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Numerical homogenization of the viscoplastic behavior of snow based on X-ray tomography images.

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Response to Anonymous Referee's comments (RC1)

We gratefully acknowledge the anonymous reviewer for his comments to clarify some points and improve the quality of our manuscript.
All the reviewers' comments have been taken into account to provide a revised version of our manuscript. The major modifications of the manuscript consist in:

- an enriched introduction containing an improved state of the art and a clearer statement of the objectives and the scope of our manuscript.
- a more detailed assessment of both the elastic and viscous material parameters on the homogenized viscous behavior of snow. A slight change in the postprocessing procedure has been made by introducing a characteristic time.

Based on the review, we understand that the main criticisms made by the reviewer on the content of our article are that:

- our model does not represent all the physical processes at stake at the microscale that modify locally the microstructure
- the parameters of our macroscopic Abouaf's model do not incorporate explicitly microstructure information
- the choice to use FEM instead of DEM is not well explained, as DEM is able to take into account grain boundary sliding and particle rearrangement.

We would like to discuss on these three points and explain more in details the main contributions of our work. In the following, we recall the reviewer's comments followed by our answer.

1. *The authors indicate that the macroscopic mechanical behavior of snow strongly depends on its microstructure, which is mainly characterized by its density, topology, external loading (elastic, visco-plastic, brittle-failure) temperature and applied strain-rate. What is missing from the list are the mechanical processes that occur at the microscale (e.g., grain boundary sliding, bond breakage, bond sintering and re-sintering after breakage and particle rearrangement; ignoring metamorphic changes) that are critical to define the evolution of the snow under defined loading (or strain rate) conditions in the RVE. Knowledge of the microstructure is only the starting point to define an evolution function. It is the explicit microscale mechanical processes that determine how snow deformation evolves within the*

RVE and the model described in the current work does not, and cannot, accurately represent the microscale mechanical processes necessary to develop and RVE evolution function.

Reply: We acknowledge the fact that our model does not take into account all the physical phenomena acting at the microscale. In the present work, we propose to determine the 3D macroscopic behavior of snow resulting from the viscous deformation (secondary creep) of the ice skeleton only. Our main objective is to formulate a 3D macroscopic law by performing 3D FEM simulations on real 3D images of snow, in order to underline the microstructure influence. This is a first step towards a more complete modeling of the snow behaviour from 3D images, by introducing more mechanisms and by simulating metamorphism changes.

2. *The Abouaf (1985) viscoplastic model used in the current work appears to be primarily porosity dependent and the authors do not provide a clear description of how the homogenized snow microstructure is incorporated into the model. The free parameters of the model include four parameters in two porosity dependent functions, neither of which appear to incorporate microscale mechanical processes. There are continuum scale models specifically developed to incorporate observed microscale measurement that might prove more suitable to this effort, if an appropriate evolution function can be determined.*

Reply: Thanks to the use of 3D images of different snow types, the particular topology of the snow microstructure is taken into account in the snow mechanical response. Indeed, the shape and size of iso-mechanical dissipation surfaces results from the strong coupling between the microstructure and the non-linear behavior of the ice under consideration. In our case, it appears that the viscous isodissipation curves can be fitted by ellipses in the plane of the two first strain and stress invariants. As a result, an Abouaf model seems to be relevant to account for snow visco-plasticity. In this model, for a given value of the exponent n, all the microstructure information is encapsulated in the two parameters f and c which depends, at the first order, on the porosity only. In the past, different expressions of the material functions $f(\phi)$ and $c(\phi)$ have been proposed to describe the densification of granular materials or matrix with spherical voids based on experimental data on metal powders [1, 2, 7], micromechanical modelling (cell model) [6] or numerical simulations on simple microstructures [14]. All these functions, which have been identified in a restricted range of porosity (typically $\phi < 0.4$) on materials with strongly evolving microstructures, point out that the porosity remains, at the first order, the main microstructural parameter influencing their macroscopic viscoplastic behavior. The results obtained in our manuscript are consistent with these observations, however simulations on snow samples with very different microstructures and similar densities (but still isotropic) would help validate this assumption. Finally, let us remark that due to the strong coupling between the microstructure and the non-linear behavior of the ice under consideration, these material functions are valid for a given value of n. In the revised version of the paper, these functions have been identified for 3 values of n (2, 3 and 4.5) (see our response to the comments of M. Montagnat). Probably, micromechanical approaches may lead to more microscopic information on the definition of these functions, but this is not straightfoward at this stage when the porosity is very large.

3. *The finite element method applied in the paper is not capable of accurately simulating the important micro-scale mechanical processes involved in the RVE deformation (primarily,* *grain boundary sliding and particle rearrangement). The only techniques that have demonstrated the ability to represent the micro-scale processes in snow to-date are the discrete element method (DEM) (Johnson and Hopkins, 2005; Hagenmuller et al., 2015) and the deformable DEM (Johnson, 1998). The authors mistakenly indicate that DEM models are only useful for simulating grain breakage and grain rearrangement. With appropriate interaction rules at the grain level the DEM is capable of modeling pretty much any physical process limited only by computational capability, which continue to improve at a rapid rate.*

Reply: We are aware that DEM is also a powerful tool to bridge the gap between micro and macro scale, and in the revised version of our manuscript the relative pros and cons of DEM and FEM applied on X-ray tomography images are discussed more in details (see the introduction). This analysis governed our choice to use a FEM approach.

- In the FEM method, the complex 3D snow skeleton observed by X-ray microtomography can be meshed without loosing any information on the microstructure and different mechanical behavior of the polycrystalline ice can be considered. In the last decade, most of the studies were dedicated to the elastic behavior of snow [13, 11, 15, 10, 18, 16], possibly up to a brittle failure [8]. Concerning the modeling of more complex snow constitutive behaviors, the proposed approaches mainly focus on the modeling of uniaxial compression tests. For instance, [17] has proposed a beam network model based on 3D images to simulate creep of snow whereas [5] used an elasto-plastic constitutive law for ice in order to determine the failure envelope.

- In the DEM method, the snow skeleton is viewed as an assemblage of ice grains interacting between each other through contact points. As mentioned by the reviewer, this method is well suited to model complex interactions taking place at the interface between snow grains such as elasto-viscoplastic contact deformation, grain sintering and bond breakage or sliding possibly leading to grains rearrangement. This method has been already used on 3D idealized assemblages of ice grains [9] to identify microstructural deformation mechanisms of snow and to simulate creep densification process. However, the application of the DEM directly on 3D images obtained by X-ray tomography is not straightforward, since every ice grain constituting the skeleton must be identified. Recently, DEM simulations taking into account cohesion and friction at the contact between grains have been performed on more realistic 3D assemblages of grains deduced from X-ray tomography. However the shape of each ice grain is approximated by a clump of spheres. Moreover, all these simulations have been performed without taking into account the crystalline orientation of each ice grain. These orientations can play an important role on the viscous deformation mechanisms (secondary creep) at the contact between two grains as it has been recently shown by [4, 3]. The granular structure of the snow (grain shape and crystal orientations) can be determined on real 3D images of snow by X-ray Diffraction Contrast Tomography (DCT) images [12]. However, the application of this technique is not straightforward and very few images are now available.

Both methods applied on X-ray tomography present some advantages and limitations, but we believe that these two approaches can still bring some interesting and complementary results in snow mechanics. Finally, we would like to mention that the proposed methodology (based on isodissipation curves) to identify and formulate the 3D viscoplastic behavior of snow can be also applied by performing DEM simulations or by using a micromechanical approach.

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