Dear Prof. Pattyn,

on behalf of my co-authors I am submitting the revised version of our manuscript entitled "Comparison of hybrid schemes for the combination of Shallow Approximations in numerical simulations of the Antarctic Ice Sheet."

Please, find below the point-by-point responses (in blue) to the referees' comments (in black), which include the changes made in text when relevant. Furthermore, please find at the end a marked-up version of our manuscript displaying all changes made to the originally submitted version.

Thank you for considering this manuscript for publication. I appreciate your time and look forward to your response.

Yours faithfully,

Jorge Bernales

Responses to review 1 of the manuscript “Comparison of hybrid schemes for the combination of Shallow Approximations in numerical simulations of the Antarctic Ice Sheet ”:

General comments

This paper describes a study of 4 hybrid schemes, combining the Shallow Ice and Shelfy Ice approximations for Antarctic Ice Sheet simulations. The 4 schemes are implemented into the open source ice sheet model Sicopolis (Greve, 1997), and a number of simulations are made for the Antarctic Ice Sheet with each and the resulting basal sliding coefficients compared. Analysis of the calibrated sliding parameter is done and runs where average values from all 4 schemes is applied, as well as the
results from other schemes (swapped).

We thank the reviewer for their very constructive comments that helped to improve the clarity of the manuscript overall. Our replies are provided in blue.

The paper is clear but the conclusions are not clearly specified and reader is left with wondering if authors have been able to conclude which scheme is preferred and how they will continue working with one, or all, of the schemes. A clearer message and conclusions from the study, as well as more concise analysis of the results (e.g. Figures 5 and 6) would improve the paper.

We agree. We have modified the text to include our conclusions regarding the relative performance and the potential for future applicability of each scheme. Our evaluation criteria do not only include the fit of the results to observations, but also the numerical stability of the schemes. Our experiments at higher grid resolutions show that the HS-1 and HS-2b schemes become numerically unstable, due to large gradients in basal sliding coefficients arising from the use of basal velocities as boundary conditions for the SIA solutions in conjunction with the calibration of sliding coefficients. The HS-2a and HS-3, which utilize the SStA as a sliding law, are numerically more stable to variations in model parameters and changes in grid resolution (especially HS-2a, which rarely produces simulation crashes). The drawback of the HS-2a is in its limited ability to influence the fit between the modeled and observed ice thickness in ice sheet sectors where the SStA velocities are low (<100 m/yr), which is the case over large tracts of the Antarctic interior. The HS-3 overcomes this limitation by accounting for the SStA contribution everywhere, but so it does with the SIA velocities, which in certain areas such as, e.g., the steep ice sheet margins are excessively high, as shown in Figure 3 of the original version of the manuscript for the SoS. To improve the performance of both schemes, our future work will reconcile the drawbacks of HS-2a with the advantages of HS-3, providing a very stable and flexible hybrid scheme. Following the suggestions of the reviewer, we have tried to make the results section more concise (see the related point below).

The structure of the paper could improve by adding a separate data section, before the
Methods section, the text in lines 4-22 on page 8 would be better located in a separate Data section and a more detailed information about the data used for this modeling approach would be beneficial for the paper. Also lines 19-24 on page 9 would be better located in a data section.

Done as suggested. The data sets are now described in a greater detail in a separate section, including their respective uncertainties. To keep the flow of the text, we have located this new section after the description of the model and before the description of the experimental setup.

The wording of the method used and the results is confusing, in this study a forward method is used to determine or rather calibrate Co , the basal sliding coefficient, and therefore it is confusing to call it inversion technique (e.g. line 23 page 8) or inversion (e.g. line 28, page 8), suggest to call it iterative technique or calibration, see further comments below (as in line 5 in the abstract: “the model is calibrated using an iterative technique . . .”)

Done. All instances have been replaced as suggested.

It is not clearly explained why authors are swapping the determined Co values be- tween the calibrated models, why is this useful? Is the following ice sheet adjustment indicating model consistency? As stated in the paper, each method has its own way of combining the approximated term and not clear why swapping of the determined (or calibrated) Co values would give any insight in the model behavior or result. It is dis- cussed that the different methods, and the results of Pollard and DeConto (2012) give qualitative similar results, with high/low values in same regions, but the numerical value of Co is method/model dependent and not clear what swapping of the results is useful for.

These experiments are performed to show the effects of prescribing a distribution of sliding coefficients derived from one ice model (with a specific scheme/approximation for the stresses) into a different ice model. Thus, these results are indicative of the degree of inter-model consistency. This concerns not only differences between the basal sliding approaches implemented in each model, but also models using the same sliding law as part of different hybrid schemes. Given an increasing number of studies attempting to quantify the basal conditions under ice sheets through a variety of methods including ice flow models, our experiments show that one need to be careful when using these results as input data sets in glaciological models. The text of the Results and Discussion section has been modified to clarify this.

The text on page 15 is not clear and the two figures 5 and 6 are not discussed in satisfying manner, what information about the schemes and the results can we draw from this analysis?

The text has been modified to clarify this (see previous point). To provide a clearer and more concise analysis of our results, we have removed figures 5 and 6, since they do not provide the information that is not already included in the main text.

Specific comments:

The wording in abstract and introduction is confusing, the Shallow Ice approximation is a zeroth order approximation assuming the thickness of the ice is much smaller than the length scale and thereby horizontal stress gradients omitted. This approximation has no assumptions for the sliding law and a full system model would equally have to assume some sliding approximation, or shelfy ice solution to account for sliding. Line 2 in the abstract should be reworded (SIA is not applicable . . . where basal sliding operates) and line 6 (minimal sliding) on page 2. Consider to rewrite also lines 2-5 on page 2, ("neglecting terms" and "simplest and most commonly used"): the Shallow Ice Approximation is a zeroth order approximation of the momentum balance equation assuming the H/L is very small.

We have reworded all instances in the text as suggested.

The wording in the abstract in lines 9 and 11 is not clear and should be rewritten for clarity: "averaged and swapped" cannot be understood until reading the main text of the paper and therefore needs some clarification in the abstract. ".. this requirement for internal consistency" – is not clear in this context and needs more explanation.

Done as suggested. The text as been modified to clarify this.

I find missing an overview figure indicating the location of the areas that are named in the paper, such as Dronning Maud Land, Coats Land, Siple Coast etc., this would be useful for readers not familiar with place names in Antarctica and makes the paper easier to read.

The location map has been added as suggested.

Note that “Blatter-Pattyn models” were developed much before the asymptotic analysis by Schoof and Hindmarsh (2010) and therefore it would be appropriate to reference the earlier papers with these model developments.

Done.

Lines 27-29 on page 2 are not clear, more information and detailed explanation of what authors mean is needed here, why would it be necessary to look for further explanations when two models yield similar result under similar forcing?

We mean that if the ice models are different (e.g. as in the hybrid schemes presented in this study), each model can independently calibrate its parameters to arrive at a similar solution (as shown in our results). Since the idea provided by this line is discussed later in the main text, we have removed it from the introduction to avoid confusion.

Model resolution is very low, were any tests made to assess the sensitivity of the method to grid resolution?

Following the concerns of the reviewer, we have managed to rerun all experiments at a higher resolution of 20km. Additionally, we have performed some sensitivity tests at higher resolutions of 15 and 10km, which show that the modeled ice flow near the ice sheet margins benefits from a denser grid due to the added flexibility (i.e. more grid points) provided to the calibration procedure, further (but slightly compared to the change from 40 to 20km) decreasing the misfit between the modelled and observed fields, representing a quantitative improvement that does not impact our conclusions.

- or to potential errors/inaccuracies in the topographic data? How good is the observed topography?

The sensitivity of the method to various model and data uncertainties (including grid resolution) was assessed by Pollard and DeConto (2012). For the topographic data, they perturbed the bedrock elevations using Gaussian noise with different noise amplitudes, finding that widespread errors of ~400m or more are necessary to considerably affect their results. In BEDMAP2, this level of uncertainty is found primarily in two regions: Between the Recovery and Support Force glaciers, and in Princess Elizabeth land (Fretwell et al., 2013), which are characterized by unrealistically smooth topography. In the former region, we attribute to this smoothness the overestimation of ice thickness found in all the applied hybrid schemes, which are not able to reproduce the observed ice streams without the proper topographic routing.

The quality of English is generally good, but in many places the wording is strange and needs some editing, it would be beneficial for the paper to have a thorough editing of all the text. Below are a number of places indicated where rewriting/editing would improve the quality of the text.

The revised manuscript will undergo the suggested check. We have applied all the corrections suggested by the reviewer.

Technical corrections:

Page 1, line 2, suggest to replace “ice dynamics” with “stress within the ice”

Done.

Page 2, line 10, suggest to add “at the base” after “no friction”

Done.

Page 2, line 21, suggest to rewrite, models do not “detect”, replace with something like “the algorithm
used to identify . . .”

Done.

Page 2, line 23, suggest to replace “versus” with “compared to”'

Done.

Page 2, line 25, suggest to replace “superposition” with “combination”

Done.

Page 2, line 31, suggest to rewrite, replace “these result from ..” with “ these are . . .”

Done.

Page 2, line 32-33 reword sentence: “Mechanical properties . . .. may serve as an example of . . . parameters” does not make sense.

Done.

These limitations include the scarcity of observational data needed to reduce the errors introduced by poorly constrained model parameters, e.g., the distribution of water-saturated sediments at the ice sheet base and their potential to enhance basal sliding.

Page 2, line 34, something missing “widespread misfit” of what? Elevation? - do all models show this type of misfit? A reference to a study showing this would be useful here.

The text has been modified to clarify this:

This is currently considered to be a major source of large, widespread misfits between the observed and modelled elevations of the AIS (e.g., de Boer et al., 2015) .

Page 3 Line 11, missing information and reference, what observational data sets are used?

The text has been modified to clarify this:

[...] using state-of-the-art observational data sets of ice thickness (Fretwell et al., 2013) and ice surface velocities (Rignot et al., 2011).

Line 14, add “the” between by and different

Done

Line 19, see comment above, replace “inverse” with “iterative”

Done.

Line 29, the model does not “consider” or is “keeping track” of temperature, the algorithm or post processing does, suggest to rewrite. Suggest to add “computed” after “ice is”

The text has been modified to clarify this:

SICOPOLIS is able to model polythermal ice sheets, i.e., explicitly identifies potential temperate regions in which the modeled ice temperature is at the pressure-melting point (Greve, 1997).

Page 5 Line 1, see comment above, “detect” seems a strange selection of word, suggest to replace with “determine” or “identify”

Done.

Line 14, not clear wording “a consistent use of inverted distributions of Co” - (replace inverted with determined, or calibrated)

Sentence deleted (this info is a duplicate of the beginning of paragraph).

Line 21, not clear wording: “enters the computation of SStA velocities” – suggest some- thing like “and SStA velocities are computed for this point”

Done. The text has been modified as suggested:

If r is larger than the threshold, that grid point is flagged as streaming ice and SStA velocities are computed for that point.

Line 25, suggest to delete “which is assigned to the SStA” this is explained in next sentence (and add velocity after SstA)

Done.

Page 6, Line 2, suggest to add “SIA and SstA” before “Velocities are . . .”

Done.

Line 16-17, unclear wording, suggest something like “It is rather used to determine. . .. . . the computed SStA contribution should partly or completely replace sliding”

Reworded as suggested:
It is rather used to determine how much the computed SStA contribution should replace the basal velocities used to compute the SIA solution.

Line 24, sentence is not clear, suggest to rewrite, something like “In the continental interior the modelled ice flow is dominated by the SIA solution

Modified as suggested:

This approach is based on the assumption that on ice shelves the SIA contribution is negligible due to low surface gradients, and therefore the modelled ice flow is dominated by the SStA solution, whereas in the continental interior the modeled ice flow is dominated by the SIA solution.

Line 29 see comment above, replace “inversion” with “determination or calibration”

Done.

Line 30, replace “infer” with “determine”

Done.

Page 7, Line 11, see comment above, “easier activation of the inversion procedure” suggest to rewrite to something like “more frequent computation of sliding velocity” - what does “slightly” mean here?

The text has been modified to clarify this:

In contrast to previous studies using SICOPOLIS where γ=1K, this parameter is set to 3K, which allows for a more frequent calibration of the sliding coefficients (Pollard and DeConto, 2012).

Line 14, suggest to delete “similarly, and”

Done.

Line 16, suggest to replace “speed” with “velocity”, not clear what “local adjustment” means here, elevation, or Co? suggest to replace “keeps the inversion” with “prevents the method”

Done. The text has been modified to clarify this:

Additionally, we implemented the following condition: When the computed surface ice velocity reaches an ancillary speed limit at a certain grid point, the adjustment of C_0 for that point is halted.

Line 17 suggest to replace “over-adjustment of” with “over-adjusting”, replace “speed” with “velocity”

Done.

Line 18, suggest to delete “the” before “numerical”

Done.

Line 20, replace “This” with “These” (plural of values)

Done.

Section in lines 22-29 is not clear, and needs rewriting for clarification what is the time in the iterative method or calibration?

Done. We have modified the paragraph to clarify this. The new version reads:

The iterative technique involves an additional limiting condition that prevents over-adjustments of C_0. At each time step and individually for each grid point, if the adjustment implemented at the previous time step reduces the difference between the modelled and observed ice thickness, the adjustment is skipped for the current time step. This allows previous adjustments to fully develop their effects over the following time steps and prevents the technique from adding unnecessary extra adjustments that can result in overshoots. The calibration is reactivated when the time derivative of the modeled ice thickness becomes zero (i.e. the difference between modelled and observed is not reduced anymore) or the misfit starts increasing (e.g. due to increased influx from surrounding areas). Our experiments have shown that this additional feature enables the use of a smaller Δt_{inv} (50 years here compared to 500--10,000 years in Pollard and

DeConto (2012)), because further adjustments will only be applied when and where strictly necessary. A further benefit is that it indirectly allows non-local adjustments of C_0 influence the local ice dynamics: If an adjustment applied in the vicinity of a grid point reduces its misfit, further adjustments at that grid point will still be halted.

Line 24, “time derivative of the ice thickness”, do you mean observed or modelled?

We mean modeled. We have modified the paragraph to clarify this (see previous point)

why suspend adjustment if previous time step reduced the difference?

Because the previous adjustment can still affect the evolution of the ice sheet during the next time steps. In other words, it could contain the exact amount of adjustment needed for the best fit, requiring just a few extra steps to minimize the difference. Without this, the algorithm would only check for the magnitude of the difference, adding a potentially unnecessary extra adjustment that could result in an overshoot. If the previous adjustment is not enough, the small time step used (50 years, which is not possible without our algorithm) will ensure a prompt correction. We have modified the paragraph to clarify this (see previous points).

(line 24): What process?

We mean the calibration. We have modified the paragraph to clarify this:

The calibration is reactivated when [...]

Line 26 replace “overshoot prevention” with “over-adjustment of Co”

Done.

line 27, suggest to replace “lets” with “allows” and delete “to” before “influence”

Done.

line 31, add “the” before “fringing”

Done.

Page 8 See comment above, move lines 6-18 to a Data description section

Done.

Line 6, re- place “which” with “that”

Done.

Line 19 suggest to add “is used to” before “account”, replace “changes” with “discrepancies” , replace “by” with “with”

Done.

Line 23, see comment above, replace “inversion” with “iterative” or calibration

Done.

Line 24, this is forward method, suggest to replace “inversion” with “iterative”

Done.

Line 28, replace “inversion” with “calibration” see above

Done.

Line 30, suggest to delete “one-to-one”

Done.

Line 33, suggest to replace “not accounted for” with “is not included in the simulations”

Done.

Page 9 Line 1, see above, replace “inversion” with “calibration”

Done.

Line 12, replace “simulations” with “calibration run”

Done.

Line 13, see above, replace “inverse technique” with “iterative” or calibration

Done.

Line 27 what do you mean by “glacier flux gate” do you mean an outlet glacier?

We mean the area where an ice stream reaches the grounding line, usually exhibiting very fast ice flow. We have changed the text to clarify this:

These mainly occur close to the ice sheet margins where an ice stream reaching the grounding line is often represented by only one grid cell at low resolution

Line 31 not clear text “which overlap at their interface” needs clarification here

We have changed the text to clarify this:

11 equidistant grid points for temperate ice and 81 grid points for ``cold'' ice densifying towards the base, sharing the grid point at their interface.

Page 10 Line 8, suggest to add “a” before “new”

Done.

Line 12-13, this sentence is not clear and needs rewriting (what is internal operation of the hybrid schemes?)

We have changed the text to clarify this:

In addition, the influence of variations in the parameters controlling how each scheme combines the SIA and SStA velocities is assessed for a wide range of parameter values.

Line 17, what does “quasi-equilibrium” mean here?

Replaced by “equilibrium”, used and defined below (see next point)

Line 19, what is “negligible” in this context? A percentage or some value would be useful here

We consider negligible a change over a prolonged time (>10000 years) smaller than 0.01 %. We have added this to the text to make it clear.

Line 24, suggest to replace “prevents” with “does not require”

Done.

Page 11 Line 9, the smallest error of 49.9m is according to the table using HS-1, is there an error in the table? If not then this section should be rewritten to reflect that.

We have updated the table to reflect the increase in grid resolution and modified the paragraph accordingly.

Line 21, replace “independently” with “independent

Done.

- here some discussion would be appropriate about if this is related to the common SMB forcing or geothermal heat flux?

Done. We also included the potential influence of the uncertainties in the topographic data, as described above (see related specific comment).

Line 22, add “simulated” before “frozen”

Done.

Line 24, suggest to edit, change to something like: “far below the pressure melting point” and delete “the” before “white coloured”

Done as suggested.

Line 25, edit the text, delete “on the other hand”, suggest to write “Areas where ice is underestimated . . .” – but what does “sparsely distributed” mean?

Done. We have changed the text to clarify this:

Areas where ice thickness is underestimated are mainly located at and around the ice margins […]

Line 27 replace “inversion” with “determination” or “computation”

Done.

Line 31, replace “inversion” with “iterative” or calibration

Done.

Page 12 Line 2, replace “inverted” with “applied” or “computed”

Done.

Line 4-5, same comment, suggest to write “region where Co is not applied”

Done.

Line 6, replace “inversion” with “calibration”

Done.

Line 21, delete “also”

Done.

Line 26, what do you mean here? What significant modification of Pollard and DeConto (2012) scheme would result in similar values of Co?

We mean that if Pollard and DeConto (2012) used a different hybrid scheme, we would expect a similar degree of variation in their calibrated values of C_0. This line duplicates what is implied in the first sentence of the paragraph, and therefore it has been deleted to avoid confusion.

suggest to replace “perturbation” with “distribution” or “pattern” and delete “inverted”

Done (see previous point).

Line 30 add “the” before hybrid

Done.

Line 33 replace “small” with “low” and add “the” after “near”

Done.

Line 35, what are “high velocity flanks”? suggest to add “sheet” before “margins” and at the end of line, after “ice”

We mean the fast flowing ice streams reaching the grounding line. We have modified the text to clarify this and added the corrections:

distinguishing between ice sheet areas with low velocities near the ice divides and fast flowing ice streams reaching the ice sheet margins

Page 13 Line 2, strange wording, suggest to replace “contaminated by” with “characterized with”

Done.

Line 6, add “the” before hybrid

Done.

Line 7, suggest to replace “in” with “at”

Done.

Line 8, “flux gates” - not clear, is this a specific location?

We mean the portion of the glacier closest to the grounding line. We have modified the text to clarify this:

Furthermore, modelled surface velocities are generally overestimated at the grounding zone of most outlet glaciers

Line 11, delete “inferred” and delete “On the other hand”

Done.

Line 14, delete “inferred”

Done.

Line 19, delete “a” before slow ice motion, replace “flow speed” with “velocity” and “predicted” by “simulated”

Done.

Line 21, delete “Arguably”

Done.

Line 22, replace “stagnated” with “stagnant”

Done.

Line 25, not clear wording “deviated to either side” - “pushed to merge” are you referring to modelled or observed velocity? – what side of what?, what is pushing what?, suggest to replace “deficiencies” with “errors”

Done. We have modified the text to clarify this:

In other cases, the modelled rapid ice flow follows a different route compared to observations, sometimes merging with adjacent ice streams. These shifts may originate from local errors in the bedrock topography data accentuated by its projection onto the coarse horizontal grid we use here.

Line 28, according to the table, it is HS-1 that has the minimum misfit for the ice thickness

Done. The text has been modified to fix this.

Line 30, see above, replace “inversion” with “calibration”

Done.

Line 31, suggest to delete “in the modelled surface velocities”

Done.

Line 33, what do you mean by “opposite ends”?

We mean that their degrees of fit to observations is very different (keeping in mind the limitations of the method, as explained in the manuscript), with SoS performing the poorest and HS-2b representing one of the best fits in terms of final ice sheet geometry. We have modified the text to clarify this:

Although the results of the HS-2b simulation presented in Section 5 are in many aspects similar to those from the SoS, their respective skills in reproducing observations are very different [...]

Line 34, suggest to replace “enabling” with “adding”

Done.

Page 14 Line 4, suggest to replace “imply” with “control”

Done.

Line 5, what does “internal operation of the hybrid scheme” mean?

We have modified the text to clarify this:

In order to provide a deeper insight into how each hybrid scheme combines the SIA and SStA velocities, [...]

Line 11, suggest to replace “the velocity field from the observational data set” with “ the observed velocity” Line 12, suggest to re- place “simple differentiation” with “transition”

Done.

Line 17, reword, “tends to prevent” to “can prevent” and delete “s” in causes (and cause underestimation..)

Done.

Line 24, what is a “cursory comparison” ? clarification is needed

We mean that is readily visible from the figures. We have modified the text to avoid confusion:

At a first glance it may seem that overestimations are caused by an excessive contribution of the SStA, but a comparison with the SoS scatter plot shows that this is not necessarily the case.

Line 29-30 suggest to rewrite (delete Here we attempt) and write: To isolate the influence . . .. we plot averaged errors . . .

Done.

Line 35, replace “inferred” with “resulting” or delete “inferred”

Done.

Page 15 Line 1 add “the” before different

Done.

Lines 2-5 this sentence is not clear and does not explain the difference between the 3 figures in Fig.5 clarification is needed here

Figure 5 has been removed (see previous points).

Line 9, suggest to replace “looking” with “comparing”

Done.

Line 19 delete “the” before “period”

Done.

Line 26 – this line is not clear, what do you mean by “generalizing” – see comment above, it is not clear what information about the model can be drawn from Figures 5 and 6

We have modified the text and removed Figures 5 and 6 (see previous points).

Line 34, delete “On the other hand,”

Done.

Page 16 Line 2 suggest to add “distributions of Co from” before the HS-1 and HS-3

Done.

Lines 1-6, it is not clear from this text what this analysis gives for useful information about the different schemes used in the study

The text has been modified to reflect our answer to the related general comment.

Line 8, add “The” at beginning of line

Done.

Line 11, rewrite: “allows us” is a strange wording here

Done.

In order to explore the sensitivity of the results to parameter variations within this parameter space, we perform an additional series of experiments where we vary the somewhat arbitrary threshold and reference quantities used by some of the hybrid schemes.

Line 23, what is the criteria for selecting lower and upper limits for each scheme?

For the HS-1 values outside the [0,1] interval are invalid. For HS-2a and HS-2b, our tests showed that higher or lower values produce no noticeable differences compared to the representative limits we chose. We have modified the text to clarify this:

Here we test parameter values within a range that contain almost every possible scenario, either because values outside the range are unphysical (HS-1) or they exhibit no noticeable differences compared to the range limits (HS-2a and HS-2b).

Line 31, suggest to replace “decrease” with “get worse”

Done.

Page 17 Line 4, delete “by” before 2.5%
Done.

Line 24, delete “,” after “enables”

Done.

Line 27-29 suggest to rewrite, something like “differ in the way ways the 1) relative contributions . . . are computed 2) areas where . . . is applied is determined, and 3) basal sliding is accounted for.

Done.

Line 30, see above, replace “inverse” with “iterative” or “calibration”

Done.

Line 31, delete “all” or add “the applied” after “all”

Done.

Line 32, add “the” before “schemes”, replace “below” with “less than” and add “the” before “total”

Done.

Page 18 Line 3, delete one "can"

Done.

Line 5, add "the hybrid" after "all"

Done.

Line 7, add "the" before hybrid

Done.

Line 8, suggest to replace "develops" with "exists"

Done.
Line 15 What particular scheme is discussed here? It is not clear

Done. We have modified the text to clarify this:

We have found that the HS-2a tends to increase basal sliding coefficients in an attempt to compensate for an insufficient sliding that would otherwise lead to a larger misfit.

Line 19 add "," after Here

Done.

Line 27, what does "the neglect of paleoclimate signal" mean here – can applied geothermal heat flux, or applied SMB play a role here?

We mean the neglect of transient temperature effects from previous glaciations and the glacial isostatic adjustments in the bedrock. In the areas mentioned in the text, the geothermal heat flux mostly allows the schemes to calibrate the sliding coefficients, providing the good fit mentioned in the text. These processes were tested by Pollard and DeConto (2012), finding only a small on the results. A dryer climate forcing over these areas would in fact reduce the velocities, because less mass would need to be removed in order to obtain the same fit. We have modified the text to include these points:

Such misfits might originate from other factors such as, e.g., the lower model resolution, the assumption of an ice sheet in equilibrium, or biases in the model-based geothermal heat flux and/or climatic forcing data sets (Pollard and DeConto, 2012).

Line 33, does the scheme affect the way the temperature is computed?

Not explicitly. However, due to the coupling of the evolution equations the computed velocities do affect the ice temperature. From the four hybrid schemes, only the HS-1 shows a noticeable difference in the basal temperature field, due to its particular threshold-based selection of grid points where the combination of SIA and SStA is applied. We have added a discussion of this point in the conclusions, which contributes to our evaluation of the applicability of each hybrid scheme in future work (see first point)

Line 34, does the scheme allow determination of hard or soft bed? – it can only indicate high or low value for Co, or what?

We agree. The calibration of the sliding parameter is just an attempt to quantify the combined effect of several processes influencing whether and how the ice slides over the bed. We have modified the text to clarify this:

Furthermore, there is a qualitative agreement in the patterns of low vs. high values of C_0 obtained from each calibration run.

Page 19 Line 3, suggest to replace “inferred” with “calibrated”

Done.

- not clear how high variability can “provide an opportunity to quantify the effects of the uncertainty” – suggest to rewrite to clarify what is meant here.

We have modified the text to clarify this:

We assessed the effects of the high variability in the calibrated parameter distributions derived from different hybrid schemes by performing additional experiments in which averaged and swapped distributions of basal sliding coefficients are prescribed as external data sets.

Line 8, see comments above, replace “inverse method” with “iterative technique”

Done.

Line 9, what is meant with “internal consistency required to avoid misfit” suggest to rewrite to clarify.

Done (see answer to related general comment)

Suggest that the concluding sentence of the paper will state the main results of the study and how it can be useful for further modelling approaches.

Done (see answer to related general comment)

Figures and tables

Table 2, suggest that the caption include some explanation, referring to text, what the different lines stand for (HS-1 etc).

Done.

See comments above, the lines may have got mixed up, since the text states that the minimum difference for elevation is for HS-2, but 49.9 m is in line HS-1

Done, we have checked this (see answer to comments above)

Figure 2, is this at the end of the simulation?

Yes. We have modified the caption to clarify this:

Comparison of the equilibrium ice sheet states derived from different schemes at the end of the simulations. [...]

Figure 4, suggest to replace “grid cell” with “point” for the right hand column figures

Done.

Figure 5, the figure caption is not clear and needs editing. What does “quantify different ice flow regimes” mean here? The y-axis label is mean velocity ratio, but the text states Surface velocity error, clarify what is shown here. Suggest to replace “by” with “as” before ratios.

This figure as been removed (see previous points)

Figure 6, as discussed above it is not clear how this figure is useful. What information can be gained from this analysis? “retrieved distributions of basal sliding coefficients” is not clear, do you mean standard deviation of the determined values with each scheme in each grid point?

This figure as been removed (see previous points)

Figure 7, what does this figure tell us? What meaning does the median of the inferred distributions of Co have? Is this a useful quantity?

Please see answer to related general comment

Figure 8, what do the bumps in the lines of HS-1 with r-thr=0.0 and HS-2b for v-ref=1000 m/yr shortly after 50 kyr mean? Is this instability in the simulations?

Yes, it corresponds to instabilities in the simulations. We discuss this in the conclusions (see also answer to first general comment)

Figure 9 “throughout the simulations” – do you mean at the end of simulations?

Yes. We have modified the caption to clarify this:

Mean differences between the modelled and observed ice thickness at the end of the simulations, […]

Cited studies:

Fretwell, P., Pritchard, H. D., Vaughan, D. G., Bamber, J. L., Barrand, N. E., Bell, R., ... & Catania, G. (2013). Bedmap2: improved ice bed, surface and thickness datasets for Antarctica. The Cryosphere, 7(1).

Pollard, D., & DeConto, R. M. (2012). A simple inverse method for the distribution of basal sliding coefficients under ice sheets, applied to Antarctica. *The Cryosphere*, *6*(5), 953-971.

Responses to review 2 of the manuscript “Comparison of hybrid schemes for the combination of Shallow Approximations in numerical simulations of the Antarctic Ice Sheet ”:

The manuscript by Bernales et al uses an ice-sheet model for Antarctica to test different schemes for the calculating of the ice velocities. The main focus is on combining the shallow ice and shelfy stream approximations and use different hybrid schemes to combine the two. Four different schemes are implemented in the model and used to inversely determine the basal sliding coefficient C0 by looking at the difference between modelled and observed ice thickness. Differences between the four schemes are explained clearly, through how the spatially varying basal sliding coefficient differ for each scheme, how grounded ice volume evolves and how ice thicknesses vary relative to the observations. Model performance is also checked against observations of ice surface velocities. Methods and results are explained clearly. The two main concerns I have are that the manuscript does miss a real strong final conclusion and that uncertainties in the methodology should be discussed in more detail.

We thank the reviewer for their very constructive comments that helped to strengthen the manuscript. Our replies can be found below.

Main comments

1. Uncertainties in methodology

Paragraph starting on Page 2, Line 30 to Page 3, Line 8:

You should here also mention that there are more parameters and/or processes involved that are poorly constrained (or less known) such as bedrock uplift due to post- glacial rebound (GIA), response of the bed itself to current changes in ice, heat flux from the bed, properties of ice flow (flow parameter) or the bedrock height uncertainty (see Bedmap2 paper). Following on the uncertainties, an important point to make using this method is that by solving for the basal stress you actually compensate/correct for other (some of them mentioned above) uncertainties in the boundary conditions and uncertainties/errors in the model itself. This is I think a key point to keep in mind when using the methods described in the manuscript. This should be mentioned here and discussed thoroughly in the final discussion/conclusion as well!

Following the suggestion of the reviewer, we have added a discussion of potential effects of these poorly constrained processes and boundary conditions on the modeling results. We have also added a separate Data section where the uncertainty in the bedrock topography data is spelled out. The possibility of canceling errors is a clear limitation of the method, since at present there is no easy way to differentiate what could be an artificial compensation of, e.g., an insufficient model representation of physical processes or deficiencies in the input data sets from the actual conditions at the base of the ice sheet. This shortcoming will hopefully be overcome in the future once the necessary observational data sets and more sophisticated ice models become available. For now -and for the purposes of this study-, the risk of error cancellation is over-passed by the dramatic reduction in the misfit between the modeled and observed ice thickness and surface velocities. This limitation is now discussed in the method description and in the final discussion sections.

2. Rewording of basal velocities and SIA

Starting on Page 4, Line 16:

Throughout the manuscript it is mentioned that SIA has a basal velocity. This is mostly a matter of rewording some of the text, not so much an error but rather how this is actually computed. I think the SIA itself does not have a basal velocity, but rather that the basal velocity is a basal boundary condition to calculating the SIA, which could also be set to zero if you do not include sliding at all. This is actually nicely illustrated in Fig. 1 of Bueler and Brown (JGR, 2009). I think this could be rewritten in some parts of the manuscript. In the general comments below some of these lines are mentioned.

We agree. All instances have been rewritten accordingly.

3. Final conclusions

On page 19:

The sentence “This suggests .. Stokes equations.” seems quite obvious but good to mention. However in the final conclusions I do miss a discussion on two things particularly: - Is a distribution of basal parameter C0 derived using present-day forcing and observations also applicable for long-term paleoclimate simulations, can it be used for other time periods and climate regimes?

This is an interesting and challenging question, since the current availability and the accuracy of the “observational” data for past periods is not sufficient for testing the performance of the reconstructed coefficients under paleoclimate scenarios. Basal thermal regimes of continental-scale ice sheets and thus the areas where model calibration procedures can be applied do not only depend on the external thermal forcing that remains nearly constant in time (e.g., geothermal heat flux) but also on climate conditions and ice sheet geometrical settings that had undergone significant changes in the past (Bentley et al., 2014). There are regions where basal sliding is not identified by the method under the present-day conditions, where paleo-ice streams could have developed under a different climate regime, and vice versa. Thus, the derived distribution of C_0 is not necessarily applicable everywhere. However, we believe that it is a good first-order approximation, and is definitely a better guess than commonly used single, homogeneous values of C_0 over the entire domain. The method used to derive C_0 could be potentially extended to the use for other time periods/climate regimes (or even adapted for the transitions between largely dissimilar regimes), providing that the necessary topographic and climate data are available. Looking at the recent efforts to reconstruct the past geometries of the Antarctic Ice Sheet (Mackintosh et al., 2014, Bentley et al., 2014), this may become well possible in the upcoming decades. We have added this point in the final discussion.

- What is the best scheme to be used here? Is there a scheme that simulates the observations best, also considering the lowest over- and underestimations, i.e. the lowest absolute difference in ice thickness.

We have evaluated the performance of each scheme based on both its fit to observations and numerical stability for a range of model setups. We have found that at higher grid resolutions the HS-1 and HS-2b schemes become numerically unstable, due to large gradients in basal sliding coefficients arising from the use of basal velocities as boundary conditions for the SIA solutions in conjunction with the calibration of sliding coefficients. The HS-2a and HS-3, which utilize the SStA as a sliding law, are numerically more stable to variations in model parameters and changes in grid resolution (especially HS-2a, which rarely produces simulation crashes). The drawback of the HS-2a is in its limited ability to influence the fit between the modeled and observed ice thickness in ice sheet sectors where the SStA velocities are low (<100 m/yr), which is the case over large tracts of the Antarctic interior. The HS-3 overcomes this limitation by accounting for the SStA contribution everywhere, but so it does with the SIA velocities, which in certain areas such as, e.g., the steep ice sheet margins are excessively high, as shown in Figure 3 of the original version of the manuscript for the SoS. To improve the performance of both schemes, our future work will reconcile the drawbacks of HS-2a with the advantages of HS-3, providing a very stable and flexible hybrid scheme. We have added this point in the final discussion.

General comments

Page 1
Line 1: Replace introduced with used or implemented

Done.

L 5: What do you mean with realistic scenario? Do you mean like present day climate forcing? Please be specific.

We mean a non-synthetic/simplified ice sheet geometry. We have modified the text to avoid confusion:

Here, we implement four different hybrid schemes into a model of the Antarctic Ice Sheet in order to compare their performance under present-day conditions.

L 8: Remove comma after Despite this

Done.

L 8: Robust agreement with what, or of which variable? Again please be specific.

We have modified the text to clarify this:

[…] we observe a robust agreement in the reconstructed patterns of basal sliding parameters.

L 17: Change to: However, the time scales over which an ice sheet builds up and disintegrates

Done.

Page 2
L 9: Remove commas before membrane and after important

Done.

L 7-14: You could first introduce the SSA (L10-14) and then discuss the ice streams (L7-10).

Done as suggested.

L 15: Instead of mentioning highly dynamic regions, actually state here something like: The limitations of SIA models for calculating the highly dynamics ice streams have ..

We have modified the text as follows:

The limitations of SIA models for reproducing the dynamics of ice streams have prompted the development of [...]

L30: Rephrase realistic scenarios, same as in the abstract, present-day climate forcing?

We have modified the text as suggested:

Despite the above differences among models, all of them are subject to common limitations when applied to the present-day Antarctic Ice Sheet

Page 3
L 17: No new paragraph, start sentence as: First, the ice-sheet model

Done.

Page 4
L 16: Please rewrite, the SIA itself does not have a basal velocity, but rather U-b is here the basal boundary condition to calculating the SIA.

We have modified the text as suggested:

At the base of the grounded ice sectors, bedrock stress conditions and the associated potential for sliding are linked to the basal velocity, u_b, used as a boundary condition for the computation of the SIA velocities, through an empirical Weertman-type sliding

L 18: Eexplain here how the effective basal pressure Nb is calculated.

Done. The description reads:

N_b is the effective basal pressure, computed as N_b = rho_{ice} g H - rho_{sw} g H_{sw}, where rho_{ice} and rho_{sw} are the density of ice and sea water, respectively, g is the gravitational acceleration, H is the modelled ice thickness, and H_{sw} is the difference between the mean sea level and the ice base topography.

L 22: Is Tm an actual temperature or a temperature difference. Explain this in the text.

We have modified the text in page 4, line 5 to clarify this:

[...] T_m is the temperature difference relative to the pressure-melting point, [...]

Page 5

L 19: Rewrite to something like: where ub is the Weertman sliding velocity (Eq. (2)) and us the surface SIA velocity, respectively.

We have modified the text as suggested:

where u_b is the Weertman sliding velocity (Eq. 2) and u_s is the surface SIA velocity.

Page 6
L 11-16: Do not mention sliding-SIA but rather SIA including (Weertman) sliding. Here it could also be discussed the carefulness of using Weertman sliding, as discussed by Bueler and Brow (JGR, 2009; see also their Appendix B) it leads to a discontinuities in the velocity field.

We have modified the text as suggested:

As described in Section 2.1, the SIA solution in SICOPOLIS is computed using the Weertman sliding component coming from Eq. 2 as a boundary condition. To assess the influence of a SIA solution including Weertman sliding, we have divided this hybrid scheme into two: A sub-scheme (HS-2a) that replicates the idea of Bueler and Brown (2009) and prescribes no basal velocities in the computation of the SIA velocities, and a sub-scheme (HS-2b) that keeps the Weertman sliding component and uses it to compute a slightly modified weight [...]

We have mentioned the discussion on Weertman-type sliding in the description of the model with the respective reference to Bueler and Brown (2009).

L 16: change to: where ub is the basal sliding velocity as in ..

Done.

L 28: At the end of section 2.2 a table could be added that summarises the 4 schemes with the column show something like : 1) name, 2) sliding 3) reference to equations, 4) reference to studies

Done. We have added the table as suggested.

Page 7
L 23-24: Can you explain here and perhaps clarify in the text why you stop the adjustment when the difference between modelled and observed ice thickness is reduced?? A reduction of this difference is actually what you want right?

Because the previous adjustment can still affect the evolution of the ice sheet during the next time steps. In other words, it could contain the exact amount of adjustment needed for the best fit, requiring just a few extra steps to minimize the difference. Without this, the algorithm would only check for the magnitude of the difference, adding a potentially unnecessary extra adjustment that could result in an overshoot. If the previous adjustment is not enough, the small time step used (50 years, which is not possible without our algorithm) will ensure a prompt correction. We have modified the paragraph to clarify this (see previous points).

The text has been modified to clarify this:

The iterative technique involves an additional limiting condition that prevents over-adjustments of C_0. At each time step and individually for each grid point, if the adjustment implemented at the previous time step reduces the difference between the modelled and observed ice thickness, the adjustment is skipped for the current time step. This allows previous adjustments to fully develop their effects over the following time steps and prevents the technique from adding unnecessary extra adjustments that can result in overshoots. The calibration is reactivated when the time derivative of the modeled ice thickness becomes zero (i.e. the difference between modelled and observed is not reduced anymore) or the misfit starts increasing (e.g. due to increased influx from surrounding areas). Our experiments have shown that this additional feature enables the use of a smaller Δt_inv (50 years here compared to 500--10,000 years in Pollard and DeConto (2012)), because further adjustments will only be applied when and where strictly necessary. A further benefit is that it indirectly allows non-local adjustments of C_0 influence the local ice dynamics: If an adjustment applied in the vicinity of a grid point reduces its misfit, further adjustments at that grid point will still be halted.

L 22-29: This paragraph could be replaced to Section 3.

We prefer to keep this paragraph here, since it is an important part of the calibration algorithm and follows the description of other constraints in the method.

Page 8
L 13: On spin up procedures for ice-sheet models see also this paper, you might want to refer to this. Fyke, J. G. and Sacks, W. J. and Lipscomb, W. H., A technique for generating consistent ice sheet initial conditions for coupled ice sheet/climate models. Geoscientific Model Development, 7, 1183 – 1195, 2014.

We have added the reference as suggested.

Page 9
L 15-16: Reword: a simulation time of 100.000 years (100 kyr)

Done.

L 21-22: Spanning 100 kyr (mention exact length of your experiments)

Done.

Page 10
L 1: Remove -and get insight into-

Done.

L 4: Be specific: observed ice-sheet thickness and surface velocities as a measure ..

Done.

L 19: .. the variations of the total grounded ..

Done.

L 26: Numerically more or equally stable, and also a similar computational time com- pared to the SoS?

As explained in the paragraph. All hybrid schemes take longer than the SoS due to the calculation of SStA velocities over a larger domain. However, the hybrid schemes require less of these (in this case, very long) iterations to get convergence in the solution, which makes them less prone to numerical instabilities compared to SoS. We have modified the paragraph to clarify this:

Compared to the SoS, the computation of SStA velocities in grounded ice sectors implies an extra computational effort for each iteration in the numerical solvers, with the computing time increasing by a factor of ~4 for the applied hybrid schemes. The computing time of the HS-1 is somewhat shorter relative to the other hybrid schemes (but still longer than for the SoS) due to the prognostic identification of ice streams that does not require a computation of SStA velocities over the entire ice sheet. However, we have observed that the iterative solvers in the model require a substantially smaller number of iterations when the hybrid schemes are used, making them numerically more stable compared to the SoS.

Page 11
L 1: Change to: that include basal sliding with the SIA solution

Done.

L 4: Remove -high frequency-

Done.

L 7: The inset of Figure 1

Done.

L 13: Change to: that include basal sliding with the SIA.

Done.

L 20-21: The large overestimation of ice thickness between Shakleton Range and the Pensacola Mountains as you state is seen in all panels of Fig. 2. Could you give a more clear explanation why this particular area is overestimated. Also note that this is a region of large uncertainties in ice thickness (see Bedmap2 paper).

As you have pointed out, this area has one of the largest uncertainties in the BEDMAP2 data. The lack of observational data produced an unrealistically smooth bedrock topography (Fretwell et al., 2013). We attribute to this uncertainty the absence of topographically driven ice streams that fosters the accumulation of ice at the interior. We have added this point to the text.

L 28, 32: I suggest not to refer to Fig. 1a but to: the inset of Fig. 1.

Done.

Page 12
L 2: Change to: over the areas where there is no sliding and C0 is not inverted.

Done.

L 32: Change to: .. that are unresolved by the model due to its..

Done.

L 34: Remove comma after divides.

Done.

Page 13
L 7: Remove commas before and after for example

Done.

L 26: Change to: .. projection onto the course horizontal grid we use here.

Done.
L 29: Remove comma after simulation

Done.

L 33: Change to: Equation (10) uses the basal velocities from equation (2) to compute..

Done.

L 34-35: Where sliding velocities from equation (2) are high.

Done.

Page 14
L 1: Here also perhaps mention how H-2a compares to H-2b?

Done.

L 5: are implemented

Done.

L 5: Change to: .. hybrid schemes, Fig. 4 illustrates the

Done.

L 22: Remove comma after general

Done.

L 23-24: At a first glance it may

Done.

L 25: Change to full stop: the case.

Done.

Page 16
L 2: Change to: The misfit for Hs-1 increases

Done.

L 4: The Hs-3 experiment shows an ..

Done.

L 19: Change to: the basal velocity in the Hs-2b experiments

Done.

Page 17
L 24: Remove commas before and after continental-scale.

Done.

L 25: Remove entire

Done.

Page 18
L 1: Also mention the range of the ice thickness errors not only the means.

Done.

L 17: Change to: which adds up SIA (without sliding) and SstA.

Done.

L 29: Also mention the uncertainties in ice thickness and surface velocities (see their respective references).

Done.

Tables and Figures

Table 2:
Should the sum of the last two columns not be 100%? Please clarify and explain.

The columns show the percentage of area where the flow is predominantly dominated either by the SIA or the SStA, defined as the area where the weight w is <0.25 (SIA-dominated) or >0.75 (SstA dominated). Areas where w is between 0.25 and 0.75 are excluded from these percentages, since they present a closer mix of both velocity solutions. We have clarified this in the caption.

Fig. 1:
- Label a and b can be removed, rather refer to fig. 1 and inset of fig. 1.

Done.

- Instead of a coloured bar for the pre-initialization the names of the periods could also be placed above (pre-initialization period and automatic calibration).

Done.

- In the legend of the inset the difference between solid and dashed should be clearer.

Done.

Fig. 4:
It is not really clear which scheme represents the surface velocities the best. Could you also add a correlation/R-square value to the scatter plots? Also use the correlation in the discussion.

Thanks for the suggestion. We have added the root-mean-square error to the scatter plots, which provides a simple way to clarify this. This information is also used in the results and final discussion sections to aid the determination of the "best" scheme overall.

Fig 8: Hs-2a always seems to be have a misfit, whereas the other scheme should have a possible solution which closely fits the observations, for a particular (probably not the same) set of parameters. Please explain and discuss this in the conclusions.

HS-2a prescribes zero basal velocity before the computation of the SIA. Sliding comes from the SStA, but only in those regions where the SStA velocities are high (see the weights w in Figure 4). Thus, at the continental interior the contribution from the SStA is small and not enough to prevent the overestimations of ice thickness that produce the misfit observed in Figure 8. The HS-3 does not scale the SStA contribution with any weight, and thus is able to prevent the overestimations. This has been added to the final discussion (see also our reply to the third main comment).

Cited studies:

Bentley, M. J., Cofaigh, C. Ó., Anderson, J. B., Conway, H., Davies, B., Graham, A. G., ... & Mackintosh, A. (2014). A community-based geological reconstruction of Antarctic Ice Sheet deglaciation since the Last Glacial Maximum. Quaternary Science Reviews, 100, 1-9.

Bueler, E., & Brown, J. (2009). Shallow shelf approximation as a "sliding law" in a thermomechanically coupled ice sheet model. Journal of Geophysical Research: Earth Surface, 114(F3).

Fretwell, P., Pritchard, H. D., Vaughan, D. G., Bamber, J. L., Barrand, N. E., Bell, R., ... & Catania, G. (2013). Bedmap2: improved ice bed, surface and thickness datasets for Antarctica. The Cryosphere, 7(1).

Mackintosh, A. N., Verleyen, E., O'Brien, P. E., White, D. A., Jones, R. S., McKay, R., ... & Miura, H. (2014). Retreat history of the East Antarctic Ice Sheet since the last glacial maximum. Quaternary Science Reviews, 100, 10-30.

Comparison of hybrid schemes for the combination of Shallow Approximations in numerical simulations of the Antarctic Ice Sheet

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Abstract. The Shallow Ice Approximation (SIA) is commonly ~~introduced~~ used in ice-sheet models to simplify the ~~equations describing icedynamics~~force balance within the ice. However, the SIA ~~is not applicable in fast flowing regions where basal sliding operates~~alone is not able to adequately reproduce the dynamics of the fast flowing ice streams usually found at the margins of ice sheets. To overcome this limitation, recent studies have introduced heuristic, hybrid combinations of the SIA and the Shelfy Stream Approximation. Here, we implement four different hybrid schemes into a model of the Antarctic Ice Sheet in order to compare their performance under ~~a realistic scenario~~present-day conditions. For each scheme, the model is calibrated using an iterative technique to infer the spatial variability in basal sliding parameters. Model results are validated against topographic and velocity data. Our analysis shows that the calibration compensates for the differences between the schemes, producing similar ice sheet configurations through quantitatively different parameter distributions. Despite this ~~,~~ we observe a robust agreement in the reconstructed patterns of ~~hard vs.soft basal conditions. We use averaged and swapped parameter distributions~~ basal sliding parameters. We exchange the retrieved parameter distributions between the schemes to demonstrate that the results of the model calibration cannot be straightforwardly transferred to models based on different approximations of ice dynamics. However, this ~~requirement for internal consistency can be fulfilled~~ limitation can be overcome through the implementation of easily adaptable calibration techniques, as shown in this study.

1 Introduction

Accurate projections of ice-sheet–driven sea level changes require the use of numerical models that are capable of capturing the dynamics of rapidly flowing regions and grounding-line zones (Pattyn et al., 2013). This requirement can be best accommodated using the most complete models which are currently available for modelling the ice dynamics, referred to as Full Stokes models (FS) ~~(e.g., ?)~~ (e.g., Gagliardini et al., 2013) . However, the time scales over which ~~ice sheets build up and disintegrate~~ an ice sheet builds up and disintegrates in response to variations in the climatic forcing typically involve ~~tens of~~ many thousands of years. Numerical experiments over such time spans are necessary ~~, e.g.,~~ to separate the long-term transient component from relatively fast fluctuations in the ice volume during the observational record. These long-term, continental-scale paleo-simulations are currently ~~unfeasible~~ infeasible using FS models due to the computational expenses triggered by the non-linearity of the model equations and the complex interdependence of the involved quantities.

To overcome the contemporary spatio-temporal limitations of FS models, a hierarchy of approximations has been developed over the last decades ~~. These approximations build upon neglecting terms in the momentum balance equations, and are categorised according to the degree of simplification~~ (e.g., Hindmarsh, 2004). The ~~simplest and most commonly used approximation for long-term simulations of ice sheet dynamics is the~~ Shallow Ice Approximation (SIA) (e.g., Hutter, 1983) ~~. This approximation~~ is a zeroth-order approximation of the momentum balance equations that keeps only the gravity-driven vertical shear stress, predicting reasonably well the behaviour of grounded ice masses which are characterised by a thickness much smaller than their horizontal ~~dimensions, smooth bed topography, and minimal sliding~~ length scales. Ice floating in the sea water experiences almost no friction at the base~~. However, this is not the case for areas of rapid ice flow, known as ice streams. Such areas serve as a bridge~~ , and its behaviour is usually described by the Shallow Shelf Approximation (SSA) (Morland, 1987) , which omits the vertical shear stress from the FS equations and neglects the basal drag. The transition between grounded and ~~marine portions of an ice sheet,~~ floating ice regions exhibits areas where ice flow is often enhanced by basal conditions favourable for sliding, generating rapid ice flow features known as ice streams. In these ice sheet sectors ~~,~~ membrane stresses become increasingly important ~~,~~ sharing many similarities with the floating ~~extensions of ice sheets known as ice shelves. Ice floating in the sea water experiences almost no friction, and its behaviour is usually described by the Shallow Shelf Approximation (SSA) (Morland, 1987) , which omits the vertical shear stress from the FS equations and neglects the basal drag~~ice shelves, and thus the SIA is no longer appropriate to describe the ice dynamics. It is also important to note that the absence of a membrane stress transfer in the SIA renders this approximation invalid for modelling the grounding-line migration, that is, the migration of an interface between grounded and floating ice sectors (Pattyn et al., 2012).

~~The limitations of SIA models in these highly dynamical regions have prompted the development of more sophisticated methods that~~ More sophisticated methods have been designed to overcome ~~these limitations , while still aiming~~ the limitations of SIA models when reproducing the dynamics of ice streams, which still aim at low computational costs. The approaches used by this new generation of continental-scale ice-sheet models include depth-integrated Blatter–Pattyn models (Blatter, 1995; Pattyn, 2003) ba on the asymptotic analysis by Schoof and Hindmarsh (2010), algorithms that detect and apply the SIA where it is valid and use the FS elsewhere (Ahlkrona et al., 2015), and so-called hybrid models utilising heuristic combinations of the SIA and the Shelfy Stream Approximation (SStA, which is the SSA including basal drag) (Bueler and Brown, 2009; Winkelmann et al., 2011; Pollard and DeConto, 2012a). These hybrid models ~~detect~~ utilise a number of algorithms to identify zones of potential fast flow where ice streams may operate and heuristically weight contributions of each approximation based on predefined criteria. The use of hybrid models enables simulations over hundreds of thousands of years on continental scales, yet showing a reasonable performance ~~versus~~ compared to higher-order models in idealised scenarios and inter-comparison tests (e.g., Pattyn et al., 2013; Feldmann et al., 2014). Since the ~~superposition~~ combination of the SIA and the SStA is based on heuristics, the approaches used to combine the two approximations vary from model to model, ranging from weighted averages of both velocity solutions to a simple summation over the entire domain. ~~Since in most cases the resulting velocity fields will be different, the possibility of having two different models yielding similar results under similar forcing could be attributed to other factors,~~

~~such as the selection of additional model parameters, model calibration and initialisation, and a range of model-dependent biases.~~

Despite the above differences among existing models, all of them are subject to common limitations when applied to ~~realistic scenarios. These result from~~ the present-day Antarctic Ice Sheet (AIS). These limitations include the scarcity of observational data needed to reduce the errors introduced by poorly constrained model parameters ~~. Mechanical properties of the subglacial bedrock surface and the associated distribution of zones where sliding can be enhanced by soft-bed conditions may serve as an example of such poorly constrained parameters. This~~ and boundary conditions, e.g., the flow enhancement factors introduced to account for the anisotropy of ice flow, geothermal heat flux, the glacial isostatic adjustment, and the distribution of water-saturated sediments at the ice sheet base. The latter has the potential to enhance basal sliding and is currently considered to be a major source of large, widespread misfits ~~(several hundreds of metres)~~ between the observed and modelled elevations of the ~~Antarctic Ice Sheet (AIS)~~AIS (e.g. de Boer et al., 2015) . Recent studies have attempted to quantify potential distributions of these intrinsic bed properties using sophisticated inverse methods (e.g., Joughin et al., 2009; Morlighem et al., 2010; Arthern and Gudmundsson, 2010; Pralong and Gudmundsson, 2011; Arthern et al., 2015). These diagnostic methods focus mainly on the fit between the modelled and observed ice velocities. Pollard and DeConto (2012b) presented a much simpler algorithm, aiming to fit the observed surface elevations instead of velocities. The prognostic model is run forward in time, and the local elevation error is used to periodically adjust the basal sliding parameters until the best fit between the observed and modelled elevations is attained. This procedure has the ability to drastically reduce large elevation errors during the calibration and initialisation of ice-sheet models, which is an important requirement for ~~paleo-simulations~~ simulations that would otherwise be undermined by poor parameter choices.

In this paper, we evaluate the performance of four different hybrid approaches implemented as part of the same continental-scale ice-sheet model, applied to the entire AIS. To this end, a calibration procedure based on the aforementioned iterative technique of Pollard and DeConto (2012b) is applied to each hybrid scheme using ~~state-of-the-art observational data sets as constraints~~the observed ice thickness (Fretwell et al., 2013) . For comparison purposes, the same procedure is carried out using a SIA-only model. The results of these experiments are validated against an independent, observational data set of surface ice velocities (Rignot et al., 2011) . Additionally, we ~~shed light onto the ways in which~~ provide insights into the relative contributions of the shallow approximations ~~are derived by~~ in different hybrid schemes. ~~Discrepancies between~~ By exchanging the inferred distributions of basal sliding parameters ~~are used to quantify the effects of the high uncertainty in basal mechanical properties~~between the applied hybrid schemes, we test the applicability of the model calibration results in different types of ice flow models. For hybrid approaches involving adjustable parameters, we also explore the sensitivity of the results to parameter variations.

~~This work is structured as follows.~~ First, the ice-sheet model and the hybrid schemes are described in Section 2, where we also detail the adapted ~~inverse~~ iterative technique for the calibration of the basal sliding parameters. The ~~setup of the~~ observational and model-based data sets used in our simulations are described in Section 3. The set-up of the numerical experiments can be found in Section 4. The results are presented and discussed in Section 5, followed by the summary and conclusions provided in Section 6.

2 Methods

2.1 Model overview

In this study, the simulations of the AIS are carried out using the open source, three-dimensional, thermo-mechanical ice sheet-shelf model SICOPOLIS (SImulation COde for POLythermal Ice Sheets) version 3.2-dev, revision 619 (Greve, 1997; Greve and Blatter, 2009; Sato and Greve, 2012). It uses finite differences to solve the numerically approximated SIA and SSA equations for grounded and floating ice, respectively. Relevant modifications introduced in the model specifically for this study are presented in a greater detail in Sections 2.2 and 2.3.

~~The model considers ice sheetsas polythermal~~SICOPOLIS is able to model polythermal ice sheets, i.e., ~~explicitly keeps track of~~ it explicitly identifies potential temperate regions in which ~~ice~~ the modelled ice temperature is at the pressure-melting point (Greve, 1997). Within these regions, ice and small amounts of liquid water can coexist, and the water content is used as an additional input for a ~~regularized~~ regularised Glen's flow law (Glen, 1955) utilised in our experiments, following Greve and Blatter (2009), in the form

$$\eta(T_\mathrm{m},\sigma_\mathrm{e}) = \frac{1}{2EA(\sigma_\mathrm{e}^{n-1}+\sigma_0^{n-1})}, \qquad (1)$$

where η denotes ice shear viscosity, T_m is the temperature ~~below~~ difference relative to the pressure-melting point, σ_e is the effective shear stress, $\sigma_0 = 10\,\mathrm{kPa}$ is a small constant used to prevent singularities when σ_e is very small, $n = 3$ is the flow law exponent, and A is a temperature- and pressure-dependent rate factor (Cuffey and Patterson, 2010). In ~~the~~ temperate ice regions, A is modified to account for the aforementioned water content, following Lliboutry and Duval (1985). The empirical coefficient E is a flow enhancement factor, which is introduced to account for the effects of anisotropic ice fabric. The value of E depends on the deformation regime of ice, ~~thus depending on whether it is being applied on~~ which is different for grounded ice, where horizontal shear prevails, ~~or~~ and floating ice, dominated by longitudinal stretching (Ma et al., 2010). In general, large-scale marine ice-sheet models use a homogeneous, constant value ranging between 1 and 8 for the grounded ice, and between 0.2 and 1 for ice shelves (e.g., de Boer et al., 2015). Within these ranges, the computed age of ~~the~~ ice is often used to assign different values for glacial and interglacial ice. Here, we use $E = 1$ and $E = 0.5$ for grounded and floating ice, respectively. These values are smaller than those chosen in previous studies using SICOPOLIS (e.g., Sato and Greve, 2012), and are based on our initial tests and the sensitivity analysis by Pollard and DeConto (2012b).

At the base of the grounded ice sectors, bedrock stress conditions and the associated potential for sliding are linked to the basal ~~SIA~~ velocity, $\boldsymbol{u}_b$, used as a boundary condition for the computation of the SIA velocities, through an empirical Weertman-type sliding law (Weertman, 1964; Dunse et al., 2011), in the form

$$\boldsymbol{u}_\mathrm{b} = -\frac{C_\mathrm{b}}{N_\mathrm{b}^q}|\boldsymbol{\tau}_\mathrm{b}|^{p-1}\boldsymbol{\tau}_\mathrm{b}, \qquad (2)$$

where $\boldsymbol{\tau}_\mathrm{b}$ is the basal shear stress, and $p = 3$ and $q = 2$ are the sliding law exponents. N_b is the effective basal pressure, ~~and $p = 3$ and $q = 2$ are the sliding law exponents~~computed as

$$N_\mathrm{b} = \rho_\mathrm{ice} g H - \rho_\mathrm{sw} g H_\mathrm{sw}, \qquad (3)$$

where $\rho_{\rm ice}$ and $\rho_{\rm sw}$ are the density of ice and sea water, respectively, g is the gravitational acceleration, H is the modelled ice thickness, and $H_{\rm sw}$ is the difference between the mean sea level and the ice base topography. The parameter $C_{\rm b}$ depends on the basal temperature and pressure conditions:

$$C_{\rm b} = C_0 e^{T_{\rm m}/\gamma}, \tag{4}$$

where the exponential function controls the amount of sliding, depending on the temperature below the pressure-melting point $T_{\rm m}$ and the sub-melt-sliding parameter γ, ensuring a smooth transition across different basal thermal ~~conditions~~regimes and thus, prohibiting discontinuities in the velocity field (Bueler and Brown, 2009) . A spatially varying factor C_0 is introduced to account for differences in the bedrock material properties affecting sliding (e.g. hard bedrock vs.~~water-saturated~~ soft sediments). Potential distributions of C_0 have been explored using different iterative and inverse approaches and a variety of sliding laws, aiming to find spatially varying values that minimise the ~~discrepancies~~ discrepancy between the modelled and observed quantities such as ice thickness, ice surface velocity and elevation change (e.g., Joughin et al., 2009; Morlighem et al., 2010; Arthern and Gudmundsson, 2010; Pralong and Gudmundsson, 2011; Pollard and DeConto, 2012b; Arthern et al., 2015)~~)~~. A particular ~~inverse~~ iterative technique implemented in our model is described in Section 2.3. Other model components include evolution equations for ice temperature and ice thickness, with the latter forced by independent modules for the computation of the surface ~~mass balance, basal melt, and changes in bedrock elevation~~ and basal mass balances (Greve and Blatter, 2009; Sato and Greve, 2012). A summary of the model parameters used in this study is provided in Table 1.

2.2 Hybrid schemes

For this study, four hybrid approaches have been implemented into the model. Each of them offers a different way to ~~detect~~ identify the fast flowing zones and combine the horizontal SIA velocity, $\boldsymbol{u}$, and the horizontal SStA velocity, $\boldsymbol{v}$. For each scheme, individual velocity solutions from ~~each shallow approximation~~ the shallow approximations are calculated independently. It is important to note that SStA velocities in grounded ice regions include basal drag. This implies a major difference from SSA velocities computed in the floating ice shelf sectors, for which the friction at the ice-ocean interface is negligible.

For consistency, the basal drag term that enters the SStA equations is computed using the same sliding law as described in Section 2.1 (Eq. (2)), in the form

$$\boldsymbol{\tau}_{\rm b} = -\beta_{\rm drag}\boldsymbol{v}_{\rm b}, \tag{5}$$

where $\boldsymbol{v}_{\rm b}$ is the the basal SStA velocity and the drag coefficient, $\beta_{\rm drag}$, is computed as

$$\beta_{\rm drag} = \frac{N_{\rm b}^{\frac{q}{p}}}{C_{\rm b}^{\frac{1}{p}}} \left(\frac{1}{\sqrt{v_{\rm b_x}^2 + v_{\rm b_y}^2 + v_0^2}} \right)^{1-\frac{1}{p}}. \tag{6}$$

Here, $v_{\rm b_x}$ and $v_{\rm b_y}$ are the horizontal components of the SStA velocity at the ice base, and $v_0 = 0.01\,\mathrm{m\,a^{-1}}$ is a small regularisation quantity introduced to prevent singularities at the locations where there is no basal sliding (Bueler and Brown, 2009). ~~Equation allows for a consistent use of the inverted distributions of C_0 during the computation and combination of the SIA and SStA velocities in the grounded ice zones (Section 2.3).~~

The first hybrid scheme (henceforth HS-1) is the original implementation in SICOPOLIS v3.2-dev (revision 619) based on the slip ratio of grounded ice, computed as

$$r = \frac{|\boldsymbol{u}_\mathrm{b}|}{|\boldsymbol{u}_\mathrm{s}|}, \tag{7}$$

where $\boldsymbol{u}_\mathrm{b}$ is the Weertman sliding velocity (Eq. (2)) and $\boldsymbol{u}_\mathrm{s}$ are the basal and surface SIA velocities, respectivelyis the surface SIA velocity. At each iteration, and for each velocity component, the local slip ratio r is compared to a prescribed threshold r_thr ranging from 0 to 1 (Section 4). If r is larger than the threshold, that the grid point is flagged as a streaming point and enters the computation of SStA velocities streaming ice where the SStA velocities should be computed. Once SStA velocities are computed, the individual contributions from of the SIA and SStA velocities at each streaming grid point are determined using the weight

$$w(r) = \frac{r - r_\mathrm{thr}}{1 - r_\mathrm{thr}}, \tag{8}$$

which is assigned to the SStA. Then, for each streaming grid point, the hybrid horizontal velocity $\boldsymbol{U}$ is computed as

$$\boldsymbol{U} = w \cdot \boldsymbol{v} + (1 - w) \cdot \boldsymbol{u}, \tag{9}$$

recalling that $\boldsymbol{u}$ and $\boldsymbol{v}$ are the horizontal SIA and SStA velocities, respectively.

The second approach (henceforth HS-2) is based on the idea by Bueler and Brown (2009), in which SStA velocities are calculated over the entire ice sheet and used as a sliding velocity complementing a non-sliding SIA model. Velocities are computed SIA and SStA velocities are combined as in Eq. (9), and the weighting function is adopted from Bueler and Brown (2009, Eq. (22))

$$w(|\boldsymbol{v}|) = \frac{2}{\pi}\arctan\left(\frac{|\boldsymbol{v}|^2}{v_\mathrm{ref}^2}\right), \tag{10}$$

where v_ref is a reference ice velocity (Section 4). The velocity v_ref marks the point for which the SIA and SStA contributions are equally weighted, i.e., the resulting hybrid velocity is a standard mean of both solutions. The weighting function is smooth, monotone, and its value converges towards 0 for small velocities and towards 1 when $\boldsymbol{v}$ is large compared to the reference velocity v_ref. As in the HS-1, w is used to compute respective contributions from the SIA and SStA, with the difference that Eq. (10) uses v_ref as the only criterion to determine the SStA contribution. The SStA velocities are calculated over the entire ice sheet, and an a priori identification of fast flow zones is not required.

As described in Section 2.1, the SIA solution in SICOPOLIS already contains a sliding component coming from is computed using the Weertman sliding law (Eq. (2)) as a boundary condition. To assess the influence of a sliding SIA SIA solution including the Weertman sliding, we have divided split this hybrid scheme into two: A non-sliding-SIA sub-scheme (HS-2a) that replicates the idea of Bueler and Brown (2009) with no basal velocity prescribed in the computation of the SIA, and a sliding-SIA sub-scheme (HS-2b) that keeps the sliding-SIA has a Weertman sliding component and uses it to compute a slightly modified weight:

$$w(|\boldsymbol{u}_\mathrm{b}|) = \frac{2}{\pi}\arctan\left(\frac{|\boldsymbol{u}_\mathrm{b}|^2}{v_\mathrm{ref}^2}\right), \tag{11}$$

where $\boldsymbol{u}_{\mathrm{b}}$ is the basal SIA sliding velocity as in Eq. (2). Thus, in the HS-2b the SStA solution is not does not serve as a replacement of a sliding law. The latter It is rather used to determine when and where the corresponding SStA contribution should partly or completely replace itto which extent the computed SStA velocity should replace the basal velocity used to compute the SIA solution.

The third approach (henceforth HS-3) simply adds up the non-sliding SIA and SStA solutions:

$$\boldsymbol{U} = \boldsymbol{u} + \boldsymbol{v} \tag{12}$$

This superposition of approximations has been employed in recent studies using non-sliding SIA models complemented SIA models in combination with a SStA solution as a sliding law (e.g., Winkelmann et al., 2011). It bypasses the need for additional free parameters, such as r_{thr} and v_{ref} in the HS-1 and HS-2, respectively. This approach is based on the assumption that on ice shelves the SIA contribution is negligible due to low surface gradients, and therefore the modelled ice flow is dominated by the SStA solution. In , whereas in the continental interior , the sectors of the ice sheet where basal drag is high enough for vertical shear to dominate are controlled the modelled ice flow is dominated by the SIA solution (Winkelmann et al., 2011). Since the SIA and SStA solutions are computed over the entire domain, their superposition enables a smooth transition across different flow regimes, ranging from slow ice motion in the interior to a characteristic fast flow of ice shelves, thereby allowing for stress transmission across the grounding line. As in the HS-2, an identification of fast flowing zones is purely diagnostic and not required during the computation of $\boldsymbol{U}$. Table 2 presents a summary of the hybrid schemes applied in this study.

2.3 Inversion Calibration of basal sliding coefficients

We have implemented an iterative method following Pollard and DeConto (2012b) in order to infer determine the distribution of sliding coefficients C_0 that minimises the difference between the modelled and observed ice thickness. The method starts from a spatially uniform guess value for the distribution of C_0 and runs the model forward in time, as described in Section 4. At a given time step, Δt_{inv}, the method uses the basal temperature below the pressure-melting point, T_{m}, to identify grounded grid points where basal sliding may occur. If the absolute value of T_{m} is smaller than the parameter γ from Eq. (4), i.e., close to the pressure-melting point, the method computes the difference between the modelled and observed ice thickness, which is then used to locally adjust C_0 at this grid point according to

$$C_0^* = C_0 10^{\Delta H}, \tag{13}$$

where C_0^* is an updated sliding coefficient and $\Delta H = (H - H_{\mathrm{obs}})/H_{\mathrm{inv}}$. Here, ΔH is the difference between the modelled and observed ice thickness, scaled by a factor, H_{inv}, in order to prevent overshoots. For the same reason and following the implementation by Pollard and DeConto (2012b), variations in the value of the multiplicative factor $10^{\Delta H}$ are further limited by a range of $\sim .03$ to 30. The parameter γ is set to 3 K, in In contrast to previous studies using SICOPOLIS where $\gamma = 1\,\mathrm{K}$. Based on our initial tests, this choice provides an easier activation of the inversion procedure, slightly improving the results, here we set this parameter to a value of 3 K, allowing for a more frequent calibration of the sliding coefficients (Pollard and DeConto, 2012b) .

Studies using this iterative technique and other independent inversion methods have shown that potential distributions of sliding coefficients C_0 are highly heterogeneous, with values spanning several orders of magnitude (e.g., Pollard and DeConto, 2012b; Arthern et al., 2015). Similarly, and to To ensure numerical stability, we limit our inferred values to a range of 1 to $10^5\,\mathrm{m\,yr^{-1}\,Pa^{-1}}$ during the calibration procedure. Additionally, we have implemented the following condition: When the computed surface ice speed velocity reaches an ancillary speed limit at a certain grid point, the local adjustment adjustment of C_0 for that point is halted. This keeps the inversion from over-adjustment of the prevents the method from over-adjusting the sliding coefficients when the speed velocity limit has been reached and no noticeable changes occur in response to further adjustments of C_0. This additional constraint is applied in order to ensure the numerical stability and keep the modelled ice velocities within the range of observations. For the experiments presented in this study, the lower speed limit is defined as $0.1\,\mathrm{m\,a^{-1}}$, whereas the upper limit is set to $4000\,\mathrm{m\,a^{-1}}$. This These values are based on the observed surface velocities of Rignot et al. (2011).

Our adaptation of the The iterative technique involves an additional limiting condition that takes into account temporal variations in the ice thickness. In each prevents over-adjustments of C_0. For each individual grid point, if the adjustment implemented at the previous time step reduces the difference between the modelled and observed ice thickness , further adjustments are suspended. The process is reactivated when reduced at the previous time step, the adjustment at the current time step is deactivated. This allows previous adjustments to fully develop their effects over the following time steps and prevents the technique from adding unnecessary extra adjustments that often result in overshoots. The calibration is activated again as soon as the time derivative of the ice thickness becomes zero modelled ice thickness drops to zero (i.e., the difference between the modelled and observed ice thickness is not reduced anymore) or the misfit starts increasing (e.g. due to increased influx from surrounding areas). Our experiments have shown that this additional feature improves the overshoot prevention and enables the use of a smaller Δt_{inv} (50 years used here compared to 500–10,000 years in Pollard and DeConto (2012b)), because further adjustments will only be applied when and where strictly necessary. A further benefit is that it indirectly lets allows non-local adjustments of C_0 to influence the local ice dynamics: If an adjustment applied at the previous time step in the vicinity of a given grid point causes the local misfitto decrease, further local adjustments grid point reduces the misfit, further adjustments at this grid point will still be halted, and vice versa.

3 Experimental setupData sets

The modified version of SICOPOLIS described in Section 2 is applied to the entire AIS and fringing ice shelves. The experiments performed during the calibration of the ice sheet-shelf system have their aim to quantify the differences and similarities between the hybrid approaches. Default values for the parameters controlling the hybrid schemes are $r_{\mathrm{thr}} = 0.5$ (HS-1), and $v_{\mathrm{ref}} = 100\,\mathrm{m\,a^{-1}}$ (HS-2a and HS-2b). As mentioned in Section 2.2, the HS-3 does not include any free parameters. Additionally, and for comparison purposes, the same experiments are performed using a SIA-only scheme (henceforth SoS).

The calibration procedure takes an advantage of the improving improved quality of the modern, continental-scale Antarctic data sets, such as climatic forcing (Van Wessem et al., 2014) , topography (Fretwell et al., 2013) , and surface velocities

~~(Rignot et al., 2011)~~ and topography (Fretwell et al., 2013) . The forcing data serve as time-invariant boundary conditions for our ~~model simulations, which run until a thermal and dynamic quasi-equilibrium is reached~~equilibrium (steady-state) model simulations. It should be noted that the modern AIS is ~~not necessarily in~~ unlikely in a steady state, and that a transient simulation of, e.g., the entire last glacial cycle would provide a more realistic scenario for the calibration procedure. However, existing reconstructions of the Antarctic paleo-climate and past ice sheet configurations still contain large uncertainties, with in-situ data being ~~scattered in both~~ sparse in space and time. Keeping this in mind, we believe that using modern data sets and assuming equilibrium conditions is a valuable first-order approximation to a more complex model calibration. Furthermore, such equilibrium ~~setup~~ set-up can serve as an initial guess for transient deglaciation simulations, which include time-dependent processes not considered here (Fyke et al., 2014) .

Initial modern conditions for surface topography, ice shelf thickness, and bedrock elevations relative to the present-day sea level are derived from the BEDMAP2 data set (Fretwell et al., 2013). BEDMAP2 is a compilation of 24.8 million ice thickness data points obtained from a variety of sources including airborne and over-snow radar surveys, satellite altimetry, seismic sounding data, and satellite gravimetry (Fretwell et al., 2013) . This compilation is complemented by surface elevation data from several digital elevation models to derive previously unknown bedrock features, and allows for a detailed modelling of the AIS. Figure 1 shows the bedrock topography data from BEDMAP2, together with the locations mentioned in the text. The main sources of uncertainty in the ice thickness and bedrock elevation maps are the errors in surface digital elevation models and ice thickness measurements, as well as the applied regridding, which produce overall uncertainties ranging from 59 m across areas with smooth landscapes to 1000 m in regions where only gravimetric data are available (e.g., south of Coats Land).

At the base of the thermal bedrock, geothermal heat flux is prescribed according to the ~~study~~ map of Shapiro and Ritzwoller (2004). This map is derived from a global seismic model of the upper mantle and the crust that assuming a relation between seismic velocities and mantle temperatures and uses observations from regions with similar structures to infer heat-flow probability distributions where such observations are scarce or non-existent. In the inferred heat-flux map West Antarctica is characterised by an average heat flow that is nearly three times higher than in East Antarctica. Although the resulting map depends on the accuracy of available observations as well as on the choice of the seismic model and similarity functional, the inferred distributions are robust to internal parameter changes, especially for continental areas such as Antarctica (Shapiro and Ritzwoller, 2 However, the method is not able to reproduce small-scale patterns caused by local variations in crustal heat production, which may exert an important control on the dynamics of the Antarctic ice streams.

Boundary conditions at the surface include ~~observational and model-based~~ Antarctic accumulation rates and near-surface air temperature data from the regional climate model ~~RACMO (Lenaerts et al., 2012; Van Wessem et al., 2014)~~ RACMO2.3 (Van Wessem et al., 2014) , averaged over the period of 1979 to 2010. RACMO is forced at its boundaries by the reanalysis data from ERA-Interim (Dee et al., 2013) . In the interior of the domain the Antarctic climate conditions are modelled with a horizontal resolution of 27 km and 40 levels in the vertical direction. RACMO contains modules that are specifically developed for glaciated regions, including a multilayer snow model. The model compares well with 3234 in situ observations of surface mass balance used for its validation, displaying a correlation of $r^2 = 0.77$ and a particularly good fit for the dry East Antarctic plateau (Van Wessem et al., 2014) .

A simple lapse-rate correction of $0.008\,^{\circ}\mathrm{C\,m^{-1}}$ accounts for changes in surface elevationis used to account for the discrepancies between the modelled and observed surface elevations. Surface melt is computed by with a positive degree-day (PDD) scheme (Reeh, 1991; Calov and Greve, 2005) using the factors $\alpha_{\mathrm{ice}} = 8\,\mathrm{mm\,d^{-1}\,^{\circ}C^{-1}}$ and $\alpha_{\mathrm{snow}} = 3\,\mathrm{mm\,d^{-1}\,^{\circ}C^{-1}}$ (ice equivalent) for ice and snow, respectively (Ritz et al., 2001), and a standard deviation of $\alpha_{\mathrm{std}} = 5\,^{\circ}\mathrm{C}$ for the statistical fluctuations of in air temperature.

All input fields are projected onto a regular, rectangular, polar stereographic grid covering the entire Antarctic continent and the surrounding Southern Ocean, with a nominal horizontal resolution of $20\,\mathrm{km}$, corresponding to 301×301 grid points. Our choice of resolution has been motivated by the large number of experiments presented in Section 5, each spanning $400,000$ years, and the fact that Pollard and DeConto (2012b) have shown that the results remain essentially unchanged when the horizontal resolution is increased to $10\,\mathrm{km}$ (in a nested simulation), even in rapidly flowing sectors. However, our tests using an increasingly higher resolution have identified areas where the modelled ice flow is more sensitive to the resolution used. These mainly occur close to the ice sheet margins where the grounding zone of a glacier is often represented by only one grid cell at lower resolution. In such regions the use of a finer grid allows for a more detailed treatment of the topographically constrained glacial flow. In the context of this comparison study, however, these limitations equally affect the performance of all hybrid schemes and do not impact our conclusions. In the vertical direction, ice columns consist of 91 layers (11 equidistant grid points for temperate ice and 81 grid points for "cold" ice densifying towards the base, sharing the grid point at their interface), mapped to a $[0,1]$ interval using a σ transformation (Greve and Blatter, 2009) .

4 Experimental setup

The modified version of SICOPOLIS described in Section 2 is applied to the entire AIS and the fringing ice shelves. The experiments performed during the calibration of the ice sheet-shelf system aim at quantifying the differences and similarities between the hybrid approaches. Default values for the parameters of the hybrid schemes are $r_{\mathrm{thr}} = 0.5$ (HS-1), and $v_{\mathrm{ref}} = 100\,\mathrm{m\,a^{-1}}$ (HS-2a and HS-2b). As mentioned in Section 2.2, the HS-3 does not include any free parameters. Additionally, and for comparison purposes, the same experiments are performed using a SIA-only scheme for grounded ice (henceforth SoS).

The bulk of the calibration consists of the inversion technique for an iterative technique used to derive the distribution of sliding coefficients C_0 (Section 2.3) that exerts a dominant control on the resulting ice distribution and its fit to the observational data. The inversion observations. The calibration procedure starts from a homogeneous guess value of $C_0 = 1\,\mathrm{m\,yr^{-1}\,Pa^{-1}}$, which is equal to the lower limit of the considered range (see Section 2.3). The time step between adjustments and the scaling factor in Eq. (13) are set to $\Delta t_{\mathrm{inv}} = 50$ years and $H_{\mathrm{inv}} = 5000$ metres, respectively. We follow the method by Pollard and DeConto (2012b) and allow for a free evolution of both the ice sheet and ice shelf thickness, but their interface (the grounding line) is kept at its present-day observed position. Free evolution is needed because the inversion calibration of the sliding coefficients requires an evolving ice thickness that will be routinely compared to observations. The reasons for a constrained grounding line are twofold: ~~Such~~ This approach 1) ~~ensures a one-to-one~~ enables a comparison with observations in coastal regions during the calibration and 2) prevents artificial transitions between grounded and floating areas caused by equally artificial effects of ~~the~~ unrealistic initial thermal regimes and C_0 distributions that evolve from ~~the~~ initial guess values. Our tests show that such artefacts can produce feedbacks that are difficult or impossible to reverse. For the same reason, glacial isostatic adjustment is not ~~accounted for~~included in the simulations, and ice shelf fronts are constrained to the observed locations. As mentioned above, the ice shelf thickness is allowed to evolve, but basal melt rates are adjusted at each time step in order to keep the modelled ice shelves as close as possible to observations. This ensures a consistent computation of mass fluxes across all flow regimes, and does not overlap with the ~~inversion~~ calibration of sliding coefficients, since these are not applied in floating ice sectors.

In this study, the calibration procedure is divided into ~~two steps. First~~four steps. In the first three steps, a relaxation scheme is applied to the evolution of the modelled ice thickness, H. Here, the difference between the current solution of the ice thickness equations, H_{new}, and the solution from the previous time step, H_{old}, is scaled at every time step by a factor h_{rlx} ranging between 0 and 1, as ~~following~~follows:

$$H = H_{\mathrm{old}} + h_{\mathrm{rlx}}(H_{\mathrm{new}} - H_{\mathrm{old}}). \tag{14}$$

For a time-invariant forcing, our tests have shown that different values of h_{rlx} will result in very similar equilibrium states. The relaxation simply delays the time point at which this state is reached. However, such relaxation procedure allows for bigger time steps ~~for the computation of~~ at which the topography evolution is computed, without affecting the internal temperature evolution. In this way, an equilibrium with the boundary conditions will be reached faster by the temperature field than by the topography. This effectively minimises transient effects in the closely associated ice thickness and velocity fields, especially at the beginning of the ~~simulations~~ calibration runs when model parameters follow the initial guess values. More importantly, it allows us to simultaneously apply the ~~inverse~~ iterative calibration technique described in Section 2.3, in contrast to approaches in which the topography is fixed (e.g., Sato and Greve, 2012) . This ensures that the modelled ice thickness distribution ~~stays as close as possible~~ has the closest possible match to the initial observed distribution throughout the initial calibration stages. The value of the relaxation factor is set to $h_{\mathrm{rlx}} = 0.001$ ~~throughout this~~ in the first stage of the calibration driven over a simulation time of ~~100 thousands of years (kyr~~100,000 years (100 kyr), using a time step of ~~10~~ 5 years. The ~~second stage of the calibration is activated once the thermal equilibrium has been reached. It deactivates the relaxation~~ following second and third stages replicate the first one, but with larger scaling factors of $h_{\mathrm{rlx}} = 0.01$ and ~~applies~~ $h_{\mathrm{rlx}} = 0.1$, respectively, for an additional simulation time of 100 kyr in each stage. In the final stage, the relaxation is deactivated and a smaller time step of 1 year ~~over a~~ is applied over an additional simulation time of ~~200 kyr~~100 kyr, which provides a ~~time-span long enough~~ long enough time span to attain a dynamic equilibrium.

~~All input fields are projected onto a regular, rectangular, polar stereographic grid covering the entire Antarctic continent and the surrounding Southern Ocean, with a nominal horizontal resolution of 40 km, corresponding to 151 × 151 grid points. Our choice of a rather coarse resolution has been motivated by the large number of experiments presented in Section 5, spanning hundreds of thousands of years, and the fact that Pollard and DeConto (2012b) have shown that the results remain essentially~~

unchanged when the horizontal resolution is increased to 20 and 10 km (the latter in a regional, nested simulation), even in rapidly flowing sectors. Moreover, fixing the grounding line at its observed position prevents unrealistic migrations that would otherwise arise from the use of low resolution. However, our tests using an increasingly higher resolution have identified areas where the modelled ice flow is more sensitive to the resolution used. These mainly occur close to the ice sheet margins where a glacier flux gate is often represented by only one grid cell at low resolution. In such regions the use of a finer grid allows for a more detailed treatment of the topographically constrained glacial flow. In the context of this comparison study, however, these limitations equally affect the performance of all hybrid schemes and do not impact our conclusions. In the vertical direction, ice columns consist of 91 layers (11 equidistant grid points for temperate ice and 81 grid points for "cold" ice densifying towards the base, which overlap at their interface), mapped to a 0, 1interval using a sigma transformation (Greve and Blatter, 2009) .

5 Results and discussion

In this chapter we present an ensemble of simulations of the AIS that aim to comprehensively evaluate and get insight into different hybrid schemes combining the SIA and SStA. Our evaluation uses the degree of agreement between the modelled and observed ice sheet geometries and flow patterns surface velocities as a measure of their performance, allowing for a point-by-point quantification of the model errors.

Keeping in mind that each hybrid scheme builds upon the SIA solution, partially or entirely replacing it at variable locations with the SStA solution, we have also included the results from the SIA-only scheme in our comparison. In particular, this enables a qualitative separation of relative contributions of the SIA and SStA, providing a new insight into the internal differences between the schemes and their applicability to ice sheet areas with diverse dynamical characteristics.

By applying an automated model calibration against the observed ice thickness to each of the hybrid schemes we infer spatial distributions of poorly constrained sliding coefficients as a proxy for mechanical conditions at the ice-bedrock interface and assess their sensitivity to the choice of a particular hybrid scheme. In addition, the influence of variations in parameters controlling the internal operation of the hybrid schemes the parameters controlling relative contributions of SIA and SStA velocities in each scheme is assessed for a wide range of parameter values.

5.1 Comparison of equilibrium states

As described in Section 43, our experiments use the BEDMAP2 observational data set (Fretwell et al., 2013) as an initial ice sheet configuration, running the model forward in time under a relaxation scheme (Eq. (14)) until the temperature distribution within the ice sheet reaches a quasi-equilibriuman equilibrium state. After this initialisation, the relaxation is deactivated and the model runs under an automated calibration procedure (Section 2.3) until a full thermal and dynamic equilibrium is attained. This equilibria are attained. The equilibrium is defined as the point in time in which the variation of variations of the total grounded ice volume becomes negligibleover a prolonged time ($> 10,000$ years) are smaller than 0.01%.

Starting from the initialised states, the time required to reach an equilibrium varies from scheme to scheme (Fig. 2b). The HS-1attains , HS-2a and HS-3 attain a virtually invariable state after only 50 kyr20 kyr, as opposed to the HS-2a that requires about

~~150 kyr. The length of calibration for other schemes falls within this range~~SoS and HS-2b that display significant oscillations in the total volume around a mean equilibrium value even after a period of 100 kyr, suggesting unstable equilibrium states. Compared to the SoS, the computation of SStA velocities in the grounded ice sectors ~~implies~~ requires an extra computational effort for each iteration in the numerical solvers, with the computing time increasing by a factor of ~ 4 for the applied hybrid schemes. The characteristic computing time of the HS-1 is somewhat shorter than that of the other hybrid schemes (but still longer than for the SoS) due to the prognostic identification of ice streams that ~~prevents~~ obviates the need for the computation of SStA velocities over the entire ice sheet. However, we have observed that the iterative solvers in the model require a substantially smaller number of iterations when the hybrid schemes are used, making them numerically more stable compared to the SoS.

The calibration procedure applied to all schemes yields total grounded ice volumes which are in a close agreement with the reference value of ~~$2.55\times10^{7}\,\mathrm{km}^3$~~ $\sim 2.56\times10^{7}\,\mathrm{km}^3$ from the observational data (Fretwell et al., 2013), with the maximum deviation being below ~~2 %~~1.5 %. Individual values of the modelled ice volumes and their respective deviations for most of our experiments are summarised in Table 3. The best fit to observations is obtained using the HS-2b, which corresponds to an ~~underestimation~~ overestimation of the total grounded ice volume by ~~less than 0.3 %~~0.1 %. The other schemes produce relatively larger misfits, with the HS-2a simulation yielding the greatest deviation of ~~1.88 %~~ 1.46 % arising from an overestimation of the total ice volume. ~~It can be observed that the schemes that include basal sliding in the computation of the SIA solution (HS-1, HS-2b, and SoS) tend to produce smaller ice sheets than those using the SStA as a sliding law (HS-2a and HS-3).~~ The smallest ice sheet is produced by the SoS that underestimates the total grounded ice volume by ~~1.55 %. The evolution of the total grounded ice volume depicted in Fig. 2b also shows different types of signals: The SoS and HS-1 solutions contain high-frequency variations even after an equilibrium is reached. These are contrasted by the HS-2b solution that displays low-frequency oscillations and the smooth, noise-free curves obtained from the HS-2a and HS-3 schemes.~~

0.72 %.

The inset of Figure 2 ~~a shows~~ shows the total ice thickness errors for all schemes. The errors are computed as an average of the absolute ~~values of the misfit~~ misfit values in all grounded grid points and remain below ~~60~~ 50 metres for all hybrid schemes (Table 3). Among these, the largest error of ~~59.8 m~~ 46.6 m is produced by the HS-2a, while the smallest error of ~~49.9 m~~ 40.0 m is obtained using the HS-2b. For the former, an 83 % of the misfit is due to overestimations of ice thickness, while the latter shows ~~a nearly even split between~~ nearly even contributions from under- and overestimations. This is in accordance with the results shown in Fig. 2~~b~~, where the misfits obtained from the two schemes using the SStA as a sliding law are dominated by an excessive ice thickness. ~~As mentioned above, the opposite is generally not the case for the~~ The hybrid schemes that include a ~~sliding component within the SIA~~Weertman basal sliding component during the computation of SIA velocities are not necessarily dominated by underestimations of the ice thickness. For example, ~~only~~ less than a quarter of the misfit produced by the HS-1 can be attributed to areas with an ice thickness deficit. ~~In~~ For comparison, the total error produced by the SoS is ~~73.1 m, with 72 %~~ 52.4 m, with 65 % of the misfit coming from ~~the underestimation of~~ an underestimation of the ice thickness.

The results depicted in Fig. 2 reflect only a generalised, time-dependent information that can be inferred from each run. In order to have a ~~precise~~ more detailed overview of the results, Fig. 3 (left column) shows the corresponding spatial maps of ice thickness errors. All schemes provide a reasonably good fit to the observational data in the continental interior, with larger discrepancies mainly occurring at the ice sheet marginsmargin. It can be readily observed that the modelled ice sheet is too thick over the mountainous regions and in across the region between the Shakleton Range and the Pensacola Mountains , which according to observations is not characterised by steep bedrock topography gradients(south of Coats Land), which has some of the largest uncertainties in the topographic data (Fretwell et al., 2013) . These are common features for all schemes, independently regardless of the approach chosen for the sliding component. This can be explained by frozen conditions at the ice base, which prevent the initiation of the basal sliding, thereby restricting the influence of the calibration procedure, and could originate from insufficient geothermal flux, too high precipitation rates, and/or an unrealistic smoothness of the bedrock in this area that prevents the formation of topographically driven ice streams. The distribution of zones where the modelled basal temperatures are far from the temperature at below the pressure-melting point are depicted as the white-coloured areas in Fig. 3 (bottom rowright column). In general, these areas coincide with the locations where the largest ice thickness errors occur. On the other hand, underestimations of Areas where the ice thickness are sparsely distributed is underestimated are mainly located at and around the ice margins. In many of these areas, however, sliding is identified and the inversion calibration of C_0 is performedreaches its lower limit, implying that the surface mass input is insufficient to minimise the misfit. The SoS produces too thin ice along most of the ice sheet marginsmargin, which explains the high percentage of the ice thickness error errors arising from underestimations (inset of Fig. 2a). In contrast, the hybrid schemes produce error patterns that differ only slightly between each other. The only exception is the HS-2b, which resembles the patterns from is closest to the SoS, albeit exhibiting smaller underestimations at the ice sheet margins.

Furthermore, we evaluate the performance of the inversion calibration procedure over the areas where it is applied directly. For this purpose we have calculated averaged absolute errors in the ice thickness across regions where basal sliding operates (inset of Fig. 2a). It should be kept in mind that the calibration procedure also affects the ice masses located in the immediate proximity to sliding areas through surface elevation gradients and/or stress transmission. Mean ice thickness errors over the sliding areas are smaller than those estimated for the whole ice sheet, although the degree of relative improvement varies from scheme to scheme (Table 3). For example, the misfit produced by the SoS decreases only by $\sim 10\,\%$ by only $\sim 20\,\%$ if calculated over the sliding ice sheet sectors, while more than 50% 60% of the errors resulting from the HS-2a occur over the areas where no sliding is recovered and C_0 is not invertedcalibrated. For all hybrid schemes, the percentage of the errors associated with an underestimation of the ice thickness increases substantially when their performance is assessed across areas of the retrieved basal sliding, supporting the observation that the modelled ice is excessively thick mainly in the regions where the calibration procedure does not operatein regions where C_0 is not calibrated.

The inferred distributions of C_0 are shown in Fig. 3 (right column). In general, the areas where the inversion calibration is performed are similar for all schemes (Table 3), although there is a significant spread in the inverted values. Conditions favourable for sliding are predicted over more than a half of the ice-covered area (Table 3). retrieved values. The HS-1 scheme predicts a minimum corresponding to 53 % 45 % of the total grounded ice area, contrasted by the corresponding higher percentages for the other schemes ranging between 60 % and 66 %54 % and 62 %. The upper limit of $10^5\,\mathrm{m\,yr^{-1}\,Pa^{-1}}$ for the sliding coefficient is reached by all schemes across up to 5 % 18 % of the ice-covered land, with their highest concentration occurring in the Siple Coast region, where ice streams flow rapidly over a smooth and deformable bed provided by strong lubrication from water saturated subglacial sediments (e.g., Blankenship et al., 1986; Alley et al., 1987; Kamb, 2001). The upper limit is also reached in ~~the~~ Coats, MacRobertson, and Ellsworth Lands (Fig. 1). The C_0 values placed at the lower limit of $1\,\mathrm{m\,yr^{-1}\,Pa^{-1}}$ are also present in the estimates from all runs, with a relatively higher coverage ranging between ~~$\sim 8\,\%$ and $\sim 30\,\%$~~ $\sim 10\,\%$ and $\sim 32\,\%$ of the ice sheet area. The SoS and HS-2b infer this value throughout most of the West AIS and over vast parts of the East AIS, particularly at the ice margins. Its incidence is relatively lower for the other hybrid schemes, especially for the HS-1 and HS-2a. Quantitatively, the good agreement between the inferred coefficient distributions is mainly restricted to the areas where C_0 reaches an upper or lower limit. In other areas the values generally vary across orders of magnitude.

Direct comparison of our results to those from Pollard and DeConto (2012b) is hindered by the differences in the sliding laws and hybrid schemes. Nevertheless, we have ~~also~~ found a good qualitative agreement in the inferred distributions of C_0, with similar patterns of high vs. low values. For instance, both studies identify the Siple Coast and Coats Land regions as areas where the ice streaming flow is driven by basal conditions favourable for sliding. A similar agreement is found for the Thwaites and Pine Island Glacier areas, as well as in the MacRobertson Land (Fig. 1). Low values of C_0 are predicted in the continental interior of West Antarctica and over most of the East AIS. ~~Despite the differences between our model and that of Pollard and DeConto (2012b) we expect that significant modifications to their hybrid scheme would also result in similar perturbations in the inverted values of C_0.~~

5.2 Analysis of the SIA and SStA contributions

As described in Section 2.2, different hybrid approaches are not expected to produce exactly the same equilibrated ice velocity fields. We demonstrate this in Fig. 4, where the modelled steady-state surface velocities derived from the SoS and the hybrid schemes are compared to a continental-scale observational data set (Rignot et al., 2011). This high-resolution (900 m) data set contains many small-scale features that are ~~barely resolved~~ unresolved by the model ~~,~~ due to its lower resolution. Nevertheless, all schemes are able to reproduce the observed range of ice flow regimes, distinguishing between the ice sheet areas with ~~small velocities near ice divides, and high velocity flanks around the ice~~ low velocities near the ice divides and fast flowing ice streams reaching the ice sheet margins. In the transition zones between the continental interior and the ice sheet margins, all schemes reproduce to some extent the fast flowing ice streams identified by observations. However, in contrast with the results from the hybrid schemes, surface ice velocities derived from the SoS are ~~contaminated by~~ characterised with noise-like patterns, which are especially visible in the areas of rapid ice flow. We attribute these artefacts to a combination of lacking stress transmission in the SIA, which allows for steep gradients in the modelled velocities, and the calibration procedure, which can potentially amplify these gradients through local adjustments of C_0.

Although the overall character of the observed surface ice velocities is qualitatively well reproduced by all the hybrid schemes, modelled ice flow is clearly too fast at and around several ice stream locations, such as ~~, for example , in~~ for example at the Siple Coast. Furthermore, modelled surface velocities are generally overestimated ~~at the flux gates~~ close to the grounding lines of most outlet glaciers, in ~~line with~~ particular due to the resolution-related limitations ~~mentioned~~ discussed in Section ~~4~~3. These overestimations are particularly large in the SoS simulation, even though the discrepancies between the modelled and observed ice thickness are small (Fig. 3, left column)~~, owing to the values of C_0 inferred from the calibration procedure (Fig. 3, right column). On the other hand, overestimations~~ . Overestimations of the ice flow velocity by the hybrid schemes at the ~~flux gates~~ grounding zone are smaller than those derived from the SoS simulation, but they cover a wider area and reach further upstream. Similar observations have been made by Pollard and DeConto (2012b) using a different hybrid ice sheet-shelf model and a different set of topographic ice observations and external forcing. They proposed that the ~~inferred~~ overestimation of surface velocities in these areas may be caused by a coarse model resolution, exaggerated snowfall rates or an excessive internal deformation compared to sliding near the ice margins.

In order to quantitatively evaluate the model fit to observations, we have calculated point-by-point ratios between the modelled and observed surface velocities (Fig. 4, right column). As mentioned above, the modelled velocities are in a good agreement with observations in the continental interior characterised by ~~a~~ slow ice motion. In contrast, at the margins the ice ~~flow speed predicted~~ velocity simulated by the hybrid schemes sometimes reaches values that are several hundred times higher than in the observational data set~~, but~~ . Yet, this mostly happens across areas where our model generates a non-existent fast flow. ~~Arguably, one~~ One of the best examples of such model artefacts is the former Ice Stream C in the Siple Coast, which ~~has been stagnated~~ is predicted by the model but has been stagnant for ~ 150 years in reality (Hulbe and Fahnestock, 2007; Engelhardt and Kamb, 2013). In some cases, the locations of the modelled ice streams are shifted relative to the observed ones, thereby generating adjacent zones of under- and overestimations of the surface velocity. In other cases, the modelled rapid ice flow ~~deviates to either side and is pushed to merge with adjacent~~ follows a different route compared to observations, sometimes merging with neighbouring ice streams. These shifts may originate from local ~~deficiencies~~ errors in the bedrock topography data accentuated by its projection onto ~~a~~ the coarse horizontal grid we use here.

The mean errors in the absolute surface ice velocity fall within the range of ~~~ 30 to $\sim 80\,\mathrm{m\,yr^{-1}}$~~ ~ 16 to $\sim 55\,\mathrm{m\,yr^{-1}}$ (Table 3), with the HS-2b and SoS producing the minimum and maximum misfits, respectively, analogously to the results for the mean ice thickness ~~error~~ errors discussed in Section 5.1. In the SoS simulation ~~,~~ a general underestimation of the ice thickness near the ice sheet margins coincides with areas where sliding coefficients tend to reach the lower limit prescribed for the ~~inversion~~calibration. In turn, the use of such low values triggers a slowdown of the ice flow in the ~~modelled surface velocities in the~~ transition zone.

Although the results of the HS-2b simulation presented in Section 5.1 are in many aspects similar to those from the SoS, their ~~respective skills in reproducing observations are at the opposite ends. Equation 11~~ abilities to reproduce the observed ice flow patterns are very different. The HS-2b uses the basal velocity from ~~the SIA~~ Equation (2) (utilised by the SoS) to compute the weights of relative contributions of the shallow approximations, thereby ~~enabling~~ adding the SStA contribution where sliding velocities from ~~the SIA~~ Equation (2) are high. In these rapidly flowing sectors, we attribute ~~the~~ a better performance of the HS-2b compared to the SoS to the inclusion of the stress transmission by the SStA. It also fosters a SStA-dominated modelled ice flow in the surroundings of ice streams, particularly upstream, thereby improving the overall fit to observations. Compared to the HS-2b, the HS-2a has a similar performance with the second smallest mean velocity error of $17.6\,\mathrm{m\,yr^{-1}}$, followed by higher mean errors of 23.2 and $29.7\,\mathrm{m\,yr^{-1}}$ produced by the HS-3 and HS-1, respectively. Thus, the use of hybrid schemes allows for a two- to threefold reduction in the surface velocity misfit.

As described in Section 2.2, the hybrid schemes used in this study mainly differ in how the SStA weight, w, is computed. This does not only ~~imply the~~ control relative contributions from the shallow approximations, but also the locations where such ~~combination is~~ combinations are implemented. In order to provide a deeper insight into ~~the internal operation of the hybrid schemes~~how each hybrid scheme combines the SIA and SStA velocities, Fig. 5 ~~compares~~ illustrates the equilibrated distributions of w. It can be immediately observed that the distributions of w produced by the hybrid schemes are very different, both in ~~the~~ spatial coverage and ~~individual contributions of the SIA and SStA~~inferred values. For example, the percentage of the grounded ice area where the modelled ice flow is dominated by the SStA ($w > .75$) is $\sim 30\,\%$ for the HS-1, but only ~~$\sim 3\,\%$ and $\sim 8\,\%$~~ $2.4\,\%$ and $8.5\,\%$ for the HS-2a and HS-2b, respectively (Table 3). Furthermore, the transition between the SIA- and SStA-dominated ~~regimes of the modelled ice flow~~ ice sheet sectors appears patchy in the HS-1, whereas it is smooth and collocated with the present-day ice streams in the HS-2a, thereby resembling the ~~velocity field from the observational data set. The transitions~~ observed surface velocities. Such transitions are sharp in the HS-2b~~are sharp~~, implying a simple differentiation between fast and slow ice flow areas. This particular scheme exhibits a SIA-dominated ice flow regime ($w < .25$) over ~~$81\,\%$~~ $82\,\%$ of the grounded ice area that may ~~help explain the~~ explain a high degree of similarity with the SoS, especially keeping in mind that it also includes basal sliding in the SIA. In contrast, the HS-2a ~~uses a non-sliding SIA , but still produces~~ sets basal ice velocities to zero in the computation of the SIA velocities, still producing a similar percentage of ~~ice-sheet–covered areadominated by the SIA~~SIA-dominated area. In these sectors, differences in the ice thickness derived from the HS-2a and HS-2b (Fig. 3, left column) can be attributed to ~~the~~ a presence or absence of ~~the~~ basal sliding in the ~~SIA that tends to~~ computation of the SIA solution that may prevent overestimations, e.g., in interior East Antarctica ~~, and causes underestimations in some~~ and cause underestimations in other sectors, such as, for example, the surroundings of Dronning Maud and MacRobertson Lands.

~~The disparity in the ways how different hybrid schemes measure relative SIA and SStA contributions may explain the nature of the misfits~~ It is clear that the degree of agreement between the modelled ~~ice flow and observations~~and observed ice flow is sensitive to the choice of a particular hybrid scheme and the way it measures relative SIA and SStA contributions. This can be ~~visualized~~ best visualised using scatter plots of the modelled vs. observed surface velocities for each scheme, color-coded for the values of the corresponding w distributions (Fig. 5). In general ~~,~~ underestimations of the modelled velocities occur across slowly flowing ($< 10\,\mathrm{m\,yr^{-1}}$) areas, which are dominated by the SIA, while the areas of fast flow dominated by the SStA are responsible for most of the overestimation. At ~~first glance,~~ a first glance it may seem that overestimations are caused by an excessive contribution of the SStA, but a ~~cursory~~ comparison with the SoS scatter plot shows that this is not necessarily the case~~:~~. The largest overestimations occur in the SoS simulation, with surface velocities reaching the upper permitted limit clustered in the upper part of the scatter plot (Fig. 5). As mentioned above, the SoS and HS-2b share many similarities, and a comparison between their respective scatter plots shows that the use of the hybrid scheme reduces both underestimations in the lower velocity range and overestimations in the fast flowing areas.

~~Here we attempt to isolate the influence of relative SIA and SStA contributions on different flow regimes, which can be characterised by weights between $w = 0$ (slow, SIA-dominated flow) and $w = 1$ (fast, SStA-dominated flow). For different intervals within this range, we computed averaged errors arising from under- and overestimation of the surface velocity (Fig. ??). As has been demonstrated in Fig. 5, overestimations in the modelled velocities escalate for all schemes as w~~

approaches 1. It is readily observed that the departures of the modelled surface velocities produced by the SoS simulation (which discounts the SStA contribution) from observations are larger than those from the hybrid schemes, in terms of both under- and overestimations and for all types of flow regimes. This supports our assertion that the inferred overestimations in the fast flowing regions are not necessarily caused by an excessive contribution from the SStA solution. Overall, the performance of different hybrid schemes is quantitatively similar, with no striking outliers occurring when they are validated against observational data. The HS-1 shows larger velocity misfits across the slow ice sheet sectors if evaluated against the distributions of w from the HS-2a and HS-2b. This rise in velocity misfits is caused by an increased use of the SStA by the HS-1 across the areas where the HS-2a and HS-2b favour a SIA-dominated ice flow (Fig. 5).

5.3 Intercomparison of the inferred basal sliding coefficients

Although the performance of different the hybrid schemes is quantitatively similar when evaluated against observations, the inferred values of basal sliding coefficients (C_0) vary by orders of magnitude in many regions of Antarctica (Fig. 3, right column). We quantify this variability by looking at the standard deviation of the distributions of C_0 (Fig. ??). Approximately 35 % of the total area where the calibration procedure is activated is characterised by a standard deviation of $100\,\mathrm{m\,yr^{-1}\,Pa^{-1}}$ or higher. At the other end, only a quarter of the sliding area displays a standard deviation of $1\,\mathrm{m\,yr^{-1}\,Pa^{-1}}$ or less. Of this latter fraction, almost two thirds occur across areas where the adjusted value of C_0 reaches either the lower or the upper limit prescribed for the inversion procedure. This observation suggests a possibility of even higher variability in the distribution of C_0 if, for example, the upper limit were to increase. It is important to keep in mind that these large differences between the inferred distributions of C_0 arise solely from the differences in the hybrid schemes, namely in their ways to combine the shallow approximations, since all other model components are exactly the same for all experiments.

To demonstrate the effects arising from the variability in the inferred distributions of prescribing a distribution of C_0 derived from one ice model (using a specific hybrid scheme) into a different ice model (using a different hybrid scheme or other level of approximation), we performed additional an additional set of experiments, which start from utilise the equilibrium states described in Section 5.1 as initial conditions. In these experiments, we exchange the distributions of C_0 inferred from the HS-2a and HS-2b, and then run the model over the a period of 100 kyr. As described in Section 2.2, these schemes slightly differ in how they compute the SStA contribution, mainly due to different techniques used to account for basal sliding. Although both schemes identify similar locations of rapidly flowing ice (Section 5.2), their inferred distributions of basal sliding coefficients contain the highest variability among all hybrid schemes implemented in this study. At the end of the these additional 100kyr runs, the mean misfit between the modelled and observed ice thickness is above 200 m for the HS-2a, and above 300 m almost 350 m for the HS-2b (not shown). This represents increments of $\sim 250\,\%$ and $\sim 550\,\%$ $\sim 330\,\%$ and $\sim 775\,\%$ in the deviations from observations, respectively.

We generalize these experiments and To further exemplify the significance of the associated uncertainties in the retrieved basal sliding parametersby prescribing a , we prescribe the median of the inferred distributions of C_0, computed from all hybrid schemes, at the base of the modelled ice sheet. Our choice of a median over an average is motivated by our initial tests in which generally larger values of C_0 inferred from the HS-2a tend to produce an average biased towards this particular distribution.

Figure 6 shows differences between the modelled and observed ice thickness at the end of these additional runs. Comparison with Fig. 3 reveals a general degradation of the fit between the model and observations. The HS-2a and HS-2b exhibit the largest sensitivity to the change in the basal sliding parameters, with a significant amplification of over- and underestimations of the ice thickness occurring across most of the ice sheet, respectively. For the HS-2a, an absolute ice thickness misfit increases by ~~$\sim 150\,\%$~~ $\sim 180\,\%$ to a mean value of ~~$150\,\mathrm{m}$~~ $131\,\mathrm{m}$ (Table 4), of which ~~$97\,\%$~~ $95\,\%$ is due to overestimations. ~~On the other hand, the~~ The misfit increases by ~~$\sim 230\,\%$~~ $\sim 355\,\%$ to a mean value of ~~$172\,\mathrm{m}$~~ $182\,\mathrm{m}$ in the HS-2b simulation, with underestimations of the ice thickness accounting for ~~$89\,\%$~~ $88\,\%$ of the total difference. This degree of degradation is less pronounced when the HS-1 and HS-3 are used. The misfit ~~from the former~~ for the HS-1 increases by only ~~$\sim 30\,\%$~~ $\sim 50\,\%$, displaying a mixture of areas where the modelled ice thickness is either too large or too small relative to observations. The ~~latter~~ HS-3 experiment shows an intermediate degree of sensitivity to a change in basal parameters and exhibits similar misfit patterns as the HS-2b, albeit the magnitude of the underestimation is smaller and the mean absolute error in the ice thickness remains ~~below~~ around $100\,\mathrm{m}$. These experiments show that care is needed when using quantifications of the basal conditions obtained from external sources (e.g., Joughin et al., 2009; Morlighem et al., 2010; Arthern and Gudmundsson, 2010; Pralong and Gudmundsson, 2011; Pollard and DeCo input in ice flow models. This concerns not only differences between the basal sliding approaches implemented in each model, but also models using the same sliding law as part of different hybrid schemes, as shown in this study.

5.4 Exploration of the hybrid parameter space

~~Disparity~~ The disparity in the results from the hybrid schemes presented in the previous sections showcases the impacts of slight changes in the model representation of fast flowing zones of ice sheets, even though it corresponds to a small fraction of the parameter space in ice-sheet models. ~~Within~~ In order to explore the sensitivity of the results to parameter variations within this parameter space, ~~the use of~~ we perform an additional series of experiments where we vary the somewhat arbitrary threshold and reference quantities used by some of the hybrid schemes~~allows us to perform an additional series of experiments in which we explore the sensitivity of the results to variations in these parameters~~.

As described in Section 2.2, the hybrid schemes HS-1, HS-2a, and HS-2b use the weight w to calculate relative contributions of the SIA and the SStA. For the HS-1, the computation of w involves a prescribed threshold for the slip ratio of grounded ice (Eq. (8)), ~~which~~ that determines the locations where the hybrid velocity is calculated. The experiments discussed in the previous sections use a default value of $r_{\mathrm{thr}} = 0.5$, meaning that the SIA and SStA solutions are combined only in areas where the basal velocity is at least half of the surface velocity. In contrast, the HS-2a and HS-2b combine both shallow approximations everywhere, and w is computed using a prescribed reference velocity (Eqs. ~~10 and 11~~(10) and (11)), using a default value of $v_{\mathrm{ref}} = 100\,\mathrm{m\,yr^{-1}}$. If the SStA velocity in the HS-2a or the basal ~~SIA~~ velocity in the HS-2b reaches this reference value, the schemes assign equal weights to the SIA and the SStA. In general, higher values of r_{thr} or v_{ref} imply less contribution of the SStA solution, and vice versa.

The effects of variations in these parameters on the evolution of the total grounded ice volume during the calibration procedure are demonstrated in Fig. 7. Here we test ~~values within the range of what can be considered lower and upper limits for~~

~~each hybrid scheme~~parameter values within a range that contains almost every possible scenario, either because values outside the range are non-physical (HS-1) or they exhibit no noticeable differences compared to the range limits (HS-2a and HS-2b).

For the HS-1, the upper limit for the slip ratio threshold, $r_{\mathrm{thr}} = 1$, implies the use of a SIA-only solution, whereas the lower limit, $r_{\mathrm{thr}} = 0$, implies that a combination of SIA and SStA solutions is applied everywhere, determined by the weight w computed using the slip ratio. Within the range of tested values, maximum deviations from the observed grounded ice volume are below ~~1.5 %~~1.2 %. Only the use of $r_{\mathrm{thr}} = 1$ produces an ice sheet that is smaller than observed. An analysis of the mean absolute ~~difference~~ differences between the modelled and observed ice thickness (Fig. 8) reveals that the use of ~~values below $r_{\mathrm{thr}} = 0.5$~~ larger values of r_{thr} leads to a larger misfit with observations, ~~which is also the case for the upper limit,~~ with the maximum error corresponding to $r_{\mathrm{thr}} = 1$~~. However, $r_{\mathrm{thr}} = 0.75$ produces a slightly improved result, suggesting that at some higher value the fit will start to decrease~~.

In the HS-2a, all tested values produce an ice sheet that is larger than ~~observations~~observed. This is caused by a general overestimation of the ice thickness ~~at~~ in the continental interior. Here, the modelled ice flow is slow and dominated by the ~~non-sliding~~ SIA. Thus, the weights w are small, implying a negligible contribution of the SStA, independently of the value of v_{ref}. Values smaller than $v_{\mathrm{ref}} = 100\,\mathrm{m\,yr^{-1}}$ are found to produce total grounded ice volumes and mean ice thickness misfits that are close to the results of the reference run ($v_{\mathrm{ref}} = 100\,\mathrm{m\,yr^{-1}}$), although with a slight improvement ~~of~~ in the model fit to observations. Values of $v_{\mathrm{ref}} > 100\,\mathrm{m\,yr^{-1}}$ lead to a larger misfit, with the highest tested value of $v_{\mathrm{ref}} = 1000\,\mathrm{m\,yr^{-1}}$ producing a grounded ice volume that ~~deviates by 2.5 %~~ displays a deviation of 2.3 % from observations. The use of even larger values of v_{ref} asymptotically decreases the contribution of the SStA. It is important to note, however, that since the HS-2a does not include sliding in the SIA component, its solutions will never approach that of the SoS.

In contrast, the HS-2b activates the sliding law even in the regions where ice streams have not been identified by the scheme, as long as the conditions for sliding described in Section 2.1 are fulfilled. This may explain the similarity between the grounded ice volumes produced by the SoS and the HS-2b solutions, using a reference velocity set to the upper limit of $v_{\mathrm{ref}} = 1000\,\mathrm{m\,yr^{-1}}$. This parameter value produces a deviation of ~~2%~~ −0.9 % from the observed ice volume, ~~similarly~~ which is similar (but with opposite signs) to the misfit obtained from the ~~simulating~~ simulation using the lower limit of $v_{\mathrm{ref}} = 5\,\mathrm{m\,yr^{-1}}$ tested in this study. For values higher than the default value of $v_{\mathrm{ref}} = 100\,\mathrm{m\,yr^{-1}}$, the simulations produce an ice sheet that is smaller than observed. The use of $v_{\mathrm{ref}} = 60\,\mathrm{m\,yr^{-1}}$ leads to the best fit between the modelled and observed ice ~~volumes. Here, the misfit of 0.12 % in the total ice volume is accompanied by the the second smallest misfit in ice~~ thickness among all schemes and parameter values, reaching ~~$\sim 49\,\mathrm{m}$. The smallest misfit overall is produced by the HS-2b, using a reference velocity of $v_{\mathrm{ref}} = 30\,\mathrm{m\,yr^{-1}}$, although here the deviation of 0.66 % in the~~ $\sim 39\,\mathrm{m}$, with a small deviation from the observed grounded ice volume ~~is higher~~of 0.27 %.

However, it is important to keep in mind that the calibration procedure has a limited power to reduce the misfits between the model and observations in areas where no sliding occurs. This is why some deviations are expected from our numerical experiments, in particular due to overestimations of the ice volume over mountainous regions where the calibration procedure does not operate. Therefore, a perfect fit to the observed ice volume obtained by any of the schemes would likely involve underestimations of ice thickness in other regions.

6 Summary and conclusions

We implemented and compared the performance of four hybrid schemes for the combination of the Shallow Ice and Shelfy Stream Approximations ~~into~~ as part of the ice-sheet model SICOPOLIS. The use of shallow approximations enables ~~,~~ continental-scale, long-term paleo-simulations of ~~entire~~ ice sheets, preventing the restrictive computational expenses of more complex models. Moreover, hybrid schemes overcome the limitations of simpler SIA-only models in regions characterised by rapid ice flow driven by basal sliding. The hybrid schemes in this study differ in ~~their ways to:~~ the ways 1) ~~compute~~ relative contributions of the shallow approximations are computed, 2) ~~identify~~ areas where a combination of shallow approximations is applied are identified, and 3) ~~account for basal sliding~~ the basal sliding is accounted for.

By adapting a simple ~~inverse~~ iterative technique to infer the distribution of basal sliding parameters from observational topographic data sets (Pollard and DeConto, 2012b), we show that all the applied hybrid schemes produce dynamic equilibrium states that are in a good agreement with observations. For all the schemes, mean ice thickness misfits ~~are below $\sim 80\,\mathrm{m}$ and~~ range from 40 to $52.4\,\mathrm{m}$, and the total grounded ice volume deviations do not exceed $2.5\,\%$ for a wide range of the associated parameter uncertainties. For optimal parameter choices, mean errors in the ice thickness remain below ~~50m~~ $40\,\mathrm{m}$. For comparison, present-day simulations in continental-scale ice-sheet models typically produce widespread errors in the ice thickness reaching hundreds of metres, with deviations in total grounded ice volume that can ~~can~~ exceed $10\,\%$ (e.g., de Boer et al., 2015). A clear limitation of the calibration method presented here is the possibility of an error cancellation during the adjustment of C_0. At present there is no easy way to differentiate between the effects of an artificial compensation of, for example, the lacking model physics or errors in the input data sets through the model calibration and the model parameters that are representative of the actual conditions at the base of the ice sheet. This shortcoming will hopefully be overcome in the future once the necessary observational data sets become available and the use of more sophisticated ice models more feasible. The degree of applicability of the calibrated C_0 distributions for long-term paleoclimate simulations is currently unclear, and its quantification requires further modelling studies of the past ice sheet geometries. However, we believe that the retrieved variability in basal sliding coefficients is a good first-order approximation, and is certainly a better guess than commonly used spatially uniform values of C_0 over the entire domain. We believe that the method used to derive C_0 could be potentially extended to the use for other time periods or climate regimes (or even adapted for the transitions between largely dissimilar regimes), providing that the necessary topographic and climate data are available. Considering the recent efforts to reconstruct the past geometries of the AIS (Mackintosh et al., 2014; Bentley et al., 2014) , this may well become possible in the upcoming decades.

We also computed a non-hybrid, SIA-only solution to allow for a qualitative separation of relative contributions of the SIA and the SStA. Such direct comparison is possible because all the hybrid schemes analysed in this study share identical model components. Although the calibration procedure applied to the SIA-only scheme also produces a reasonable fit to observations, the misfits are overall larger than those from the hybrid schemes. Moreover, the modelled surface velocities exhibit noise-like patterns at and around locations where rapid ice flow ~~develops~~exists. Since these patterns are not present in the hybrid solutions, we attribute their occurrence to the lack of stress transmission in the SIA flow.

We find that individual weights assigned to the SIA and SStA solutions vary significantly from scheme to scheme. For the schemes in which the SIA/SStA weights are computed, the modelled ice flow is dominated by the SIA over ~~50 % to 90 %~~ $\sim 50\,\%$ to $\sim 80\,\%$ of the AIS, with the predominant contribution of the SStA generally limited to the locations and surroundings of the observed ice streams. ~~An inclusion of a sliding component in the SIA tends to prevent an excessive accumulation of ice in the continental interior of the ice sheet, as opposed to the scheme that uses the SStA solution as a sliding law but assigns relatively small weights to it across these areas. We have found that this particular scheme tends to increase basal sliding coefficients in an attempt to compensate for an insufficient sliding that would otherwise lead to a larger misfit. The scheme adopted by Winkelmann et al. (2011) , which adds up non-sliding SIA and SStA solutions instead of using the SIA/SStA weights, to some extent reduces overestimations of the ice thickness in the continental interior, and thus the values of basal sliding coefficients.~~

~~Here we use the fact that large portions of the AIS are dominated by the SIA to compare a hybrid scheme including basal sliding in the SIA solution with the SIA-only scheme. The latter generates a general underestimation of the ice thickness at the ice sheet margins, while the former effectively reduces them. This improvement can be primarily attributed to the inclusion of the stress transmission in the SStA at and around the fast flowing regions, where the assumptions behind the SIA are violated.~~

Comparison of our results with an independent, observational data set of surface ice velocities reveals a reasonable agreement in the continental interior of Antarctica. However, the modelled ice flow appears to be too fast at the ice sheet margins in all hybrid schemes, especially close to the observed ice stream locations. These misfits between the modelled and observed surface velocities are contrasted by relatively small differences between the modelled and observed ice thickness in these areas. Such misfits ~~might~~ may originate from other factors such as, e.g., ~~the neglect of the paleoclimate signal within the AIS implied by~~ uncertainties in the observational ice thickness and surface velocity data sets (Fretwell et al., 2013; Rignot et al., 2011, see Section 3) , the limited model resolution, the assumption of an ~~equilibrium, the lower model resolution, anddeficiencies in the forcing data sets~~ ice sheet equilibrium with present-day climate conditions, and/or errors in the model-based geothermal heat flux and/or climatic forcing (Pollard and DeConto, 2012b).

Although all hybrid schemes produce a comparably good fit to the observational topographic data set, the inferred values of basal sliding coefficients largely differ between different schemes. Therefore, the discrepancies in the representation of ~~ice dynamics in~~ the ice dynamics by different hybrid schemes are compensated ~~by a high degree of uncertainty associated with~~ through adjustments of this poorly constrained parameter. Nevertheless, the schemes mostly agree on the areas where the ice base is at or close to the local pressure-melting point. Furthermore, there is a ~~general~~ qualitative agreement in the patterns of ~~hard~~ low vs. ~~soft basal conditions~~ high values of C_0 obtained from each calibration run. Although any attempt to quantify a local potential for sliding would require new and improved observational data sets ~~on~~ describing basal conditions, the robustness of the inferred patterns provides an initial guess for their real distribution.

~~The high variability in the inferred parameters provided an opportunity to quantify the effects of~~

Furthermore, we have evaluated the performance of each hybrid scheme based on both its fit to observations and its numerical stability for a range of model set-ups and have found that at higher grid resolutions the HS-1 and HS-2b become numerically unstable, owing to large gradients in basal sliding coefficients arising from the use of basal velocities as boundary conditions for the SIA solutions in conjunction with the calibration of sliding coefficients. In addition, variations in the free parameter

r_{thr} of the uncertainty in basal conditions. By prescribing averaged and swapped distributions of basal sliding coefficients derived from different hybrid schemes , we show that they HS-1 generate artefacts in the resulting basal temperature field. The HS-2a and HS-3, which utilise the SStA as a sliding law are numerically more stable to variations in model parameters and changes in grid resolution. The drawback of the HS-2a is its limited ability to influence the fit between the modelled and observed ice thickness in ice sheet sectors where the SStA velocities are low (<100 m/yr), which is the case for large parts of the Antarctic interior. This limitation is independent of the choice of the parameter v_{ref} (Figure 7). The HS-3 overcomes this limitation by accounting for the SStA contribution everywhere. However, the HS-3 also utilises the SIA velocities over the entire ice sheet, which in certain areas are excessively high, such as the steep ice sheet margins (Figure 4), leading to an increased root-mean-square error compared to HS-2a (Figure 5). To improve the performance of both schemes, our future work will reconcile the drawbacks of HS-2a with the advantages of HS-3, providing a very stable and flexible hybrid scheme.

Finally, we assessed the effects of differences between the calibrated parameter distributions derived from the four hybrid schemes by performing additional experiments in which the inferred distributions of C_0 were either averaged or exchanged between the schemes. Our results show that the parameter distributions are not exchangeable between the schemes, since their use this leads to a strong degradation of the fit between the model and model fit to observations. This suggests that results of a model calibration and/or initialisation cannot be straightforwardly transferred to a model that uses a different level of approximation to of the Stokes equations. Nevertheless, this paper shows that a simple inverse method for the distribution of C_0 can be easily adapted into Given an increasing number of studies attempting to quantify the basal conditions under ice sheets through a variety of modelling approaches. We believe that such implementations can provide the internal consistency required to avoid the aforementioned misfits between the model and observationsmethods including ice flow models, our experiments show that one need to be careful when using these results as input data sets in glaciological models.

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Table 1. Symbols and values for the model parameters used in this study.

Symbol	Description	Units	Value
g	gravitational acceleration	$\mathrm{m\,s^{-2}}$	9.81
ρ	density of ice	$\mathrm{kg\,m^{-3}}$	910
T	absolute temperature of ice	K	
T_m	temperature below pressure-melting point	K	
κ	heat conductivity of ice	$\mathrm{W\,m^{-1}\,K^{-1}}$	$9.828\mathrm{e}^{-0.0057T}$
c	specific heat of ice	$\mathrm{J\,kg^{-1}\,K^{-1}}$	$146.3+7.253T$
L	latent heat of ice	$\mathrm{kJ\,kg^{-1}}$	335
β	Clausius–Clapeyron gradient	$\mathrm{K\,m^{-1}}$	8.7×10^{-4}
R	universal gas constant	$\mathrm{J\,mol^{-1}\,K^{-1}}$	8.314
κ_r	heat conductivity of the lithosphere	$\mathrm{W\,m^{-1}\,K^{-1}}$	3
α_ice, α_snow	PDD factors for ice and snow	$\mathrm{mm\,d^{-1}\,{}^\circ C^{-1}}$	8, 3
α_std	PDD standard deviation	$^\circ\mathrm{C}$	5
σ_e	effective shear stress	Pa	
σ_0	residual stress	kPa	10
E_SIA, E_SSA	enhancement factor for the SIA and SSA		1, 0.5
A	ice rate factor	$\mathrm{s^{-1}\,Pa^{-3}}$	
n	Glen flow law exponent		3
$\boldsymbol{u}$	horizontal SIA velocity	$\mathrm{m\,s^{-1}}$	
$\boldsymbol{v}$	horizontal SStA velocity	$\mathrm{m\,s^{-1}}$	
$\boldsymbol{U}$	horizontal hybrid velocity	$\mathrm{m\,s^{-1}}$	
v_0	regularisation speed in SStA equations	$\mathrm{m\,yr^{-1}}$	0.01
$\boldsymbol{\tau}_\mathrm{b}$	basal shear stress	Pa	
N_b	effective basal pressure	Pa	
p, q	sliding law exponents		3, 2
γ	sub-melt-sliding parameter	K	3
C_0	~~inverted~~ calibrated basal sliding parameter	$\mathrm{m\,yr^{-1}\,Pa^{-1}}$	
Δt_inv	time step for inversion of C_0	yr	50
H_inv	scaling factor for ~~inversion~~ calibration of C_0	m	5000
r	slip ratio of grounded ice		
r_thr	default threshold value of r		0.5
v_ref	default reference value of $\lvert\boldsymbol{v}\rvert$	$\mathrm{m\,yr^{-1}}$	100
w	weighting function in hybrid schemes		
h_rlx	scaling factor for relaxation procedure		0.001

Table 2. Summary of the results at the end of the calibration procedure, including: Total grounded ice volume V_{grd} (km^3); deviation from total grounded ice volume ΔV_{grd} (%); mean absolute error hybrid schemes implemented in the ice thickness $\bar{\Delta} H$ (m); Fraction of the total area where basal sliding is activated A_{sld} (%); mean ice thickness error only where basal sliding operates $\bar{\Delta}_{sld} H$ (m); Mean surface velocity error $\bar{\Delta} v_s$ (m yr^{-1}); fraction of the grounded ice area dominated by the SIA ($w < 0.25$) $A_{SIA.25}$ (%); and fraction of the grounded ice area dominated by the SStA ($w > 0.75$) $A_{SStA.75}$ (%)this study.

Scheme	Basal sliding	Reference in text	References
HS-1	Weertman-type sliding	Equation (8)	This study
HS-2a	SStA velocities	Equation (10)	Bueler and Brown (2009)
HS-2b	Weertman-type sliding	Equation (11)	This study
HS-3	SStA velocities	Equation (12)	Winkelmann et al. (2011)

Table 3. Summary of the results at the end of the calibration procedure for each of the applied hybrid schemes (HS, see Section 2.2), including: The total grounded ice volume, V_{grd} (km^3); a deviation from the total grounded ice volume, ΔV_{grd} (%); a mean absolute error in the ice thickness, $\bar{\Delta} H$ (m); a fraction of the total area where basal sliding occurs, A_{sld} (%); a mean ice thickness error only where basal sliding operates, $\bar{\Delta}_{sld} H$ (m); a mean surface velocity error, $\bar{\Delta} v_s$ (m yr^{-1}); a fraction of the grounded ice area dominated by the SIA (only where $w < 0.25$), $A_{SIA.25}$ (%); and a fraction of the grounded ice area dominated by the SStA (only where $w > 0.75$), $A_{SStA.75}$ (%).

Scheme	V_{grd} [km^3]	ΔV_{grd}	$\bar{\Delta} H$ [m]	A_{sld}	$\bar{\Delta}_{sld} H$ [m]	$\bar{\Delta} v_s$ [m yr^{-1}]	$A_{SIA.25}$	A_{SSt}
HS-1	2.58×10^7	+1.22 % +1.20 %	49.9 44.6	53.3 % 45.2 %	28.0 20.8	51.0 29.7	44.3 % 47.9 %	30.2 %
HS-2a	2.60×10^7 2.59×10^7	+1.88 % +1.46 %	59.8 46.6	60.4 % 54.2 %	26.9 17.6	42.5 17.6	80.2 % 82.0 %	3.2 %
HS-2b	2.54×10^7 2.56×10^7	−0.25 % +0.10 %	52.0 40.0	65.6 % 62.0 %	35.9 25.4	32.9 16.0	81.0 % 82.3 %	8.5
HS-3	2.58×10^7 2.57×10^7	+1.23 % +0.77 %	54.3 42.6	63.5 % 58.7 %	31.4 22.3	46.2 23.2	-	-
SoS	2.51×10^7 2.53×10^7	−1.55 % −0.72 %	73.1 52.4	61.2 % 58.9 %	65.9 41.3	80.5 55.3	100 %	0

Table 4. Summary of the results at the end of additional 100kyr runs with a prescribed median of the inferred basal sliding coefficients, including: Total The total grounded ice volume, V_{grd} (km^3); a deviation from total grounded ice volume, ΔV_{grd} (%); a mean absolute error in the ice thickness, $\bar{\Delta} H$ (m); and a mean ice thickness error only where basal sliding operates, $\bar{\Delta}_{sld} H$ (m).

Scheme	V_{grd} [km^3]	ΔV_{grd}	$\bar{\Delta} H$ [m]	$\bar{\Delta}_{sld} H$ [m]
HS-1	2.58×10^7 2.60×10^7	+1.08 % +1.95 %	64.0 67.0	50.6 45.2
HS-2a	2.72×10^7 2.70×10^7	+6.65 % +5.58 %	150.0 131.2	119.0 110.1
HS-2b	2.39×10^7 2.37×10^7	−6.39 % −7.41 %	172.0 182.1	120.7 167.6
HS-3	2.51×10^7 2.48×10^7	−1.57 % −2.97 %	93.3 105.0	83.6 104.8

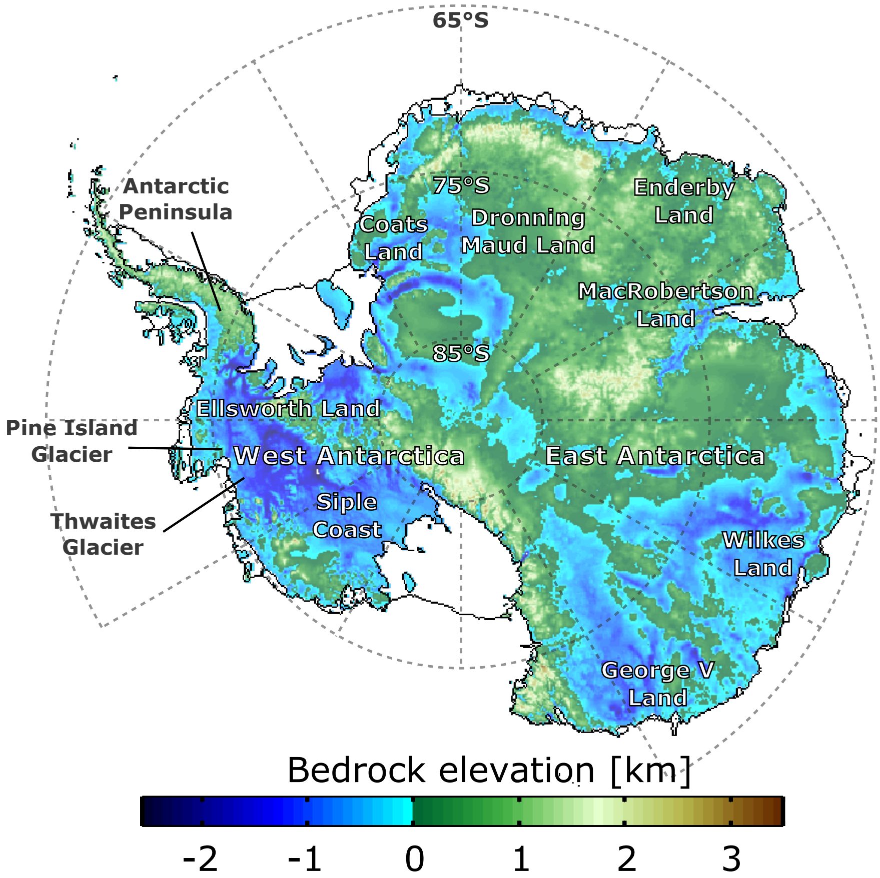

Figure 1. ~~Overview of the calibration procedure. **(a)** Mean differences between the modelled and observed ice thickness at the end~~ A bedrock elevation map of Antarctica obtained from the ~~simulations, in m. For each scheme~~ BEDMAP2 data set (Fretwell et al., 2013) , ~~a mean error is calculated for the entire ice sheet (left bars) and separately over~~ including the ~~areas where basal sliding is identified (right bars). Fractions~~ location of the ~~mean error arising from under- and overestimations are shown~~ sites mentioned in ~~blue and red, respectively. **(b)** The evolution of~~ the ~~total grounded ice volume during the calibration procedure, in $\times 10^7$ km^3~~ text.

~~Surface velocity error as a function of the SStA weight. The errors are represented by ratios of modelled to observed velocities (Fig. 4). To enable a comparison of all schemes, the distributions of w derived by HS-1, HS-2a, and HS-2b are used to quantify different ice flow regimes. Each circle spans a weight interval of 0.1 and represents an average of velocity ratios computed using grid points with w values within this interval (Fig. 5). The averaging procedure is performed separately for under- and overestimations.~~

~~**(a)** Standard deviation of the retrieved distributions of basal sliding coefficients, C_0, in $\mathrm{m\,yr^{-1}\,Pa^{-1}}$, from four hybrid schemes. **(b)** Fraction of the total sliding area, in %, characterised by different magnitude ranges of the standard deviation of C_0.~~

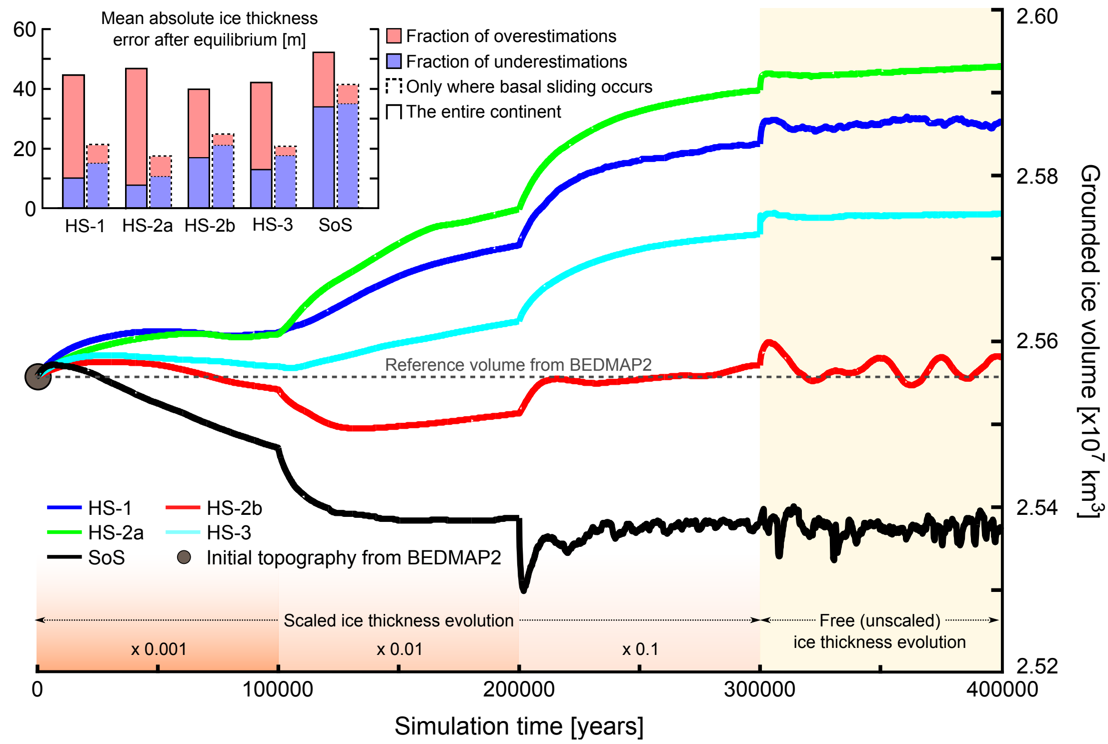

Figure 2. Overview of the calibration procedure. **Main:** The evolution of the total grounded ice volume during the calibration procedure, in $\times 10^7\,\mathrm{km}^3$. **Inset:** Mean differences between the modelled and observed ice thickness at the end of the simulations, in m. For each scheme, a mean error is calculated for the entire ice sheet (left bars) and separately over the areas where basal sliding is identified (right bars). Fractions of the mean error arising from under- and overestimations are shown in blue and pink, respectively.

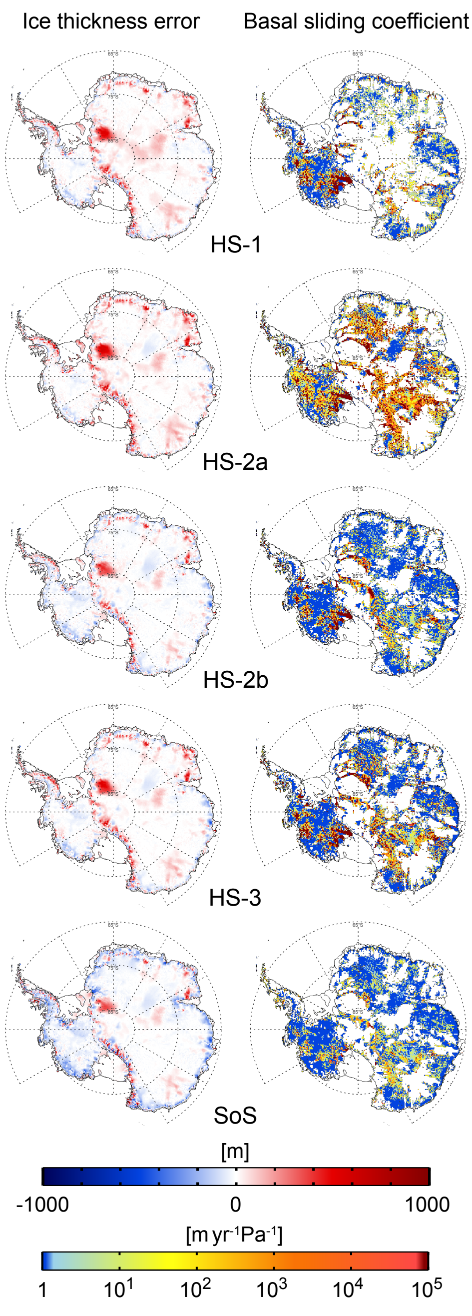

Figure 3. Comparison of the ~~equilibrium~~ ice sheet states derived from different schemes at the end of equilibrium simulations. **Left column:** Differences between the modelled and observed ice thickness, in m. **Right column:** Inferred distributions of basal sliding coefficients, in $\mathrm{m\,yr^{-1}\,Pa^{-1}}$. Non-coloured areas mark the locations where basal sliding is not identified and the calibration procedure does not operate. Color-code saturates at the upper and lower limits allowed for the ~~inversion~~ calibration procedure.

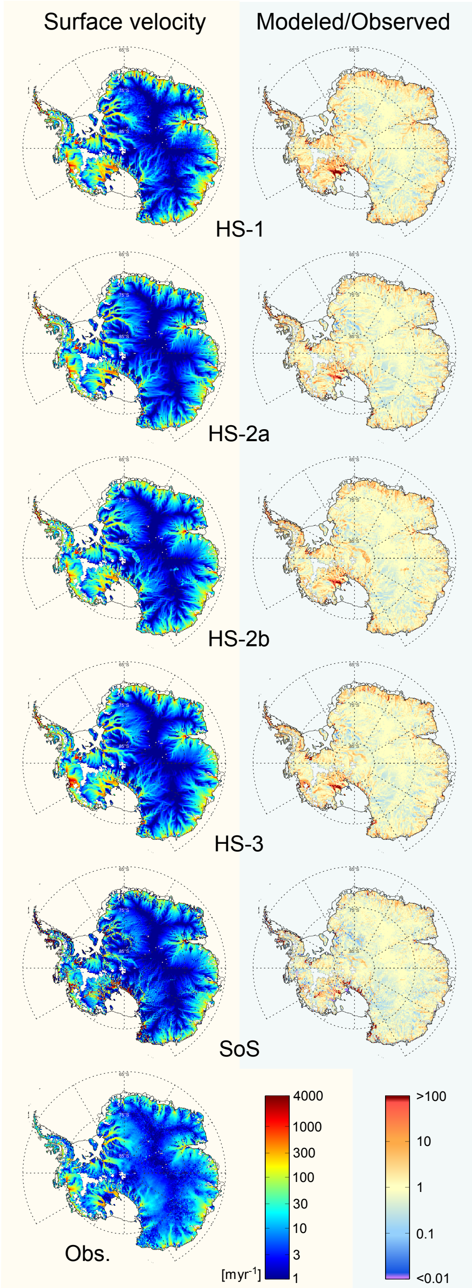

Figure 4. Left column: Equilibrium surface velocities across the grounded ice areas derived from different schemes, in $\mathrm{m\,yr^{-1}}$, compared to the observational data set from Rignot et al. (2011) (bottom), regridded to the model resolution of ~~40 km~~ 20 km. **Right column:** Ratios of the modelled to observed surface velocities, plotted on a logarithmic scale. Velocities smaller than $2\,\mathrm{m\,yr^{-1}}$ are excluded. Color-code saturates at ratios larger than 100 or smaller than 0.01.

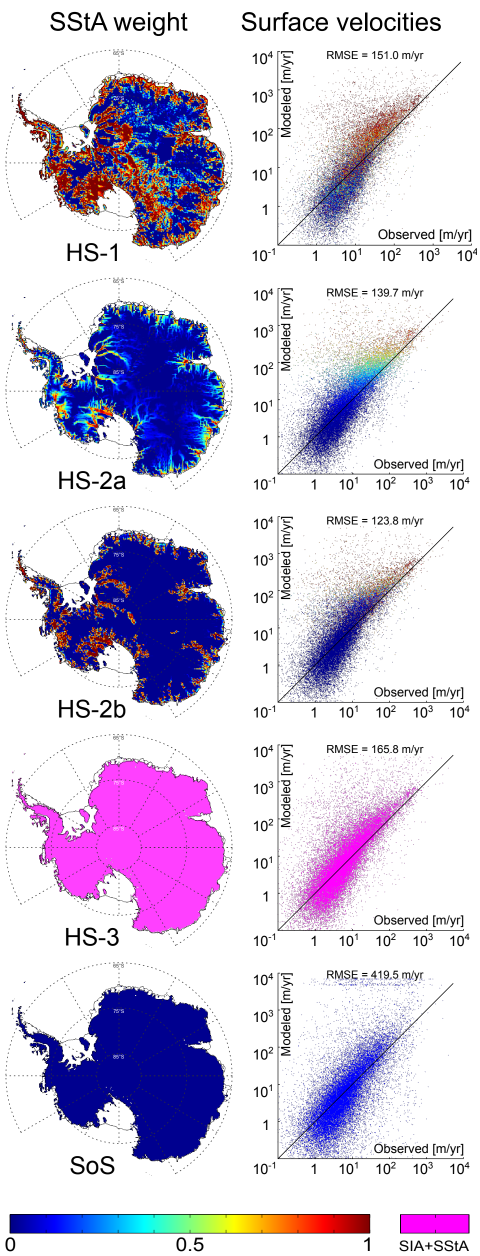

Figure 5. Left column: Equilibrium SStA weights derived from different schemes. The HS-3 does not use a weighting function and simply adds velocities derived from both shallow approximations. **Right column:** Scatter plots of modelled vs. observed surface velocities, in $\mathrm{m\,yr^{-1}}$. Each ~~grid cell~~ point is color-coded according to the corresponding SStA weight.

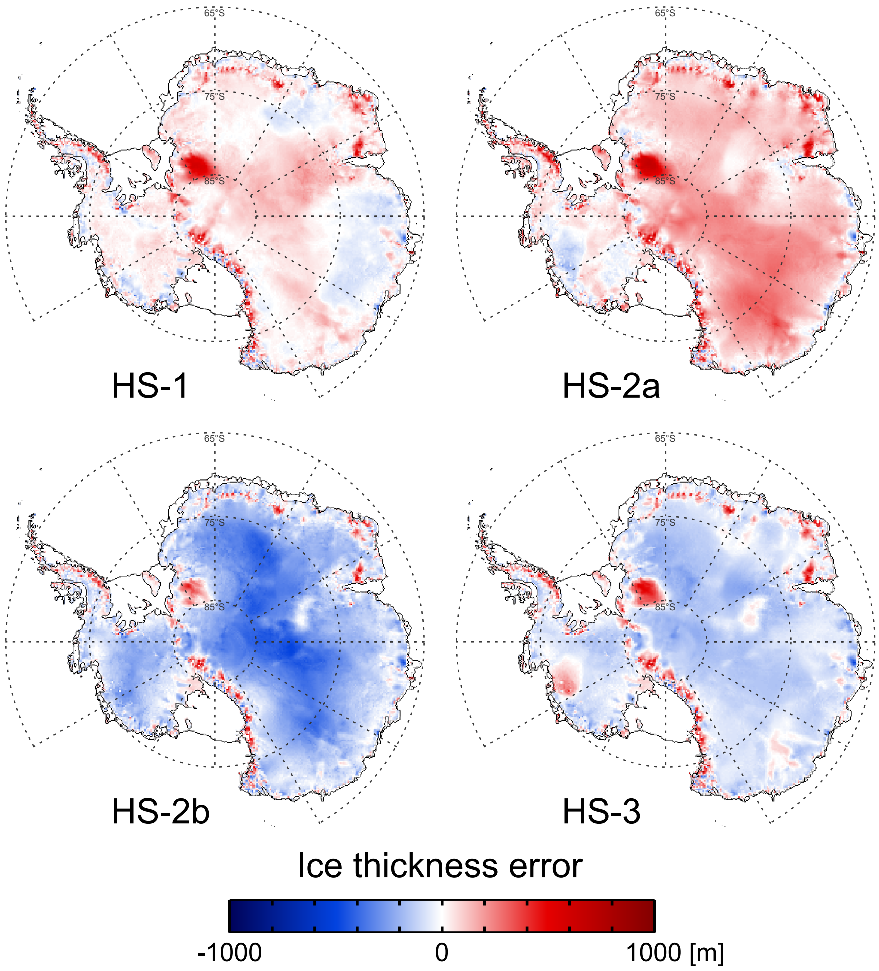

Figure 6. Difference between the modelled and observed ice thickness for all hybrid schemes, in m, after ~~an~~ additional $100\,\mathrm{kyr}$ ~~run~~ runs in which a median of the inferred distributions of C_0 is prescribed at the ice sheet base. The values of C_0 derived from the SoS are not included in the median distribution.

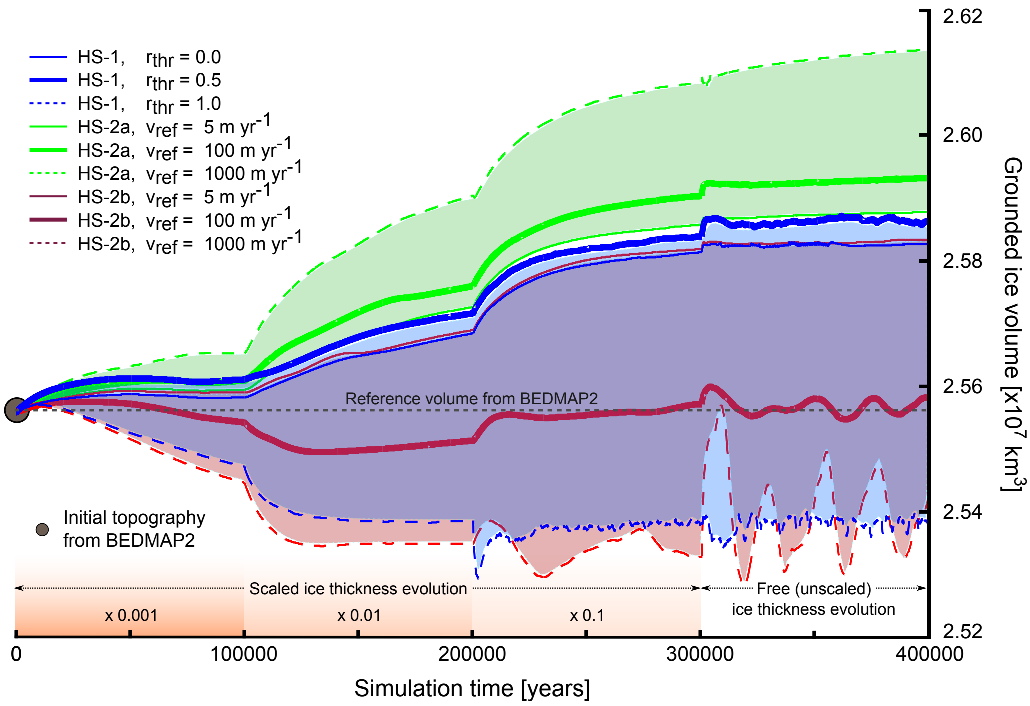

Figure 7. The evolution of the total grounded ice volume during the calibration procedure, in km^3, for different values of free parameters included in the computation of the SStA weight. The volume spread contains the reference values used for the experiments discussed in Sections 5.1–5.3 (solid thick lines), and values that are representative of the lower and upper limits of possible parameter ranges (thin solid and dashed lines, respectively).

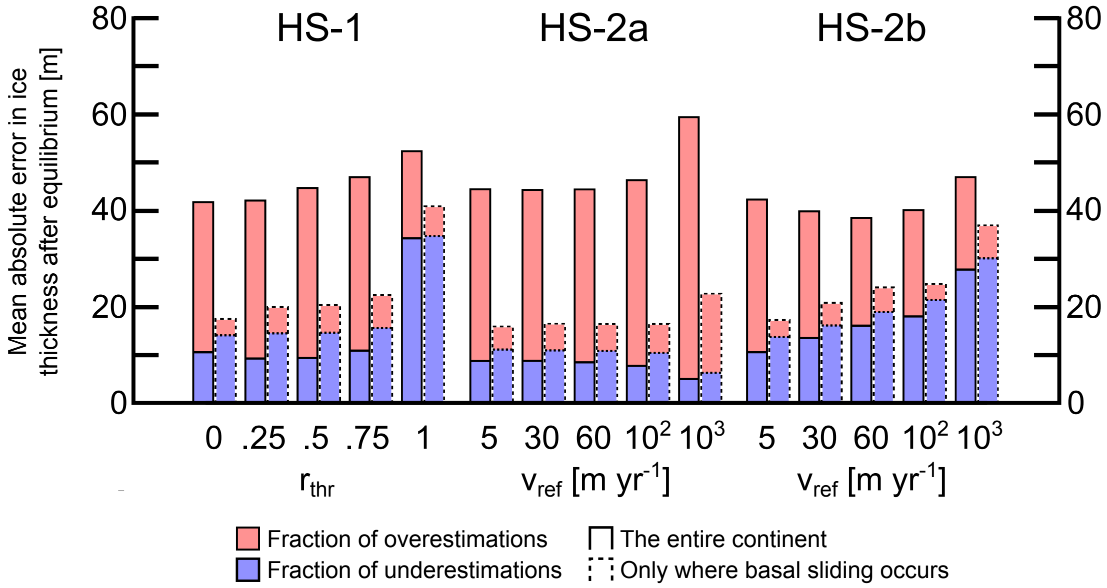

Figure 8. Mean differences between the modelled and observed ice thickness ~~throughout~~ at the end of the equilibrium simulations, in $\times 10^7\,\mathrm{km}^3$, for different values of free parameters included in the computation of the SStA weight. As in Fig. 2, a mean error is calculated for the entire ice sheet (left bars), and separately over the areas where basal sliding is identified (right bars). Each bar is divided into ~~fractions of the error~~ relative errors arising from under- (blue) and overestimations (red).