

Interactive comment on “Theoretical framework for estimating snow distribution through point measurements” by E. Trujillo and M. Lehning

Anonymous Referee #2

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Trujillo and Lehning present a theoretical model for quantifying errors/uncertainty in snow measurements (in this case snow depth) and demonstrate the validity of the model using LiDAR data from the CLPX project (Colorado, USA) at a forested site and at a wind-influenced area. This work provides an important contribution to snow science, as there has been minimal attention to errors associated with the design of snow surveys. To my knowledge, the framework appears to have no technical issues.

The authors apply it for a number of different examples that will be of interest to snow field studies. I recommend for publication in The Cryosphere after attention to a suite of (mostly minor) comments below.

Response: *We would like to thank the reviewer for his insightful and encouraging comments. We hope to clarify all concerns and address all suggestions below. The suggestions have been very useful to improve the quality of the revised manuscript.*

GENERAL COMMENTS

- The mathematical framework is likely to be new material to some snow scientists, so I would argue that additional citations and an expanded background section should be included to provide resources for learning about the methodological framework. I suspect some may be unfamiliar with first/second order stationary processes, and most equations (after equation 3) may not be obvious to all readers. This would help to maximize comprehension of the theoretical framework.

Response: *Thank you, we have now added some additional explicit clarification/references in the Background section where appropriate in response to this comment and the recommendations from Reviewer 1. The original manuscript refers to the main implications of first and second order stationarity where the assumption is mentioned in the section, and we have added references to textbooks where the concept is fully explained. Also, we have included reference to this assumption in the “Summary and Conclusions” section (see response to first Specific Comment). The new references are:*

Journel, A. G. and C. J. Huijbregts (1978). Mining Geostatistics. Academic Press, London.

Cressie, N. (1993). Statistics for spatial data. John Wiley & Sons, Inc, USA.

- The study was conducted with LiDAR data near peak snow conditions (assuming nearly full snow cover). How might the analysis and the theoretical framework change in a situation with patchy or incomplete snow cover, such as during the middle of the snowmelt season? Please discuss.

RE: *Let us point out that the methodology is not really suitable for patchy/discontinuous snow covers because the spatial probability distribution of snow depth (and SWE for that matter) would now take the form of a truncated distribution with a discrete non-zero probability (or density if probability density function) at zero equal to the area with no snow cover. Such situation requires a different approach in*

which, for example, samples from the areas with no snow covered can be considered as censored samples, and as such would need a different development not considered here. It is however an interesting question, but not addressed in the context of this article. We added a discussion to this in the “Summary and Conclusions” of the revised manuscript.

SPECIFIC COMMENTS

- The development of the framework (section 2) would be better supported if the authors cited prior works that demonstrate snow distributions can be approximated as 1st or 2nd order stationary processes and with exponential decay (hence providing the basis for their hypothesis in the theoretical framework).

RE: *Unfortunately, there are not many examples that refer to this in the context of snow distribution in the literature, but we have added some text to discuss this approximation in the “Summary and Conclusions” section of the revised manuscript. The text reads:*

“Here, we should highlight some of the implications of the assumptions made in the model. In simplified terms, the second-order stationarity assumption implies that the mean and the variance of the process/variable (e.g., snow depth) are independent of the spatial location, and that the covariance is dependent only on the separation vector (i.e., lag). Although these assumptions may not be as adequate over larger scales (e.g., greater than 100 m), at smaller scales the assumption in the context of the model application to snow depth should be valid. We present these examples to show how the error can be quantified with good accuracy around such smaller scales. Application of such types of approaches at larger scales will require additional evaluations with particular attention as to what the specific demands of the application are”.

- Does the theoretical model match the observations only because the exponential decay exponent was fitted to the data (e.g., page 14, lines 2-7)? Were independent data used to fit the exponent? If not, then there is a problem here where the same LiDAR data are used for both fitting and for model evaluation. Please clarify and rectify this issue.

RE: *The model requires prior assumption of a correlation/covariance model and estimates of the parameters of this model (e.g., the decay exponent for the exponential case). In the implementation we indeed use the LIDAR data for the parameter estimation, which we have done to illustrate the applicability of the model and its ability to estimate the error using real snow depth data. We have added a discussion of this in the “Summary and Conclusions” section. We clarify how the model can be implemented if prior information is not available, which directly addresses this comment. And to help clarify why this implementation is possible, we would like to make the following remarks that support the applicability of the model for snow depth:*

1) The sensitivity of the error to the exponential decay exponent shown throughout the article illustrates that even in the event of under or over estimation of the value of the decay exponent, the bias in the error estimation and optimal locations is small, which shows quantitatively that the model is robust for the application illustrated here. And with regard to this, we intend to test the model using multiple other datasets as they become available, using environments with different characteristics to further illustrate the applicability of the model in several conditions. These tests are not

included as part of this article as the length and extend of the article would be much longer than it already is, the main focus of this manuscript is to present the approach in detail in hope that the community builds on its applicability, aside from our own application to other datasets after this presentation.

2) Snow distribution in mountain environments has been shown to be consistent within season and from year to year because the controlling processes are relatively consistent from season to season (Deems et al., 2008; Sturm and Wagner, 2010; Schirmer et al., 2011; Melvold and Skaugen, 2013; Helfrich et al., 2014). For example, predominant wind directions lead to the formation of snow drifts in locations consistent from year to year according to these predominant directions and the relative orientation of topographic features. Similarly, in forested environments, canopy interception is a dominant factor affecting the spatial patterns of distribution of snow (Trujillo et al., 2007; Trujillo et al., 2009), which in the absence of major disturbances, remains a dominant process from season to season, and also within the season. These observations suggest that the correlation/covariance model should also be consistent, as well as the parameters of the model. These parameters can be estimated via a dense survey of one or more small plots of a size similar to that that is aimed to be represented, or via Laser scanning. These surveys would not have to be repeated as the parameters and covariance models should be preserved. We will be working on further illustrating this with further implementation of the method here.

- It was not clear to me why the three-point profile analysis (section 4.2) and the fivepoint two-dimensional analysis (section 5.2) were conducted. These types of manual snow surveys may be somewhat unconventional, as it is more common to have 10 or more measurements made in a single profile. The main example that I can recall that had 3-5 point types of surveys is the CLPX experiment (see Elder et al. 2009), which was a specialized measurement campaign, so I think it is necessary to cite this study (and any other studies with similar snow surveys) in order to clarify the motivation for the analyses in section 4.2 and 5.3. The other two analyses have clear relevance for snow depth stations (one point) and more typical snow surveys (n points), but the other analyses needs more definitive motivation.

RE: *Thank you for pointing this out, and we hope to clarify this here. We are including these cases for the following reasons: the two cases with a fixed number of measurements (1D/3-point, 2D/5-point) provide the opportunity to illustrate how the methodology can be used to determine an optimal location on the basis of error minimization. The 3-point example is used to show how an explicit analytic solution can be achieved, and how such analytical solution does indeed correspond to the observations using the case of snow depth. The 5-point example is then an extension of this, although for this 2D case, no explicit analytical solution can be achieved, and the methodology can only be applied numerically. The former case presents a more formal and elegant solution, and the latter presents a more practical solution, and one that will most likely be faced in practice. Also, let us mention that there are indeed a number of examples in the literature that do use these 3-point and 5-point pattern, in some cases without addressing the issue of the representativeness of the measurements (e.g., Molotch, 2009; Jepsen et al., 2012)*

- If researchers were planning a snow survey (1-d profile instead of 2-d grid), how might they estimate the exponential decay exponent (v) in order to estimate a_{optimal} ? (page 16) The v values considered here vary by an order of magnitude and are demonstrated to have a substantial impact on the normalized squared error (Fig

10). The authors suggest (page 18, lines 5-7) that LIDAR or dense manual measurements can characterize the covariance/correlation characteristics, but what can be done if resources are unavailable for application of these detailed methods (a realistic scenario that is acknowledged by the authors, page 25, lines 1-3)? If resources are available for LiDAR surveys, then this eliminates the need for manual snow measurements in the first place (and determination of the v parameter), so this issue needs more attention in the text.

RE: *Thank you again as this is indeed an important point. As mentioned above, one issue in favor of the approach is the consistency of the spatial patterns of distribution of snow depth. This implies that because of this consistency, the parameters to characterize the spatial memory do not change as much to require such repeated measurements. For example, a detail survey can be conducted under different conditions to characterize the range of the correlation parameters. With these parameters, not only the parameter that could be used for a particular set of conditions (e.g., after a snow storm, or close to peak accumulation) can be estimated, but also the range in the error and/or optimal locations can be estimated. Another possibility is to perform such surveys (e.g., with TLS) once for several conditions, and characterize the range of possible values for the correlation function parameters. Also here, we should point out that although we show results for a wide range of the exponential decay exponent values, we are finding that most of the values that we have observed are in the lower range of those presented (e.g., $0.1-0.2 \text{ m}^{-1}$). Hence, the biases in the estimated error and the survey design remain small. We would like to find out more about this using more comprehensive dataset including areas with spatial fields repeated throughout a season and multiple seasons. However, this type of testing will require a significant effort and we would not be able to include such results in the extent of just one manuscript.*

- Why is the error less sensitive to the exponential decay exponent in the two-dimensional case relative to the one-dimensional case (page 23, lines 18-21)? Does this suggest that snow surveys should always be done in a gridded fashion instead of a profile? I think this is a very useful result and so it would be helpful if the authors explained better why this result emerged.

RE: *Thank you. First, let us point out that the number of measurements increases as a function of the square of the number of points along each axis (e.g., 1, 4, 16, ..., N^2), which partly explains why the error decreases so rapidly. Beyond a certain number of measurements regularly distributed in the area, the measurements gather enough information such that there is only a very minor gain with adding any new measurements, regardless of the exponent value. Also, the little sensitivity to v comes from the fact that very rapidly, the increase in the number of points is such that much more information about the surrounding area is gathered by the points. As the reviewer points out, this type of stratified sampling is very efficient at reducing the error and capturing information within an area. If the objective is to only represent a profile section, the stratified sampling provides also a rapid decrease in the error with N , although if the profile were to provide information of the surrounding area (not just the line), a profile would not be as representative as a 2D distributed stratified grid. We added a short reference to this in section 5.3.*

- It would be helpful to have more information about the LiDAR data in Section 3. From what is presented in this section, it is unclear what spatial resolution is achieved with the LiDAR (though this is identified in the abstract and later in the text at page

18, line 12) and also there is no discussion of the uncertainty in the LiDAR measurements (e.g., vegetation-induced errors, slope-induced errors, errors in the snow-on vs snow-off scenes, etc.). How does uncertainty in the LiDAR measurements impact the robustness of the analysis?

RE: *We have added some clarification addressing this comment in the Data section (3). Unfortunately, the manual snow surveys during CLPX were performed about two weeks earlier than when the flights were performed and little could be said about the uncertainty of the LIDAR snow depths. However, the dataset has proven to be highly valuable to address some important scientific questions and we believe that despite of this question, it provides us with an extensively used example. Further testing with other datasets will build on the implementation shown here.*

TECHNICAL COMMENTS/CORRECTIONS

- Page 4, Lines 20-21: It sounds redundant to say “. . . use point measurements to represent snow distribution from point measurements”. I recommend deleting one of the phrases with “point measurements”.

RE: *Thank you. We have eliminated the second one.*

- Page 7, Line 5: Add “a” before “function”.

RE: *Corrected.*

- Page 8, Line 23: delete “at” after “represented” (“at” is already included before “which”).

RE: *Corrected.*

- Page 9, Line 26: Please clarify what the “x and y” directions represent. Is x east-west, y north-south?

RE: *Yes. We added “(east)” and “(north)” after x and y, respectively.*

- Page 9, Line 29: The phrasing “overestimate in absolute value terms” is somewhat confusing because it no longer directly conveys that the point values tend to have severe underestimation of snow depth for cases of snow depth below the mean value. Consider rephrasing.

RE: *We realize that the wording might be a bit confusing here. Perhaps we can express this better here. Ultimately, let us consider here two snow depth profiles, one with the snow depths at the nominal scale (~1 m), and a second one with a moving average (MA) of the first one with an averaging window equal to the sampling spacing. Ultimately, the variance/standard deviation of the first profile (~1 m) is larger than that of the MA, with a distribution that reflects these differences. The samples from the first profile will reflect a larger variance than that of the samples from the MA profile as they are drawn from these distributions, and this is what is reflected in the figures to which the text refers to here. We have included this clarification in the text in section 4 as suggested.*

- Page 12, Line 5 (Eq 6): I do not think that “C” has been defined. I assumed it is the covariance?

RE: *Thank you, this is correct. The equation has been corrected to “COV”.*

- Page 13, Line 11: It should read “shorter” instead of “sorter”.

RE: *Corrected.*

- Page 13, Line 27: It should be "consists" instead of "consist".

RE: *Corrected.*

- Page 14, Line 21: It should read “three measurements” (plural) in the subsection title.

RE: *Corrected.*

- Page 15, Line 14: This should either read “as an advantage” or “as advantageous”.

RE: *Corrected.*

- Page 16, Line 6: Delete “to” after “estimation of”.

RE: *Corrected.*

- Page 17, Line 3: It should read “N measurements” (plural) in the subsection title.

RE: *Corrected.*

- Page 18, Lines 3-4: I recommend adding Sturm and Wagner (2010) to the list of citations here.

RE: *Thank you, we have included the reference.*

- Page 23, Lines 8-10: These results are for the two-dimensional case at RW, but how does the model perform at RW?

RE: *We have focused on the isotropic case for the two-dimensional examples. The reason for this is mainly for simplicity as we present the method in the context of snow depth. We intend to build on this as we extend the developments to the anisotropic cases, which would be required for the case of RW. The snow depth field in FS has been shown to exhibit little directionality and low anisotropy versus the field in RW (Trujillo et al., 2007). This is however an important point by the reviewer, and this analysis will need to complement further application of this method, especially for open environments with strong influence of wind redistribution of snow and/or preferential deposition driven by the interaction with topography.*

TABLE AND FIGURE COMMENTS

- Figure S1 (supplement) – This figure appears to be missing. There are supposed to be three figures in the supplement, but only two are provided.

RE: *We apologize if this is the case. However, in the pdf that we can still download from the website we see the three supplementary figures. Figure S1 illustrates the balance between the different terms of the equation. There may have been a problem with the pdf, in which case we would be happy to provide a new version.*

CITATIONS

Elder, K., Cline, D., Liston, G. E., & Armstrong, R. (2009). NASA Cold Land Processes Experiment (CLPX 2002/03): Field Measurements of Snowpack Properties and Soil Moisture. *Journal of Hydrometeorology*, 10(1), 320–329. doi:10.1175/2008JHM877.1

Sturm, M., and A. M. Wagner, 2010: Using repeated patterns in snow distribution modeling: An Arctic example. *Water Resour. Res.*, 46, 1–15, doi:10.1029/2010WR009434.

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- Trujillo, E., Ramirez, J. A., and Elder, K. J.: Scaling properties and spatial organization of snow depth fields in sub-alpine forest and alpine tundra, *Hydrol. Processes*, 23, 1575–1590, doi: 10.1002/hyp.7270, 2009.

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2 THEORETICAL ANALYSIS OF ERRORS WHEN ESTIMATING SNOW DISTRIBUTION

3 THROUGH POINT MEASUREMENTS

4

5 By

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ESTIMATING SNOW DISTRIBUTION THROUGH
POINT MEASUREMENTS

Abstract

In recent years, marked improvements in our knowledge of the statistical properties of the spatial distribution of snow properties have been achieved thanks to improvements in measuring technologies (e.g., LIDAR, TLS, and GPR). Despite of this, objective and quantitative frameworks for the evaluation of errors and extrapolations in snow measurements have been lacking. Here, we present a theoretical framework for quantitative evaluations of the uncertainty of point measurements of snow depth when used to represent the average depth over a profile section or an area. The error is defined as the expected value of the squared difference between the real mean of the profile/field and the sample mean from a limited number of measurements. The model is tested for one and two dimensional survey designs that range from a single measurement to an increasing number of regularly-spaced measurements. Using high-resolution ($\sim 1\text{m}$) LIDAR snow depths at two locations in Colorado, we show that the sample errors follow the theoretical behavior. Furthermore, we show how the determination of the spatial location of the measurements can be reduced to an optimization problem for the case of the predefined number of measurements, or to the designation of an acceptable uncertainty level to determine the total number of regularly-spaced measurements required to achieve such error. On this basis, a series of figures are presented that can be used to aid in the determination of the survey design under the conditions described, and under the assumption of prior knowledge of the spatial covariance/correlation properties. With this methodology, better objective survey designs can be accomplished, tailored to the specific applications for which the measurements are going to be used. The theoretical framework can be extended to other spatially distributed snow variables (e.g., SWE) whose statistical properties are comparable to those of snow depth.

47 The assessment of uncertainties of snow measurements remains a challenging problem in
48 snow sciences. Snow cover properties are highly heterogeneous over space and time and the
49 representativeness of measurements of snow stage variables (e.g., snow depth, snow density, and
50 snow water equivalent (SWE)) is often overlooked due to difficulties associated with the
51 assessment of such uncertainties. This has been, at least in part, due to the limited knowledge of
52 the characteristics of the spatial statistical properties of variables such as snow depth and SWE,
53 particularly at the small-scales (sub-meter to tens of meters). However, a turning point has been
54 reached in recent years thanks to improvements in remote sensing of snow (e.g., light detection
55 and ranging (LiDAR) and Radar technologies), which have allowed significant progress in the
56 quantitative understanding of the small-scale heterogeneity of snow covers in different
57 environments, with resolutions and areas of coverage previously unresolved with the standard
58 methods of measurement (e.g., Trujillo et al., 2007; Trujillo et al., 2009; Mott et al., 2011).

59 Point or local measurements of snow properties will continue to be necessary for purposes
60 that range from inexpensive evaluation of the amount of snow over a particular area, to
61 validation of models and remote sensing measurements. Such measurements have a footprint
62 representative of a very small area surrounding the measurement location (i.e., support,
63 following the nomenclature proposed by Blöschl (1999)), and the integration of several
64 measurements is necessary for a better representation of the snow variable in question over a
65 given area. Because of this, tools for quantitative evaluations of the representativeness and
66 uncertainty of measurements need to be introduced, and the uncertainty of such measurements
67 should be more widely discussed in the field of snow sciences.

68 Currently, efforts to assess the reliability and uncertainty of snow measurements have
69 focused on statistical analyses using point measurements (e.g., Yang and Woo, 1999; Watson et
70 al., 2006; Rice and Bales, 2010; Lopez-Moreno et al., 2011; Meromy et al., 2013) or
71 synthetically generated fields in a Monte Carlo framework (e.g., Kronholm and Birkeland, 2007;
72 Shea and Jamieson, 2010), and comparisons between remotely sensed and ground data (Chang et
73 al., 2005; Grünwald and Lehning, 2014). These studies have been useful to empirically quantify
74 uncertainties associated with point measurements; However, these type of approaches do not
75 provide a quantitative framework for the assessment of uncertainties associated with a particular
76 sampling design, they do not allow for an optimal sampling strategy (e.g., selecting the number
77 of points and locations for a desired accuracy level), and they do not take advantage of the
78 increased knowledge of the characteristics of the heterogeneity of snow cover properties.

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79 Another possible approach is one in which the expected error in the estimation of a particular
80 statistical moment of a field over a defined domain (e.g., areal mean or standard deviation from a
81 finite number of measurements) is determined on the basis of known statistical properties of the
82 field in question. Such approach uses geostatistical principles that have been proposed by
83 Matheron (1955; 1970) and others, and that have been applied in mining geostatistics (Journel
84 and Huijbregts, 1978), the analysis of uncertainties when measuring precipitation (Rodríguez-
85 Iturbe and Mejia, 1974), and for a more general analysis of the effects of sampling of random
86 fields as examples of environmental variables (e.g., Skøien and Blöschl, 2006), among others.
87 Despite of these examples, there is to the authors' knowledge no attempt of implementing such
88 type of approach in snow sciences, tailoring the methodology to the particular analysis of
89 uncertainties when measuring snow variables such as snow depth. Such an implementation
90 appears to be lacking in numerous studies that use point measurements to represent snow

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93 | distribution, addressing the spatial extrapolation of such point measurements as the “true” spatial
94 | distribution of snow depth when evaluating the performance of interpolation methodologies,
95 | regressions trees, and hydrological models. These comparisons ignore the intrinsic error incurred
96 | when extrapolating the original point measurements, leaving a proportion of uncertainty that can
97 | be significant unaccounted for. This is the principal motivation of the present study, with the
98 | intention of spreading the use of more objective and quantitative methodologies for error
99 | evaluation in snow sciences. Also, the approach that is presented below can be used for objective
100 | survey design to estimate snow distribution from point measurements. We do not intend to
101 | present our approach as novel in the general geostatistical sense; instead, we present the
102 | derivation with the specific application for snow sciences in mind. However, because of the
103 | general nature of the random fields’ theory that the development is based on, similar
104 | developments can indeed be applied to other environmental variables that can be described as a
105 | random field.

106 | On this basis, the error in the estimation of spatial means from point measurements over a
107 | particular domain (e.g., a profile, or an area) can be quantified as the expected value of the
108 | squared difference between the real mean and the sample mean obtained from a limited number
109 | of point measurements. Such an approach, as it will be shown here, uses spatial statistical
110 | properties of snow depth fields in a way that allows for an objective evaluation of the estimation
111 | error for snow depth measurements. The sections below illustrate the use of such methodology
112 | for optimal design of sample strategies in the specific context of snow depth. However, the
113 | methodology can also be implemented for other snow variables such as snow water equivalent,
114 | given that similar geostatistics can be used to characterize their spatial organization.

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Deleted: he development of the expression to evaluate the error presented here includes some additional terms that appear in the derivation of the expression that are not accounted for in developments. These terms are needed to accurately predict the behavior that is observed when measuring snow depth from a limited number of measurements.

2 Background

Let $Z(\mathbf{x})$ denote a random field function of the coordinates \mathbf{x} in the n -dimensional space \mathbb{R}^n . Bold letters represent a location vector from hereon. In our case, the field can represent e.g.: snow depth or snow water equivalent (SWE) at a given time of the year. The mean of the process over a domain A (e.g., a profile section or an area) is defined as:

$$\mu_z(A) = \frac{1}{A} \int_A z(\mathbf{x}) d\mathbf{x} \quad (1)$$

In practice, the mean is often obtained from the arithmetic average of measurements at a finite number of locations, N , within the domain:

$$\bar{Z} = \frac{1}{N} \sum_{i=1}^N z(\mathbf{x}_i) \quad (2)$$

The performance of the estimator \bar{Z} can be evaluated by calculating the expected value of the square difference between the estimator \bar{Z} and the true mean $\mu_z(A)$

$$\sigma_{\bar{Z}}^2(A) = E \left[\left(\frac{1}{N} \sum_{i=1}^N z(\mathbf{x}_i) - \frac{1}{A} \int_A z(\mathbf{x}) d\mathbf{x} \right)^2 \right] \quad (3)$$

For a 1st order stationary process (i.e., the mean independent of location; e.g., Cressie (1993), section 2; and Journel and Huijbregts (1978), section 2), (3) can be expressed as

$$\begin{aligned} \sigma_{\bar{Z}}^2(A) = & \frac{1}{N^2} \sum_{i=1}^N VAR[z(\mathbf{x}_i)] + \frac{2}{N^2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N COV[z(\mathbf{x}_i)z(\mathbf{x}_j)] \\ & - \frac{2}{N \cdot A} \sum_{i=1}^N \int_A COV[z(\mathbf{x}_i)z(\mathbf{x}_j)] d\mathbf{x}_j \\ & + \frac{1}{A^2} \int_A \int_A COV[z(\mathbf{x}_i)z(\mathbf{x}_j)] d\mathbf{x}_i d\mathbf{x}_j \end{aligned} \quad (4)$$

140 where $\text{VAR}[\]$ and $\text{COV}[\]$ are the variance and the covariance, respectively. If we further
 141 assume that the process is second order stationary (e.g., Cressie (1993), section 2; and Journel
 142 and Huijbregts (1978), section 2), that is, if the mean and the variance are independent of the
 143 location, and the covariance function depends only on the vector difference $\mathbf{x}_i - \mathbf{x}_j$, (3) can be
 144 expressed as

$$\sigma_z^2(A) = \sigma_p^2 \left[\frac{1}{N} + \frac{2}{N^2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \text{CORR}[\mathbf{x}_i - \mathbf{x}_j] - \frac{2}{NA} \sum_{i=1}^N \int_A \text{CORR}[\mathbf{x}_i - \mathbf{x}_j] d\mathbf{x}_j + \frac{1}{A^2} \int_A \int_A \text{CORR}[\mathbf{x}_i - \mathbf{x}_j] d\mathbf{x}_i d\mathbf{x}_j \right] \quad (5)$$

146 where $\text{CORR}[\]$ is the correlation function, and σ_p^2 is the variance of the point process.

147 The first two terms in (5) are the total sum of the covariances (or correlation as σ_p^2 has been
 148 factored out) between all point locations $i = 1, \dots, N$ (e.g., measurement locations). The first of
 149 the two terms is only a function of the number of points, while the second is a function of the
 150 number of points, N , and the correlations between the locations. Such correlations are themselves
 151 a function of the separation vectors (both in magnitude and direction), and the parameters of the
 152 correlation function. These two terms are independent of the size of the area A , and can be
 153 thought of as the portion of the error caused by the correlation between the point processes at the
 154 locations $i = 1, \dots, N$ (e.g., measurement locations). Term 3 accounts for the correlation between
 155 the measurement locations and the continuous process over the domain A . This term can be seen
 156 as a negative contribution to the total error assuming that the sum of the integrals is positive. The
 157 term is a function of the number of points, N , the domain area, A , the location of the points and

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the correlation structure, characterized using the parameters of the correlation function. Lastly, term 4 is the contribution to the error caused by the intrinsic correlation structure of the continuous process over the domain. This term is a function of the domain (e.g., size and shape of A) and the correlation structure (e.g., parameters of the correlation function).

3 Data

For the analyses and tests of the methodology [presented](#) here, Light Detection and Ranging (LIDAR) snow depths obtained as part of the NASA's Cold Land Processes Experiment (CLPX) will be used (Cline et al., 2009). The dataset consists of spatially distributed snow depths for 1-km x 1-km areas (Intensive Study Areas - ISAs) in the Colorado Rocky Mountains close to maximum snow accumulation in April, 2003. [The data were processed from snow-on \(8-9 April, 2013\) and snow-off \(18-19 September, 2013\) LIDAR elevation returns with an average horizontal spacing of 1.5 m and vertical tolerance of 0.05 m. The final CLPX snow depth contour product \(0.10 m vertical spacing\) was generated from these returns. This product was used to generate gridded snow depth surfaces with 1024x1024 elements over the ISAs, for a grid resolution of 0.977 m.](#) For this study two areas will be used: the Fraser – St Louis Creek ISA (FS) and the Rabbit Ears – Walton Creek ISA (RW) (Figure 1). The FS ISA is covered by a moderate density coniferous (lodgepole pine) forest on a flat aspect with low relief. The RW ISA is characterized by a broad meadow interspersed with small, dense stands of coniferous forest and with low rolling topography. The snow depth distributions in these ISAs show differences that are relevant for the analysis of the methodology introduced here. At the FS ISA, the snow depth distribution is relatively isotropic (Figure 1b), with short spatial correlation memory and little variations in the spatial scaling properties (i.e., power-spectral exponents and scaling breaks) with direction (Trujillo et al., 2007). On the other hand, the spatial distribution of snow

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depth in the RW ISA is more anisotropic (Figure 1c), with longer spatial correlation memory along a principal direction aligned with the predominant wind direction versus shorter memory along the perpendicular direction, and with variations in the power-spectral exponents and scaling breaks according to the predominant wind directions (Trujillo et al., 2007).

4 One-dimensional process

The spatial representation of the snow cover requires a basic assumption on the scale or resolution at which a field or profile is going to be represented. This relies on the spatial support of the measurements. For the case of snow depths, point measurements from local surveys using a snow depth probe are frequently used for this representation. Generally, there are additional sources of uncertainty associated with these types of measurements, such as the accuracy of the position of the measurement in space or deviations in the vertical angle of penetration of the probe through the snow pack. These uncertainties are additional to any of the uncertainties estimated using the methodology discussed here.

The one-dimensional case provides a good opportunity to illustrate the limitations of point measurements. Consider the case of a snow depth profile that is measured using a snow depth probe at a regular spacing “d”. Each of these point measurements is meant to represent the mean snow depth over a particular distance surrounding the measurement, and the question is: over what distance is such assumption valid? In this case, the intrinsic assumption is that the measurement is representative over the distance “d”, but at this point the validity of such assumption is not proven.

The answer to this question is conditioned to how variable the profile is and over what distances. To look at this, let us look at two snow depth profiles, one in a forested environment

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(FS) and another in an open environment (RW) in the Colorado Rocky Mountains (Figure 2a and Figure 3a, respectively). The variability in the profiles is markedly different, with variations over shorter distances in the forested area, and a smoother profile in the open and wind influenced environment. This is reflected in the spatial correlation structure of these snow depth profiles, with stronger correlations over longer distances in open and wind-influenced environments with respect to that in forested environments (Trujillo et al., 2007; Trujillo et al., 2009). These differences should be considered when selecting the sampling frequency required to capture the variability and accurately represent the mean conditions within a particular sampling spacing. This is illustrated by comparing the mean snow depth for a particular resolution to the point value at the center of the interval (Figure 2b in a forested environment and Figure 3b in an open and wind-influenced environment). In the Figures, average versus point values at several sampling intervals are compared for normalized profiles ($\mu = 0, \sigma = 1$) separated every 30 m in both the x (east) and y (north) directions and for an area of 500 m by 500 m. The 30-m separation between profiles is chosen to reduce the spatial correlation between them. Firstly, the resulting comparison shows that the point values generally overestimate the variability in mean snow depths if we replace the mean snow depth distribution by its point sample. To clarify this, let us consider here two snow depth profiles, one with the snow depths at the nominal scale (~1 m), and a second one with a moving average (MA) of the first one with an averaging window equal to the sampling spacing. Ultimately, the variance/standard deviation of the first profile (~1 m) is larger than that of the MA, with a distribution that reflects these differences. The samples drawn from the first profile will reflect a larger variance than that of the samples from the MA profile as they are drawn from these distributions, and this is what is reflected in Figure 2 and Figure 3. The degree of overestimation can be quantified through the slope of the regression line (in red in

228 Figure 2b and Figure 3b). In the forested environment (Figure 2b), the slopes range between 0.8
229 and 0.13, with decreasing slopes with increasing spacing. These slopes indicate that, on average,
230 the mean values are 0.8 times the point values for the 5 m spacing and 0.1 times the point values
231 for the 100 m spacing. In the open and wind-dominated environment, the slopes are higher and
232 range between 0.97 and 0.23 from 5 m spacing and 100 m spacing, respectively. A clear
233 difference emerges: forested environments require shorter separation between single
234 measurements if the snow depth profile is to be accurately captured by the measurements. The
235 variability within the size of the interval determines the degree of uncertainty associated with the
236 point measurements, as the sub-interval variability is related to the degree of overestimation of
237 the mean value within the interval. Secondly, the differences between average and point values
238 for each spacing distance are generally more scattered in the forested environment than in the
239 open environment, and in both environments the degree of scattering increases with spacing
240 (Figure 2c and Figure 3c). However, it is important to note here that we are comparing
241 normalized profiles ($\mu = 0$, $\sigma = 1$), allowing us to focus on the rescaled spatial variations. What is
242 highlighted is the relevance of the spatial structure of the profile rather than the absolute
243 variance. This spatial structure can be quantified by, for example, the spatial
244 covariance/correlation function.

245 Additionally to the differences in the correlation structure, there are also differences in the
246 absolute variability in snow depth in these environments (Figure 4). As opposed to the
247 normalized snow depth discussed above, the subinterval standard deviation as a function of
248 interval size along the profiles is higher in the open and wind-influenced environment at RW
249 versus the forested environment at FS (Figure 4a). Mean standard deviation values in the open
250 environment are twice as large, as those at the forested environment towards the larger interval

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253 sizes (~100 m). The standard deviation increases with interval size in both environments, with
254 the steepest increase at the lower interval sizes. Furthermore, the standard deviation tends to
255 stabilize more rapidly in the forested environments, with an increase of only 1.8 cm between 30
256 m and 100 m. On the other hand, the standard deviation continues to increase in the open
257 environment at RW, with less of an asymptotical behavior for the scales analyzed.
258 Complementary, the shaded areas (25% to 75% quantiles) give an idea of the variability of
259 standard deviation values, with a much wider range in RW versus FS, and an increase in the
260 range between quantiles with interval size in RW.

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261 Consistent with the standard deviation, the sub-interval mean range (range defined as the
262 difference between the maximum and minimum snow depths within an interval) increases with
263 interval size in both FS and RW (Figure 4b). However, the mean range is larger in the open
264 environment at RW and the rate of increase with interval size is also steeper. Similarly, the
265 shaded areas indicate wider distribution of range values in the open environment at RW, while
266 relatively uniformly distributed around the mean across interval sizes in the forested environment
267 at FS. The results in Figure 2-Figure 4 illustrate this contrasting behavior between the snow
268 covers in these environments and their influence on measurement strategies: that is, the forested
269 environments requires shorter separation between measurements for accurate representation of
270 the snow cover, however, in the wind-influence and open environment, the subinterval
271 variability is higher indicating wider variations around any sampled measurement within the
272 interval.

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273 Ultimately, the number and distance between measurements and the specific arrangement of
274 the measurements are all conditioned to what the measurements are needed for. Hydrologic
275 applications may not require a highly detail representation of a snow depth profile (or a field),

281 and representing the average conditions over a given distance (or area) is sufficient, but small-
 282 scale process-based studies may require a more detailed characterization over shorter distances
 283 (or smaller areas). This implies that the decision depends on the particular use that the
 284 measurements will support. In the following sections, the equations presented in the Background
 285 (section 2) will be applied to evaluate the uncertainty associated with multiple measurement
 286 designs for profiles and fields of snow depth.

287 4.1 Case 1: Single measurement along a profile section

288 Equation (2) can be used to evaluate the uncertainty of a single measurement along a profile
 289 section of length L . For this case, as well as for the following cases in this article, an exponential
 290 covariance with a decay exponent ν ($\nu > 0$) will be assumed:

$$291 \quad COV(\mathbf{h}, \sigma, \nu) = \sigma^2 \exp(-\nu \|\mathbf{h}\|) \quad \text{for } \sigma^2 > 0, \text{ and } \nu > 0 \quad (6)$$

292 where σ^2 is the variance, and $\|\mathbf{h}\|$ is the length of the vector \mathbf{h} . For this one-dimensional case
 293 and combining (6) and (5), the following expression is obtained:

$$294 \quad \sigma_z^2(x, L, \nu) / \sigma_p^2 = 1 - \frac{2}{Lv} \left[2 - \exp(-\nu x) - \exp(-\nu \cdot [L - x]) \right] + \frac{1}{L^2 \nu} \left[2L + \frac{2}{\nu} \exp(-\nu L) - \frac{2}{\nu} \right]$$

295 (7)

296 where x is the distance from one extreme of the section to the location of the measurement
 297 (Figure 5a). The normalized squared error $\sigma_z^2(x, L, \nu) / \sigma_p^2$ is minimized at x equal to half of the
 298 section length, $L/2$, regardless of ν . The existence of a correlation in the profile leads to this
 299 solution, as the middle location contains more information about its surroundings. Also, this
 300 solution is different from the solution for an uncorrelated profile (e.g., white noise), for which

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304 the squared error would be equal to the variance, independent of the location of the
305 measurement.

306 The results here are confirmed with an analysis of LIDAR snow depths profiles in FS and
307 RW (Figure 6). The analysis consists of calculating the difference between the mean and the
308 point value for sections of a given length (varied between 10 m – 50 m) and for x (Figure 5a)
309 between 0 and L along the profile sections. Each sample section of length L will provide a single
310 difference for each of the x values. These sample differences are then used to calculate the mean
311 normalized squared error for each x , and the same is repeated for each section length L . The
312 results indicate that the real snow depth profiles behave as predicted by the model of the error,
313 with a minimum error at x equal to half of the section length. Another difference highlighted by
314 these results is the difference between the sample errors in the forested environment (FS) versus
315 the open environment (RW) for the larger interval sizes (e.g., 50 m). The sampled normalized
316 squared error in the forested environment shows only a mild decrease in the square error to
317 around 0.7-0.8 towards the inside of the section length. However, this decrease is achieved for
318 the measurement along most of the interval length with the exception of the extremes. This can
319 be explained by the relationship between the spatial memory of snow depth (e.g., the correlation
320 function) and the section length. Densely forested environments exhibit correlation lengths that
321 are shorter than those in open and wind influenced environments (e.g., Trujillo et al., 2007;
322 Trujillo et al., 2009). As the section length increases beyond such correlation lengths, a
323 measurement location towards the middle of the interval contains less information of the
324 surrounding snow depths in a forested environment (e.g., FS) versus an open and wind
325 influenced environment (e.g., RW). This is observed in Figure 6c versus Figure 6f, with the
326 results in RW showing a more clear minimum towards the center of the profile section. The

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330 results also show a poorer performance of the model in RW versus FS, as the exponential
331 correlation model has a poorer fit in RW at the shorter-lag range; However, model performance
332 is improved for longer section lengths (e.g., [Figure 6c](#) and [f](#))

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333 Model and sampled results thus support that the measurement location can be fixed in the
334 middle of the interval, and the normalized squared error can then be described as a function of
335 both, the exponential decay exponent, ν , and the length of the section, L ([Figure 7a](#)). The
336 normalized squared error increases with interval length, with a steeper increase for larger
337 exponential decay exponents, for which the squared error approaches that of an uncorrelated
338 field more rapidly. The theoretical model is tested on the snow depth fields at FS and RW. The
339 test consists of calculating the sampled normalized squared error as the average of all squared-
340 differences between the mid-section snow depth and the mean from all LIDAR grid-points
341 within each interval of length L . This is done for profiles separated every 30 m, similar to the
342 analysis above, and for profiles along the x and y directions. The theoretical normalized squared
343 error is estimated from (7) using the exponential decay exponent from the model fitted to the
344 sampled correlation function. The results show that the theoretical model reproduces the sampled
345 squared error remarkably well, even reproducing the anisotropic properties of the correlograms,
346 represented by the different exponents of the exponential model along x and y directions ([Figure](#)
347 [7b](#) and [c](#)). The model also reproduces the different behavior of the squared error between both
348 fields (i.e., FS and RW), showing that the normalized squared error increases more rapidly and is
349 larger in the forested environment ([Figure 7b](#)) versus the open environment ([Figure 7c](#)).
350 However, it should be noted here that as the error is normalized, and as the variance of the field in
351 the open environment is larger ([Figure 4a](#)), the absolute squared error could reach higher values
352 in the open environment (RW). In this regard, one feature to discuss here is the assumption that

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the point variance of snow depth in these environments has been estimated as the spatial variance over the entire study area, as it is generally practiced in time series analysis and geostatistics. In practice, this is the only possible approach because there is limited information to estimate the point variance from multiple realizations of the process at each spatial location, as inter- and intra- annual snow depth fields are not available, not only for these areas, but for almost any area where this methodology may be applied.

4.2 Case 2: Three measurements along a profile section

From (5) it is also evident that increasing the number of measurements will reduce the squared error. In the case of three measurements separated by a distance 'a', with the middle measurement centered in the section of length L (Figure 5b), and for an exponential covariance function with parameter v, (5) leads to the following expression for this particular case:

$$\begin{aligned} \sigma_z^2(a, L, v) / \sigma_p^2 = & \frac{1}{3} + \frac{2}{9} [2 \exp(-va) - \exp(-2va)] \\ & - \frac{4}{3Lv} \left[3 - \exp\left(-\frac{vL}{2}\right) (1 + \exp(-va) + \exp(va)) \right] \\ & + \frac{1}{L^2 v} \left[2L + \frac{2}{v} \exp(-vL) - \frac{2}{v} \right] \end{aligned} \quad (8)$$

Equation (8) can be minimized to determine the optimal separation distance between points, a, as a function of L and v:

$$a_{optimal} = -\frac{1}{v} \ln(t) \quad (9)$$

where

$$t = \frac{B + \sqrt{B^2 - 4AB}}{2A}$$

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$$A = \frac{4\nu}{9}$$

378

$$\text{and } B = -\frac{4}{3L} \exp\left(-\frac{\nu L}{2}\right)$$

379

The combination of (8) and (9) can be used to determine the normalized squared error,

380

σ_z^2 / σ_p^2 , and the optimal distance, $a_{optimal}$, for the measurement pattern in Figure 5b. The model

381

predicts that the normalized squared error is minimized at an intermediate location between 0

382

and $L/2$ (black lines in Figure 8a and b). The results show an increase in the error with interval

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size, L , as well as little sensitivity of $a_{optimal}$ to ν . This latter feature can be seen as an advantage

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since small biases in the estimation of ν will not result in significant biases in the estimation of

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$a_{optimal}$. One could almost assume a value of $a_{optimal}$ without prior knowledge of the exponential

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decay exponent, selecting $a_{optimal}$ within the range of values indicated by the model for a range of

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possible exponential decay exponents. Note that $a_{optimal}$ is located close to the 60% distance from

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the center towards the outer boundary of the profile section for all section lengths (Figure 8a and

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b). On the other hand, the measurement error displays a higher sensitivity to ν around $a_{optimal}$,

390

indicating that biases in the estimation of ν would have a more noticeable effect on the

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estimation of the measurement error. This is further clarified in Figure 8c, in which the

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normalized error (not squared) and $a_{optimal}$ can be obtained for corresponding profile section

393

lengths (L) and exponential decay exponents (ν) based on the isolines shown. For example, for a

394

profile section of 30 m, and an exponential decay exponent of 0.2 m^{-1} , the normalized error is

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0.32 and $a_{optimal}$ is 9.63 m (see intersect of the two isolines in Figure 8c). The normalized error in

396

Figure 8c is not squared, highlighting the sensitivity of the measurement error to ν , which

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400 represents the degree of spatial correlation of the profile in this case (e.g., lower values indicate
401 stronger spatial memory/correlation, hence lower measurement errors).

402 The performance of the model is tested against the normalized squared error obtained from
403 the same snow depth profiles in FS and RW. The test consists of estimating the normalized
404 squared error for profiles sections of length between 10 m and 80 m, with a being varied between
405 0 and $L/2$ (Figure 9). For each value of a , the normalized squared error is estimated based on the
406 means obtained using the three snow depth samples for each section. All squared differences are
407 then averaged to obtain the values presented in the Figure. Sampled and modeled errors follow
408 the same trend across all a values and for the different L values in Figure 9. The minimum error
409 is also reproduced by the model proving the applicability of the model for estimating the optimal
410 separation between measurements. The model does perform better in the forested environment of
411 FS versus RW, particularly for lower a values. This can be justified as the exponential
412 covariance model displays a better fit in FS over RW, particularly over the lower range of lag
413 values. Also, note that both the modeled and sampled normalized squared errors are lower for the
414 snow depth profiles at RW because of the longer spatial memory of the snow depth distribution
415 in this environment (higher spatial correlations) when compared to that in FS.

416 | 4.3 Case 3: N measurements along a profile section

417 As stated above, the measurement error can be reduced by increasing the number of
418 measurements taken over a given section of length L . Let us focus on the case of stratified
419 sampling where N regularly spaced measurements are taken over the interval (Figure 5c), and to
420 quantify this reduction we can use (5) and the exponential covariance model. Equation (5) can
421 then be reduced to:

$$\sigma_z^2(N, L, \nu) / \sigma_p^2 = \frac{1}{N} + \frac{2}{N^2} \sum_{k=1}^{N-1} k \exp\left(-\nu \left[L - kL/N\right]\right) - \frac{4}{Lv} \left[1 - \frac{1}{N} \sum_{k=1}^N \exp\left(-\nu \frac{L}{N} \left[N - k + \frac{1}{2}\right]\right)\right] - \frac{2}{L^2 \nu^2} [1 - Lv - \exp(-\nu L)] \quad (10)$$

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The normalized squared error (σ_z^2 / σ_p^2) obtained with (10) for profiles sections of lengths between 10 and 80 shows a steep decrease with N (Figure 10), with a steeper decrease for higher exponential decay exponents. For the longer profile sections (e.g., 80, Figure 10d), little reductions are achieved in the squared error beyond only a few measurements (e.g., $N = 16$). Equation (10) and the results in Figure 10 can be used to determine the number of measurements necessary to achieve a desired accuracy level. One could, for example, design a survey to sample a snow depth profile with a mean value every 10 m. The number of measurements required to achieve a desired level of accuracy can be obtained from Figure 10a, based on previous knowledge of the sample estimate of the exponential decay exponent. This can be achieved thanks to the intra-annual and inter-annual persistence of the spatial patterns, and hence, the spatial statistical properties of snow depth fields in mountain environments, as shown in previous studies using both manual surveys and LIDAR measurements (e.g., Deems et al., 2008; Sturm and Wagner, 2010; Schirmer et al., 2011; Melvold and Skaugen, 2013; Helfrich et al., 2014). A detailed spatial survey (e.g., dense manual measurements or TLS), sampling different portions of an area can be used to determine the covariance/correlation characteristics of the snow depth distribution, with which the model for the error can be applied. An a priori estimate of the exponential decay exponent may also be possible and will be tested in future applications of the framework, given the relative insensitivity of the error with respect to ν .

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443 Following the method described in the previous section, we test the performance of the
444 model against the normalized squared error obtained from the same snow depth profiles in FS
445 and RW. In this case, the sampled squared error is estimated based on the N regularly-spaced
446 measurements distributed along the profile sections of length L . As the snow depth fields are
447 gridded at ~ 1 -m resolution, the location of the measurements is approximated to the closest
448 coordinate in the profile section following the pattern in Figure 5c. Once again, sampled and
449 modeled errors follow closely the same trend for the different L values in both FS and RW
450 (Figure 11). The error decreases with N , with a rapid decay at the lower N values, illustrating that
451 the error can be drastically reduced by simply increasing the number of measurements by a small
452 amount. The normalized squared error across all N values is lower for RW than for FS,
453 consistent with the higher spatial correlations observed in the snow depth fields of RW versus
454 FS. Once again, there are some differences between the sampled and modeled normalized
455 squared error in RW for the shorter profile lengths and for small N values: a consequence of the
456 poorer fit of the exponential model for the shorter lag range in RW. However, the model is still
457 able to reproduce the error in both fields, and the applicability of the model is illustrated even
458 when the fit of the correlation model can be improved.

459 **5 Two-dimensional process**

460 Similar to the one-dimensional process, equation (5) can be formulated to calculate the
461 squared error in the two-dimensional space. To exemplify this, we apply the methodology to an
462 isotropic process over the x - y plane for three cases in a square area: (a) one single measurement
463 in the center of the area, (b) five measurements radiating out from the center (Figure 12a), and
464 (c) N by N measurements regularly spaced in the x and y directions (Figure 12b).

465 For the isotropic case, the covariance/correlation function is only dependent on the
 466 magnitude of the lag vector,

$$467 \quad h_{i,j} = |\mathbf{x}_i - \mathbf{x}_j| \quad (11)$$

468

469 and, consequently, the error is represented by,

$$470 \quad \sigma_z^2(A) = \sigma_p^2 \left[\begin{aligned} & \frac{1}{N} + \frac{2}{N^2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N CORR[h_{i,j}] \\ & - \frac{2}{NA} \sum_{i=1}^N \int_A CORR[h_{i,j}] d\mathbf{x}_j \\ & + \frac{1}{A^2} \int_A \int_A CORR[h_{i,j}] d\mathbf{x}_i d\mathbf{x}_j \end{aligned} \right] \quad (12)$$

471

472 The exponential correlation function for the isotropic case takes the following form:

$$473 \quad CORR(h, v) = \exp(-vh) \quad (13)$$

474 where h is the magnitude of the lag vector. Replacing into the expression for σ_z^2 , we obtain,

$$475 \quad \sigma_z^2 = \sigma_p^2 \left[\begin{aligned} & \frac{1}{N} + \frac{2}{N^2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \exp(-v|\mathbf{x}_i - \mathbf{x}_j|) \\ & - \frac{2}{NA} \sum_{i=1}^N \int_A \exp(-v|\mathbf{x}_i - \mathbf{x}_j|) d\mathbf{x}_j \\ & + \frac{1}{A^2} \int_A \int_A \exp(-v|\mathbf{x}_i - \mathbf{x}_j|) d\mathbf{x}_j d\mathbf{x}_i \end{aligned} \right] \quad (14)$$

476 For the case of a rectangular area of side dimension L_x and L_y in the corresponding x and y
 477 directions, the equation becomes,

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$$\sigma_Z^2 = \sigma_p^2 \left[\begin{aligned} & \frac{1}{N} + \frac{2}{N^2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \exp \left(-v \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)^{1/2} \right) \\ & - \frac{2}{NA} \sum_{i=1}^N \int_0^{L_y} \int_0^{L_x} \exp \left(-v \left((x_i - x)^2 + (y_i - y)^2 \right)^{1/2} \right) dx dy \\ & + \frac{1}{A^2} \int_0^{L_y} \int_0^{L_x} \int_0^{L_y} \int_0^{L_x} \exp \left(-v \left((x' - x)^2 + (y' - y)^2 \right)^{1/2} \right) dx dy dx' dy' \end{aligned} \right] \quad (15)$$

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The limits of the integrals can be changed depending on the desired location of the origin. In this case, the origin is located at the lower-left corner.

As discussed earlier, the first term is only a function of N , such that the base error is the variance of the point process divided by the number of points. The second term is a function of N , the location of the points, and the decay rate v . The third term is a function of N , A , the location of the points, and the decay rate v . The fourth term is a function of A and v , but is independent of the location of the points and N (i.e., independent of the survey design, and only a function of the correlation structure of the continuous process).

5.1 Case 1: Single measurement in the center of the area

In this case, we focus on a single measurement in the middle of a square area of side dimension L . Numerical solution of (15) shows that the normalized squared error increases rapidly with L , with a steeper increase for higher exponential decay exponents (Figure 13a), which approach a normalized squared error of 1 for L values less than 10 (e.g., $1 \leq v \leq 5$). The theoretical results in Figure 13a can be used to determine the discrepancy between a single measurement in the middle of an area and the areal mean for a second order stationary and anisotropic process with an exponential covariance/correlation function. Comparison of the modeled and sampled normalized square errors for the FS snow depth field indicate very good agreement between modeled and sample errors (Figure 13b). The sample error is estimated

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498 following the same procedure explained for the one-dimensional cases, although in the two-
499 dimensional space. Both sampled and modeled errors show the same behavior across L values
500 between 1 m and 100 m, although the scatter in the sampled error increases for larger L values.
501 This can be explained by the smaller number of samples to estimate the mean normalized
502 squared error and the fact that the correlation structure decays rapidly and a single sample
503 becomes less correlated to the surrounding area for these larger areas. The model introduced here
504 can then be used to assess the representativeness of a single measurement over an area
505 objectively and accurately, and it can be extended for other covariance/correlation functions as
506 needed.

507 5.2 Case 2: Five measurements radiating out from the center of the area

508 The case five measurements radiating out from the center (Figure 12a), with a point in the
509 middle of the area and four points separated by a distance a from the center leads to a similar
510 optimization problem as illustrated in case 2 of the one-dimensional examples (section 4.2). In
511 the two-dimensional case, (15) does not have an explicit solution for a , and numerical
512 implementation is required. The equation can be solved by simply replacing the point
513 coordinates and the correlation function parameters. Following this approach, the normalized
514 squared error can be obtained for areas of varying sizes (Figure 14). Similar to the one-
515 dimensional example (case 2, section 4.2), σ_z^2/σ_p^2 decreases with a , reaching a minimum at an
516 intermediate distance from the middle point outwards. The decay in σ_z^2/σ_p^2 is more rapid for the
517 least correlated processes (i.e., higher decay exponents) reaching a value close to the base
518 normalized square error that is a function of the number of points (i.e., $1/N = 1/5$ in this case). An
519 extended analysis of the effect of each of the terms in the equation is included in the

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Supplementary Information. The error, as shown in Figure 14, is minimized as a consequence of two balancing terms that lead to this intermediate solution. The optimal solution is a balance between reducing the correlation between the individual measurements (e.g., increasing the separation between the location of the measurements) but increasing the correlation between the measurements and the surrounding area (e.g., locating the measurements closer to the middle of the area). These two competing effects lead to an optimization problem based on the location of the point measurements. For the least correlated processes, the error behaves closer to the behavior of an uncorrelated field once the measurements become effectively decorrelated (e.g., $a > 1$ in Figure 14b for $\nu = 5$). Figure 14 exemplifies how (15) can be used to determine the optimal measurement location for areas of different sizes, and to determine the associated error with configurations other than the optimal.

The performance of the model is tested against the normalized squared error obtained from the snow depth field in FS. The test consists of estimating the normalized squared error for square areas of side dimension (L) between 10 m and 79 m, with a being varied between 0 and $L/2$ (Figure 15). For each value of a , the normalized squared error is estimated based on the means obtained using the five snow depth samples for each section. All squared differences are then averaged to obtain the values presented in the figure. Once again, the sampled and modeled errors follow the same trend across all a values and for the different L values. The minimum error and $a_{optimal}$ are also reproduced closely by the model, and as the area size increases, the sampled and modeled error approach the error for an uncorrelated field at larger separations (i.e., 0.2). These results illustrate that the performance of the model in the two-dimensional space is remarkable, similar to what was observed in the one-dimensional case.

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5.3 Case 3: N by N measurements regularly spaced in the x and y directions

Similarly to the one-dimensional case, the two-dimensional case of N by N regularly spaced measurements (Figure 12b) leads to a decreasing normalized squared error with N (Figure 16). There is a sharp decrease in the error with just increasing the number of measurements in the lower range of N . The analysis illustrates that stratified sampling, as the one shown here, is an excellent approach to minimizing the error. For example, for the area of 10 by 10, increasing N to 4 ($N^2 = 16$) reduces the [normalized](#) squared error to less than 0.05. It is also worth noting here that for this two-dimensional case, the error is less sensitive to the value of the exponential decay exponent (ν) for the higher N values as the mean is accurately captured regardless of the correlation of the field. [Beyond a certain number of measurements regularly distributed in the area, the measurements gather enough information such that there are only very minor improvements with the addition of new measurements, regardless of the exponent value.](#) Figure 16 serves as an example of how the methodology can be used for objective selection of the number of measurements necessary to achieve a desired accuracy level using prior knowledge of the spatial covariance function.

The performance of the model is tested again for square areas of side dimension (L) between 10 m and 79 m using the snow depth field in FS, and for an increasing number of rows/columns of measurements leading to a total number of measurements of N^2 (Figure 17). The results illustrate again the accurate performance of the theoretical model, with sampled and model errors following closely the same squared errors. Both sampled and modeled errors increase as the size of the area increases, as expected. These results complete the model performance tests for the two-dimensional isotropic case.

6 Summary and Conclusions

A methodology for an objective evaluation of the error in capturing mean snow depths from point measurements is presented based on the expected value of the squared difference between the real average snow depth and the mean of a finite number of snow depth samples within a defined domain (e.g., a profile section or an area). The model can be used for assisting the design of survey strategies such that the error is minimized in the case of a limited and predetermined number of measurements, or such that the desired number of measurements is determined based on a predefined acceptable uncertainty level. The model is applied to one- and two-dimensional survey examples using LIDAR snow depths collected in the Colorado Rockies. The results confirm that the model is capable of reproducing the estimation error of the mean from a finite number of samples for real snow depth fields.

Here, we should highlight some of the implications of the assumptions made in the model. In simplified terms, the second-order stationarity assumption implies that the mean and the variance of the process/variable (e.g., snow depth) are independent of the spatial location, and that the covariance is dependent only on the separation vector (i.e., lag). Although these assumptions may not be as adequate over larger scales (e.g., greater than 100 m), at smaller scales the assumption in the context of the model application to snow depth should be valid. We present these examples to show how the error can be quantified with good accuracy around such smaller scales. Application of such types of approaches at larger scales will require additional evaluations with particular attention as to what the specific demands of the application are. Also, the methodology as presented here is not suitable for discontinuous snow covers if both snow-covered and snow-free areas are considered in the error estimation. This case has not been considered in the development here.

591 Implementation of the model in practice requires prior assumption of a
592 correlation/covariance model and estimates of the parameters of this model (e.g., the decay
593 exponent for the exponential case). In the examples here we use LIDAR data for the parameter
594 estimation, which we have done to illustrate the applicability of the model and its ability to
595 estimate the error using real snow depth data. Snow distribution in mountain environments has
596 been shown to be consistent intra- and inter-annually because the controlling processes are
597 relatively consistent during the season and from season to season. Such consistency suggests that
598 the correlation/covariance model should also be consistent, as well as the parameters of the
599 model. These parameters can be estimated via a dense survey either manually or with TLS of one
600 or more small plots of a size similar to the size that is aimed to be represented. These surveys
601 would not necessarily have to be repeated as the parameters and covariance models should be
602 preserved. Detailed surveys can be conducted under different conditions to characterize the range
603 of the correlation models and parameters (e.g., after a snow storm, or close to peak
604 accumulation). Also here, we should point out that although we show results for a wide range of
605 the exponential decay exponent values, we are finding that most of the values that we have
606 observed are in the lower range of those presented (e.g., $0.1-0.2\text{ m}^{-1}$). Hence, the biases in the
607 estimated error and the survey design remain small.

608 Currently, remote sensing technologies (e.g., TLS, Airborne LiDAR, and ground penetrating
609 radar) are allowing for the characterization of snow cover properties at increasing resolutions in
610 both space and time. Such improvements can be utilized in the context presented here providing
611 information about the range of best fitting covariance/correlation models and parameters for
612 different conditions, supporting the application of methodologies such as the one presented here.
613 With such improvements, survey designs can be optimized such that estimation errors can be

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626 explicitly addressed and accounted for, particularly when extrapolating a limited number of
627 measurements to estimate the spatial distribution of snow. Such applications will continue to be
628 relevant despite of the aforementioned improvements, as access to these technologies is limited
629 | by their cost and the expertise that is required for their application.

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Figures

638 Figure 1. (a) Location of the Fraser and Rabbit Ears study areas in the state of Colorado (in
639 grey). (b) LIDAR Snow depth distributions on April 8, 2003, at the Saint Louis Creek Intensive
640 Study Area (ISA) and (c) on April 9 at the Rabbit Ears ISA.

641 Figure 2. (a) Sample normalized snow depth profile (mean = 0, standard deviation = 1) in a
642 forested environment from LIDAR (1-m resolution) at the Fraser – St. Louis Creek (FS)
643 intensive study area (ISA) of the Cold Land Processes eXperiment (CLPX) (Trujillo et al., 2007;
644 Cline et al., 2009). The profile is sampled with regular separations (spacing) of 5 m, 10 m, 25 m,
645 50 m, and 100 m (from top to bottom, respectively). (b) Average values within sampling
646 intervals (same as in (a)) versus point samples for normalized snow depth profiles in the FS ISA.
647 The red line is a linear regression fit, with slope β and r^2 as indicated in each plot. (c) Histograms
648 of the difference between the point and average values for each of the sampling intervals. The
649 vertical red line marks the mean difference.

650 Figure 3. (a) As Figure 2 but for an open and wind **influenced** environment at the Rabbit Ears
651 – Walton Creek (RW) ISA of the CLPX (Trujillo et al., 2007; Cline et al., 2009). (b) Average
652 values within sampling intervals (same as in (a)) versus point samples for normalized snow depth
653 profiles in the RW ISA. The red line is a linear regression fit, with slope β and r^2 as indicated in
654 each plot. (c) Histograms of the difference between the point and average values for each of the
655 sampling intervals. The vertical red line marks the mean difference.

656 Figure 4. Sub-interval standard deviation (a) and range (b) for varying interval lengths for
657 profiles of snow depth in a forested environment (FS) and an open and wind-influenced
658 environment (RW) in the Colorado Rocky Mountains (same regions as those in Figure 2 and
659 Figure 3). The mean standard deviation and mean range for the study areas are shown by the
660 solid lines, while the shaded areas cover the quantiles between 25% and 75% of the values for all
661 the intervals in these areas.

662 Figure 5. Survey designs for the sampling of a snow profile.

663 Figure 6. Comparison of the theoretical and sampled normalized squared error (σ_z^2/σ_p^2) for
664 the case of a single measurement along a profile section of length L , as in Figure 5a. The survey
665 case applied to profiles in FS and RW along the x and y directions. Solid lines are the theoretical
666 error using exponential decay exponents derived from the functions fitted to the sampled
667 correlation functions of the two surfaces in the x and y directions.

668 Figure 7. (a) Theoretical normalized squared error for a single measurement in the middle of a
669 section of length, L , and for an exponential correlation function with a decay exponent, v . (b) and
670 (c) Comparison of the theoretical and sampled normalized squared error for the same survey case
671 applied to profiles in FS and RW along the x and y directions. Dashed lines are the theoretical
672 error from (7) using exponential decay exponents derived from the functions fitted to the
673 sampled correlation functions of the two surfaces in the x and y directions.

674 Figure 8. (a) and (b) Theoretical normalized squared error for the three-point pattern along a
675 profile section in Figure 5b, and for profile section lengths (L) of 1 (a) and 25 (b). Each of the

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colored lines corresponds to a specific decay exponent, ν , and the black line marks the theoretical solution for $a_{optimal}$. (c) Theoretical normalized error and $a_{optimal}$ for isolines of profile section lengths (L) and exponential decay exponents (ν) for the three-point pattern along a profile section of length L in Figure 5b.

Figure 9. Theoretical and sampled normalized squared error (σ_z^2/σ_p^2) for the three-point pattern along a profile section in Figure 5b, and for profile section lengths (L) between 10 m and 80 m in FS and RW. The solid lines are the theoretical error from (8) using exponential decay exponents derived from the functions fitted to the sampled correlation functions of the two surfaces in the x and y directions, while the dots correspond to the sampled error for profiles in FS (a-d) and RW (e-h).

Figure 10. Theoretical normalized squared error (σ_z^2/σ_p^2) for the N -point pattern along a profile section in Figure 5c, and for profile section lengths (L) between 10 and 80 obtained from (10).

Figure 11. Theoretical and sampled normalized squared error (σ_z^2/σ_p^2) for the N -point pattern along a profile section in Figure 5c, and for profile section lengths (L) between 10 m and 80 m in FS and RW. The solid point markers are the theoretical error from (10) using exponential decay exponents derived from the functions fitted to the sampled correlograms of the two surfaces in the x and y directions, while the circle markers with the dotted lines correspond to the sampled error for profiles in FS (a-d) and RW (e-h).

Figure 12. Sample survey designs with (a) a 5-point pattern centered in the area, and (b) a regularly spaced pattern. For the 5-point pattern, a can vary between 0 and $L/2$, while for the $N \times N$ points pattern, the separation between the measurements is determined by the number of points.

Figure 13. (a) Theoretical normalized squared error (σ_z^2/σ_p^2) for the two-dimensional case with a single measurement in the middle of a square area with side dimension L . (b) Theoretical and sampled normalized squared error for the same two-dimensional survey applied to the snow depth field in FS. The dashed line is the theoretical error derived for an exponential decay exponent of 0.17 derived from the sampled correlation function of snow depth in FS, while the solid line is the sampled normalized squared error for the snow cover in FS.

Figure 14. Theoretical normalized squared error (σ_z^2/σ_p^2) as a function of the distance a from the center of the area for square areas of side dimensions (L) between 10 and 80. Each curve corresponds to an exponential decay (ν) between 0.1 and 5.

Figure 15. Theoretical and sampled normalized squared error (σ_z^2/σ_p^2) for the 5-point pattern in Figure 12a over square areas of side dimensions (L) between 10.7 m and 79.1 m. The separation distance (a) is varied from the center outwards. The solid line is the theoretical error derived for an exponential decay exponent of 0.17 derived from the sampled correlation function of snow depth in FS, while the solid red point markers are the sampled normalized squared error for the snow cover in FS.

715 Figure 16. Theoretical normalized squared error (σ_z^2/σ_p^2) for the N by N point pattern in
716 Figure 12b, and for areas of side dimension (L) between 10 and 80. The exponential exponent is
717 varied between 0.1 and 5.

718 Figure 17. Theoretical and sampled normalized squared error (σ_z^2/σ_p^2) for the N by N point
719 pattern in Figure 12b, and over square areas of side dimensions (L) between 10.7 m and 79.1 m.
720 The solid black point markers are the theoretical error for an exponential decay exponent of 0.17
721 derived from the sampled correlogram of snow depth in FS. The dotted red lines with circle
722 markers are the sampled normalized squared error for the snow cover in FS.

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