Reply to Referee 1:

Many thanks for your careful reading and your helpful comments and suggestions. Please find below point-by-point replies (in black) to your comments and questions (which are reprinted in blue). To give you an overview of all the changes in the paper, we also provide a diff-document that highlights the changes between the initial submission and this re-submission.

Major Comments:

1. This manuscript presents a new model that carry out inversions for the basal geothermal heat flux from surface velocity observations. This is the first piece of work I am aware of to present such a model, and does so very clearly. The manuscript is extremely well written and I would say it is almost ready to be published as it is. Not being an expert in numerics I am probably not the best person to assess the discussion in section 4, but hopefully another reviewer can analyse this section more carefully.

   We thank the reviewer for appreciating our work and the effort that went into it.

   The one major thing I question about this work is the applicability of it to real data. Given uncertainties surrounding other parameters, could such an inversion really give us sensible predictions for the geothermal heat flux? This question does not take away from the interest of the paper as a mathematical exercise, but since it is for a cryospheric journal it would be good if the potential of the model (or lack of) was discussed a bit further in this paper. To address this point I suggest you add in a section before the conclusions about applicability of the method and future work. You say in the first paragraph of the introduction that you want to study the prospects for, and limitations of, inferring the geothermal heat flux form surface ice velocities, but where really is any discussion of this? Your assessment of the ability to invert depending on length scales of heat flux and the noise level in velocity observations is obviously very relevant but how does this correspond to what we expect from real data?

   The main point of our paper is the presentation of a (to the best of our knowledge) new formulation for inference of the basal geothermal heat flux from velocity data, and the presentation of a scalable algorithm for the solution of this problem. We have extended a paragraph on the applicability of the method and the assumptions we are making on page 2. While we use model problems in this present paper, the computational methods are designed such that they will scale to realistic large-scale problems and work, in principle, with real data. Note that while being simple, the model problems we use are realistic in terms of dimensions and geometry, constitutive parameters and boundary conditions. While real world problems are without a doubt more complex and challenging, we believe that our model problems are useful and the qualitative results regarding the inversion of geothermal heat flux variations are relevant also for problems with real data and geometry.

   Minor Comments:

2. Abstract, lines 17-19 Long sentence. Split into two.

   In the revised manuscript, we have split this sentence into two.

3. Page 6, line 15. Aren't there some more recent estimates for geothermal heat flux in certain areas of Antarctica? e.g. Fisher et al, 2015, Science.

   We thank the reviewer for pointing out this reference. We incorporated it into our introduction.

4. Equations 19-23 Could we have a table of variables etc to reference? Had to spend some time going back to remind myself of what e.g. B^* is.

   In the revised manuscript, we have added a new table (see Table 1) that lists the variables used in the forward and adjoint problems. We have also added a description of the adjoint operator B^* below Equation (25).

5. Equations 19-23 explain why some terms in blue.

   Does the reviewer refer to Equation 31-34? The blue terms in these equations correspond to terms in the Hessian expression that involve the adjoint variable, which are neglected for a Gauss-Newton approximation of the Hessian. We explain this on page 11, lines 6-14.

6. page 15, line 5 Figure 3 referenced before Figure 2 (page 16, line 10). Swap order of figures.

   We removed the first (unnecessary) reference to Figure 3 such that now they appear in the correct order.

7. Figure 4 Legend label overlapping with surrounding box.

   The legend box has been removed from both Figure 4 and Figure 5.