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Discussions

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Interactive comment on "Radiative transfer model for contaminated slabs: experimental validations" by F. Andrieu et al.

F. Andrieu et al.

francois.andrieu@u-psud.fr

Dear referee, Thank you for the attention you paid to this work. Your comments were greatly appreciated and carefully treated.

Every comment, remark and question has been treated. The answers appear in bold (if it survives the editor, as the italics did not...), and are indicated by a "A:" after the comments or questions. If no clear answer is provided below a given remark, please directly refer to the modifications. The modifications to be done in the manuscript appear in a second part.

General comments

This paper presents an evaluation of an approximated radiative transfer model based

on laboratory measurements. The spectral reflectance of pure ice slabs laid on snow was measured in the visible and near infrared (NIR) with a spectro-goniometer. A model for uncertainties propagation based on the bayesian inference is used to retrieve the probability density functions of ice slab thickness and snow grain size from these measurements. The paper is well written and relatively easy to follow. However, some rapid interpretations or inadequate discussions of the results do penalize the evaluation of the model and the results are not as convincing as expected. Also, the interest of the study from a geophysical point of view is poorly demonstrated and should be pointed out for TC readers. In particular, it would be worth replacing this study it in its scientific context, highlighting more explicitly how it differs from previous works. Hence I recommend this paper be published only after the following points (some of them being critical) are addressed.

Specific comments

1) Although the text is easy to follow, it is hard to understand what the exact objective of the paper is.

The abstract and introduction suggest that the objective is to validate a radiative transfer model. Practically, two parts of the model are validated: 1) the representation of surface roughness and 2) the 2 media radiative transfer. From a geophysical point of view, it might be more interesting to evaluate the retrieval method, which constitutes, in addition, a consequent part of the study. The success of the method would suggest that this model can be applied to satellite data analysis, which is a real TC topic. This point does not imply significant changes in the manuscript, except in the definition of the objectives. More generally, the introduction is certainly too short and lacks of relevant references concerning the topic that is studied. For instance there are no references to studies of lake ice (e.g. Mullen and Warren, 1988) or sea ice spectral reflectance (e.g. Perovich 1996), that might be relevant. References to studies in planetary science for which the study might be most relevant are scarce, as well. Hence it is hard to know what is new and original with regards to the literature. Practically, do these laboratory

experiments (a few mm of ice on snow) correspond to situations encountered on Earth, on other planets? Why is it relevant to study that kind of 2 layer media? Why are the surface roughness and ice thickness so important to study?

A: These remarks are very pertinent. Various modifications have been done to take them into account. More precisely, the abstract and introduction has been modified, and the objectives have been changed in order to give more space to the methodology. With this goal, a section dedicated to numerically evaluate the quality of the retrieval has been added. This section also bring key insights to the major issues on grain-size retrieval you pointed out. We show that the method is able the retrieve with negligible uncertainties the thickness and grain-size for low noise data (i.e. lower than 0.5 %). We show that the *a posteriori* uncertainties on the grain-size increase with the noise and with thickness of the slab and that at some point, the grain-size cannot be determined. On the contrary, the thickness of the layer is correctly determined even with a ultra high noise (10 %), regardless of the grain-size of the snow. In this case, the uncertainty on the result is about 30 %. In the experiments case, considering a 2 % noise, *a posteriori* uncertainties ranges from 2 % to 5 % on the thickness, and from 15 % to 80 % for the grain-size, (see attached graphs).

We confirm the expected trend showing that the thicker the slab, the harder it is to get the grain-size. We furthermore obtain comparable levels of *a posteriori* uncertainties in the numerical simulations and real data inversions.

The abstract should detail the main results of the study. Instead, it essentially describes the content of the paper, without giving any quantitative results. It is somehow vague while it should be very straightforward and self-sufficient. Give numbers, percentages, wavelengths... to make it more concrete for the readers.

To sum up, the objective of the paper should also be clearly identified as soon as possible, rather than later in disparate places (see technical comments). It will substantially

help the reader to follow the rest of the paper.

A: The abstract has been modified.

2) Since the evaluated model was published in another journal, it might be worth to detail more its mutliple scattering component, and in particular the meaning of the term isotropization (does it mean isotropic scattering?) which is not sufficiently defined. In fact most of the description is dedicated to the surface roughness and inclusions (the latter being unnecessary, see below), while ice thickness and snow grain size retrieval essentially depend on th treatment of multiple scattering. Concerning this point, as highlighted by the authors, the model used is approximated (p.5154, l.12). In particular I'm quite confused with the choice of isotropic scattering for snow (and probably inclusions) while it's been pointed out decades ago that snow, as any scattering particulate medium with particles large than the wavelenght, is highly anisotropic (e.g. Barkstrom 1972, Bohren 1983). Assuming isotropic scattering is thus a critical shortcoming (even more for BRDF than directional-hemispherical reflectance for instance) and can hardly be used to retrieve snow grain size. This shortcoming has a negative impact on the whole model, unless the ice thickness is such that the snow is not seen by the radiation (in which case the model is reduced to Fresnel reflection). Practically, it is not difficult (nor computationnaly demanding) to handle anisotropic scattering. The asymmetry parameter g has to be accounted for in complement to the single scattering albedo. In the adding-doubling model, it might be suffcient to define the albedo of the underlying snow layer, using for instance analytic expressions including g (e.g. Kokhanovsky and Zege, 2004; Libois et al., 2013). Anyways, the snow grain size retrieval (2 microns) is unrealistic, even in the case of hoar crystals formation. These results cannot remain as is (ie without more discussion) in a paper aimed at validating a radiative transfer model. Considering anisotropic scattering (although the flux will be nearly isotropic due to multiple scattering and weak absorption) may also improve the model and the comparison with experiments.

A: Details on the multiple scattering have been added in the description. This

model is dedicated to the study of the slab layer. The choice of a lambertian bottom condition may be considered a critical shortcoming for snow, as it has indeed been shown decades ago that snow was a very directive material, but this is not the case for planetary regolith, that usually show a weak directivity and are often assumed to be lambertian (e.g. Vincendon 2007). In this work, we tested the model in unfavourable conditions, and showed that this shortcoming had no impact on the reliability of the results concerning the slab layer. Moreover, snow is particularly directive at the wavelengths for which it is the most absorbent, but the radiation at these wavelengths is absorbed by thin slab. On the contrary, for less absorbent wavelengths, the snow is less directive. In the end, this combination strongly flatten the BRDF. For planetary applications, the typical case that will be discussed will be ice on top of granular regolith.

You pointed out that the grain-size retrieval was unrealistic. Clearly the grain-size inversion results for sample 2 and 3 were not satisfactory, and we interpreted it as a grain-size in disagreement with fundamental hypothesis of the model. But this interpretation seems also unsatisfactory. We thus made further numerical testing to investigate this point, and it appeared that in fact, the grain-size of the underlying snow cannot be determined. A section 4.4 has been added on that point, and the results interpretations have been changed (see technical comments and modifications).

3) The choice of the title is very misleading because the reader expects a study on impurities. In fact, the slabs (the nature of the slab is not detailed in the title) are made of pure ice as stated p.5140 l.9.l. As a consequence, many details about the ice inclusions add noise to the paper and should be removed to keep only the version of the model used in the present study (see also technical comments).

A: The title has been changed, and many unnecessary details on the inclusions have been removed.

4) Several conclusions are drawn too quickly (eg surface roughness equivalent to average slope, 2 μm snow grain size retrieval, ice absorption masking the snow below, validation of isotropization...) and should be argued more rigorously (see technical comments).

A: See technical comments

5) The "Discussion and conclusion" part looks more like a conclusion only (or summary of results) because it does not bring much new physical insight. In case a Discussion is really wished, it might be relevant to elect one or two topics and really discuss them. Is the accuracy of the model enough for the objectives? What are its limits? How could it be improved? To which satellite data this could be applied?...

A: A section dedicated to the discussion has been added.

6) The figures captions are too detailed. They should be purely descriptive. Instead, most of the text should be placed within the main text (mostly in the Results part). This is the case for Figs. 3, 4, 5, 7, 8, 9, 10, 11. For instance, "the model reproduces the data well" is not supposed to be found in a figure caption.

A: Captions have been changed

Technical comments (italic indicates suggestions for replacement)

A: Too bad the italics did not survive the editor!

Title

The title is fuzzy and poorly describes the content of the paper. First because the kind of slabs is not precised (ice slabs on top of snow) and the "contaminated" does not detail if it means by air bubbles, dust, BC. Practically it is not contaminated at all in the study, making the term very inappropriate in the title.

A: This remark is very pertinent. We used this title to enhance the link between the two papers, but this was at the price of being confusing for the present study,

so the title has been changed.

Abstract

It is more usual (?) to have the abstract in a single paragraph, making it more consistent and self-sufficient. The objective should stand before the description of experiments and model, not l.11. More generally, the abstract is too general and would be more appropriate for the last paragraph of the introduction aimed at describing the different parts of the paper. Details and reference l.4-5 are useless at this stagel. l.5-6 are not clear because it refers to a second interface while the snow substrate has not been ontroduced so far. It is not important to point out the isotropization there. Keep references for the introduction. l.10 : phase angles is not a standard term, would incidence angles fit (you use it later)? l.13 : the retrieved quantities appear in parenthesis, while this is to me the main result of the paper.

A: All theses remarks have been taken into account: the abstract has been replaced, and the introduction modified.

Introduction

p. 5139, l. 5-6 : ice snow covered don't lead to albedo changes. Changes in snow cover do. You do not mention ice at this stage, only snow, which sounds weird for a model dedicated to ice slabs.

p. 5139, l.8 and 11 : snow grain size and specific surface area are essentially the same thing

l. 13 : please provde references of snow properties retrieval (Zege et al. 2008, Negi and Kokhanovsky 2011)

l.20 homogeneous surface is misleading because it may refer to surface roughness while you supposedly mean vertically homogenous. When the medium is not vertically homogeneous. l.28 "owing to" is awkward. As suggested by the long path lengths measured...

p.5140, l.1 : please provide reference for "several decimeters"

p.5140, l.3 what does contaminated mean here? l. 5 : give some examples of satellite providing this kind of measurements l.7 : impurity content put forward but in fact not used. l.15 : remove "real"

A: See the modifications

Description of the model

p.5140, l.22 : aren't the inclusions assumed actually spherical in your model?

A: The inclusions are supposed to be statistically spherical. This means that they are not supposed to be perfect spheres, but their mean shape is a sphere.

p.5140, l.22-24 : the content of the experiment (pure ice slab overlying snow) should be presented in the introduction with relevant natural cases where this situation occurs.

A: It has been added

p.5141, l.8 : it's the specular contribution in the model (not measurement) that is more likely to be estimated from the roughness

p.5141, l.10-15 : merge this with p.5140, l.24 and avoid repetitions

p.5141, l.20 : reaches

p.5141, l.21 : is valid reaches

p.5141, l.28 : how is the single scattering albedo of snow computed?

A: It is a free parameter of the model, that leads to the grain-size (knowing the optical constants)

p.5142, l.2 : please provide reference for adding-doubling formulas

p.5142, l.18 : what is the size (horizontal extent) of the sample?

A: The sample is a 20cm in diameter cylinder

p.5142, l.19 : how thick is the snow substrate? Are you sure that it is optically thick at all wavelengths?

A: It is 20cm thick. It is considered that a granular media constituted of transparent grains is optically thick after 100 layers. In our case, the grains are submillimetric, and thus it is clearly optically thick at all wavelengths.

p.5142, l.23 : what do you mean by the evolution of snow grain size? Increasing or decreasing? Is it a quantitative statement or was it actually measured?

A: For the grain-size, no measurement was done. For the thickness, a quantitative measurement showed a decrease of 0.343mm per day.

p.5143, l.2 : what is the rotation speed?

A: Rotation speed is 10s per full rotation. The acquisition of one wavelength at one geometry lasts also 10s.

p.5143, l.6 : the title should be more explicit. Maybe Specular reflectance at 1.5 m.

p.5143, l.14-15 : it is not clear whether these measurements were obtained on ice only or with the snow substrate.

p.5143, l.23 : To remain consistent from a title to another, maybe indicate for 3.2.2 on which material the reflectance was measured

p.5143, l.24-25 : this objective should appear earlier, probably in the introduction if it is indeed a main objective of the paper

p.5144, l.1 : what is the reflectance factor? What is phase angle?

p.5144, l.6-10 : how do these data support the isotropization hypothesis? Be more rigorous in your explanation

p.5144, l.8 : what is an isotropic layer? Is scattering isotropic, or is the material geo-

metrically isotropic (or both)?

Method

p.5145, l.10 : it might be useful to define here the actual model parameters to make the model more concrete

p.5145, l.10 : reflectance observations

p.5145, l.21 : the independency of the measurements is not straightforward for the reader. In fact, how are the measurements performed? One geometry, all wavelengths, or one wavelength all geometries? This could make a difference.

A: The independency is actually a hypothesis. It makes sense because the measures are done separately, but they could be dependant (for example, the temperature of the captor may evolve during the experiment). The independency between the different wavelength is a very common hypothesis, but it is perfectly true that its relevance depend on how the experiments are done and what technology is used. In our, case, each wavelength is measured one after the other at one geometry, and them another geometry is taken. We can thus consider that they are independent. In the case of an array of detectors that measures a whole spectrum at once, the signal at one wavelength may be influenced by the neighbouring ones.

p.5145, l.21 : please detail how potential bias is treated, or argue why measurements are unbiased

p.5145, l.23-24 : it is confusing to say that the uniform prior distribution of the parameters is equal to that of the data (whose prior is actually not uniform).

A: That must have been unclear. The prior information is any information on the parameters of the model without any data. It is by definition independent of the data. No prior information means that without seeing the data, the parameter can be anything within its definition range.

p.5146, l.1 : This is actually Bayes's theorem.

p.5146, l.3 : moving integrand at the end might be more standard

p.5146, l.3 : the general formula seems useful given that you then assume a perfect model. Eq.3 could come directly, avoiding unnecessary details.

A: Eq. 1 and 2 seem useful to us...

p.5147, l.9 : It is not clear why there is a specular lobe expected. Is it due to the surface roughness, to the scattering by inclusions, or to something else?

A: The specular lobe (in the model) is due to the roughness of the surface. Its defines the distribution of the slopes and the the proportion of the surface that is in specular conditions for a given observation geometry. In the model, the specular lobe is only due to the first interface reflexion.

p.5147, l.20 : Maybe precise the penetration depth of 1.5 m radiation to support the statement.

A: The penetration depth is <1mm

p.5148, l.9 : please provide quantitative uncertainties for the reflectance measurements

p.5148, l.12 : the original errors propagation is meant to propagate pdf, so you should not summarize the pdf by its mean and std, unless you show it is close to Gaussian, in which case this becomes relevant. Otherwise, why using such a complicated (but very worth) method?

A: Yes that is true, it is the whole point of the method. Still, in this study, *a posteriori* PDF for thickness are close to gaussian for every inversion, and thus we decided to describe these pdf with only mean and standard deviation. The advantage is that it allows to give an expression for uncertainties on the results. Note that we did not express the result this way for the snow grain-size, as it show more complex distributions.

C3133

p.5149, l.4 : here the reader discovers that there are no impurities in the ice slabs. This is quite contradictory with the title of the paper, and makes useless many details provided earlier in the study. Please give this essential detail in the introduction and remove all unnecessary details on inclusions and impurities.

A: Major modifications have been done on that point.

p.5149, l.5 : are you sure this spectral resolution is required? The spectral variations of ice optical index are not that strong.

A: It is true that it is not necessary for water ice... We did it at this resolution because it is needed for CO2 ice, that is the next application of the method. As the method is fast, it did not bother us.

p.5149, l.4-15 : this part is very similar to the one parameter model. Is it really necessary to provide all the details that are substantially similar to the previous model?

A: It seems useful to us

Results

p.5152, l.2 : I do not really understand your point. Is the sample perfectly smooth (no roughness) but tilted? If yes, then why do you observe a lobe? And why does your model (which assumes micro-scale roughness) fit your observations?

A: We consider that the roughness is probably much lower that the observed value. So yes, to a certain extent, we can assume than what we suggest is that the surface is perfectly smooth, but tilted. It is true that in this case, you would not expect to see a lobe, but a ellipsoïd. We actually see a lobe because of the large (4.1°) aperture of the captor : when close to the specular geometry, it collect the whole ellipoïd.

p.5152, l.11 : the title should be more explicit

p.5152, l.11 : it might be useful to add a short description of the content of this section

C3134

because the reader is lost among the various inversion approaches. 1) example for individual geometries 2) results for 39 geometries 3) BRDF or something similar. Moving some text from the figures captions to here might help.

A: This has been done.

p.5152, l.12 : if the roughness found for the previous sample is due to the average slope, how can it be used for the other samples that do not have the same slope?

A: The point is that for such very low roughnesses (<0.5°), the value has negligible impact on the diffuse reflectance. This is the reason why it can be inverted independently. It is the same results as if we considered a perfectly smooth surface.

p.5152, l.16 : what quantity is shown in Fig.7? Which method is used for the best match? All angles together, or 39 angles individually, or a single angle?

p.5152, l.23 : relatively sharp : be more quantitative

p.5152, l.27 : indeed, it would have been useful to get snow grain size characteristics, especially because relevant instruments are used in other laboratories in Grenoble.

p.5152, l.27 : you could get a rough estimate of the snow grain size from the BRDF measurements on snow using any analytical reflectance formula from Hapke or Kokhanovsky.

A: we have that kind of estimates, and we even have a microscope photography of the snow but only before putting ice on top of it. We do not have records after.

p.5153, l.1 : where do you see a decreasing trend in your data? Is this decreasing trend expected from snow physics considerations? Domine et al. (2009) may provide useful discussion.

A: Is is expected to decrease from physics considerations, but as you rightfully stress, Domine et al, 2009 provide counter examples. This consideration have

been removed.

p.5153, l.18-19 : Again, this sentence is not easy to understand because the meaning of isotropization is not clear, hence the argument sounds weak.

A: The argument was indeed weak, as we have no data to support it. It has been removed.

p.5154, l.4 : above all, such snow grains (2 μm) do not exist.

p.5154, l.5-8 : no physical process can produce such small grains, hence the problem is more likely due to the model

A: What happens is probably that (1) for such thicknesses, the signal coming from the snow is weak, and more weakened by the noise, and thus the grain-size cannot be determined (see added numerical tests section), and (2) the data is not compatible with the grain-sizes defined in the model. It seems least uncompatible with ultra low grain-sizes (that are not consistent with the fundamental hypothesis of the model, and thus shall not be interpreted as such). This means that the model cannot reproduce the data and compensates by putting the lowest possible grain-size. It shall be interpreted as an undetermined grain-size. The discussions on that point have been modified.

Another point to take into account is that the model can produce a spectrum for 1μm grain-sizes, but this spectrum is not expected to correspond to the reality as fundamental hypothesis are violated. What is modelled for a 1μm grain-size may correspond to a 1 μm, or maybe to a 20μm grain-size in reality, or even to something radically different.

You can see on the provided supplementary figures 1, 2 and 3 three different *a posteriori* pdf for the grain-size, when the ice layer is 10mm thick, for three different grain-sizes modelled: 1μm, 50μm and 100μm. For these graphs, we created a synthetic LUT, and tried to invert spectra directly extracted from it, to

which we added a noise corresponding to the experiment (Brissaud et al., 2004, approximately 2 % , see the added section 4.4).The pdf represented in these supplementary figures are not stacks as represented in figure 4 and 5 (in the added section about numerical validations of the method) but single runs, and at one geometry (nadir). Every different run corresponds to a pseudo-random generation of noise, and leads to a different *a posteriori* pdf. The plots given here are not outliers, and the same behaviour is observed on other geometries of on the BRDF). It is important to keep in mind that the value that was modelled, and that should be found by the inversion does not correspond to a zero probability, so the method is not invalidated. Still, the standard deviation of the *a posteriori* pdf can be under-estimated due to an edge effect. More plots can be provided on synthetic tests if needed. We can deduce from the numerical testing that the pdf of the grain-size calculated for sample 2 and 3 shall not be interpreted as results. The dispersions observed in the numerical tests for inversion on single geometries are compatible with the one observed in the experiments for single geometries: the supplementary figures are compatible with figure 11 (previously figure 8), and the new figure 3 is also compatible with figure 11 (previously figure 8).

On the other hand, the dispersions of the pdf of the grain-size for inversions of the BRDF on numerical tests and experiment are not compatible with each other (see supplementary figure 4: 10mm slab thickness, 100μm grain-size, 2 % noise, same geometries as the experiment, inversion of the BRDF). This means that the model cannot reproduce properly the data, and that the "least bad" is at the edge of the definition range. It can be interpreted as an incapability of the model to reproduce the data, combined to an edge effect. This stress the importance of numerical testing beforehand. Your remarks made us see this important methodological aspect and greatly improved this paper.

Discussion and conclusion

p.5154, l.12 : see remarks above about the isotropization

p.5154, l.10-18 : this first paragraph does not provide new information (compared to the Results section) as expected in a Discussion.

p.5154, l.27 : please justify "as expected". In fact, the extinction coefficient of ice at 800 nm is 2.1 m-1 (47 cm penetration depth), so that the influence of the snow substrate is major in your measurements. You could set a substrate with albedo varying from 0 to 1 in your model to show the sensitivity to the substrate.

p.5155, l.1 : please give quantitative facts to support your statement

p.5155, l.7-9 : the snow impurities argument is not very convincing, in particular because in the NIR range the impact of most impurities is almost insignificant. You should support your statement with spectral analysis to identify other species than pure ice in the snow.

p.5155, l.20-29 : it might not be appropriate to finish on such a technical note about scientific computing in a paper submitted to The cryosphere. An opening on the possibilities offered by the model for the interpretation of satellite data on icy planets would be for instance much more appropriate.

A: All these remarks have been taken into account in the re-writing of two separate sections "Discussion" and "Conclusion"

Figures

Fig.2 : in the surface medium

Fig.2 : title above graphs is redundant with caption

Fig.3 : what are reflectance factor and phase angle? Measures – measurements. Last 3 sentences should be moved to Results

Fig.4 : title above graphs is redundant with caption

Fig.5 : title above graphs is redundant with caption

Fig.6 : title above graphs is redundant with caption

Fig.7 : set all figures in a row rather than in one column Last 3 sentences should be moved to Results

Fig.8 : Last 2 sentences should be moved to Results

Fig.9 : measure - measurement Last 3 sentences should be moved to Results

Fig.10 : title above graphs is redundant with caption Last 3 sentences should be moved to Results

Fig.11 : the difference with Fig.8 is not clear. Precise that it is the result of all angles/wls inversion All caption (except first sentence) should be moved to Results

A: The required modifications have been done.

References

Barkstrom, B. R. (1972). Some effects of multiple scattering on the distribution of solar radiation in snow and ice. Journal of Glaciology, 11, 357-368.

Bohren, C. F. (1983). Colors of snow, frozen waterfalls, and icebergs. JOSA, 73(12), 1646-1652.

Domine, F., Taillandier, A. S., Cabanes, A., Douglas, T. A., Sturm, M. (2009). Three examples where the specific surface area of snow increased over time. The Cryosphere, 3(1), 31-39.

Kokhanovsky, A. A., Zege, E. P. (2004). Scattering optics of snow. Applied Optics, 43(7), 1589-1602.

Libois, Q., Picard, G., France, J. L., Arnaud, L., Dumont, M., Carmagnola, C. M., King, M. D. (2013). Influence of grain shape on light penetration in snow. The Cryosphere, 7(6), 1803-1818.

Mullen, P. C., Warren, S. G. (1988). Theory of the optical properties of lake ice. Journal of Geophysical Research: Atmospheres (1984–2012), 93(D7), 8403-8414.

Negi, H. S., Kokhanovsky, A. (2011). Retrieval of snow albedo and grain size using reflectance measurements in Himalayan basin. The Cryosphere, 5(1), 203-217.

Perovich, D. K. (1996). The Optical Properties of Sea Ice (No. MONO-96-1). COLD REGIONS RESEARCH AND ENGINEERING LAB HANOVER NH.

Zege, E., Katsev, I., Malinka, A., Prikhach, A., Polonsky, I. (2008). New algorithm to retrieve the effective snow grain size and pollution amount from satellite data. Annals of Glaciology, 49(1), 139-144.

Modifications to be done:

Replace the title of the paper by: "Retrieving the characteristics of slab ice covering snow by remote sensing"

Replace the abstract by:

We present an effort to validate a radiative transfer model previously developed, and an innovative bayesian inversion method designed to retrieve the properties of slab ice covered surfaces. This retrieval method is adapted to satellite data, and is able to provide uncertainties on the results of the inversions. We focused in this study on surfaces composed of a pure slab of water ice covering an optically thick layer of snow. We see sought to retrieve the roughness of the ice/air interface, the thickness of the slab layer and the mean grain-size of the underlying snow. Numerical validations have been conducted on the method, and showed that if the thickness of the slab layer is above 5 mm and the noise on the signal is above 3%, then it is not possible to invert the grain-size of the snow. On the contrary, the roughness and the thickness of the slab can be determined even with ultra high levels of noise up to 20%. Experimental validations have been conducted on spectra collected from laboratory samples of water ice on snow using a specro-gonio-radiometer. The results are in agreement with the numerical validations, and show that a grain-size can be correctly retrieved for low slab thicknesses, but not for bigger ones, and that the roughness and thickness are correctly inverted in every case.

Introduction

p. 5139, l. 4-6: Change "Ice- and snow-covered areas have a strong impact on planetary climate dynamics, as they can lead to significant regional-scale albedo changes at the surface and surface–atmosphere volatiles interactions" into "Ice- and snow-covered areas have a strong impact on planetary climate dynamics, as changes in their characteristics or coverage can lead to significant regional-scale albedo changes at the surface and surface–atmosphere volatiles interactions"

p. 5139, l.8-9: change (Dozier et al., 2009; Negi and Kokhanovsky, 2011) into ((Dozier et al., 2009; Negi and Kokhanovsky, 2011, Picard et al., 2009; Mary et al., 2013)

p. 5139, l.10-11: remove ", or on the specific surface area (Picard et al., 2009; Mary et al., 2013)"

p. 5139, l.13 : add "(Zege et al. 2008, Negi and Kokhanovsky 2011)" after "retrieving such properties"

p.5139, l.14-16: replace the sentence "Ray-tracing algorithms, such as those described in Picard et al. (2009) for snow or Pilorget et al. (2013) for compact polycrystalline ice, simulate the complex path of millions of rays into the surface." by "Ray-tracing algorithms, such as those described in Picard et al. (2009) for snow, Pilorget et al. (2013) for compact polycrystalline ice or Muinonen et al. (2009) for particulate media such as rough ice grains in an atmosphere, simulate the complex path of millions of rays into the surface."

p.5139,lL16-17: replace sentence: "They provide very accurate simulations but have the weakness of being time consuming." by: "Such modelings are experiencing a golden era due to the positive comparison between models and exact calculations (e.g. Muinonnen et al. 2012, Mishchenko et al., 2015)."

p. 5139, l.19: replace "inspired from" by "inspired by"

p. 5139, l.20: replace "as homogeneous" by "statistically as a mono-layer"

p. 5139, l.28: replace "owing to" by "as suggested by".

p.5140, l.1 : replace "several decimeters" by "several centimeters to decimeters (Eluszkiewicz 1993, Quirico et al. 1999, Douté et al. 1999, Douté et al., 2001)".

p.5140, l.1-17: replace the text from "Radiative transfer" on line 1 to the end of the introduction by the following text: "Compact slabs have very different radiative properties from close packed granular media, and radiative transfer models have been developed

to study their characteristics (e.g. Mullen and Warren 1988, Jin Stamnes 1994, Perovich 1996, Jin 2006) in the case of sea or lake ices. We developed an approximated model (Andrieu et al., 2015) model that has the interest of being able to model a layer of ice covering a surface with radically different optical properties, for instance a refractive index, unlike it predecessors. It was designed to study planetary ice slabs, with a fast numerical implementation, which has already been numerically validated and aims at the analysis of massive spectro-imaging planetary data such as the OMEGA (Bibring et al. 2004) or CRISM (Murchie et al., 2007) datasets for the study of Mars icy surface and seasonal cycle, NIMS (Carlson et al., 1992) dataset for SO_2 on Io or RALPH (Reuter et al., 2009) data for the ices of Pluto. For this purpose, it is semi-analytic and implemented to optimize the computation time.

In the present article, we will test the accuracy of this approximated model on laboratory spectroscopic measurements of pure water ice on top of snow bidirectional reflectance distribution function (BRDF). The slabs that will be studied thus contain no impurity, and the surface properties we will seek to retrieve will be the thickness of the ice, the roughness of the surface and the grain-size of the underlaying snow. The main goals of this work are thus (i) to test the ability of the model to reproduce reality and (ii) to propose an inversion framework to retrieve surface ice properties, including uncertainties, in order to demonstrate the applicability of the approach to satellite data.

We presents a set of spectro-goniometric measurements of different water ice samples put on top of snow using the spectro-radiogoniometer described in Brissaud et al. (2004). Three kind of experiments were conducted. First, the BRDF was measured for a snow layer only, and then measured again after adding a slab ice layer at the top. The objective was to test the effect of an ice layer at the top on the directivity of the surface. Second, the specular spot was closely investigated, at high angular resolution, at the wavelength of 1.5 μm, where ice behaves as a very absorbing media. Finally, the bidirectional reflectance was sampled at various geometries on 61 wavelengths ranging from 0.8 to 2.0 μm. In order to validate the model, we made qualitative tests

to demonstrate the relative isotropization of the flux. We also conducted quantitative assessments by using a Bayesian inversion method in order to estimate the sample thickness, surface roughness and snow grain-size from the radiative measurements only. A simple comparison between the retrieved parameters and the direct independent measurements allowed us to validate the model.

The inversion algorithm that will be tested is based on lookup tables that minimize the computation time of the direct model. The solution is then formulated as a probability density function, using bayesian formalism. This strategy will be very useful for analysing hyperspectral images. The thickness of ice estimated from the inversion will be compared to real direct measurements. In addition, the specular lobe will be adjusted to demonstrate that the model is able to reasonably fit the data with a coherent roughness value."

Description of the model

p.5140, l.27-p5141, l5: Remove text from "Within the slab" to "even bubbles"

p.5141, l.8 : replace "of a measurement" by "in the model"

p.5141, l.10-15 : remove "The surface is considered to be constituted of many unresolved facets, whose orientations follow the defined probability density function. The specular reflectance is obtained integrating every reflection on the different facets."

p.5141, l.20 : replace "reach" by "reaches"

p.5141, l.21 : replace "are" by "is"

p.5141,l.22-23: remove the following sentence: "This means that the size of the inclusions and the thickness of the slab layer must be larger than the considered wavelength."

p.5141,l.23-24: replace "The inclusions inside the matrix are close to spherical and homogeneously distributed." by "If the matrix is contaminated with inclusions, unlike in this

work, then these inclusions are supposed to be close to spherical and homogeneously distributed inside the matrix."

p.5142, l.2 : add references (Stamnes et al., 1988, Douté et al. 1998, Van de Hulst, 2012) after "formulas

p.5142, l.18 : replace "sections" by "circular sections of 20 cm in diameter"

p.5143, l.2 : add after "the measurement." The sample complete a full rotation (10s) during the measurement of the reflectance at one wavelength and one geometry.

p.5143, l.6 : change "contribution" into "reflectance"

p.5143, l.13: replace title by "Ice on snow diffuse reflectance spectra"

p.5144, l.1 : add "(the ratio between the bidirectionnal reflectance I/F of the surface and the reflectance of a perfectly lambertian surface)" after "factor"

p.5144, l.1: add "(angle between incident and emergent directions)" after "phase angle"

p.5144, l.5 : add "as the dependance of the reflectance on the phase angle is almost killed by the addition of the ice layer" after "matrix".

p.5144, l.8 : replace "isotropic" by "lambertian"

p.5144, l.24; replace "inverse" by "invert"

Method

p.5145, l.10 : add (i.e. the slab thickness, the roughness parameter, the snow grain-size) after "model parameters m"

p.5145, l.10 : add "(the reflectance simulations)" after "F(m)"

p.5145, l.11 : add "(the reflectance observations)" after "the data d"

p.5145, l.11: replace "it" by "this problem"

C3145

p.5145, l.12: replace "each element" by "each quantity"

p.5145, l.21 : replace "The covariance matrix C is assumed here to be diagonal since measurements at a given geometry/wavelength are independent of the other measurements. The " by "The measurements at any given wavelength/geometry are supposed to be independent with each other, as each measurement of one wavelength, at one geometry is done individually. The matrix C is thus assumed to be diagonal and its"

p.5145, l.22 : add after "each measurement." the sentence: "We also consider that the atmosphere contribution is negligible, and that the instrument is well calibrated. In the case of satellite measurements, uncertainties or systematic bias can be taken into account, by the mean of a non diagonal matrix, which eigenvectors represent the different sources of bias or uncertainties (C must remain a positive-definite matrix)."

p.5145, l.26: replace "Tarantola and Valette (1982) is" by "Bayes's theorem (Tarantola and Valette,1982) is"

p.5146, l.3 : move the integrand "dd" at the rigth.

p.5146, l.11: replace the period at the end of the line by a comma

p.5146, l.12: add at the beginning of the line "where ^{t} is the transpose operator that applies to $(F(m) - d_{mes})$.

p.5147, l.17: We chose to focus on the 1.5 μm wavelength, as it showed a penetration depth lower than 1 mm and thus much lower than the thickness of the used sample"

p.5147, l.22: Add after "negligible." "Indeed, the penetration depth inside a water ice slab at the 1.5μm wavelength is lower than one millimetre."

p.5148, l.1: remove the comma after i.e.

p.5148, l.2: replace "every possible variability" by "the variability of the model according to its parameters"

C3146

p.5148, l.9 : replace "Brissaud at al. (2004)." by: " Brissaud at al. (2004). It *a posteriori*corresponds at this wavelength to 2 % of the signal.

p.5148, l.12 : replace "The full PDF" by: "If the PDF is close to a Gaussian, then it"

p.5149, l.5 : remove "as required by the variability in the optical constants of water ice,"

p.5149, l.20 : replace "wavelengths" by "wavelength"

p.5151, l.1: Add a section "4.4: Numerical validations of the inversion method"

In order to numerically validate the inversion method described above, two kind of tests were conducted. First, we applied a gaussian noise and inverted every spectrum in the synthetic spectral database. We show that with a negligible noise, the parameters are always correctly retrieved with negligible uncertainties, and as the level of noise on the data increases, so do the uncertainties on the results. Secondly, we generated spectra for parameters that were not sampled in the database and tried to recover successfully their characteristics.

On Figure 4 each curve corresponds to a stack of 1000 *a posteriori* PDF for the grain size of the underlying snow resulting from 1000 random noise draws of the same 2% level. Figure 4a is obtained for a low slab thickness of 1mm. In this case, the grain size of the snow can be correctly estimated: the PDF are centred on the correct value and the dispersion suggests an *a posteriori* uncertainty lower than the retrieved value. When the thickness of the slab layer increases, so does the *a posteriori* uncertainty on the estimation of the grain-size. For a slab thickness of 5mm (figure 4b), the *a posteriori* uncertainty is of the same order than the estimated value, meaning that the grain-size cannot be retrieved. The grain-size of the snow thus cannot be retrieved for slab thicknesses greater than 5 mm.

Figure 5a represents the stack of 1000 *a posteriori* PDF for the thickness of the ice layer. These PDF do not depend on the grain-size of the snow, but only on the thickness itself and the level of noise. It shows that the thickness can be estimated, in the

experimental conditions (2 % noise level) with an uncertainty of 2 % for lowest thicknesses to 5 % for highest ones. All obtained *a posteriori* PDF for the thickness were very close to gaussian. We were thus able sum them up by their means and standard deviations, allowing us to plot for example the uncertainty on the thickness estimation as a function (figure 5b) of the thickness that we want to estimate and (figure 6) of the level of noise on the data. Figure 5b show the uncertainty (at 2σ) on the estimation of the thickness of the slab layer as a function of the thickness itself, in the experimental conditions described by Brissaud et al, 2004, that means a 2 % noise level on the signal. This relative uncertainty does not depend on the thickness in the range of values tested. The low values for thicknesses below 1 mm is an effect of the discretisation in the LUT: the thickness has been sampled every 0.1mm. Below 1mm, this sampling step is large relatively to the values itself and ranges from 10 % to 100 %. The relative uncertainty that we expect to be about 5 % is then no longer measurable, and the value drops to 0.

Figure 6 shows the evolution of the *a posteriori* uncertainties for the estimations of thicknesses and grain-sizes as a function of the noise level. For the grain-sizes, a slab thickness of 2 mm has been used. The results show that with very low noise i.e. lower than 0.5 %, the *a posteriori* uncertainties on the results are of the same order of magnitude, even for the grain-size. When the level of noise increase, the uncertainties on the thicknesses estimations increase in the same proportions (figure 6b), unlike the uncertainties on grain-sizes (figure 6a) that increase drastically with the noise level. The uncertainties on the grain-sizes seem to saturate for high noises. This effect is only an edge effect due to the size of the LUT: the dispersion of the *a posteriori* PDF cannot get bigger than the range of values tested.

With the level of noise at 2 % as expected for the measured spectra (Brissaud et al., 2004), *a posteriori* uncertainties are expected to be about 5 % on the thickness, and should be lower than 50 % for the grain-size for low thicknesses. This means that the method should be able to retrieve thicknesses with an uncertainty that correspond to

the level of noise, but cannot retrieve grain-sizes of the snow when the ice layer above is thicker than 5mm.

Captions for new figures:

Figure 4: Normalized stacks of 1000 *a posteriori* PDF for the grain-size of the snow, when conducting the inversion on synthetic data, with added random noise. The legends indicate the value for the grain-size used to create the synthetic data. (a) The ice layer is 1 mm thick. (b) The ice layer is 5 mm thick.

Figure 5: (a) Normalized stacks of 1000 *a posteriori* PDF for the thickness of the slab ice layer, when conducting the inversion on synthetic data, with added random noise. The legends indicate the value for the thickness used to create the synthetic data. (b) *a posteriori* uncertainty (at 2σ) on the thickness estimation as a function of the slab thickness.

Figure 6: (a) *A posteriori* uncertainties at 2 /sigma on the grain-size as a function of the noise standard deviation, for a 2 mm thick ice layer. (b) *A posteriori* uncertainties at 2 /sigma on the thickness as a function of the noise standard deviation.

Results

p.5151, l.2 : Change "Specular lobe" into "Specular lobe: roughness retreival"

p.5151, l.10 :Replace "of the measure" by "of the recorded measurement geometries"

p.5152, l.11 : Change "Diffuse" into "Diffuse reflectance: thickness and grain-size retrieval"

p.5152, l.11 : Divide 5.2 into three subsubsections: 5.2.1 Example for individual geometries 5.2.2 Results for 39 geometries 5.2.3 Full BRDF inversion. Paragraph from p.5152, l.12 to p.5153, l.2 will be 5.2.1.; Paragraph from p.5153, l.3 to p.5153, l.14 (after "(sample 3).") will be 5.2.2.; Paragraph from p.5153, l.14 to p.5154, l.8 will be 5.2.3.

p.5152, l.16 :Replace the sentence "Figure 7 represents examples of the best matches

we obtained for the three measured samples at various geometries." by the following: "Figure 7 represents three examples of measured and best simulated reflectance spectra for three different geometries."

p.5152, l.23 : remove "relatively".

p.5153, l.1: Replace sentence "The general trend of decreasing grain size seems to be in agreement with visual assessment." by "As predicted by the numerical tests, the snow grain-size is not be accessible for slab thicknesses above 5mm. The *a posteriori* PDF for samples 2 and three then are not to be interpreted.

p.5153, l.10 : replace "inversions" by "inversion"

p.5153, l.18-19 : Again, this sentence is not easy to understand because the meaning of isotropization is not clear, hence the argument sounds weak.

p.5153, l.23-p.5154, l.8 Remove text from "The grain sizes returned" to "at the bottom of the slab layer." and replace it by: "The grain-size returned (see Figure 11b) for sample one is lower, but compatible with the one given by independent measurements. For samples 2 and 3, the pdf are not interpreted, as the grain-size cannot be constrained by the method."

p.5154, l.8: Add a section 6 named "Discussion"

The two main goals of this work were (i) to develop and validate an inversion method that is adapted to the treatment of massive and complex datasets such as satellite hyperspectral datasets, and (ii) to partially validate a previously developed radiative transfer model.

The first criterion is the speed of the whole method, including the direct computation of the LUT and the inversion. The lookup tables used for this project were computed in 150 s for the roughness study (1763 wavelengths sampled, 30933 spectra) and 2.5h for the thickness and grain size study (33186 wavelengths sampled, 666 315 spectra). The inversions themselves were performed in less than one-tenth of a second for spec-

ular lobe and independent spectral inversions, and 2s for BRDF-as-a-whole inversions. Every calculation was computed on one Intel CPU with 4 GB RAM. It has to be noted that once the lookup table has been created, an unlimited number of inversions can be conducted. This means that this method satisfies the speed criterion for the study of massive and complex datasets. For inversions over very large databases, the code has been adapted to GPU parallelization. It is also possible to increase the speed of the calculation of the lookup tables by means of multi-CPU computing.

A second aspect is the reliability of the inversion method, regardless of the direct model. Indeed, as any model makes assumptions, the method should allow the user to know how to interpret the result obtained. The bayesian statistics in our method allowed us to determine that the thicknesses that we estimated in this work were reliable, with a 5 % uncertainty. Moreover, for the radiative transfer model used in this work (see section 2), and in the experimental conditions described in section 3, we could determine on synthetic cases (see section 4.4) that a 5 % uncertainty should be expected on ice thickness estimation, and that the grain-size of the underlying snow could not be determined for ice thicknesses higher than 5 mm. The experimental results on the thickness were in agreement with these estimations.

The last point to be discussed is the capability of the model to reproduce the reality. Section 5 showed that every thickness estimation was in agreement with independent measurements. This means that the modelling of the ice layer is radiative transfer model is satisfactory, and that this quantity can be determined only using spectral measurements. However, this is not the case for the estimations of the grain-size of the snow. Indeed, when the ice layer is thicker than 5 mm, our synthetic study predicts that it cannot be retrieved. Still, the results obtained on experimental data for slab thicknesses greater than 5 mm (blue and green curves in figures 11 and 14) showed *a posteriori* PDF for the grain-sizes with surprisingly low standard deviations compared to what was obtained on synthetic data. The experimental results favour situations in which the geometrical optics hypothesis that is fundamental in the radiative transfer

model is no longer valid. This shall not be interpreted as a result on the grain-size, as the synthetic test showed that it was unaccessible. These low *a posteriori* uncertainties shall rather be interpreted as a compensation effect: a behaviour that cannot be reproduced by the model may be approached by the most extreme values tested. In our case, small grain-size, even if they are not realistic, or in agreement with the model's hypothesis, will produce an effect in the simulation that reproduces the data better than in the other cases.

Discussion and conclusion

p.5154, l.9: Replace this section by a section "Conclusion"

The aim of this present work is to validate an approximate radiative transfer model developed in Andrieu et al. (2015) using several assumptions. The most debated one is that the radiation become lambertian when it reaches the substrate. We first qualitatively validated this assumption with snow and ice data. We then quantitatively tested and validated our method using a pure slab ice with various thicknesses and snow as a bottom condition. The thicknesses retrieved by the inversion are compatible with the measurements for every geometry, demonstrating the robustness of this method to retrieve the slab thickness from spectroscopy only. The result given by the inversion of the whole data set is also compatible with the measurements. We also validate the angular response of such slabs in the specular lobe. Unfortunately, it was not possible to measure the micro-topography in detail to compare with the retrieved data. Nevertheless, we found a very good agreement between the simulation and the data. In future work, an experimental validation of the specular lobe and roughness should be addressed.

The large uncertainties in the grain size inversion demonstrate that the bottom condition is less important than the slab for the radiation field at first order, as predicted by the synthetic tests conducted. The inconsistency between the *a posteriori* PDF on the grain-sizes for experimental data and numerical tests stresses that synthetic tests

must be performed in order to determine which quantities can be retrieved or not in the context of the study, and to precise the expected uncertainties.

The comparison of the *a posteriori* uncertainties in the thickness of the slab and the grain size of the snow substrate illustrates the fact that those uncertainties depend both on the constraint brought by the model itself and the uncertainty introduced into the measurement, which only the Bayesian approach can handle. The use of Bayesian formalism is thus very powerful in comparison with traditional minimization techniques. We propose here a fast and innovative method aiming at massive inversions, and we demonstrated that it is adapted remote sensing spectro-imaging data analysis. The radiative transfer model used in this study was proven appropriate to study the superior slab layer, but not the bottom one, unless the top layer is thin (thinner than 5mm in our case). The whole method is thus adapted to study the top slab layer of a planetary surface using satellite hyperspectral data, for instance Martian seasonal deposits, that are constituted of a slab CO_2 ice layer resting directly on the regolith.

Figures

Fig.1: In the figure, replace "Granular subtrate" by "Granular substrate"

Fig.2 : add "medium" after "in the surface" (line 1)

Fig.3 : remove title above graphs

Fig.3: change caption into: " (a) Reflectance factor at a wavelength of = 1.4μm vs. phase angle for snow only (black crosses) and the same snow but covered with a 1.420.27mm water ice slab (red squares). (b) Same data but normalized by the value at a phase angle $\alpha = 20°$."

Fig.4 : remove title above graphs

Fig.4: remove sentence "The sample may be slightly misadjusted resulting in a general drift on the observation."

Fig.4: remove last sentence "With this adjustments, the model reproduces the data well."

Fig.5 : remove title above graphs

Fig.5: remove last sentence "The shape and the intensity of the specular lobe are well reproduced."

Fig.6 : remove title above graph

Fig.6: remove sentence "This function is very sharp and thus the parameter θ is well constrained."

Fig.7 : set all figures in a row rather than in one column

Fig.7 : remove sentences "The simulated spectra well reproduce the data within the range of a priori uncertainties. For sample 3 (c), the reflectances in the 0.8–1.0 μm range are not very well reproduced. The model cannot match the high levels of the measurement. This could be explained by a change in the experimental protocol, leading to the condensation of very fine frost at the bottom of the slab layer."

Fig.8 : Replace "Marginal probability density functions *a posteriori*" by "Marginal *a posteriori* probability density functions"

Fig.8 : remove sentences "The functions are very sharp and very close to Gaussian for the thickness of the slab (a) but are broad for the grain size of the substrate (b). The thickness is well constrained by the inversion, whereas the grain size of the substrate cannot be determined with high precision."

Fig.9 : l.1 change "measures" into "measurement"

Fig.9: remove "The thicknesses retrieved and measured are compatible. "

Fig.9: remove "The geometry appears to have an impact on the result for sample 1 and 2. The thickness estimated seems to increase with incidence and decrease with

azimuth. The geometrical effect disappears for large thicknesses. "

Fig.10 : remove title above graphs

Fig.10 : remove "The simulation reasonably well reproduces, if not perfectly, the geometrical behavior of the surfaces. The quality of the geometrical simulation seems to increase with the thickness of the slab. This is consistent with the isotropization effect of a slab, which will increase with the thickness."

Fig.11 : the difference with Fig.8 is not clear. Precise that it is the result of all angles/wls inversion All caption (except first sentence) should be moved to Results

Fig.11 : Replace caption by "Marginal probability density functions *a posteriori*" by "Marginal *a posteriori* probability density functions when conducting the inversion on the whole BRDF"

Fig.11: remove "The functions are very sharp and very close to Gaussian for the thickness of the slab (a). The *a posteriori* uncertainties in the results are much smaller than the previous ones, because the data set is larger and thus more constraining. Still, these uncertainties are not fully reliable, as the model cannot perfectly reproduce the BRDF within the a priori uncertainties (see Fig. 10). (b) The grain size can be determined on sample 1, and is consistent with the results on inver- sions of single spectra (see Fig. 8). However, they cannot be inverted for sample 2 and 3, as the returned probability density function is close to a Dirac delta function at the boundary of the definition range."

Add to references:

Bibring, J.-P.; Langevin, Y.; Mustard, J. F.; Poulet, F.; Arvidson, R.; Gendrin, A.; Gondet, B.; Mangold, N.; Pinet, P.; Forget, F.; the OMEGA team; Berthé, M.; Bibring, J.-P.; Gendrin, A.; Gomez, C.; Gondet, B.; Jouglet, D.; Poulet, F.; Soufflot, A.; Vincendon, M.; Combes, M.; Drossart, P.; Encrenaz, T.; Fouchet, T.; Merchiorri, R.; Belluci, G.; Altieri, F.; Formisano, V.; Capaccioni, F.; Cerroni, P.; Coradini, A.; Fonti, S.; Korablev,

O.; Kottsov, V.; Ignatiev, N.; Moroz, V.; Titov, D.; Zasova, L.; Loiseau, D.; Mangold, N.; Pinet, P.; Douté, S.; Schmitt, B.; Sotin, C.; Hauber, E.; Hoffmann, H.; Jaumann, R.; Keller, U.; Arvidson, R.; Mustard, J. F.; Duxbury, T.; Forget, F. Neukum, G., Global Mineralogical and Aqueous Mars History Derived from OMEGA/Mars Express Data, Science, 2006, 312, 400-404

Carlson, R.; Weissman, P.; Smythe, W. Mahoney, J. Russell, C. (Ed.) Near-Infrared Mapping Spectrometer Experiment on Galileo, The Galileo Mission, Springer Netherlands, 1992, 457-502

Douté, S.; Schmitt, B.; Quirico, E.; Owen, T.; Cruikshank, D.; de Bergh, C.; Geballe, T. Roush, T., Evidence for Methane Segregation at the Surface of Pluto, Icarus, 1999, 142, 421-444

Douté S., S.; Schmitt, B.; Lopes-Gautier, R.; Carlson, R.; Soderblom, L.; Shirley, J. the Galileo NIMS Team, Mapping SO2 Frost on Io by the Modeling of NIMS Hyperspectral Images, Icarus, 2001, 149, 107-132

Eluszkiewicz, J., On the Microphysical State of the Martian Seasonal Caps, Icarus, 1993, 103, 43-48

Jin, Z., K. Stamnes, and W.F. Weeks (1994) The effect of sea ice on the solar energy budget in the atmosphere-sea ice-ocean system: A model study. Journal of Geophysical Research, 99(C12), 25281–25294.

Jin, Z.; Charlock, T. P.; Rutledge, K.; Stamnes, K. Wang, Y. Analytical solution of radiative transfer in the coupled atmosphere-ocean system with a rough surface Appl. Opt., Applied Optics, OSA, 2006, 45, 7443-7455

Mishchenko, M. I.; Dlugach, J. M.; Chowdhary, J. Zakharova, N. T. Polarized bidirectional reflectance of optically thick sparse particulate layers: An efficient numerically exact radiative-transfer solution, Journal of Quantitative Spectroscopy and Radiative Transfer, 2015, 156, 97-108

Muinonen, K., Nousiainen, T., Lindqvist, H., Munoz, O., and Videen, G. (2009). Light scattering by Gaussian particles with internal inclusions and roughened surfaces using ray optics. Journal of Quantitative Spectroscopy and Radiative Transfer 110, 1628-1639.

Muinonen, K., Mishchenko, M. I., Dlugach, J. M., Zubko, E., Penttilna, A., and Videen, G. (2012). Coherent backscattering numerically verified for a finite volume of spherical particles. Astrophysical Journal 760, 118, 11p. (doi:10.1088/0004-637X/760/2/118).

Mullen, P. C., Warren, S. G. (1988). Theory of the optical properties of lake ice. Journal of Geophysical Research: Atmospheres (1984–2012), 93(D7), 8403-8414.

Murchie, S.; Arvidson, R.; Bedini, P.; Beisser, K.; Bibring, J.-P.; Bishop, J.; Boldt, J.; Cavender, P.; Choo, T.; Clancy, R. T.; Darlington, E. H.; Des Marais, D.; Espiritu, R.; Fort, D.; Green, R.; Guinness, E.; Hayes, J.; Hash, C.; Heffernan, K.; Hemmler, J.; Heyler, G.; Humm, D.; Hutcheson, J.; Izenberg, N.; Lee, R.; Lees, J.; Lohr, D.; Malaret, E.; Martin, T.; McGovern, J. A.; McGuire, P.; Morris, R.; Mustard, J.; Pelkey, S.; Rhodes, E.; Robinson, M.; Roush, T.; Schaefer, E.; Seagrave, G.; Seelos, F.; Silverglate, P.; Slavney, S.; Smith, M.; Shyong, W.-J.; Strohbehn, K.; Taylor, H.; Thompson, P.; Tossman, B.; Wirzburger, M. Wolff, M. Compact Reconnaissance Imaging Spectrometer for Mars (CRISM) on Mars Reconnaissance Orbiter (MRO), J. Geophys. Res., AGU, 2007, 112, E05S03-

Perovich, D. K. (1996). The Optical Properties of Sea Ice (No. MONO-96-1). Cold regions research and engineering lab Hanover NH.

Quirico, E.; Douté, S.; Schmitt, B.; de Bergh, C.; Cruikshank, D. P.; Owen, T. C.; Geballe, T. R. Roush, T. L., Composition, Physical State, and Distribution of Ices at the Surface of Triton, Icarus, 1999, 139, 159-178.

Reuter, D.; Stern, S.; Scherrer, J.; Jennings, D.; Baer, J.; Hanley, J.; Hardaway, L.; Lunsford, A.; McMuldroch, S.; Moore, J.; Olkin, C.; Parizek, R.; Reitsma, H.; Sabatke,

D.; Spencer, J.; Stone, J.; Throop, H.; Van Cleve, J.; Weigle, G. Young, L. Russell, C. (Ed.), Ralph: A Visible/Infrared Imager for the New Horizons Pluto/Kuiper Belt Mission, New Horizons, Springer New York, 2009, 129-154.
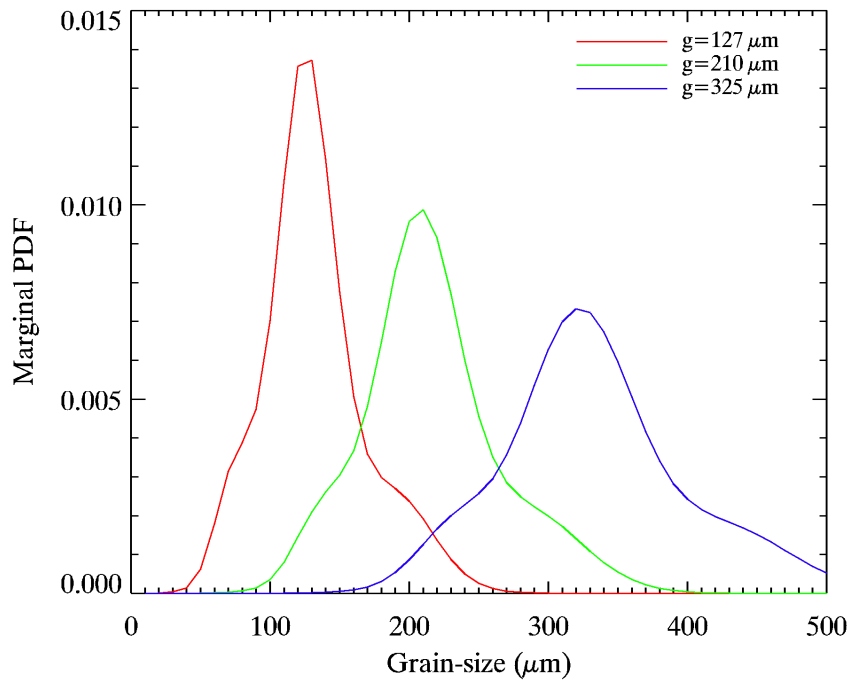
Van de Hulst, H. C., Multiple light scattering: tables, formulas, and applications, Elsevier, 2012

Zege, E., Katsev, I., Malinka, A., Prikhach, A., Polonsky, I. (2008). New algorithm to retrieve the effective snow grain size and pollution amount from satellite data. Annals of Glaciology, 49(1), 139-144.
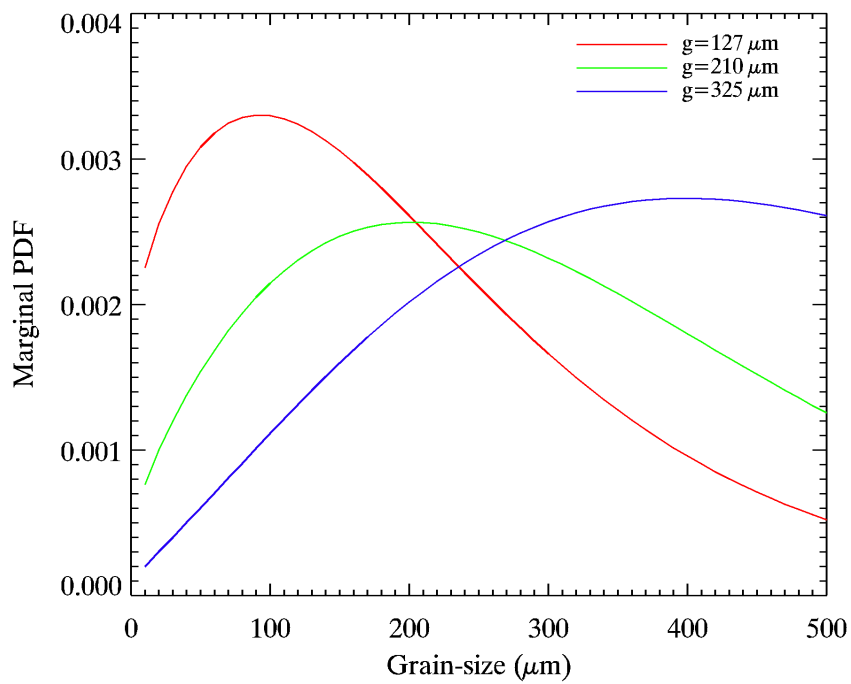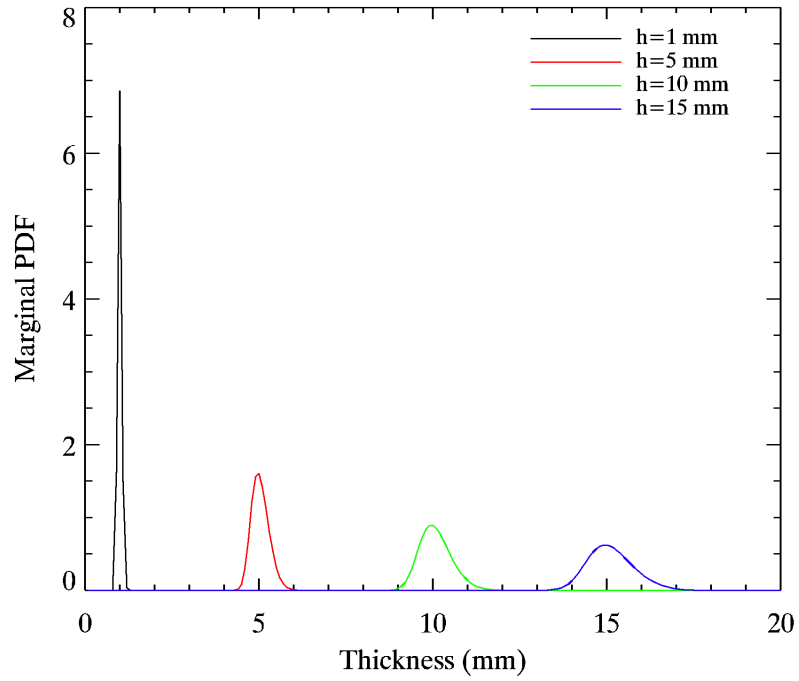
Fig. 1. Figure 4a: Normalized stacks of 1000 a posteriori PDF for the grain-size of the snow for a 1mm thick ice layer, when conducting the inversion on synthetic data, with added random noise.

C3159

Fig. 2. Figure 4b: Normalized stacks of 1000 a posteriori PDF for the grain-size of the snow for a 5mm thick ice layer, when conducting the inversion on synthetic data, with added random noise.

C3160

Fig. 3. Figure 5a: Normalized stacks of 1000 a posteriori PDF for the thickness of the slab ice layer, when conducting the inversion on synthetic data, with added random noise.

C3161
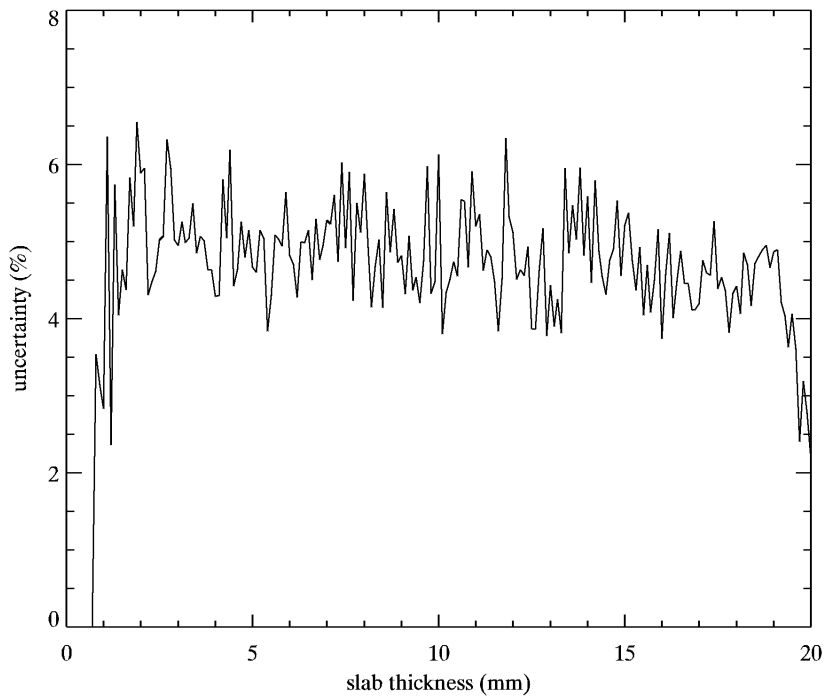
Fig. 4. Figure 5b: A posteriori uncertainty (at 2 \sigma) on the thickness estimation as a function of the slab thickness.
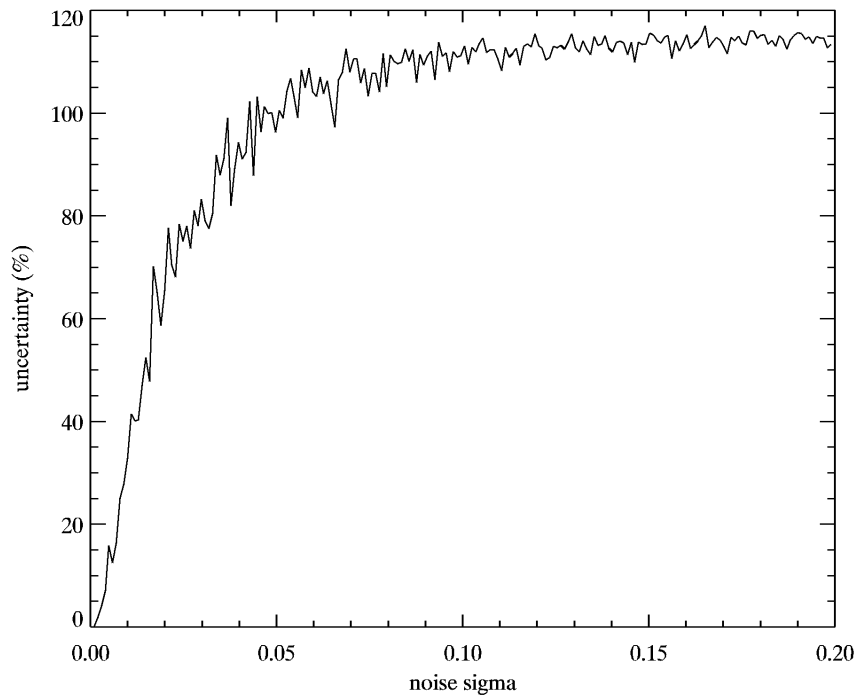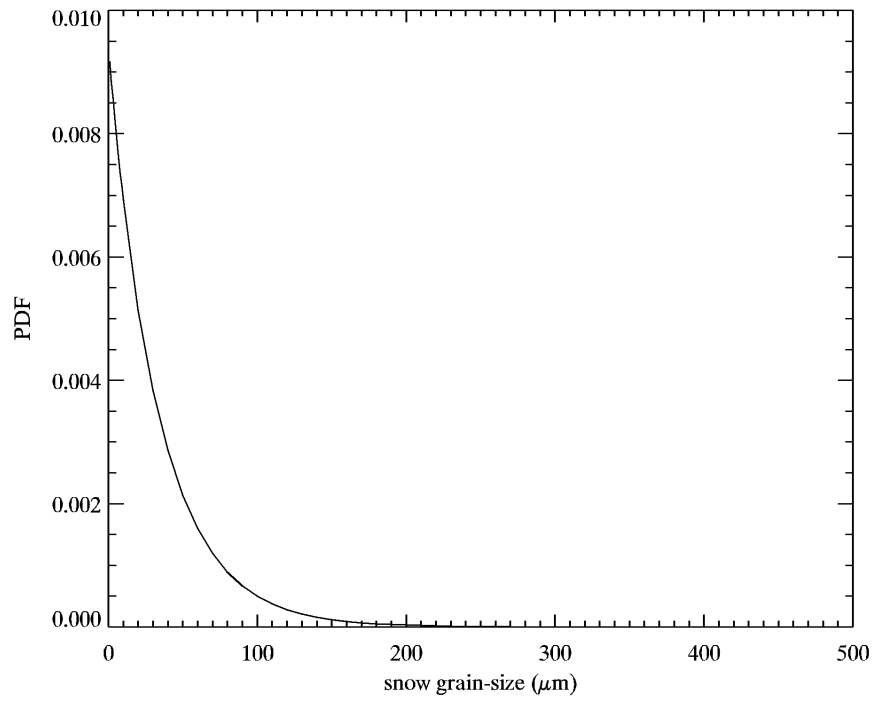
Fig. 5. Figure 6a: A posteriori uncertainties at 2 /sigma on the grain-size as a function of the noise standard deviation, for a 2 mm thick ice layer.
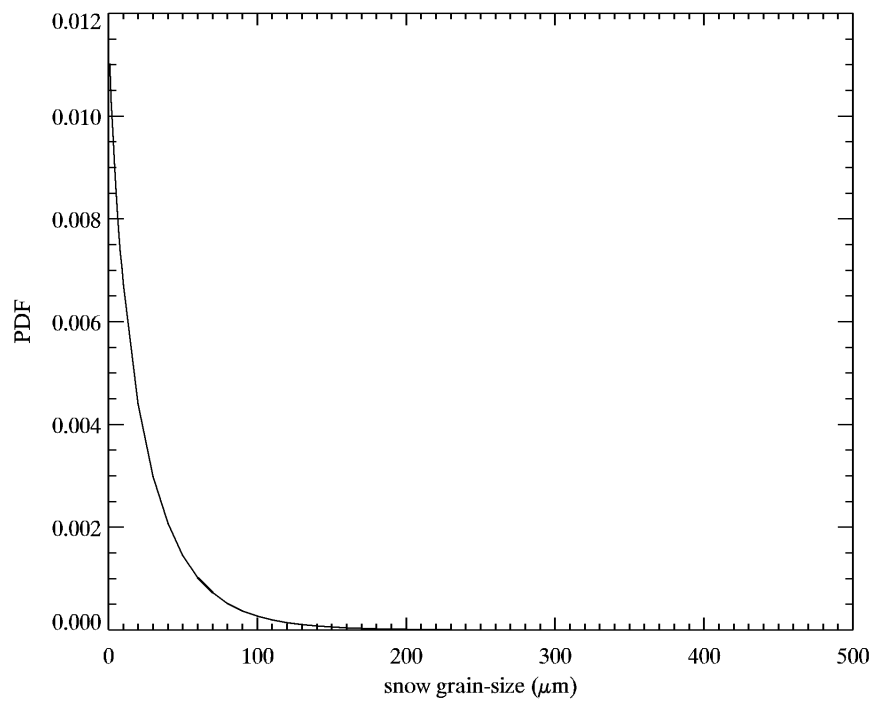
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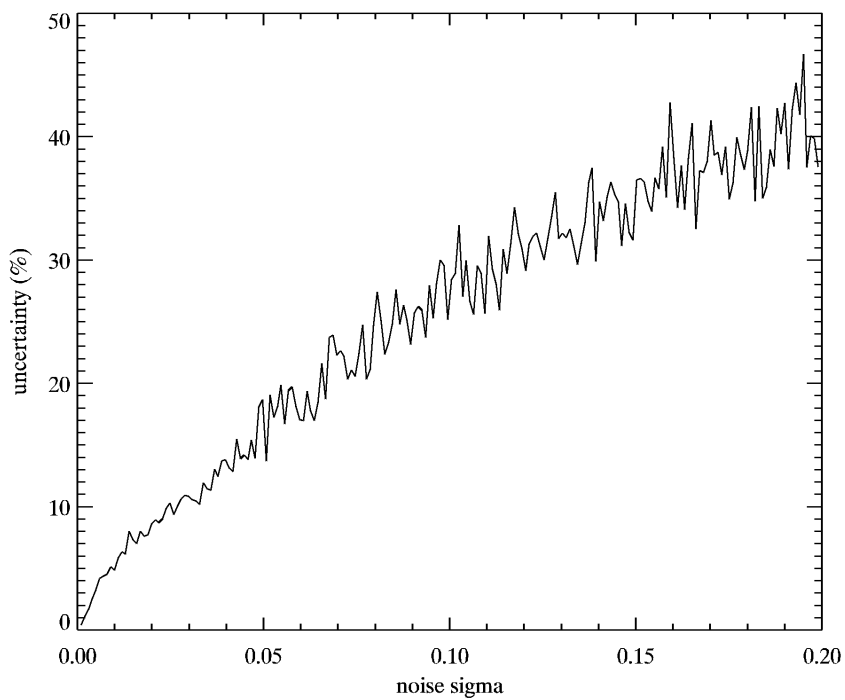
Fig. 6. Figure 6b: A posteriori uncertainties at 2 /sigma on the thickness as a function of the noise standard deviation.
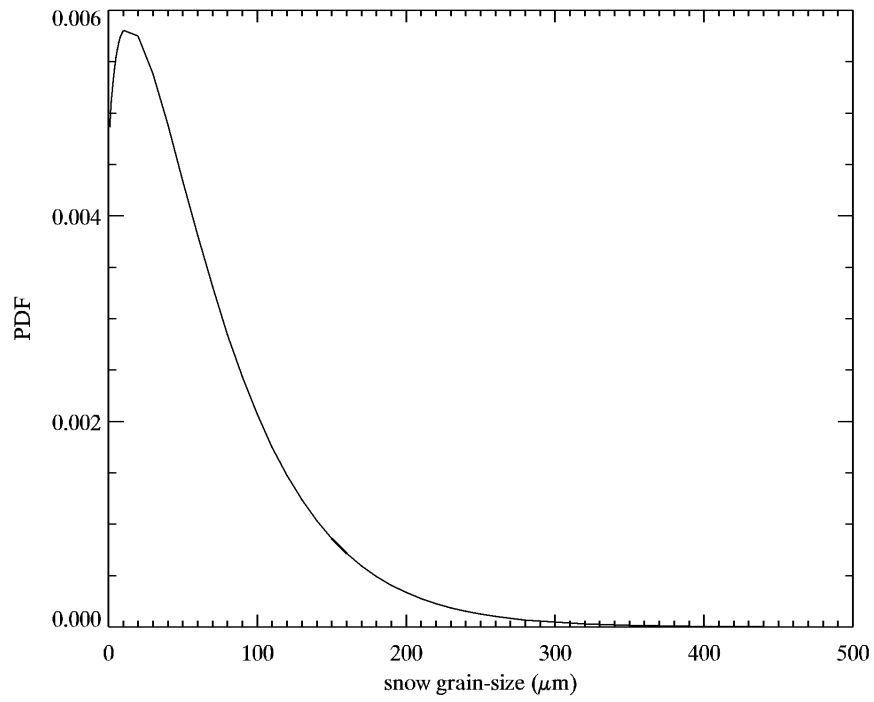
Fig. 7. Supplementary Figure 1: a posteriori pdf for the grain-size, when conducting the inversion on the nadir geometry. The ice layer is 10mm thick and the grain-size is 1μm
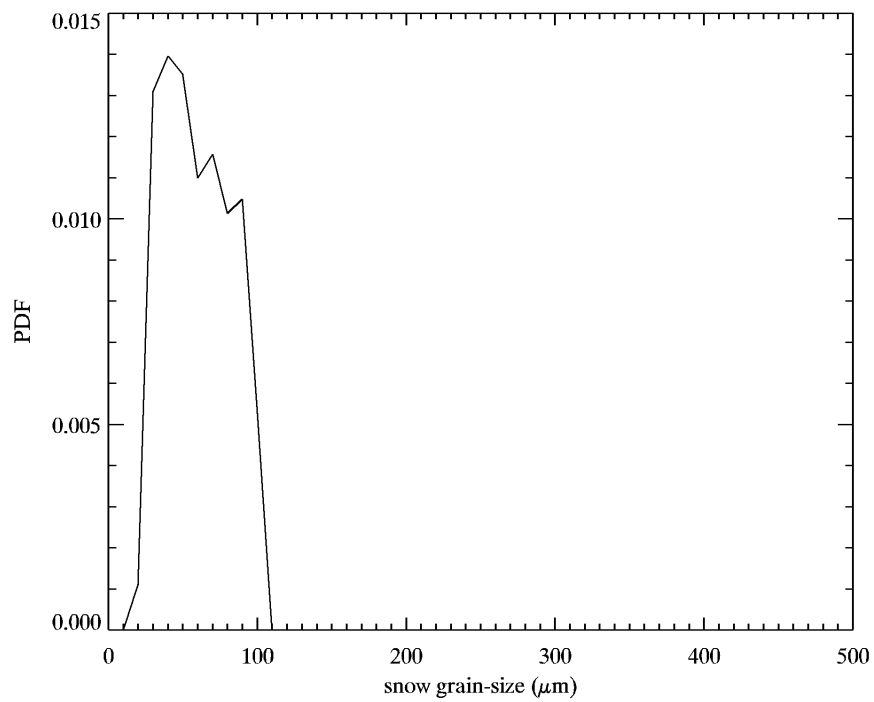
Fig. 8. Supplementary Figure 2: a posteriori pdf for the grain-size, when conducting the inversion on the nadir geometry. The ice layer is 10mm thick and the grain-size is 50μm

Fig. 9. Supplementary Figure 3: a posteriori pdf for the grain-size, when conducting the inversion on the nadir geometry. The ice layer is 10mm thick and the grain-size is 100μm

Fig. 10. Supplementary Figure 4: a posteriori pdf for the grain-size, when conducting the inversion on the BRDF. The ice layer is 10mm thick and the grain-size is 100μm