Manuscript tc-2016-272

Numerical homogenization of the viscoplastic behavior of snow based on X-ray tomography images.

Antoine Wautier, Christian Geindreau, Frédéric Flin

March 15, 2017

Response to Maurine Montagnat's comments (SC1)

We gratefully acknowledge Maurine Montagnat for her comments to improve the quality of our manuscript.

All the reviewers' comments have been taken into account to provide a revised version of our manuscript. The major modifications of the manuscript consist in:

- an enriched introduction containing an improved state of the art and a clearer statement of the objectives and the scope of our manuscript.
- a more detailed assessment of both the elastic and viscous material parameters on the homogenized viscous behavior of snow. A slight change in the postprocessing procedure has been made by introducing a characteristic time.

The specific influence of the material parameters of the ice on their numerical results is discussed below.

1. *As already discussed with one of the authors, I think the values of the Young modulus and stress exponent taken for ice in this manuscript can be questioned.*
Schulson and Duval 2009 (Creep and Fracture of Ice, chapter 4, part 4.2.2) give a clear explanation about the difficulties arising when trying to estimate the Young modulus from a "classical" mechanical test, since plasticity is activated very soon. They mention the work done by Gammon et al (1983 for instance), based on acoustic wave propagation as being the only one to give "robust" data of Young modulus for ice. In the "ice community", it is now accepted that the value of the Young modulus, from Gammon et al work, is 9.33 GPa, very far from the 325 MPa taken here...
This value of about 9 GPa is also the one taken by Theile et al. 2011 (Acta Mat) for instance in order to estimate the Young modulus of snow with FE simulations from microCT images of natural snow.

Reply: Even if the value of about 9 GPa seems to be the most widely accepted value for the Young's modulus within the ice community, reported values in the literature cover 2 orders of magnitude from 0.2 GPa to 9.5 GPa as mentioned in [1]. As a result, we chose a value of 325 MPa based on our experimental results. However in the revised version of our manuscript, the value of 9 GPa was also considered. The typical stress response of a snow sample under a constant given strain rate is illustrated on Figure 1 for the snow sample *RG_1600* for the cases $(n,\ E) = (4.5,\ 325\ \text{MPa})$ and $(n,\ E) = (4.5,\ 9\ \text{GPa})$. The mechanical response is characterized by a transient regime driven by the elastic properties followed by a permanent regime dominated by the viscoplastic behavior. Because of the change in the Young'modulus value, the duration of the transient regime is changed. In order to compare the time responses of the two aforementioned cases in Figure 1, a characteristic time τ is defined as the ratio between the ice viscosity $\eta(\dot{\mathrm{E}}_{\mathrm{ref}}) = (\dot{\mathrm{E}}_{\mathrm{ref}}/A)^{1/n}/\dot{\mathrm{E}}_{\mathrm{ref}}$ and the Young modulus E

$$\tau = \frac{\eta(\dot{\mathrm{E}}_{\mathrm{ref}})}{E} = \frac{1}{E}\left(A^{-\frac{1}{n}}\,\dot{\mathrm{E}}_{\mathrm{ref}}^{\frac{1-n}{n}}\right). \quad (1)$$

In Figure 1, the responses $S_1(t/\tau)$ and $\bar{S}_2(t/\tau)$ are found independent of the Young's modulus value chosen. As a result, the homogenization approach presented in our manuscript and, more precisely the macroscopic 3D viscoplastic behavior of snow deduced from ou numerical simluations is completely independent of the Young's modulus value chosen.
The revised version of the paper has been modified accordingly. The second step of postprocessing procedure is slightly modified (in the form) and the initial and final asymptotes are computed with respect to t/τ and not t.

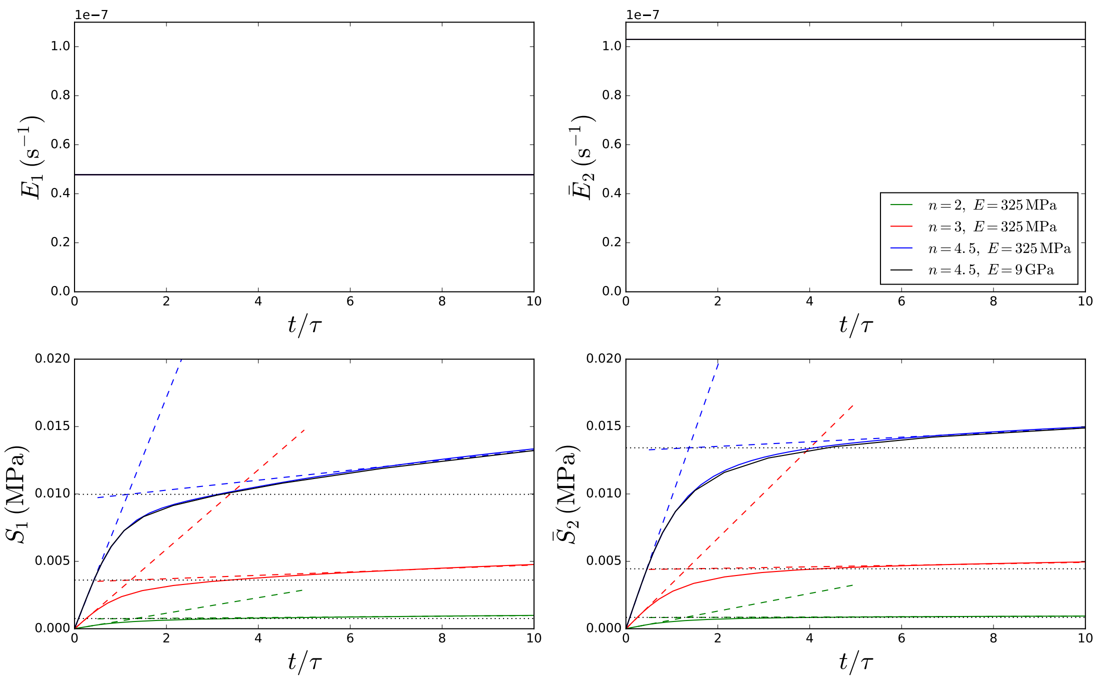

Figure 1: Imposed strain rate (top) and stress response (bottom) versus dimensionless time (t/τ) for the sample *RG_1600*. Loading strain rate is characterized by $\theta = 65°$ in equation (21). Two Young's moduli and three values of n are considered.

2. *Another question concerns the value of the stress exponent. The stress exponent at secondary creep in ice is known to be 3, based on a strong experimental work (well summarized in Schulson and Duval 2009). Values close to 5 (or 4.5) were found from experiments that were pushed up to the tertiary creep, when dynamic recrystallization (or micro fracturation)*

comes into play. I guess that this type of behavior is not relevant for the conditions mentioned here?
Maybe these values do not play a significant role in the main results of the paper... nevertheless, to use well documented values could maybe enhance the quality of this work?

Reply: Concerning the visco-plastic parameters used, even if the most commonly used value for the exponent n of the Norton Hoff's law is 3, it is found to vary between 1.8 and 4.6 under usual loading (strain rate, stress) and temperature conditions [3, 5, 4]. The value of 4.5 initially chosen was based on a relaxation test performed on an ice cylinder. In the revised version of the manuscript, two other values of n ($n = 2$ and $n = 3$) are considered to show its influence on the material parameters f and c.
As expected, the Figure 1 shows that for a given imposed strain rate and a given sample, the macroscopic responses $S_1(t/\tau)$ and $\bar{S}_2(t/\tau)$ depend on the value of n. The viscous stress increases with increasing n, since the ice viscosity $\eta(\dot{\mathrm{E}}_{\mathrm{ref}})$ increases. Figure 2 shows the influence of n onto the isodissipation curves for the particular snow sample *MF_522*. Similar results have been obtained on the other samples. As expected, for a given value $\mathcal{P}_{\mathrm{v}}^{\circ}$, the size of the isodissipation curves increases with n (since the ice viscosity $\eta(\dot{\mathrm{E}}_{\mathrm{ref}})$ increases) but their shape remains unchanged. They can be deduced from each other by simple dilation.
The values for $f(\phi)$ and (ϕ) for $n \in \{2, 3, 4.5\}$ deduced from these simulations are summarized in Table 1 and reported in Figure 3. By contrast to the Young's modulus, the exponent n has a noticeable influence on the parameters f and c and consequently on the parameters a, b, p and q given in Table 2. The influence of n on these parameters is illustrated in Figure 4.

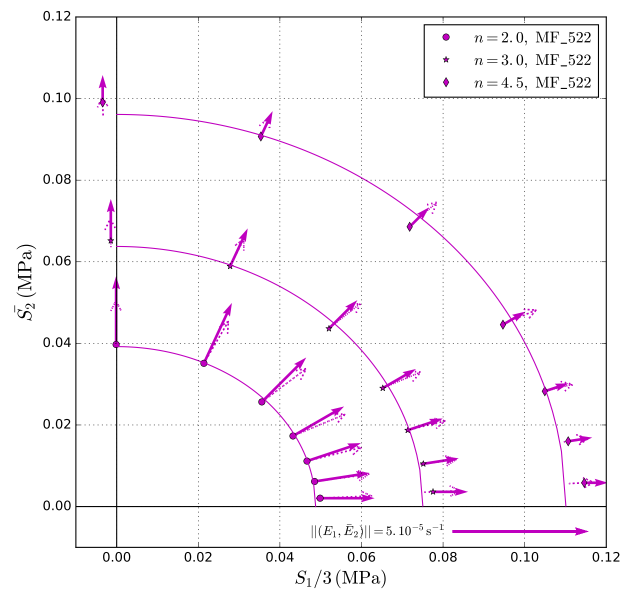

Figure 2: Influence of the exponent n onto the isodissipation curves ($\mathcal{P}_{\mathrm{v}} = \mathcal{P}_{\mathrm{v}}^{\circ}$) for the particular snow sample *MF_522*. The associated strain flow vectors $(E_1, \bar{E}_2)$ are represented by solid arrows. Abouaf models are fitted to the numerical points (solid lines) and theoretical values of strain flow vectors are shown (dashed arrows).

Table 1: Optimal values for the parameters f and c of the Abouaf's equivalent stress (25) for three values of n.

		$n = 2$		$n = 3$		$n = 4.5$	
Sample name	Porosity	f	c	f	c	f	c
PP_123kg_600	0.87	36.0	150	79.7	336	146	628
RG_172kg_600	0.81	16.3	75.7	33.5	156	58.3	277.4
RG_256kg_512	0.72	4.05	20.5	6.98	34.7	10.5	52.3
RG_1600	0.64	2.07	11.0	3.32	17.0	4.70	24.0
RG_430kg_651	0.53	0.915	6.38	1.40	9.12	1.89	12.1
MF_522kg_542	0.43	0.354	3.32	0.503	4.26	0.630	5.07

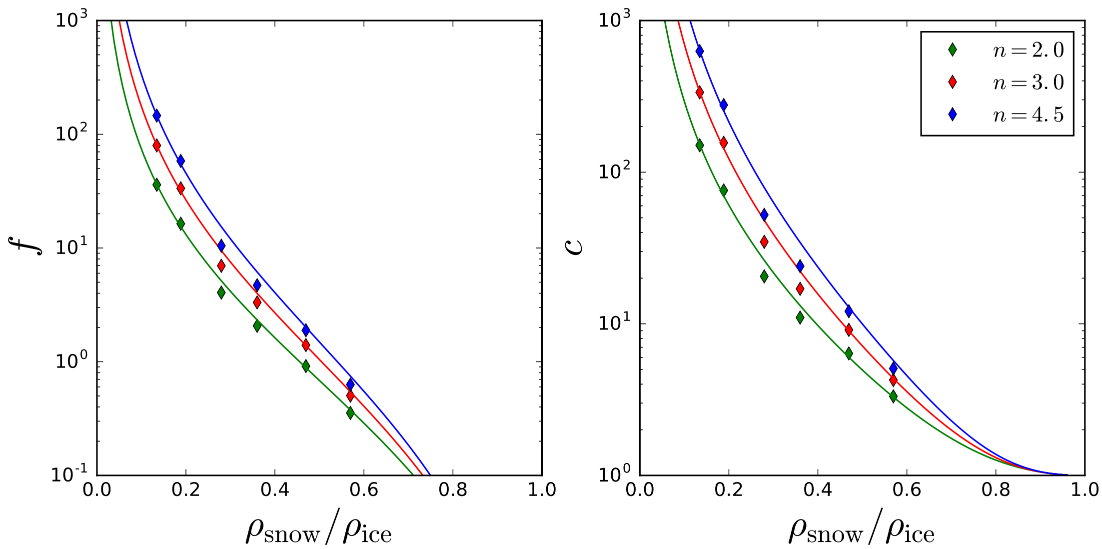

Figure 3: Evolution of the Abouaf coefficients f and c (numerical results as diamond points, functions (29) as solid lines) with respect to the snow compacity for different n values.

All these new results have been incorporated in the revised version of the manuscript. In addition the Figures 6, 8 and 11 of the paper were updated to show the influence of the exponent n on the results. The exponent n doesn't have any influence on the shape of the isodissipation curves but only on the stress level leading to a given isodissipation. Finally, the evolution of the ratio Σ_{rr}/Σ_{zz} in the case of an oedometer test is similar to the one measured by [2] and [6] on metallic powders. This ratio is almost independent of the exponent n, which is consistent with the experimental results of [6].

Table 2: Optimal parameters chosen for the expressions (29) for different n values.

	$n = 2$	$n = 3$	$n = 4.5$
a	0.68	1.0	1.5
p	2.1	2.3	2.5
b	4.0	6.1	8.9
q	2.0	2.2	2.3

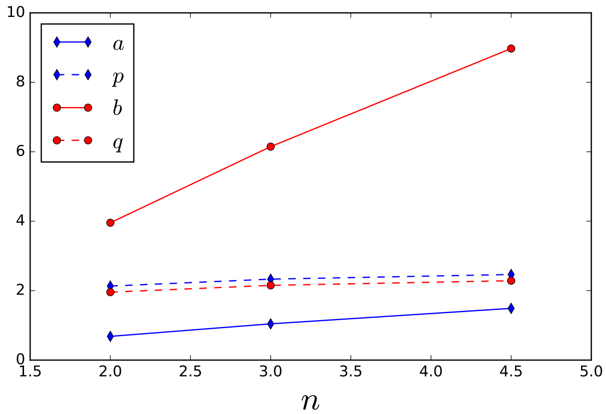

Figure 4: Evolution of the fitted coefficients a, p, b and q with respect to the exponent n of the ice Norton Hoff constitutive behavior.

*

References

[1] Chaman Chandel, Praveen K Srivastava, and P Mahajan. Micromechanical analysis of deformation of snow using X-ray tomography. *Cold Regions Science and Technology*, 101:14–23, 2014.

[2] C. Geindreau, D. Bouvard, and P. Doremus. Constitutive behaviour of metal powder during hot forming. part i : Experimental investigation with lead powder as a simulation material. *Eur. J. Mech. A/Solids*, 18:581–596, 1999.

[3] Carlo Scapozza and Perry Bartelt. Triaxial tests on snow at low strain rate. part ii. constitutive behaviour. *Journal of Glaciology*, 49(164):91–101, 2003.

[4] Stefan Schleef, Henning Löwe, and Martin Schneebeli. Hot-pressure sintering of low-density snow analyzed by X-ray microtomography and in situ microcompression. *Acta Materialia*, 71:185–194, 2014.

[5] Erland M Schulson, Paul Duval, et al. *Creep and fracture of ice*. Cambridge University Press Cambridge, 2009.

[6] P. Viot and P. Stutz. Nouveau dispositif expérimental pour l'étude du comportement viscoplastique des poudres métalliques à hautes températures : application à une poudre de cuivre. *C. R. Mecanique.*, 330:653–659, 2002.