V. Danserau et al.: Maxwell-Elasto-Brittle rheology for sea ice modelling

In this article, the authors present a viscoelastic damage model of sea ice mechanics. In contrast to existing models, this model takes into account transient elastic deformations and dynamic stress redistributions. Furthermore, a numerical solution of the model is applied to a simple test geometry, and the scaling behavior with respect to time and space discretizations is investigated. In my opinion, this is a very smart generalization of the modeling frameworks existing today. The concept is promising and deserves to be published.

Nevertheless, I think that the article still requires a number of changes and improvements in order to make it stand out. There are mainly three points which I am concerned about, and which the authors should revise/discuss more in detail:

The **literature discussion** in the introduction is very much focused on sea ice modeling. Damage mechanics has been a vital topic within the glacier and ice sheet modeling community in recent years though, and – even though length, time and stress scales may differ by several orders of magnitude – the models used in this context are conceptually not fundamentally different from those used for sea ice. Particularly, the pioneering work by Pralong et al. (2006) (and several other articles by the same authors) might be important; but also later contributions, such as Duddu and Waisman (2013), could potentially be relevant.

Damage framework: In the damage mechanics literature, a number of different damage measures have been proposed. They differ in how they are included in the constitutive framework and in their level of anisotropy. The damage evolution has to be prescribed depending on the choice of the damage measure. I think the authors should explain their choice in more detail! Why does d_{crit}, and thus the stress, enter the damage evolution equation linearly? The level of anisotropy present in a damage theory is reflected by the tensorial structure of the damage measure. In this light, using a scalar damage measure in a framework which is supposed to model induced anisotropy should carefully be justified. Furthermore, damage healing is known to be delicate with respect to the entropy principle (see also Pralong's article cited above). I am not sure to which extent this issue affects the healing parameterization given in this work, but at least I think it should be

addressed as a possible problem.

Language: As far as I can tell (English not being my first language), the language of the article frequently sounds a little clumsy, or at least significantly deviates from usual conventions of scientific English. I suggest it to be revised by a native speaker of English.

Apart from that, I would like to suggest a number of minor changes (which the authors are free to accept or not). A list of them is attached below.

Zurich, March 7, 2016 Arne Keller

Suggested corrections/changes:

- p.2 l.6 (and many other places): "constitutive relationship:" Even though admittedly a constitutive relationship sounds very romantic, I guess the more common terminology in this context is "constitutive relation."
- p.3 l.25: "physically consistent with observed behavior:" Is it physically consistent, or consistent with observed behavior, or both? In either case, that should be made clear.
- p.5 l.9: "processing... method:" I am not sure whether "processing" is the right word for this.
- p.5 l.11: "continuum solid:" Shouldn't this be a "continuous solid?"
- p.5 l.25-p6l2: Maybe make two sentences out of this in order to make it more readable.
- p.6 l.5: "material's velocity" rather "the velocity of the material" (as the material is not a person).
- p.6 l.13f: "the ration of ... and of the time": drop second "of"
- p.6 l.25: add commma after "However"
- p.7 l.1: "i.e. .. forcing" what do you mean by this?
- p.7 l.5 what does "such a model" refer to?

- p.7 l.20ff: I do not quite understand why the use of a viscoelastic constitutive relation has to be justified from rock mechanics? There is a large literature about viscoelasticity of ice.

- p.8 l.26ff: Even if I feel bad about this self promotion, but a very similar model for glacier ice has been proposed by Keller and Hutter (2014).

- p.9 l.23: I guess the strain rate tensor is the symmetric gradient of the velocity? Please state this explicitly. Furthermore, I think something should be said about how strain rates and strains are related (the notation suggests that the strain rate tensor is the rate of the strain tensor, which is generally not the case).

- p.10 l.20: Maybe write this as equation.

- p.11 l.4: The principal stresses are eigenvalues, not components.

- p.13 2nd paragraph: The physical interpretation of the damage variable crucially depends on how it affects the stress-strain relation. Therefore, I think this should be discussed right when defining the damage measure.

- p.13 l.22f: Please define h.

- p.14 l.22ff: "Time steps" and "elements" are concepts of numerical methods. One should be careful with using them for motivating the governing equations of a model. In physics, space and time are not discrete.

- p.15 l.1: Equation numbers missing? Please give a clear definition of ϵ (particularly in the context mentioned above, strains and strain rates).

- p.15 l.6: Again, principal stresses are not stress components (they do not have the transformation properties of tensor or vector components).

- p.17 l.10: It may be overly rigorous, but I think it is not a proper use of the Landau notation to write $\mathcal{O}$(some constant number)

- p.17 l.19: "modelling," vs. e.g. p.16 l.15, "parameterizations:" It is better style to consistently use either British or American spelling.

- p.18 l.3: "the the"

- p.18 l.23: "... are entirely defined by ... :" Either they are well-defined or not, there is nothing in between, so no need for this emphasis. Maybe better rephrase to "... only depend on"

- p.18 l.24: "the constitutive equation..." please specify which one (the constitutive framework consists of more than one equation).

- p.19 Eqs. 12-13: I think this regularization technique would be worth a more concise discussion: Is this purely a method to ensure convergence of the numerics? (In this case I'd suggest not to mix this with the physical governing equations). Or is this a conceptual problem? Finally, this is a known problem in continuum damage mechanics that the more damage is increasing the weaker its physical interpretation becomes.

- p.19 l.17f: "we take this approach.... but it had really no impact on our results.... :" This sounds very sloppy (and the switch from present to past tense is somewhat random).

- p.20 l.1: "a 2-dimensional plate ... and a constant healing rate ..." What are the consequences of the two-dimensional plate assumption? Probably a simpler velocity field? Apart from that, it sounds odd to me to squeeze those two assumptions (concerning completely different parts of the model) into one sentence.

- p.20 l.6: Not sure whether "assimilate" is the right word for this.

- p.20 l.9f: What does "internal stress" refer to, Cauchy stress? And shouldn't it rather be distributed "over" the depth? What is the advantage of keeping the entire stress tensor?

- p.20 l.14: Definition of the ice concentration?

- p.20 l.17ff: Rather make two sentences out of this.

- p.21 l.3: What is the reason not to write $A = 1$?

- p.21 l.6: "c*" is the * a typo?

- p.21 l.20f: "the dynamical system... read:" should be singular ("reads").

- p.22 l.11: "...parameters must evolve within..." so, they evolve while the model is running? This would change the dynamical equations. Otherwise, rather rephrase this.

- p.22 l.14: "characteristic time for damaging" Rather write "damage evolution" instead of "damaging" (idem in several other passages).

- p.22 l.23: typo "*One* the one hand"

- p.23 l.6: "propagation of the damage:" drop the article.

- p.23 l.13ff: "This separation.... calculations:" Somewhat weird semantics and ambiguous syntax in this sentence. The content absolutely makes sense though. Maybe rephrase this (and split into two sentences).

- p.23 l.17: "Considering the estimates ... aforementioned:" Either write "the aforementioned estimates" or "the estimates ... mentioned before."

- p.24 l.4: "over undamaged... areas:" rather "in" (idem l14).

- p.24 l.13: typo "euqation"

- p.24 l.15: "To get round this problem:" sounds very sloppy....

- p.24 l.21: ".. function, unnecessary:" no comma (maybe rather write "is unnecessary").

- p.25 l.5: "... transmitting the damage information within the material:" Not sure whether "within" is the appropriate preposition.

- p.25 l.6ff: I don't quite understand this logic... If the waves are not resolved, why should they be filtered out of the solution? Furthermore, replace "the model's solution" by "the solution of the model" (A model *can* be a person, just in this case for sure it is not ;-))

- p.25 l.25: Rather "*in* a material..."

- p.26 l.4: "time derivative for the Cauchy stress..." derivative *of*

- p.26/27, description of the test geometry: I don't quite understand the boundary conditions. Is the velocity on the lower (short) edge set to $(0, 0)$ or may there be a non-zero component in x-direction? Concerning the lateral boundary, in the text it says "no confinement is applied on the lateral sides" (that is, the boundary may move freely?), whereas in the sketch (Fig. 4) it says $\mathbf{u}(x, y) = 0$, thus the velocity is fixed. Please clarify these ambiguities! Furthermore, if the lateral boundary *is* fixed, what happens to the inflowing ice mass if the ice thickness is kept constant?

- p.27 l.26f: "so that to be representative:" the conjunction "so that" should not be followed by an infinitive.

- p.28 l.1: drop the comma.

- p.28 l.10-26: Maybe move this to the literature discussion?

- p.29 l.8: "... is its strong anisotropy" would this finding not call for the use of a tensor damage variable? Please explain why a scalar damage model is sufficient.

- p.30 l.15 "over undamaged parts" rather "in undamaged parts?"

- p.30 l.19: "spatial distribution of the damage criteria" isn't the damage criterion the same everywhere? Only its parameters vary.

- p.32 l.6: "we use the output of strain rate fields from simulations..." isn't it rather the output of simulations, not that of strain rate fields?

- p.32 l.24: "....and clusters in space ..." This seems somehow syntactically lost in the sentence?

- p.34 l.9: "of the discrete failure events:" drop the article.

- p.34 l. 25: "... the of damage rate are anti-correlated" something missing here?

- p.35 l.6: "mean total deformation $\langle \dot{\epsilon}_{\mathrm{tot}} \rangle_t$" that's a deformation rate, not a deformation.

- p.36 l.7: "permanent deformations *within the material*" There are no deformations outside of the material, so no need to state this.

- p.36 l.9 "show the Maxwell-EB model simulates...." maybe make the beginning of the subordinate clause clear by using "that". Or even better split into two sentences.

- p.36 l.26: "internal stress within the material" why not just "the Cauchy stress"? Or even "the stress?" I guess there are neither external stresses, nor stresses outside of the material....

- p. 37 l.1: "of the material" instead of "the material's"

- p.37 l.20: "carrying numerical experiments" rather "carrying *out*"

- p.39 l.20: typo "shear faaults"

- p.50, Fig. 6b,c: Excessive use of colored plots. Except for the yellow one, they all look more or less the same to me. I am probably not the only one who will have trouble distinguishing those plots: statistically, you can expect that about one out of ten male readers has a similar color vision deficiency. So it probably makes sense to use dash patterns instead, or to add plot labels.

- p.51, Fig. 7a (and various other figures): Are the units dimensionless? If so, this should be made clear, e.g. with the tilde notation used in the text.

References

Duddu, R. and H. Waisman, 2013: A nonlocal continuum damage mechanics approach to simulation of creep fracture in ice sheets. *Computational Mechanics* **51**(6), 961–974.

Keller, A. and K. Hutter, 2014: A viscoelastic damage model for polycrystalline ice, inspired by weibull-distributed fiber bundle models. part I: Constitutive models. *Continuum Mechanics and Thermodynamics*, 1–16.

Pralong, A., K. Hutter, and M. Funk, 2006: Anisotropic Damage Mechanics for Viscoelastic Ice. *Continuum Mechanics and Thermodynamics* **17**(5), 387–408 doi:10.1007/s00161-005-0002-5.