow on the basis of basal melt rates alone (cf. Fahnestock et al., 2001; Greve, 2005).

The question now becomes: what happens with the water produced by frictional heating? There are two possibilities, not mutually exclusive:

- 1) The water drains away, initially along a film which may evolve into a dispersed drainage system and further into a channelized drainage system (e.g., Weertman and Birchfield, 1983). Generally water will drain away in the

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direction of ice flow and ultimately discharge from the ice sheet, thus representing overall mass loss. Any thermal gradient at the base will be continuously smoothed by the advective heat transport of the flowing water. As a result, the entire basal environment (basal ice, water and top of bedrock) may reach thermal equilibrium at the pressure melting point.

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2) If water cannot drain away freely, the water will remain under pressure. Water may move upwards through the temperate ice layer to the cold-temperate-boundary (CTB). Here it will refreeze and release its latent heat; this heat will warm up the cold ice just above the CTB, and thus thicken the temperate layer, as further explained below.

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3 Growing and maintaining a temperate ice layer

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The growth and continuance of a temperate ice layer below cold ice is an interesting problem in its own right. The problems associated with this are two-fold (Fig. 2):

i) because cold ice above the CTB moves (by internal deformation) towards the margin, there is a strong component of horizontal thermal advection, isotherms are compressed and the effective thermal gradient at the base of the cold ice just above the CTB is steep, much steeper than can be maintained by heat conduction alone (Paterson, 1994; Dahl-Jensen, 1989; Funk et al., 1994). In Borehole D north of Jakobshavn Isbrae, this thermal gradient has been measured as $0.05\text{ }^{\circ}\text{C m}^{-1}$ ($= 50\text{ }^{\circ}\text{C km}^{-1}$) (Lüthi et al., 2002). There is thus the tendency to cool and hence shrink the temperate layer. For a temperate layer to exist and grow, energy must thus be added to the CTB (e.g., Clarke et al., 1977; Blatter and Hutter, 1990);

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ii) a temperate ice layer has no bulk thermal gradient (it has arguably a very small negative gradient, but this is ignored here), so no heat can be conducted through it; it forms a near-ideal thermal barrier (e.g. Aschwanden and Blatter, 2005).

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Several mechanisms can be invoked to add energy to the CTB, despite the absence of a thermal gradient (Fig. 2):

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- 1) Bedrock highs can conduct heat: if these penetrate the CTB, heat can be conducted into cold ice. This mechanism is limited by the heat conductivity of rock and the height of the obstacle; it cannot explain a CTB above a bedrock high;
- 2) Temperate ice layer may locally thicken by internal deformation, i.e. by folding or thrusting for instance where basal ice flow is heterogeneous near obstacles (e.g. Bell et al. 2014). However, this can only redistribute temperate ice and move it into cold ice, rather than lead to an overall thickening of the temperate layer itself.
- 3) Strain heating within deforming cold ice above the CTB. Given that this zone is subject to high strain this maybe significant (e.g., Clarke et al., 1977; Iken et al., 1993). However, the sharp nick in the temperature profile in Borehole D (Fig. 2) suggests that most heat is transferred across the CTB, rather than generated within the cold ice above the CTB;

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4) Transport of water through the temperate layer. If water moves upward through the temperate layer and crosses the CTB, it freezes, releases its latent heat and warms the cold ice just above the CTB. As the ice temperature rises to T_m , the CTB moves upwards and the temperate layer thickens. A water flux through the temperate layer therefore equates with a heat flow. An intergranular vein network of water likely exists along grain boundaries in temperate ice (Nye and Frank, 1973; Mader, 1992). If (locally) $P_w > P_i$ because of a poorly connected subglacial drainage system (high P_w has been observed in drill hole C in Greenland; Iken et al. 1993) then water can migrate upwards against gravity, either by percolation or possibly by hydrauling fracturing. Water may also migrate as bubbles through ductile deformation, as suggested by the experiments of Wilson et al. (1996). Lovell et al. (2015) suggested that the 'dispersed' basal ice facies found close to the base in surge glaciers may form by shear deformation and partial melting along grain boundaries, resulting in an upward flux of liquid and gas along grain boundaries. Although temperate ice has a low permeability (Lliboutry, 1971), even a small water flux is very effective in transporting 'heat' because of the large latent heat of melting compared to the specific heat capacity of ice: 1 kg of freezing water can heat 160 kg of ice by 1 °C (Paterson, 1994). To maintain the CTB with a thermal gradient of 0.05 °C m⁻¹ just above it (e.g., Lüthi et al., 2002), an energy flow of 0.105 W m⁻² is required, about twice the normal geothermal heat flow (see Appendix). This in turn requires a flux of water through the temperate layer of ~23 mm yr⁻¹ (see Appendix), well within the range of water production by frictional melting at moderate to high sliding velocities. This latter mechanism is probably the most important to grow and maintain a temperate layer.

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Deleted: Although temperate ice has a low permeability (Lliboutry, 1971), but likely to have a intergranular vein network (Nye and Frank).

Deleted: Although the water flow is opposite to gravity, this can be overcome if the subglacial drainage system is poorly connected and locally $P_w > P_i$; (high P_w has been observed in drill hole C in Greenland (Iken et al. 1993)). In that case, water can also move upward by hydrauling fracturing.

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4 The creep component in temperate ice

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In Weertman's (1957) model, stress concentrations build up around an obstacle according to:

$$(3) \sigma_{stoss} = \frac{1}{6} \tau \lambda^2 (wh)^{-1}$$

where σ_{stoss} is the normal stress on the stoss side, acting horizontally; τ is the overall shear stress; λ is the spacing between obstacles; and w and h the width and height of the obstacle, respectively (Fig. 5). Weertman (1957) assumed that the creep component of ice flowing around a hard obstacle worked with a rheology according to 'Glen's Flow law', enhanced by stress concentrations around the obstacle. This law concerns the general relation between imposed deviatoric stress and resulting strain rate. The temperature dependence follows the Arrhenius relation, so that the relationship between strain rate $\dot{\epsilon}$, deviatoric stress σ and temperature is typically described in one dimension as:

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$$(4) \dot{\epsilon} = A \sigma^n e^{-\frac{Q_a}{RT}}$$

where A is a constant, R the gas constant, n the stress component and Q_a the activation energy (Glen, 1955; Paterson, 1994; Alley, 1992). This general power law is applicable at appropriate conditions to many crystalline materials such as quartz, olivine and metals (e.g., Poirier, 1985). For ice, experiments suggest that $n \sim 3$ and $Q_a \sim 80-120$ kJ mol⁻¹ for $T > -10$ °C (e.g., Barnes et al., 1971; Duval et al., 1983; Alley, 1992; Paterson, 1994). Comparisons with borehole tilt deformation studies

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suggest that this describes the rheology of clean ice reasonably well (e.g., Dahl-Jensen and Gundestrup, 1987; Lüthi et al., 2002). Note that strain rate $\dot{\epsilon}$ increases exponentially with temperature. The rate-controlling deformation mechanism of power-law creep in ice and other crystalline materials such as quartz and olivine is normally regarded to be intracrystalline creep, mainly by dislocation glide along basal planes (e.g., Duval et al., 1983; Poirier, 1985; Alley, 1992). The question is whether temperate ice behaves according to the same power law, has the same rate-controlling deformation mechanism (e.g., Hooke, 1981; Parizek and Alley, 2004) and is applicable to temperate ice flow around hard obstacles.

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Experimental data compiled by Morgan (1991), all performed under constant stress (1 bar = 10^5 Pa), are plotted in Fig. 3 to illustrate the effect of temperature on the strain rate. The natural logarithmic of strain is plotted against the reciprocal of temperature, so that a straight line would confirm the Arrhenius relations within the power law. For temperatures between -5 and -0.5 °C, the data plot on a straight line, the gradient of which equals $(-Q_a/R)$, confirming the Arrhenius relation in equation (4) over this temperature interval. However, at about -0.02 °C there is a sharp nick in the trend, with strain rates increasing by up to a factor of ten as the melting temperature is approached. Thus, at constant stress, ice above c. -0.2°C shows a sudden weakening of a factor 5 to 10, evidently not described by the Arrhenius relation of equation (4) and suggesting that power-law creep is not dominant close to the melting temperature (see also Barnes et al. 1971; Colbeck and Evans, 1973; Morgan, 1991).

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The stress-strain rate dependency (the value of n in equation (4)) for temperate ice is ambiguous. Experiments at constant temperature, but varying stress ($T \sim -0.01$ °C, $\sigma = 10-100$ kPa) by Colbeck and Evans (1973) suggest $n = 1.3$; experiments of creep at the melting temperature past a sphere by Byers et al. (2012) also suggest $n < 1.5$. By analysing bulk stress and strain rate using borehole tilt measurements in a 3D borehole network in the temperate Worthington Glacier, Marshall et al. (2002) noted $n \sim 1$ at low stresses, but, interestingly, a change to $n \sim 4$ at higher stresses (>1.8 kPa). De La Chapelle et al. (1999) noted a similar change from $n \sim 1.8$ at low stresses to $n \sim 3$ at high stress (>250 kPa) in experiments of pure ice in the presence of a brine (i.e. not temperate ice *sensu stricto*). Analysis of bulk stress and strain rate of the temperate Glacier de Tsanfleuron, however, suggest $n \sim 1$ (Chandler et al., 2008). At the base of the same glacier, Tison and Hubbard (2000) documented large grain sizes (5-20 mm) and a well-developed crystallographic fabric in basal deforming ice.

Deleted: These data show that: (i) Power Law Creep (equation 7) does not adequately describe ice creep above -0.2 °C and is invalid for temperate ice (see also Barnes et al., 1971), and (ii) temperate ice is 5-10 times weaker than cold ice just below the nick point (Colbeck and Evans, 1973; Morgan, 1991).

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It thus appears that deforming temperate ice behaves fundamentally different from deforming cold ice and cannot be reliably modelled with a standard power law or Glen's flow law. The sharp transition just below the melting temperature suggests that this difference is largely caused by the presence of water. Duval (1977) noted, in temperate ice experiments that a rise in water content up to c. 0.8% leads to a 5-8 times strain rate increase. Temperate ice in Alpine glaciers can contain 1-2 % water (Vallon et al., 1976), which has been observed in experiments to gather along grain triple junctions (Barnes and Tabor, 1966; Wilson et al., 1996), so that it is likely that a vein network along triple junctions exists (Nye and Frank, 1973; Mader, 1992). Partial melting of a deforming temperate layer is furthermore suggested by the formation of bubble-free ice, both in experiments (Barnes and Tabor, 1966) as well as in Alpine and surging Svalbard glaciers (Tison and Hubbard, 2000, Lovell

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et al. 2015). The dominant deformation mechanism for temperate ice, however, is uncertain and it is possible that different deformation mechanisms operate simultaneously. Possible deformation mechanisms and their potential enhancement by the presence of water are suggested below.

- 1) Diffusion creep is enhanced by the presence of liquid along grain boundaries, since that liquid functions like a fast diffusion path (Pharr and Ashby, 1983; Raj, 1982; Goldsby and Kohlstedt, 2001).
- 2) Dislocation creep is also enhanced by liquid (Duval, 1977; De la Chapelle et al. 1995; 1999). Water along grain boundaries decreases the surface area of grain-to-grain contacts and cause an increase in grain-to-grain contact stresses; this will enhance dislocation creep (De La Chapelle et al., 1999) but also other deformation mechanisms. Liquid may also suppress strain hardening and enhance easy intracrystalline basal slip (Duval, 1977; De La Chapelle et al. 1999).
- 3) Dynamic recrystallization and grain growth is rapid in deforming temperate ice (e.g., Duval et al. 1983; Wilson, 1986). Dynamic recrystallization aids dislocation creep as it grows crystals with orientations favourable for easy basal slip and suppresses strain hardening (e.g. Duval et al. 1983). Dynamic recrystallization results in a coarse grain size and should aid development of a crystal fabric.
- 4) Grain boundary sliding is commonly invoked to explain weakening in ductily deforming materials (superplasticity). Superplasticity has been experimentally achieved in ice at very low temperatures (-30 to -80°) and very small (3-40µm) grain sizes (Goldsby and Kohlstedt, 1997, 2001; Goldsby and Swainson, 2005), rather different than the temperate ice under discussion here. Whether grain boundary sliding in ice leads to the formation or destruction of a crystallographic fabric appears debatable (Goldsby and Kohlstedt, 2001; 2002; Duval and Montagnat, 2003; Goldsby and Swainson, 2005).
- 5) Grain boundary melting (or 'internal pressure melting') has been observed in ice deformation experiments with indentors by Barnes and Tabor (1966), Barnes et al. (1971) and Wilson et al. (1996). The principle is that ice melts at highly stressed grain boundaries, liquid is transported to lesser stressed grain boundaries where it refreezes – or water may escape if the intergranular vein network is efficient. Either way, this leads to strain. For ice (in contrast to almost all other materials), it is important to emphasise that grain boundary melting involves a negative volume change upon melting ($\Delta V = -9\%$), which makes grain boundary melting under elevated stresses a thermodynamically favourable mechanism. The distance of heat transport to the stressed grain boundaries necessary to sustain grain boundary melting is half the grain size (Fig. 4), some three orders of magnitude smaller than the size of most bedrock obstacles. It further requires that ice and water are in thermal equilibrium, and might thus not be observed in experiments with cold ice and where the liquid is a brine (cf. De La Chapelle, 1995; 1999). Grain boundary melting is supported by the formation of bubble-poor ice at the base of temperate glaciers: both Tison and Hubbard (2000) and Lovell et al. (2015) show that such ice not formed by direct freeze-on (regelation ice), but by a metamorphic process involving partial melting. Grain boundary melting is loosely analogous to

pressure solution (solution-precipitation creep) observed in salts and limestone, in that material changes from solid to liquid or vice-versa along grain boundaries in different stress states (Pharr and Asby, 1983; McClay, 1977; Rutter, 1983), but differs in that grain boundary melting creates its own liquid.

Which of these deformation mechanisms is dominant is difficult to establish. Given the very different grain sizes and temperatures at which grain boundary sliding has been shown to occur (Goldsby and Kohlstedt, 1997, 2001), and the development of a crystallographic fabric in temperate ice (Tison and Hubbard, 2000), grain boundary sliding is probably not significant. Diffusion creep, grain boundary sliding and grain boundary melting all work on grain boundaries and are grain-size sensitive: they are favoured by a small grain size and the presence of a liquid; these mechanisms normally result in $n < 3$, and thus could explain the $n \sim 1$ behaviour seen in some experiments and natural glaciers. Considering the near-unique pressure-melting behaviour of H_2O , grain boundary melting is worthy of further study. However, all grain-size sensitive mechanisms are at odds with the large grain sizes observed and can, on their own, not explain well-developed fabrics. Well-developed fabrics potentially attest to dislocation creep, but is in turn at odds with the $n \sim 1$ behaviour commonly observed. Altogether there is no clear evidence of a single dominant deformation mechanism, and all deformation mechanisms mentioned above may contribute. The change in the stress-dependency as observed in the Worthington Glacier (Marshall et al. 2002) as well as in some experiments (De La Chapelle et al. 1999) suggests that the dominant deformation mechanism in temperate ice depends on the magnitude of stress. For the moment the rather non-generic term 'melt-assisted creep' is used herein. A strong crystallographic fabric and concentrations of dust or silt particles are known to significantly weaken cold ice in simple shear (e.g., Lile, 1978; Paterson, 1984, 1991; Dahl-Jensen and Gundestrup, 1987; Azuma, 1994), but whether this leads to further weakening of temperate ice is not known. There is still much unknown about creep in temperate ice; regardless of the actual mechanism all experiments show that temperate ice is significantly weaker than, and behaves very differently from, cold ice.

5 The pressure melting component

In the classic Weertman's model, the stress concentration on the stoss side that build up according to equation (3) causes a lowering of the melting temperature ΔT_m according to:

$$(5) \quad \Delta T_m = -C\sigma_{stoss}$$

where C is the pressure melting constant ($7.4 \cdot 10^{-8} \text{ K Pa}^{-1}$). As an example, with $w = 1 \text{ m}$, $h = 1 \text{ m}$ and $\tau = 124 \text{ kPa}$, similar to the parameters used by Weertman, (1957), this would result in a stoss-side normal stress $\sigma_{stoss} = 330 \text{ kPa}$ (see Appendix), causing a lowering of the melting point of $\Delta T_m = -0.025 \text{ }^\circ\text{C}$. Assuming the deviatoric stress on the lee-side is equal but opposite, the melting temperature at the lee side is higher by an equal amount, so that $\Delta T_m = +0.025 \text{ }^\circ\text{C}$. Weertman (1957) envisaged that water freezing on the lee-side released latent heat; this excess heat was regarded to be transported through the obstacle towards the stoss side where it caused further melting. The resultant heat flux through the obstacle Q_{ob} is then

Deleted: herein Comparisons with other materials are no help, as water is (almost) unique in that its solid phase is less dense than its liquid phase Experiments with an indenter pressing into temperate ice led Barnes et al. (1971) to suggest that internal pressure melting was significant (see also Wilson et al. 1996). To distinguish this mechanism from the large-scale stoss-side melting, this mechanism is termed here *grain boundary pressure melting* (GBPS). GBPS works by the principle that grain boundaries orientated at high angles to the maximum deviatoric stress are under higher stress than grain other boundaries (Fig. 4). Pressure melting thus occurs along these grain boundaries, and liquid migrates away (Wilson et al. 1996). Water may refreeze on grain boundaries that are under low stress (e.g., oriented at high angles to the minimum deviatoric stress); alternatively, excess water may percolate away through the intragranular network. The net effect is strain. The distance of heat transport to the stressed grain boundaries necessary to sustain GBPS is half the grain size (Fig. 4), some three orders of magnitude smaller than the size of most bedrock obstacles. In their experiments, Barnes et al. (1971) noted the clear presence of liquid on grain boundaries and specifically at triple junctions in support of GBPS; high water content in deforming temperate ice was { ...

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controlled by the total temperature difference between the lee and stoss side ($2\Delta T_m = 0.05^\circ\text{C}$ in this example), the thermal conductivity of rock K_r , and the length l of the obstacle :

$$(6) \quad Q_{ob} = K_r 2\Delta T_m l^{-1}$$

This heat flux was regarded to control the amount of thermal energy available at the stoss side for melting to proceed and hence as controlling the velocity by melting V_{pm} :

$$(7) \quad V_{pm} = Q_{ob} H_{ice}^{-1} \rho_{ice}^{-1} = K_r 2\Delta T_m l^{-1} H_{ice}^{-1} \rho_{ice}^{-1} \propto l^{-1}$$

where ρ_{ice} the density of ice. This model has a number of problems. Firstly, if cavitation occurs it is unclear how heat can be transported to the stoss-side (an issue also addressed by Lliboutry, 1993). Secondly, V_{pm} is inversely proportional to the length of the obstacle, following equation (4). This implies that ice melting around an obstacle that is, say, four times longer than another obstacle (Fig. 5b), would be four times slower, even though this obstacle is more streamlined (having a longer aspect ratio). This result contradicts most observed geomorphology (Stokes and Clark, 1999; Bradwell et al., 2008) and supports the notion that pressure melting is not dominant for large obstacles.

6 Stoss-side pressure melting in temperate ice

What are the rate controlling factors for stoss-side pressure melting in a layer of temperate ice? In the conceptual model here (Fig. 5c), the temperate layer is thicker than the height of the obstacle. Water is continually produced by frictional heating, there is a net-melting environment with an excess of water, and water pressure on the ice-bed contact will be high. Depending on the basal melt rate and the amount of water flowing along the ice-rock interface, heat advection by flowing water may well be more efficient than heat conduction through rock or ice. In that case, no significant thermal gradients through bedrock obstacles can build up, and the entire basal system (temperate ice, water, and top rock) is kept at thermal equilibrium at T_m . The only exception is the stoss-side of a bedrock obstacle, where the melting temperature is continually depressed as a result of the concentrated deviatoric stress acting onto it (Fig. 5c). Thus, the problem of stoss-side pressure melting is reduced to a single 'cool spot' at the stoss side, with a temperature ΔT_{stoss} below the ambient T_m ($\Delta T_{stoss} = -0.025^\circ\text{C}$ in the example, with similar parameter values as Weertman, 1957). To sustain stoss-side pressure melting, heat needs to be transported only a short distance towards this cool spot; anywhere else is at T_m . Although total ΔT is half compared to the Weertman model (there being no 'warm spot' at the lee side), the transport distance is much smaller. Most obstacles will be wider than high, in which case the critical transport distance is $0.5 h$. As a consequence the heat flow towards the stoss side will be greater than in Weertman's model, and it is independent of the length of the obstacle. Cavitation may occur, as this system is not dependent on regelation. The process can work without any regelation in an overall melting environment, which is compatible with the observation of continuous net basal melting at the base of ice sheets (e.g., Fahnestock et al., 2001). If regelation occurs on the lee side (because of lower stress) of one obstacle, any excess heat will be advected by flowing water to the stoss-side of the next obstacle. In this process, the rate controlling factors are the height of the obstacle (but not the width or the length) and the efficiency of the heat advection by flowing water. The process is not limited by heat

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flow through the obstacle (cf. Weertman, 1957; Kamb, 1970), and the length of the obstacle becomes irrelevant. This type of stoss-side melting will be faster than in Weertman's model for all but the shortest obstacles, and certainly so for obstacles with $l > h$, which is the case for most observed bedrock bedforms in even the roughest of deglaciated terrains (Bradwell, 2013; Roberts and Long, 2005).

7 Effect of surface water input on temperate ice on a rough bed,

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Influx of surface melt water represent addition of thermal energy and can further aid stoss-side melting and hence basal motion. Consider the dramatic influx of surface melt water by the sudden drainage of supraglacial meltwater lakes in West Greenland (e.g., Das et al., 2008). This water is relatively warm (c. 1° C; Tedesco et al., 2012) and such an influx thus represents a significant addition of thermal energy to the base. Sudden influx of warm melt water may have the following consequences:

a) Increase of basal water pressure P_w , resulting in a drop in effective pressure P_e , lowering the friction on flat surfaces. Frictional heating and drag on the flats will drop, as long as P_w remains high. On the other hand, because there is less drag on the flat surfaces, the normal stress σ_{stoss} onto the stoss side of obstacles, increases (also temporarily), enhancing stoss-side melting as well as creep.

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b) The basal system is flushed with water that is well above the ambient T_m . Given the very fast recorded flow of large amounts of water (Das et al., 2008), it is assumed here that little heat is lost during englacial transport. Water entering the basal system at 1 °C is 1.5 °C above T_m and 1.525 °C above the stoss-side melting point. There is thus potentially a steeper thermal gradient between the warm water and the cold spot at stoss-side and more thermal energy is available. This may lead to accelerated stoss-side melting, which would continue until all the water temperature has cooled to T_m .

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c) A more longer-term effect is that if warm water is added to the base and cannot drain away freely, the additional thermal energy will lead to a thickening of the temperate layer (upwards migration of the CTB), in essence cryo-hydrological warming of the basal system (cf. Phillips et al. 2010).

The rate controlling factors of this enhanced stoss-side melting are (i) the flux of surface melt water; (ii) the temperature of this water, (iii) the dissipation of the extra heat, for instance by further melting of ice above the flat surfaces.

In West Greenland, sudden supraglacial meltwater drainage events are accompanied by (a) an immediate (hour time scale) speeding up of surface velocity, with a total horizontal displacement of < 1 m, and vertical uplift of the ice surface on a centimetre scale, followed by (b) a longer period (days) of decelerating but still above-average ice velocity (e.g., Das et al., 2008; Shepherd et al., 2009; Hoffman et al., 2011). The centimetre-scale ice uplift is clearly insufficient to lift basal ice over 1-10 m high obstacles that are likely to exist at the base of the Greenland ice sheet, given its gneiss-dominated bedrock (Roberts and Long, 2005; Krabbendam and Bradwell, 2014). The sudden, short jump is probably due to true sliding as basal ice is pushed higher onto (but not over) sloping obstacles due to an increase of P_w . However, the longer-term (days) increase

in ice velocity may well be caused by accelerated stoss-side melting as described above. It is remarkable that the dramatic lake drainage events have a rather muted effect on ice velocity, strongly suggesting that the basal ice in West Greenland is 'stuck' on the stoss sides of pronounced bedrock obstacles. The term 'lubrication' (e.g., Parizek and Alley, 2004; Shannon et al. 2013) is inappropriate to describe the latter process: it is enhanced melting-regelation, rather than a lowering of friction, that leads to the speed-up.

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8 Critical obstacle size

Weertman (1957) introduced the notion of the 'critical obstacle size'. Because pressure melting and enhanced creep have different dependencies on the obstacle size, melting and regelation should be the dominant mechanism for small obstacles, whereas enhanced creep should be dominant for larger obstacles. For his chosen parameters, Weertman arrived at a critical obstacle size of ~ 1 cm (Fig. 6). Replacing the controlling mechanisms with those for temperate ice, but otherwise applying the same parameters, this would result in approximately a 5-10 times increase in creep velocity and a doubling of the pressure melting velocity, because the controlling height is 0.5h. This means that the critical obstacle size becomes even smaller, e.g. c. 0.5 cm (Fig. 6). However, in subglacial observations and experiments Kamb and La Chapelle (1964) noted that melting/regelation was dominant at much larger obstacles and suggested a critical obstacle size of about 1 metre. They suggested that whilst the qualitative idea of a critical obstacle size of Weertman (1957) was correct, the quantification was incorrect. On the scale of typical bedrock obstacles such as roche moutonnées ($l \sim 5-50$ m, $h \sim 1-10$ m), enhanced creep will be dominant and, as discussed above, will in temperate ice be controlled by some form of melt-assisted creep. Melting/regelation may not be important for bedrock obstacles on a rough bed, but can be important for ice flow around cobble-sized debris.

9 Discussion

9.1 Summary of rate-controlling mechanisms

In summary, ice flow around a bedrock obstacle in temperate ice is accommodated by stoss-side pressure melting or by enhanced creep, with creep being more important for larger obstacles. In temperate ice, the enhanced creep component of basal motion operates close to or at the melting temperature, the rheology of this melt-assisted creep is poorly constrained, but does not behave according to standard power-law creep, is up to one magnitude faster than power-law creep in cold ice, and may involve a component of grain boundary melting. The effects of strain softening and dust particles are uncertain; more laboratory experiments on the deformation of temperate ice may help to better understand creep in temperate ice.

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If a temperate layer exists that is thicker than the height of bedrock obstacles, it is proposed here that stoss-side pressure melting is constrained by:

- Stress concentration on the stoss-side, which depends on the surface area of the stoss-side with respect to the spacing of obstacles, and also to the slope of the obstacles, something not considered here;
 - Height of the obstacle, which is critical to the transport distance of heat transport;
 - Frictional heating on the flat surfaces, which causes the production of excess meltwater;
- 5
- The efficiency and possible localisation of the local drainage network;
 - Input of surface meltwater.

In contrast to the classic Weertman model, stoss-side pressure melting is *not* constrained by heat flow through the rock obstacle, cavitation is possible, and the length of the obstacle is not relevant.

10 9.2 Basal sliding regimes throughout an ice sheet

The proposed model has implications for ice sheet behaviour and the modelling thereof. Consider a hypothetical half-ice sheet (Fig. 7), based on the thermo-mechanical model by Dahl-Jensen (1989) and with a passing resemblance to the Greenland Ice Sheet. Thermal gradients are strongly affected by horizontal motions as ice velocity exceeds the rate of heat conduction, so that a cold 'tongue' occurs within the ice sheet (e.g., Dahl-Jensen, 1989; Iken et al., 1993). The bed is regarded as rough and hard. In such a model, three thermomechanical basal regimes can occur, with a potential fourth operating seasonally.

1) In the cold-based regime, the thermal gradient crosses T_m well into bedrock. All geothermal heat is conducted upwards through the ice. No sliding occurs, and frictional heating is zero. All deformation is internal and can be described by power-law creep.

2) At some point the thermal gradient crosses T_m at the base of the ice sheet, and the base is at the pressure melting point. Sliding starts and frictional heating kicks in. In this regime, the friction coefficient will be highly variable as some patches will be frozen, with very high static friction coefficient (Barnes et al., 1971; Budd et al., 1979; Zoet et al., 2013) but low sliding velocities, whereas other patches will be wet, with lower friction coefficient but higher sliding velocities. Excess heat may still be conducted away by the overlying cold ice and water may well regelate onto lee-sides and on overlying ice, thus limiting the amount of water present. In this regime, basal sliding by some form of classic Weertman sliding is likely. Almost all creep still occurs in cold ice and can be described by power-law creep.

3) Continuous frictional heating overwhelms the heat conducting capacity of the overlying ice and a thick temperate layer develops. Where or when this happens depends in part on the thermal structure of the ice sheet, and in particular the thermal gradient near the base: a steeper gradient requires more frictional heating to develop a temperate basal layer. Once a thick temperate ice layer has developed, basal motion occurs according to the processes described above: (i) on the flat surfaces frictional sliding occurs; (ii) ductile flow will occur by melt-assisted creep, much faster than power-law creep; (iii) stoss-side melting will be faster than in Weertman Sliding.

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4) if large amounts of surface meltwater can drain to the base of the ice sheet, for instance by periodical drainage of supraglacial lakes, a different temporal thermo-mechanical regime develops. Influx of surface meltwater adds thermal energy to the basal environment; this thermal energy is available in part to further accelerate stoss-side pressure melting.

5 9.3 Relevance for ice streaming

The corollary of the processes described herein is that if a thick temperate layer is present, basal motion over a hard bed with bedrock humps provides less drag than ~~ice modelled with cold ice properties.~~ This finding is relevant to ice-sheet modelling. For instance, Peltier et al. (2000) argue that the Laurentide ice Sheet cannot be adequately modelled using a standard ‘Glen’s flow law’ rheology; instead they suggest a different, weaker bulk rheological behaviour. This solution, however, is at odds with findings that the bulk rheology of ice in boreholes can be adequately described as ~~power-law creep.~~ Instead, it may be more realistic to invoke fast, weak basal motion, even on hard beds, in ice-sheet models. How may this relate to ice-streaming mechanisms? A thick temperate layer has been observed in boreholes adjacent to the Jakobshavn Isbrae and inferred within its centre (Lüthi et al., 2002), whereas boreholes in non-ice streaming parts of the Greenland ice sheet show an absence of a temperate layer (Ryser et al., 2014). Ice streams are widespread and their locations appear to be controlled by a range of factors, of which topographic steering and the presence of soft, deforming sediment bed are seen as the most important (e.g., Winsborrow et al., 2010). There are, however, numerous palaeo-ice streams that neither portray strong topographic channelling, nor have soft-sediment (till) at their base (Bradwell et al., 2008; Bradwell, 2013; Eyles, 2012; Krabbendam et al., 2016).

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A modern example of such an ice stream maybe the Northeast Greenland Ice Stream (NEGIS) (Joughin et al., 2001; Fahnestock et al., 2001). While there is topographic steering near its outlet glaciers, topographic steering is weak over much of its length (Joughin et al., 2001; Christianson et al. 2014). ~~Seismic and radio-echo-sounding have shown the base of the NEGIS to be rough (e.g. Christianson et al. 2014) and also suggests the possible presence of water-rich till. However, the presence of soft, deformable till below the NEGIS is not proven and, if existent, may not be the rate-controlling factor, as deforming till will not help in moving ice over bedrock humps.~~ The bedrock geology of Greenland is dominated by

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Precambrian gneisses, and such rocks almost certainly underlie most of the inland track of the NEGIS. Deglaciated areas of Precambrian gneisses in Canada, Scotland, Scandinavia and Greenland generally show a lack of till, extensively exposed bedrock and a rough landscape of rock knolls and rock basins (Roberts and Long, 2005; Krabbendam and Bradwell, 2014).

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Using high-resolution radar profiles to reconstruct an isochron stratigraphy, Fahnestock et al. (2001) reported large areas of ‘missing’ basal ice below the NEGIS, and attributed this to very high, long-term basal melt rates (up to c. 100 mm yr⁻¹). These high basal melt rates were thought to be caused by anomalously high (10 times above normal values) geothermal heat flow. The geophysical evidence for such anomalously high geothermal flow, however, is ~~localised and~~ non-unique, and does not cover the track of the NEGIS; moreover, the contribution of frictional heating to high basal melting rates was not taken

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into account by Fahnestock et al. (2001). Instead, I suggest here that the NEGIS may possess a substantial basal layer of temperate ice, formed and maintained largely by frictional heating. Basal melting rates of 100 mm yr⁻¹ are possible in areas of high friction and/or high ice velocity (Fig. 1). In the inversion technique employed by Joughin et al. (2001), a weak basal temperate ice layer would potentially give a similarly low basal drag as a soft, deformable bed.

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The existence and distribution of a temperate basal layer below cold ice is potentially an important factor controlling ice dynamics and ice streaming. The occurrence of a basal temperate layer and estimates of water content can be detected remotely by using radio-echo sounding at certain frequencies (Björnsson et al., 1996; Murray et al., 2000, 2007). Future challenges to improve dynamic ice sheet modelling include a better knowledge of the rheology of temperate ice, basal friction and frictional heating, basal roughness and distribution and thickness of the basal temperate layer.

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10 10 Conclusions

Basal motion of ice past hard-bed obstacles involves a competition between stoss-side melting and enhanced creep. In a basal layer of temperate ice, stoss-side melting is not controlled by heat flow through the obstacle, but instead by the thickness of the temperate layer, the availability and flux of basal meltwater and the height of the obstacle. Creep in temperate ice is up to ten times faster than in cold ice, suggesting a switch in deformation mechanism to melt-assisted creep. Melt-assisted creep probably comprises several deformation mechanisms triggered or enhanced by the presence of water: grain boundary melting, fast dynamic recrystallisation and enhanced dislocation creep. Together, this suggest that basal motion in temperate ice over a rough, hard bed provides low drag, allowing the possibility of fast ice flow over hard, rough beds. Three different thermo-mechanical regimes control basal sliding: (i) cold-based regime, (ii) warm-based but with a thin temperate layer and (iii) warm-based with a substantial temperate layer. The onset zones of (palaeo)ice streams may coincide with a minimum thickness of the temperate layer, and a thick basal temperate ice layer can explain ice streaming over rough, hard beds, possibly including the Northeast Greenland Ice Stream.

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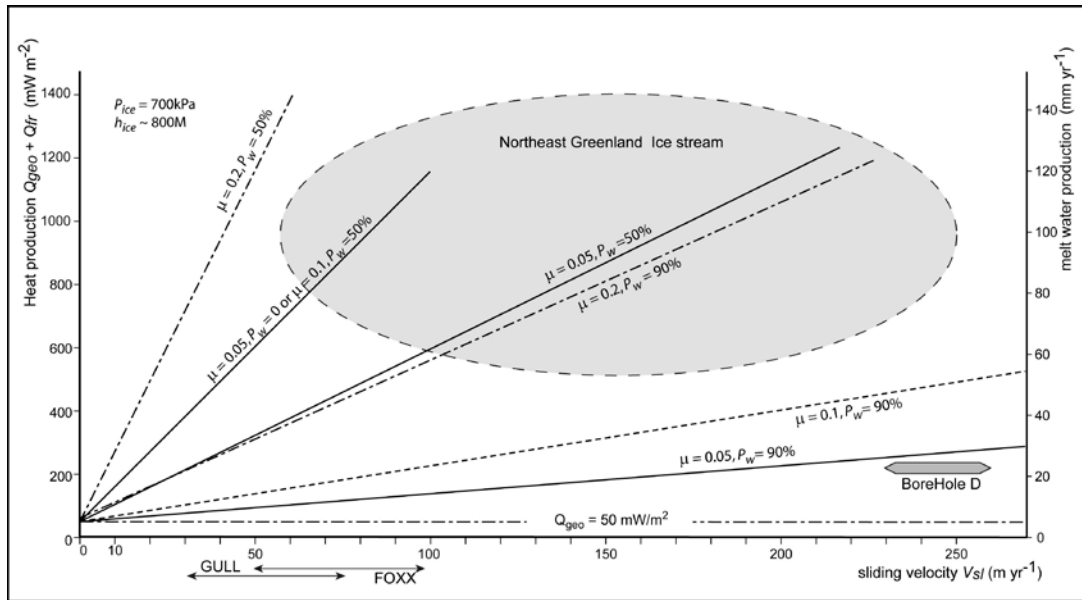
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Figures



5 **Figure 1.** Basal heat production (left) caused by geothermal heat flux and frictional heating as a function of basal sliding velocity, for different values of friction coefficient μ and water pressure P_w as a percentage of overburden pressure. On the right hand side the rate of melt water production, assuming all heat is taken up by melting. Sliding velocities of Borehole D (adjacent to Jakobshavn Isbrae) after Lüthi et al. (2002); melt rate calculated on basis of thermal gradient – see text. Sliding velocities and basal melt production of Northeast Greenland Ice stream after Joughin et al. (2001) and Fahnestock et al. (2001). GULL, and FOXX show seasonal range of basal ice velocities, deduced from borehole data near Swiss Camp, West Greenland (Ryser et al. 2014).

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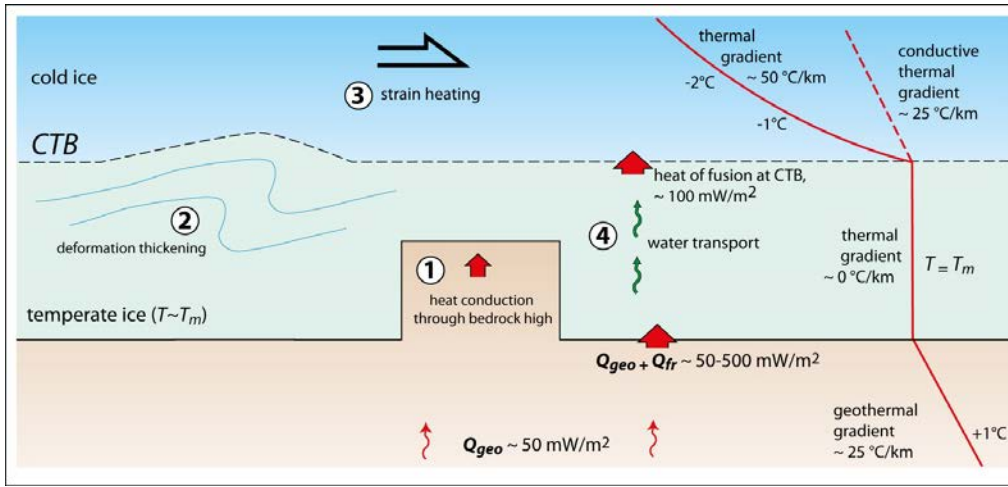


Figure 2. Constraints on the thermal growth of the temperate layer, see text. Schematic thermal gradient of Borehole D is indicated in red line (after Lüthi et al., 2002), CTB = cold-temperate boundary; Q_{geo} = geothermal heat flow; Q_{fr} = heat production by frictional heating.

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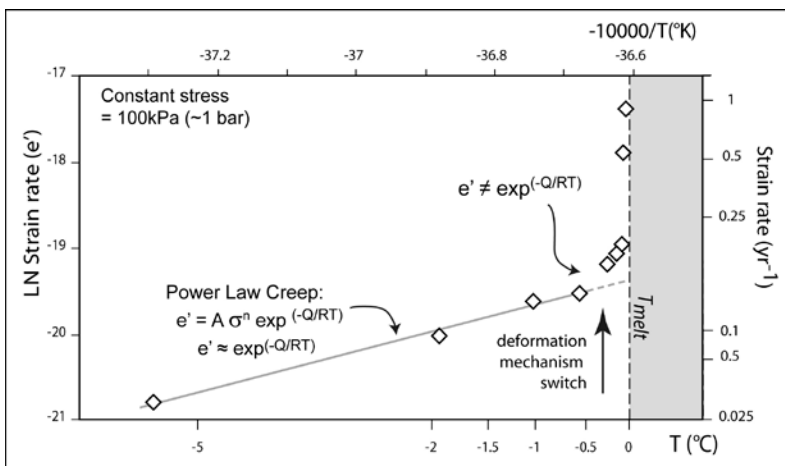


Figure 3. Strain rate against temperature, for experiments performed at 100 kPa, replotted after Morgan (1991). X-axis: reciprocal of temperature; Y-axis: natural logarithm of strain rate. Points following the Arrhenius relation within the power law should appear on a straight line.

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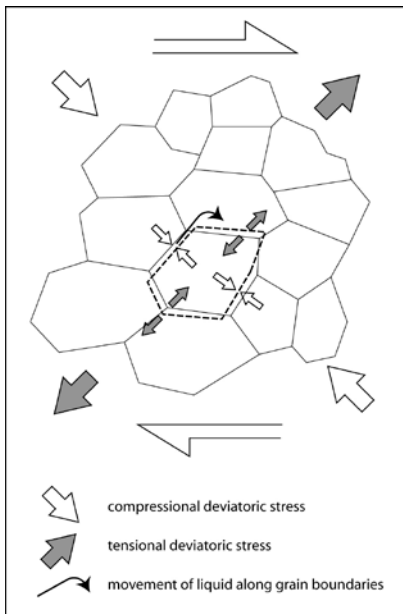
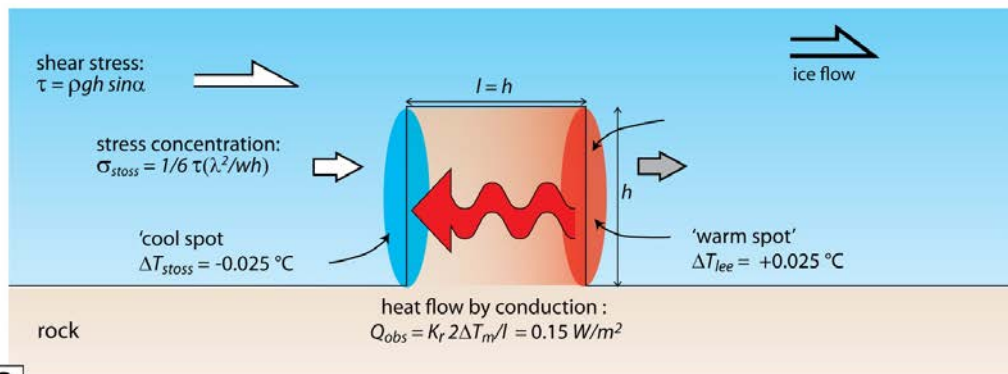


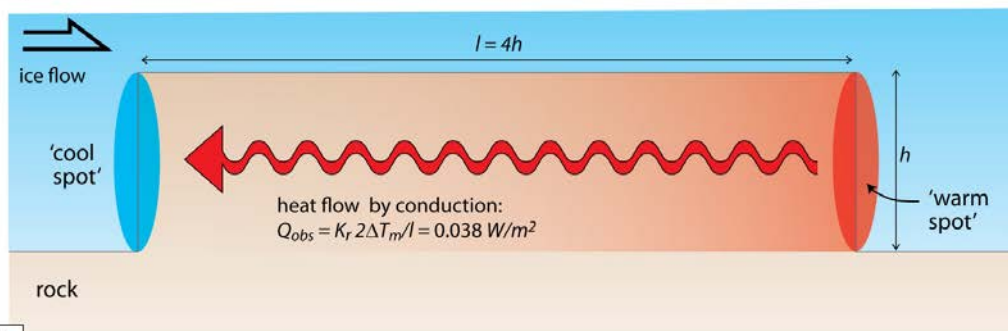
Figure 4. Schematic illustration of grain boundary melting under simple shear. Melting occurs at grain contacts under high stress (compressional deviatoric stress); regelation may occur at grain contacts under low stress (tensional deviatoric stress).

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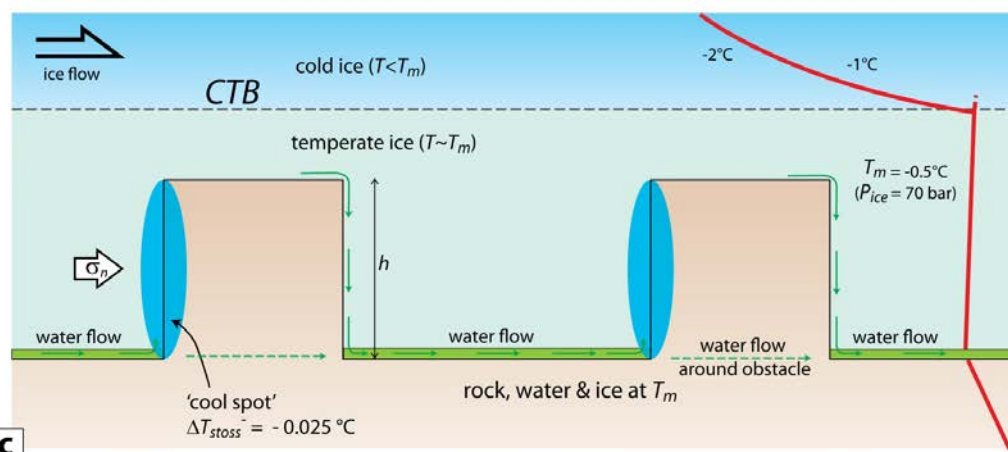
- Liquid water moves along grain boundaries.



a



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c

Figure 5. (a) Basic Weertman sliding model, illustrating components of pressure melting. Thermal gradient through bedrock obstacle indicated by red arrow; (b) Weertman sliding pressure melting with an elongate obstacle, all other parameters are the same; (c) Pressure melting, water and heat transport in a temperate basal layer with significant meltwater flow: a thermal equilibrium occurs everywhere by heat advection by flowing water except at the 'cool spot' of the stoss side. CTB = cold-temperate ice boundary. Schematic thermal gradient of Borehole D is indicated (after Lüthi et al., 2002).

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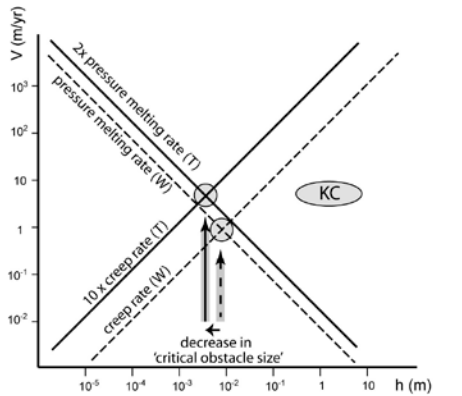


Figure 6. Sliding velocity due to pressure melting and enhanced creep (logarithmic scale) as a function of height of obstacle (Weertman, 1957). Intersection represents the critical obstacle size. (W) = velocities following Weertman; (T) = velocities in temperate ice; KC = velocity and critical obstacle size of observations of Kamb and LaChapelle (1964).

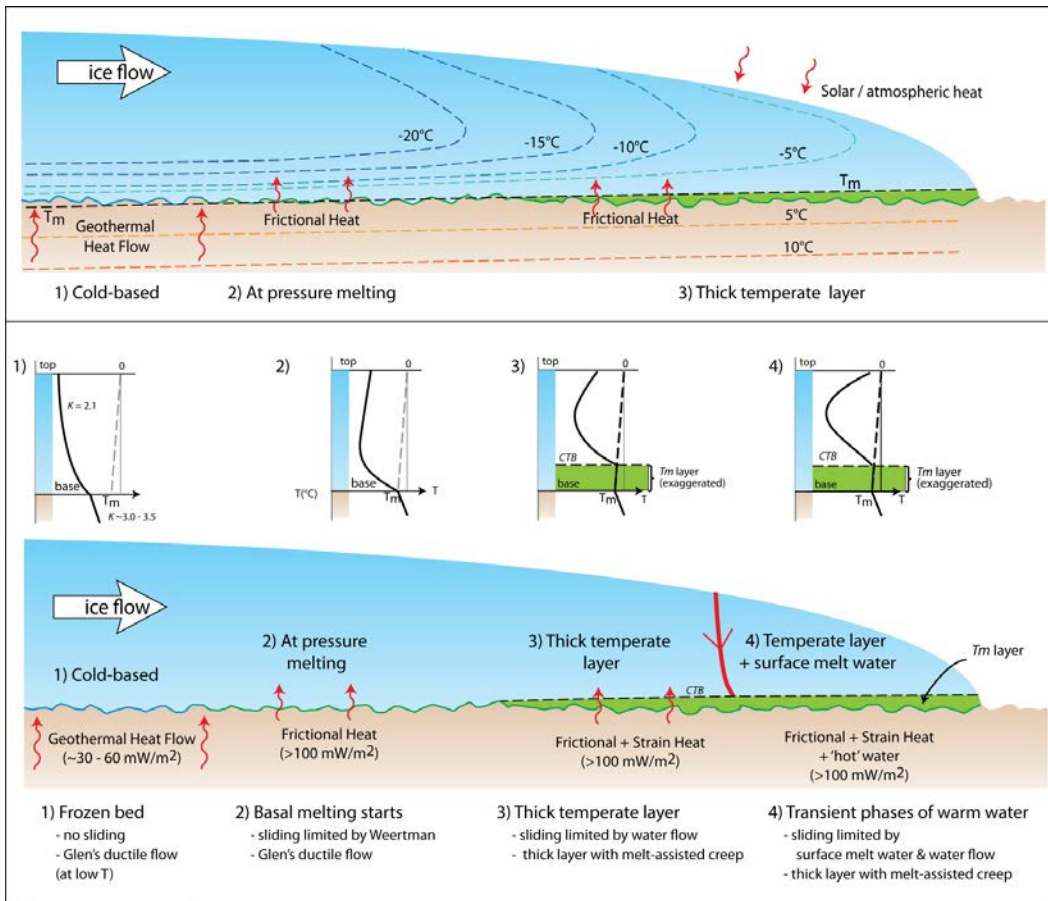


Figure 7. Hypothetical half-ice-sheet (e.g., Dahl-Jensen 1989), with different thermal regimes, further explained in the text.

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CTB = cold-temperate boundary; K = thermal conductivity, in $\text{Wm}^{-1}\text{K}^{-1}$, T_m = melting temperature. Numbers correspond to different thermal regimes described in text.

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Table 1

Constant	Symbol	Value	
Density Ice	ρ_{ice}	910 kg m ⁻³	
Thermal conductivity ice	K_{ice}	2.10 Wm ⁻¹ K ⁻¹	Deleted: Gravitational constant
Thermal conductivity rock	K_r	3.0 Wm ⁻¹ K ⁻¹	
Heat of fusion ice	H_{ice}	334 · 10 ³ J kg ⁻¹	
Pressure-melting constant	C	7.4 · 10 ⁻⁸ K Pa ⁻¹	Deleted: Year
			Deleted: Gas constant
Variable	Symbol	Unit	
Overall shear stress near base	τ	Pa	
Normal stress on stoss side (horizontal)	σ_{stoss}	Pa	Deleted: "
Normal stress on base (vertical)	σ_e	Pa	Deleted: "
Heat flow through obstacle	Q_{ob}	W m ⁻²	
Heat production by frictional heating	Q_{fr}	W m ⁻²	Deleted: (per surface area)
Melt rate (by mass)	M_r	kg s ⁻¹ m ⁻²	
Melt rate (by volume)	M_r	mm yr ⁻¹	
Pure sliding velocity	V_{sl}	m s ⁻¹ or m yr ⁻¹	
Velocity component by pressure melting	V_{pm}	m s ⁻¹ or m yr ⁻¹	
Velocity component by enhanced creep	V_{cr}	m s ⁻¹ or m yr ⁻¹	
Water flow across CTB	F_w	kg s ⁻¹ m ⁻²	
Parameter	Symbol	Value	
Ice thickness	h_{ice}	800 m	
Surface slope	α	1°	
Height obstacle ⁽¹⁾	h	1 m	
Width obstacle ⁽¹⁾	w	1 m	
Spacing obstacles ⁽¹⁾	λ	4m	
Geothermal heat flow ⁽²⁾	Q_{geo}	0.05 W m ⁻²	

⁽¹⁾ same as Weertman (1957)

⁽²⁾ typical range: 0.03 – 0.07 W m⁻² = 30 - 70 mW m⁻²

Table 1. Constants, variables and parameters with chosen values.

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Appendix

Frictional heating is in general given by:

$$(A1) \quad Q_{fr} = \mu \sigma_{nv} V_{sl} \quad \text{in [W m}^{-2}\text{]}$$

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At the base of an ice sheet this can be expressed as:

$$(A2) \quad Q_{fr} = \mu(P_i - P_w) V_{sl} \quad \text{in [W m}^{-2}\text{]}$$

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Assuming all heat is taken up by melting ice, the melt rate is given by:

$$(A3) \quad M_{rm} = \frac{(Q_{geo} + Q_{fr})}{H_{ice}} \quad \text{in [kg s}^{-1} \text{m}^{-2}\text{]}$$

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Recalculated in mm yr^{-1} , the melt rate becomes :

$$(A4) \quad M_{rv} = \frac{1000}{\rho_{ice}} * 3.15 \cdot 10^7 \frac{(Q_{geo} + Q_{fr})}{H_{ice}} \quad \text{in [mm yr}^{-1}\text{]}$$

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To maintain a temperate layer below cold ice with a steep thermal gradient requires a heat flow through the cold-temperate boundary (CTB) of:

$$(A5) \quad Q_{CTB} = K_{ice}(dT/dz) \quad \text{in [W m}^{-2}\text{]}$$

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where K_{ice} is the thermal conductivity of ice and dT/dz the thermal gradient just above the CTB. In the case of borehole D near Jakoshavn Isbrae, the thermal gradient is $0.05 \text{ } ^\circ\text{C m}^{-1}$ (Lüthi et al., 2002), requiring Q_{CTB} becomes $2.1 * 0.05 = 0.105 \text{ W m}^{-2}$, about twice the normal geothermal heat flow (compare with Fig. 3).

The transport of energy by water flow through the temperate layer Q_{LAT} that freezes above the CTB is given by:

$$(A6) \quad Q_{LAT} = F_w H_{ice} \quad \text{in [W m}^{-2}\text{]}$$

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where F_w is the mass flux of water across the CTB (in $\text{kg m}^{-2} \text{ s}^{-1}$), and H_{ice} the heat of fusion of ice. A temperate layer will maintain its thickness if $Q_{LAT} = Q_{CTB}$, and will thicken if $Q_{LAT} > Q_{CTB}$. The required mass flux of water thus becomes:

$$(A7) \quad F_w = K_{ice}(dT/dz) / H_{ice} \quad \text{in [kg m}^{-2} \text{s}^{-1}\text{]}$$

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In the example of Borehole D, the required mass flux $F_w = 6.6 \cdot 10^{-7} \text{ kg m}^{-2} \text{ s}^{-1}$.

Expressed as a melt rate, using equation A4, $M_{rv} = 23 \text{ mm yr}^{-1}$.

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Pressure melting temperature is given by:

$$(A8) \quad \Delta T_m = -C\Delta P$$

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The overall pressure is:

$$(A9) \quad P_{ice} = \rho_{ice}gh$$

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With $h_{ice} = 800 \text{ m}$, it follows that $P_{ice} = 7.13 \cdot 10^6 \text{ Pa}$, so that $\Delta T_m = -0.52 \text{ } ^\circ\text{C}$.

Shear stress is given by:

$$(A10) \quad \tau = \rho gh \sin \alpha$$

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With surface slope $\alpha = 1^\circ$ and ice thickness $h_{ice} = 800$ m, the shear stress $\tau = 1.24 \cdot 10^5$ Pa or 124 kPa.

The concentration of horizontal normal stress acting onto a vertical stoss side is given by Weertman (1957):

(A11) $\sigma_{stoss} = \frac{1}{6} \tau \left(\frac{\lambda^2}{wh} \right)$ in [Pa]

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Using the same parameters as Weertman (1957): $h = 1$ m, $w = 1$ m, $\lambda = 4$ m and the overall shear stress from equation (A10),

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5 $\tau = 124$ kPa, it follows that: $\sigma_{stoss} = 330$ kPa.

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The pressure melting point at the stoss side is given by:

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(A12) $\Delta T_{stoss} = -C \sigma_{stoss}^n$ in [$^\circ$ C]

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Taking the value of σ_{stoss}^n from Eq. (A4) it follows that: $\Delta T_{stoss} = -0.024$ $^\circ$ C. This is the pressure melting point depression below ambient T_m ; so that the $T_{stoss} = -0.544$ $^\circ$ C.

10 Heat flow through obstacle is given by Weertman (1957):

(A13) $Q_{ob} = K_r 2 \Delta T_m / l$ in [$W m^{-2}$]

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For an obstacle 1 m long, and taking the value of ΔT_{stoss} from (A12), $Q_{ob} = 0.15$ $W m^{-2}$. For an obstacle 4 m long, but with all other parameters the same, the heat flow becomes: $Q_{ob} = 0.038$ $W m^{-2}$

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