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The Cryosphere Discussions

Interactive comment on "Radiative transfer model for contaminated slabs: experimental validations" *by* F. Andrieu et al.

F. Andrieu et al.

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Dear referee,

Thank you for the attention you paid to this work. Your comments were greatly appreciated and carefully treated.

Every comment, remark and question has been treated. The answers appear in bold, and are indicated by a "A:" after the comments or questions. If no clear answer is provided below a given remark, please directly refer to the modifications. The modifications to be done in the manuscript appear in a second part. They are a combination of what was asked be both of the reviewers.

REVIEW

F. Andriu et al.: Radiative transfer model for contaminated slabs: experimental validations, submitted to Cryosphere, TCD 9, 5137–5169, 2015

The present manuscript describes an effort to validate a theoretical radiative-transfer model published earlier in Applied Optics (referenced in the manuscript). It further describes new Bayesian inverse methods for the retrieval of the model parameters from the experimental measurements. The manuscript can become publishable in Cryosphere after it is revised according to the following comments. In studies of close-packed particulate media, the radiative-transfer-type models are experiencing a golden era due to the recent quantitative, positive comparison between radiative-transfer-type and exact electromagnetic computations for the same particulate systems published in

Muinonen, K., Mishchenko, M. I., Dlugach, J. M., Zubko, E., Penttiln"a, A., and Videen, G. (2012). Coherent backscattering numerically verified for a finite volume of spherical particles. Astrophysical Journal 760, 118, 11p. (doi:10.1088/0004-637X/760/2/118).

A: References to Muinonen et al. 2012 and Mishchenko et al. 2015 have been added in the introduction

Concerning the radiative-transfer coherent-backscattering part of this comparison, see,in particular:

Muinonen, K. (2004). Coherent backscattering of light by complex random media of spherical scatterers: Numerical solution. Waves in Random Media 14(3), 365-388.)

A: We do not model coherent backscattering in this model. There is no evidence that a slab medium produces such effect, as a particulate media does. We therefore chose not to model it, and thus consider that this reference, even if it reinforces the idea of positive comparisons between RT models and exact computations, could be misleading to the reader.

The authors seem not to be aware of these works that, indeed, strengthen their science case and need to be referenced in the present article. Furthermore, the authors seem to be unaware of

Muinonen, K., Nousiainen, T., Lindqvist, H., Munoz, O., and Videen, G. (2009). Light scattering by Gaussian particles with internal inclusions and roughened surfaces using ray optics. Journal of Quantitative Spectroscopy and Radiative Transfer 110, 1628-1639.

that comes very close to their work in having a Monte Carlo radiative-transfer solution within a host medium. As the Monte Carlo ray-tracing solutions are highly parallelizable, the computational feasibility is no longer an obstacle in developing increasingly realistic theoretical models. It would be prudent for the authors to realize this fact and not to stress the slowness of Monte Carlo approaches (Abstract, p. 5138, l. 18).

A: The final aim of our model is massive datasets inversion, and thus we need to be able to simulate in a reasonable time dozens of millions of spectra (that may contain thousands of wavelengths). For that, MC model, even if they are highly parallelizables, are still too time consuming. Still, we agree with reviewer 1 that the sentence lacked arguments, so we chose to change it, and added the suggested reference.

As to the Bayesian inversion of the experiments for the theoretical model parameters, this is a step most welcome in radiative-transfer analyses.

Continuing into more detailed comments, on page 5141, line 1, the authors discuss spherical inclusions. It is not evident from the present manuscript what such inclusions are and why they are spherical. Presumably, these have been discussed in the earlier paper (Andriu et al. 2015, Applied Optics) but the present paper should include sufficient description about the theoretical modeling to be validated. Later, on line 21, what do the authors mean by "every following transit is considered isotropic"? Again, mode description is needed.

Then, on page 5144, line 9, the authors state that a thin coverage of slab ice is enough

to strongly flatten the BRDF. In terms of geometric optics, what is the reason for this flattening? The well-known divergence of rays when they refract back from the medium through the interface of ice and air?

A: It is due to the properties of water ice: snow is particularly directive at the wavelengths for which it is the most absorbent. The radiation at these wavelengths is quickly absorbed by the slab. On the contrary, for less absorbent wavelengths, the snow is less directive. In the end, this combination strongly flatten the BRDF. For planetary applications, the typical case that will be discussed will be ice on top of granular regolith. Planetary regolith are known to show a smooth BRDF, and this hypothesis will not be challenged.

On page 5145, line 12, what is the so-called "element"? It is difficult to understand on the basis of the present description so describe more thoroughly.

A: “each quantity”

On page 5146, something is wrong with the a posteriori probability density function in Eq. 5: First, what is the quantity "x" transposed in the equation? Second, there should be a transpose of the column vector $(F(m) - d_mes)$ multiplying $C - 1$. The problem repeats itself in Eq. 8.

A: This is only a matter of mathematical standards! The “x” is the multiplicative operator, the transpose operator apply to what follows it, that is (F(m)-dmes). This solves the problem. A note has been added in the text to clarify this point.

On page 5151, the authors address the specular lobe and its maximum. Whereas the authors are rightfully carrying out an assessment of the orientation of their sample for a better match between the theoretical and measured lobe, they need to become aware of the fact that the angular position of the maximum can depend on the asymmetric diffraction patterns of the surface elements. For further information, see, for example,

Muinonen, K. (1989). Scattering of light by crystals: a modified Kirchhoff approxima-

tion. Applied Optics 28, 3044-3050.

A: We do agree on that point. Still, as the sample is put into rotation during the experiment (a full rotation is done for each separate wavelength measurement), we believe that we can neglect this effect.

More description is required for the challenges in the actual inversion encountered on page 5153, line 29: if the parameter definition domain is too narrow, why not make it wider to obtain more realistic a posteriori distributions?

A: More description has been added. See for example a new section about a numerical validation of the method. There are two important points: (i) we cannot make the parameters definition domain wider if it contradicts one fundamental hypothesis of the model and (ii) there are conditions for which we know that the inversion process will not be able to conclude.

Detailed comments, mostly on the language:

1) page 5138 line 2: approximated → approximate (also at 5140, 2 and 5140, 8) line 9: media → medium line 20: density probability → probability density

2) page 5140 line 19: inspired from → inspired by

3) page 5144 line 24: inverse → invert

4) page 5148 line 2: variability → configuration of the parameters?

5) page 5149 line 20: wavelengths → wavelength

6) page 5151 line 10: what is meant by "of the measure"?

7) page 5153 line 10: inversions points → inversion points

8) page 5154 line 1: what is meant by "the measure of sample 1"? I assume this does not refer to the size of sample 1 but the measurements of sample 1.

9) page 5154 line 15: various thickness → varying thickness

10) page 5159 Figure 1: subtrate → substrate

11) page 5166 Figure 8: Marginal probability density functions a posteriori → Marginal a posteriori probability density functions

12) page 5169 Figure 11: Marginal probability density functions a posteriori → Marginal a posteriori probability density functionn

A:These comments have been taken into account (see the modifications)

Modifications to be done:

Replace the title of the paper by: “Retrieving the characteristics of slab ice covering snow by remote sensing”

Replace the abstract by:

We present an effort to validate a radiative transfer model previously developed, and an innovative bayesian inversion method designed to retrieve the properties of slab ice covered surfaces. This retrieval method is adapted to satellite data, and is able to provide uncertainties on the results of the inversions. We focused in this study on surfaces composed of a pure slab of water ice covering an optically thick layer of snow. We see sought to retrieve the roughness of the ice/air interface, the thickness of the slab layer and the mean grain-size of the underlying snow. Numerical validations have been conducted on the method, and showed that if the thickness of the slab layer is above 5 mm and the noise on the signal is above 3%, then it is not possible to invert the grain-size of the snow. On the contrary, the roughness and the thickness of the slab can be determined even with ultra high levels of noise up to 20%. Experimental validations have been conducted on spectra collected from laboratory samples of water ice on snow using a specro-gonio-radiometer. The results are in agreement with the numerical validations, and show that a grain-size can be correctly retrieved for low slab thicknesses, but not for bigger ones, and that the roughness and thickness are correctly inverted in every case.

Introduction

p. 5139, l. 4-6: Change “Ice- and snow-covered areas have a strong impact on planetary climate dynamics, as they can lead to significant regional-scale albedo changes at the surface and surface–atmosphere volatiles interactions” into “Ice- and snow-covered areas have a strong impact on planetary climate dynamics, as changes in their characteristics or coverage can lead to significant regional-scale albedo changes at the surface and surface–atmosphere volatiles interactions”

p. 5139, l.8-9: change (Dozier et al., 2009; Negi and Kokhanovsky, 2011) into ((Dozier et al., 2009; Negi and Kokhanovsky, 2011, Picard et al., 2009; Mary et al., 2013)

p. 5139, l.10-11: remove “, or on the specific surface area (Picard et al., 2009; Mary et al., 2013)”

p. 5139, l.13 : add “(Zege et al. 2008, Negi and Kokhanovsky 2011)” after “retrieving such properties”

p.5139, l.14-16: replace the sentence “Ray-tracing algorithms, such as those described in Picard et al. (2009) for snow or Pilorget et al. (2013) for compact polycrystalline ice, simulate the complex path of millions of rays into the surface.” by “Ray-tracing algorithms, such as those described in Picard et al. (2009) for snow, Pilorget et al. (2013) for compact polycrystalline ice or Muinonen et al. (2009) for particulate media such as rough ice grains in an atmosphere, simulate the complex path of millions of rays into the surface.”

p.5139,lL16-17: replace sentence: “They provide very accurate simulations but have the weakness of being time consuming.” by: “Such modelings are experiencing a golden era due to the positive comparison between models and exact calculations (e.g. Muinonnen et al. 2012, Mishchenko et al., 2015).”

p. 5139, l.19: replace “inspired from” by “inspired by”

p. 5139, l.20: replace “as homogeneous” by “statistically as a mono-layer”

p. 5139, l.28: replace “owing to” by “as suggested by”.

p.5140, l.1 : replace “several decimeters” by “several centimeters to decimeters (Eluszkiewicz 1993, Quirico et al. 1999, Douté et al. 1999, Douté et al., 2001)”.

p.5140, l.1-17: replace the text from “Radiative transfer” on line 1 to the end of the introduction by the following text: “Compact slabs have very different radiative properties from close packed granular media, and radiative transfer models have been developed to study their characteristics (e.g. Mullen and Warren 1988, Jin Stamnes 1994, Perovich 1996, Jin 2006) in the case of sea or lake ices. We developed an approximated model (Andrieu et al., 2015) model that has the interest of being able to model a layer of ice covering a surface with radically different optical properties, for instance a refractive index, unlike it predecessors. It was designed to study planetary ice slabs, with a fast numerical implementation, which has already been numerically validated and aims at the analysis of massive spectro-imaging planetary data such as the OMEGA (Bibring et al. 2004) or CRISM (Murchie et al., 2007) datasets for the study of Mars icy surface and seasonal cycle, NIMS (Carlson et al., 1992) dataset for SO_2 on Io or RALPH (Reuter et al., 2009) data for the ices of Pluto. For this purpose, it is semi-analytic and implemented to optimize the computation time.

In the present article, we will test the accuracy of this approximated model on laboratory spectroscopic measurements of pure water ice on top of snow bidirectional reflectance distribution function (BRDF). The slabs that will be studied thus contain no impurity, and the surface properties we will seek to retrieve will be the thickness of the ice, the roughness of the surface and the grain-size of the underlaying snow. The main goals of this work are thus (i) to test the ability of the model to reproduce reality and (ii) to propose an inversion framework to retrieve surface ice properties, including uncertainties, in order to demonstrate the applicability of the approach to satellite data.

We presents a set of spectro-goniometric measurements of different water ice samples put on top of snow using the spectro-radiogoniometer described in Brissaud et al. (2004). Three kind of experiments were conducted. First, the BRDF was measured for a snow layer only, and then measured again after adding a slab ice layer at the top. The objective was to test the effect of an ice layer at the top on the directivity of the surface. Second, the specular spot was closely investigated, at high angular resolution, at the wavelength of 1.5 μm, where ice behaves as a very absorbing media. Finally, the bidirectional reflectance was sampled at various geometries on 61 wavelengths ranging from 0.8 to 2.0 μm. In order to validate the model, we made qualitative tests

to demonstrate the relative isotropization of the flux. We also conducted quantitative assessments by using a Bayesian inversion method in order to estimate the sample thickness, surface roughness and snow grain-size from the radiative measurements only. A simple comparison between the retrieved parameters and the direct independent measurements allowed us to validate the model.

The inversion algorithm that will be tested is based on lookup tables that minimize the computation time of the direct model. The solution is then formulated as a probability density function, using bayesian formalism. This strategy will be very useful for analysing hyperspectral images. The thickness of ice estimated from the inversion will be compared to real direct measurements. In addition, the specular lobe will be adjusted to demonstrate that the model is able to reasonably fit the data with a coherent roughness value."

Description of the model

p.5140, l.27-p5141, l5: Remove text from "Within the slab" to "even bubbles"

p.5141, l.8 : replace "of a measurement" by "in the model"

p.5141, l.10-15 : remove "The surface is considered to be constituted of many unresolved facets, whose orientations follow the defined probability density function. The specular reflectance is obtained integrating every reflection on the different facets."

p.5141, l.20 : replace "reach" by "reaches"

p.5141, l.21 : replace "are" by "is"

p.5141,l.22-23: remove the following sentence: "This means that the size of the inclusions and the thickness of the slab layer must be larger than the considered wavelength."

p.5141,l.23-24: replace "The inclusions inside the matrix are close to spherical and homogeneously distributed." by "If the matrix is contaminated with inclusions, unlike in this work, then these inclusions are supposed to be close to spherical and homogeneously distributed inside the matrix."

p.5142, l.2 : add references (Stamnes et al., 1988, Douté et al. 1998, Van de Hulst, 2012) after "formulas

p.5142, l.18 : replace "sections" by "circular sections of 20 cm in diameter"

p.5143, l.2 : add after "the measurement." The sample complete a full rotation (10s) during the measurement of the reflectance at one wavelength and one geometry.

p.5143, l.6 : change "contribution" into "reflectance"

p.5143, l.13: replace title by "Ice on snow diffuse reflectance spectra"

p.5144, l.1 : add "(the ratio between the bidirectionnal reflectance I/F of the surface and the reflectance of a perfectly lambertian surface)" after "factor"

p.5144, l.1: add "(angle between incident and emergent directions)" after "phase angle"

p.5144, l.5 : add "as the dependance of the reflectance on the phase angle is almost killed by the addition of the ice layer" after "matrix".

p.5144, l.8 : replace "isotropic" by "lambertian"

p.5144, l.24; replace "inverse" by "invert"

Method

p.5145, l.10 : add (i.e. the slab thickness, the roughness parameter, the snow grain-size) after "model parameters m"

p.5145, l.10 : add "(the reflectance simulations)" after "F(m)"

p.5145, l.11 : add "(the reflectance observations)" after "the data d"

p.5145, l.11: replace "it" by "this problem"

p.5145, l.12: replace "each element" by "each quantity"

p.5145, l.21 : replace "The covariance matrix C is assumed here to be diagonal since measurements at a given geometry/wavelength are independent of the other measurements. The " by "The measurements at any given wavelength/geometry are supposed to be independent with each other, as each measurement of one wavelength, at one geometry is done individually. The matrix C is thus assumed to be diagonal and its"

p.5145, l.22 : add after "each measurement." the sentence: "We also consider that the atmosphere contribution is negligible, and that the instrument is well calibrated. In the case of satellite measurements, uncertainties or systematic bias can be taken into account, by the mean of a non diagonal matrix, which eigenvectors represent the different sources of bias or uncertainties (C must remain a positive-definite matrix)."

p.5145, l.26: replace "Tarantola and Valette (1982) is" by "Bayes's theorem (Tarantola and Valette,1982) is"

p.5146, l.3 : move the integrand "dd" at the rigth.

p.5146, l.11: replace the period at the end of the line by a comma

p.5146, l.12: add at the beginning of the line "where ^{t} is the transpose operator that applies to $(F(m) - d_{mes})$.

p.5147, l.17: We chose to focus on the 1.5 μm wavelength, as it showed a penetration depth lower than 1 mm and thus much lower than the thickness of the used sample"

p.5147, l.22: Add after "negligible." "Indeed, the penetration depth inside a water ice slab at the 1.5μm wavelength is lower than one millimetre."

p.5148, l.1: remove the comma after i.e.

p.5148, l.2: replace "every possible variability" by "the variability of the model according to its parameters" p.5148, l.9 : replace “Brissaud at al. (2004).” by: “ Brissaud at al. (2004). It *a posteriori*corresponds at this wavelength to 2 % of the signal.

p.5148, l.12 : replace “The full PDF” by: “If the PDF is close to a Gaussian, then it”

p.5149, l.5 : remove “as required by the variability in the optical constants of water ice,”

p.5149, l.20 : replace “wavelengths” by “wavelength”

p.5151, l.1: Add a section “4.4: Numerical validations of the inversion method”

In order to numerically validate the inversion method described above, two kind of tests were conducted. First, we applied a gaussian noise and inverted every spectrum in the synthetic spectral database. We show that with a negligible noise, the parameters are always correctly retrieved with negligible uncertainties, and as the level of noise on the data increases, so do the uncertainties on the results. Secondly, we generated spectra for parameters that were not sampled in the database and tried to recover successfully their characteristics.

On Figure 4 each curve corresponds to a stack of 1000 *a posteriori* PDF for the grain size of the underlying snow resulting from 1000 random noise draws of the same 2% level. Figure 4a is obtained for a low slab thickness of 1mm. In this case, the grain size of the snow can be correctly estimated: the PDF are centred on the correct value and the dispersion suggests an *a posteriori* uncertainty lower than the retrieved value. When the thickness of the slab layer increases, so does the *a posteriori* uncertainty on the estimation of the grain-size. For a slab thickness of 5mm (figure 4b), the *a posteriori* uncertainty is of the same order than the estimated value, meaning that the grain-size cannot be retrieved. The grain-size of the snow thus cannot be retrieved for slab thicknesses greater than 5 mm.

Figure 5a represents the stack of 1000 *a posteriori* PDF for the thickness of the ice layer. These PDF do not depend on the grain-size of the snow, but only on the thickness itself and the level of noise. It shows that the thickness can be estimated, in the

experimental conditions (2 % noise level) with an uncertainty of 2 % for lowest thicknesses to 5 % for highest ones. All obtained *a posteriori* PDF for the thickness were very close to gaussian. We were thus able sum them up by their means and standard deviations, allowing us to plot for example the uncertainty on the thickness estimation as a function (figure 5b) of the thickness that we want to estimate and (figure 6) of the level of noise on the data. Figure 5b show the uncertainty (at 2σ) on the estimation of the thickness of the slab layer as a function of the thickness itself, in the experimental conditions described by Brissaud et al, 2004, that means a 2 % noise level on the signal. This relative uncertainty does not depend on the thickness in the range of values tested. The low values for thicknesses below 1 mm is an effect of the discretisation in the LUT: the thickness has been sampled every 0.1mm. Below 1mm, this sampling step is large relatively to the values itself and ranges from 10 % to 100 %. The relative uncertainty that we expect to be about 5 % is then no longer measurable, and the value drops to 0.

Figure 6 shows the evolution of the *a posteriori* uncertainties for the estimations of thicknesses and grain-sizes as a function of the noise level. For the grain-sizes, a slab thickness of 2 mm has been used. The results show that with very low noise i.e. lower than 0.5 %, the *a posteriori* uncertainties on the results are of the same order of magnitude, even for the grain-size. When the level of noise increase, the uncertainties on the thicknesses estimations increase in the same proportions (figure 6b), unlike the uncertainties on grain-sizes (figure 6a) that increase drastically with the noise level. The uncertainties on the grain-sizes seem to saturate for high noises. This effect is only an edge effect due to the size of the LUT: the dispersion of the *a posteriori* PDF cannot get bigger than the range of values tested.

With the level of noise at 2 % as expected for the measured spectra (Brissaud et al., 2004), *a posteriori* uncertainties are expected to be about 5 % on the thickness, and should be lower than 50 % for the grain-size for low thicknesses. This means that the method should be able to retrieve thicknesses with an uncertainty that correspond to the level of noise, but cannot retrieve grain-sizes of the snow when the ice layer above is thicker than 5mm.

Captions for new figures:

Figure 4: Normalized stacks of 1000 *a posteriori* PDF for the grain-size of the snow, when conducting the inversion on synthetic data, with added random noise. The legends indicate the value for the grain-size used to create the synthetic data. (a) The ice layer is 1 mm thick. (b) The ice layer is 5 mm thick.

Figure 5: (a) Normalized stacks of 1000 *a posteriori* PDF for the thickness of the slab ice layer, when conducting the inversion on synthetic data, with added random noise. The legends indicate the value for the thickness used to create the synthetic data. (b) *a posteriori* uncertainty (at 2σ) on the thickness estimation as a function of the slab thickness.

Figure 6: (a) *A posteriori* uncertainties at 2 /sigma on the grain-size as a function of the noise standard deviation, for a 2 mm thick ice layer. (b) *A posteriori* uncertainties at 2 /sigma on the thickness as a function of the noise standard deviation.

Results

p.5151, l.2 : Change “Specular lobe” into “Specular lobe: roughness retreival”

p.5151, l.10 :Replace “of the measure” by “of the recorded measurement geometries”

p.5152, l.11 : Change “Diffuse” into “Diffuse reflectance: thickness and grain-size retrieval”

p.5152, l.11 : Divide 5.2 into three subsubsections: 5.2.1 Example for individual geometries 5.2.2 Results for 39 geometries 5.2.3 Full BRDF inversion. Paragraph from p.5152, l.12 to p.5153, l.2 will be 5.2.1.; Paragraph from p.5153, l.3 to p.5153, l.14 (after “(sample 3).”) will be 5.2.2.; Paragraph from p.5153, l.14 to p.5154, l.8 will be 5.2.3.

p.5152, l.16 :Replace the sentence “Figure 7 represents examples of the best matches

we obtained for the three measured samples at various geometries.” by the following: “Figure 7 represents three examples of measured and best simulated reflectance spectra for three different geometries.”

p.5152, l.23 : remove “relatively”.

p.5153, l.1: Replace sentence “The general trend of decreasing grain size seems to be in agreement with visual assessment.” by “As predicted by the numerical tests, the snow grain-size is not be accessible for slab thicknesses above 5mm. The *a posteriori* PDF for samples 2 and three then are not to be interpreted.

p.5153, l.10 : replace “inversions” by “inversion”

p.5153, l.18-19 : Again, this sentence is not easy to understand because the meaning of isotropization is not clear, hence the argument sounds weak.

p.5153, l.23-p.5154, l.8 Remove text from “The grain sizes returned” to “at the bottom of the slab layer.” and replace it by: “The grain-size returned (see Figure 11b) for sample one is lower, but compatible with the one given by independent measurements. For samples 2 and 3, the pdf are not interpreted, as the grain-size cannot be constrained by the method.”

p.5154, l.8: Add a section 6 named “Discussion”

The two main goals of this work were (i) to develop and validate an inversion method that is adapted to the treatment of massive and complex datasets such as satellite hyperspectral datasets, and (ii) to partially validate a previously developed radiative transfer model.

The first criterion is the speed of the whole method, including the direct computation of the LUT and the inversion. The lookup tables used for this project were computed in 150 s for the roughness study (1763 wavelengths sampled, 30933 spectra) and 2.5h for the thickness and grain size study (33186 wavelengths sampled, 666 315 spectra). The inversions themselves were performed in less than one-tenth of a second for specular lobe and independent spectral inversions, and 2s for BRDF-as-a-whole inversions. Every calculation was computed on one Intel CPU with 4 GB RAM. It has to be noted that once the lookup table has been created, an unlimited number of inversions can be conducted. This means that this method satisfies the speed criterion for the study of massive and complex datasets. For inversions over very large databases, the code has been adapted to GPU parallelization. It is also possible to increase the speed of the calculation of the lookup tables by means of multi-CPU computing.

A second aspect is the reliability of the inversion method, regardless of the direct model. Indeed, as any model makes assumptions, the method should allow the user to know how to interpret the result obtained. The bayesian statistics in our method allowed us to determine that the thicknesses that we estimated in this work were reliable, with a 5 % uncertainty. Moreover, for the radiative transfer model used in this work (see section 2), and in the experimental conditions described in section 3, we could determine on synthetic cases (see section 4.4) that a 5 % uncertainty should be expected on ice thickness estimation, and that the grain-size of the underlying snow could not be determined for ice thicknesses higher than 5 mm. The experimental results on the thickness were in agreement with these estimations.

The last point to be discussed is the capability of the model to reproduce the reality. Section 5 showed that every thickness estimation was in agreement with independent measurements. This means that the modelling of the ice layer is radiative transfer model is satisfactory, and that this quantity can be determined only using spectral measurements. However, this is not the case for the estimations of the grain-size of the snow. Indeed, when the ice layer is thicker than 5 mm, our synthetic study predicts that it cannot be retrieved. Still, the results obtained on experimental data for slab thicknesses greater than 5 mm (blue and green curves in figures 11 and 14) showed *a posteriori* PDF for the grain-sizes with surprisingly low standard deviations compared to what was obtained on synthetic data. The experimental results favour situations in which the geometrical optics hypothesis that is fundamental in the radiative transfer

model is no longer valid. This shall not be interpreted as a result on the grain-size, as the synthetic test showed that it was unaccessible. These low *a posteriori* uncertainties shall rather be interpreted as a compensation effect: a behaviour that cannot be reproduced by the model may be approached by the most extreme values tested. In our case, small grain-size, even if they are not realistic, or in agreement with the model's hypothesis, will produce an effect in the simulation that reproduces the data better than in the other cases.

Discussion and conclusion

p.5154, l.9: Replace this section by a section “Conclusion”

The aim of this present work is to validate an approximate radiative transfer model developed in Andrieu et al. (2015) using several assumptions. The most debated one is that the radiation become lambertian when it reaches the substrate. We first qualitatively validated this assumption with snow and ice data. We then quantitatively tested and validated our method using a pure slab ice with various thicknesses and snow as a bottom condition. The thicknesses retrieved by the inversion are compatible with the measurements for every geometry, demonstrating the robustness of this method to retrieve the slab thickness from spectroscopy only. The result given by the inversion of the whole data set is also compatible with the measurements. We also validate the angular response of such slabs in the specular lobe. Unfortunately, it was not possible to measure the micro-topography in detail to compare with the retrieved data. Nevertheless, we found a very good agreement between the simulation and the data. In future work, an experimental validation of the specular lobe and roughness should be addressed.

The large uncertainties in the grain size inversion demonstrate that the bottom condition is less important than the slab for the radiation field at first order, as predicted by the synthetic tests conducted. The inconsistency between the *a posteriori* PDF on the grain-sizes for experimental data and numerical tests stresses that synthetic tests must be performed in order to determine which quantities can be retrieved or not in the context of the study, and to precise the expected uncertainties.

The comparison of the *a posteriori* uncertainties in the thickness of the slab and the grain size of the snow substrate illustrates the fact that those uncertainties depend both on the constraint brought by the model itself and the uncertainty introduced into the measurement, which only the Bayesian approach can handle. The use of Bayesian formalism is thus very powerful in comparison with traditional minimization techniques. We propose here a fast and innovative method aiming at massive inversions, and we demonstrated that it is adapted remote sensing spectro-imaging data analysis. The radiative transfer model used in this study was proven appropriate to study the superior slab layer, but not the bottom one, unless the top layer is thin (thinner than 5mm in our case). The whole method is thus adapted to study the top slab layer of a planetary surface using satellite hyperspectral data, for instance Martian seasonal deposits, that are constituted of a slab CO_2 ice layer resting directly on the regolith.

Figures

Fig.1: In the figure, replace “Granular subtrate” by “Granular substrate”

Fig.2 : add “medium” after “in the surface” (line 1)

Fig.3 : remove title above graphs

Fig.3: change caption into: “ (a) Reflectance factor at a wavelength of = 1.4μm vs. phase angle for snow only (black crosses) and the same snow but covered with a 1.420.27mm water ice slab (red squares). (b) Same data but normalized by the value at a phase angle $\alpha = 20°.''$

Fig.4 : remove title above graphs

Fig.4: remove sentence “The sample may be slightly misadjusted resulting in a general drift on the observation.”

Fig.4: remove last sentence “With this adjustments, the model reproduces the data well.“

Fig.5 : remove title above graphs

Fig.5: remove last sentence “The shape and the intensity of the specular lobe are well reproduced.”

Fig.6 : remove title above graph

Fig.6: remove sentence “This function is very sharp and thus the parameter θ is well constrained.”

Fig.7 : set all figures in a row rather than in one column

Fig.7 : remove sentences “The simulated spectra well reproduce the data within the range of a priori uncertainties. For sample 3 (c), the reflectances in the 0.8–1.0 μm range are not very well reproduced. The model cannot match the high levels of the measurement. This could be explained by a change in the experimental protocol, leading to the condensation of very fine frost at the bottom of the slab layer.”

Fig.8 : Replace “Marginal probability density functions *a posteriori*” by “Marginal *a posteriori* probability density functions”

Fig.8 : remove sentences “The functions are very sharp and very close to Gaussian for the thickness of the slab (a) but are broad for the grain size of the substrate (b). The thickness is well constrained by the inversion, whereas the grain size of the substrate cannot be determined with high precision.”

Fig.9 : l.1 change “measures” into “measurement”

Fig.9: remove “The thicknesses retrieved and measured are compatible. “

Fig.9: remove “The geometry appears to have an impact on the result for sample 1 and 2. The thickness estimated seems to increase with incidence and decrease with

azimuth. The geometrical effect disappears for large thicknesses. “

Fig.10 : remove title above graphs

Fig.10 : remove “The simulation reasonably well reproduces, if not perfectly, the geometrical behavior of the surfaces. The quality of the geometrical simulation seems to increase with the thickness of the slab. This is consistent with the isotropization effect of a slab, which will increase with the thickness.”

Fig.11 : the difference with Fig.8 is not clear. Precise that it is the result of all angles/wls inversion All caption (except first sentence) should be moved to Results

Fig.11 : Replace caption by “Marginal probability density functions *a posteriori*” by “Marginal *a posteriori* probability density functions when conducting the inversion on the whole BRDF”

Fig.11: remove “The functions are very sharp and very close to Gaussian for the thickness of the slab (a). The *a posteriori* uncertainties in the results are much smaller than the previous ones, because the data set is larger and thus more constraining. Still, these uncertainties are not fully reliable, as the model cannot perfectly reproduce the BRDF within the a priori uncertainties (see Fig. 10). (b) The grain size can be determined on sample 1, and is consistent with the results on inver- sions of single spectra (see Fig. 8). However, they cannot be inverted for sample 2 and 3, as the returned probability density function is close to a Dirac delta function at the boundary of the definition range.”

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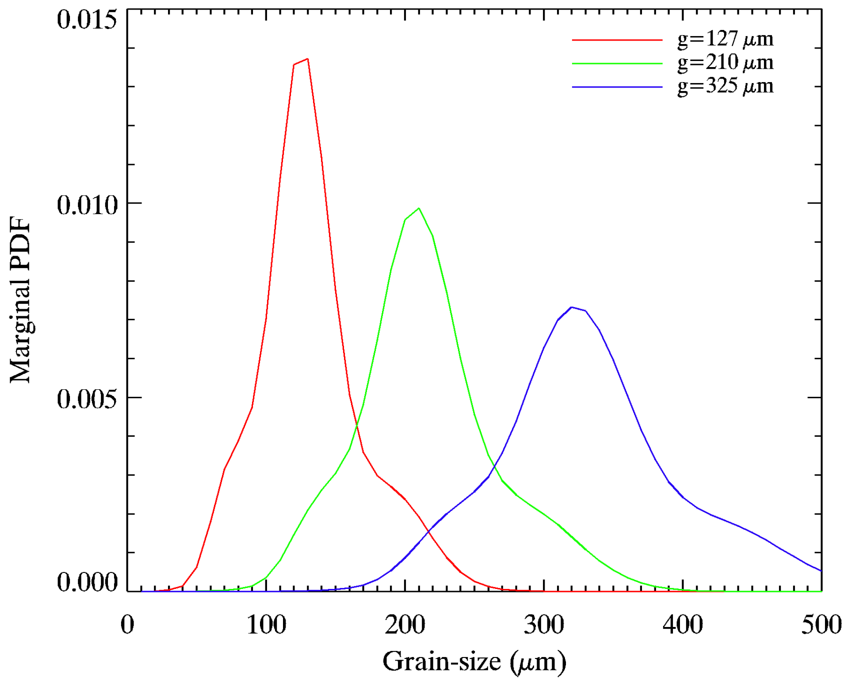

Fig. 1. Figure 4a: Normalized stacks of 1000 a posteriori PDF for the grain-size of the snow for a 1mm thick ice layer, when conducting the inversion on synthetic data, with added random noise.

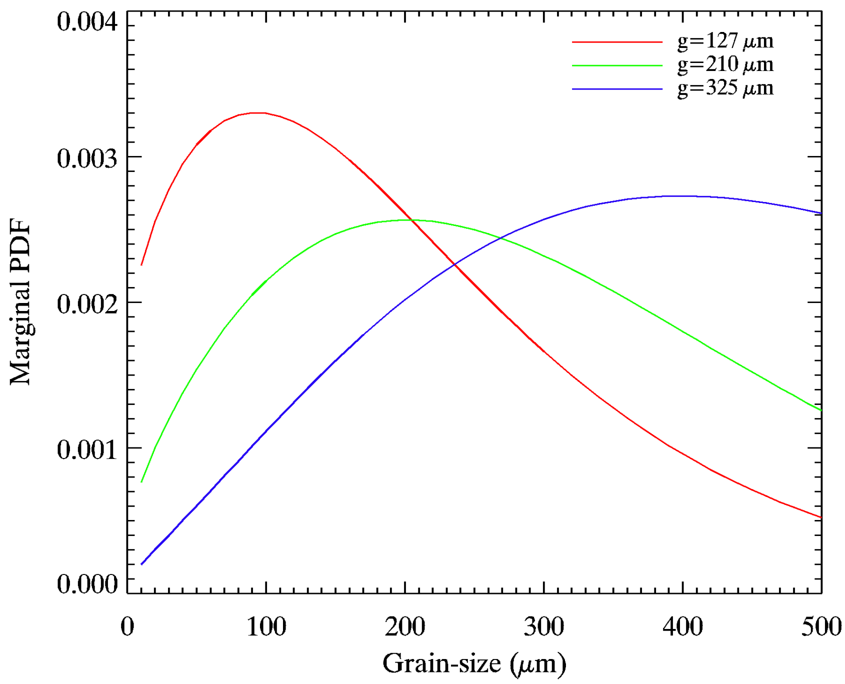

Fig. 2. Figure 4b: Normalized stacks of 1000 a posteriori PDF for the grain-size of the snow for a 5mm thick ice layer, when conducting the inversion on synthetic data, with added random noise.

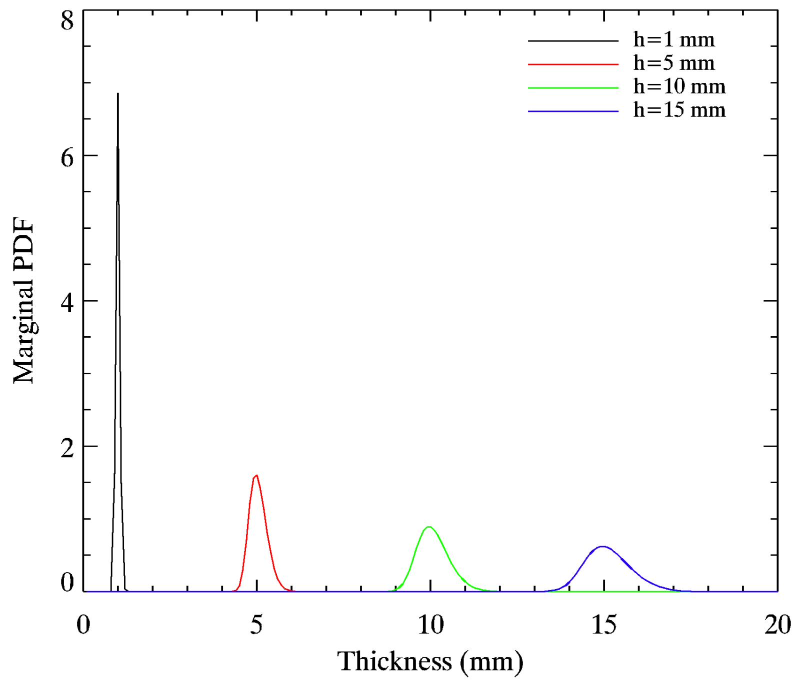

Fig. 3. Figure 5a: Normalized stacks of 1000 a posteriori PDF for the thickness of the slab ice layer, when conducting the inversion on synthetic data, with added random noise.

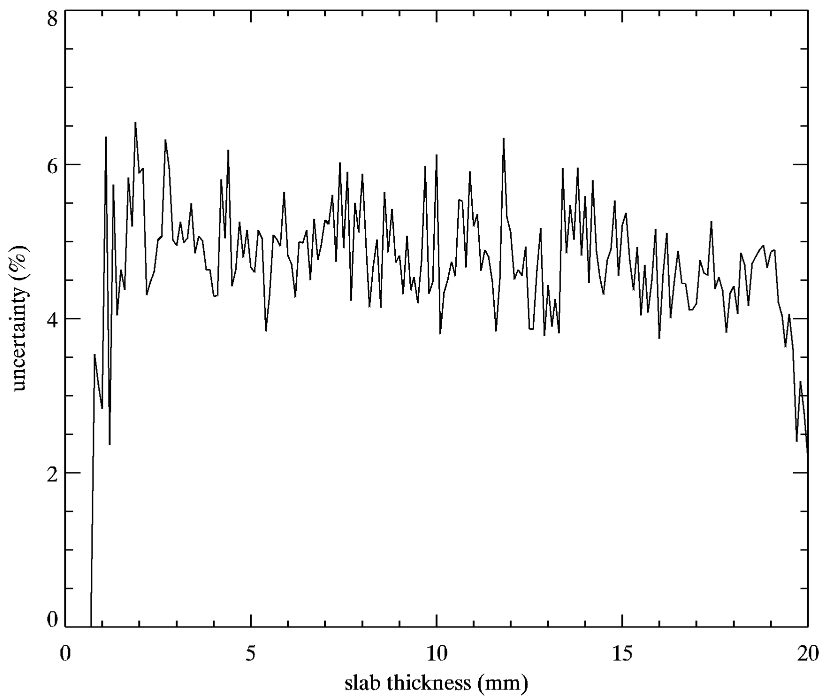

Fig. 4. Figure 5b: A posteriori uncertainty (at 2 sigma) on the thickness estimation as a function of the slab thickness.

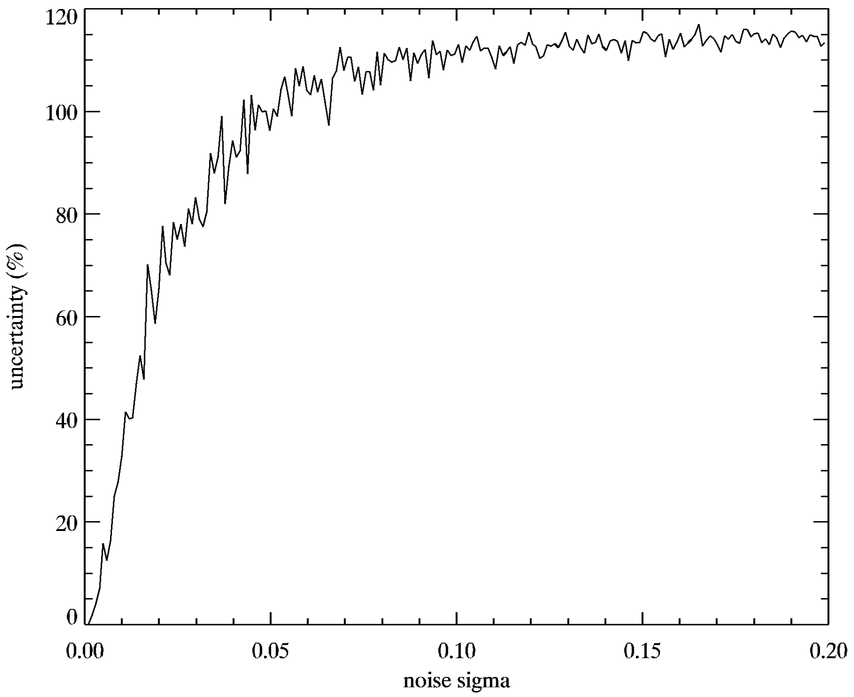

Fig. 5. Figure 6a: A posteriori uncertainties at 2 sigma on the grain-size as a function of the noise standard deviation, for a 2 mm thick ice layer.

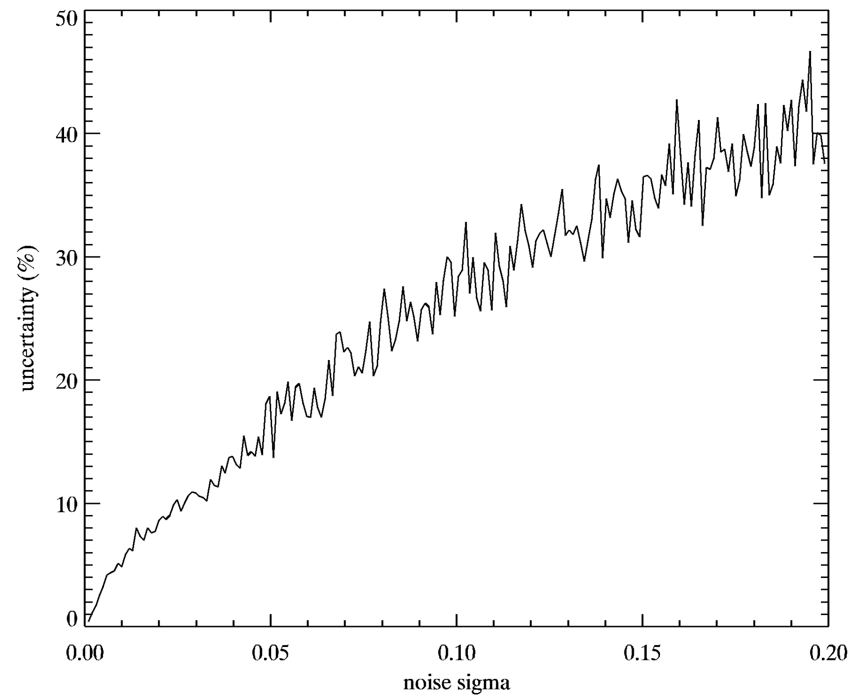

Fig. 6. Figure 6b: A posteriori uncertainties at 2 sigma on the thickness as a function of the noise standard deviation.

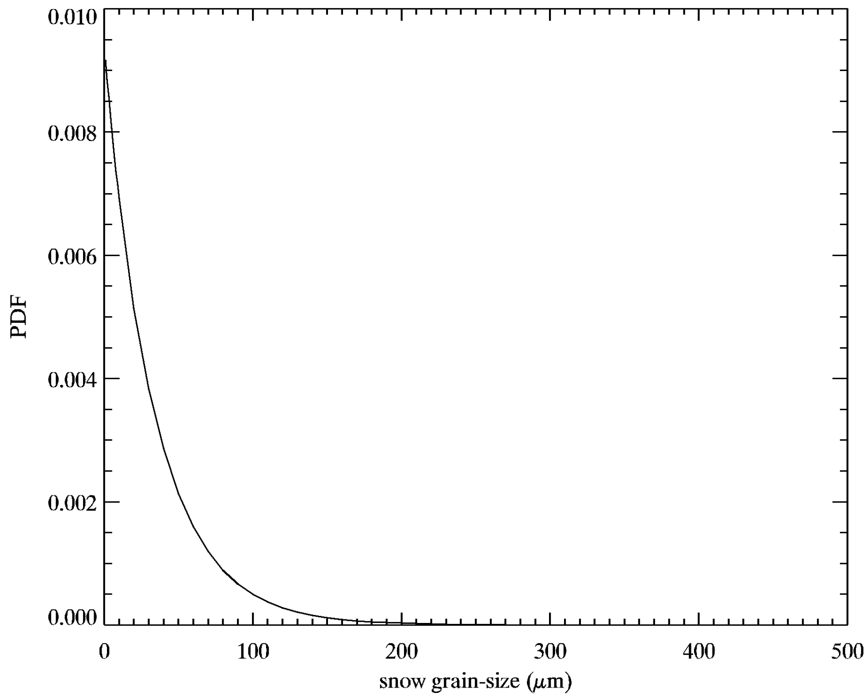

Fig. 7. Supplementary Figure 1: a posteriori pdf for the grain-size, when conducting the inversion on the nadir geometry. The ice layer is 10mm thick and the grain-size is 1μm

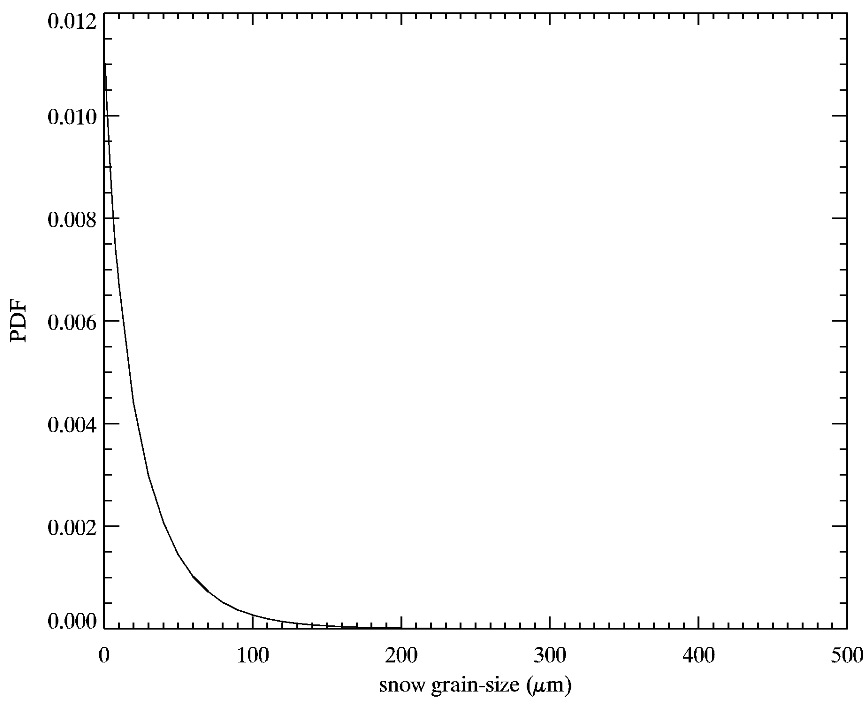

Fig. 8. Supplementary Figure 2: a posteriori pdf for the grain-size, when conducting the inversion on the nadir geometry. The ice layer is 10mm thick and the grain-size is 50μm

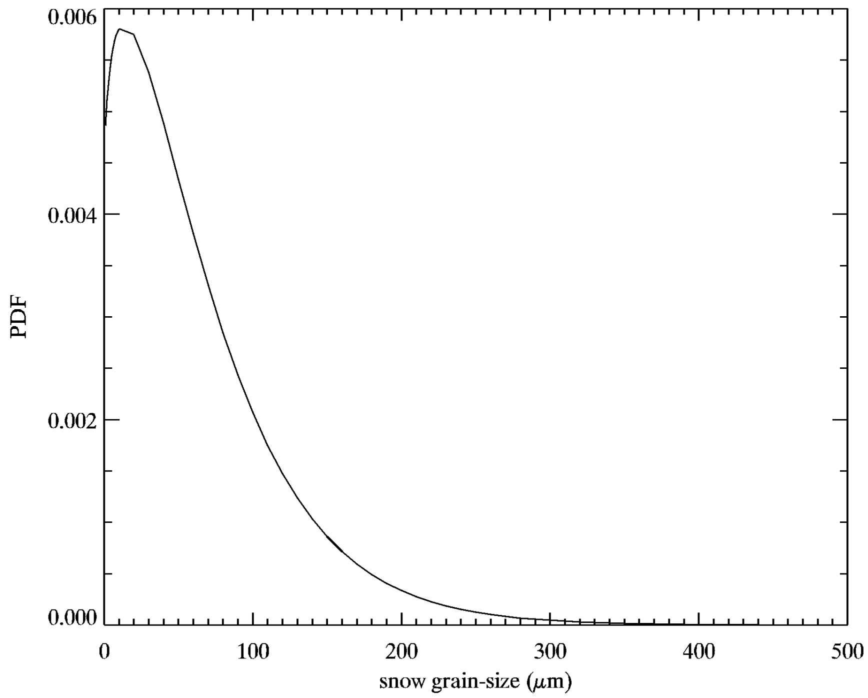

Fig. 9. Supplementary Figure 3: a posteriori pdf for the grain-size, when conducting the inversion on the nadir geometry. The ice layer is 10mm thick and the grain-size is 100μm

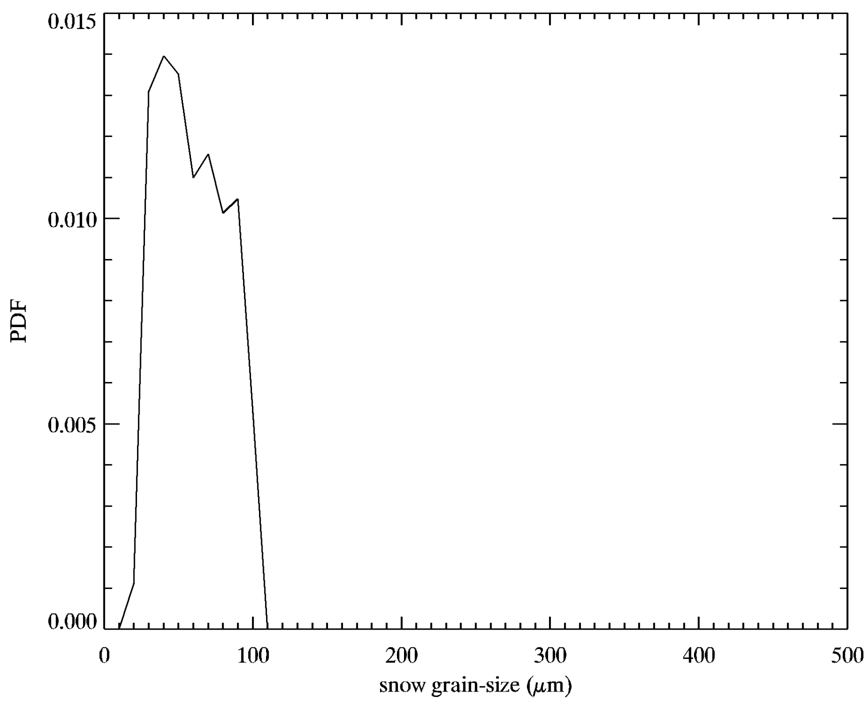

Fig. 10. Supplementary Figure 4: a posteriori pdf for the grain-size, when conducting the inversion on the BRDF. The ice layer is 10mm thick and the grain-size is 100μm