

Self-affine subglacial roughness: consequences for radar scattering and basal **thaw**water discrimination in northern Greenland

Thomas M. Jordan¹, Michael A. Cooper¹, Dustin M. Schroeder², Christopher N. Williams¹, John D. Paden³, Martin J. Siegert⁴, and Jonathan L. Bamber¹

¹Bristol Glaciology Centre, School of Geographical Sciences, University of Bristol, Bristol, UK.

²Department of Geophysics, Stanford University, Stanford, California, USA.

³Center for Remote Sensing of Ice-Sheets, University of Kansas, Lawrence, Kansas, USA.

⁴Grantham Institute and Department of Earth Science and Engineering, Imperial College, London, UK.

Correspondence to: T. M. Jordan (tom.jordan@bris.ac.uk).

The abstract been revised. In particular, we are explicit about how our study impacts upon the prior work of Oswald and Gogineni (2008, 2012). The rest of the abstract has also been edited to make room for the extra content and for improved accessibility.

5 **Abstract.** Subglacial roughness can be determined at variety of length-scales from radio-echo sound-
ing (RES) data; either via statistical analysis of topography, or inferred from basal radar scattering.
Past studies have demonstrated that subglacial terrain exhibits self-affine (power-law) roughness
scaling behaviour, but existing radar scattering models don't take this into account. Here, using RES
data from northern Greenland, we introduce a self-affine statistical framework that enables a consis-
10 tent integration of topographic-scale roughness with the electromagnetic theory of radar scattering.
We demonstrate that the degree of radar scattering, quantified using the waveform abruptness (pulse
peakiness), is topographically controlled by the Hurst (roughness power-law) exponent. Notably;
specular bed reflections are associated with a lower Hurst exponent, with diffuse scattering asso-
ciated with a higher Hurst exponent. Abrupt waveforms (specular reflections) have previously been
15 used as a RES diagnostic for basal water, and to test this assumption we compare our radar scattering
map with a recent prediction for the basal thermal state. We demonstrate that the majority of thawed
regions (above pressure melting point) exhibit a diffuse scattering signature which is in contradiction
with the prior approach. Self-affine statistics provide a generalised model for subglacial terrain, and
can improve our understanding of the relationship between basal properties and ice-sheet dynamics.
20 ~~Subglacial roughness can be determined at variety of length-scales from radio-echo sounding (RES)
data; either via statistical analysis of along-track topography, or inferred from basal radar scattering.
Past studies have demonstrated that subglacial terrain exhibits self-affine (fractal) scaling behaviour,
where vertical roughness has a power-law relationship with the horizontal length-scale. A self-affine~~

25 statistical framework, which enables a consistent integration of topographic roughness and radar
scattering, has yet to be applied to RES. Here we do this for recent RES data from northern Greenland,
and demonstrate that subglacial topography exhibits pronounced spatial variation in the Hurst (roughness
power-law) exponent. A radar scattering model then enables us to explain how the Hurst exponent
exerts strong topographic control upon radar scattering, which we map using the waveform abruptness
(pulse peakiness) parameter. Notably, lower abruptness (associated with diffuse scattering) occurs
30 for regions with a higher Hurst exponent, and higher abruptness (associated with specular reflections)
occurs for regions with a lower Hurst exponent. Finally, we compare the RES-derived data with an
independent prediction for the subglacial thermal state of northern Greenland. This analysis shows
that the majority of predicted thawed regions do not have the specular RES scattering signature of
deep subglacial lakes, and instead have a diffuse scattering signature.

35 1 Introduction

With the development of the newest generation of thermomechanical ice-sheet models, there has
been a growing awareness that better constraining the physical properties of the glacier bed is essen-
tial for improving their predictive capability (e.g. Price et al. (2011); Seroussi et al. (2013); Nowicki
et al. (2013); Shannon et al. (2013); Sergienko et al. (2014); Ritz et al. (2015); Cornford et al. (2015)).
40 Notably, the basal traction parameterisation - which encapsulates the thermal state, basal roughness,
and lithology - is potentially the largest single geophysical uncertainty in projections of the response
of ice-sheets to climate change (Ritz et al., 2015). Distinction between frozen and thawed regions of
the glacier bed is particularly important in constraining ice dynamics, since appreciable basal mo-
tion can only occur in regions where the glacier bed is wet (Weertman, 1957; Nye, 1970; Macgregor
45 et al., 2016). Airborne radio-echo sounding (RES) is the only existing remote sensing technique that
can acquire bed data with sufficient spatial coverage to enable subglacial information to be obtained
across the ice-sheets (refer to Pritchard (2014) and Bamber et al. (2013a) for recent Antarctic and
Greenland coverage maps). Often, however, there is great ambiguity in RES-derived subglacial in-
formation (Matsuoka, 2011), or RES-derived information is sub-optimal for directly applicability in
50 ice-sheet models (Wilkens et al., 2015). Subsequently, data analysis methods which seek to improve
the clarity and glaciological utility of RES-derived subglacial information are undergoing a period of
rapid development (e.g. Oswald and Gogineni (2008); Li et al. (2010); Fujita et al. (2012); Wolovick
et al. (2013); Schroeder et al. (2013, 2016); Jordan et al. (2016)).

RES data analysis methods for determining subglacial physical properties can be categorised in
55 two ways; those which determine bulk properties (including the discrimination of basal water), and
those which determine interfacial properties (subglacial roughness). Bulk material properties of the
glacier bed can, in principle, be determined using the basal reflection coefficient (Bogorodsky et al.,
1983; Peters et al., 2005; Jacobel et al., 2009; Schroeder et al., 2016). Performing basal reflection

analysis on ice-sheet-wide scale is, however, greatly limited by uncertainty and spatial variation in
60 englacial radar attenuation (Matsuoka, 2011; Matsuoka et al., 2012; Macgregor et al., 2012, 2015;
Jordan et al., 2016). In contrast with bulk properties, subglacial roughness analysis methods are
(near) independent of radar attenuation. Subglacial roughness can be determined either via statis-
tical analysis of topography (typically spectral analysis) (Taylor et al., 2004; Siegert et al., 2005;
Bingham and Siegert, 2009; Li et al., 2010; Rippin, 2013), or inferred from the electromagnetic
65 scattering properties of the radar pulse (Oswald and Gogineni, 2008; Schroeder et al., 2014; Young
et al., 2016)). Spectral analysis can provide valuable insight toward aspects past ice dynamics and
landscape formation (Siegert et al., 2005; Bingham and Siegert, 2009; Rippin et al., 2014). However,
since the technique is limited to investigating length-scales greater than the horizontal resolution
(typically ~ 30 m or greater), the relevance of the method in informing contemporary basal sliding
70 physics - which requires metre-scale roughness information (Weertman, 1957; Nye, 1970; Hubbard
et al., 2000; Fowler, 2011) - remains unclear. Radar scattering is sensitive to the length-scale of the
electromagnetic wave (Shepard and Campbell, 1999) (~ 1 -5 m in ice for the majority of airborne
sounders), and can potentially reveal finer scale roughness information, including the geometry of
subglacial hydrological systems (Oswald and Gogineni, 2008; Schroeder et al., 2013, 2015; Young
75 et al., 2016). High reflection specularity, such as occurs for deep (> 10 m) subglacial lakes (Oswald
and Robin, 1973; Gorman and Siegert, 1999; Palmer et al., 2013), has [has been proposed to be a
generic RES diagnostic for basal water \(Oswald and Gogineni, 2008, 2012\).](#)

Degrees of radar scattering can be mapped using either the waveform properties of the bed-echo
- e.g. the [waveform](#) abruptness (pulse-peakiness) (Oswald and Gogineni, 2008, 2012), or by con-
80 straining the angular distribution of scattered energy - e.g. the specularity content (Schroeder et al.,
2013; Young et al., 2016). Maps of both scattering parameters indicate defined spatial patterns, but,
to date, have not been integrated with topographic-scale roughness analysis (horizontal length-scales
 ~ 10 s of metres and upwards). As such, there is a knowledge gap regarding the topographic control
upon radar scattering. Observations indicate that subglacial roughness exhibits self-affine (fractal)
85 scaling behaviour over length-scales from $\sim 10^{-3}$ m to $\sim 10^2$ m (Hubbard et al., 2000; Macgregor
et al., 2013). Self-affine scaling corresponds to when the vertical roughness increases at a fixed
slower rate than the horizontal length-scale; following a power-law relationship that is parameterised
by the Hurst exponent (Malinverno, 1990; Shepard et al., 2001). It is observed for a wide variety of
natural terrain (Smith, 2014), including: the surface of Mars (Orosei et al., 2003); volcanic lava
90 (Morris et al., 2008); and alluvial channels (Robert, 1988). If widely present, the self-affinity of
subglacial roughness poses a challenge for integrating topographic roughness with existing glacial
radar scattering models (Berry, 1973; Peters et al., 2005; Macgregor et al., 2013; Schroeder et al.,
2015). This is because these [are](#) statistically stationary models [which](#) assume that roughness is inde-
pendent of horizontal length-scale, and [hence](#) an artificial scale-separation between high-frequency
95 roughness and low-frequency topography is present (Berry, 1973). Radar scattering models with

non-stationary, self-affine statistics naturally incorporate the multi-scale dependence of roughness, and are in widespread use in other fields of radar geophysics (e.g. Shepard and Campbell (1999); Franceschetti et al. (1999); Campbell and Shepard (2003); Oleschko et al. (2003)).

The paragraph below now been re-written, to make the purpose of each section clearer (in particular 5.3

In this study, we explore the connection between self-affine subglacial roughness and radar scattering using recent airborne Operation Ice Bridge (OIB) RES data from the north western Greenland Ice-Sheet (GrIS). Firstly we review the theory of self-affine roughness statistics, using examples from ice-penetrating radargrams and bed elevation profiles to demonstrate its applicability to subglacial terrain (Sect. 2). We then outline analysis methods that enable topographic roughness and radar scattering (quantified using the waveform abruptness) to be extracted from RES flight-track data (Sect. 3). A self-affine radar scattering model, adapted from planetary radar (Shepard and Campbell, 1999; Campbell and Shepard, 2003), is then used to predict the relationship between the Hurst exponent and waveform abruptness (Sect. 4). We then present maps of the RES-derived roughness and scattering data for the northern GrIS, and compare the spatial distribution with bed topography (Bamber et al., 2013a) and a recent prediction for the basal thermal state (Macgregor et al., 2016) (Sect. 5.1). The radar scattering model is then used to quantify self-affine topographic control upon radar scattering, via the Hurst exponent (Sect. 5.2). The statistics of the RES-derived data in predicted thawed and frozen regions of the glacier bed is then analysed (Sect. 5.3), with the purpose of testing the validity of the basal water discrimination method by Oswald and Gogineni (2008, 2012). Finally, we discuss the wider consequences of our study, including: subglacial landscape classification, the relationship between bed properties and ice-sheet dynamics, basal thaw/water discrimination, and radar scattering theory applied to RES (Sect 6).

2 Self-affine subglacial roughness

To improve the presentation of the profile data we now use OIB radargrams

2.1 Overview

Statistical methods to calculate the Hurst exponent, and thus to quantify self-affine scaling behaviour, are well established in the earth and planetary science literature (Malinverno, 1990; Shepard et al., 2001; Kulatilake et al., 1998; Orosei et al., 2003). These space-domain methods extract the Hurst exponent using the variogram (roughness verses profile length) and devioqram (roughness versus horizontal lag). Our motivation for use of these methods, rather than the spectral (frequency-domain) methods previously applied in studies of subglacial roughness (Taylor et al., 2004; Siegert et al., 2005; Bingham and Siegert, 2009; Li et al., 2010; Rippin, 2013)), is that they better reveal self-affine scaling behaviour (Turcotte, 1992; Shepard et al., 1995, 2001). Since the theory of self-affine

130 roughness and related space-domain methods are not widely discussed in the glaciological literature
 - the only example being Macgregor et al. (2013) - we now provide a review of the key concepts. **In
 order to illustrate the validity of the statistical model to subglacial terrain, we use recent OIB-RES
 data from northern Greenland (described fully in Sect. 3.1).**

2.2 Interfacial roughness parameters

135 Topographic roughness can be measured by means of statistical parameters that are, in general,
 a function of horizontal length-scale (Shepard et al., 2001; Smith, 2014). Two different interfacial
 roughness parameters - the root mean square (rms) height and rms deviation - are typically employed
 in self-affine roughness statistics (Shepard et al., 2001). The rms height is given by

$$\xi(L) = \left[\frac{1}{N-1} \sum_{i=1}^N (z(x_i) - \bar{z})^2 \right]^{\frac{1}{2}}, \quad (1)$$

140 where N is the number of sample points within the profile window of length L , $z(x_i)$ is the bed ele-
 vation at point x_i , and \bar{z} is the mean bed elevation of the profile. ξ represents the standard deviation in
 bed elevation about a mean surface, and **models the topographic roughness as a Gaussian-distributed
 random variable (Orosei et al., 2003).** The rms deviation is given by

$$\nu(\Delta x) = \left[\frac{1}{N} \sum_{i=1}^N [(z(x_i) - z(x_i + \Delta x))]^2 \right]^{\frac{1}{2}}, \quad (2)$$

145 where Δx is the horizontal step size (lag). ν has a particular significance in the parameterisation
 of radar scattering models with self-affine statistics (Shepard and Campbell, 1999; Campbell and
 Shepard, 2003), and **we focus upon this roughness parameter when integrating topographic-scale
 roughness with radar waveform data.** The rms slope, which is proportional to the rms deviation, is
 also widely used in self-affine statistics, but we do not do so here.

150 2.3 Self-affine scaling behaviour and the role of the Hurst exponent

Self-affine scaling is a sub-class of fractal scaling behaviour and can be parameterised using the
 Hurst exponent, H (Malinverno, 1990; Shepard et al., 1995, 2001). H quantifies the rate at which
 roughness in the vertical direction increases relative to the horizontal length-scale (and is defined for
 $0 \leq H \leq 1$). For a self-affine interface the following power-law relationships hold:

$$155 \quad \xi(L) = \xi(L_0) \left(\frac{L}{L_0} \right)^H, \quad (3)$$

and

$$\nu(\Delta x) = \nu(\Delta x_0) \left(\frac{\Delta x}{\Delta x_0} \right)^H, \quad (4)$$

where where L_0 is a reference profile length, and Δx_0 is a reference horizontal lag (Shepard and
 Campbell, 1999; Shepard et al., 2001). Three limiting cases of self-affine scaling are typically dis-
 160 cussed (Shepard and Campbell, 1999). Terrain with $H = 1$ (where the roughness in the vertical

direction increases at the same rate as the horizontal length-scale) is referred to as ‘self-similar’. Terrain with $H = 0.5$, (where the roughness in the vertical direction increases with the square root of horizontal length-scale) is referred to as ‘Brownian’. Terrain with $H = 0$ (where the roughness in the vertical direction is independent of horizontal length-scale) is referred to as ‘stationary’. For a stationary ($H = 0$) interface it follows from Eq. (3) and Eq. (4) that ξ and ν are independent of L and Δx respectively (i.e. the roughness parameters are independent of horizontal length-scale).

We will later demonstrate that subglacial terrain exhibits near-ubiquitous self-affine scaling behaviour with pronounced spatial structure and variation for H . Examples of OIB ice-penetrating radargrams (Z-scopes) (Paden, 2015) and associated bed elevation profiles, for terrain with different H are shown in Fig. 1. Four examples of 10 km along-track bed elevation profiles which encompass the range of observed H values are shown in Fig. 1a, along with a zoom window to a 1 km track section in Fig. 1b. Clear differences are apparent between the different terrain examples. The black ($H \approx 0.9$) terrain (Fig. 1a) and red ($H \approx 0.7$) terrain (Fig. 1b) are between Brownian and self-similar scaling behaviour. This terrain exhibits ‘persistent trends’ (where neighbouring measurements tend to follow a general trend of increasing or decreasing elevation) (refer to Shepard and Campbell (1999) for a full discussion). A feature of terrain with higher H is that it tends to appear relatively rough at larger length-scales (low frequency) and smooth at smaller length-scales (high-frequency). By contrast, the green ($H \approx 0.3$) terrain (Fig. 1d) is in the sub-Brownian scaling regime, and exhibits ‘anti-persistent trends’ (where neighbouring measurements tend to alternate between increasing and decreasing elevation). A feature of lower H terrain such as this example, is that it tends to have similar roughness across length-scales. The blue ($H \approx 0.5$) terrain (Fig. 1c) is close to an ideal Brownian surface, and exhibits no overall persistence (with some sections of the profile following an increasing/decreasing elevation trend, and other sections alternating). The 10 km profile windows in Fig. 1 represent the length of flight-track data over which H is calculated (see Sect. 3.2).

2.4 Calculation of the Hurst exponent using the variogram and devioqram

In order to estimate/calculate H , and identify the scale regime over which glacial terrain exhibits self-affine behavior, ξ and ν are plotted as a function of L and Δx respectively on double-logarithmic scale plots; referred to as the variogram and devioqram (Kulatilake et al., 1998; Shepard et al., 2001). Variogram and devioqram plots for $\xi(L)$ and $\nu(\Delta x)$ for the four terrain examples in Fig. 1 are shown in Fig. 2a and Fig. 2b respectively. It follows from Eq. (3) and Eq. (4) that, upon this double-logarithmic scale, a straight line relationship is predicted for glacial terrain that is self-affine with the gradient equal to H . In practice, a single self-affine relationship only holds over a limited scale regime and a ‘break-point’ transition is often observed (Shepard et al., 2001). We describe how we assess the break points for glacial terrain in Sect. 3.2, along with further details regarding the application of the variogram and devioqram to along-track RES data. Fig. 2 clearly demonstrates the significance of the Hurst exponent and horizontal length-scale when assessing the relative roughness

of different terrain. For example, the black ($H \approx 0.9$) terrain is rougher than the red ($H \approx 0.7$) terrain at larger length-scales, but is smoother at smaller length-scales. ~~with the green terrain ($H \approx 0.3$) becoming rougher relative to the other terrain at the smaller scales.~~

200 The space-domain variogram and devioqram have an (approximate) correspondence to the frequency-domain power spectrum (Turcotte, 1992; Shepard et al., 1995, 2001). In frequency-space, self-affine scaling occurs when the power spectrum, S , has a relationship of the form $S(k) \propto k^{-\beta}$ where k is the spatial frequency and $-\beta$ is the spectral slope. The relationship between β and H is dimensionally dependent, and for along-track data is given by $H = \frac{1}{2}(\beta - 1)$ (Turcotte, 1992). Despite this
205 correspondence, the space-domain methods are recommended to ~~estimate~~calculate H as they are less noisy and less likely to bias slope estimates than the power spectrum method (Shepard et al., 1995). The study by Hubbard et al. (2000) observed self-affine scaling in the roughness power spectrum over length-scales from $\sim 10^{-3}$ m to ~ 10 m for different sites across recently deglaciated terrain in the immediate foreground of Glacier de Tsanfleuron, Switzerland. Their range for measured values
210 of β corresponds to $2.27 < \beta < 2.48$, which implies $H \approx 0.7$.

3 Analysis of RES data

3.1 Ice penetrating radar system and coverage region

We have removed the frozen-thawed mask from the data coverage map (and moved the related text to the results. Extra information on LIB pre-processing has also been added here.

215

The airborne RES data used in this study were collected by the Center for Remote Sensing of ice-sheets (CRISIS) within the Operation IceBridge (OIB) project, over the months March-May in years 2011 and 2014. For all measurements the radar instrument, the Multichannel Coherent Radar Depth Sounder (MCoRDS), was installed upon a NASA P-3B Orion aircraft. The sounder has
220 a frequency range from 180 to 210 MHz, corresponding to a centre wavelength ~ 0.87 m in ice. After accounting for pulse shaping and windowing, this results in a depth-range resolution in ice of ~ 4.3 m (Rodriguez-Morales et al., 2014; Paden, 2015). For the flight lines considered, the along-track resolution after synthetic aperture radar (SAR) processing and multi-looking is ~ 30 m with an along-track-sample spacing of ~ 15 m (Gogineni et al., 2014). The 2011 and 2014 field seasons
225 were used since they have a higher along-track resolution than other recent field seasons, and hence enable a clearer connection to be made between radar scattering and topographic-scale roughness.

The study focused upon flight-track data from north western Greenland, and encompassed measurements close to three deep ice cores: Camp Century, NEEM and NorthGRIP (Fig. 3). The first reason for selection of this region is that the data coverage for the 2011 and 2014 field seasons is
230 of high density relative to most other regions of the ice-sheet. The second reason is that confidence

regarding the basal thermal state is high near to the ice cores, and thus enables the validity of the basal water RES analysis by Oswald and Gogineni (2008, 2012) to be tested.

Measurements from MCoRDS are supplied as data products with different levels of additional processing (Paden, 2015). Level 2 data corresponds to ice thickness, ice surface, and bed elevation data, and is used to calculate topographic-scale roughness and the Hurst exponent (Sect. 3.2). Details regarding the semi-manual picking procedure are described by Paden (2015), and only the highest quality picks were used. Level 1B data corresponds to radar echo strength profiles, and is used to extract the waveform abruptness parameter from the bed-echo (Sect. 3.3). Basal reflection values can also be extracted from Level 1B but we do not do this here, since we do not wish to bias our interpretation due to uncertainty in radar attenuation. The pre-processing of the combined channel Level 1B data is also described by Paden (2015). Sequentially this involves: channel compensation between each of the antenna phase centers; pulse compression (using a 20% Turkey window in the time domain); coherent-averaging of the channels; SAR processing with along-track frequency window; channel combination; waveform combination.

3.2 Determination of topographic roughness and Hurst exponent from Level 2 data

The along-track spacing (~ 15 m) of the Level 2 data is half the horizontal resolution (~ 30 m), and the horizontal resolution which represents the spacing at which bed elevation measurements are considered as independent. Therefore, to remove local correlation bias, the Level 2 data were down-sampled, considering every second data point (corresponding to a ~ 30 m along-track spacing). Each flight-track was then divided into 10 km along-track profile windows, as was shown in the examples in Fig. 1a. The windows overlap with a sample spacing of 1 km, with the centre of each window defined to be the point at which H and the roughness parameters are geolocated to. This ‘moving window’ approach was employed as it enables greater continuity in the estimates for H . Prior to estimating H , $\xi(L)$ and $\nu(\Delta x)$ were computed following Eq. (1) and Eq. (2) respectively. These calculations used the ‘interleaving’ sampling method described in (Shepard et al., 2001), which enables all of the data points to be sampled effectively. The ~~Our~~ windowing method is similar to that described in Orosei et al. (2003) for the self-affine characterisation of Martian topography, where a non-overlapping 30 km window was assumed. The ~~Our~~ choice of 10 km for the profile window and 1 km for the effective resolution, represent a good trade-off between resolution and the smoothness of the derived data fields.

In this study we are interested in calculating H at the length-scale of the Fresnel zone (~ 100 m), since this enables the most accurate parameterisation of the radar scattering model described in Sect. 4. Additionally, the focus on smaller length-scales is a robust approach to calculate H due to the break point transitions that occur at larger length-scales (Shepard et al., 2001). For the data we consider, the lower bounds of the horizontal length-scales are ~ 90 m for $\xi(L)$ (since three elevation measurements are the minimum required to calculate $\xi(L)$ using Eq. (1)) and ~ 30 m for

$\nu(\Delta x)$. $\nu(\Delta x)$ therefore better enables the estimation of H at smaller length-scales and we primarily focused upon the devioqram method, Fig. 2b., ~~(although we also used the variogram, Fig. 2a, for comparative purposes)~~. Additionally, as suggested in Fig. 2, the relationships for $\nu(\Delta x)$ are, in general, significantly smoother than $\xi(L)$. The upper length-scales in the devioqram and variogram were set to be $\Delta x = 1$ km and $L = 1$ km respectively, which follows from the recommendation by Shepard et al. (2001) that at least 10 independent sections of track are used in the calculations. As shown in Fig. 2, the gradients (H) were calculated using the first five data points (which, for the devioqram, is over the range $\Delta x \sim 30$ -150 m). Self-affine scaling behaviour often extends beyond these smaller length-scales and we estimated the break points for $\xi(L)$ and $\nu(\Delta x)$ using a segmented linear regression procedure. Briefly, this involved firstly calculating the gradient (H) for the first five data points. Additional data points at increasing length-scales were then added into each linear regression model, and the gradient was recalculated. A stopping criteria was then applied where if the new gradient was found to exceed a specified tolerance from the original estimate, then a break point was identified.

3.3 Determination of waveform abruptness from Level 1B data

We have now added extra information about how L1B pre-processing relates to the abruptness signal.

The post-processing of the Level 1B data (analysis of the basal waveform) uses the procedure described in Jordan et al. (2016), which, in turn, is largely based upon Oswald and Gogineni (2008). Firstly, this involved performing an along-track average of the basal waveform, where adjacent basal waveforms are stacked about their peak power values and arithmetically averaged. This averaging approach is phase-incoherent and acts to smooth power fluctuations due to electromagnetic interference (Oswald and Gogineni, 2008). The size of the averaging window varies as a function of Fresnel zone radius, and subsequently each along-track averaged waveform ~~to~~ corresponds to approximately a separately illuminated region of the glacier bed (see Jordan et al. (2016) for details). The degree of radar scattering is quantified using the waveform abruptness

$$A = \frac{P_{peak}}{P_{agg}}, \quad (5)$$

where P_{peak} is the peak power of the bed-echo and P_{agg} is the aggregated power, which is calculated by a discrete summation of the bed-echo power measurements in each depth range bin. P_{agg} was introduced by Oswald and Gogineni (2008) since, based upon energy conservation arguments, it is argued to be more directly related to the predicted (specular) reflection coefficients than equivalent peak power values. In radar altimetry, the waveform abruptness is commonly called ‘pulse peakiness’ (e.g. Peacock and Laxon (2004); Zyguntowska et al. (2013)).

Observed values of A range from ~ 0.03 to 0.60, and in Sect. 4.3 we theoretically constrain the maximum value to be 0.65. Three examples of basal waveforms, along with their corresponding A

values are shown in Fig. 4. Higher A values are associated with specular reflections from smoother regions of the glacier bed (e.g. the blue waveform), whilst lower A values are associated with diffuse reflections from rougher regions (e.g. the green waveform) (Oswald and Gogineni, 2008). The positions of the peak power were established by firstly using Level 2 data picks, then applying a local re-tracker to centre over the peak power. When calculating the summation for P_{agg} , (both fore and aft of the peak power so as to best capture the energy contained in the echo envelope), a signal-noise-ratio threshold was implemented by testing for decay of the peak power to specified percentage above the noise floor. Thresholds of 1, 2, and 5 % were considered and 2 % was found to give the best coverage, whilst excluding obvious anomalies. Due to this quality filtering step there are therefore sometimes small gaps in the along-track A data.

As glacial RES employs a nadir-facing sounder, the scattering contribution toward the waveform abruptness is mainly from coherent reflection (as opposed to side-looking SAR instruments which would be mainly diffuse scattering). Whether coherent pre-processing (either coherent pre-summing of Doppler focusing) of the raw data acts to increase or decrease the value of A depends upon the exact character and roughness of the surface. As a first example; if the specular/nadir component of the echo is assumed to be coherent, whilst the diffuse/off-nadir component is assumed to be incoherent (e.g. Grima et al. (2014)), then coherent processing would cause the specular component of the signal to increase with coherent gain but not the diffuse (incoherent) signal. Therefore the measured A value would decrease with gain. As a second example; if both the specular/nadir and diffuse/off-nadir components of the echo are assumed to be coherent (e.g. Schroeder et al. (2013, 2015)) then for small SAR processing angles (coherent pre-summing) the waveform abruptness should be largely unaffected. However, for larger angles (exceeding the angle spanned by the specular component of the echo in the scattering function) the A value will decrease with coherent pre-processing.

The basal waveform (and hence the calculated values of A), results from a superposition of along-track and cross track energy (Young et al., 2016). Subsequently, the anisotropy of radar scattering (and inferences regarding the anisotropy of subglacial roughness), is not explicitly revealed by A . Hence, the studies of Oswald and Gogineni (2008, 2012) treat A as an isotropic parameter, and we follow this approach here.

4 Radar scattering model for self-affine roughness

We have now added extra information about volume scattering.

4.1 Overview

The waveform abruptness has previously been discussed without reference to roughness statistics, and here we do this using a self-affine radar scattering model. Radar scattering models from natural

terrain fall into two different categories: ‘coherent’ which incorporate deterministic phase interference and ‘incoherent’ which incorporate random phase interference (Ulaby et al., 1982; Campbell and Shepard, 2003; Grima et al., 2014). Coherent scattering models are applicable where the reflecting region is orientated near-perpendicular to the incident pulse (the nadir regime) and the reflecting region is fairly smooth at the scale of the illuminating wavelength (Campbell and Shepard, 2003), which is normally assumed to be a good approximation for the RES of glacier beds (e.g. Peters et al. (2005); Macgregor et al. (2013); Schroeder et al. (2015)). **Volume (Mie) scattering, is typically neglected from basal RES scattering analysis and would hypothetically require scatterer dimensions of the order of the radar wavelength ($\sim 0.5\text{-}5$ m dependent on the bed dielectric and radar system).**

This neglect of of volume scattering is justified given the $\sim 10^{-6}$ m to 10^{-3} m scale of water pore-radii in typical bed materials (Nimmo, 2004). Moreover, even in the extreme case of planetary ice regoliths (which are colder than terrestrial ice and will therefore sustain larger heterogeneities), scatterer dimensions are $\sim 10^{-3}$ to 10^{-2} and volume scattering losses are small (Aglyamov et al., 2017).

Below we describe and adapt a coherent scattering model, first developed for the nadir regime of planetary radar sounding measurements, which incorporates self-affine roughness statistics (Shepard and Campbell, 1999; Campbell and Shepard, 2003). The model is parameterised using the Hurst exponent values derived from the subglacial topography (Sect. 3.2), and thus enables a connection to be made between the topographic roughness and radar scattering. Coherent scattering models can be used to model a decrease in specularly reflected power as a function of rms roughness (Berry, 1973; Peters et al., 2005), and this is the central aspect of the model which we focus upon here. Specifically, we show that, under assumptions of energy conservation, this power decrease can be used to **theoretically** predict the relationship between the Hurst exponent and waveform abruptness.

4.2 Modeling the coherent power

The physical assumptions behind the self-affine scattering model are summarised in Shepard and Campbell (1999). The central assumption that differentiates the model from coherent stationary ($H=0$) models (Berry, 1973; Peters et al., 2005; Macgregor et al., 2013; Grima et al., 2014; Schroeder et al., 2015), is that the rms height increases as a function of radius, r , about any given point, following the self-affine relationship

$$\xi(r) = \frac{1}{\sqrt{2}} \nu_\lambda \left(\frac{r}{\lambda} \right)^H, \quad (6)$$

where $\nu_\lambda = \nu(\Delta x = \lambda)$ is the wavelength-scale rms deviation. Equation (6) assumes radial isotropy for H and ξ , and, since we are focusing upon constraining the (near) isotropic abruptness parameter, is a justifiable approximation. The statistical distribution for $\xi(r)$ is assumed to be Gaussian, which is similar to most $H=0$ models (but with an additional radial dependence.) Via ν_λ , the self-affine model is explicitly formulated with respect to the horizontal-scale of rms roughness. The radio wave-

length of MCoRDS in ice is ~ 0.87 m (Paden, 2015), and hence wavelength-scale rms deviation is approximately equivalent to metre scale rms deviation. An unavoidable caveat to the parameterisation of the radar scattering model using Eq. (6) is that the H values derived from the topography (length-scale ~ 30 -150 m) are extrapolated downwards to the wavelength-scale.

375 An expression for the radar backscatter coefficient (radar cross-section per unit area) is then derived by considering a phase variation, $\frac{4\pi\xi(r)}{\lambda}$, integrated across the Fresnel zone (Shepard and Campbell, 1999; Campbell and Shepard, 2003). For nadir reflection the radar backscatter coefficient is given by

$$\sigma_0 = \frac{16\pi^2 R_e^2}{\hat{r}_{max}^2} \left(\int_0^{\hat{r}_{max}} \exp \left[-\frac{4\pi^2}{\lambda^2} \nu_\lambda^2 \hat{r}^{2H} \right] \hat{r} d\hat{r} \right)^2, \quad (7)$$

380 where $\hat{r} = \frac{r}{\lambda}$ is the wavenumberscaled radius, \hat{r}_{max} is the wavenumberscaled radius of the illuminated area (the Fresnel zone), and R_e is the reflection coefficient for the electric field (Campbell and Shepard, 2003). The coherent power, P , can then be obtained by dividing Eq. (7) by $4\pi^2 \hat{r}_{max}^2$ (a geometric factor which follows from the backscatter coefficient of a flat conducting plate (Ulaby et al., 1982)) to obtain

$$385 \quad P = \frac{4R_e^2}{\hat{r}_{max}^4} \left(\int_0^{\hat{r}_{max}} \exp \left[-\frac{4\pi^2}{\lambda^2} \nu_\lambda^2 \hat{r}^{2H} \right] \hat{r} d\hat{r} \right)^2. \quad (8)$$

For the case where $H=0$, $\xi(r)$ in Eq. (6) is independent of radius. It follows that $\xi^2 = \frac{1}{2}\nu_\lambda^2$ and the exponent in Eq. (8) is also independent of radius which gives

$$P = R_e^2 \exp \left(-\frac{16\pi^2}{\lambda^2} \xi^2 \right). \quad (9)$$

Equation (9) is the same power decay formula as coherent $H=0$ models (Peters et al., 2005; Macgregor et al., 2013; Grima et al., 2014; Schroeder et al., 2015), where it is sometimes multiplied by a first order Bessel function (which enables some of the incoherent energy contribution to be captured (Macgregor et al., 2013)). Thus the stationary limit of the self-affine model that we use is consistent with previous glacial basal scattering models. It is clear that the coherent power for the self-affine model, Eq. (8), has two roughness degrees of freedom: H and ν_λ , which can be conceptually related to the gradient and the intercept of the devioqram (Fig. 2). This contrasts with the stationary model, Eq. (9), which has one degree of freedom: ξ .

395

4.3 Predicted relationship between the Hurst exponent and waveform abruptness

The utility of the waveform abruptness in quantifying different degrees of scattering, rests upon the assumption that the majority of the overall energy is contained within the echo envelope (Oswald and Gogineni, 2008). In other words, it is assumed that, for reflection from the same bulk material, the aggregated/integrated power from a rough interface ($\nu_\lambda > 0$) is equivalent to the peak power from a

400

given smooth interface: i.e. $P_{agg} \approx P(\nu_\lambda = 0)$. This energy equivalence was demonstrated to hold well for the waveform processing procedure and Greenland RES systems by Oswald and Gogineni (2008). It follows from this energy equivalence that the abruptness, A , can be expressed in terms of
 405 the coherent power, Eq. (8), as

$$A = \frac{P_{peak}}{P_{agg}} = C \frac{P(\nu_\lambda)}{P(\nu_\lambda = 0)}, \quad (10)$$

where C is a proportionality constant that corresponds to the theoretical maximum abruptness value, which occurs when the radar pulse is specularly reflected and $P_{agg} = P_{peak}$. For a perfectly specular reflection the pulse is the shape of compressed chirp with the pulse width determined by the signal
 410 bandwidth, and C can be estimated from the number of range cells which fit in the depth-range resolution (~ 0.65). Finally, substituting Eq. (8) into Eq. (10) gives

$$A = \frac{4C}{\hat{r}_{max}^4} \left(\int_0^{\hat{r}_{max}} \exp \left[-\frac{4\pi^2}{\lambda^2} \nu_\lambda^2 \hat{r}^{2H} \right] \hat{r} d\hat{r} \right)^2. \quad (11)$$

As is the case for P in Eq. (8), Eq. (11) has two roughness degrees of freedom: H and ν_λ . Shepard and Campbell (1999) note that the primary dependence for P , (and hence A), is upon H ; with a
 415 weaker secondary dependence upon ν_λ . In order to illustrate this dependency, we firstly consider the relationship between A and H for fixed ν_λ (Fig. 5a), and secondly the relationship between A and ν_λ for fixed H (Fig. 5b). Fig. 5a demonstrates that higher values of ν_λ (the black curve), result in negligible A for all but the lowest values of H . Intermediate values of ν_λ (the red and blue curves), exhibit a sharp transition from higher to lower values of A as H increases. Low ν_λ (the green curve)
 420 has high A for all H . Fig. 5b demonstrates a monotonic decrease in A with ν_λ for each value of H , with the decay length decreasing rapidly with increasing H .

This paragraph has been moved from discussion

It is important to note that the predictions of the self-affine radar scattering model are consistent with the specular RES scattering signature that we would expect from electrically deep subglacial
 425 lakes. Under the self-affine roughness framework, a large geometrically flat feature such as a lake would have a negligible value of H and ν_λ . This scenario occurs for the low H limit of the green curve in Fig. 5a, where predicted values for A are ~ 0.65 (corresponding to a perfectly specular reflection).

The physical explanation for the strong dependence of the coherent power upon H , and the relationships which we observe in Fig. 5, is discussed by Shepard and Campbell (1999) and Campbell and Shepard (2003). It relates to the fact that significant coherent returns can only occur from annular regions where $\xi(r) < \left(\frac{\lambda}{8}\right)$ (the Rayleigh criterion). It follows from Eq. (6) that high values of H lead to a rapid increase in roughness with radius that rapidly exceeds this threshold. Subsequently, for high H interfaces, the roughness at the wavelength-scale, ν_λ , must be a couple orders of mag-
 435 nitude smaller than the Rayleigh criterion to enable significant coherent returns (i.e. non-negligible

A). The curves in Fig. 5 assume $\hat{r}_{max}=100$ (corresponding to a Fresnel zone radius ~ 115 m for the ice wavelength ~ 0.87 m). In general, the relationships in Fig. 5 are insensitive to this choice of radius. This is because the radii of the coherent annular regions are typically significantly less than the Fresnel zone, and thus act as the dominant length-scale for the integration limit in Eq. (11).

440 5 Results

Following the reviewer comments the presentation of the results section has been substantially revised (although we would like to stress that none of the results are actually any different than before). Key changes include: (i) Comparing the RES-derived data with the DEM and broadening the geographical interpretation of the data maps in 5.1. (ii) placing the uncertainty analysis at the end of
445 *the results (thus making the presentation more accessible), (iii) Better separating the presentation of topographic and thermal control of radar scattering.*

Firstly, we describe maps for the rms deviation and Hurst exponent (topographic-scale roughness), and the waveform abruptness (radar scattering) in the northern Greenland (Sect. 5.1). In this analysis
450 we compare the RES-derived data with the Greenland bed digital elevation map (DEM) (Bamber et al., 2013a) and the predicted basal thermal state (Macgregor et al., 2016). Secondly, by comparing the theoretical predictions of the self-affine radar scattering model with the observed relationship between the Hurst exponent and waveform abruptness, we quantitatively assess topographic control upon radar scattering (5.2). Thirdly, we perform a statistical analysis of the RES-derived
455 data in predicted thawed and frozen regions of the glacier bed (Sect. 5.3), which enables us to assess the validity of the basal water discrimination algorithm in Oswald and Gogineni (2008, 2012). Finally, we present uncertainty estimates for the RES-derived data (Sect. 5.4). ~~Since our primary focus is exploring the relationship between the Hurst exponent and waveform abruptness (which we demonstrate to be near isotropic), we choose not to investigate roughness anisotropy in detail.~~

460 5.1 Maps for self-affine roughness and radar scattering in northern Greenland

In Fig. 6 flight-track maps for the RES-derived roughness and scattering data, are compared with the Greenland bed DEM (Bamber et al., 2013a) and the predicted basal thermal state (Fig. 11 in Macgregor et al. (2016)). The flight-track maps all demonstrate a high degree of spatial structure, with some notable correlations present (both between each other and the DEM). There is a clear
465 inverse relationship between the rms deviation, ν , (shown at two different length-scales in Fig. 6a and Fig. 6b) and the waveform abruptness, A , (Fig. 6c), with higher abruptness (specular reflections) present in smoother regions of the ice-sheet bed and lower abruptness (diffuse scattering) present in rougher regions. For example, smoother regions (lower ν , higher A) occur for flight-tracks in the region inland from the settlement of Qanaq and around Camp Century (including the green profile,

470 Fig. 1d); in the interior around the NorthGrip borehole; and a region ~ 150 km ENE of the NEEM ice
core. Whilst these smoother regions are at a range of bed elevations, (ranging from ~ 800 m NE of
Qanaq to around sea level in the interior), they are all spatially correlated with flatter bed topography
(Fig. 6e). Correspondingly, many rougher regions (higher ν , lower A) are spatially correlated with
475 more complex topography - e.g. the region of the ice-sheet inland from the Melville Bugt coast
(including the red profile, Fig. 1b). However, some rougher regions of the bed have a less obvious
correlation with higher contour gradients - e.g. the flatter regions inland from the Humboldt glacier.

Pronounced spatial variation in the Hurst exponent, H , is evident in Fig. 6d. H also has a inverse
relationship with A and spatially correlates with the bed topography in a similar manner to ν . In
other words, lower H is associated with higher A and flatter regions of the bed - e.g. near Camp
480 Century, whilst higher H is associated with lower A and generally more complex bed topography -
e.g. inland from the Melville Bugt coast and inland from Ryder glacier (including the black profile,
Fig. 1a). In Sect. 5.2 a quantitative assessment of this relationship is made using the radar scattering
model. The simple notion that, at the topographic-scale, 'rougher regions of the bed correspond to
higher H ' can be related back to the power-law scaling relationship in the devioqram (Fig. 2b). The
485 length-scales for the rms deviation maps $\nu(\Delta x = 30$ m) in Fig. 6a and $\nu(\Delta x = 150$ m) in Fig. 6b are
chosen as they are the lower and upper bounds in the devioqram calculation for H . It is interesting
that, despite the clear spatial variation in H in Fig. 6d, the overall spatial distributions for $\nu(\Delta x =$
30 m) and $\nu(\Delta x = 150$ m) are remarkably similar. Thus, from a purely visual inspection of ν at
different length-scales, the pronounced spatial variation in H is not immediately apparent.

490 *The context regarding the basal thermal state mask (previously in 3.1) has now been added here*

The basal thermal state prediction by Macgregor et al. (2016) (Fig. 6f) represents an up-to-date
best estimate for the GrIS at a 5 km resolution. It is based upon a trinary classification: likely
thawed/above pressure melting point (red), likely frozen/below pressure melting point (blue), uncer-
tain (grey). The mask was determined using four independent methods: thermomechanical model-
495 ing of basal temperature, basal melting inferred from radiostratigraphy, surface velocity, and surface
texture. The mask is therefore independent of our RES-derived data fields. There are some obvious
correlations between the basal thermal state prediction and the RES-derived roughness and scatter-
ing data. For example many predicted thawed regions toward the margins - e.g. the region of the
ice-sheet inland from the Melville Bugt - correspond to rougher terrain (higher H and ν) and diffuse
500 scattering (lower A). However, there are regions of predicted thaw that demonstrate the opposite
behaviour (lower H and ν , and higher A); for example the two interior regions previously identified
as smooth around the NorthGrip borehole and the region ENE of NEEM. The scattering signature
of predicted thawed regions is therefore non-distinct and can be either specular or diffuse. Predicted
frozen regions tend to be smoother with specular reflections (higher A), although it clear that spa-
505 tial variation is present with some regions exhibiting more diffuse scattering (lower A). Sect. 5.3
provides a more detailed statistical analysis.

There some clear discontinuities in the flight-track maps for ν and H in Fig. 6. These can be explained by either roughness-anisotropy, or due the self-affine terrain model breaking down in certain regions (e.g. a sharp terrain discontinuity such as a subglacial cliff). By contrast the map for A is smoother, which is consistent with its interpretation as an isotropic scattering parameter.

5.2 Statistics for topographic control upon radar scattering and comparison with scattering model

Before we consider a quantitative comparison between the predictions of the radar scattering model and the RES-derived data, we firstly summarise the statistics for the Hurst exponent, H . The total frequency distribution for H , corresponding to the flight-track data in Fig. 6d, is shown in Fig. 7a. The distribution is divided into three categories: (i) $H > 0.75$ ('High' H); (ii) $0.5 < H \leq 0.75$ ('Medium H '); (iii) $H \leq 0.5$ ('Low' H), which we later use to compare with the radar scattering model predictions. These categories correspond to approximately 30 %, 50 % and 20 % of the total data respectively. Approximately 0.1% of the H estimates are > 1 and none of the H estimates are < 0 , representing near-ubiquitous self-affine scaling behaviour, ($0 < H < 1$). A overall negative skew is observed with a mean value of 0.65, indicating that the majority of the subglacial terrain along the flight-tracks lies between Brownian ($H = 0.5$) and self-similar ($H = 1$) scaling regimes. The spatial coverage of the radar flight tracks in Fig. 6d is, however, more comprehensive in regions of higher H . Thus the mean value and skew of H in Fig. 7a are likely overestimates and underestimates of true (equal area) averaged values for the region.

The self-affine coherent scattering model (Sect. 4) predicts that there are two roughness degrees of freedom that control A : H (the primary control) and ν_λ (the secondary control). Given the primary dependence of A upon H that is predicted by the model, a natural starting point is to compare with the observed relationship between A and H (Fig. 6). This approach assumes a downward extrapolation of H from the topographic-scale to the wavelength-scale in the radar scattering model. At metre-scale, ν_λ is significantly smaller than the along-track resolution (~ 30 m), and therefore cannot be observed directly. Based upon the assumption that ν_λ varies spatially, a statistically-distributed inverse relationship between H and A is predicted which corresponds to the family of predicted curves in H - A space in Fig. 5. Since higher values of A (specular reflections) are predicted to be suppressed for higher H , we would expect there to be a statistically lower value of A for the High H than the Low H category in Fig. 7a. In other words; a statistically-distributed inverse relationship is predicted.

In order to test this prediction, we considered the statistics of three separate A distributions for each H category, which are shown for: High H in Fig. 7b, Medium H in Fig. 7c, and Low H in 7d. A nearest neighbour interpolation was used to pair each A value (~ 100 -150 m along-track spacing) with each H value (1 km along-track spacing). The lowest mean value, smallest variance, and

strongest positive skew is observed for the High H category. This supports the general prediction in Fig. 5 that higher A values (specular reflections) are suppressed in regions of higher H , with lower A values (diffuse scattering) being more probable. The highest mean value, greatest variance, and weakest positive skew is observed for the Low H category. Again, this supports the prediction in Fig. 5 that A is less constrained in regions of lower H , with a tendency toward higher values (specular reflections). As would be expected, the A -distribution statistics for the Medium H category lie between the High H and Low H categories with intermediate mean values, variance, and skewness. Finally, the observed values of A in Fig.7 range from ~ 0.03 to 0.60 which is in agreement with the theoretically constrained maximum value of 0.65.

~~The next section has been removed since it adds little to the overall results or conclusions. We repeated the above analysis for H values that were estimated using the variogram for $\xi(L)$. The distributions for the three H categories are qualitatively similar to Fig. 7b-d but with less pronounced quantitative differences between H categories (e.g. the mean values for A are 0.201, 0.243 and 0.254 for High, Medium and Low H categories respectively). This is as expected, since the length-scales at which H is estimated using the devigram are more directly related to radar scattering than the variogram.~~

5.3 Statistics in thawed and frozen regions

~~To better introduce the frozen-thawed statistics we have now rewritten the first paragraph (by moving some of the material about the O+G algorithm that was originally in the discussion to this section)~~

~~The spatial distributions in Fig. 6 indicate that predicted thawed regions...ete~~ Here we summarise the statistics of the RES-derived roughness and scattering data in predicted thawed and frozen regions of the glacier bed, with an overall purpose of testing the basal water discrimination algorithm by Oswald and Gogineni (2008, 2012). Conceptually, their approach assumes that water in thawed regions has a similar RES signature to deep subglacial lakes which exhibit brighter and more specular reflections than surrounding regions (e.g. Oswald and Robin (1973); Gorman and Siegert (1999); Palmer et al. (2013)). In their algorithm regions of thaw are discriminated if: (i) the relative bed reflectivity is above a threshold, (using an attenuation model where the attenuation rate has an inverse relationship with surface elevation); (ii) the abruptness is also above a threshold (around 0.3). Thus, in their approach, high abruptness (specular reflections) is a necessary, but not sufficient, criteria for identifying basal water. A further feature of their approach is that spatial continuity for water is imposed: i.e. only larger-scale regions (~ 100 s km² and upwards) are considered.

The distributions for all RES-derived data exhibit pronounced statistical differences between thawed and frozen regions (Fig. 8). The mean value for H in thawed regions is 0.74 with a strong negative skew (Fig. 8a), whereas the mean value for H in frozen regions is 0.54 with a weak negative skew (Fig. 8b). The mean value for $\nu(\Delta x=30$ m) in thawed regions is 6.36 m, which is over double

the mean value of 2.80 m in frozen regions. A qualitatively similar distinction between thawed and frozen regions is also present for $\nu(\Delta x=150\text{ m})$, with a mean value of 21.7 m in thawed regions and 7.2 m in frozen regions (not shown). The thawed distribution for A is similar to the high H category in Fig. 7b, with a mean A value of 0.165 and strong positive skew. The frozen distribution is similar to the low H category in Fig. 7d with a mean A value of 0.264 and a weak positive skew. These statistics demonstrate a contradiction with the basal water discrimination algorithm of Oswald and Gogineni (2008, 2012). Lower abruptness (diffuse scattering) is more common in thawed regions where basal water is likely to be present. Moreover, the necessary high abruptness (specular reflections) condition for water is generally not satisfied (particularly at the larger spatial scales that were considered by Oswald and Gogineni (2008, 2012) when mapping basal water).

~~In Fig. 8 an anti-correlation between the skewness of the H and A distributions is observed (i.e. in thawed regions a strong negative skew for H is paired with a strong positive skew for A , and in frozen regions a weak negative skew for H is paired with a weak positive skew for A). This provides further observational evidence for the self-affine radar scattering model (Sect. 4), which predicts a statistically distributed inverse relationship between H and A .~~

5.4 Uncertainty and consistency of RES-derived data

~~The majority of this new section has been moved from 5.1. When calculating H using the devioqram (over the horizontal length-scale $\Delta x \sim 30\text{-}150\text{ m}$ as described in Sect. 3.2) a mean r^2 value for the linear regression model of ~ 0.99 was obtained. This correlation is, however, likely to be artificially high due to the smoothing of the data that occurs when applying the windowing/averaging of the data.~~ In RES data analysis, cross-over distributions at flight-track intersections can give an indication of uncertainty based upon internal consistency (e.g. Macgregor et al. (2015); Jordan et al. (2016)). However, due to the anisotropy in Fig. 6d, cross-over analysis for $H(\nu(\Delta x))$ cannot be applied directly. Hence repeat estimates were made using the variogram to calculate $H(\xi(L))$ (i.e. calculating H using rms height). The map for $H(\xi(L))$ (not shown) has a similar spatial distribution as Fig. 6d but with greater high-frequency noise apparent. Differencing the estimates as $H(\nu(\Delta x)) - H(\xi(L))$ and performing cross-over analysis gives a mean bias of -0.026 and a standard deviation of 0.10 (10% of the parameter range). The small mean bias is potentially explained by the variogram estimates being at a slightly larger length-scale ($L \sim 90\text{-}210\text{ m}$). Additional cross-over analysis using different profile window sizes (e.g. 15 km) confirms that 0.10 serves a reasonable estimate for the uncertainty of H . Since A is assumed to be isotropic, the uncertainty can be estimated via cross-over analysis of flight-track intersections. This gives a cross-over standard of ~ 0.05 (again $\sim 10\%$ the parameter range).

As part of the analysis we also considered estimation of the breakpoint transitions for $H(\xi(L))$ and $H(\nu(\Delta x))$ using the segmented linear regression procedure described Sect. 3.2. The exact values of the breakpoints depend upon how strict the stopping criteria is, so here we just discuss some general

615 trends. Firstly, the self-affine scaling relationships often extend over a much greater length-scale than the upper length-scale used in the calculation of H (often over 500 m as occurs in Fig. 2). Secondly the breakpoints for $H(\nu(\Delta x))$ generally occur at greater length-scales than for $H(\xi(L))$. Thirdly, the break points for both $H(\nu(\Delta x))$ and $H(\xi(L))$ tend to be greater toward the ice-sheet margins where H is higher.

620 **6 Discussion**

We have re-ordered and edited some of the content in the discussion.

Our results demonstrate that self-affine scaling behaviour is a near-ubiquitous property of the subglacial topography of northern Greenland. Moreover, there is both spatial structure and variability in the Hurst exponent, which can range from being near-self similar ($H = 1$) to sub-Brownian ($H < 0.5$). The Hurst exponent is valuable as it provides a way to integrate maps of topographic-scale roughness metrics (e.g. rms height and rms deviation) and maps of radar scattering parameters (e.g. the waveform abruptness), which provide finer-scale roughness information. Notably, theoretical predictions and observations both demonstrate that higher values of the abruptness (specular reflections) are suppressed in rougher regions of the bed with a higher Hurst exponent. Additionally, extended continuous regions of higher abruptness are generally limited to occur in smoother regions with a lower Hurst exponent. This finding implies that maps of radar scattering information - including both the waveform abruptness parameter discussed in here and in Oswald and Gogineni (2008) and Oswald and Gogineni (2012), and the specular content in Schroeder et al. (2013) and Young et al. (2016) - will, in future, benefit from analysis that incorporate self-affine topographic control.

635 The Hurst exponent provides information about the relationship that exists between vertical roughness and the horizontal length-scale. Whilst it is related to the slope of the roughness power spectrum, past spectral analysis of glaciological terrain tends to obscure this information (since an integrated ‘total roughness’ metric is typically used) (Taylor et al., 2004; Siegert et al., 2005; Bingham and Siegert, 2009; Li et al., 2010; Rippin, 2013). Subsequently, the Hurst exponent represents new subglacial roughness information, that could potentially be utilised much more widely than our current application in constraining radar scattering. For example, planetary scientists have previously employed the Hurst exponent in a geostatistical classification of Martian terrain (Orosei et al., 2003). Interestingly, the spatial distribution of the Hurst exponent for the Martian surface, has a similar level of spatial variation and coherence to what we observe for glacial terrain. Additionally, the distribution of H for Martian terrain is skewed toward higher, self-similar, values with near-continuous regions of lower H limited to mid-latitude plains. For Greenland, this self-affine statistical landscape classification could be integrated with existing knowledge of geology (e.g. Henriksen (2008)) and larger-scale landscape features including subglacial drainage networks (Cooper et al., 2016; Chu

et al., 2016; Livingstone et al., 2017) and paleofluvial canyons (such the 'mega canyon' feature, 650 observed in Fig. 6e which has Petermann glacier as its modern day terminus) (Bamber et al., 2013b).

The Hurst exponent has previously been shown to play a dynamical role in the flow resistance of alluvial channels (Robert, 1988). Whilst basal sliding - modulated by enhanced plastic flow and regulation (Weertman, 1957; Nye, 1970; Hubbard et al., 2000; Fowler, 2011) - is clearly a different physical phenomena, it is possible that the Hurst exponent itself may provide a useful radar-derived 655 parameter for informing our understanding of geometric control upon this process. In Sect. 5.1 and Sect. 5.3 we observed that toward the ice-sheet margins such inland from the Melville Bugt coast, predicted thawed regions are characterised by higher (often near self-similar) values of H , whereas frozen regions are characterised by lower values (normally Brownian, or sub-Brownian). One could therefore conjecture that the persistent behaviour associated with high H interfaces (neighbouring 660 points follow a similar elevation trend, Sect. 2.3) could act to promote basal sliding, whereas the anti-persistent behaviour (neighboring points alternate in elevation trend) associated with low H interfaces could act to inhibit basal sliding. However, as is widely acknowledged, attributing a direct link between subglacial roughness and contemporary ice dynamics is a complex topic (Siegert et al., 2005; Bingham and Siegert, 2009; Rippin et al., 2014). Therefore, as with other measures or 665 basal roughness, the spatial variation in the Hurst exponent is likely to also originate from different glaciological processes at a variety of spatial scales, including erosion and deposition. Additionally, we recommend that future works which investigate the connection between the Hurst exponent and glaciological processes should be discussed with reference to anisotropy and flow direction.

The statistical analysis of the waveform abruptness in predicted frozen and thawed regions (Sect. 670 5.3) demonstrates that, overall, very different RES scattering signatures are likely to be present than assumed by Oswald and Gogineni (2008, 2012). Firstly, the majority of the predicted thawed regions have lower abruptness (diffuse scattering). In their algorithm, this would correspond to false-negative detection of basal thaw (since the necessary high abruptness condition for thaw is not satisfied). Secondly, high abruptness is often present in predicted frozen regions, many of which are interpreted as 675 thawed by Oswald and Gogineni (2008, 2012) (e.g. some of the region of higher abruptness near to the Camp Century ice core, which at high bed elevation is likely to correspond to harder bedrock). It is, however, important to note that some of smoother regions discriminated as wet by Oswald and Gogineni (2008, 2012) are consistent with basal thermal state prediction by Macgregor et al. (2016) (e.g. near NorthGRIP). Radar bed reflectivity was also used by Oswald and Gogineni (2008, 680 2012) in their discrimination of thawed beds. Since these original studies, the role that uncertainty in radar attenuation has in biasing the spatial distribution of radar bed reflectivity has become much better understood (Matsuoka, 2011; Macgregor et al., 2012; Jordan et al., 2016). For example, if an attenuation model has a constant systematic bias in attenuation rate, then there will be an ice thickness - correlated bias in estimated bed reflectivity (Jordan et al., 2016). Thus, spatially-correlated 685 bias in the attenuation model is one explanation for why elevated reflectivity was observed in some

predicted frozen regions. Additionally, geological transitions, between less reflective sediment and more reflective bedrock (see Bogorodsky et al. (1983); Peters et al. (2005) for reflectivity values) could also play a role in complicating the analysis.

690 Subglacial hydrological systems are understood to be capable of more complex and variable scattering signatures than the specular lake-like reflection assumed by (Oswald and Gogineni, 2008, 2012). For example, concentrated hydrological channels act as an anisotropic rough surface capable of orientation-dependent scattering (Schroeder et al., 2013; Young et al., 2016). Additionally, due to scattering from the lake bottom and related interference effects, shallower (depth < 10 m) subglacial lakes can appear to produce diffuse scattering, (refer to Gorman and Siegert (1999) for a full
695 discussion). Channelised drainage networks have recently been identified beneath Humbolt glacier (Livingstone et al., 2017), in agreement with the generally diffuse scattering that we observe in this region. Whilst the majority of the thawed regions have lower abruptness, there are some smaller, localised, patches of higher abruptness present in Fig. 6c: for example, in the main trunk of Petermann glacier. These regions are consistent with there being deep lake-like water present (in the sense that
700 specular reflections are observed in a region predicted to be above pressure melting point). However, because the frozen abruptness distribution in Fig. 8f indicates that basal water is not required to produce highly specular reflections, it is not possible to confirm that this is the case without additional analysis. ~~This is because the frozen abruptness distribution in Fig. 8f indicates that basal water is not required to produce highly specular reflections, and thus smooth regions of bedrock may be responsible for the high abruptness.~~ The presence of ~~at least~~ some localised patches of high
705 abruptness in thawed regions is consistent with the recent discovery of two small subglacial lakes in north western Greenland of $\sim 8 \text{ km}^2$ and $\sim 10 \text{ km}^2$ in extent (Palmer et al., 2013). More generally, however, the relative rarity of high abruptness in thawed regions is in agreement with hydrological potential analysis (Livingstone et al., 2013), which predicts that deep subglacial lakes are both rare
710 and small in the northwest of the GrIS.

The content has been re-ordered in the next paragraph. We have also integrated the paragraph on metre-scale roughness here.

The anisotropy of the Hurst exponent was not considered in the radar scattering model, which was justifiable because we were interested in understanding how the Hurst exponent relates to the (near)
715 isotropic waveform abruptness. However, in certain regions of the ice-sheets, basal radar scattering is known to be highly anisotropic; as revealed by maps of the specularity content for Thwaites glacier (Schroeder et al., 2013) and Byrd Glacier (Young et al., 2016). Thus a clear direction of future research would be to modify the self-affine radar scattering model (Sect. 4) to take into account anisotropy in H , and then to compare this model with maps for the specularity content. The
720 pronounced spatial heterogeneity for H implies that estimation of roughness statistics from $H = 0$ radar scattering models (Eq. (9) and (Berry, 1973; Ulaby et al., 1982; Peters et al., 2005; Grima et al., 2014)) may give erroneous results; particularly when comparing the overall spatial distribu-

tion between regions with different H values. Additionally, the radar scattering model is formulated with respect to wavelength-scale (approximately metre-scale) roughness, and thus provides a way to estimate metre-scale roughness (i.e. given A and H obtain an estimate for ν_λ in accordance with the curves in Fig. 5). This could have important glaciological consequences, since the physical processes which influence basal sliding operate at the metre-scale (Weertman, 1957; Nye, 1970; Hubbard et al., 2000; Fowler, 2011).

Finally, geostatistically-based interpolation methods which employ aspects of self-affine statistics (Goff and Jordan, 1988), have found recent application in generating synthetic subglacial topography (Goff et al., 2014). The self-affine characterisation of subglacial topography described here informs such techniques, and, in turn, could be used to inform the ice-sheet-wide interpolation of future Greenland (Bamber et al., 2013a; Morlighem et al., 2014) and Antarctic (Fretwell et al., 2013) subglacial digital elevation maps.

~~This has been moved to the radar scattering model section It is important to note that both the predictions of the self-affine radar scattering model (Sect. 4), and the observed relationship between the abruptness and Hurst exponent (Sect. 5.1, Sect. 5.2), are consistent with the specular RES scattering signature that we would expect from electrically deep subglacial lakes. Under the self-affine roughness framework, a large geometrically flat feature such as a lake would have a negligible value of H and ν_λ . This scenario occurs for the low H limit of the green curve in Fig. 5a, where predicted values for A are ~ 0.65 (corresponding to a perfectly specular reflection).~~

~~The following paragraph has been integrated with others in the discussion (some of the content also repeated the parts of the introduction and was therefore redundant~~

~~Whilst topographic (spectral) roughness analysis has already been performed for topography beneath ice-sheets...~~

7 Summary and conclusions

We have performed a general edit and re-emphasised the significance of our study for basal thaw discrimination

In this study we used recent OIB RES data to demonstrate that subglacial roughness in northern Greenland exhibits self-affine scaling behaviour, with pronounced spatial variation in the Hurst (roughness power-law) exponent. We used a self-affine radar scattering model to predict how the Hurst exponent exerts control upon basal radar scattering, which we parameterised using the waveform abruptness. We then demonstrated an agreement between the predictions of the radar scattering model and the statistically-distributed inverse relationship that is observed between the Hurst exponent and waveform abruptness. This enables us to conclude that self-affine statistics provide a valuable framework in understanding the topographic control which influences ice-penetrating radar scattering from glacier beds. Self-affine statistics provide a generalised model for subglacial terrain,

and in the future could be used to further explore the relationship between bed properties, ice-sheet dynamics, and landscape formation.

760 An additional glaciological motivation behind our study was to establish if waveform abruptness could be used to aid in the discrimination of thawed beds (and to test the prior assumption that subglacial hydrological systems in Greenland produce abrupt bed-echoes (Oswald and Gogineni, 2008, 2012)). To do this we compared our RES-derived data fields with a recent basal thermal state prediction for northern Greenland (Macgregor et al., 2016). The analysis demonstrated that thawed
765 regions of the glacier bed have statistically lower values of the waveform abruptness than frozen regions (more diffuse scattering). From the viewpoint of topographic control upon radar scattering, the simple explanation is that many thawed regions are relatively rough with a higher Hurst exponent, whilst many frozen regions are relatively smooth with a lower Hurst exponent. This finding should not be viewed as a new RES diagnostic for basal thaw/water (since deep subglacial lakes do have
770 the specular signature proposed by (Oswald and Gogineni, 2008, 2012)). However, it indicates that the algorithm in Oswald and Gogineni (2008, 2012) is likely to yield both false-negatives (failing to identify water in rougher regions) and false-positives (identifying some smoother frozen regions as wet). Hence, whilst high reflection specularity is a typical RES signature of subglacial lakes, it is not a typical RES signature of thawed regions of the Greenlandic subglacial interface. We therefore
775 conclude that RES methods which seek to discriminate thawed regions, as distinct from deep (>10 m) subglacial lakes or other electrically deep water bodies, should not use high waveform abruptness (high reflection specularity) as a necessary criteria for discrimination of a thawed bed.

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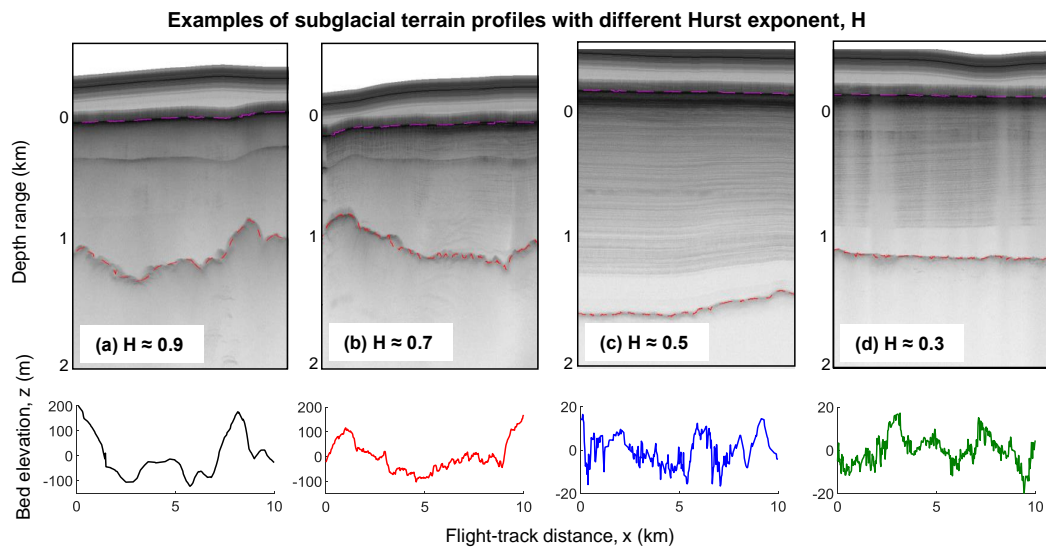


Figure 1. Example radargrams (top panel) and 10 km bed elevation profiles (bottom panel) for subglacial terrain with different Hurst exponent, H : (a) $H \approx 0.9$ (near self-similar), (b) $H \approx 0.7$ (between Brownian and self-similar), (c) $H \approx 0.5$ (Brownian), (d) $H \approx 0.3$ (sub-Brownian). The location of the profiles are shown in Fig. 3. Evident in the radargrams are the surface reflection (pink line), the bed reflection (red line), and reflections from internal layers in ice. The bed elevation profiles are linearly detrended about zero and have horizontal resolution ~ 30 m. The horizontal-vertical aspect ratio of the bottom panels differ between (a),(b) and (c),(d). Examples of along-track bed elevation profiles for subglacial terrain with different Hurst exponent, H . (a) 10 km profile window. (b) Zoom to 1 km window with sample points indicated (corresponding to ~ 30 m along-track resolution). The bed elevation is linearly detrended for each 10 km window. The aspect ratio differs between subplots.

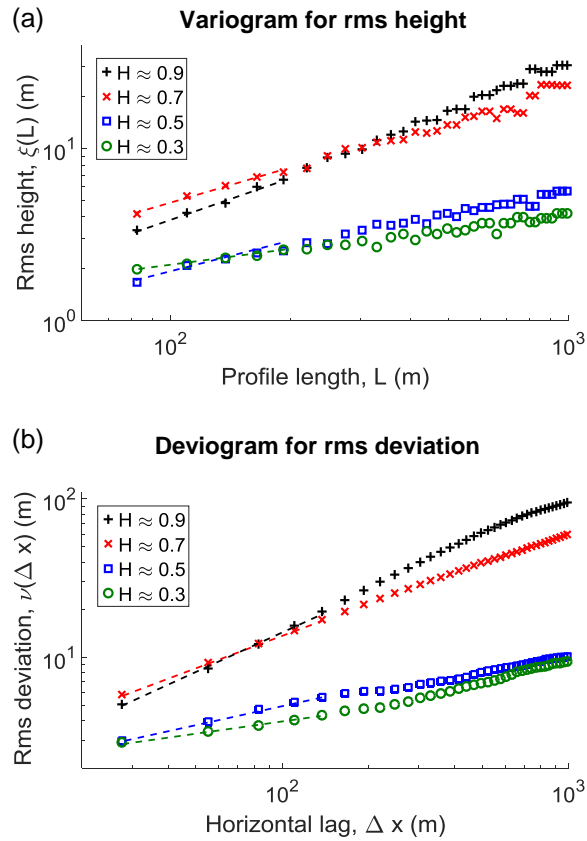


Figure 2. (a) Variogram for rms height, ξ , versus profile length L (log-log scale). (b) Deviogram for rms deviation, ν , versus horizontal lag, Δx (log-log scale). The plots correspond to subglacial terrain profiles in Fig. 1a. The Hurst exponent is estimated from the linear gradient of the first five data points (indicated by dashed lines). These space-domain plots are (approximate) equivalents to frequency-domain roughness power spectra, and smaller lengthscales correspond to higher frequencies.

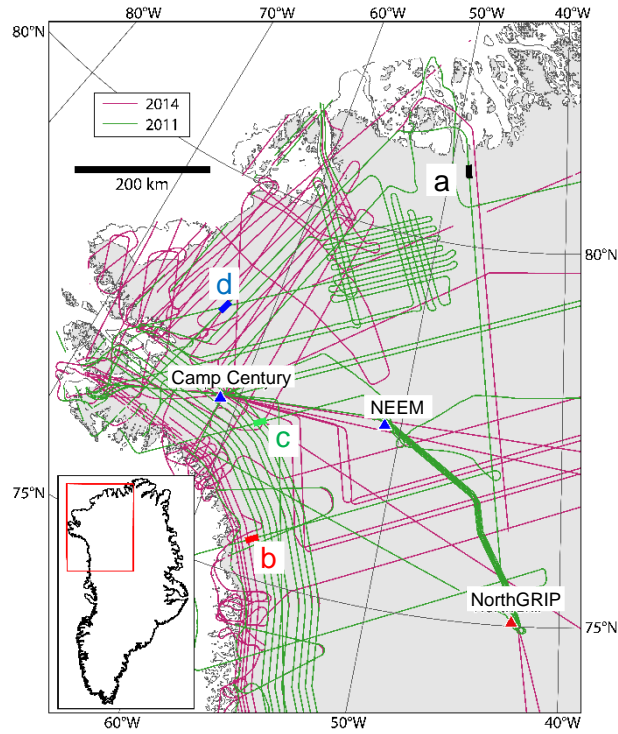


Figure 3. Data coverage map for OIB flight-tracks and region of interest. The locations of the Camp Century, NEEM and NorthGRIP ice cores are indicated, along with the terrain profile sections in Fig. 1.

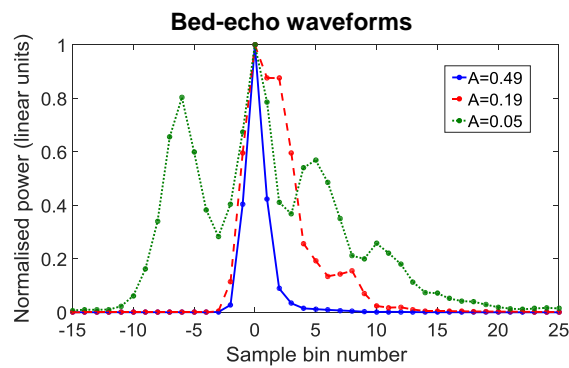


Figure 4. Examples of bed-echo waveforms and their abruptness (pulse peakiness). Observed values for A range from ~ 0.03 (associated with diffuse scattering) to ~ 0.60 (associated with specular reflection). For the purpose of comparative plotting, the waveforms are normalised about their peak power values with the sample bin of the peak power set to zero. The sample bin spacing corresponds to a depth-range spacing of ~ 2.81 m.

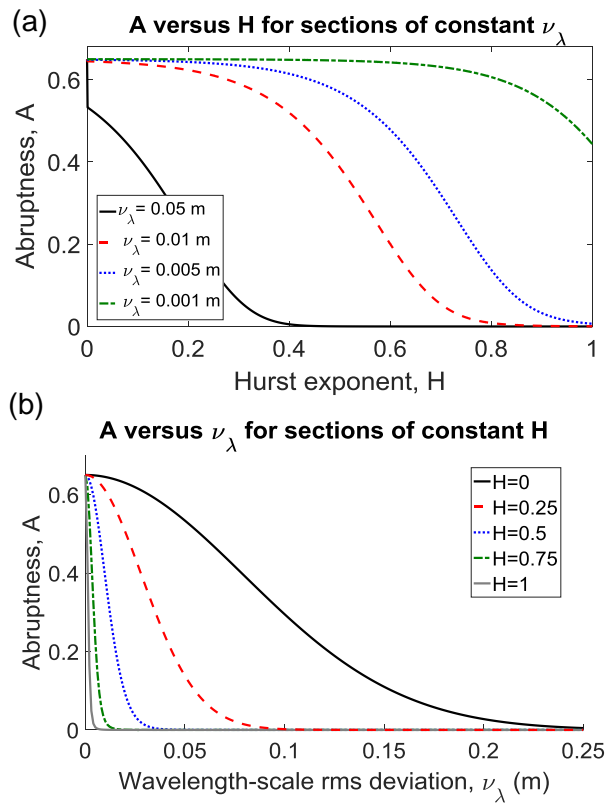


Figure 5. Parametric dependence of the self-affine radar scattering model. (a) Abruptness, A , as a function of the Hurst exponent, H , for sections of constant wavelength-scale rms deviation, ν_λ . (b) A as a function of ν_λ for sections of constant of H . The plots illustrate primary dependence for A upon H , and secondary dependence for A upon ν_λ . High A is suppressed for high H , except in the case of exceptionally small ν_λ .

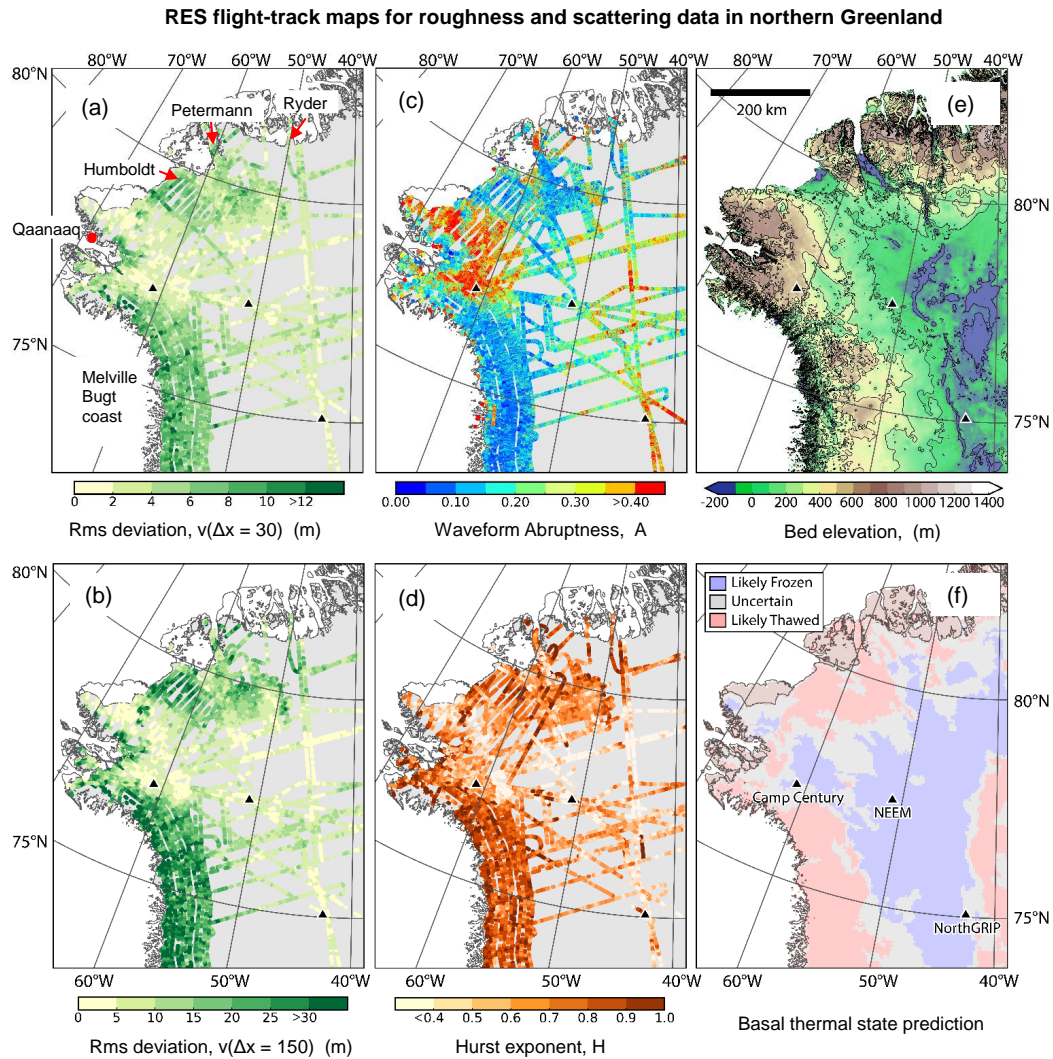


Figure 6. Data maps for the northern GrIS. (a) Hurst exponent, H . (b) Rms deviation (topographic roughness) at ~ 30 m lag, $\nu(\Delta x=30$ m). (c) Rms deviation at ~ 150 m lag, $\nu(\Delta x=150$ m). (d) Waveform abruptness (degree of radar scattering), A . (e) Greenland bed DEM (black countour lines at 200 m), (Bamber et al., 2013a). (f) Predicted basal thermal state mask (Macgregor et al., 2016). Higher values of A in (d) indicate more specular reflections.

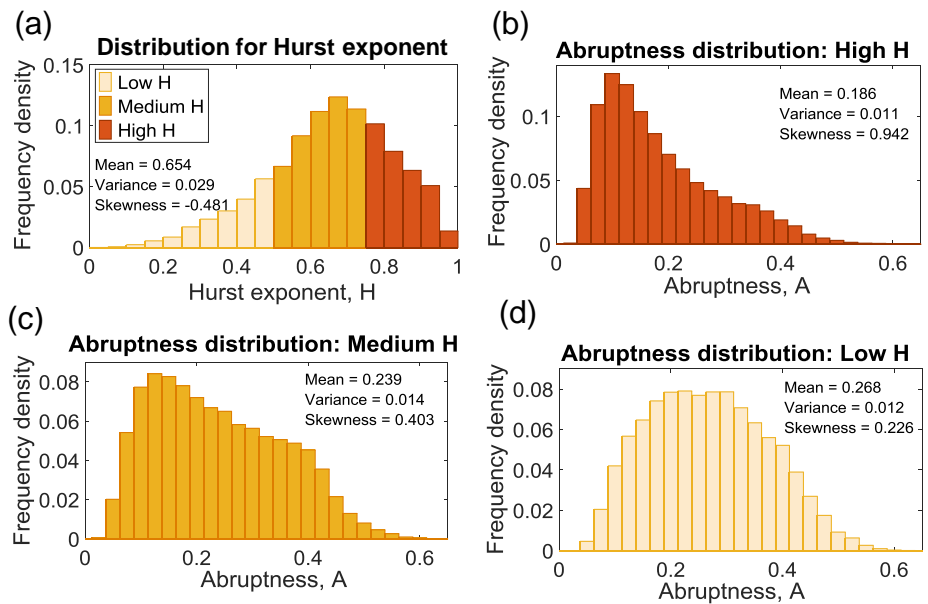


Figure 7. Relationship between Hurst exponent, H , and waveform abruptness, A , (corresponding to flight-track data in Fig. 6). (a) Total distribution for Hurst exponent. (b) Abruptness distribution for High H , ($H > 0.75$). (c) Abruptness distribution for Medium H , ($0.5 < H \leq 0.75$). (d) Abruptness distribution for Low H , ($H \leq 0.5$). The observed distributions in (b), (c) and (d) confirm the theoretical prediction of the self-affine radar scattering model that a statistically-distributed inverse relationship exists between H and A . **The analysis is performed independent of the thawed-frozen statistics in Sect. 5.3.**

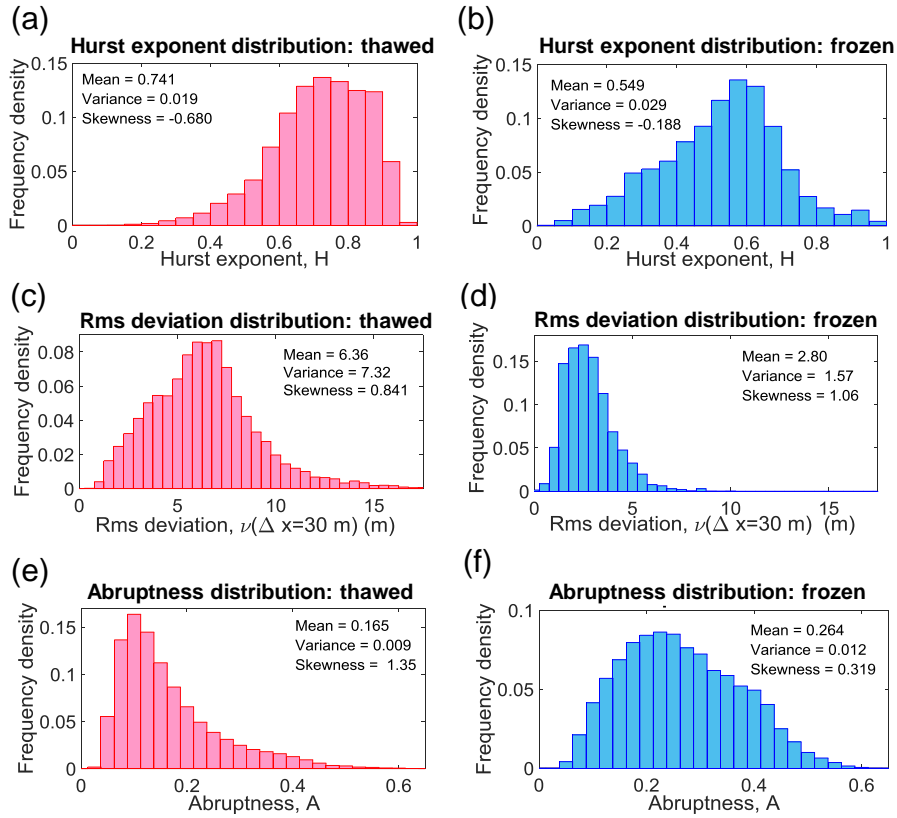


Figure 8. Distributions from basal RES analysis in thawed and frozen regions of the northern GrIS (corresponding to flight-track data in Fig. 6). (a) Hurst exponent, H , in thawed regions, (b) H in frozen regions. (c) Rms deviation, $\nu(\Delta x=30 \text{ m})$, in thawed regions. (d) $\nu(\Delta x=30 \text{ m})$ in frozen regions. (e) Abruptness, A , in thawed regions. (f) A in frozen regions. The data subsets correspond to the red (thawed) and blue (frozen) regions of the map in Fig. 6f.