

tc-2016-267-RC1 - Response to Anonymous Referee #1

Interactive comment on “Uncertainty budget in snow thickness and snow water equivalent estimation using GPR and TDR techniques” by Federico Di Paolo et al.

We would like to apologize to the Associated Editor and Reviewer 1 regarding Fig.4 which, as highlighted by the referee, had an incorrect X-scale due to a mistake made by the code when it generated the picture. This mistake has been cause of confusion. The following Figure is the correct one and, as can be seen, it is in agreement with Figure 6 and the overall results presented in the original manuscript.

Furthermore, we wish to thank the referee for his/her efforts, even though we strongly disagree with the overall assessment and with several specific comments, as discussed below. The reviewer’s comments are given in red, followed by our replies in black.

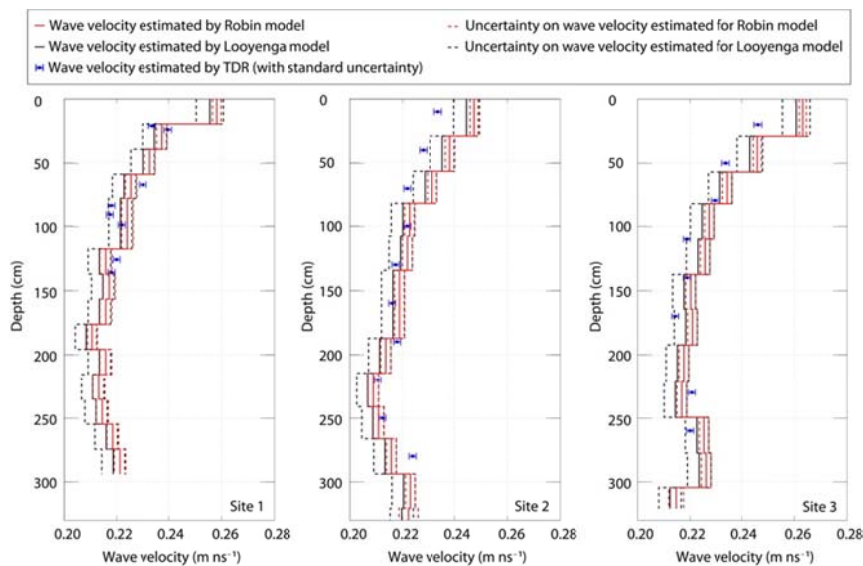


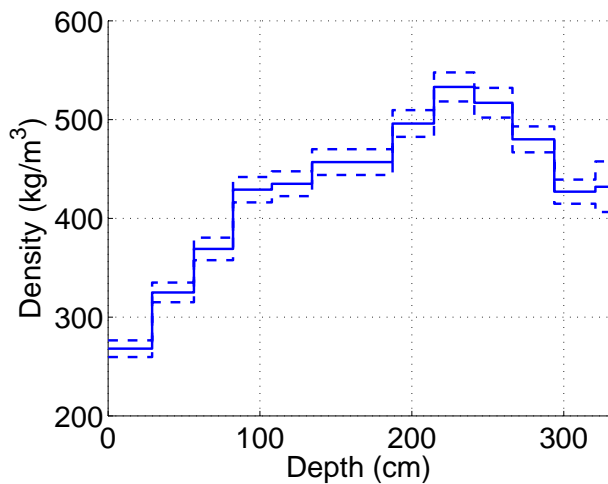
Fig.4 – corrected version

⇒ Wave velocity values in Figure 4

Wave velocity values presented in Figure 4 are mostly below 0.2 m/ns for all three snow pit sites. TDR based velocity calculations have even lower values for the top 0.7m of the test sites 2 and 3. The values presented in Figure 4 correspond either to very dense dry snow (more than 500 kg/m³) or to wetsnow conditions with a dry snow density of around 350-400 kg/m³ and an average volumetric liquid water content of 0.02 to 0.04 (see Fig. 1 within the review). When I recalculated the wave velocity on the SWE values presented in Table 3 and the values for snow depth at the profiles given on page 7, I obtain densities of 390-430 kgm⁻³ which with the Eq by Robin and Looyenga result into wave velocities of ~ 0.22 m/ns. I cannot reproduce these average values based on Figure 4 in your manuscript. The values correspond better to Figure 6 in your manuscript. If, however, the values of Figure 4 are the correct ones, the conclusions are that your snow was wet. As a consequence your simple mixing model is not applicable and has to be replaced by more sophisticated mixing models (Denoth, 1980; Denoth, 1994; Roth et al., 1990). This will influence the TDR measurements and consequently the used approach of calibrating your GPR data and calculating SWE are not valid and thus large parts of the manuscript cannot be published.

Reply: As mentioned above the figure was incorrect, thus the comments of the reviewer on this specific issue do not apply to our field conditions. In fact, from the correct values of the wave velocity we can infer that the snow is essentially dry. As requested by Referee 1, using the formula reported in *Denoth, 1994* the

snow water content is always lower than 1%, as is clear comparing the plot made by the referee with the one of the density reported below.



⇒ **Concept of measurement**

Unfortunately you made several severe mistakes while sampling:

- **The use of a 0.5m resolution for your hand-probed snow height ruins the entire analysis.**

⇒ **Concept of estimating uncertainty**

It remains unclear, why the authors have chosen in some cases to use their definition of Type B uncertainties, while it would have been possible to stay with a Type A data uncertainty by e.g. sampling density or snow height several times in close proximity.

Ref. comment (page 18, line 26): This fact reduces your whole discussion on uncertainty to absurdity. The uncertainty of your probe reading is too large from the very beginning.

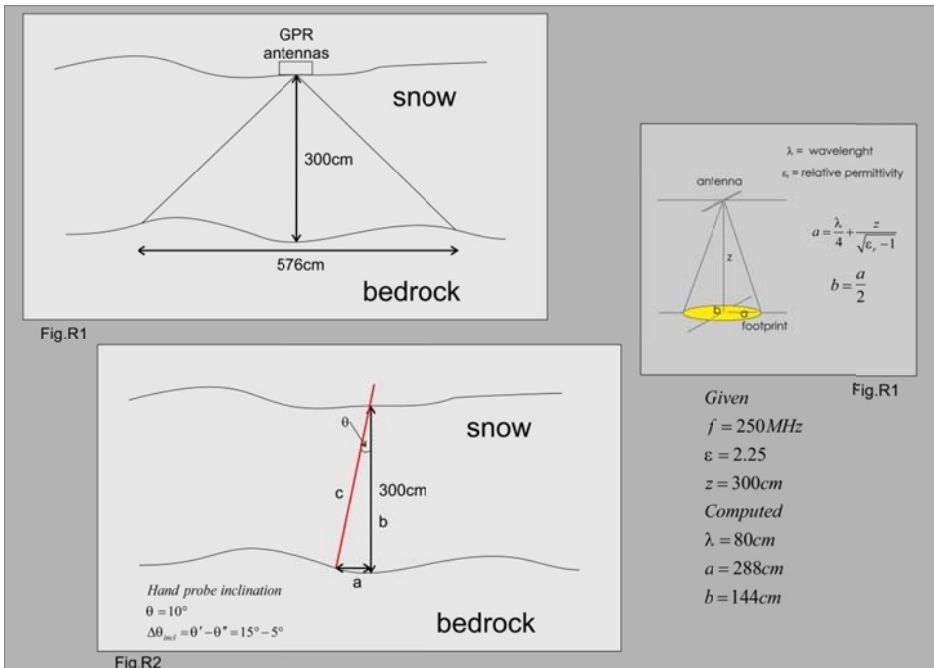
Ref. comment (page 18, line 26): I totally disagree! If you were right, all laser-scan based measurements would be useless! If you really believe in this argument, you must prove it with literature or examples.

Ref. comment (page 12, line 6): This is a very weak point in your analysis. The measured snow height with the probe somehow represents your target variable and has the lowest resolution. Why did you not combine the 0.5m resolution with a standard ruler so that you end up with a cm resolution?

Ref. comment (page 10, line 1): not true! Depends on your method! In addition it totally depends on the used scale! If you assume that the snow height on almost flat terrain, you can easily compute 10 probe measurements in an array of 10 by 10 cm which will result in a Type A evaluation of your uncertainty, if you define a 100cm² foot print, which should be fine for your radar beam. Your Type B is again totally driven by the scale of your measurement device, in your case by the very, very coarse setting (0.5m) of your probe.

Reply: We did not introduce any “personal” method to compute the uncertainties but we followed the most up to date procedure. See for example: <http://www.bipm.org> or <https://www.nist.gov/>. However, in the following we better explain why our procedure is rigorous and correct and why using a hand probe with a much higher resolution scale would not appreciably improve our analysis.

According to GUM, the overall uncertainty on each quantity should be computed considering all possible sources of uncertainties, that is Type A + Type B to estimate the so called uncertainty budget. In the paper, for the hand probe measurements, we have applied a simplified procedure estimating the overall uncertainty using the standard deviation of a uniform distribution (type B uncertainty). In the following we demonstrate that the chosen value of uncertainty is only 1.5 times the uncertainty computed using a “more refined” scale on the hand probe (e.g., 2 cm scale ruler) but taking into account other possible sources of uncertainty. The computation has been performed for the 250 MHz antennas, but can be extended to the other antennas.



We first computed the footprint of the antenna on the bedrock for a snow height of 300 cm (see Fig.R1a and R1b). For each measurement the footprint is $(576 \times 288) \text{ cm}^2$ (i.e., much larger than $10 \times 10 \text{ cm}$ - compare to Ref. comment: Page 10, line 1). In such a large area, and considering the site (alpine region), we can assume bedrock surface irregularities of at least 20 cm, so in the antenna footprint the snow height interval is 290 - 310 cm (see Fig.2R). Furthermore, for simplicity we will neglect the undulation of the snow.

We consider now some of the possible source of errors that can affect our measurements (see for example: A Beginner's Guide to Uncertainty of Measurement, by Stephanie Bell –National Physical Laboratories, 1999):

a) List of possible errors due to hand probe

- 1) Is the hand probe calibrated? What is the uncertainty on calibration?
- 2) Has the hand probe been modified by the use?
- 3) What is the hand probe resolution?

b) List of possible errors associated to snow height

- 1) Are the top and the bottom of the snow layer well defined and constant?

c) List of possible errors due to measuring process

- 1) How well do the hand probe starting points (i.e., the tip at the bottom) coincide with the bottom of the snow layer?
- 2) Is the rod perpendicular to the snow surface at the measurement point (rod parallel to the snow height)?
- 3) How repeatable are the measurements?

In our field conditions, as these questions are not easy to quantify, we will simplify our computation as follows, bearing in mind that such simplification will certainly introduce an underestimation of the overall uncertainty.

We assume to measure the snow with a hand probe having marks every 2 cm. Thus the uncertainty on reading (Type B uncertainty assuming a uniform distribution) is:

$$2a = 2cm$$

$$\sigma_{read} = \frac{a}{\sqrt{3}} = 0.6cm$$

Now, we randomly generated 10 measurements of the snow height (according to the range of height variability given above that is ± 10 cm). The measurements are:

300, 302, 310, 295, 308, 291, 298, 304, 290, 305

Note that these readings are taken assuming a resolution of 1 cm (half of the marks distance on the scale). From these data we can estimate the Type A uncertainty averaging the measurements and the standard deviation uncertainties:

$$\bar{x} = 300.3cm$$

$$\sigma_x = 6.8cm$$

$$\sigma_{\bar{x}} = 2.2cm$$

From this calculation we see that the uncertainty associated with a single measurement (and any other $n+1, n+2, n+...$ measurements that can be made with the hand probe) is about 7 cm, that is over 3 times the divisions on our hypothetical hand probe scale (i.e., 2 cm). This means that we are using a “system” that is too refined for our measurement and the uncertainty is much larger than the scale division. In other words, the spread of the data due to bedrock (or bottom layer) irregularities is much larger than the sensitivity of the instrument.

Furthermore, we can try to compute the bias error associated with the inclination of the probe (see Fig.R2). If we assume that we insert the probe with an average angle of 10° with respect to the vertical and we also assume that the uncertainty on this inclination is 10° (between 15° and 5°) we can compute for 300 cm snow height a bias error (BE) as follows:

$$c = \frac{b}{\cos \theta} = 304.6cm$$

$$BE = 4.6cm$$

The uncertainty on this error is:

$$c' = \frac{b}{\cos \theta'} = 310.6cm$$

$$c'' = \frac{b}{\cos \theta''} = 301.1cm$$

$$\Delta c = 9.5cm$$

Now assuming a uniform distribution, the type B uncertainty for the inclination is:

$$\sigma_{incl} = \frac{9.5}{2\sqrt{3}} = 2.7cm$$

As the bias error is, by definition, always there and overestimates the snow height, it should be subtracted from the average value:

$$SH_{corr} = 300.3 - 4.6 = 295.7 \text{ cm}$$

The combined standard uncertainty associated to this value is:

$$\sqrt{\sigma_{\bar{x}}^2 + \sigma_{read}^2 + \sigma_{incl}^2} = \sqrt{2.2^2 + 0.6^2 + 2.7^2} = 3.5 \text{ cm}$$

This standard uncertainty represents 68% of probability that the measurement falls between ± 3.5 cm. If we use a coverage factor, e.g., $k=2$ (as suggested by GUM), we will have that about 95% of our measurements will fall in the interval ± 7.1 cm.

We can conclude that the uncertainty on our measurement is at best about 7 cm. We should remember however that we are underestimating the uncertainty, as several items in points **a)**, **b)** and **c)** above weren't considered because they are too difficult to estimate in the present case. For example, it is not easy to estimate the effect on snow height measurements of rocks or ice lenses present in the snowpack. As a consequence is quite possible that, in our case, the overall uncertainty is ≥ 10 cm.

In measurement theory (and practice) the best choice of instrument scale division is driven by the overall (or maximum) uncertainty, thus to optimize our calibration in our specific field conditions, we should have used a hand probe with marks divisions computed using the combined standard uncertainty computed above (i.e., 10 cm). From this value we can compute the suitable scale division assuming a uniform distribution as follows: $2a = 2 \times 10 \times \sqrt{3} = 34.6 \text{ cm}$. In our work we used a probe with 50 cm distance between the marks, so in analogy with the example discussed above, with a resolution of 25 cm, which for a uniform distribution gives a type B uncertainty of 14 cm. This is an uncertainty of about 1.5 times the value we have found using a hand probe with only 2 cm marks distance.

If we use for the hand probe readings a uncertainty of 10 cm (assuming that, as we will demonstrate below, the uncertainties on the TDR are correct) we found a $u_{twt_{HP}} = 0.91 \text{ ns}$ which does not change the results presented in the original manuscript (cf. Table 2 in the manuscript).

We can conclude that, in the specific field conditions of our study, the use of a more precise technique like laser, which samples a very small area [see e.g., Lee et al., 2015], would not significantly improve the GPR calibration due to all sources of errors present in the large radar footprint.

⇒ Concept of measurement

Unfortunately you made several severe mistakes while sampling:

• I believe that your analysis of the TDR travel time is inaccurate. Please provide more details how you processed the data including figures.

⇒ Concept of estimating uncertainty

In addition, I have the feeling that Type B uncertainties are purely driven by the units given by the chosen estimated values. E.g. with TDR small differences in estimating t_2 may result in large differences for the permittivity. I want to see a broader discussion on that topic.

Ref. comment (page 11, line 6): Provide a Figure on the signal and explain exactly how you picked the derivate. Since snow tends to give a weak signal, the slope indicating the end of your sensor tends to be flat which introduces again a source of uncertainty, since you have to pick a certain time of your

signal. Please comment on that and make statements on how you could eventually prevent this source of error for future investigations.

Ref. comment (page 18, line 7): In my opinion not shown and biased by your chosen parameters for u.

Reply: The comments made by the referee require some explanation about TDR working principle and its applicability to snow measurements. In the following we summarize such information, including an example of the analysis of a waveform collected during the measurement campaign. Time domain reflectometry (TDR) is a valuable method for measuring the electromagnetic (e.m.) wave propagation velocity in solid, granular and liquid materials. Our TDR system consists of an open ended three-prong line (hereafter referred to as probe line) that can be embedded in the material; the instrument generates a stepwise signal with a 200ps rise time [Tektronix 1502 cable tester]. The output of the TDR system consists essentially of the signal recorded at the feeder line input, which displays the results of the reflection processes taking place in the system. A first partial reflection occurs when the wave front from the generator comes across the impedance discontinuity at the probe line input. In fact, the probe line is designed in a way to exhibit an impedance mismatch at the generator feeder line. A second reflection takes place at the end of the probe, with a unit reflection coefficient as the probe is open-ended. The wave velocity is estimated from the signal two-way travel time; a complete discussion about the multiple reflections is given by *Yanuka et al. (1988)* and *Topp et al. (1988)*. An example of the TDR trace acquired during the survey in the Ortles-Cevedale group is reported in Fig. R3. The first and second arrows indicate the reflection at the beginning and the end of the probe line. As we can see, the two reflections are very clear and can be used to determine the two way travel time of the signal along the TDR probe: in fact, the two way travel time appears in the TDR trace as the time interval between the reflections at the probe edges. For non-magnetic materials (like snow), this time is given by $\Delta t = 2L\sqrt{\varepsilon_a} / c$, where L is the length of the probe, c the light speed in a vacuum, and ε_a is the apparent relative permittivity given by [Von Hippel, 1994]

$$\varepsilon_a = \varepsilon' \frac{(1 + \sqrt{1 + \varepsilon''^2 / \varepsilon'^2})}{2},$$

where ε' and ε'' are the real and imaginary part of the permittivity. Therefore, the two way travel time can be used to compute the apparent permittivity or, alternatively, the velocity of the em signal in the medium under test (i.e., snow), by using:

$$\varepsilon_a = (c\Delta t / 2L)^2$$

$$v = 2L / \Delta t$$

Note that ε_a reduces to the real part of permittivity if losses are negligible.

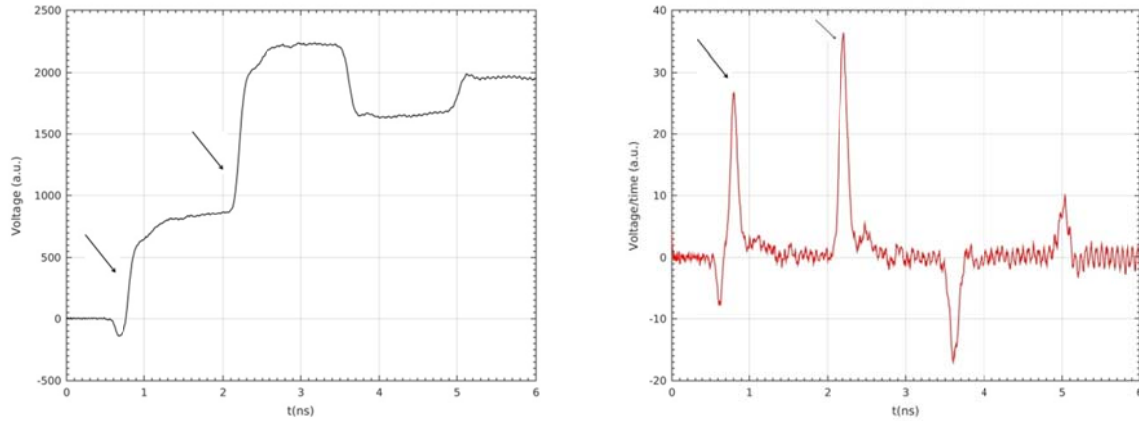


Fig. R3 Left panel represents the TDR response; right panel represents the derivative of the TDR response.

To pick the t_1 and t_2 times (which correspond to the beginning and the end of the probe, respectively), we used the derivative method which is widely discussed in Robinson et al. (2003), Robinson et al. (2005) and Mattei et al. (2006) and is briefly described in the following. An example of the time derivative of the TDR response is reported in the right panel of Fig. R3: the time derivative of the TDR response can be modelled by a series of Gaussian functions

$$\dot{r}(t) = \sum_{j=1}^n A_j \exp[(t - t_j) / \sigma_j]^2,$$

where the index j runs over the successive reflections present in the TDR trace. We applied a minimization procedure to the waveform shown in Fig.R3 (right panel) up to $j=2$ to evaluate the values of Δt from the time difference $t_{j+1} - t_j$.

We use the time sampling of the Tektronix cable tester (0.0041ns) to estimate the uncertainty on each t_j value assuming a uniform distribution (Type B uncertainty).

Furthermore, we would like to reply to this comment: **“In addition, I have the feeling that Type B uncertainties are purely driven by the units given by the chosen estimated values. E.g. with TDR small differences in estimating t_2 may result in large differences for the permittivity. I want to see a broader discussion on that topic”**.

As shown by the waveform (see Fig.3R), the measurement of t_2 is well constrained, being an open-ended probe, that is the impedance is $Z=\infty$ and the reflection coefficient $\Gamma=1$. As a consequence, the effect of t_2 on the permittivity given by:

$$\frac{\Delta \varepsilon}{\varepsilon} \propto 2 \frac{\Delta t_2}{t_2}$$

is substantially irrelevant.

Ref. comment (page 6, line 18): For some reason you miss to report the state of the art on this combination made by several colleagues in the paragraph (see General Remarks).

Ref. comment (page 7, line 4): Again, you miss to relate this in a broader context. What about upward-looking radar systems in combination with GPS?

Ref. comment (page 19, line 1): If you plan to resubmit, please discuss this in a broader context by using a broader spectrum of existing literature. Especially Schmid et al. (2014) will be very helpful for you.

Reply: We are well aware about the very interesting application of upward looking GPR published in several papers. However, we did not mention such papers in our manuscript because the field conditions and the rationale of the measurement are totally different and hardly comparable; consequently we compared our results with those reported in similar experiments. In particular, upward looking GPR has been employed at fixed locations monitoring the time evolution of the snow layer (surface). In contrast, in our case we have a strong spatial variability in the properties of the basal reflector. Therefore the two methodologies are affected by different phenomena and error sources.

References:

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