Response
We thank the Referees for their critical and constructive comments.
Reviewers' comments are in indented blocks and in italic, followed by thy authors response.
Referee #3

> *This manuscript addresses submarine melting at tidewater glaciers using simple plume models. This topic is of great current interest due to its potential importance for Greenland ice sheet dynamics, and these simple plume models are now a common tool for examining ice-ocean interaction processes. The results in this manuscript can be split into three sections: the first describing key factors driving variability in submarine melt rate as calculated using the plume model, the second comparing the plume models to similar studies using general circulation models, and the third comparing melt rate obtained using the plume models with field estimates from the literature.*
>
> *I am in agreement with the previous reviewers of this manuscript, and am not convinced that this revised version of the manuscript has adequately addressed their concerns, hence I have raised some of these again. But I also have a significant number of additional concerns. At present, there are a large number of grammatical and typographical errors, numerical inconsistencies and references used incorrectly. Furthermore, in many places the results of this manuscript are poorly discussed. I am sorry for the length of this review but there are errors in many aspects of the manuscript.*
>
> *I have concerns with each of the results sections, in some cases serious. Some parts of the first section are not particularly novel, and the parts which are novel could be better presented/discussed. The comparison to GCMs in the second section feels rather superficial as the authors are not able to offer any insight into perceived model disagreement (particularly whether it is real disagreement or just different set-ups), so I am not sure what the reader is supposed to take from this. My most serious concerns are with the final section: at present I believe this is presented in an inappropriate fashion, and that the conclusion reached is overly simplistic and far too bold. This conclusion is reached based on matching highly uncertain field estimates with a plume model where parameters are pushed to their limits; in short such a comparison has huge uncertainties and is not a basis on which to reach the strong conclusions stated here. What is particularly lacking is an appreciation or discussion of whether a line plume is even appropriate for the subglacial hydrology we are beginning to uncover at tidewater glaciers, or whether there might be substantial melting outside of plumes. At the moment it feels as if the authors are just matching two uncertain numbers without consideration of process at all.*
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> *I found the appendix to be perhaps the best part of this manuscript, with some elegant and insightful derivations. I think there is a good case for moving some of this material into the main manuscript.*
>
> *I have detailed my concerns in detail below. This manuscript might be able to make a contribution to the literature, but in my opinion significant revisions are needed to improve the quality of the manuscript, to bring out the novel elements and to undertake the model-observation comparison in a more appropriate manner and with more discussion of process.*

We appreciate reviewer's comments and suggestions. At the same time we would like to note that most of reviewer's criticism is based on premise that our paper is aimed on comprehensive modeling of ocean – glacier interaction in Greenland fjords. While we fully agree that better understanding of such interaction is crucial for further improvements of comprehensive Greenland glacial system models, this is not the aim of our paper. The aim of our paper is to test whether simple turbulent plume models can be used to simulate impact of global warming on the entire Greenland ice sheets (GrIS) mass loss with some useful skill on centennial and even longer time scales. Obviously, high resolution ocean GCMs cannot be used at present to this end and not only because of prohibiting computational cost but also because needed information (e.g. fjords bathymetry, spatial distribution of subglacial runoff) for 200+ outlet Greenland glaciers and respective fjords is still not available. This is why in our paper we explore a possibility to use a much simpler and much less demanding approach which is still superior compare to several previous attempts to quantify "dynamical component" of GrIS contribution to future sea level rise. At the same time, we fully agree with the reviewer that the manuscript can and should be improved in presentation and by correcting unfortunate typos and other technical mistakes. This has been done. Below is our response to specific reviewer's comments.

> *Major comments:*

Plume model sensitivities: Sensitivity to subglacial discharge and ambient fjord conditions have already received much attention (e.g. Jenkins, 2011; Xu et al., 2012; Sciascia et al., 2013; Xu et al., 2013; Cowton et al., 2015; Slater et al., 2016), and this manuscript reaches many of the same conclusions with the same method as some of the quoted papers, so I am not sure what is new here. This first section is most novel in its presentation of sensitivities to entrainment rate and glacier front angle, but the presentation of these sections could be much improved (see suggestions below) with more detailed discussion/analysis. Perhaps some of the interesting material from the appendix could be brought into these sections, replacing material which is similar to previous work?

We agree that sensitivity of simple plume models to model parameters and forcing have been performed in several studies although not in a comprehensive manner. Following reviewer's suggestion we have reduced this section to just two pages leaving only results which are important for understanding of the main finding presented in the manuscript. We improved the discussion of entrainment parameter and glacier front angle with reference to the appendix. We prefer to include a summary of the important findings of the appendix into the main section rather than moving the full appendix to the main section.

Comparison of plume models to GCMs: Where the plume models disagree with the GCMs the authors are not able to offer any insight into why this might be; in particular it is not clear whether the disagreement arises from genuine disagreement in plume dynamics, or is just due to model set-up, or whether the disagreement arises because the GCMs are simulating processes beyond those in the plume model. For example, GCMs often simulate elevated melt rates just outside the plume where water is being entrained into the plume, a process which is not included in the plume model melt rates. Xu et al. (2013) also included "background" heat and salt transfer coefficients which may affect their results (see the supplementary information to their paper). The higher discharge results in Xu et al. 2013 also use a low and wide subglacial channel so that the plume might be better compared with a line plume than cone plume. None of these issues are critically evaluated here, and perhaps it is not possible to do so without detailed interrogation of the GCMs. Therefore I do not find the conclusions on your scaling factors (3.4, 2.46 etc) to be very insightful, and I am not sure what the reader gains from this section. I also don't understand why you have to apply a scaling factor of 0.48 to match the results of Sciascia et al. (2013), when Sciascia et al. (2013) obtain much better agreement with a line plume model in their Figure 5.

Our paper is not aimed at understanding why a very simple model produces results which are quantitatively somewhat different from the results of incomparably much more complex and computationally expensive GCMs. The general answer to this question is rather obvious and is given by the reviewer: *"because the GCMs are simulating processes beyond those in the plume model".* This is probably more relevant for the cone plume, which explains a large disagreement between cone plume model and respective GCMs experiment as compared to the line plume. The high resolution 3-D models and 1-D simple plume models serve completely different purposes. The first are aimed at understanding physical processes associated with the glacier-ocean interaction on very short temporal and spatial scales but cannot be applied to the entire Greenland glacial system to simulate its contribution to the future see level rise due to prohibiting computational cost. At present, this can only be done using much simpler models and this is the rationale for our work which is a part of a large-scale project aiming on modeling of the Greenland glacial system response to global warming on the time scales of 100 years and longer. Since simple plume models are based on a number of simplifications and assumptions, it is natural first to compare them with the results of physically based models before applying them to the real world. The reviewer asks *what the reader gains from this section.* We believe there are several important conclusions can be made based on our results. First, the linear plume model simulates the vertical profile of submarine melt in reasonable agreement with GCMs, although tends to overestimate total melt rate by factor 1.5 to 2. As far as the cone model is concerned, the simple cone model is in reasonable agreement with GCMs as far as the dependence of total submarine melt on subglacial discharge and on the number of channels but underestimate cumulative melt rate by factor 2.5. These findings justify the application of simple line and cone models to the real world but suggest that additional tuning of model parameters for individual glaciers is needed. As far as the apparent discrepancy with Sciascia et al. (2013) is concerned, the reason (based on personal communication) is that Sciasa accidentally shifted the ambient temperature profile by the freezing temperature when using them in Jenkins' Matlab-plume model code.

Comparison of plume models to observations: This section culminates in the final line of the abstract: "Our results show that the line plume model is more appropriate than the cone plume model for simulating submarine melt rates of real glaciers in Greenland". This is a sweeping, bold *and overly-simplistic conclusion. There are so many uncertainties in comparisons between modelled and observed melt rates that any conclusions should be drawn very cautiously. Uncertainties in the estimation of melt rates from fjord flux gates have been analysed by Jackson & Straneo (2016) and I think the points made therein should receive greater respect/discussion here. Some of the glaciers discussed in this manuscript are known to have large distinct channels for which a cone plume model (or narrow line plume model) is presumably more appropriate than the full-glacier width line plume model (e.g. to use the abbreviations from Table 6, KS has several channels (Fried et al., 2015), KAS and ST also have distinct channels (Rignot et al., 2015)). In fact applying just a line plume model to KS will clearly not capture the many discrete channels inferred to lead to substantial heterogeneity in melt rate. Stevens et al (2016) considers a different glacier to those in this paper, but provides another example of a glacier where a glacier-wide line plume would not be the most appropriate.*

I agree that when spatially averaged over the calving front, melt rates from a few cone plumes are much smaller than the observations (e.g. Slater et al., 2015; Fried et al., 2015; Carroll et al., 2016), so that (taking the observations at face value) front-wide melting may be dominated by melting outside of these cone plumes. But this does not necessarily mean that we should apply a line plume to the whole calving front. Melting outside of the cone plumes might be controlled by entrainment of water into the cone plumes, or by other fjord circulation processes which we have not yet understood. Or it might be that we should apply a low discharge line plume to these regions, we just don't know yet.

We do not understand why the reviewer characterized a rather technical statement "Our results show that the line plume model is more appropriate than the cone plume model for simulating submarine melt rates of real glaciers in Greenland" as "*sweeping, bold and overly-simplistic conclusion".* By this sentence we simply stated that in the situation when the only information available about glaciers and fjords are basic geometrical parameters (glacier width, depth of grounding line) and the total subglacial discharge, the line plume model gives much more realistic results than the cone plume model assuming that the entire subglacial discharge is distributed through one or several channels. We want to use a simple model to estimate average melt rate along the glacier front, and processes like undercutting. We now refer to Jackson & Straneo (2016) and changed the last sentence of the abstract from "simulating submarine melt rates" into to "simulating average submarine melt rates".

Needless to say that we are aware that reality is much more complex than this is assumed in the line plume model. However, there is a strong rationale to apply simple line plume model to the real world. Indeed, it is likely that in the real world the total submarine melt is a sum of melt associated with several large plumes, a network of small ones and a background melt in the areas where there is no significant subglacial discharge. The melt rate associated to each of these three components has dependences on ambient water temperature similar to the LP model and both LP and CP models have a similar power law dependence (to the power 1/3) on the total subglacial discharge. Moreover, Slater et al. 2015 show that for a large number of channels, or small distance between channels, the entrainment of the plume changes and the melt rate cannot be calculated by simply adding the individual CP -melt rates, but is asymptotically equivalent to LP melt rate. Concerning Stevens et al. (2016), they demonstrate that oceanographic properties of the distinct water mass observed at a certain depth in one of the Greenland fjords is consistent with the result of cone plume model simulation. However, they made no attempt to compare the melt rate simulated for a single cone plume with the total observed melt rate at the glacier front. Therefore their results cannot be used to argue against using the LP model instead of the CP model.

A further issue is that in order to make the line plume-modelled and observed melt rates match, the entrainment parameter takes a very low value (0.036) and the heat transfer coefficient is doubled. I am not sure there is much support for the use of this low value of entrainment coefficient for near-vertical plumes (I think this low value was originally artificially lowered to crudely represent the effect of Coriolis on ice shelf plumes – see Jenkins (1991) and more comments below). Of course, there is so little data on plumes at tidewater glaciers that the values of parameters adopted might ultimately prove to be about right, but they are not a basis on which to reach the strong conclusions presented in this paper.

We treat entrainment parameter as a tunable model parameter. We found that the lowest value of E0 allows us to obtain simulated submarine melt in a better agreement with empirical estimates (see table 1.4). Furthermore, E0=0.036 matches best the line plume experiment of Slater et al. 2015. Therefore, if we would use a more "canonical" values of E, we would need to use a larger factor for scaling up simulated discharge to match observational data.

In my opinion then, this section needs substantial rethinking to take account of what is known of the subglacial hydrology at the glaciers presented, to acknowledge the potential for melting outside of plumes, and to respect the very many uncertainties currently present in comparisons of model and observation.

We do not agree with the reviewers opinion.
We showed that for the purpose of deriving future melt scenarios for Greenland glaciers – essentially characterized by increased ocean temperature and subglacial discharge, for which the LP model has appropriate dynamics, as argued above - the LP model properly calibrated for each individual fjord is a reasonable tool. An alternative approach – using of high resolution GCMs – apart from prohibiting computational cost, would also require detailed information about spatial distribution of subglacial channels and seasonal variability of subglacial discharge in each channel. And even if such information would be available for present day, no one can be sure that it still will be valid in 100 years from now.

Minor comments:

P1L3-5: Suggest toning this down a little – the importance of submarine melting is not yet clear – maybe change "submarine melt plays a crucial role" to "submarine melt may play a crucial role" and "submarine melt will increase and outlet glaciers will retreat, contributing…" to "submarine melt will increase, potentially driving outlet glacier retreat and contributing…"

We adjusted the sentence as proposed by the reviewer.

P2L16-17: "These large uncertainties are associated with the parameterisation of the rate of submarine melt" – surely there are many more reasons for the uncertainty e.g. the representation of calving and basal sliding, lack of resolution – it would be good to acknowledge here that representation of submarine melting is not the only reason for uncertainty in future projections.

We agree that this sentence is misleading. We inserted ", among other factors," to get rid of the impression of the submarine melt rate parameterization being the only factor of uncertainty.

P2L25-26: "the influence of fjord circulation … were investigated with the 3D models" – do you mean the influence of fjord circulation on submarine melting? I don't think any of the 3D papers you cited looked at this. Maybe remove this line?

Thanks for spotting that. We meant the other way around. We changed the sentence to "Additionally the influence of subglacial discharge on the fjord circulation, which connects outlet glaciers with the surrounding ocean, were investigated with 3D models."

P2L29: for clarity, maybe change "the second one is localized (the…" to "the second one has localized subglacial discharge (the…"

Adapted the reviewers suggestion

P2L32: "these simulations" – which simulations do you mean? It's not clear at the moment – please clarify.

Clarified to: "All of the above mentioned 2D and 3D model simulations"

P3L2: “where geostrophic flow becomes significant” – the importance of Coriolis is not fundamentally what drives the quadratic dependence, so this statement is misplaced. The quadratic dependence comes from the fact that mixed layer velocity increases significantly with ocean temperature (Holland et al., 2007), which is not the case for plumes driven by large volumes of subglacial discharge (e.g. Slater et al., 2016). Perhaps either remove the reference to geostrophic flow or summarise the reason for the linear-quadratic dependence difference.

We agree and removed the reference to geostrophic flow.

P3L8-9: “Simulations with 3D models, which differ with respect to boundary conditions and turbulence parameters” – I presume that “differ” refers to the difference between 3D and 2D models? I don’t think that 2D and 3D models differ fundamentally in their boundary conditions and turbulence parameters – please clarify and be more specific.

Changed to:
“Simulations with 3D models, which differ with respect to input parameter i.e. temperature profiles, subglacial discharge and turbulence parameters or resolution and grid type, show a variety of CP melt rate profiles.

P3L9-11: I don’t think there is much reason to bring up the differing melt profiles unless you outline why the profiles differ. I’d suggest either removing this section or explaining why the profiles differ (presumably it’s because Kimura et al., 2014 assumed an unstratified fjord while the other two studies assumed a stratified fjord).

We followed the reviewer suggestion and changed the sentence to:
“Simulations with 3D models show the strong dependency of CP melt rate on stratification or other environmental factors, with maximum melt rate near the surface (e.g. Kimura et al., 2014, unstratified) or close to the bottom (e.g. Slater et al., 2015; Xu et al., 2013, stratified).”

P3L15 (and throughout the manuscript): I am not sure I would call the plume model a “parameterization”. Perhaps this is a matter of opinion, but I would imagine a “parameterization” to be even simpler than a plume model. I’d maybe consider referring to the plume model by its name rather than calling it a “parameterization”. If you agree, you’ll need to change it throughout the manuscript. E.g. on P3L16 could change “parameterization” to “a method”.

Agreed, we changed “parameterization” to “simple model” and adapted the title and all sentences accordingly.

P3L16: “in a 1D ice stream models” – this should be corrected to “in 1D ice stream models”. But I would also argue that plume models could be used in any form of ice flow model (not just 1D), so you could generalise a bit more here if wanted.

We inserted “e.g.”. to demonstrate that this is only one possibility but also mention the 1D ice stream model since it serves our future purpose. It now reads: . “Such a plume model can then be used to calculate submarine melt in e.g. 1D ice stream models.”

P3L24: I find the use of “ice tongue” here a bit odd. I think “geometry of the calving front” would be more appropriate.

Agreed, we adapted the expression

P3L30: For clarity, I’d suggest changing “additional melting … glacier front, and” to “submarine melting of the ice-ocean interface, and”.

Agreed, we adapted the expression.

P4L9: For clarity, I’d suggest changing “along the glacier” to “along the calving front”.

Agreed, we adapted the expression.

P4L11: For clarity, I'd suggest changing "along the glacier front" to "up the glacier front", because two lines above, you used "along" to refer to the across-glacier direction.

We clarified the sentence as: "Both models are formulated in one dimension, x, which is the distance from the grounding line upwards along the glacier front,or under the ice shelf, and depends on the glacier shape, described by its slope α (Fig. 1)."

P4L11-14: It'd be good to insert a reference to Fig. 1 in this paragraph.

Agreed, done.

P4L20 and L28: The notation here is a bit confusing (or the equation is missing a factor) because Δρ should be divided by a reference density (see e.g. Jenkins, 2011; his Eq. 2). In line 28 it looks like you equate Δρ=β_S (S_a-S)-β_T (T_a-T) but really it should be Δρ/ρ_0=β_S (S_a-S)-β_T (T_a-T), again see Jenkins, 2011, his Eq. 5. Dividing by the reference density is needed to make your Eq. 2 dimensionally correct.

Indeed, we now divided by the reference density.

P5L4-5: I don't think it says anywhere what your values for Ti and Si are – maybe you could include these in Table 1?

Agreed ,done.

P5L13: what exactly do you mean by this "freezing point"? Do you mean equation (7) with Sb set to 0, or equation (7) with Sb set to the ambient value Sa? I think Jenkins (2011) uses the latter? Please clarify.

We always mean the plume freezing point, which is determined by the plume salinity which in turn can vary from 0 to Sa. We now refer to "plume freezing" point for clarity.

P5L17: "dependence of the melt rate on plume velocity" – surely the melt rate is always linearly dependent on the plume velocity (see your equations 5 and 6). Do you mean something else here?

Yes, we use M0 later in the appendix an wanted it to be mentioned here. We now write:
"We do not use this approximation in our calculation, but this is nevertheless helpful to interpret some of the results
presented in our manuscript, in particular in quantifying the amount of melt rate and simplifying the melt rate dependence on temperature and subglacial discharge (Appendix A)"

P5L25: As for the line plume equations (see comment above), I believe you may be missing a factor of ρ_0 in the momentum equation.

Yes, done.

P6L6: "for glaciers with floating tongues" – I think sin(alpha) can vary without the glacier having a full floating tongue – for example if it is undercut. I'd suggest removing this phrase.

Removed 'floating tongues'

P6L28 and 30: In these expressions, the density difference ρ_0 should again be divided by a reference density so that the dimensions work out.

Yes, done.

P7L7: Even for the highest velocities the melt rate is still lowest at the grounding line so arguably the highest velocities might lead to a toe too. I'd suggest rephrasing this.

Yes, rephrased the sentence. Now it reads:
"Our sensitivity tests show that initial velocities higher than U*0 lead to maximum melting close above the grounding line of the glacier ('undercutting') while for lower velocities the melt rate increases with height and maximum melting is located

further up the calving front (Fig. 2b)."

P7L14-120: In the present position, this paragraph is a bit confusing because you define a value for qsg but then use a different value just below. It might be better to move this paragraph to below section 2.5 as it would avoid confusion and it leads naturally into section 3.

Agreed, adapted the manuscript to authors suggestion and rearranged the paragraphs-.

P7L22-26: this paragraph concerns the comparison of line and conical plume models, so it would perhaps be better in the next section (2.5).

Agreed.

P7L22: "defined per unit length" – would be clearer as "defined per unit width of grounding line"

Agreed, adapted.

P7L28: "local melt rate is higher in the CP model than in the LP model practically for all depths" – Fig. 4 shows this is true for the parameters you have picked, but is it true regardless of Q and for all stratifications? This section would be more meaningful if it could be said that this statement holds more generally than the single example you have plotted.

This particular statement is a description of the figure for this particular setting, indeed, and we did not check whether it is always the case. In fact, we are more interested in the cumulative melt, and this section highlights the role of surface area in determining the cumulative melt rate, regardless of particularities of the local melt rate. The larger cumulative melt for LP than CP was verified in various parts of the manuscript, with a number of experiment settings (e.g. Fig. 7, Fig. 8, Table 4). We changed this part of the sentence to: "local melt rate can be higher in the CP model than in the LP model".

P8L10-12: it would be worth citing Magorrian & Wells, 2016 here and including the phrase "melt-driven convection"

Agreed, and adapted to the reviewers suggestion.

P8L13-15: I find these sentences very difficult to follow: please explain more explicitly/carefully and check grammar/typos. For example, you suggest that the melt rate is linearly dependent on the plume velocity in a well-mixed fjord. But this is true in any situation (due to the form of the three-equation melt rate parameterisation).

We agree that this section on subglacial discharge was not very clear, and we rewrote much of it. We believe it is easier to follow now.

P*8L19-21: If it's to be included, this section needs expanding (i.e. explain more explicitly what Table 2 actually shows). There are a number of problems with Table 2, which I've detailed elsewhere in this review.*
We now explain more on

P8L29-31: At present, this sentence says that all of the cited studies derived values for the entrainment coefficient from laboratory results. They didn't, so this needs rephrasing.

We do not agree with the reviewers opinion since it says "Laboratory experiments [...] and model studies[...]".

P8L30: I am not sure that the value 0.036 is appropriate as quoted. The value 0.036 comes from Jenkins (1991), and to quote from that paper: "adopting the value for E0 of 0.072 given by Bo Pederson (1980) will lead to an overestimate of entrainment. This is because the Coriolis force, which is not incorporated in this simple one-dimensional treatment, will tend to deflect the flow across the basal slope, hence reducing the sinθ term. To compensate for this effect, the value of E0 used is half the figure quoted above." In other words, the value 0.036 is artificially small to try to *account for Coriolis effects which become important beneath broad ice shelves. Since your manuscript mostly looks at vertical plumes, the Coriolis force is not important (Kimura et al., 2014), and a value 0.036 is probably not appropriate. For the vertical plumes studied here I think it would be more appropriate to use values thought to be more suited to vertical plumes (e.g. 0.07 to 0.16 according to Kimura et al., 2014). I think this issue is particularly relevant to section 5, which I have discussed in the major comments above. Of course, I acknowledge there is much uncertainty in the value of E0 due to lack of field measurements, and therefore using 0.036 is not wrong per se (it is interesting to include it in the sensitivity analysis), but the above issue should be discussed. In particular, the reaching of the strong conclusions for vertical plumes in section 5 based on the value E0=0.036 is, in my opinion, not appropriate.*

Please see response to major comments.

P9L1-9: I think this section could be expanded with the results explained in more detail. At present Fig. 6 (which has a great deal of information on it) is only briefly mentioned. Fig. 7 is referred to before Fig. 6 so these figures should be swapped.

In the rewritten sections 3.2, Fig. 6 is referred first, so this point is fixed. The figures are now referred to more explicitly (last paragraph of section 3.2).

P9L13-14: The effect of temperature variation on melt rate has been looked at using methods other than 3d circulation models (i.e. with 2d circulation models and plume theory) – it'd be good to acknowledge that here, and include citations to relevant papers.

Agreed, and cited some examples. It now reads: "Previous experiments with 2D,3D ocean models and analytical solutions i.e. (Jenkins, 2011;Sciascia et al., 2013; Carroll et al., 2015; Jenkins, 2011; Magorrian and Wells, 2016; Slater et al., 2016; Carroll et al., 2015) demonstrated the behavior of the cumulative melt rate as a function of the ambient temperature Ta ."

P9L20-21: It'd be good to refer to Table 3 here, and to change q to qsg for consistency of notation.

Adapted, reviewers suggestions.

P9L22: "the cone plume model seems not to show this change in power law for analytical solutions". As far as I can tell from Table 3, this statement is referenced to Slater et al., 2016. This is inappropriate as Slater et al., 2016 only considered the high discharge regime (i.e. that in which melting does not matter for the dynamics of the plume), and did not consider the low discharge regime at all. A number of other issues with Table 3 are outlined elsewhere.

Yes we agree. We got misled by the figures of Slater 2016, that show always a discharge range starting from $Q=0m^3/s$.

P9L29-32: This discussion of the impact of glacier front angle is poor and needs to give much more insight into how the front angle affects plume dynamics and melting. The Magorrian & Wells (2016) results only apply for very low (negligible) discharge, so are not readily applicable to your results.

We attached some further explanations on the cause of the accelerating plume. The whole subsection has been extended (see now section 3.4. in the new manuscript). Furthermore some explanations on the front angle are included in section 3.2 Entrainment rate.

P10L6: "strongly overestimate plume velocity and melt rate" – can you provide a reference for this statement?

We show this in the referenced chapter on Petermann.

P10L11: "the models contain the right physics to simulate plume dynamics" – this statement is a bit odd as simple plume models also contain the "right physics". I'd suggest removing this phrase.

Rephrased to: "These models are much more complex than our simplified 1D equations, which enable them to simulate plume processes in greater details. On the other hand, they require multi-dimensional grids with

high spatial resolution, which is computationally prohibitive for our purpose of simulating a large number of Greenland glaciers.
”

> *P10L16: Xu et al 2013 will not have resolved all of the turbulence, just a bit more than the coarser resolution models. It'd be good to clarify this here.*

Rephrased to: Xu et al 2013 used a high spatial resolution in order to reduce the amount sub-grid processes.

> *P10L24: "this simulation" – it's not clear whether this refers to the Sciascia et al 2013 simulations or your plume model – please clarify.*

Rephrase to: "For our simulation..."

> *P11L4-5: this sentence implies that the results of Xu et al., 2013 agree well with plume theory. They may do, but I don't think Xu et al., 2013 did this comparison. Does this statement come from your results? If so, it might be better to leave it till later.*

Agreed, deleted that statement.

> *P12L3-5: this is a very inadequate (and incorrect) description of the issue of reliability of fjord heat flux estimates of submarine melting. Please improve and expand.*

We changed the statement, it now reads: "However, the results have to be observed with caution since a single temperature profile does not necessarily represent monthly or even annual temperature profile. As Jackson et al. (2014) shows, for Sermilik Fjord and Kangerdlugssuaq Fjord in the winter months the properties including heat content can undergo great variability within time scales of three to ten days (Jackson et al., 2014)."

> *P12L12: "velocity measurements and mass balance" – I think "ice flux divergence" is clearer.*

Adapted.

> *P12L7-23: some discussion of across-fjord variability would add to this section. Is the 'observed' melt rate which is the blue line in Fig. 12 an across-fjord average? Is it representative of the channels you mention or the ice in between?*

We added the following description:
"Our calculated melt rates were compared to the width-averaged melt rate derived by Rignot et al 2008, which is mostly dominated by the 4 channels that have maximal melt rates of 30 m/d."

> *Section 5.2: I don't think you say where your discharge estimates come from – please add.*

Added.

> *P13L10: "a shelf of 3km length" – I'm a bit confused by this as I don't think Kangerlussuup Sermia has a floating tongue (according to Fried et al 2015 it is undercut but only by a few hundred metres).*

Thanks for spotting it. It does not have a floating tongue indeed. It was an issue in our processing of the Morlighem data set. Nevertheless, we didn't use that data set in the end so that we deleted that sentence.

> *P14L9-12: you need to explain these methods more clearly – at the moment this is very confusing.*

We improved the sentence but we also refer to the original methods such that -if wished -more details can be found.

> P*15L4: would be clearer as "future changes in subglacial discharge and fjord temperature".*

Agreed, added.

> *P15L26: "such discrepancy is not surprising given the highly simplified parameterization of the LP and CP models compared to GCMs" – I disagree strongly with this statement. The plume models*

and the GCMs are simulating the same phenomenon with the same physics. If they disagree it is because of how the models are set up or run rather than because one is simpler than the other.

Agreed, deleted that sentence.

P15L31: "due to the missing Coriolis force in the plume models" – I think this statement needs to be backed up with a reference or some analysis (see similar comment above).

Please, see major comments.

Appendix:

P16L11: I think analytical solutions for the line plume model have been presented by Linden et al (1990), Straneo & Cenedese (2015) and Slater et al (2016) – it might be worth including these references.

Agreed, we mention now the references stated in the comment.

P18L5-7: I wasn't quite able to follow these derivations. Could you add some more detail/intermediate steps to make it more obvious?

We added some more explanations, hopefully it is clearer now.

P18L16: expression for b – as for my other comments above, I think this needs a factor of the reference density in the denominator.

Fixed, thanks!

Eq A15: I couldn't see where this expression comes from. Could you explain more?

More explanations were added.

Eq A20: I couldn't quite follow the final integration – could you add an intermediate step or explain? In the discussion which follows it would be good to refer to Figure 5.

After Equation A20, we inserted an explanation on our calculation steps, and expanded the calculation for the cases of high and low discharge (new equations A21 and A22)

P20L15: "background melting" – I think this is more often called "melt-driven convection" – it would be good to add this phrase.

We inserted the term melt-driven convection.

We adapted or changed all figure and table errors and corrected for grammatical errors and typos as mentioned below.

Figures and Tables:

Figure 1: "ice shelf an slope" should presumably by "ice shelf at slope"? "Melting mdot occurs on the glacier…. salinity Sb" – this sentence doesn't make sense at the moment – consider rephrasing. In the second last line of the caption you have two "ands". In general this whole caption could be clearer.

Figure 2: "U0 = 3.5 ms-3" – I guess this should be "U0 = 3.5 x 10-3 ms-1"?

*Figure 3: the y-axis label is confusing, suggest changing to "cumulative melt in % of melt when U0=U*0".*

Figure 4: change "total discharge occur through" to "total discharge is delivered through". In the fourth line of the caption, "acrosss" should be "across". Use of "entire glacier" in the last two lines – I

think it would be better to say “across the full width W”.

*Figure 5: I guess the legend should say “Qsg1/3” rather than “**1/3”? First and second line of caption: no need for “(a)”. Change “for red line” to “the red line”. The equation in the caption is not the same as the line on the plot (maybe it’s the units?). If I take Qsg = 1 m2/s then according to your plot I would get a melt rate of ~0.04 m3/d but according to your equation I would get 7.2 x 10-5 so something is wrong. Are the units on the y-axis correct? Should they not be m2/s?*

Figure 7: Change “in dependence of” to “as a function of”. Figure legend should say Ta rather than T. You quote a discharge of 10-3 m2/s – is this right? In Fig. 6a, where I presume Ta = 4, the line with a discharge 10-3 m2/s shows a cumulative melt of ~600 m2/d for E0=0.1 whereas for the same parameters, this figure gives a cumulative melt of ~1600 m2/d. So it looks to me like Fig. 6 and 7 are inconsistent?

Figure 10: Legend: last entry should presumably be 1.1° rather than 0.02?

Figure 11: plots labels (a) and (b) need to be swapped. “Same temperature profile as Xu” – presumably also the same salinity profile? Might be better then to say “same stratification”?

Figure 12: Legends and caption: in the text (P12L14) you say the maximum E0 value is 0.08 but here you say 0.16 – needs fixing. In the text (P12L14) you quota a discharge 10-4 but here you say 10-5 – please fix. Line 2 of caption: “entrainement” should be “entrainment”.

Figure 13: The units are a bit odd on the temperature plot. It might also be worth plotting the salinity profile here?

Figure 14: “Sutherland et al” should be “Sutherland and Straneo, 2012”? In the caption, “with and” should be “with a”.

Figure 15: Caption line 1: remove “in”, and “sublgacial” should be “subglacial”. “Motyka model” and “Gade model” – I think these are better referred to as “methods”. Line 2: I think “melt rate estimates” would be better than “melt rate profiles”. Line 3: insert “line” after “red dotted” and “blue dotted”.

Figure 16: How have you calculated the 1-sigma uncertainty range? It looks rather narrow compared to the spread in the data. Could you quote what the value of the “scaling coefficient” in line 2 is?

Table 1: 4th line of table: typo: “inital” should be “initial”. 6th line of table – I presume this should be the initial plume salinity rather than the ambient salinity?

Table 2: Grammar: replace “of melt rate on discharge Q” with “in”. Typo: “seperate” should be “separate”. I believe you may have mixed up the Qc values – I think in Xu et al., 2013, Qc = 4.34 m3/s?

Table 3: I have serious concerns about this table. Firstly, the grammar and typos need fixing in the caption. Second, I do not understand the separation into “experimental” and “theoretical” – all the quoted studies including yours are models of some sort. Most seriously, you have split discharge values into high and low, and then attributed the quoted studies into these categories. But as far as I know most of these studies (Jenkins, 2011; Magorrian & Wells, (2016); Slater et al., (2016)) make no distinction between high and low discharge, and even if they did they wouldn’t have the same discharge boundaries as your results, so the attribution of results from the literature in this table is very questionable. For example, Slater et al., 2016 have no results on low discharge cases, so I don’t understand where the central “1e” entry in the table comes from. I also don’t think Magorrian & Wells (2016) considered high discharges, so I don’t understand where that entry comes from either. Lastly, I think the Xu et al (2013) discharge boundary is wrong again – it should be 4.34 m3/s right? This table needs a complete rethink, probably by removing all of the literature references and just sticking to results from this manuscript. There could maybe be a discussion in the text comparing results from this manuscript to the literature, but the categorisation of other papers in this table is, I believe, incorrect.

We now describe the values as numerically and analytically determined. Magorrian & Wells consider low discharge ranges while Slater 2016 shows a solution for high discharge ranges. We just cite the literature since we believe our numerical experiment are close to the theoretical consideration which explains why these values are close to another.

Table 6: I think "Estimated subglacial discharge" is more appropriate than "measured subglacial discharge". Line 3: typo: "sublacial" should be "subglacial". GammaT and GammaS values need "x 10-2" and units.

Typos:

There are typos on P1L10 (suit), P1L19 (0.33 ± 8 mm/yr – the uncertainty value here must be a typo?), P3L16 (submarime), P4L7 and P4L17 and P6L19 (Fig. 1 should presumably be Fig. 1a), P4L14 (should 0.9 x 10-9 actually be 0.9 x 10-6?), P6L9 (euqation), P9L3 (stong), P9L26 (plume thoery), P11L19 (intoroduced), P11L20 (malt), P12L18 (domintad), P13L16 (averged), P13L19 (closet), P13L31 (Semerlik), P14L11 (coloumn), P14L11 (tempreature), P14L15 (accomodate), P14L27 (obervations), P15L14 (explenation), P16L5 (futue), P16L15 (greenladic), P17L3 (salininty), P20L10 (wit).

Grammar, numerical inconsistencies, incorrect references:

P1L1: "Two hundreds of marine-terminating" should be "Two hundred marine-terminating"
P1L5: "is hampered" should be "are hampered"
P1L5-7: rethink the structure of this sentence – it doesn't read well at the moment
P1L9: change "using" to "the use"
P1L10: change "parameterization" to "a parameterization"
P2L4: remove "of" from "most of Greenland"
P2L10-11: "Nick et al (2013), using the same flowline model implemented" reads better as "Using the same flowline model, Nick et al (2013) implemented"
P2L19: "taken to estimate" is better than "derived to calculate"
P2L20: Rignot et al., 2015 did not present estimates of submarine melt rate from ocean data – I presume this should be Rignot et al., 2010, Nature Geoscience?
P2L21: Holland et al., 2008a is not a general circulation modelling paper – I presume you mean Holland et al., 2007, Journal of Climate?
P2L22: Sciascia et al., 2013 is a 2d general circulation model, not a 3d model
P2L23: I think Holland et al., 2008b should actually be Holland et al., 2007, Journal of Climate?
P2L23-24: I think you have mixed up hydrostatic and non-hydrostatic. The Holland and Little papers were hydrostatic, Sciascia et al., 2013 was non-hydrostatic.
P2L25: "pattern, vertical" should be "pattern and vertical"
P3L25: "3D ocean models" – at the moment you also compare with 2D models (your section 4.2)
P3L22: "There we" would be better as "We then"
P3L31: "They act to" would be better as "These processes act to"
P4L3: change "during summer season" to "during the summer season"
P4L8: change "they can be a number of them" to "there can be a number of them"
P4L24: "in lateral direction" should be "in the lateral direction"
P4L11: change "Solving for equations" to "Solving equations"
P5L15 and 17: I presume "Annex" should be "Appendix"
P7L16: "maximal melting conditions for Greenlands fjord" – I presume you mean that these ambient conditions are the fjord waters found in Greenland which would give the highest melt rates. Maybe state this more explicitly as it's not very clear at the moment. Or at least change "Greenlands fjord" to "Greenlandic fjords".
P7L18-19: "Greenland fjords, most of them do not have a floating tongue (tidewater glaciers) and we therrfore generally perform…" – this sentence has many errors. I'd suggest changing to "glaciers in Greenland, most of which do not have a floating tongue, and we therefore generally…"
P7L29: "(i.e. integral of the melt rate across entire surface area of the glacier front, of width W)" should be "(i.e the integral of the melt rate across the entire surface area of the glacier front, of width W)". But I also find this confusing because it sort of suggests that the conical plume covers the whole width W, so it might be better to say what the cumulative melt rate is separately for the line and conical plumes.
P8L3: "there are more than one channel" should be "there is more than one channel"
P8L8: "can already be determined by the look on the balance velocities" reads better as "is already suggested by the form of the balance velocities"
P8L24-25: "Slower velocity as a result negatively affects melting" would be better as "Reduced velocity in turn reduces melting"
P10L12-13: grammar: "in order not to resolve the small-scale turbulences" would be better as "in order to represent the small-scale turbulence which is not resolved"
P10L19-20: insert "the" before "simple plume parameterization" in line 19 and "our" before "plume parameterization" in line 20
P10L23: insert "fjord" before "with a resolution"
P11L8: change "to Store" to "of Store" and in line 8 insert "the" before "same"

P11L21: "without a background melting and 1.7 with background melting" – it's not clear what you mean here – please be more explicit.
P12L8: "of along the floating tongue" should be something like "incised into the underside of the floating tongue"
P12L22: "but correction for Coriolis effect" should be "but a correction for the Coriolis effect"
P13L9: add "the" before "previously"
P13L16: change "2 magnitudes" to "2 orders of magnitude"
P13L28-29: "might be diluted…" – this sentence says that the derivation might be diluted, but actually you mean that the CTD profile might be diluted, so this needs reforming.
P13L33: I don't think "section 7" exists?
P15L6: "marine-terminated" should be "marine-terminating"
P15L9: I think the correct reference is Slater et al (2015). Change "At last" to "Lastly".
P15L17: "that was used parameterization of the turbulence of the plume" should be something like "that was used to parameterize turbulent entrainment into the plume".
P16L19: insert "slowly varying" before "ice temperature"

References (only those not cited in the manuscript)

Holland, P. R., A. Jenkins, and D. M. Holland (2007), The response of ice shelf basal melting to variations in ocean temperature, Journal of Climate, doi: 10.1175/2007JCLI1909.1.

Linden, P. F., G. F. Lane-Serff, and D. A. Smeed (1990), Emptying filling boxes: the fluid mechanics of natural ventilation, Journal of Fluid Mechanics, 212, 309–335, doi: 10.1017/S0022112090001987.

Pedersen, F. B. (1980), Dense bottom currents in rotating ocean, Journal of the Hydraulics Division, 106, 1291–1308.

Stevens, L. A., F. Straneo, S. B. Das, A. J. Plueddemann, A. L. Kukulya, and M. Morlighem (2016), Linking glacially modified waters to catchment-scale subglacial discharge using autonomous underwater vehicle observations, The Cryosphere, 10 (1), 417–432, doi: 10.5194/tc-10-417-2016.

Straneo, F., and C. Cenedese (2015), The dynamics of greenland's glacial fjords and their role in climate, Annual Reviews of Marine Science, 7 (1), 89–112, doi: 10.1146/annurev-marine-010213-135133

Reviewer 4

> *Overall the authors have done a good job at responding to the earlier reviewer's comments, and the manuscript has clearly improved as a result. However, there remain a few areas where further work could add to the clarity of the presentation:*
>
> *1) The use of the term "parameterisation" confused me a little. It's not a strict definition, but "parameterisation" normally refers to an algorithm that operates on a model grid to reproduce the effects of processes that cannot be fully resolved by that grid. What this paper describes is a sub-model that operates on its own grid, and includes its own parameterisation of such processes as turbulent mixing. So I think a more appropriate title would be something like "Simple models for the simulation of submarine melt in a Greenland glacial system". Further analogous modifications would be needed in the text, which should refer to "plume models" rather than "plume parameterisations".*

We agreed with the reviewer and changed the term "parameterisation" to "model". Thus we changed the title and certain sentences accordingly.

> *2) On page 4, lines 6 to 9, the LP and CP models are briefly introduced. It is implied that the CP model naturally follows from the assumption of channelized outflow. That is not quite true. A further assumption that is implicitly made is that as the plume develops it maintains a self-similar half-conical form. It is far from obvious that a plume rising above a channelized outflow will do that. The isolated, point sources of buoyancy in quiescent environments, considered in classical plume studies, do indeed produce conical plumes, but in those cases there is nothing to produce an asymmetry. It seems likely that the poorer agreement obtained between the CP model and observations is a result of the imposed half-conical geometry being inappropriate.*

We agree that out sentence was inaccurate. We added a sentence on the CP geometry.

3) I'm not sure how the quoted values of M0 (page 5, line 14) were obtained. The value given in the Appendix (page 16, line24) lies outside of that range. Using the numbers in Jenkins (2011), I get a range of 6-7x10^(-6).

There was a mistake in the M0 range in the previous version of this manuscript, which should read 7-12x10^(-6). Our range obtained numerically is higher than when using the formulation by Jenkins (2011) in Table 2. This is due to the use of the simplified equation (10) in Jenkins (2011). We now write this more explicitly in the manuscript.

4) What do you mean by "regulate the regular grid size" (page 6, lines 4-5)? Do you adapt the grid size to ensure convergence?

In fact, we simply use a constant step size. It is now stated more clearly in the text. We also simplified other aspects of the description of the numerical procedure.

5) In the response to the earlier reviews you justified the use of T0=0 (page 6, line 16). I think that should be included here, because I also cannot see why you would not use the pressure freezing point given by equation (7) with Sb set to zero. That would be the most thermodynamically consistent choice.

We added the explanation in the text: "We choose T 0 = 0°C since the temperature of subglacial water is unknown, but for obvious reasons it cannot deviate significantly from 0 °C. For conditions typical for the Greenlandic environment, we did not find any significant change in melt rate when using the pressure melting point instead of T 0 = 0 ° C, since the plume temperature rapidly converges to a balance temperature close to ambient water temperature (Appendix, Figure A3)"

6) The paragraph on page 7, lines 21-26, would be better placed in sub-section 2.5, immediately after (rather than before) the sub-heading.

We agree and rearranged the paragraphs.

7) The discussion of melt rate sensitivity to discharge seems slightly misleading, as "high" and "low" discharge or not properly quantified. The quantification of the departure from a cubic root dependence as discharge tends to zero is a new result. The earlier analyses of Jenkins (2011) and Slater et al (2016) did not consider that extreme. However, both those earlier studies effectively found departures from the cube root dependence at high discharge. The reason is the growth of the physical length required for the temperature to adjust to its equilibrium value as the discharge increases. The cube root scaling is appropriate only when the plume has reached equilibrium, so if the adjustment phase becomes a significant fraction of the domain, the melt rate dependence on discharge will depart from the cube root scaling.

We rewrote large parts of section 3.1 for clarity. Following the reviewer's suggestion, we included an analysis of what "high" and "low" discharge mean, by mean of a critical discharge (Eq. A21-A23) that separates the two asymptotic regimes (derivation in the annex, mention in the main text). Moreover, we now mention this other interesting point of possible deviation from cubic root scaling when the plume is not equilibrated (large discharge, shallow fjord).

:

8) On pages 9 to 10, I'm not sure that I picked up the explanation of why the melt rate dependence on E0 changes sign at high versus low slopes. Isn't it because higher E0 always increases the plume temperature, but for high slope it also increases drag and slows the plume down? On low slopes entrainment has little impact on drag, which is dominated by friction at the solid ice-ocean interface, so the temperature effect wins. Maybe that's what you said, but I couldn't see that in your text.

Yes, an increase in E0 always increases the plume temperature but to a greater extent for long floating tongues than for tidewater glaciers (since in tidewater glacier equilibrated plume temperature is already close to ambient temperature, i.e. its maximum potential for melting). On the other hand the plume velocity increases with decreasing E0 but substantially for tidewater glaciers and to a lesser extent for long floating tongues (because of the effect mentioned by the reviewer, in which the total drag Cd + E0 sina becomes less sensitive to changes in E0 when sina is small, i.e. long floating tongue). We rewritten the section 3.2 of the manuscript to better describe these mechanisms, and we believe it is much clearer now.

9) You give the Rossby number as the key parameter determining the appropriateness or otherwise of neglecting the Coriolis force. But what about the Ekman number? That's effectively what Jenkins (2011) used to define the rotational length scales, isn't it? The argument put forward there is that given the plume scales, the Ekman number provides the stronger control. That is, the plume remains thin enough that friction dominates over rotation well beyond one Rossby radius from the grounding line.

Yes we agree with the reviewer and insert that the plume thickness is limited by the Ekman layer depth (Jenkins, 2011).

10) On the subject of rotation, it is interesting that you find a value of 0.036 for E0 to give the best results overall. Jenkins (1991) justified the use of such a low value (used to tune the model to match observation) as a way of compensating for the lack of the Coriolis force in the model. It is interesting that you find the same tuning improves the match with observation in very different cases, where the absence of the Coriolis force cannot be the reason such tuning is required. Although it is not well constrained, if a lower value of E0 is a universal requirement, presumably that result is telling us something about how the solid, melting interface affects the turbulent mixing?

Here, we can only speculate why the smaller entrainment factor E0 leads to overall better result when comparing to observational data. If the turbulent mixing is influenced by the solid ice material or maybe more by the fjord circulation is an interesting question that would need further investigation with detailed observational data close to the glacier front.

11) I was confused by the discussion of the geometry of Kangerlussuup Sermia (Page 13, lines 8-29). At one point you mention a 3 km floating tongue, but later you talk about a 200 m undercut at an angle of 77 degrees (from the vertical or horizontal?). I don't see how these are consistent.

Yes, we removed that part of the floating tongue and think it is much clearer now.

12) Is the equation for the buoyancy flux missing at the bottom of page 17?

Yes, the sentence of the buoyancy flux was misplaced and thus we deleted it.

Finally the manuscript is littered with typographical errors and minor grammatical and spelling mistakes. The revised version requires careful proof-reading before it is resubmitted.

We hope we improved to the reviewers satisfaction.

~~Submarine~~ Simple models for the simulation of submarine melt ~~parameterization~~ for a Greenland glacial system model

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Abstract. Two ~~hundreds~~ hundred of marine-terminating Greenland outlet glaciers deliver more than half of the annually accumulated ice into the ocean and play an important role in the Greenland ice sheet mass loss observed since the mid 1990s. Submarine melt ~~plays~~ may play a crucial role in the mass balance and position of the grounding line of these outlet glaciers. As the ocean warms, it is expected that submarine melt will increase~~and outlet glaciers will retreat ,~~, potentially driving outlet glaciers retreat and contributing to sea level rise. Projections of the future contribution of outlet glaciers to sea level rise ~~is~~ are hampered by the necessity to use models with extremely high resolution of the order of a few hundred meters~~both for modelling of the outlet glaciers and as well as~~. That requirement in not only demanded when modelling outlet glaciers as a stand alone model but also when coupling them with high resolution 3D ocean models. In addition fjord bathymetry data are mostly missing or are inaccurate (errors of several 100s of meters), which questions the benefit of using computationally expensive 3D models for future predictions. Here we propose an alternative approach ~~based on using of computationally efficient parameterization~~ built on the use of a computationally efficient simple model of submarine melt based on turbulent plume theory. We show that such ~~parameterization is in a~~ simple model is in reasonable agreement with several available modeling studies. We performed a ~~suit~~ suite of experiments to ~~analyse~~ analyze sensitivity of these ~~parameterizations~~ simple models to model parameters and climate characteristics. We found that the computationally cheap plume model demonstrates qualitatively similar behaviour as 3D general circulation models. To match results of the 3D models in a quantitative manner, a scaling factor in the order of one is needed for the plume models. We applied this approach to model submarine melt for six representative Greenland glaciers and found that the ~~parameterization~~ application of a line plume can produce submarine melt compatible with observational data. Our results show that the line plume model is more appropriate than the cone plume model for simulating the average submarine melting of real glaciers in Greenland.

1 Introduction

Since the 1990s the decadal loss of ice mass by the Greenland ice sheet (GrIS) has quadrupled (Straneo and Heimbach, 2013), with an average 1993-2010 contribution of 0.33 $\pm$ ~~8~~ 0.08 mm yr^{-1}, which is about 10 % of the observed sea level rise during this period (Church and White, 2011; Church et al., 2013). This acceleration of the GrIS mass loss is attributed to increase of surface melt due to atmospheric warming (Khan et al., 2014) and speedup of the marine-terminating outlet glaciers (Rignot and Kanagaratnam, 2006). The latter has been related, among other factors, to enhanced submarine melting, which in turn is caused by warming of the surrounding ocean (Straneo et al., 2012) and, probably, by increased subglacial water discharge (Straneo and Heimbach, 2013). While ice-ocean interaction potentially plays an important role in recent and future mass balance changes of the GrIS, the understanding of this interaction remains rather poor and represents one of the main source of the uncertainties in future sea level rise projection (Church et al., 2013).

The ice sheet models used for the study of GrIS response to global warming and its contribution to sea level rise typically have resolution of 5 to 10 kilometers (Bindschadler et al., 2013), which is too coarse to resolve most ~~of~~ Greenland outlet glaciers. Instead, regional modelling at higher resolution is better suited to capture glacier dynamics. As an alternative to costly three-dimensional models, one-dimensional flowline models were convincingly applied to several major outlet glaciers (Nick et al., 2012, 2013; Lea et al., 2014; Carr et al., 2015). In particular, Nick et al. (2012) simulated with a flowline model the dynamical response of the Petermann glacier to the abrupt break up of its floating tongue in 2010 and investigated the influence of increased submarine melting on future stability of the glacier. They demonstrated the strong influence of increased submarine melt rate to the glacier's mass loss. In this study, submarine melt rate was prescribed and held constant. ~~Nick et al. (2013) using~~ Using the same flowline model, Nick et al. (2013) implemented submarine melt proportional to the ocean temperature outside of the fjord. This study was performed for the four largest outlet glaciers. Under the assumption that the result of the four largest glaciers can be scaled up for the remaining glaciers, Nick et al. (2013) estimate a total contribution of the Greenland outlet glaciers to global sea level rise of up to 5 cm during the 21st century or about 50% of the maximum expected GrIS contribution due to changes in surface mass balance. For the same period of time but using a three-dimensional ice sheet model, Fürst et al. (2015) estimated the contribution of enhanced ice discharge through outlet glaciers to be 20% to 40% of the total mass loss. These large uncertainties are associated, among other factors, with the parameterization of the ~~rate of submarine melt~~ submarine melt rate. Note that in Fürst et al. (2015) the effect of ocean warming was parameterized through enhanced basal sliding rather than explicit treatment of submarine melt.

Different approaches have been ~~derived to calculate~~ taken to estimate submarine melt rates of outlet glaciers by using empirical data ~~(Motyka et al., 2013; Rignot et al., 2015)~~(Motyka et al., 2013; Rignot et al., 2010); simplified one dimensional models of line plumes (Jenkins, 1991, 2011), axisymmetric plume models (Cowton et al., 2015; Turner, 1973) and numerical two- and three-dimensional ocean models (3D models) ~~(?Little et al., 2009; Sciascia et al., 2013; Xu et al., 2013; Slater et al., 2015)~~(Holland et al., 2 Note that 2D and 3D modelling efforts also differ with respect to model formulation, in particular some authors use ~~non-hydrostatic~~ hydrostatic models (e.g. Holland et al., 2008; Little et al., 2009), while others use ~~hydrostatic~~ non-hydrostatic models (e.g. Sciascia et al., 2013). The experiments studied submarine melt with respect to subglacial discharge and its spatial pattern ~~,~~ and vertical ocean temperature and salinity profiles. Additionally the influence of subglacial discharge on the fjord circulation, which connects outlet glaciers with the surrounding ocean, were investigated with ~~the~~ 3D models (Cowton et al., 2015; Carroll et al., 2015). Different authors considered two main types of subglacial discharge. The first one is uniformly distributed along the grounding line (referred hereafter as 'line plume', LP) (Jenkins, 1991, 2011; Sciascia et al., 2013; Slater et al., 2015; Xu et al., 2012) while the second one ~~is localized~~ has localized subglacial discharge (the axisymmetric plume, referred hereafter as 'cone plume', CP) (Cowton et al., 2015; Turner, 1973; Slater et al., 2015; Xu et al., 2013). The CP approach is motivated by the observations that a significant fraction of subglacial discharge during the melt season emerges through one or several channels underneath the glacier (Rignot et al., 2015; Stevens et al., 2016; Sole et al., 2011). ~~These simulations~~ All of the above mentioned 2D and 3D model simulations show, in agreement with previous theoretical studies, ~~show~~ that submarine melt strongly depends both on the ambient water temperature and the magnitude of subglacial discharge. However different modeling studies revealed the complex dependence of submarine melting on temperature. Sciascia et al. (2013) investigated tidewater glaciers and found a linear dependence of the submarine melt rate on ambient water temperature above freezing point. On the other hand, Holland et al. (2008) and Little et al. (2009) found a quadratic dependence on temperature under large ice shelves~~, where geostrophic flow becomes significant~~. Xu et al. (2013) detected that this relationship of melt rate to thermal forcing depends on the amount of subglacial discharge released through a single channel at a tidewater glacier: the melt rate dependence to temperature has a power of 1.76 for small discharges and is lower for higher discharge. Slater et al. (2016) found a power law dependence of melt rate on discharge, with the exponent $\frac{1}{3}$ for both the CP and the LP models in a uniform stratification. For a linear stratification their study shows that the exponent enlarges to $\frac{3}{4}$ for the CP model and to $\frac{2}{3}$ for the LP model. A change in power law could also be detected by Xu et al. (2013). They determined an exponent of 0.5 at high and 0.85 at low discharge for the CP. Simulations with 3D models ~~, which differ with respect to boundary conditions and turbulence parameters, show a variety~~ show the strong dependency of CP melt rate ~~profiles: Kimura et al. (2014) showed a melt rate profile of the CP that reaches its maximum near the water surface while Slater et al. (2015) and Xu et al. (2013) found a CP melt rate profile with the maximum located near~~ on stratification or other environmental factors, with maximum melt rate near the surface (e.g. Kimura et al., 2014, unstratified) or close to the bottom ~~.~~

(e.g. Slater et al., 2015; Xu et al., 2013, stratified). While experiments with high-resolution (several to ten meters) ~~nonhydrostatic~~ non-hydrostatic 3D ocean models demonstrate their potential to simulate rather realistically turbulent plumes and melt rates of marine-based glaciers, such models are too computationally expensive for modeling of the entire Greenland glacial system response to climate change at centennial time scale. An alternative is to use a ~~parameterization of~~ method for submarine melt based on a simplified plume model (Jenkins, 2011; Cowton et al., 2015). Such ~~parameterization~~ a plume model can then be used to calculate ~~submarime melt in a~~ submarine melt in e.g. 1D ice stream models. This would represent a step forward compared to a rather simplistic treatment of submarine melt used in previous works (e.g., Nick et al., 2013).

The main purpose of this paper is to investigate the applicability of the simple plume ~~parameterizations~~ models to simulation of melt rate of real glaciers in Greenland. To this end we first compared both cone and linear plume ~~parameterizations~~ models with the available results of simulations from high resolution 3D ocean models. Then we compare results of plume ~~parameterizations~~ models with the empirical estimates of submarine melt from several Greenland glaciers.

The paper is organized as follows. The two versions of plume model are described in the section 2. ~~There we~~ We then study the plume models sensitivity of simulated submarine melt rate to ocean temperature and salinity, the amount of subglacial discharge and to the ~~ice tongue~~ geometry of the calving front of the glacier itself. Results of simulations with the simple plume ~~parameterizations~~ models are compared to results of numerical experiments with 3D and 2D ocean models in section 4. In section 5 we compare our simulations to empirically estimated submarine melt rates for several selected Greenland glaciers. Finally, in the section 6 we discuss the applicability of the plume ~~parameterization~~ model for the purpose of developing a comprehensive Greenland glacial system model.

2 The plume models

A plume model describes buoyancy-driven rise of subglacial meltwater after it exits subglacial channels, until it reaches neutral buoyancy near the surface. Two counteracting processes control its evolution, which are (a) ~~additional melting~~ submarine melting of the ice-ocean interface under the floating tongue (if any) and ~~along the glacier~~ upwards along the calving front, and (b) turbulent entrainment and mixing of surrounding fjord water. ~~They~~ These processes act to maintain, or reduce, plume buoyancy, respectively.

Subglacial meltwater discharge Q_{sg} for a glacier can be estimated from surface runoff and basal melt over the catchment area of the glacier. How this discharge is distributed along the grounding line, however, is in general not known. It is believed that at least during the summer season, most of the subglacial discharge occurs through a network of channels (Chauche, 2016; Rignot and Steffen, 2008; Rignot et al., 2015; Schoof, 2010) but their precise number for different glaciers and relative importance is not known and can change throughout the season.

We investigate two situations. The line plume (LP) model corresponds to the simplest assumption that Q_{sg} is uniformly distributed along the grounding line (Fig. 1a), while the cone plume (CP) assumes point-wise release of meltwater (Fig. 1b), i.e. from a channel whose dimensions are small compared to the plume diameter. ~~Note there need not be only one CP: they~~ Furthermore, the CP model assumes that a self-similar half-conical form is maintained. Note that there can be a number of ~~them discretely distributed~~ discretely distributed plumes along the glacier, implying numerous CPs.

2.1 Model equations

Both models are formulated in one dimension, x, which is the distance from the grounding line upwards along the glacier front~~from the grounding line~~ , or under the ice shelf, and depends on the glacier shape, described by its slope α (Fig. 1). The model equations are written under the assumption that the plume is in equilibrium and therefore do not explicitly account for time. All model parameters and their description are listed in Table 1.

2.1.1 Line plume

The LP model after Jenkins (2011) accounts for a uniformly distributed subglacial discharge along the grounding line of a glacier (Fig. 1a). Far enough from the lateral boundaries, it assumes invariance by translation along the grounding line, so that the resulting equations only depend on x with $\frac{d()}{dx} = ()'$:

$$q' = \dot{e} + \dot{m} \tag{1}$$

$$(qU)' = D\underline{\Delta\rho}\frac{\Delta\rho}{\rho_0} g sin(\alpha) - C_d U^2 \tag{2}$$

$$(qT)' = \dot{e}T_a + \dot{m}T_b - C_d^{\frac{1}{2}} U \Gamma_T (T - T_b) \tag{3}$$

$$(qS)' = \dot{e}S_a + \dot{m}S_b - C_d^{\frac{1}{2}} U \Gamma_S (S - S_b) \tag{4}$$

where the plume state variables D, U, T and S stand for its thickness, velocity in the ~~x-direction~~x-direction, temperature and salinity, all dependent on ~~x~~x. Equation (1) describes the conservation of volume flux $q = DU$ (expressed per unit length in the lateral direction, i.e. $\mathrm{m}^2\mathrm{s}^{-1}$), which can increase by the entrainment of ambient seawater $\dot{e}$ and by melting $\dot{m}$ of ice from the glacier front. The momentum flux (Eq. 2), is based on the balance between buoyancy flux and the drag $C_d U^2$ of the glacier front. The buoyancy flux is proportional to the ~~density contrast $\Delta\rho$~~ relative density contrast $\frac{\Delta\rho}{\rho_0}$ between plume water and ambient water in the fjord (subscript a), parameterized in linear form as $\beta_S(S_a - S) - \beta_T(T_a - T)$, with coefficient β_S and β_T indicated in Table 1. The drag also results in a turbulent boundary layer (~~subscribt~~ subscript b) at the ice-water interface, where melting occurs, and heat and salt is exchanged by (turbulent) conduction-diffusion. The Equations for T and S (Eq. 3,4) account for the entrainment of ambient water and the addition of meltwater, as well as for conduction fluxes at the ice-water interface (i.e. between boundary layer and plume). The entrainment rate is calculated as $\dot{e} = E_0 U \sin(\alpha)$, proportional to plume velocity and glacier slope, with coefficient E_0. The melt rate is calculated by solving ~~for~~ heat and salt conservation at the ice-water boundary ($\dot{m}$, T_b and S_b are unknown):

$$\dot{m}L + \dot{m}c_i(T_b - T_i) = cC_d^{\frac{1}{2}}U\Gamma_T(T - T_b) \tag{5}$$

$$\dot{m}(S_b - S_i) = C_d^{\frac{1}{2}}U\Gamma_S(S - S_b) \tag{6}$$

where the subscript i for temperature and salinity refers to the inner ice, and c is the specific heat capacity. The system is closed by an expression of the freezing temperature T_b, which can be linearly approximated as a function of depth Z ($Z < 0$) and salinity of the boundary layer S_b:

$$T_b = \lambda_1 S_b + \lambda_2 + \lambda_3 Z \tag{7}$$

with coefficients λ_i listed in Table 1. For a straight wall, $Z = Z_0 + x \cdot sin(\alpha)$, where Z_0 is the negative depth at the grounding line ($x = 0$). Solving for equations (5-7) yields a second order polynomial equation for the melt rate $\dot{m}$, as a function of plume state variables. Note that Jenkins (2011) also uses an approximation of the melt rate equations, which resolves in $\dot{m} = M_0 U(T - T_f)$, where $T - T_f$ is the plume temperature above the plume freezing point, and M_0 is a slowly varying function of ice temperature below plume freezing point. ~~Numerically~~After Jenkins (2011), with a simplified formulation for the heat balance at the ice ocean interface, M_0 varies from ~~$2.9 \cdot 10^{-6}$ to $0.910^{-9}(°C)^{-1}$ over a $T_i - T_f$~~ $6 \cdot 10^{-6}$ to $7 \cdot 10^{-6}(°C)^{-1}$ over a T_i range from $-20°C$ to ~~$0°C$, respectively, and the freezing temperature is roughly~~ $-2°C$ and a plume freezing temperature calculated with equation (7) for the plume salinity varying from 0 to 35 psu (fast entrainment of ambient salinity), resulting roughly in $0°C$ to $-2°C$~~(Annex A~~. Numerically, by calculating M_0 with $M_0 = \dot{m} \cdot (U(T - T_f))^{-1}$ from our experiments (fixed $T_i = -10°C$), M_0 is slightly higher and can vary from $7 \cdot 10^{-6}$ to $12 \cdot 10^{-6}(°C)^{-1}$ (Fig. A1). We do not use this approximation in our calculation, but this is nevertheless helpful to interpret some of the results presented in our manuscript, in particular ~~the dependence of~~ in quantifying the amount of melt rate and simplifying the melt rate ~~on plume velocity (Annex~~ dependence on temperature and subglacial discharge (Appendix A).

2.1.2 Cone plume

The second plume model investigated in this paper is the CP model (Cowton et al., 2015). It differs from the LP model by the geometry of the plume, which resembles the half of an upside-down cone (Fig. 1b). In that case, the plume has definite dimensions and fluxes are expressed in full units ($\mathrm{m^3 s^{-1}}$). A cross-section of the plume is half a disk with area $\frac{\pi}{2}D^2$ where the length scale D is here the cone radius at a given x. The equations (1)-(4) now reform for the CP model by considering melting on the diameter $2D$ and entrainment around the arc πD:

$$Q' = (\pi D)\dot{e} + (2D)\dot{m} \tag{8}$$

$$(QU)' = (\frac{\pi}{2}D^2)\cancel{\Delta\rho}\frac{\Delta\rho}{\rho_0}g\sin(\alpha) - (2D)C_d U^2 \tag{9}$$

$$(QT)' = (\pi D)\dot{e}T_a + (2D)\dot{m}T_b - (2D)C_d^{\frac{1}{2}}U\Gamma_T(T - T_b) \tag{10}$$

$$(QS)' = (\pi D)\dot{e}S_a + (2D)\dot{m}S_b - (2D)C_d^{\frac{1}{2}}U\Gamma_S(S - S_b) \tag{11}$$

where variables, parameters and equations have the same meaning as for the LP model, and the volume flux $Q = \frac{\pi D^2}{2}U$ is expressed in cubic meters per second.

2.2 Numerics

For the differential equation system of (1)-(4) and (8)-(11) we choose a classical Runge-Kutta-scheme ~~in which we can regulate the regular grid~~ with constant step size Δx. ~~Thus we have control of the numerical calculation time and can easily vary $T_a(Z)$ and $S_a(Z)$ for a stratified environment. Furthermore for glaciers with floating tongues~~ Note that T_a, S_a and $\sin(\alpha)$ can vary as a function of X ~~and therefore~~, so that the model can ~~adjust dynamically to the glacier in a coupled glacier-plume version. With these initial conditions for the plume T, S, U, D at $x = 0$ we solve the~~ be applied on any glacier geometry. Initial conditions for D, U, T, S are needed at $X = 0$. Then at every step, we first solve the boundary layer equations (5)-(7) ~~and firstly determine the melt rate $\dot{m}|_{x_0}$ and the boundary conditions $S_b|_{x_0}$ and $T_b|_{x_0}$. These determined variables serve as the input parameters for the differential euqation system to determine the plume properties at the next step $X_{i+1} = X_i + \Delta x$. This routine is continued to determine the melt rate as a function of x~~, then compute the increments in the differential equation system of the LP (1)-(4) or CP (8)-(11) model. The procedure is continued until the plume reaches zero velocity or the water surface. The code is written in Python and Fortran for future coupling.

2.3 Initial conditions and balance velocity

In the rest of the manuscript, for simplicity, we refer to the boundary condition at $x = 0$ as "initial conditions" although the model equations are not time dependent. Since subglacial discharge consists of melt water, the salinity and temperature of subglacial discharge water can be set to zero ($S_0 = 0$ and $T_0 = 0$). We choose $T_0 = 0$ since the temperature of subglacial water is unknown, but for obvious reasons it cannot deviate significantly from $0°C$. For conditions typical for the Greenlandic environment, we did not find any significant change in melt rate when using the pressure melting point instead of $T_0 = 0°C$, since the plume temperature rapidly converges to a balance temperature close to ambient water temperature (Appendix, Figure A3). For both LP and CP models, initial dimensions (radius or thickness) D_0 and velocity U_0 are not known, but they are tied by subglacial discharge. In the CP case $Q_{sg} = \frac{\pi}{2} D_0^2 U_0$ (Fig. 1b) while for the the LP case, the subglacial discharge per glacier width W enters the model equations: $q_{sg} = \frac{Q_{sg}}{W} = U_0 D_0$ (Fig. 1~~).~~

a). It turns out that for a given subglacial discharge, simulated velocity rapidly adjusts to a "balance" velocity, regardless of the initial velocity (Fig. 2a), as already noticed by Dallaston et al. (2015). Analytically, the balance velocity (noted $U_\star(x)$ below) is solution of the plume equations (1)-(2) and (8)-(9) when the transient term U' is neglected. The fast adjustment around $x = 0$ (where plume dimension is small) can be explained by some rearranging into a form analogous to a first order linear differential equation for U^2 (see Appendix section A2.2). The balance velocity is not necessarily constant, but a simple expression for $U_{\star 0}$ (at $x = 0$) can be derived, if the plume dimension is expressed as a function of subglacial discharge, and the melt rate is neglected compared to entrainment in the volume flux equations (1) and (8) (see Appendix section A1). We obtain for the LP:

$$U_{\star 0} = \left(\underline{\frac{g \Delta\rho|_{x_0} \sin(\alpha)}{E_0 \sin(\alpha) + C_d}} \frac{g \frac{\Delta\rho}{\rho_0}|_{x_0} \sin(\alpha)}{E_0 \sin(\alpha) + C_d} q_{sg} \right)^{\frac{1}{3}} \tag{12}$$

and for the CP:

$$U_{\star 0} = \left(\frac{\pi}{2} \underline{\frac{(g \Delta\rho|_{x_0} \sin(\alpha))^2}{(\pi E_0 \sin(\alpha) + 2C_d)^2}} \frac{(g \frac{\Delta\rho}{\rho_0}|_{x_0} \sin(\alpha))^2}{(\pi E_0 \sin(\alpha) + 2C_d)^2} Q_{sg} \right)^{\frac{1}{5}} \tag{13}$$

Note that equation (12) is identical to the velocity derived by Jenkins (2011), and equation (13) is analogous to equation (5) in Slater et al. (2016), with the addition of the basal drag term. These balance solutions are only valid in the vicinity of the grounding line and velocity might then differ substantially as the plume develops, especially for small subglacial discharge (e.g. Magorrian and Wells, 2016). More detailed discussion and full, depth-dependent solution for the LP model are given in the Appendix.

Our sensitivity tests show that initial velocities higher than $U_{\star 0}$ lead to maximum melting ~~near the bottom~~ close above the grounding line of the glacier ('undercutting') while for lower velocities the melt rate increases with height and ~~would leave a so-called 'toe' at the glacier bottom~~ maximum melting is located further up the calving front (Fig. 2b). We checked that initial velocities smaller than the balance velocity yield very small difference in the cumulative melt rate (Fig. 3), although some differences occur for larger velocities. For the LP model an initial velocity ten times larger than the balance velocity gives a 10% higher melt rate while the CP model produces 25% more melting (Fig. 3).

Since the velocities of subglacial discharge are mostly unknown, these results prompted us to use the balance velocity of Eqs. (12) and (13) as initial condition in all experiments described below, unless stated otherwise.

2.4 ~~Default experimental setting~~Comparison between LP and CP models

~~In the next sections we perform a number of sensitivity studies with respect to key parameters. To that end we choose a default experimental setting as a benchmark. Unless otherwise stated, we consider a 500-m deep, well-mixed fjord with ambient temperature $T_a = 4°C$ and salinity $S_a = 34.65$ psu (maximal melting conditions for Greenlands fjord), with total subglacial water discharge of $\mathrm{q_{sg}} = 0.1\,\mathrm{m^2s^{-1}}$ for the LP model or $\mathrm{Q_{sg}} = 500\,\mathrm{m^3s^{-1}}$ for the CP model (which corresponds approximately to the discharge in August 2010 of the 5 km wide Store glacier (Xu et al., 2012)). Since we apply our model to Greenland fjords, most of them do not have a floating tongue (tidewater glaciers) and we therrfore generally perform experiments for a vertical wall ($sin(\alpha) = 1$). Default model parameters, including entrainment rate E_0, are indicated in Table 1.~~

A direct comparison between LP (defined per unit ~~length~~width of the grounding line) and CP (point-wise) models, requires an assumption about a length scale W (for LP) and the number of sources (for CP) over which subglacial discharge is distributed. For the CP model we assumed that the entire subglacial discharge occurs through one channel in the center of the glacier ($\cancel{Q_{sg}}Q_{sg} = 500m^3/s$). In the case of the LP model we assumed that the discharge is uniformly distributed over a fjord width $W = 150m$, so that $q_{sg} = 3.6m^2/s$. This width is about the maximum size of the plume in the CP model, near the surface, for this setting.

2.5 ~~Comparison between LP and CP models~~

Results in Figure 4 show that simulated, local melt rate ~~is~~ can be higher in the CP model than in the LP model~~practically for all depths~~, but cumulative melt rate (i.e. the integral of the ~~meltrate~~ melt rate from the bottom and across entire surface area of the glacier front, of width W) is much higher for the LP model because of the larger surface area over which melting occurs (roughly a factor two in our chosen setting).

We shall see later in this manuscript (sec. 3.1) that the (local) melt rate in the LP model varies less than linearly with subglacial discharge parameter q_{sg}, and thus for a given total discharge Q_{sg}, cumulative LP-induced melt increases with width. As a result, for a wide glacier (i.e. the glacier which is much wider than the maximum diameter of the CP), the LP model gives much higher cumulative melt rate compared to the CP model, when assuming the existence of a single subglacial channel. The situation when there ~~are~~ is more than one channel is discussed in section 4.3.

2.5 Default experimental setting

In the next sections we perform a number of sensitivity studies with respect to key parameters. To that end we choose a default experimental setting as a benchmark. Unless otherwise stated, we consider a 500-m deep, well-mixed fjord with ambient temperature $T_a = 4°C$ and salinity $S_a = 34.65$ psu (maximal melting conditions for Greenlandic fjord), with total subglacial water discharge of $\mathrm{q_{sg}} = 0.1\,\mathrm{m^2s^{-1}}$ for the LP model or $\mathrm{Q_{sg}} = 500\,\mathrm{m^3s^{-1}}$ for the CP model (which corresponds approximately to the discharge in August 2010 of the 5 km wide Store glacier (Xu et al., 2012)). Since we apply our model to glaciers in Greenland fjords, most of which do not have a floating tongue (tidewater glaciers), and we therefore generally perform experiments for a vertical wall ($sin(\alpha) = 1$). Default model parameters, including entrainment rate E_0, are indicated in Table 1.

3 Sensitivity experiments

3.1 Subglacial discharge

It is known that melt rate depends strongly on subglacial discharge. In agreement with previous studies (Jenkins, 2011; Slater et al., 2016) our model shows a cubic root-dependence of the cumulative melt rate on discharge for the LP and for the CP (for the high discharge range) ~~and for the CP~~ in a well-mixed environment. ~~Note that for the LP this dependence can already be determined by the look on the balance velocities $U_{\star 0}$ (Eq. 12) . However for smaller discharge in a well-mixed environment,~~ However for small discharges ($q_{sg} \rightarrow 0$) cumulative melt rate converges to a small but not insignificant value that does not obey the power law any more (Fig. 5). This value represents background melt rate, or 'melt driven convection', which does not depend on discharge and can be representative for winter melt rate when subglacial discharge is very small ~~.~~ (Magorrian and Wells, 2016).

To explain this change of power law we undertook a dimensional analysis to obtain theoretical solutions for the ~~plume model (~~LP in a well-mixed fjord (Appendix A). ~~Important is that, in a well mixed fjord,~~ The velocity, which determines the melt rate ~~is linear dependent on the velocity of the plume . This velocity is dependent on discharge and rather constant over the glacier front for big discharge or independent on sublgacial discharge and will accelerate along the x direction for small discharges (Annex **??**, Fig. A2). Therefore one can separate the plume behaviour into regimes for high and low discharges. We also derived an~~ (Eq. A1, Fig. A1), is initially controlled by subglacial discharge (see section 2.3), and subsequently accelerates as the plume develops. Our analysis shows that the plume acceleration depends on ambient hydrographic conditions, entrainment and glacier front characteristics, but is independent from subglacial discharge (Eq. A17-A19). As a consequence, for a given glacier, there is a high-end regime of purely discharge-driven convection (high discharge, high melt, near-constant velocity) and a low-end regime of purely melt-driven convection (zero discharge, low melt, pure acceleration) (Eq. A21 and A22). We defined a critical discharge q_c to characterize the transition between the two limiting regimes (Eq. A23).

q_c depends on glacier characteristics, and especially on the presence and length of a floating tongue. For tidewater glaciers, it is very low (of the order of $10^{-3} m^2/s$), so that summer discharge is sufficient to trigger a purely discharge-driven plume. In a glacier with a long floating tongue, q_c can be two orders of magnitude larger, thus melt-driven convection may contribute all year long. Our approximate analytical solution for the cumulative melt rate (~~Eg~~Eq. A21) ~~that is also~~ is displayed in Figure 5~~.~~, and is in reasonable agreement with the line plume result over the full discharge range.

~~However~~Note however that this analysis holds when the plume reaches a dynamic equilibrium. Slater et al. (2016) found deviation from the cubic root power law even for large discharge, for shallow fjord, when the adjustment length scale is large and a significant portion of the plume is not equilibrated.

In addition, as mentioned in the introduction, stratification can change this power law. We also performed experiments with stratification as in ~~(Xu et al., 2013)~~ Xu et al. (2013) for different discharges with the CP model and LP model. By assuming a melt rate equation of $\dot{m} = a \cdot Q^{\beta} + b$ as in (Xu et al., 2013), we derived β numerically for both models and listed them in (Tab. 2). The CP model shows values close to Xu et al. (2013). Both models show an increasing exponent for lower discharge~~(Tab. 2)~~.

3.2 Entrainment rate

Entrainment is the mechanism through which the volume flux of the plume increases with distance from its source, as warmer, saltier fjord water mixes into the plume. This leads to more heat ~~available~~ availability for melting, but on the other hand to decreased buoyancy - and velocity - as the plume gets saltier. ~~Slower velocity as a result negatively affects~~ Reduced velocity in turn reduces melting (Eq. 5, 6) (Carroll et al., 2016) (note the plume also becomes thicker to ~~accomodate~~ accommodate for increased volume flux and decreased velocity). In this section we investigate ~~what is~~ the net effect of these processes on melting for typical plume configurations.

In both plume models, entrainment depends on an entrainment rate parameter ~~E_0~~ and on glacier slope, as $E_0 sin\alpha$ (sec. 2.1)~~, which~~. E_0 is not accurately known and can be regarded as a tunable parameter within a certain range of values known from ~~pevious~~ previous work. Laboratory experiments for a pure vertical plume~~and model studies gives for E_0~~, model studies and theoretical considerations give a broad range for E_0, from 0.036 to 0.16 ~~(Jenkins, 2011; McConnochie and Kerr, 2016; Kaye and Linden, 20~~ (Jenkins, 2011; Kaye and Linden, 2004; Mugford and Dowdeswell, 2011; Kimura et al., 2014; Carroll et al., 2015; McConnochie and Ker Nevertheless, glacier slope $sin\alpha$ can vary by two orders of magnitude, so that regardless of the value of E_0 within the reported range, tidewater glaciers ($sin\alpha \sim 1$) are characterized by a high-entrainment regime, while glaciers with long floating tongue ($sin\alpha \ll 1$) have a low-entrainment regime.

~~It is known that the cumulative melt~~ Re-arranging the LP equations (1) and (2) shows that entrainment acts as an effective drag $C_d + E_0 sin\alpha$ (e.g. Eq. A15), with $C_d \ll E_0 sin\alpha$ for tidewater glaciers ~~decreases with increasing E_0 (Jenkins, 2011; Magorrian and W~~ ~~the reported~~ (i.e. controlled by entrainment), whereas $C_d \gg E_0 sin\alpha$ for glaciers with a long floating tongue (i.e. controlled by solid friction at the ice interface, insensitive to entrainment) (Fig. A3b). On the other hand, plume temperature depends on the mixing ratio of melt water to entrained water (eq. A9-A10), which is close to zero in tidewater glaciers, so that equilibrium plume temperature is nearly equal to ambient temperature in the full range of E_0 values~~). The melting is controlled in first order by the plume velocity and only to a lesser extent by availability of heat through mixing for stong subglacial discharge (Magorrian and Wells, 2016). Moreover the relative effect of entrainment can dominate the effect of the~~, i.e. already at its maximum potential for melt and insensitive to E_0 (Fig. A3c). In the low-entrainment regime characterizing long floating tongue, the mixing ratio of melt to entrainement is significant, so that temperature is strongly controlled by E_0 (Fig. A3c).

As a result, for the LP model, an increase in E_0 leads to less melting in tidewater glaciers (Fig. 6a), where the plume slows down, but increases the melt rate for glaciers with long floating tongues, where enhanced mixing results in more heat available for melting (Fig. 6b). This effect is particularly strong for warm ambient temperature (Fig. ~~) . E.g. A LP with low entrainment E_0 in a colder ambient temperature of $T_a = 3°C$ will result in higher~~ 7) and strong discharge (Fig. 6). LP cumulative melt rate ~~(due to its higher velocity) than for a plume with high~~ can decrease by over 42 % over the full E_0 ~~in a warmer fjord of $T_a = 4°C$.~~

~~For the CP model~~ range in a tidewater glacier (Fig. ~~6c)the dependence on~~ 7). In the CP model, the same physics applies and determines local melt rates, but entrainment also influences the plume radius, which grows faster with larger entrainment. As a result, even in tidewater glaciers where local melt rates decreases with E_0~~is opposite: cumulative melt rate increases with the~~

~~entrainment factor. This is due to the faster growing plume radiuswith higher entrainment, which leads to a~~ , the larger area of the plume in contact with the ice ~~, and thus~~ leads to more melting overall, ~~despite the lower local melt rate .~~ and *in fine* to increasing cumulative CP melt rate with entrainment factor E_0 (Fig. 6c).

3.3 Ambient temperature and stratification

Different fjords are characterized by different temperature and salinity profiles. Since the temperature of the ocean is projected to increase with global warming, dependence of melt rate on ocean temperature is crucial to study when investigating glaciers response to global warming. Previous experiments with 2D and 3D ocean models~~showed different~~ , as well as analytical solutions (Jenkins, 2011; Sciascia et al., 2013; Carroll et al., 2015; Jenkins, 2011; Magorrian and Wells, 2016; Slater et al., 2016; Carroll e the behavior of the cumulative melt rate as a function of the ambient temperature T_a. Figure 8 shows for both plume models the dependence of cumulative melt rate on temperature in a well-mixed ambient environment for different values of subglacial discharge. Both models show for small discharge a non-linear dependence of the melt rate on water temperature. If the discharge is very small, ~~ambient properties dominate~~ melt-driven convection dominates, and ambient properties determine the melting process ~~and one can speak of a 'melt driven convection' (Slater et al., 2015).~~

~~If we~~ (Slater et al., 2015). We assume a power law dependence of the cumulative melt rate per glacier area to the thermal forcing, i.e. ~~$\bar{m} \propto TF^{\beta}$, where $TF = T_a - T_{af}$~~ $\bar{m} \propto \Delta T_a^{\beta}$, where $\Delta T_a = T_a - T_{af}$ and T_{af} is the freezing temperature of the sea water at the fjord bottom~~,~~ . As listed as in table 3 we find that the exponent β increases with lower discharge. From 1.2 (high discharge ~~$q = 0.1$~~ $q_{sg} = 0.1$ m^2s^{-1}) to 1.8 ~~$q = 10^{-6}m^2s^{-1}$~~ ($q_{sg} = 10^{-6}m^2s^{-1}$) for the LP , and from 1.2 (high discharge $500\,\mathrm{m^3s^{-1}}$) to 1.5 (low discharge $Q_{sg} = 0.005\,\mathrm{m^3s^{-1}}$) for the CP. For the LP the range of this ~~increment~~ increase compares well to analytical solutions while the CP model ~~seems not to show this change in power law for analytical solutions (table~~ can only be compared to the analytical solution of high discharge range (Table 3).

~~An increment of the power law has also been detected when we use~~ The exponent also increases when using a realistic stratification (Fig. 9b). For the LP, we calculated an exponent of 1.2 for high discharge and 1.4 for low discharge, while the CP model shows a similar increase from 1.1 to 1.3. Carroll et al. (2015) showed that plume ~~thoery~~ theory gives a good approximation of the outflow height for ~~3d nonhydrostatic~~ 3D non-hydrostatic plume model but nevertheless ~~do~~ the exponents differ slightly in the experiment by Xu et al. (2013)~~(table 3)~~.

3.4 Glacier front angle for the line plume

The ~~Glacier~~ glacier front angle $\sin(\alpha)$ linearly impacts buoyancy (Eq. 2) and entrainment. For glaciers with a floating tongue, and therefore a smaller angle (~~$\sin(\alpha) << 1$~~$\sin(\alpha) \ll 1$), entrainment is reduced and so the temperature of the plume (~~10 c~~Fig. 10 b). The dependence of melt to the slope of the glacier for small discharges has been derived by Magorrian and Wells (2016)~~but note that the choice of E_0 can have a similar effect as the choice of subglacial discharge as depicted in Figure 10.~~ .

A glacier with a long floating tongue, and therefore a ~~smaller angle(i.e. $\sin(\alpha) = 0.02$)~~small angle, has a smaller average melt rate than a tidewater glacier ~~. However in this case higher E_0 leads to higher cumulative melting~~ but a higher cumulative melt rate (Fig. 6 b). These high cumulative melt rates (Fig.10) occur due to the longer distance under a floating tongue ~~in which the velocity accelerates~~ over which melting occurs. Furthermore, for long floating tongues, the plume velocity accelerates along the shelf (10 b)~~. The theroetical explanation of the evolution of U, $\dot{m}$, T is explained in the Annex (A and summarized in ??).~~ with a square root dependency on the distance (Eq. A19). This is consistent with our analysis for a LP model in a well-mixed environment, which demonstrates that glaciers with a long floating tongue have a high critical discharge (Eq. A23), and thus a larger contribution of melt-driven convection compared to tidewater glaciers.

However, for small α ~~both plume models are not~~ none of the plume models is applicable along the total shelf because they do not take into account Coriolis force and ~~therefore~~ plume thickness is limited by the Ekman layer depth (Jenkins, 2011). Therefore, for the total shelf area, the plume models likely strongly overestimate plume velocity and melt rate (see more in section 5.1).

4 Comparison with general circulation models

4.1 Background

Studies of turbulent plumes caused by subglacial discharge and their effect on submarine glacier melting have been performed using 2D and 3D non hydrostatic general circulation ocean models (GCM) (Sciascia et al., 2013; Xu et al., 2012, 2013; Kimura et al., 2014; Slater et al., 2015). ~~Although these models contain the right physics~~ These models are much more complex than our simplified 1D equations, which enable them to simulate plume ~~dynamics, the problem is that it requires very~~ processes in greater details. On the other hand, they require multi-dimensional grids with high spatial resolution, which is computationally ~~too expensive~~ prohibitive for our purpose ~~. In order not to resolve the small-scale turbulences, a parameterization for turbulent diffusivityis chosen to represent~~ of simulating a large number of Greenland glaciers.

These models typically parameterize unresolved, subgrid-scale ~~mixing~~turbulence with a turbulent diffusivity. Kimura et al. (2014) and Slater et al. (2015) tuned the diffusivity in such a way that the axisymmetric simulated plume (without ice contact) showed the same characteristics as the analytical models of Turner (1973) and Morton et al. (1956). Xu et al. (2013) used a high spatial resolution in order to ~~resolve turbulence explicitly~~reduce the amount sub-grid processes. These models were run for idealized fjord configuration with constant subglacial discharge and a vertical ice front. In most LP experiments, where subglacial discharge was uniformly distributed along the glacier grounding line, 2D settings were chosen. The melt rate in these experiments was computed using equations (5 -7). Since these models are more advanced compared to ~~simple plume parameterization~~ the simple plume model used in this study, it is informative to compare results of ~~plume parameterization~~ our plume models with these models.

4.2 Line plume simulations

We compare the melt rate profiles obtained in the experiments by Sciascia et al. (2013) with the LP model. Sciascia et al. (2013) used a 2D GCM with a single 10 m wide grid cell for the width and a 600m deep and 160 km long fjord with a resolution of 10 m $\times$ 10 m. For ~~this~~ our simulation we used the same temperature and salinity profiles as in Sciascia et al. (2013) and the same subglacial discharge per unit of glacier front ($q_{sg} = 0.43\,\mathrm{m^2 s^{-1}}$). We used an entrainment factor of E_0 =0.08 consistent with their experiments. The vertical melt rate profile of the simulated LP model resembled that of the melt rate simulated by the 2D GCM model but is systematically overestimated by the LP model. If we apply a scaling factor of 0.48 to the results of the LP model, the two profiles are in reasonable agreement. Still, there are some differences. The melt rate simulated by Sciascia et al. (2013) declines with height while the LP model simulates a constant melt rate over a broad depth interval. This is due to the fact that the plume model is not applicable in the vicinity of the fjord surface. A similar effect is seen in the 2D experiment of Xu et al. (2012) in figure 11 a). Again, the LP model overestimates the melt rate but when scaled up by a factor of 0.75, it yields reasonable agreement with the GCM results of Xu et al. (2012).

4.3 Cone plume simulations

For the channelized subglacial discharge the most recent, numerical experiments ~~(and most in agreement with plume theory)~~ by Slater et al. (2015) and Xu et al. (2013) were compared with simulations of the CP model. We used the same experimental settings (discharge, salinity and temperature profiles) as in the experiments of the 3D models, with an entrainment rate $E_0 = 0.1$. Xu et al. (2013) used results of a survey ~~to~~ of Store Glacier (500m deep and 5km wide) performed in 2010, in particular the observed temperature and salinity profile. They performed simulations of plumes for different discharge values but same diffusivity for a 150 m wide, 500 m deep fjord with a 1m resolution near the glacier. Their sensitivity study showed that uncertainty in channel width yielded 15% uncertainty in the cumulative melt rate. Fig. 11 b) shows the dependence of the cumulative melt rate on the discharge for a single plume from Xu et al. (2013) and the CP model. Both models reveal a similar dependence of melt rate on discharge, but the CP model underestimates the melt rate compared to the 3D GCM. To bring the two melt rates in better agreement a scaling factor of 3.4 needed for the CP model.

Slater et al. (2015) used a coarser resolution GCM with parameterized turbulence. They calibrated the GCM (vertical plume, without ice) against pure plume theory for each applied discharge value by adjusting the diffusivity until plume properties (temperature, salinity, thickness and velocity) matched plume theory by Turner with $E_0 = 0.1$ (personal communication from D. Slater). Turners plume theory is similar to our CP model (eq. 8-11) but omits the terms with melt rate $\dot{m}$ and drag C_d. After tuning, the GCM was applied to simulate the melt rate for the same discharge values and diffusivity for a vertical ice front. Furthermore a minimum velocity of $U_0 = 0.04ms^{-1}$ was ~~interoduced~~ introduced to create a ~~backround~~ background melting the is ~~calclutated~~ calculated with Equation (5-7). In order to simulate the the same cumulative ~~malt~~ melt rate as Slater et al. (2015) distributed by $1-10$ channels a scaling factor is needed of 2.46 without ~~a backround~~ calculating background melting and 1.7 with calculating background melting is needed. On the other hand, the result of the GCM for the same total subglacial discharge but uniformly distributed along the whole grounding line is rather close to the results of the LP model. Indeed, for this case Slater et al. (2015) received the cumulative melt rate of $3.69\,\mathrm{md^{-1}}$, while for the LP model we receive $2.42\,\mathrm{md^{-1}}$ for $E_0 = 0.1$ and $3.71\,\mathrm{md^{-1}}$ for $E_0 = 0.036$.

4.4 Conclusions

From these comparison of simple ~~parameterizations~~ models with physically-based model it appears that the LP model needs to be scaled down (except for Slater et al. (2015)) and the CP model scaled up. The scaling factor is in the order of one. Most importantly CP and LP models reveal a similar qualitative behavior to much more complex and computationally demanding GCMs as shown in Xu et al. (2012); Slater et al. (2015); Sciascia et al. (2013).

5 ~~Comparision~~ Comparison with empirical data

Few studies exist where submarine melt has been calculated directly based on field measurements. We used here the available data to test the LP and CP models against observations. However, the results have to be observed with caution since a single temperature profile does not necessarily represent ~~the average heat storage in a fjord since it can undergoe great variability (Jackson et al., 2014).~~ monthly or even annual temperature profile. As Jackson et al. (2014) shows, for Sermilik Fjord and Kangerdlugssuaq Fjord in the winter months the properties including heat content can undergo great variability within time scales of three to ten days (Jackson et al., 2014). Furthermore, uncertainties in the estimation of melt rates from fjord flux gates have been analysed in depth by **?** by considering the total heat and salt budget of the fjord. Thus, when considering the derived melt fluxes from field measurements we have to keep in mind the other possible melt rate flux contributers as sea ice or melting ice bergs.

5.1 Petermann glacier

For the the years 2002-2006 Rignot and Steffen (2008) calculated the melt rate of the floating tongue of Petermann glacier obtained from ~~velocity measurements and mass balance~~ice flux divergence. They detected 4 ~~big channels of along~~ large channels incised into the underside of the floating tongue. Due to its long floating tongue, the estimated melt rate is reliable because it is less affected by errors in estimating the calving rate as it is the case for tidewater glaciers. For modeling the melt rate of Petermann glacier we used temperature and salinity profile in the fjord in front of the floating tongue measured in the year 2003 by ~~?~~Johnson et al. (2011). We also use the data from Morlighem et al. (2014) to define the margins of the Petermann glacier and to compute average one-dimensional profile of the floating tongue. We then use a polynomial fit to smooth the profile of the floating tongue. Fig. 12 a) shows the annual mean melt rate calculated with the LP model for ~~$E_0 = 0.08$~~ $E_0 = 0.16$ and $E_0 = 0.036$. Our calculated melt rates were compared to the width-averaged melt rate derived by Rignot and Steffen (2008), which is mostly dominated by the 4 channels that have maximal melt rates of 30 m/d. Even for a minimum discharge of ~~$10^{-4}\,\mathrm{m^2 s^{-1}}$~~ $10^{-5}\,\mathrm{m^2 s^{-1}}$ (as discussed in section 3.1) and with $E_0 = 0.036$, the LP model significantly overestimates the melt rate beyond a very narrow range (few km) directly next to the grounding line. This is an expected result, because for long floating tongues at a certain length L Coriolis force becomes important for small to moderate Rossby numbers R and a (horizontal) plume velocity U if $\frac{U}{fL} < 1$. At this length scale the plume flow gets deflected (to the right here) and will be ~~domintad~~ dominated by geostrophic flow as modelled by Gladish et al. (2012). Yet this is not taken into account in our simple plume model, as discussed in section 3.4. On the other hand, when using the CP model and a large discharge given by the total runoff over the catchment area distributed over four identical subglacial channels, we receive very low melt rates (Fig. 12b). It is clear that the LP model is in better agreement than the CP model at simulating the melt rate near the grounding line of the Petermann glaciers but ~~correction for~~ a correction for the Coriolis effect is required further from the grounding line.

5.2 West Greenland glaciers

In a small fjord in West Greenland the melt rate of four glaciers was determined by measuring the fjord salinity, temperature and velocity ~~in the fjord near~~ close to the glacier fronts ~~(?)~~(Rignot et al., 2010). In Torrsukatak fjord (TOR) the average and cumulative melt represents the melt rates of both glacier fronts together (Seermeq Avangnardleq and Sermeq Kujatdleq) since the fronts are situated in the same head of the fjord branch. The two other glaciers, Kangilerngata Sermia (KANGIL) and ~~Equip Sermia (EQUIP~~Eqip Sermia (EQIP) enter different fjords. ~~Measured~~ The measured velocity in front of ~~EQUIP~~ EQIP does not show an upwelling pattern but more a right to left circulation, nevertheless we also calculated the melt rate with our plume models for ~~EQUIP~~EQIP. For all glaciers we took the total width of the glacier to determine the subglacial discharge per unit of length for the LP model and determined the average depth of the grounding line as a starting point for the LP model. The total subglacial discharge Q_{sg} was taken from the table of Rignot et al. (2010) We then compare our simulations to the average melt rate determined by ~~?~~Rignot et al. (2010). As shown in the experiment by Slater et al. (2015), a large number of channels acts like a LP but we also computed cumulative melt assuming the existence of one ~~big~~ large single CP starting at the maximum depth of the grounding line. Table 4 shows the ratio between observed and simulated melt rate for two types of plume models and two values of entrainment rate factor E_0. For KANGIL and ~~EQUIP~~ EQIP results of the LP model are in reasonable agreement with measurements, especially for the smallest E_0 value (45%-105%). Although for ~~EQUIP~~ EQIP the agreement is the best with the LP model, the lack of upwelling circulation indicates that the plume ~~parameterization~~ model may not be applicable to this glacier and therefore this agreement may be a pure coincidence. The melt rate ratio of one CP shows rather poor results (1%-5%).

We also compared our model with the data from Fried et al. (2015) for Kangerlussuup Sermia glacier which is located in West Greenland northward of the previously discussed glaciers. ~~We used data from Morlighem et al. (2014) for the glacier elevation and after averaging to a one dimensional profile we obtained a shelf of 3 km length. Note that caution is needed since the data set is averaged over 10 years and has a resolution of 300m.~~ Realistic temperature stratification can lead to maximal melting at the bottom of tidewater glaciers near the grounding line (e.g. Fig. 9a). This maximal melting at the bottom may cause so-called undercutting, which may enhance mass loss by calving (Rignot et al., 2015). Fried et al. (2015) found that 80% of the tidewater glacier is undercut by 45 meters in average. The glacier releases subglacial discharge via two ~~big~~ large channels, but their corresponding melting contributes only 15% of the total melt of the glacier front. Furthermore Carroll et al. (2016) showed that the simulated melt rate of a single cone plume is about 2 order of magnitudes lower than the spatially ~~averged~~ averaged melt rate by Fried et al. (2015). Thus we investigate whether the LP model can calculate the average melting by assuming that the 250 meter deep glacier is undercut below 50 meters depth, with an angle of 77° to achieve the observed undercutting (Fig. 13a). Bartholomaus et al. (2016) give the belonging CTD data and estimate an summer discharge. We use the CTD ~~closet~~ closest to the glacier front in Summer 2013 and the mean summer discharge ($208\mathrm{m}^3\mathrm{s}^{-1}$) per glacier width (3km) as input data for the LP model. Fried et al. (2015) find a total melt rate of 2 md^{-1} for the whole calving front. They assumed that the glacier is only undercut by submarine melting, such that the distance of grounding line to the overhang position subtracted by the glacier's velocity gives the submarine melt value. With this input data and an entrainment rate factor of $E_0 = 0.036$ we achieve an average melt rate of 1.5 md^{-1} (Fig.13). This value is close to the empirical data but this plume would not result in the mentioned undercutting depth, since it penetrates up to 10 meter below the sea surface. The entrainment factor $E_0 = 0.16 - 0.13$ lets the plume stop at 50 m depth but their melting corresponds only to 50% of the empirical data for the total melt rate. If the the LP model is correct it means, that additional fjord circulation make out 50% of the melting. Furthermore ~~deriving the melt rate by one~~ CTD profile close to the glacier might be diluted by near local surface runoff or calving an thus cooling and freshening of the surface ocean waters. Thus deriving the melt rate from a such an CTD profile can lead to high uncertainty ranges.

5.3 Helheim

Sutherland and Straneo (2012) used results of a field campaign in ~~Semerlik~~ Sermilik fjord in summer 2009 where temperatures, salinities and velocities were measured at seven stations in the fjord to calculate the melt rate of Helheim glacier. We applied the temperature and salinity profile closest to the glacier ~~(section 7)~~ for the LP model to simulate the melt rate for comparison. We assume, following Sutherland and Straneo (2012), that Helheim glacier is a tidewater glacier and has a depth of $700\,m$ and a width of 6 km and the subglacial discharge of $5.1 \pm 0.76\mathrm{km}^3\mathrm{a}^{-1}$ (summer in 2008; Andersen et al., 2010). Figure 14 shows the simulated melt rates over for different E_0 with the average subglacial discharge. Our best fit computes an average melt rate of ~~1.6~~ 1.7 md^{-1} (Sutherland 1.8 md^{-1}) with an entrainment factor $E_0 = 0.036$.

5.4 Store Glacier

Another well documented glacier is Store glacier. Xu et al. (2013) estimated an average submarine melt rate of $3.0 \pm 1.0\ \mathrm{md}^{-1}$ in summer (sec. 4) while new calculations, due to new bathymetry data reveal a melt rate of $4.5 \pm 1.5\ \mathrm{md}^{-1}$ (Chauche, 2016). Additionally, Chauche (2016) conducted a survey to determine average melt rate and subglacial discharge from November 2012 until May 2013. Two different techniques were used, which we reference as Gade (Gade, 1979) and Motyka (Motyka et al., 2003) in Figure 15 a). The Motyka technique is based on conservation of heat, salt and volume. The Gade-technique is based on the identification and quantification of different processes (i.e. submarine melting, runoff-mixing, thermal cooling, local sea ice formation) that ~~modify the water coloumn by it's tempreature and salinity in certain - for the process - typical gradients~~can be identified by their typical temperature-salinity properties in a TS-plot (i.e.Straneo et al. (2011)). We used the LP model with $E_0 = 0.036$ and an input subglacial discharge determined by Motyka and Gade with the corresponding temperature and salinity profiles, to simulate melt rates. Results from the LP model are biased low compared to the measurements (Figure 15 b), with melt rate underestimated by 75% in average (Table 5). Note that the Motyka method comes with large error bars for both subglacial discharge and corresponding melt rate, which ~~accomodate~~ accommodate for the LP model bias (Figure 15). Stated uncertainties for the Gade method are smaller and are not consistent with the LP model results.

5.5 Summary

We tested both line and cone plume models against available empirical data for melt rate, and the line plume was best suited to reproduce observations (Table 4). Table 6 and Figure 16 provide for each glacier the measured discharge and melt rate, with error bars, and corresponding range in simulated melt rate (when errors in observed discharge are taken into account as input). When default drag, heat and salt transfer coefficients are used, the simulated melt rate tends to underestimate observed melt rate, thus the best match was obtained with an entrainment rate $E_0 = 0.036$, on the lower end of our range (e.g. see Fig. 7 for how melt rate varies with E_0). Nevertheless, three glaciers (Helheim, ~~Equip~~ Eqip Sermia and Kangerlussuup Sermia) out of seven match observations within the error bars.

Varying other model parameters can change the mean but not the spread of simulated melt rate across glaciers and discharge ranges. For instance, if the heat exchange coefficient is increased to $\Gamma_T = 4.2\cdot10^{-2}$ (instead of the default $\Gamma_T = 2.2\cdot10^{-2}$), the bias can be reduced and simulated melt rates are close with ~~obervations~~ observations (Helheim and ~~Equip~~ Eqip Sermia fall out, conversely). Figure 16 shows a comparison of measured and simulated melt rate with the modified heat transfer coefficient.

Clearly, many fjord processes are not taken into account in this simplified approach. For example, the circulation in front of ~~Equip~~ Eqip Sermia was mostly horizontal ~~(?)~~(Rignot et al., 2010), instead of the vertical upwelling represented in the model. There are also issues with the measurements themselves, such as time sampling or difficulties to retrieve discharge and melt rate, as seen for the Store glacier (Fig. 15), or for Helheim, where CTD profiles for temperature and salinity were taken one year after discharge rates measurements.

Nevertheless, the simple line plume model is in general agreement with the observations (Fig. 16) - and shows a a correlation coefficient of 0.7 (16 b) with the modified heat transfer coefficient. The theoretical background and similar dependency on discharge compared to more complex models (see previous sections) make it suitable for modelling studies over a larger number of Greenland glaciers, and to investigate melt rate response to future changes in ~~discharge and~~ subglacial discharge and fjord temperature.

6 Conclusions

1) We presented two ~~parameterizations~~ simple models for simulation of the submarine melt rate of ~~marine-terminated~~ marine-terminating glaciers, the so-called cone plume and line plume models and studied sensitivity of these two models to different forcings (fjord temperature, stratification, subglacial discharge) and model parameter (entrainment parameter E_0). We also compared these models with results of experiments performed with 2D and 3D ocean GCM by ~~Slater et al. (2016)~~ Slater et al. (2015) and Xu et al. (2013). ~~At last~~ Lastly we compared the results of simulations of the LP and CP models with empirical estimates of melt rate for several Greenland glaciers.

2) We found that for small subglacial discharge, typical for winter conditions, cumulative melt does not depend on the discharge. For high discharge typical for summer conditions we found a power dependence of 1/3 of submarine melt on subglacial discharge for the LP models, ~~and a power of 2/5 for the CP model,~~ which is consistent with the previous studies. We found a theoretical ~~explenation~~ explanation of this behaviour, explained in the Annex A. Furthermore we found that the power

dependence to the ambient temperature in a well-mixed environment also is 1.7-1.8 for lower discharges and is only 1.2 for the higher discharge for both models.

3) We investigated the sensitivity of the melt rate to the entrainment parameter E_0 that was used ~~parametrization of the turbulence of~~ to parameterize turbulent entrainment into the plume. For a tidewater glacier the cumulative melt rate of the LP model increases with decreasing E_0 while it decreases for the CP model. This is explained by the fact that although in both cases higher entrainment rate slows down the plume and reduces the melt rate per unit of area, for the CP, this effect is overcompensated by the widening of plume for the higher entrainment coefficient. In general, we found a notable effect of the entrainment parameter on the melt rate for the range of entrainment parameter given in the literature. The uncertainty range of E_0 can have the same effect as $1^{\circ}\mathrm{C}$ change in ocean temperature.

4) We compared the CP and LP models to results of 3D GCM experiments, and find qualitatively similar melt rate profiles. In most cases, the LP model overestimates the results of the GCM by approximately a factor two, while the CP model underestimates melt rate from GCMs. ~~Such discrepancy is not surprising given the highly simplified parameterization of the LP and CP models compared to GCMs. Importantly~~More importantly, we find the same power law dependence of melt rate on subglacial water discharge as in Slater et al. (2016), for given ambient hydrographic conditions. As a result, with a constant scaling factor of the order of one, the simplified models can reproduce a wide range of melt rates spanning several orders of magnitude.

5) In the case of the long floating tongue, like the Petermann glacier, the LP model significantly overestimates the melt rate outside of the narrow zone along the grounding line which is probably due to the missing Coriolis force in the plume models.

6) Although it is known that in summer a part of the subglacial meltwater is delivered in the fjord through several channels, we found that the submarine melt rate associated with the discharge through the channels and better described by the CP model, makes out only a small amount of the empirically estimated total melt rate of a glacier front. Furthermore the total number of channels for every summer is unknown for different glaciers. When we compare the LP model to empirical data, it is evident that the LP model is more appropriate than the CP model for simulation of both winter and summer melt of real Greenland glaciers. However, the model has to be adjusted for individual glaciers since the scaling parameter is not the same for different glaciers. Thus, for the ~~futue~~ future we will use the tuned LP model coupled to a ~~1d~~ 1D ice flow model to determine the importance of submarine melt rate to glacier dynamics.

Code availability. The Code for the line and cone plume, written in Python, is available as supplementary material.

Appendix A: Semi-analytical solutions for the LP model

In this appendix, we analyze the LP model equations in order to derive approximate analytical solutions. This in turn helps to ~~interprete~~ interpret the results of the numerical experiments presented in this paper, performed with the more complete plume models from Jenkins (2011). First analytical solutions for the LP model were undertaken by Linden et al. (1990) and summarized in Straneo and Cenedese (2015). Slater et al. (2016) previously presented approximate analytical solutions for the ~~CP model~~ LP model for higher discharge ranges. Jenkins (2011) noticed that for strong discharge, plume velocity in the LP model does not change much with depth and is thus similar to the initial balance velocity (our equation 12). Magorrian and Wells (2016) covered the case for small discharge. The reasoning in this appendix provides a unifying solution for small and large discharge with the LP model applied at tide water glacier and glacier with long floating tongues.

We restrict the analysis to the typical conditions of a 500m deep ~~greenladic~~ Greenlandic fjord (T_a $(0-4°C)$).

A1 Simplified melt rate equation

After Jenkins (2011), the melt rate can be approximated as

$$\dot{m} \approx M_0 \cdot U \cdot \Delta T \tag{A1}$$

where $\Delta T = T - T_f$ is the temperature above freezing and M_0 is a slowly varying function of ice temperature below freezing point, which can be considered constant for the purpose of this appendix. Freezing point temperature is given by $T_f = \lambda_1 S + \lambda_2 + \lambda_3 Z$.

We run several experiments in a typical parameter range for tidewater and long floating tongue glaciers in Greenland's fjords and could confirm that the approximation is accurate for the LP model (Fig. A1a). The Parameters varied were Ta $(0-4°C)$,q_{sg} $(1 \cdot 10^{-5} - 0.1)$, E_0 (0.036-0.16) and $\sin\alpha$ (0.02-1) for constant depth of 500m, ~~$T_i = -15°C$~~ $T_i = -10°C$ and $S_a = 34.2$. With linear regression we found an average value for $M_0 = 8.8 \cdot 10^{-6}$.

Let $T_e = \frac{E_0}{M_0}\sin\alpha$, the entrainment-equivalent temperature ($°C$), be a measure of the ratio of entrainment to melting (it corresponds to the temperature for which melting equates entrainment). We have:

$$\frac{\dot{m}}{\dot{e}} \approx \frac{\Delta T}{T_e} \ll 1 \tag{A2}$$

in all these experiments (Fig. A1b), consistently with the ranges for E_0 (0.036-0.16) and $\sin\alpha$ (0.02-1), so that T_e spans two orders of magnitude, roughly $10^2 - 10^4$ $°C$.

A2 Balance regime

In Figure 2 we showed that CP velocity rapidly converges regardless of initial velocity. This has been also shown by Dallaston et al. (2015) for the LP and also holds for the plume temperature, ~~salininty~~ salinity and melt rate. Here we derive analytical solutions for these convergence values (indicated with $_\star$) and associated length scales for the our approximation of the LP model (i.e. (A1) and (A2)), by using the equation for the volume flux (1) so that:

$$(qX)' = q'X + qX' = (\dot{e} + \dot{m})X + qX' \tag{A3}$$

where $q = DU$ (the volume flux) and X can be any of the T, S or U. The convergence value $X_\star$ can be obtained by solving the corresponding equation $(qX)' = f$ (where f is the right-hand side term, e.g. (2), (3) or (4)) with $X' = 0$. Moreover, when the right-hand side term is not or weakly dependent on X (i.e. for T and S, as will be detailed below), the equation is analogous to a first order differential linear equation with convergence length scale $L_X = \frac{q}{q'} \approx \frac{q}{\dot{e}} = \frac{D}{E_0 \sin\alpha}$, i.e. with fast convergence near the grounding line, where plume thickness D is small.

A2.1 Balance temperature and salinity

Temperature and salinity equations (3) and (4) can be rewritten as an intuitive mixing law by merging in (5) and (6):

$$(qT)' = \dot{e}T_a + \dot{m}T_m \tag{A4}$$

$$(qS)' = \dot{e}S_a \tag{A5}$$

where T_m is an effective meltwater temperature, derived from (5):

$$T_m = c_i/cT_i - L/c + T_b(1 - c_i/c) \approx c_i/cT_i - L/c \tag{A6}$$

Variations of boundary layer temperature T_b around $0°C$ can be safely neglected compared to latent heat, so that we will treat T_m as a constant. If ~~$T_i = -15°C$~~ $T_i = -10°C$, we have ~~$T_m \approx -92°C$~~ $T_m \approx -89°C$. Nevertheless for completeness, note that T_b can be expressed as a function of melt rate, plume and ice temperatures from equation (5). Using our simplified melt rate equation (A1) and given that $\dot{m} \ll C_d^{\frac{1}{2}}\Gamma_T U$ by two orders of magnitude, an accurate approximation for T_b is given by:

$$T_b - T_f = \left(1 - \frac{c_i M_0 (L/c_i - T_i)}{c C_d^{\frac{1}{2}} \Gamma_T}\right)\Delta T \approx \cancel{0.3}\,0.1\Delta T \tag{A7}$$

where we verify that boundary layer temperature is somewhat closer to freezing temperature than to plume temperature. In the case of plume salinity S_b cancels out completely and $S_i = 0$ (as can be verified straightforwardly using (4) and (6)), so no other term is needed.

~~Equations and can also be combined with to obtain an expression for plume buoyancy flux:~~

By decomposing (A4) as outlined in (A3), and searching for solutions when $T' = 0$, with $\dot{m} \ll \dot{e}$, we obtain an expression for balance temperature:

$$T_\star \approx T_a + \frac{\dot{m}}{\dot{e}}(T_m - T_a) \tag{A8}$$

which can be rearranged by using (A2) in "balance" regime, so that $\frac{\dot{m}}{\dot{e}} \approx \left(\frac{\dot{m}}{\dot{e}}\right)_\star \approx \frac{\Delta T_\star}{T_e}$, and neglecting the second order T_a/T_e, into:

$$\Delta T_\star \approx \frac{\Delta T_a}{1 - T_m/T_e} \tag{A9}$$

~~so that~~ or equivalently

$$\left(\frac{\dot{m}}{\dot{e}}\right)_\star \approx \frac{\Delta T_a}{T_e - T_m} \tag{A10}$$

The ratio $-T_m/T_e$ spans about 10^{-2} to 1 in our experiments, with a maximum of 0.02 for tidewater glaciers, i.e. $\Delta T_\star \approx \Delta T_a$. Here the the freezing temperature implied by Δ should be taken for balance plume salinity, which is nearly the same as ambient salinity in first approximation (Eq. (A5), (A2), (A10)):

$$S_\star = \frac{\dot{e}}{\dot{e}+\dot{m}} S_a \approx (1-\frac{\dot{m}}{\dot{e}}) S_a \approx (1-\frac{\Delta T_a}{T_e - T_m}) S_a \approx S_a \tag{A11}$$

so that $\Delta T_\star \approx T_\star - T_{fa}$ and $\Delta T_a \approx T_a - T_{fa}$, where T_{fa} is the freezing temperature for ambient salinity.

A2.2 Balance velocity

A similar reasoning as in the previous section (using (A3) and $q' \approx \dot{e}$), (1) and (2) can be rearranged into an equation for U^2 (note the identity $(U^2)' = 2UU'$):

$$\frac{1}{2}(U^2)' + \frac{(C_d + C_e)}{D} U^2 = b \tag{A12}$$

~~where $b = \sin(\alpha) g \Delta\rho$~~ where $b = \sin(\alpha) g \frac{\Delta\rho}{\rho_0}$ and $C_e = E_0 \sin\alpha$. This highlights in one equation basic plume dynamics, buoyancy-accelerated and balanced by drag and entrainment.

Equation (A12) is analogous to a first order linear differential equation with equilibrium solution for $x >> L_u$

$$U_\star = \sqrt{\frac{b \cdot D}{C_d + C_e}} \tag{A13}$$

and length scale

$$L_u = \frac{D}{2(C_d + C_e)} \tag{A14}$$

Note that equation (A13) does not represent a strict equilibrium but a dynamic balance between velocity, plume thickness and buoyancy, which is maintained while the plume thickness and associated volume flux keeps increasing. Note that as the plume dimension increases ~~with entrainment~~ due to entrainment (or for large discharge), so does the length scale L_u, and ~~the feedback balance becomes looser~~ U may lag behind $U_\star$. At $x = 0$ for typical discharge and entrainment values L_u is less than a centimeter. Our simulations show that velocity reaches dynamic balance $U_\star$ within the first few meters after the grounding line (not shown). This shows qualitative agreement with the above analysis but ~~which~~ suggests that initial changes in plume dimension D and buoyancy b should be taken into account for more detailed analysis of the transient regime. In the present analysis we focus on the balance regime. The theoretical equilibration length scale for velocity is shorter than for temperature and salinity by a factor 2 or more, since $L_{TS}/L_U = 2(1+\frac{C_d}{E_0 sin\alpha})$, especially for long floating tongues. In the actual simulations the ratio is even larger, because the plume keeps growing with distance from its source.

~~Equation can also be~~ The balance velocity is more conveniently expressed as a function of plume's volume flux q instead of thickness D. By taking the square of equation (A13) and multiplying by $U_\star$, the identity $q \approx q_\star = DU_\star$ can be used, so that:

$$U_\star = \left(\frac{q \cdot b}{C_d + C_e}\right)^{\frac{1}{3}} \tag{A15}$$

~~with~~ This new expression shows that velocity can be written as a power law of the buoyancy flux qb. Its initial condition is known, since $q = q_{sg}$ and $b = b_0$ at $x = 0$. ~~In~~ The evolution of qb as the plume develops can also conveniently be derived from (A4) and (A5). For simplicity we limit the derivation to the case of a fjord without stratification ($T'_a = 0$ and $S'_a = 0$), ~~we have an expression for the buoyancy flux qb from and~~ where:

$$(qb)' = \dot{m} b_m \tag{A16}$$

where $b_m = g \sin\alpha(\beta_S S_a - \beta_T(T_a - T_m))$ is the meltwater buoyancy minus the heat sink required to melt the ice. Note the temperature account for about 15% of buoyancy variations. According to (A1) the melt rate is proportional to U, thus in the regime where $U \approx U_\star$, we obtain a new differential equation for $U'_\star$. By elevating (A15) at the third power and differentiating, we can use (A16) and the identity $(U^2)' = 2UU'$ to obtain:

$$(U_\star^2)' = \frac{2}{3}\frac{b_m}{C_d + C_e} M_0 \Delta T \tag{A17}$$

Equation (A17) shows that plume acceleration (in "balance" regime) does not depend on subglacial discharge. By integration,

$$U_\star^2 = U_{\star 0}^2 + \int_0^x (U_\star^2)' dx \approx U_{\star 0}^2 + (U_\star^2)' x \tag{A18}$$

where $U_{\star 0}$ is the balance velocity at $x = 0$, given by (A15), and $\Delta T \approx \Delta T_\star$ in $(U_\star^2)'$ and finally by replacing $\Delta T_\star$ with (A9), we obtain:

$$U_\star(x) \approx \sqrt{\left(\frac{q_{sg} \cdot b_0}{C_d + C_e}\right)^{\frac{2}{3}} + \frac{2}{3}\frac{b_m}{C_d + C_e} \cancel{\frac{M_0 \Delta T_a}{1 - T_m/T_e}} \frac{M_0 \Delta T_a}{1 - M_0 T_m/C_e} x} \tag{A19}$$

where $b_0 = g \sin\alpha(\beta_S S_a - \beta_T T_a)$ is the buoyancy at x=0 (equal to meltwater buoyancy). See Table A1 for a summary of the variables defined in the appendix (note we already wrote T_e in full to see the effect of entrainment more explicitly).

A2.3 Cumulative melt rate

~~By integrating~~ With equations (A1) ~~with~~ , (A9), (A17) and (A19), we ~~obtain~~ can now derive an expression for the cumulative melt rate in the LP model ~~:~~ , as:

$$M_\star(x) = \int_0^x \dot{m} dx \approx M_0 \Delta T_\star \int_0^x U_\star(x) dx = \begin{cases} M_0 \Delta T_\star U_{\star 0} x, & \text{if } U'_\star \approx 0 \\ \frac{C_d + C_e}{b_m}\left(U_\star^3(x) - U_{\star 0}^3\right), & \text{if } U'_\star > 0 \end{cases} \tag{A20}$$

(we used the fact that if $U = (A + Bx)^{1/2}$ where A and B are constant (A19), a primitive $\int U$ is $\frac{2}{3B}(A + Bx)^{3/2} = \frac{2}{3B}U^3$, provided that $B \neq 0$, i.e. that $U'_\star \neq 0$; additionally since $B = (U_\star^2)'$ is proportional to $\frac{2}{3} M_0 \Delta T_\star$ (A17), these terms cancel out)

$$M(x)=\int_0^x \dot{m}dx \approx M_0\Delta T_\star \int_0^x U_\star(x)dx = \frac{C_d+C_e}{b_m}\left(U_\star^3(x)-U_{\star 0}^3\right)$$

The error of (A21) compared to the cumulative melt rate of the LP model in the unstratified case for tidewater glaciers was 2 % for ~~big~~ large discharge ($q = 0.1m^2s^{-1}$) and 9 % for small discharge(~~$q = 1\cdot 10{-}6m^2s^{-1}$~~ $q = 1\cdot 10^-6m^2s^{-1}$). For the case of a long floating tongue and a discharge of $q = 0.1m^2s^{-1}$ the error was in the range of 10 %.

A3 ~~The role of subglacial discharge and the shape of the glacier~~

~~We investigated the plume properties and melt rate of a typical tidewater glacier and a glacier with a long floating tongue (order of Petermann glaicer). While for the tidewater glacier the plume temperature rapidly approaches the temperature of the ambient water (Fig. A3 c) the plume under a long floating tongue stays cooler since the melt-entrainment ratio becomes bigger .A look on the velocity of the plume shows an acceleration under floating tongues. Equation reveals that for a tidewater glacier, a plume starting wit a velocity~~ ~~$U_{\star 0}$~~~~, which is dependent on the subglacial discharge , will accelerate with a slope independent of the subglacial discharge. Therefore plumes with small discharges will highly accelerate while the velocity of plumes with big discharge will remain almost constant along~~ ~~Z~~ ~~(Fig. A2)as reported by Dallaston et al. (2015). That explains the different exponents of melt rate as a function of subglacial discharge in the literature. In the case of a very small discharge~~ ~~$q_{sg}\rightarrow 0$~~ ~~then~~ ~~$U_\star \rightarrow \sqrt{\frac{2}{3}\frac{b_m}{C_d+C_e}\frac{M_0\Delta T_a}{1-T_m/T_e}x}$~~ ~~and the melting becomes independent of the discharge and we speak of the background melting. Magorrian and Wells (2016) undertook a scaling analysis for the plume model for small discharges. For~~ ~~$q_{sg}\rightarrow 0$~~ ~~comparision shows that in our analysis velocity accelerates with~~ ~~$U(x)\sim\sqrt{\sin(\alpha)X}$~~ ~~equal to~~ We can identify two limiting cases for equation (A21), characterized by discharge-driven ($U_\star' \approx 0$, i.e. large discharge M_{high}) or melt-driven ($U_{\star 0}\approx 0$, i.e. small discharge M_{low}) convection (see also (A18) and (A19)). Let us pose L the full length of the ~~results of Magorrian and Wells (2016). The cumulative melt rate for small discharges in dependence to the thermal forcing~~ ~~ΔT_a~~ ~~gives a power law of 3/2, also confirmed by the work of Magorrian and Wells (2016). For tidewater glacierswith very high discharge the acceleration term can be neglected and therefore the velocity - and thus the melt rate- depends on the initial conditions of subglacial discharge with the cubic root. Also~~ plume (note that it is related to grounding line depth $Z = Lsin\alpha$). We have:

$$M_{high}\approx\left(\frac{b_0\,q_{sg}}{C_d+C_e}\right)^{\frac{1}{3}}\left(\frac{M_0\Delta T_a}{1-M_0T_m/C_e}L\right), \qquad \text{discharge-driven, } q_{sg}\gg q_c \tag{A21}$$

$$M_{low}\approx\left(\frac{b_m}{C_d+C_e}\right)^{\frac{1}{2}}\left(\frac{2}{3}\frac{M_0\Delta T_a}{1-M_0T_m/C_e}L\right)^{3/2}, \qquad \text{melt-driven, } q_{sg}\ll q_c \tag{A22}$$

where the critical discharge q_c is defined as the discharge for which $M_{high} = M_{low}$, equal to:

$$q_c = \left(\frac{2}{3}\right)^{9/2}\frac{b_m^{3/2}}{b_0(C_d+C_e)^{1/2}}\left(\frac{M_0\Delta T_a}{1-M_0T_m/C_e}L\right)^{3/2} \tag{A23}$$

For a tidewater glacier q_c is of the order of $10^{-3}m^2/s$, and can reach $10^{-1}m^2/s$ for a few 10s km floating tongue. This indicates that in glaciers with a floating tongue, melt-driven convection tends to be more important than in tidewater glaciers.

We also note from (A21) and (A22) that the dependence of melt rate on entrainment coefficient C_e (which contains both entrainment parameter E_0 and glacier slope $sin\alpha$) is not monotonic. For moderately large values of C_e ($-M_0T_m/C_e \ll 1$, e.g. tidewater glaciers), variations in the left-hand side factor ("dynamics") dominate and the melt rate is ~~then linear dependent on ΔT_a. Our approximation of $U_\star$, $T_\star$ and $m_\star = M_0 \cdot \Delta T_\star \cdot U_\star$ are displayed along the LP models results in Figure A3 and show good agreements.The approximation of the cumulative melt rate shows the biggest deviation for the floating tongue with 10 %~~a decreasing function of, but moderately sensitive to, C_e. However, for small C_e values ($-M_0T_m/C_e \sim 1$ and thus $C_e \ll C_d$, i.e. long floating tongue), where variations in the right-hand side factor ("thermodynamics") dominate, the melt rate is an increasing function of C_e. For the lowest entrainment value $E_0 = 0.036$, "small" means a glacier front angle of the order of $sin\alpha = 0.02$.

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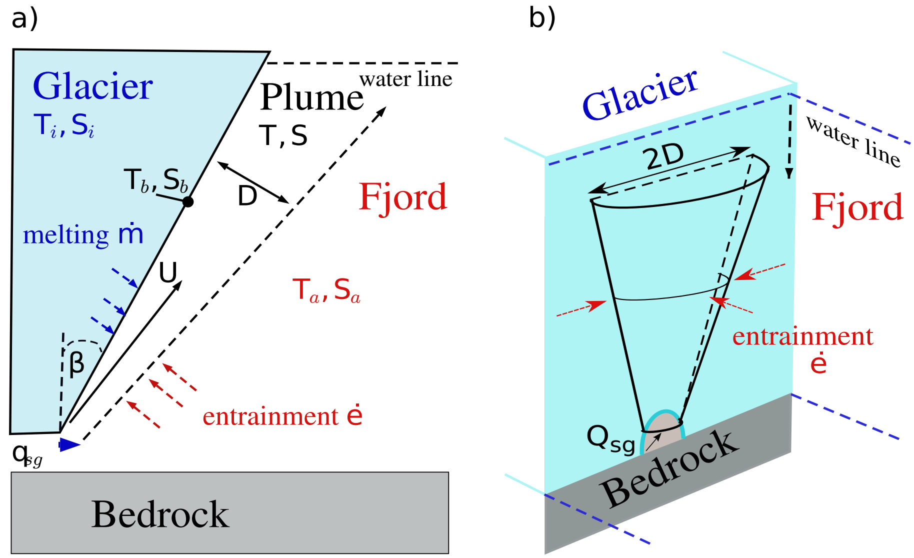

Figure 1. a) Conceptual scheme of the 1D plume model after Jenkins (2011). ~~Uniformly~~ Subglacial freshwater flux q_{sg}, which is uniformly distributed along the grounding line~~subglacial freshwater flux q_{sg}~~, enters the fjord ~~and~~ where it forms a plume that rises up due to buoyancy. The plume is described explicitly with its temperature T, salinity S, thickness D and velocity U. ~~The plume~~ It rises along the ice shelf~~an~~, following the shelfs slope $\alpha = 90^\circ - \beta$, until it either reaches the water surface or has zero velocity due to the loss ~~its~~ off buoyancy. The ambient water with salinity S_a and temperature T_a entrains into the plume with an entrainment rate $\dot{e}$. Melting $\dot{m}$ occurs on the glacier front and adds to the plume buoyancy with water of the temperature T_b and salinity S_b. b) Conceptual scheme of two-dimensional CP model modified after Jenkins (1991) and Cowton et al. (2015). Subglacial discharge enters the fjord localized, via a channel. The plume geometry is described as a half cone and ~~and~~ the entrainment occurs around the arc. The subglacial discharge is $Q_{sg} = \frac{D_0^2 U_0 \pi}{2}$ where D_0 is the initial radius and U_0 is the initial velocity

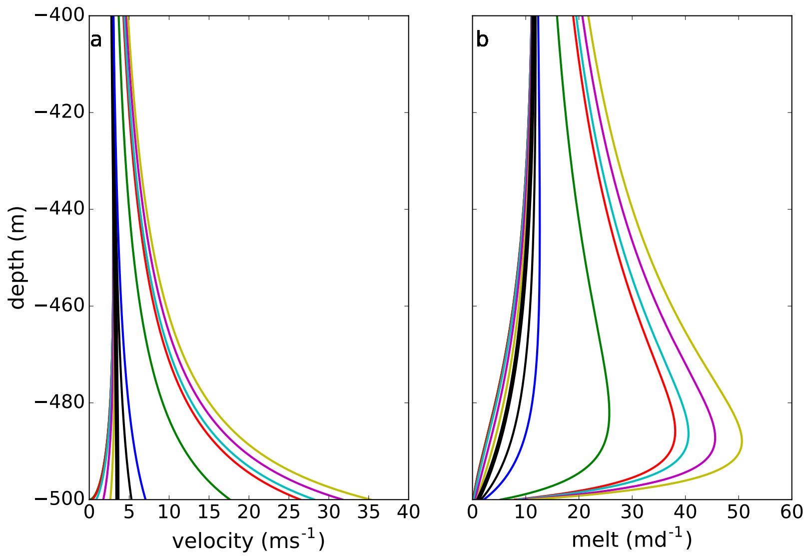

Figure 2. Different runs of the CP model for different initial velocities. Panel a) depicts the velocity profile in the first 100 m. All ~~starting~~ intital velocities converge within 100 m to the trajectory of the balance velocity $U_{\star 0} = 3.5\,\mathrm{ms}^{-1}$ (thick black line). The corresponding initial radii differ thus from 300 m (for $\cancel{U_0 = 3.5\,\mathrm{ms}^{-3}}$ $U_0 = 3.5 \cdot 10^{-3}\,\mathrm{ms}^{-1}$) to 3 m (for $U_0 = 35\,\mathrm{ms}^{-1}$). Panel b) shows the corresponding melt profile. Higher initial velocities give a maximal melt rates at deeper levels. All melt rate profiles converge to the same melt rate after a certain depth.

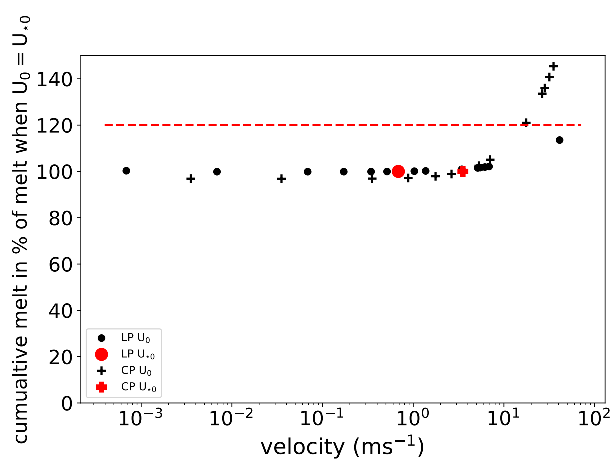

Figure 3. Sensitivity of cumulative melt rate to different initial velocities, for both plume models. Melt rate (black) is in percent of the cumulative melt achieved with initial balance velocity $\cancel{U_\star}$ $U_{\star 0}$ (red). Red dashed line shows 120 % mark. Only very high initial velocities can ~~for the CP model~~ appreciably increase the cumulative melt rate for the CP.

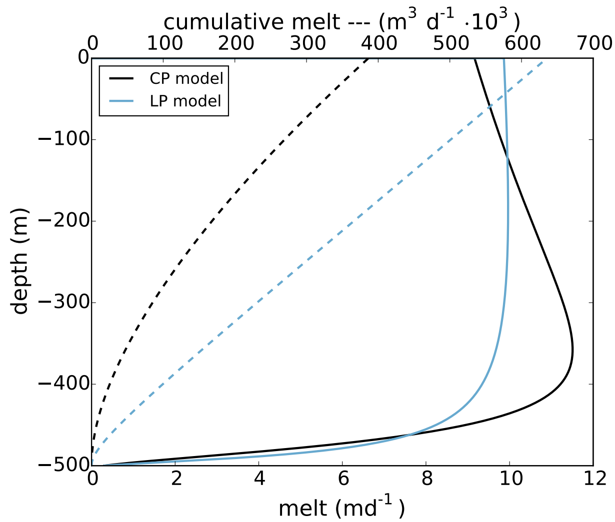

Figure 4. Melt rate profiles in a well-mixed fjord simulated by the CP model (black) and LP model (blue) for a width $W = 150\,\mathrm{m}$ and the total discharge of $Q_{sg} = 500\,\mathrm{m^3s^{-1}}$. In the case of the CP model the total discharge ~~occur~~ is delivered through one channel in the center of the glacier, in the case of LP model the discharge is uniformly distributed with the rate $q_{sg} = \frac{Q_{sg}}{W} = 3.6\,\mathrm{m^2s^{-1}}$. Both plumes start with a velocity of $U_0 = 1\frac{m}{s}$. Solid lines show melt rate averaged ~~acrosss~~ across the plume in the case of CP model and ~~over entire glacier~~ across the full width W in the case of LP model. The dashed lines shows the corresponding cumulative melt rate for the entire glacier.

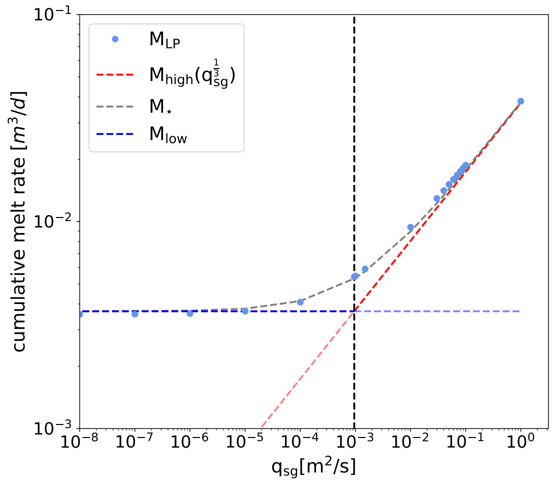

Figure 5. Cumulative melt rate of the LP model ~~(a)~~ for $E_0 = 0.1$ with different discharge values for a well-mixed environment with $T_a = 4°\mathrm{C}$ and $S_a = 34.65\,\mathrm{psu}$. ~~For red line corresponds $\dot{m} = 6.6 \cdot 10^{-5} \cdot Q_{sg}^{\frac{1}{3}} + 6.0 \cdot 10^{-6}$ for the higher discharge range.~~ The ~~grey~~ gray dashed line ~~(a)~~ is our analytical solution for the cumulative melt rate of the LP model (Eq. A21). The red dashed and blue dashed line indicate the limiting melt rate regimes M_{high} (A21) and M_{low} (A22) respectively. The transition of these regimes is defined by the critical discharge q_c (A23) depicted here with black dashed line.

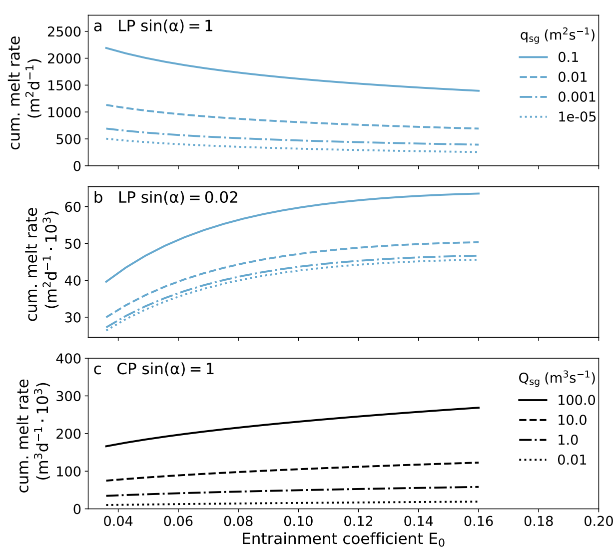

Figure 6. Cumulative melt rates of the different plume models as a function of ~~the~~ entrainment ~~rate factor~~ parameter E_0 for four different discharge values. The cumulative melt rate is depicted for a) LP of a tidewater glacier ($sin(\alpha) = 1$), b) LP of a long floating tongue and c) CP of a tidewater glacier. For the LP model for $sin(\alpha) = 1$ a higher E_0 leads to lower cumulative melting opposite to the other two cases.

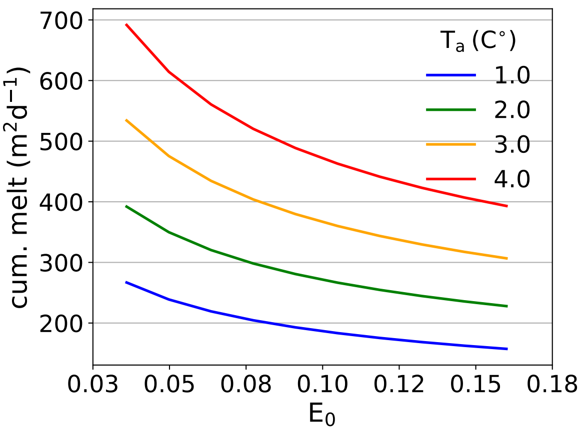

Figure 7. ~~cumulative~~ Cumulative melt rate in ~~dependence of~~ the LP as a function of entrainment parameter E_0 for $q_{sg} = 10^{-3}\,\mathrm{m^2s^{-1}}$ in four different well mixed ambient Temperatures ~~(T_a)~~, in a tidewater glacier.

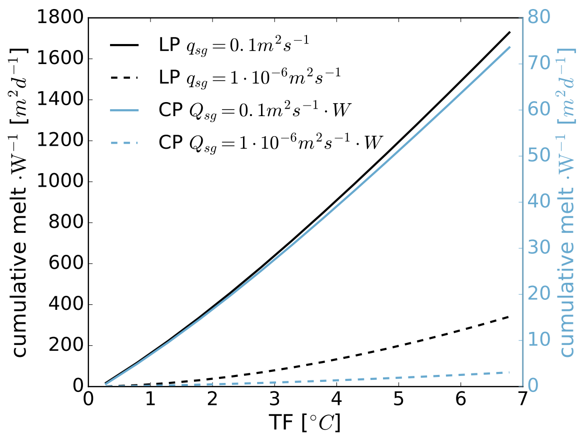

Figure 8. Cumulative melt rate per glacier width W for the LP model (black) and CP (blue) model as a function of the thermal forcing (~~$TF = T_a - T_{af}$)~~ $\Delta T_a = T_a - T_{af}$) for high (solid lines) and low (dashed lines) discharge values. The experiment is for a well-mixed, 500m deep and 5km wide tidewater glacier ($\sin(\alpha) = 1$), with $S_a = 34\,\mathrm{psu}$ and $E_0 = 0.1$.

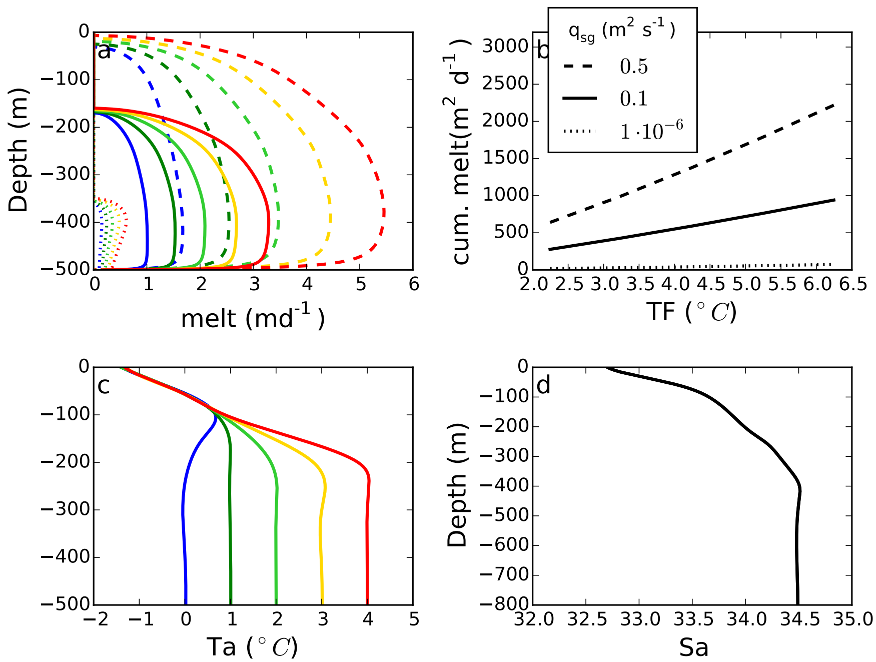

Figure 9. Influence of stratification and discharge on the melt rate profile of the LP (a) . The three different discharge values ($q_{sg} = 0.5,\, 0.1,\, 10^{-6}\,\mathrm{m^2s^{-1}}$, dashed, solid, dotted) in a stratified environment for a fixed salinity profile (d) and 5 different temperature profiles (c) result in 15 different melt rate profiles. The melt rate of the corresponding temperature profile is displayed in the same color as well as in the same style (dashed,dotted or solid) for the corresponding discharge. Note that a very high discharge ($q_{sg} = 0.5\,\mathrm{m^2s^{-1}}$) is needed for the plume to reach the surface. For each discharge value the corresponding cumulative melt rate is depicted (b) as a function of the thermal forcing ($\cancel{TF = T_a - T_b}$$\Delta T_a = T_a - T_b$, eq. 7) at the grounding line. For $\cancel{\dot{m} \sim TF^\beta}$ $\dot{m} \sim \Delta T_a^\beta$ we found β values of 1.2 for ($q_{sg} = 0.5\,\mathrm{m^2s^{-1}}$), 1.2 for ($q_{sg} = 0.1\,\mathrm{m^2s^{-1}}$) and 1.4 for ($q_{sg} = 10^{-6}\,\mathrm{m^2s^{-1}}$).

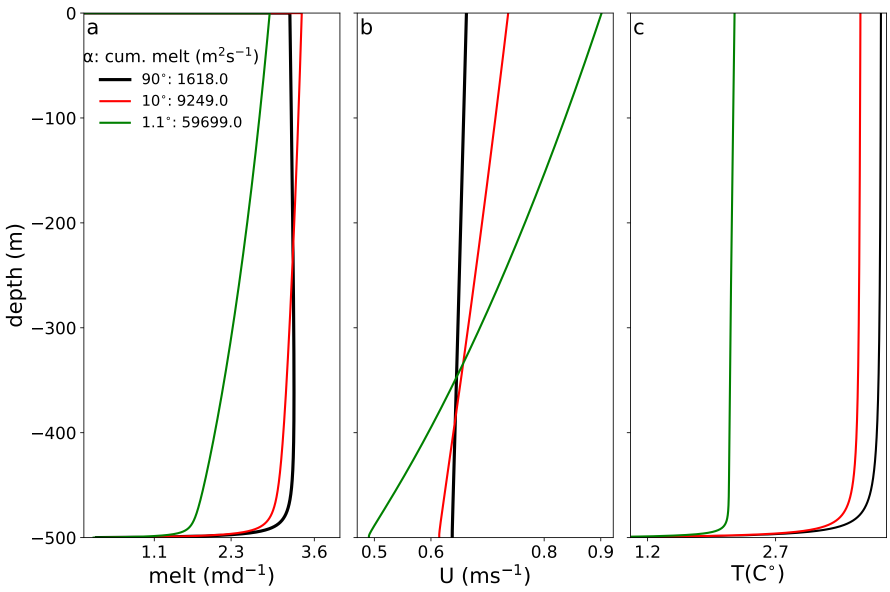

Figure 10. Melt profile (a) and corresponding plume velocity profiles (b) plume temperature (c) and salinity (d) for the LP model for different glacier types: a tide water glacier ($\alpha = 90°$), shelf glacier $\alpha = 10°$ and a shelf glacier with a long floating tongue ($\alpha = 1.1°$) of 25 km. The fjord is well mixed with $Ta = 4°\mathrm{C}$, $Sa = 34.2\,\mathrm{psu}$ and the discharge was set to $q_{sg} = 0.1\,\mathrm{m^2s^{-1}}$ with $E_0 = 0.1$. Note that the profiles of $\alpha = 90°$ and $\alpha = 10°$ are very similar but the cumulative melt rate of the shelf glacier increased ~~by~~ on 500 %. For the long floating tongue the cumulative melt rate is an order of magnitude higher.~~The grey dashed lines indicate $T_\star$ and $S_\star$ (A2.1.)~~

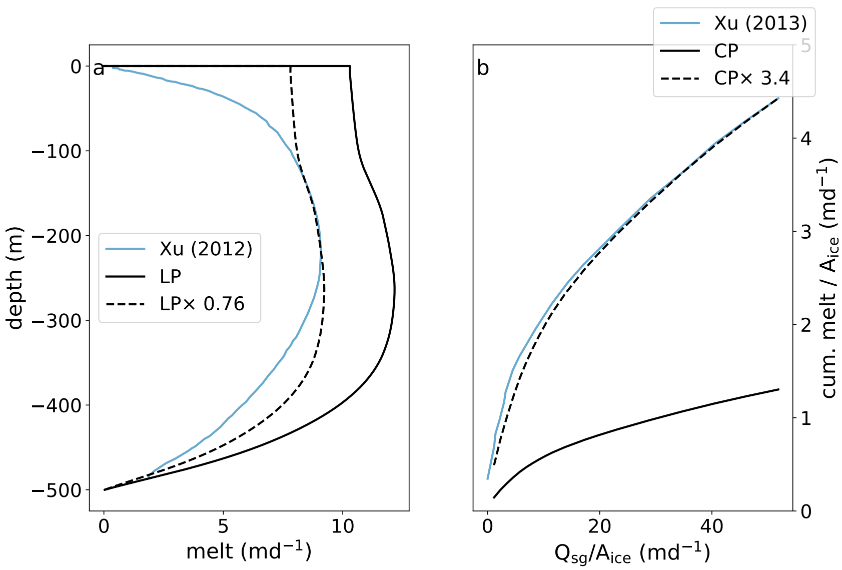

Figure 11. Comparison between LP, CP and GCM simulations. a) Experimental results from Xu et al. (2012) (blue line) and LP model (black, solid line) for ($Q_{sg} = \frac{150}{5} = 30\,\mathrm{m^2 s^{-1}}$) and $E0 = 0.07$, $U_0 = 3\,\mathrm{ms^{-1}}$ and the same ~~temperature profile~~ stratification as in Xu et al. (2012). A scaling factor of 0.74 is needed to match the two melt profiles (black, dashed line).b) Average melt rate over a 150 m wide and 500 m deep glacier part as a ~~funciton~~ function of discharge localized in one channel. Following Xu et al. (2013), for the x-axis, the discharge Q_{sg} was divided by the area of the ice face $A_{ice} = 150 * 500\,\mathrm{m}^2$ so that $q_{sq} = 50\,\mathrm{md^{-1}}$ corresponds $Q_{sg} = 43.4\,\mathrm{m^3 s^{-1}}$. The numerical results of Xu et al. (2013) are displayed with the blue line. Taking the same conditions (T_a, S_a, Q_{sg}) and an entrainment factor of $E_0 = 0.1$ the CP model gives the solid black line. To match the experiment a scaling factor of 3.40 is needed (black, dashed line).

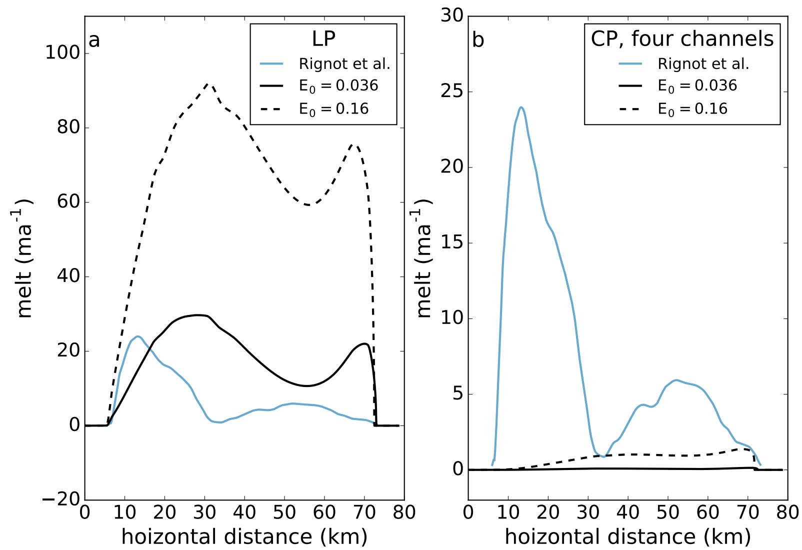

Figure 12. Melt rate of a) LP model simulated over the long floating tongue of Petermann glacier with a minimal discharge of $Q_{min} = 10^{-5}\,\mathrm{m}^2\mathrm{s}^{-1}$ for the minimal ($E_0 = 0.036$) and maximal ($E_0 = 0.16$) value (black lines) of the ~~entrainement~~ entrainment parameter. In panel b) we used the maximum discharge $Q_{sg} = 296\,\mathrm{m}^3\mathrm{s}^{-1}$ (total runoff assumed only in summer) distributed over four channels to compute the melt rate with the CP model. As forcing variables we used the fjord's temperature and salinity profile in front of the floating tongue for the year 2003 summarized by Johnson et al. (2011) and from Morlighem et al. (2014) we determined the glacier thickness and depth of the floating tongue (see 5.1 for details). For both E_0 the melt rate is highly overestimated with the LP model and underestimated with the CP model. The empirical melt rate estimated by Rignot and Steffen (2008) is displayed with the blue line. Note the different vertical scale on the panels.

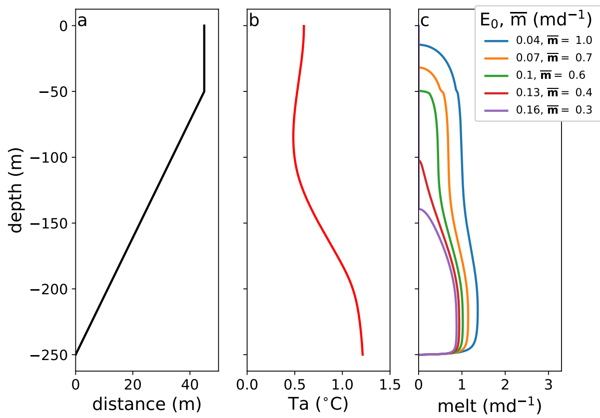

Figure 13. Kangerlussuup Sermia average undercut profile at the terminus (a) with the assumed temperature profile (b) give different melt rate profiles (c) simulated for different E_0 and same $Q_{sg} = 208\,\mathrm{m^3s^{-1}}$. All average melt rates are below the determined $2\,\mathrm{md^{-1}}$ by Fried et al. (2015).

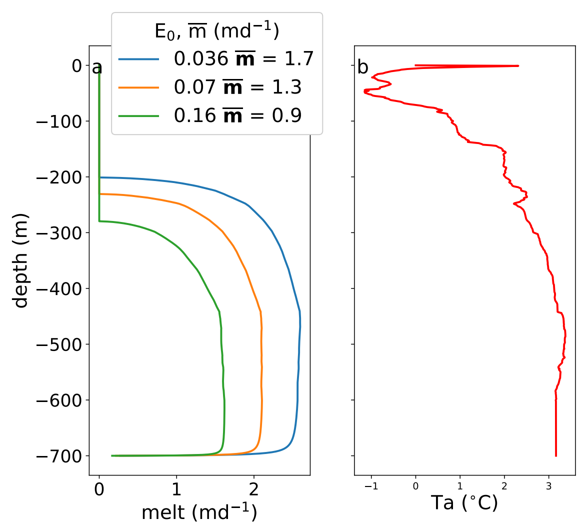

Figure 14. Three vertical melt rate profiles of the LP model (a) for three different entrainment coefficients E_0 for Helheim glacier. With ~~and~~ a discharge of $2.69 \cdot 10^{-2} m^2 s^{-1}$ and $E_0 = 0.036$ we obtain an average melt rate of ~~$\overline{m} = 1.6\mathrm{ma}^{-1}$ very~~ $\overline{m} = 1.7\mathrm{ma}^{-1}$ close to ~~Sutherland et. al~~ Sutherland and Straneo (2012) (~~1.7~~ 1.8 md^{-1}).

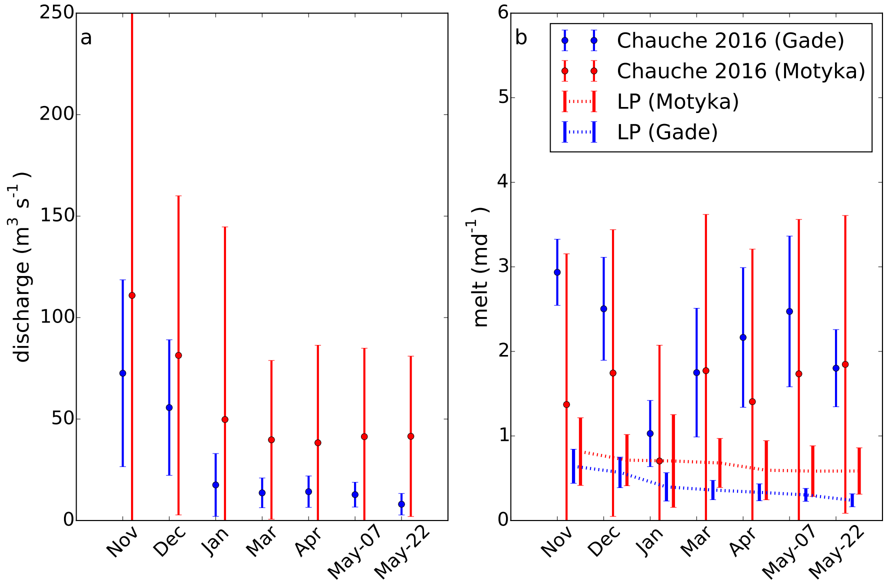

Figure 15. a) Estimated subglacial discharge of Store Glacier for winter 2012/2013 ~~in~~ from (Chauche, 2016). Red ranges give ~~sublgacial~~ subglacial discharge estimates by the Motyka ~~model~~ method and blue ranges by the Gade ~~model~~ method and b) the corresponding melt rate profiles. Simulated melt rates by the LP model with E_0=0.036 are depicted in the red dotted line (subglacial discharge from Moytka model) and blue dotted line (subglacial discharge from Gade model.)

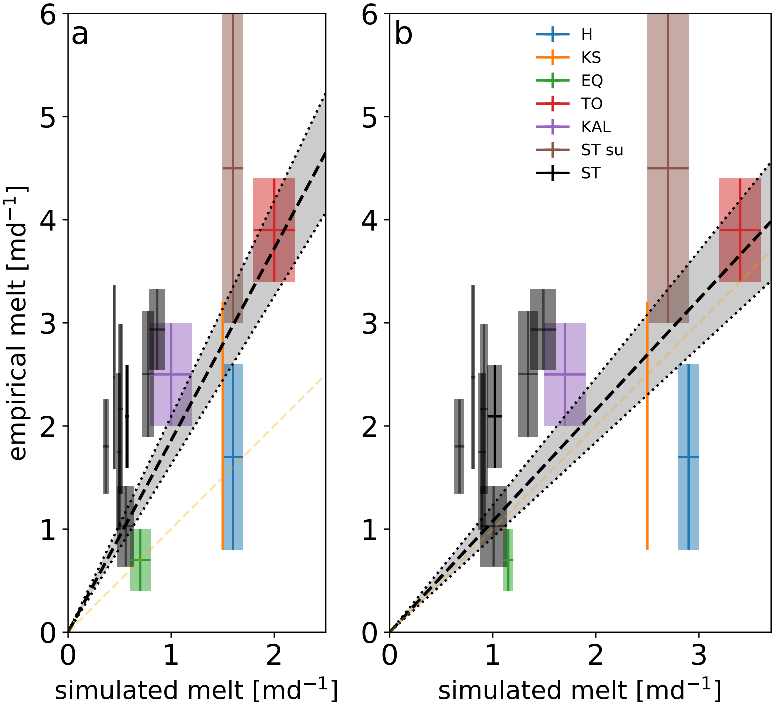

Figure 16. Measured versus simulated melt rate for a number of glaciers, for data given in Table 6. The squares represent error bars in measured and simulated melt rate. The black regression line with 1-sigma uncertainty range indicates the average scaling coefficient of 1.85 required to match observations given model parameters. In both panels, entrainment rate is 0.036. Panel (a) shows model simulations with default values for the heat transfer coefficient $\Gamma_T = 2.2 \cdot 10^{-2}$, while Panel (b) shows simulation with $\Gamma_T = 4.2 \cdot 10^{-2}$, which produces a scaling coefficient closer to one. Glacier abbreviations are shown in Table 6. Note that only one representative value for the Store glacier in Winter was used, as reported in Table 6.

Table 1. Model parameters of the LP and CP model with typical fjord default values. Note that the values may differ for specific experiments (as explicitly stated in the corresponding descriptions).

Symbol	Value	Units	Desciption
q_{sg}	0.1	$\frac{m^2}{s}$	default value for subglacial discharge for LP
Q_{sg}	500	$\frac{m^3}{s}$	default value for subglacial discharge for CP
$U_{\star 0}$	–	$\frac{m}{s}$	initial default value for plume velocity
$T\|_{x0}$	0	$^{\circ}C$	~~inital~~ initial default value for plume temperature
T_a	4	$^{\circ}C$	default value for ambient temperature
T_i	-10	$^{\circ}C$	default value inner ice temperature
$S\|_{x0}$	$1e-6$	psu	default value for ambient salinity
S_a	34.65	psu	default value for ambient salinity
S_i	0	psu	default value for inner ice salinity
E_0	$0.1[0.036-0.16]$	–	Entrainment coefficient
C_d	$2.5 \cdot 10^{-3}$	–	Drag coefficient
λ_1	$-5.73 \cdot 10^{-2}$	$^{\circ}C$	Seawater freezing point slope
λ_2	$8.32 \cdot 10^{-2}$	$^{\circ}C$	Seawater freezing point offset
λ_3	$7.61 \cdot 10^{-4}$	$^{\circ}C\, m^{-1}$	Depth dependence of freezing point
L	$3.35 \cdot 10^{5}$	$J\, kg^{-}1$	Latent heat of fusion for ice
c_i	$2.009 \cdot 10^{3}$	$J\, kg^{-}1\, K^{-1}$	Specific heat capacity for ice
c	$3.974 \cdot 10^{3}$	$J\, kg^{-}1\, K^{-1}$	Specific heat capacity for seawater
β_S	$7.86 \cdot 10^{-4}$	–	Haline contraction coefficient
β_T	$3.87 \cdot 10^{-5}$	–	Thermal expansion coefficient
g	9.81	$m\, s^{-2}$	Gravity constant
Γ_T	$2.2 \cdot 10^{-2}$	–	Thermal turbulent transfer coefficient
Γ_S	$6.2 \cdot 10^{-4}$	–	Salt turbulent transfer coefficient

Table 2. Determination of the power law β of melt rate ~~on discharge Q or~~ in the equation ~~$\dot{m} = a(b \cdot Q^{\beta} + c)$~~ $\dot{m} = a \cdot Q^{\beta} + b$. ~~Seperation~~ Separation between high ($Q > Q_c$) and low discharge ($Q < Q_c$) at a certain discharge Q_c

$\beta(Q > Q_c)$	$\beta(Q < Q_c)$	Q_c [discharge range]	Experiment
0.54	0.80	~~5.76~~ 4.34 [1-45] $\frac{m^3}{s}$	(Xu et al., 2013)
0.45	0.70	~~4.34~~ 5.76[1-45] $\frac{m^3}{s}$	CP model
0.33	0.54	$5 \cdot 10^{-5}$ [10^{-5}-1] $\frac{m^2}{s}$	LP model

Table 3. ~~Determination~~ Numerical determination of the power law β ~~of~~ for a melt rate ~~on~~ dependence of $\bar{m} \propto \Delta T_a^{\beta}$ with the thermal forcing ~~$TF = T_a$~~ glaciers. The exponent was derived for the ~~relation $\bar{m} \propto TF^{\beta}$~~ LP and CP model for ~~low~~ lower and ~~high~~ higher discharge ~~with comparision to theoretical~~ ~~are~~ A comparison to an analytically derived value of β for limiting discharge (low and high) ranges from ~~this~~ literature and our study ~~lower show results~~ cases additionally.

discharge ($\frac{m^2}{s}$ or $\frac{m^3}{s}$)	β ~~experimental~~ numerical	β ~~theoretical~~ analytical	~~Experiment~~ stratification
~~high ($q = 0.1 \frac{m^2}{s}$)~~ 0.1	1.2	1 [a,c]	~~LP, well mixed~~ well mixed
~~low ($q = 1 \cdot 10^{-6} \frac{m^2}{s}$)~~ $1 \cdot 10^{-6}$	1.8	1.5[b,c]	~~LP, well mixed~~ well mixed
~~high ($q = 0.1 \frac{m^2}{s}$)~~ 0.1	1.2	~~2 (linear stratified)~~ -	~~LP, realistic stratified~~ realistic stratified
~~low ($q = 1 \cdot 10^{-6} \frac{m^2}{s}$)~~ $1 \cdot 10^{-6}$	1.4	2(linear stratified)[b]	~~LP, realistic stratified~~ realistic stratified
~~high ($q = 500 \frac{m^3}{s}$)~~ 500	1.2	1[d]	~~CP, well mixed~~ well mixed
~~low ($q = 5 \cdot 10^{-3} \frac{m^2}{s}$)~~ $5 \cdot 10^{-3}$	1.5	~~1~~ -	~~CP, well mixed~~ well mixed
~~high ($q = 500 \frac{m^3}{s}$)~~ 500	1.1	-	~~CP, realistic stratified low ($q = 5 \cdot 10^{-3} \frac{m^2}{s}$) 1.3~~ realistic st
~~high ($q > 5.8 \frac{m^3}{s}$)~~ $5 \cdot 10^{-3}$	~~1.2~~ 1.3	-	~~3D CP, realistic stratified low ($q < 5.8 \frac{m^2}{s}$) 1.6~~ realistic str

~~[a] this study~~ [a] Jenkins (2011) ~~[c]~~ [b] Magorrian and Wells (2016) ~~[d]~~ [c] ~~chapter ??~~ section A2.3 ~~[e]~~ [d] Slater et al. (2016) ~~Xu et al. (2013)~~

Table 4. Simulated cumulative melt rate (%) of empirical estimated cumulative melt rate for different entrainment rates for three West Greenland glaciers.

TOR			KANGIL			EQIP		
model	E_0	melt ratio (%)	model	E_0	melt ratio (%)	model	E_0	melt ratio (%)
LP	0.036	57	LP	0.036	45	LP	0.036	103
CP	0.036	1	CP	0.036	2	CP	0.036	5
LP	0.16	22	LP	0.16	19	LP	0.16	44
CP	0.16	2	CP	0.16	3	CP	0.16	8

Table 5. ~~Comparision~~ Comparison of the melt rate calculated with the LP model and the empirical data obtained with the Gade and Motyka model (Chauche, 2016).

	melt (md^{-1}) (Chauche, 2016)	melt (md^{-1}) LP ($E_0 = 0.036$)
Gade	2.2 ± 0.5	0.6 ± 0.1
Motyka	1.6 ± 0.4	0.7 ± 0.3
Average	1.9 ± 0.5	0.7 ± 0.2

Table 6. ~~Measured~~ Estimated subglacial discharge Q_{emp} and melt rate m_{emp} for a number of glaciers, and corresponding melt rate m_{lp} from LP model simulations. Values of Store winter are taken from the ~~gade~~ Gade model (Fig. 15). For each glacier, local hydrography (temperature and salinity profiles) and measured ~~sublacial~~ subglacial discharge is used to drive the LP model. Ranges indicate measurement errors. Errors in subglacial discharge are propagated to errors in simulated melt rate via the LP model. For the Store glacier in Winter we only report a representative value in the table, where both mean and error were averaged. Simulated melt rate m_{LP} is obtained with $E_0 = 0.036$, $\cancel{\Gamma_T = 2.2}$ $\Gamma_T = 2.2 \cot 10^{-2}$. Melt rate $m^{\star}_{LP}$ with modified $\cancel{\Gamma_T = 4.2}$ $\Gamma_T = 4.2 \cdot 10^{-2}$ is also provided.

	$\mathrm{Q_{emp.}}$ $[\mathrm{m^3/s}]$	$\mathrm{m_{emp.}}$ $[\mathrm{m/d}]$	$\mathrm{m_{LP}}$ $[\mathrm{m/d}]$	$\mathrm{m^{\star}_{LP}}$ $[\mathrm{m/d}]$
Helheim (H)	137 -189	0.7-2.6	1.6-1.7	2.9-3.0
Kangerlussuup Sermia (KS)	208	0.8-3.2	1.5	2.5
~~Equip~~ Eqip Sermia(EQ)	101-121	0.4-1.0	0.7-0.8	1.1-1.2
Seermeq Avangnardleq and Sermeq Kujatdleq (TO)	559-679	3.4-4.4	2.0-2.2	3.2-3.6
Kangilerngata Sermia (KAL)	208-328	1.9-3.0	1.0-1.2	1.5-1.9
Store (Winter, ~~gade~~ Gade model) (ST)	8-73	1.7-2.7	0.5-0.7	0.9-1.1
Store (Summer) (ST su)	201-291	3.0-6.0	1.4-1.7	2.4-2.9

Table A1. Summary of appendix variables. Illustrative value provided for ~~$T_i = -15°C$~~$T_i = -10°C$, $T_a = 4°C$, $S_a = 34.65$ psu, $\sin\alpha = 1$ (tide water glacier), and range for $E_0 = 0.036 - 0.16$

Symbol	Definition	Interpretation	Illustrative Value
M_0	-	melt rate coefficient in (A1)	$8.2 \cdot 10^{-6}\ °C^{-1}$
b_0	$g\sin\alpha(\beta_S S_a - \beta_T T_a)$	buoyancy at $x = 0$	0.27 ms^{-2}
b_m	$g\sin\alpha(\beta_S S_a - \beta_T(T_a - T_m))$	buoyancy source term due to melting	0.23 ms^{-2}
T_m	$c_i/cT_i - L/c$	effective meltwater temperature	$-0.9 \cdot 10^2\ °C$
T_e	$\frac{E_0}{M_0}\sin\alpha$	entrainment-equivalent temperature	$4.4 \cdot 10^3 - 2.0 \cdot 10^4\ °C$
C_e	$E_0 \sin\alpha$	effective entrainment	0.036 - 0.16
ΔT_a	$T_a - T_f(S_a)$	ambient temperature above freezing	$\approx 6\ °C$

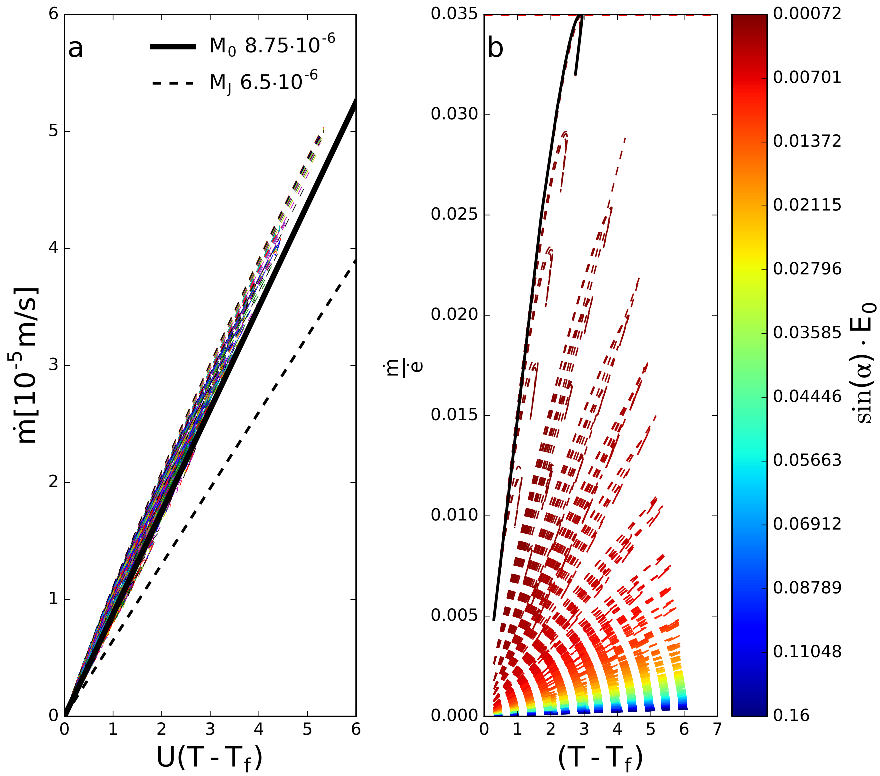

Figure A1. Investigation of melting proportion in the plume equations for different LP experiments. The plume model was run in a well mixed environment for different parameter settings: $E0[0.036-0.16]$, $sin(\alpha)[0.02-1]$, $q_{sq}[10^{-5}-0.1\frac{m^2}{s}]$, $T_a[0-4]^\circ C$. Panel a) shows the melt rate as a function of plume velocity U and plume temperature T and it's freezing temperature T_f ($\dot{m} = M_0(T - T_f)U$). The average slope of the run is $M_0 \approx 8.75 \cdot 10^{-6}$ (thick black line) while for the simplified equations of Jenkins (2011) a typical slope lies in the order of $6.5 \cdot 10^{-6}$ (black dashed line). The second panel illustrates that $\dot{m} << \dot{e}$ in this parameter range, but ~~beeing biggest~~ $\dot{m}/\dot{e}$ being largest for long floating tongues.

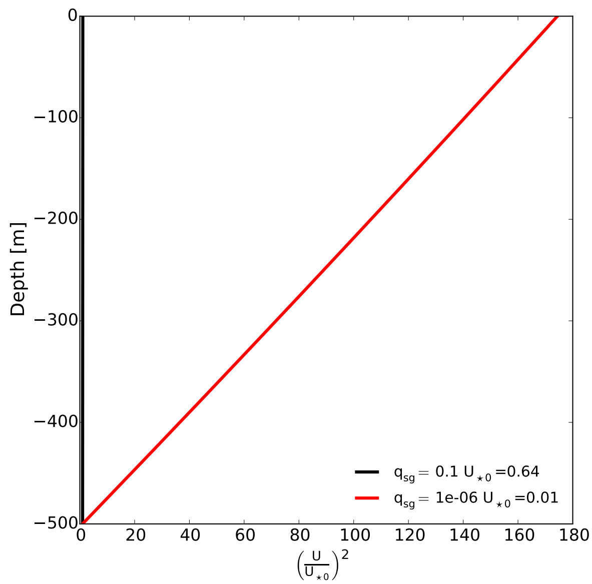

Figure A2. Evolution of U for an initial velocity of $U_\star$. The plume with a small discharge ($q_{sg} = 10^{-6}\frac{m^2}{s}$) will accelerate strongly (red line, $U_\star = 0.14\frac{m}{s}$) while the plume velocity with larger discharge remains almost constant (black line, $q_{sg} = 0.1\frac{m^2}{s}$, ~~;~~ $U_\star = 0.63\frac{m}{s}$).

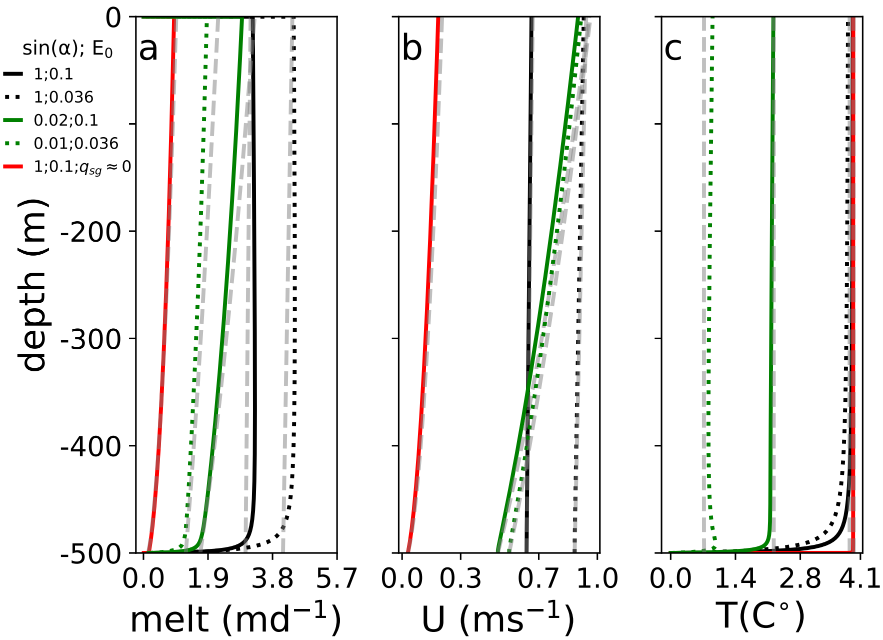

Figure A3. Evolution of m (a), U(b) and T(c) along Z for the tidewater glaciers with high discharge ($q_{sg} = 0.1\,\frac{m^2}{s}$, black line), low discharge ($q_{sg} = 10^{-5}\,\frac{m^2}{s}$,red line) and a floating tongue glacier ($q_{sg} = 0.1\,\frac{m^2}{s}$, green line). Solid lines indicate plumes with $E_0 = 0.1$ while dotted lines represent plume with $E_0 = 0.036$. The corresponding "balance" approximations ~~of~~ $m_\star = M_0 U_\star \Delta T_\star$, $U_\star$ ~~,~~ and $T_\star$ ~~(~~are overlaid as grey, dashed ~~line) show high similarities~~lines.