Response to Vieli

Thank you for your detailed, helpful comments.

>a) Advection of debris: I do not understand the formula/calculation of the near surface
>debris concentration Co, nor where it comes from: -Firstly, Co depends on the number
>of grid-cells in the vertical, which makes no sense, unless Co is the total mass of debris
>per vertical grid-cell unit but then the units do not fit.

Thank you for this observation. The wording certainly needs to be cleared up in this paragraph. Co is the concentration of debris in the surface-bounding englacial cell with units of (kg/m^3).

We have added a sentence: "Co is the mass concentration of debris in the surface-bounding cell" after the definition of Co.

>Secondly, surely the concentra-
>tion of debris in the accumulation area should at the surface depend both on debris
>deposition rate AND accumulation rate of ice (snow). For example if ice accumulation
>is increased for the same debris deposition rate dˆdot the debris concentration should
>be lower.

We agree with your statement. But because the englacial grid is coarse compared to the processes occurring at the surface of glacier (in the accumulation zone) we simply add debris into the upper most englacial cell as in the equation for Co. This model neglects the processes of firn compaction and snow metamorphism in the accumulation zone.

>Thirdly, Co does not seem to have the units of the concentration C (kg/mˆ3)
>osed lower down. So I really do not get what is done with debris concentration at the sur-
>face boundary in the accumulation area, in my opinion bz (accumulation rate) should also be
>relevant and be included! Should be clarified! Anyway, however it is done, as
>bz is constant with time (steady state case) I guess all the conclusions are qualitatively
>not really affected.

Co does have the same units as C (kg/mˆ3). Ddot [=] mm/yr, rho_rock [=] kg/m^3, dt [=] yr, H[=] m. There is a dx in the numerator and denominator that cancel out.

>Further, am I right that the concentration here is a mass concen-
>tration (kg/mˆ3) rather than a volume concentration (%), maybe should be made more
>explicit?

Concentration is the mass concentration and has been added to the text at P6434 line 23 and line 24. We have added a sentence: "Co is the mass concentration of debris in the surface-bounding cell."

>b) Advection equation for debris: I assume this equation (14) for concentration is ok,
>but I am a bit confused about it, as I thought one should be able to describe it by a
>simple advection equation. It probably is that but it is written in the zeta-coordinate
>system with vertical gridsize h_zeta changing along flow, so I am just not familiar with
>it.

We added a simple advection equation above the equation we use in our manuscript to make it clear that the equation with the zeta coordinate system is used in the model. We also expanded the explanation of this equation.

>Further, I thought that an ice parcel with a certain concentration will keep this con-
>centration all the way while it is advected, but of course it will be stretched or sheared
>or vertically extended on the way but within the parcel the concentration should stay
>constant (or am I wrong here?). This means if ice with debris of a constant concentra-
>tion is deposited over a certain area on the surface in the accumulation area, this will
>be advected through the glacier as a band of constant debris concentration, although
>this band can be thinned or extended vertically. Ice is incompressible and the debris
>particles are fixed within their ice packet thus within the band I expect constant con-
>centration (or am I wrong here?).

You are indeed correct here and we have edited the text to remove the word 'parcel' and instead discuss an englacial cell. Changed P436 line 1.

>I know that numerical diffusion can be an issue in
>advection schemes but this would be at the edge of the margin of the debris band and
>the authors seem to have accounted for that. From looking at fig. 5a) I guess eqn. 14
>seem to do what I expect it to do, but from the formulation and the text explanation I
>am not able to fully follow it, so maybe could be clarified a bit.

There is still some numerical diffusion in the model. It is just greatly reduced compared to what would occur without the diffusion correction scheme. We are not sure how to quantify the effect of numerical diffusion so we follow Smolarkiewicz's lead and note that "numerical diffusion is greatly reduced."

We added a note that numerical diffusion is greatly reduced by using the diffusion correction scheme in the implementation and numerics section:

"This iterative scheme imposes a two-step anti-diffusion correction algorithm to the advection scheme which greatly reduces numerical diffusion (Smolarkiewicz, 1983)."

>Further, and somewhat related, from methods I understood that debris deposition over
>the area d_width is constant, so near the surface debris concentration (along the sur-
>face) should be almost the same (constant), but this is not the case in Fig. 5a, it looks

>as if it has been smoothed out (or diffused). Did I miss something here?

Part of the smoothing visible in Fig. 5A is due to the contourf plotting tool in matlab. The other portion is related to a bit of numerical diffusion. Because the width of the debris deposition zone effects the response of the glacier to a very small degree we are confident that a minor amount of numerical diffusion does not effect the validity of our results.

We have adjusted the text when discussing the numerical diffusion scheme to note that it only reduces the diffusion, but does not eliminate it.

>c) Debris flux at snout: I understand the reason of the extra flux divergence term for
>debris transport at the snout (dflux_snout/dx) but I do not understand how it is technically
>implemented (also not from Appendix B). In particular I do not understand, to what
>location/area the 'snout' exactly refers to. Is it the last two gridpoints of the glacier (last
>ice covered and first ice-free?)? or is it a fixed length-area measured from terminus?
>For the former it would then be gridsize dependent (the authors may address this or a
>similar issue in the appendix A). So this should really be explained in some more detail,
>maybe in a sketch. In particular: at which locations (grid points) is eqn 16 being used
>for example and what and where exactly is the 'snout'. Clarifying this is important as
>the analysis in Appendix B (and fig B1) shows it is important for the length evolution.

Thank you for catching this. We agree that the explanation needs to be improved. We have removed the term 'snout' and replaced it with terminus wedge. We have included a new figure (A1) to explain how the terminus wedge is implemented and where the terminal debris-flux is removed from the glacier. We also note that we are implementing a terminal wedge parameterization on p6436 line 19 and reference Appendix A. Several new sentences were also added to Appendix A. We appreciate this catch!

>d) implementation and numerics there could be a bit more information on how the
>debris thickness and advection scheme is numerically solved. More specifically: -I
>assume the debris thickness equation (15 and 16) is solved in the same way as the
>ice thickness equation (1) with a second-order Runge-Kutta difference scheme -what
>is used for the debris advection scheme (eqn 14), a 'correction-method' is given here
>(Smolarkiwicz) or is this already the whole advection scheme

We use the iterative "upstream" advection scheme of Smolarkiewicz, 1983 which limits strong numerical diffusion.

We updated the manuscript text to reflect the above sentence:

"Next, we use a second-order Runge--Kutta centered difference scheme to evolve H(x,t), followed by the implementation of an iterative "upstream" debris advection scheme following Smolarkiewicz, 1983. This iterative scheme imposes a two-step anti-diffusion correction algorithm to the advection scheme which greatly reduces numerical diffusion (Smolarkiewicz, 1983)."

-what boundary condition
>has been chosen for the ice flow at the upper end of the glacier (x=0)

We added in the 'implementation and numerics' section that there is a no flux boundary condition at (x=0):
"We impose a no flux boundary at the upper end of the glacier."

>e) Figures 8 and 9(A+B): I do not find the labelling of the d-loc variation very effective,
>it is hard to see how dloc is varying, in which direction and by how much. Maybe using
>colored dots/lines wit

Great suggestion. Both figures 8 and 9A have been modified as you recommend using marker width.

>Line 3: strictly speaking it is the mass balance gradient in the ablation area, or maybe
>it is rather the '. . .ablation rates can be reduced. . .'

We have changed 'the mass balance gradient' to 'ablation rates' as you suggest.

>For introduction and discussion in general, the very recent Rowan et al (2015, Earth
>and Planetary Science Letters, 430, 427-438) maybe relevant

Thank you for the note. We have included this paper in both the introduction and discussion. We unfortunately did not see this paper before we submitted the paper but we have cited it several times in the text now!

Added citations p. 6426 line 18. We also cite the paper in reference to future research noting that the Rowan et al., 2015 paper is running a model that takes into account the planview dimension of glaciers.

>Line 94 and 95: regarding the use of SIA for modelling glacier geometry evolution the
>intercomparison study of Leysinger and Gudmundsson JGR (2004, Vol 109, F01007)
>would be relevant here as it demonstrated the validity of such a simplification on mod-
>elling glacier evolution (comparing SIA with a full system flow model).

Good suggestion. This citation has been added.

>Line 133: I guess the authors refer to exponential curve fittings here as other studies
>have used such fitting, so it would be useful to add these references. Otherwise it is
>not clear why exponential is relevant here. (similar on line 427).

We added several references for folks using an exponential curve fit: “(e.g., Konrad and Humphrey, 2000; Hagg et al., 2008)”

>Line 158: a very minor point: but these ‘other sliding relations’ have a theoretical phys-
>ical basis behind, maybe some reference to such other models could be given.

This is true. Our results here are not sensitive to the selection of sliding parameterization. We added a reference to Cuffey and Patterson, 2010 here as you suggest.

>Line 162, eqn 8: it is not clear to what ‘u’ is referring to here. Is it the vertically averaged
>velocity, the surface velocity or the basal velocity. Should be clarified.

'u' here is the vertically averaged ice velocity.

This has been added to eqn. 8 and the sentence following it.

>Line 171, eqn 10: it is not clear how u_coupling is determined/calculated, eqn 8 only
>refers to how tau_bx is modified. Is u-coupling actually used (and relevant) for calcu-
>lating the vertical velocity profile? Or is u-coupling determined from substracting udef
>from u-total?

u_coupling is determined from subtracting u_def from (utotal-u_sliding).

We added an explanation on p. 6432 line 5.

>Line 178: Is this equation referring to the deformation velocity (udef)? (see explanation
>in next point). Also not clear how u_coupling is integrated into this.

See comment above.

>Line 180-181, eqn 12: I might be wrong here, but I think w=0 is not the correct boundary
>condition is there is basal sliding on a slope, then there is vertical component from the
>along bed sliding velocity. I guess this bed parallel vertical component from sliding has
>been subtracted already here. Should be clarified.

w=0 at the bed assumes no melting. the ice has to remain in contact wight the bed.
the sliding is taken into account by the fact that it is not u but du/dx that counts in the integration, and hence any gradient in sliding generates a gradient in vertical velocity. but the bottom b.c. remains w=0.

>Line 192-197: maybe some typical values for headwall erosion could be given here.

We have included examples of headwall erosion rates as requested. “(typically ranging between 0.5 and 2 mm/yr)”

>Line 206-207: a detail on terminology, I do not think all the these debris deposition
>variables all need a dot on top, for the debris deposition RATE dˆdot I agree, but for
>d_width or d_loc it is not referring to RATES, and if the authors insist on the dots, the
>d_flux should for consistency have one as well (here it is actually a RATE).

Thank you pointing this out. The dot has been removed from debris variables that are not a rate and added to d_flux term throughout the manuscript and figures.

Line 210: where do these values of deposition rates come from???

We consider these to be viable deposition rates based on viable parameter inputs to equation (13) as noted on lines 2015-2018. We explored deposition rates up to 32 mm/yr but the results did not add to the patterns shown in the results so we limited deposition rates to 8 mm/yr.

“The deposition
rates explored in this study are viable based on headwall erosion rates (typically ranging
between 0.5 and 2 \unit{mm\,yr^{-1}}), headwall heights, and headwall slopes
for high-relief mountain environments (e.g., Heimsath and McGlynn, 2008;
Ouimet et~al., 2009; Scherler et~al., 2011; Ward and Anderson, 2011).”

Line 228 (see main comments above (a)): something odd about this definition of Co

See our comments and corrections above.

Line 250: should maybe refer to appendix B here.

The reference is added.

>Line 250-258 (see main comment (c) above): not clear to me to which
>area/location/gridpoints the ‘snout’ (and its equation 16) applies. Sketch?

See above also. A new figure as been added to the appendix and appendix A has been modified.

>Section Implementation and numerics (see main comment (d) above: some more de-
>tails on numeric needed.

See above response.

>Line 277: This is just my personal opinion, but not crucial: I find it not that useful in
>giving the location dloc as percentage of the non-debris covered glacier length as in
>nature such a length is usually not available, so maybe it would be better to relate dloc
>to the ELA position. Anyway, it does not change anything.

This is an good point. But it is not clear how we would define d_loc below the ELA without using the debris-free glacier length. We left these percentages as is.

>Line 289-290: again not that crucial: M_input is the ‘cumulative’ mass that has been
>deposited/added, so I would rather say something like ’. . .where Minput is the total rock
>mass deposited on the glacier and accumulated over time,. . .’

'and accumulated over time’ was added at line 289. Thank you for the note.

>Line 293: I guess the base run is not the most representative example for testing
>(showing) debris mass conservation as the englacial part is very small, the case of
>dloc=7% (fig 5a) maybe would have been better. But it seems the authors tested this
>for all cases anyway and the errors are still below 1%.

We agree that it would be better to show the case of dloc = 7% for the debris mass conservation plot in figure 4 but we decided to show the case from the base parameter set instead for consistency. The model does conserve debris as you note.

>Line 304 (and some figures): a small detail: not so clear to me why they use the
>letter epsilon for this debris emergence position, epsilon has already been used for
>backweathering rate. It is a position so ‘x’ with some subscript maybe more useful.

Also a good suggestion. We have changed the symbol for the epsilon for this debris emergence position to x_emergence_int.

>Line 335-345, section 4.2.2: From line 336 I take that the authors would like to investi-
>gate the relative importance of d and dwidth, which they do by an extensive sensitivity
>study in which they vary them independently. The issue is that dflux is also changing
>for variable d and dwidth. If the relative importance should really be addressed in detail
>I would keep dflux constant while varying d and dwidth (and plot it this way).

Good idea. Figure 9 has been revised and now only has 3 panels. 9B now clearly shows the effect of changing deposit width and deposition rate using color and marker size.

>Line 370-376: It maybe useful to already here mention that in the model the width does
>not vary along flow where as in reality the width in the accumulation area is often much
>wider which of course affects AAR.

We added that our ssdf glacier has an AAR of 0.5 due to no width variation along the flow. As suggested.

>Line 383: related to above: I would add here. '. . . has an AAR of 0.5, due to no width
>variation along flow.'

The sentence has been modified as you suggest.

>Line 408: here high dependence of time evolution on dfluxˆsnout is mentioned but this
>model investigation has never been presented or mentioned before in the results/text,
>it is however in the appendix B. So it should be mentioned in the results that it has
>been undertaken (but refer to appendix and fig. B1) and then here a reference to the
>appendix B and its figure B1 should be added.

Agreed. A sentence referring to these results was added in the results section p. 6437 l. 6-7 The reference to the appendix/B1 is added in line 408.

>Line 422: I guess here it should be clarified that for the '2dim-case' dloc is of secondary
>importance (I expect for 3d it may different).

A reference to the 2D case has been added in line 422.

>Line 427: again, it would be useful to add a reference of studies who have used ex-
>ponential curve fittings, otherwise why is exponential relevant here. (similar on line
>133)

A couple of references were added.

>Line 463: after '. . .removal from the toe' refer to (see Fig B1 Appendix B)

Line 463. The reference has been added.

>Line 463: remove 'a' before 'high melt rates'

The 'a' was removed from line 463.

>Appendix A (in particular lines 525-528: I struggle to understand this 'gridsize depen-
>dence', this should be explained better. What is meant by 'increasing dx from 100m to
>200m?' change if grid size or an advance. . ..???

I wrote a few sentences to help clear up what we mean here:

"When the model is allowed to evolve from the ssdf glacier to the debris-covered steady state, debris is advected into previously debris debris-free cells on the glacier surface. In our model, the debris thickness h_debris(x,t) represents a layer of equal thickness on any cell. Debris thickens slower on the finite-difference grid with a larger dx because the debris advected into a cell is spread over a longer distance (due to the larger dx). There is therefore a timescale built into the thickening of debris in a cell that is dependent on dx. Because ablation rates are sensitive to debris-cover thickness, changing dx in the model has an effect on the evolution of the glacier.

In order to test the effect of changing the grid spacing in the x-direction on the steady state debris-covered glacier length we increased dx from 100 (used in all simulations outside of this test) to 200m. This test led to differences in steady state debris-covered glacier length which were less than 200m even when d_flux was varied."

>Appendix B: again (see main comment (c) above) the 'area/location of the 'snout' is not
>clear at all, maybe explain here first and add a sketch.

We made Figure 1A and expanded Appendix A to address this comment.

>Table 1: here slopes are given in % but in fig. 11 where different slopes are
>considered in the figure ratios are used. make consistent.

We changed the slopes to percentages in figure 11 to match those in the table.

>Fig. 5: would be useful to add a fine line at the elevation of the ELA. Further, explain in
>caption what dark grey dashed vertical line is (I assume the non-debris glacier lengths
>position.

These are good catches.

The ELA line has been made thicker. The vertical dashed line is now better labeled in the figure as well with an arrow.

>Fig. 6: the scale on the right of A-C is very small and as yellow very hard to read. I
>would increase the size of this figure.

The yellow has been changed to brown and the figure size has been increased.

>Caption: 'Modelled glacier changes. . .' is very
>vague. Why not say 'Modelled changes in ice fluxes, thicknesses and velocities due
>to. . .'. Further: figures D-E are not really explained, so add after '. . .shown in
> Fig. 6.'
>(D-E) Comparison of surface velocities and ice thicknesses for the debris covered and
>debris-free cases.

The suggested changes were made in the figure 6. caption.

>Fig. 8: I do not find the labelling of the d-loc very effective. For (a) it seems ok but for
>(b) the labels are far from the arrows. Maybe using colored dots/lines with a color scale
>for dloc would be better, it is already a color figure anyway.

Figure 8 has been updated using the marker size to represent d-loc more effectively.

>Fig. 9 A+B: again the same issue as in Fig. 8, it is even harder to see to what dloc the
>different lines refer to. Maybe using colors would address the issue.

Fig. 9 A has been updated using marker size to represent d_loc. Fig. 9B in the original manuscript has been removed.

>Fig. 10: why having shifted y-axis on the left. Could one not use one axis on left and
>one on right?

One axis was shifted to the right as you suggest.

>Fig B1: not so clear what the blue arrow refers to. Does it mean from the onset of the
>arrow down no steady state is reached (continues to advance?).

This has been removed from the figure and a sentence has been placed in the caption instead.

Rowan Response

Thank you for your comments. We addressed your critiques by (1) clarifying the purpose of our study; (2) simplifying overly-complicated writing and figures; and (3) emphasizing that this study models hypothetical glaciers and simply reproduces the ‘general patterns’ from real debris-covered glaciers.

>The model is suggested to be representative of glaciers in the Himalaya, relying on data presented by Scherler et al. 2011, but only weakly represents these glaciers using a few measured parameters from this region.

It was not our intention to suggest that our simulations are representative of glaciers in the Himalaya. While we do use parameters derived from glaciers in the Himalaya (because these glaciers are extensively studied) we also use a linear bed, fixed glacier width, and steady state which implies that our simulations are hypothetical in nature. No real glacier has a linear bed or fixed width.

Furthermore, this manuscript does not rely on the ‘general trends’ documented by Scherler et al., 2011. Rather our simulations and conclusions are completely independent of Scherler’s inferences. The similarities of the ‘general trends’ brought up by Scherler et al., 2011 and our simulations are strong evidence of debris’ influence on glacier response. We go through an extensive sensitivity test to show how different parameters in the debris-glacier system effect glacier response. Scherler’s ‘general trends’ are supported by our simulations, independent of our parameter choices. So the fact that some of our base parameters loosely represent glaciers in the Khumbu region does not invalidate our study.

To alleviate this confusion, we have explicitly stated in numerous locations that our simulations are hypothetical in nature and are not meant to represent Himalayan glaciers. We also highlight that our simulations reproduce the ‘general trends’ documented by Scherler et al., 2011 and are not meant to represent the many Himalayan debris-covered glaciers. Please see the comments below.

>The model design is not sufficient to represent the behaviour of specific glaciers in a region with highly negative mass balance and instead would be more convincing as a theoretical case.

The model design is sufficient to address the goal of this manuscript: to isolate the effect of debris on glacier response. We think that this manuscript does represent theoretical glaciers. It was not our intent to model the behavior of specific glaciers in High Asia as you suggest.

Please see the comments below to see how we have worked to make this more clear.

>In particular, the model only operates with steady state simulations, whereas the mass balance of present-day glaciers in the Himalaya is clearly far from equilibrium.

The model is fully-transient but we largely present steady-state results from hypothetical, theoretical glaciers. We provide the first means to evaluate the effect of parameter choice on debris covered glacier response through a detailed definition of debris-covered glacier steady state. Scherler et al., 2011 highlight 'general trends' in their dataset and show that the trends hold independent of glaciers with stagnant ice (see below). We merely show that our simulations match these 'general trends' independent of parameter choice (like bed slope, erosion rate, debris deposition location, etc).

>Relevance of the study to present-day glaciers. The introduction section almost exclusively considers glaciers with negative mass balances where mass loss has been ongoing for several centuries.

Most of the literature has focused on debris-covered glacier response to climate change; thus it is reasonable that most of the introduction would focus on this research. However, even though nearly all studied debris-covered glaciers are experiencing periods of negative mass balance, the presence of debris cover has also perturbed each of the glaciers mentioned in the introduction. Our model highlights the urgent need to isolate debris as an essential driver of glacier response. To clarify this point, we have overhauled the introduction to precisely introduce our study and the problem we address.

>However, the work presented here does not
address glaciers in this condition, but rather those that are slowly advancing without a climatic driver.

The paper is theoretical in nature and seeks to highlight the effect of debris on glacier response. The parameters are loosely based on glaciers from the Khumbu region. We also vary the parameters to explore their effect. The results are still representative of Scherler's 'general trends.' We use a hypothetical, linear bed and a uniform width, and a steady piecewise linear mass balance profile. Our intent is not to limit studies that have addressed the effect of climate change on glacier response but rather to understand the effect of debris on glacier length so that we can then more clearly diagnose the effect of climate change of debris-covered glaciers. On page 6427 line 17-19 we note:

"This study lays the foundation for future modeling efforts exploring the response of debris-covered glaciers to climate change."

We have changed this line to better represent our intentions.

"By only assessing the effect of debris on glaciers, this study lays
the theoretical foundation for efforts exploring the response of

debris-covered glaciers to climate change.”

>The comment on P6425 at line 15 is misleading; whilst a minority of Himalayan debris-covered glaciers are advancing (which may be due to distinctive surge-type behavior, although it is not clear here which glaciers the authors refer to), the majority lose mass by surface lowering rather than terminus recession (e.g. Bolch et al. 2011, TC), so comparison of their terminus positions over time is a poor metric by which to explore glacier change.

The citations at the end of the sentence to which the reviewer refers highlight the variable response of debris-covered glacier termini. We agree that mentioning that these debris-covered glaciers are losing mass by surface lowering is important. Thank you.

This paragraph now reads:

“Debris-covered glacier termini exhibit a wide range of responses to climate change (Scherler et al., 2011a). While almost all Himalayan debris-free glaciers are retreating, Himalayan debris-covered glacier termini are not responding coherently to climate change despite a strong trend toward negative mass balance (e.g., Bolch et al., 2011; Benn et al., 2012). Some Himalayan debris-covered glacier termini are advancing, others are stationary, and yet others are retreating (e.g., Raper and Braithwaite, 2006; Scherler et al., 2011a; Benn et al., 2012; Banerjee and Shankar, 2013). This discrepancy between debris-covered glacier mass balance and terminal response highlights the pressing need to understand the sometimes counterintuitive effects of debris on glacier response.”

Instead of dismissing debris-covered terminus positions over time as a poor metric for glacier change, we would argue that they are a metric of debris-covered glacier response that is poorly understood. Why would a debris-covered glacier terminus keep advancing even when it is responding to a period of negative mass balance and experiencing surface lowering? The modeling framework we present allows us to address this question (though not in the present study).

This paper goes through an extensive sensitivity analysis, with detailed methods and justification (see the appendices) to show how we can use debris-covered glacier length for comparison between glaciers. The terminus parameterization is novel and also highlights how the processes of debris removal at the toe can have important implications for the time evolution of debris-covered glaciers.

>Implications/impact of the modelling. The authors could revise the manuscript to instead consider hypothetical glacier change rather than by attempting to match observational data, still by using mass balance/flow parameters that are representative of real glaciers.

This modeling effort does in fact address hypothetical glaciers, though we did not explicitly state that they are hypothetical in the original manuscript. Of course no real glaciers have a linear bed or uniform width. We simply compare our theoretical results to the dataset of Scherler et al., 2011 in order to reinforce the 'general trends' they highlight.' See our reference to text from the Scherler et al., 2011 paper below. In order to avoid this sort of misunderstanding we now more clearly state that we model hypothetical glaciers where we match the general observations of Scherler et al, 2011b.

We find it compelling that our theoretical, hypothetical model nonetheless, reproduces the general trend between AARs and debris cover and the general trend between relative glacier surface velocities and debris cover in Scherler's dataset. It is noteworthy that the simulations were run with no attempt to 'tune to' or 'match' Scherler's observations. Rather the comparison arose after the simulations were completed.

>The interest in this study for me is in exploring how debris-covered glaciers can advance in the absence of climatic change, transform into rock glaciers, and how these processes are observed in the geological record. Under what conditions will an advancing glacier retain sufficient supraglacial debris to significantly affect its mass balance? The authors state that these results have important implications for palaeo-climate reconstructions from glacial geology, which would be a valuable outcome from this study.

The aim of this study is to determine the effect of debris input on glacier response. This paper therefore has implications for both glaciers in the geologic record as well as implications for modern debris-covered glaciers (simply because there is debris present in and on them). The vast majority of extant glaciers are undoubtedly undergoing periods of negative mass balance. That said, it is valuable to highlight the effect of debris cover on glaciers so we can better understand the initial conditions for our numerical debris-covered glacier models and better model the response of debris-covered glaciers to climate change.

>Assumption of steady state. The main model output is change in glacier length, which is not a suitable variable for observation of debris-covered glacier mass loss when considering present-day glaciers with a generally negative mass balance, such as those in the Himalaya.

No glaciers have a linear bed or constant width so it is implied that these are hypothetical glaciers. We are not aware of a metric (besides length) for comparing the cumulative effects of debris on glaciers. We go through great effort to define and justify our steady state definition which in turn justifies our comparison of glacier lengths. We do also present AARs and the half-width velocities as model output.

We thought that the hypothetical nature of our simulations was implied because we were using a linear bed, constant width glacier with a steady climate forcing. We have emphasized why we make these choices to avoid confusion. We now highlight the hypothetical nature of our model in the abstract, introduction, and numerics/implementation section.

>Moreover, the authors should emphasize the usefulness of their steady-
state simulations to this study; for example, P6426 line 24, clarify if/why one would expect debris-covered glaciers to ever reach equilibrium.

The steady-state assumption is widely used in debris-covered and debris-free glacier modeling. It is a useful concept to establish a baseline or initial condition from which to explore a system that then responds to a climate change scenario. Konrad and Humphrey, 2000 use a steady-state debris-covered glacier model. Banerjee and Shankar, 2013 model steady-state debris-covered glaciers and their response to climate change. Rowan et al., 2015 also use a steady-state glacier to simulate a late-Holocene extent of the Khumbu glacier.

Konrad and Humphrey, 2000 highlighted the importance of terminal debris transport and terminal ablation as a key process that could limit rock glacier/debris-covered glacier extent. Our manuscript expands on this notion and has a clear theoretical foundation from several debris-covered glacier modeling studies. Our definition of steady state goes beyond previous debris-covered glacier modeling studies by explicitly defining that steady state requires steady debris, steady geometry, and steady mass balance. It is important that this model conserves debris; as far as we know, other models have not dealt with this vital issue.

Our intent is/was not to reproduce ‘real’ debris-covered glaciers rather the steady state we define is a metric by which we can compare model simulations.

> The dataset presented by Scherler et al. 2011 captures glaciers where surface lowering is sustained
and is therefore difficult to relate directly to the model results.

It is important to note that we do state this in the manuscript before revisions P6441lines 23-28:

“The Scherler dataset was collected from glaciers responding to periods of negative mass balance. Reduced surface velocities under debris cover (not necessarily stagnant) – resulting
from debris-covered glacier response to climate change – could account for the data with low debris cover percentages and low ratios of half length mean ice surface velocities (Fig. 11b).”

We are not trying to model or represent all debris-covered glaciers in High Asia or the Himalaya or 'relate directly' our results to Scherler's data. Rather, our intent is to reproduce a 'general trend' based on a suggestion that debris input perturbs AARs and shifts peak velocities up glacier. We only compare our hypothetical results to glaciers from the Scherler et al., 2011 dataset. Scherler et al., 2011 makes a compelling case for a 'general trend' in debris-covered glacier AARs and surface velocity patterns. Our analysis also shows extensively how changing h_star, bed slope, debris deposition location, and debris flux would effect the model results in the context of the Scherler et al., 2011 dataset. Despite our inability to model the specifics of all debris-covered glaciers in High Asia, we believe that our analysis quantifies the 'general trends' laid out by Scherler et al., 2011.

We are now quoting from Scherler et al., 2011 paragraph [47]:

"[47] When hillslope-derived debris is deposited in the accumulation zone of a glacier, it first becomes englacial during its transport downstream and, at higher concentrations, may reduce the amount of ice deformation [Russell, 1895; Paterson, 1994] and influence basal sliding [Iverson et al., 2003]. On the glacier surface, however, its main effect is modulating melt rates and thus mass balances. Because debris thicknesses on Himalayan glaciers are usually greater then a few centimeters [e.g., Shroder et al., 2000; Owen et al., 2003; Heimsath and McGlynn, 2008], the insulating effect dominates so that melt rates are lower compared to clean ice [Mattson et al., 1993; Kayastha et al., 2000; Mihalcea et al., 2006]. Lower melt rates allow debris-covered glaciers to grow longer for a given accumulation area, hence decreasing the accumulation area ratio (AAR; Figure 3). Because only the ablation zone grows larger, the position of the maximum velocity along a glacier's length, usually located near the ELA or the climatic snow line (Figure 6), should shift upstream as debris cover increases. This inference is supported by our velocity data (Figure 12) and results from a simple numerical model of a debris-covered glacier [Konrad and Humphrey, 2000]."

Scherler et al.'s 2011 data on AARs is presented in figure 11 A and B. Our model hypothetical/ theoretical model simply reproduces the 'general trend' laid out by Scherler et al., 2010.

and paragraph [49] from Scherler et al., 2011:

"We note that many Himalayan glaciers have been retreating and/or thinning during the past few decades [e.g., Berthier et al., 2007; Bolch et al., 2008a; Raina, 2009], and may have been doing so since ~1850 AD [Mayewski and Jeschke, 1979]. This has potential effects on the observed velocity distribution. In particular, heavily debris covered glaciers that are thinning [Bolch et al., 2008a], but not necessarily retreating [Scherler et al., 2011], could result in a gradual shift of maximum velocities upstream and exaggerate the trend we observe. However, the upstream shift of peak velocities with increasing debris cover (Figure 12a) is also

observed when excluding stagnating glaciers, suggesting
that this is a general trend."

This "general trend" is presented in the Scherler et al., 2011 data in figure 11 B and D. Our model hypothetical/ theoretical model simply reproduces this trend. The fact that our simulations in steady state can reproduce the 'general trend's documented by Scherler et al., 2011 makes the effect of debris on glacier response even more compelling.
I emphasize again that our results and conclusions are independent of the comparison to Scherler's observations. We compare the results to lend support to our conclusions and make our study more compelling.

>The impact of climatic change on debris-covered glaciers could be discussed by reference to transient simulations by Rowan et al. 2015, EPSL.

We have added reference to the Rowan et al., 2015 paper in both the introduction and discussion with reference to both the paper's transient and steady-state simulations. We only found the Rowan et al., 2015 paper after the paper was submitted. We apologize for any frustration on your part. We did not cite it because we were not aware of it before we submitted the paper.

>Relevance to Himalayan glaciers. The simulations presented here cannot be considered to represent 'real' glaciers as the model design is too simplistic to capture the key factors controlling the behavior of these glaciers, such as high relief, variable bed topography, highly variable flow velocities, and highly negative mass balances.

We do use a simplified model design because we want to understand a specific portion of debris-covered glacier complexity: the effect of debris delivery on glacier length/ dynamics, which we consider a pre-requisite to understanding glacier response to climate change. The model set up allows for a wide range of complexity without considering 'high relief, variable bed topography, highly variable flow velocities, and highly negative mass balances.' By including all of these complexities we argue that it would be more difficult to isolate the effect of debris on glacier response. As a result we chose the 'simplified, hypothetical' model framework.

The linear bed is necessary for this study because it allows us to isolate the effect of debris on glaciers. Without the linear bed our results would conflate the effects of a non-linear bed with the effects of debris on glacier response.

The introduction has been greatly modified to highlight our intent to isolate the effect of debris on glaciers. We have also added notes in the 'Implementation and numerics' section that explain why we use a linear bed.

>While the model parameterization may be more representative of Himalayan glaciers at some
point in the geological past, the assumption of steady state undermines the relevance of

the study to a complex set of glaciers in a variable climatic regime.

This study does use a hypothetical framework and it is not intended to represent any single glacier or 'real' glacier in the Himalaya or High Asia. Our intent in plotting the data from Scherler et al., 2011 with our model results is to show that the model re-produces the 'general trend' that Scherler et al., 2011 highlights. Along these lines we have better highlighted in the text that we only intend to match the broad observations laid out by Scherler et al., 2011 and emphasized that that Scherler data stems from glaciers responding to negative mass balance.

>The comparison to glaciers in the Himalaya or indeed elsewhere, does not add value to the paper
as there is no clear indication that mountain glaciers ever approach steady state over decadal–centennial timescales.

While it is true that mountain glaciers do not likely reach steady state (at any timescale) due to the effect of interannual climate variability, transience in glacier dynamics, or stochastic debris input, modeling glaciers in steady-state allows us to compare the effect of parameters on glacier response in a quantitative fashion. And it is compelling none-the-less that our steady-state simulations reproduce the broad observations of the Scherler et al., 2011. This suggests that debris has an important effect on basic glacier properties. Our manuscript quantifies that effect.

>This could be addressed by considering longer-term change over glacial cycles where small climatic fluctuations could be “averaged out” by much larger glaciers.

This is an interesting suggestion but it is well outside the scope of the current study.

>Manuscript style. The manuscript is mostly well written, but would benefit from being more accessible to a glaciological and geological audience.

We agree that the paper could be more accessible. We have revised the introduction, clarified where necessary, and removed jargon. We have improved the legibility of figures 8 and 9. We also clarified the Appendices and added an new explanatory figure in Appendix A. We have also used prose instead of variable names where appropriate to make the text more legible.

> The introduction does not really describe the specific problem considered in the study.

We have revised the introduction to address the specific problem we pose as you imply. This should improve the accessibility of the manuscript to a broader audience. Thank you for pushing us to look at the introduction again.

>Even with similar interests to the topic of this manuscript, I found the detail of text and figures
somewhat dense and difficult to follow in places.

Our manuscript presents a significant number of new quantitative approaches to modeling debris-covered glaciers. Because of the shear number of parameters and the complexity with which they interact the text will inherently be technical. Also because of the wide range of parameters explored and the number of simulations we ran our figures contain a lot of data. We feel that the number of simulations we present supports the robustness of our conclusions and is a strength of the paper.

With that said we have streamlined the text throughout. We also improved the legibility of several figures and captions. It would have been helpful if you listed the figures you struggled with and specifically listed what you found difficult.

>In particular, the relevance of different parameters noted to impact on and be affected by glacier
behaviour (debris cover, AAR, glacier velocities, etc.) should be discussed quantitatively in light of the outcomes from the modelling experiments.

We are not sure how making our results more quantitative will improve the legibility of the figures and text. Because we explore a wide parameter space adding more numbers that are dependent on the specific parameter choice will only make the manuscript less legible especially as we would then have to list all of the parameter choices that the results depended on. We have made efforts to improve and streamline this section just the same.

>Minor points Title: would better describe the study and read more readily without the colon. Suggest: “Modelling the extension of debris-covered glaciers due to steady debris input” or similar, as the model presented in this manuscript simulates this rather than all aspects of glacier change.

We feel that your suggested title is purely a style choice. Thank you for the suggestion but we are happy with the title as it reads.

>Abstract: should include more clearly quantitative results, for example, the conditions of the experiment described by “Our model and parameter selections produce two-fold increases in glacier length.” is not clear.

Because we explore a wide parameter space and use a hypothetical glacier geometry adding quantitative results that are dependent on the specific parameter choice/model set will only make the abstract less legible especially as we would then have to list all of the parameter choices that the results depended on. While “Our model and parameter selections produce two-fold increases in glacier length.” is general it is representative of the strong length enhancing effect of debris-cover.

>P6425, line 2: Scherler et al. 2011b is cited before 2011a.

This was corrected. We cite Konrad and Humphrey, 2000 Instead.

>P6425, l 5: “ablation” rather than “melt”

This is changed in the manuscript. We removed 'melt rate' and replaced it with 'ablation'

>P6425, l 14: what is meant by “almost coherently”?

“almost coherently” was removed and replaced with 'almost all'

>P6427, l 4–5: please phrase the problem more precisely, e.g. “how does the location/timing/frequency/magnitude of debris delivery and the description of the relationship between debris thickness and ablation affect change in glacier length/rate of advance/mass balance, relative to glacier morphology (e.g. size, shape, etc.)”.

We rewrote this paragraph to make the purpose of this study more clear. Because we address a number of issues linking debris and glaciers we prefer to highlight the problem in a paragraph as opposed to a sentence:

“Here we attempt to improve our understanding of the debris-glacier-climate system (and subsequently better project future glacier change) by isolating how each component (debris, glacier, and climate) in the system affects all others. While significant effort has focused on glacier-climate interaction, less research has focused on isolating the effect of debris on glacier dynamics, glacier length (e.g., Konrad and Humphrey, 2000), or glacier response to climate change. We address debris-glacier interactions by isolating the role of debris in governing basic glacier dynamics and glacier length.

We use a simple glacier model to simulate hypothetical debris-covered glaciers. This new framework allows us to isolate the effects of debris on glacier response by controlling the potentially conflating effects of a variable bed, variable glacier width, or a temporally variable climate. To isolate the effect of debris on glacier response, we start each simulation with a steady state debris-free (ssdf) glacier and impose a step change increase in debris deposition rate while holding climate steady. In many debris-covered glacier systems, debris is deposited in the accumulation zone, advected through the glacier, and emerges in the ablation zone (e.g., Boulton and Eyles, 1979; Owen and Derbyshire, 1989; Benn and Owen, 2002; Benn et al., 2012). Our new transient 2-D numerical model (x, z) links debris deposition, englacial debris advection, debris emergence, surface debris advection, debris-melt coupling, debris removal from the glacier terminus, and shallow-ice-approximation dynamics (Figs.~1 and 2). We provide a new terminus parameterization which allows for the use of glacier length as a metric for comparison between simulated debris-covered glaciers. This new framework allows us to explore the sensitivity of hypothetical debris-covered glaciers to debris thickness-melt formulations or changes in debris-input related variables like debris flux, debris deposition location, and debris deposition zone width. We compare our theory-based results to the 'general trends' documented by Scherler et al. (2011b). By only assessing the effect of debris on glaciers, this study lays the theoretical foundation for efforts exploring the more complex response of debris-covered glaciers to climate change.”

>P6428, l4: are these simulations run over thousands of years?

Yes. We are running a fully-transient model from a debris-free steady state to a debris-covered steady state, which takes thousands of years in most cases. Please see Figure 4, and Figure 12.

>P6436, l23: for Khumbu Glacier debris-covered ice mass balance, see also Benn and Lehmkuhl, 2000, Quaternary International, and references therein.

Thank you for the reference.

We make no change here because we use the Wagnon paper to guide our parameter selection.

>P6443, l24–26: this is not a helpful conclusion for those investigating palaeoclimate indicators in high mountain environments! Could your model results be used to reduce these uncertainties?

We are not sure why this is not a helpful suggestion. Debris cover can have a considerable effect (at least a two fold effect for the on glacier lengths for the parameter space we explore). So in addition to changes in precipitation and summer temperature the input of debris therefore becomes an important parameter for paleoglacier modeling. Knowing the detailed history of debris input to a specific paleo-glacier seems very difficult. From our perspective the best way to estimate paleoclimate from former glacier extents is to avoid catchments that were strongly perturbed by debris or to explore the full uncertainty associated with debris input rates and locations.

The sentence has been modified:

“The effect of debris on paleoclimate estimates can be mitigated by avoiding de-glaciated catchments with high-relief headwalls, supraglacially sourced moraine sediments, or by using a debris-glacier-climate model to estimate the effect of debris on glacier extent.”

>P6446: The “Future Work” section would be more usefully presented as “Limitation of the current study” or similar, to help the reader evaluate the strengths and weaknesses of the approach and results presented. The authors are then free to investigate these in future without asking the reader to wait to discover the value of the present study.

While we do not believe that readers will be confused by the title of this section, we do agree that it could be improved. It has been changed to:

“Potential model improvements and future research”

>P6447, l7: what is meant by “memory in the system”?

“memory in the system” refers to the fact debris-covered glaciers respond to mass wasting events that occurred in the past. So the system (the debris-covered glacier) is responding to depositional events in the past (the memory).

The sentence now reads: “Debris advection through and on a glacier can take hundreds of years, leading to memory in the system (i.e., the glacier responds to debris input from the past).”

>P6447: Some of the points presented in the conclusions could be drawn from previous work rather than the current study and can be removed to the introduction.

We state in the conclusion that that our 'simulations show that:' These conclusions can be directly drawn from our results and we are therefore comfortable keeping the conclusions as they stand. It would have been helpful if you included which conclusions should be moved to the introduction.

>Quantitative outcomes of the present study are needed in the conclusions (and the abstract)
to demonstrate where the most important sensitivities of debris-covered glaciers are.

Because of the considerable parameter search we are not sure how to present quantitative results that would be meaningful. This manuscript is meant to help improve our theoretical understanding of how glaciers respond to debris input. We therefore do not include quantitative results in the conclusion as we would also have to include the parameters used to define these results.

>Finally, the conclusions would preferably be written as continuous prose rather than bullet points.

We prefer the bulleted style because of the diversity of conclusions. It is also easier to see each of the conclusions when a reader takes a quick glance.

We did change the leading sentences in the conclusion:
“Before modeling the response of debris-covered glaciers to a warming climate, it is helpful to constrain how debris effects glaciers – independent of climate change.”

to

“It is necessary to constrain the effect of debris on glaciers so we can better predict and model the response of debris-covered glaciers to climate change.”

Summary of Changes to the Manuscript:

1) The introduction has been seriously edited to make the goal of the study more clear. We explain better why we use a linear bed slope and simplified model framework, which is to isolate the effect of debris on glacier response.

2) We emphasized the hypothetical nature of our study and note that we match the 'general trends' of Scherler et al., 2011b's dataset. We also make it more clear that our results are not dependent on the choice of base parameters from the Khumbu region.

3) The language throughout the manuscript has been smoothed to make it more readable. Special attention was paid to the results and discussion sections as well as a few of the figure captions. Our intent was to make the paper more accessible and readable. We state the names of debris related parameters more often to ease the experience for readers.

4) Overly complicated figures were simplified and unnecessary panels were removed (figs 8 and 9). Some figures were rotated for consistency with other figures (e.g. length on the y-axis; 10 and B1).

5) The terminus parameterization was described in more detail in appendix A and a new figure was added to make this parameterization more clear.

6) We added a more simple metric for comparing the effect of the different debris-related variables and included a new table (Table 2) to better summarize our results.

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Modeling debris-covered glaciers: ~~extension due~~ response to steady debris ~~input~~deposition

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Abstract

Debris-covered glaciers are common in rapidly-eroding alpine landscapes. When thicker than a few centimeters, surface debris suppresses melt rates. If continuous debris cover is present, ~~mass balance gradients can be~~ ablation rates can be significantly reduced leading to increases in glacier length. In order to quantify feedbacks in the debris-glacier-climate system, we developed a 2-D long-valley numerical glacier model that includes englacial and supraglacial debris advection. We ran 120 simulations on a linear bed profile in which a hypothetical steady state debris-free glacier responds to a step increase of surface debris deposition. Simulated glaciers advance to steady states in which ice accumulation equals ice ablation, and debris input equals debris loss from the glacier terminus. Our model and parameter selections can produce two-fold increases in glacier length. Debris flux onto the glacier and the relationship between debris thickness and melt rate strongly control glacier length. Debris deposited near the equilibrium-line altitude, where ice discharge is high, results in the greatest glacier extension when other debris related variables are held constant. Debris deposited near the equilibrium-line altitude re-emerges high in the ablation zone and therefore impacts melt rate over a greater fraction of the glacier surface. Continuous debris cover reduces ice discharge gradients, ice thickness gradients, and velocity gradients relative to initial debris-free glaciers. Debris-forced glacier extension decreases the ratio of accumulation zone to total glacier area (AAR). ~~The model reproduces first-order relationships~~ Our simulations reproduce the 'general trends' between debris cover, AARs, and glacier surface ~~velocities from glaciers in High Asia~~velocity patterns from modern debris-covered glaciers. We provide a quantitative, theoretical foundation to interpret the effect of debris cover on the moraine record, and to assess the effects of climate change on debris-covered glaciers.

1 Introduction

Glaciers erode landscapes directly by subglacial quarrying and abrasion, and indirectly by steepening hillslopes above glaciers. Oversteepened hillslopes can deliver loose rock (debris) onto glacier surfaces (Benn and Evans, 2010). Steep hillslopes and high hillslope erosion rates in alpine settings therefore tend to correspond with the occurrence of debris-covered glaciers (e.g., the Himalaya and the Alaska Range; Scherler et al., 2011b). We refer to a debris-covered glacier as any glacier with continuous debris cover across the full glacier width over a portion of the glacier (after Kirkbride, 2011).

Debris cover more than a few centimeters thick damps the melt rate ablation of underlying ice (e.g., Østrem, 1959; Shroder et al., 2000; Owen et al., 2003). If debris supply is high to a glacier surface is high, mass balance profiles can be greatly altered, leading to increases in glacier volume and length (e.g., Konrad and Humphrey, 2000; Scherler et al., 2011b2011a; Fig. 1). Thick debris cover on glaciers can also lead to low accumulation-area ratios (AARs; Scherler et al., 2011b). Paleoclimate estimates will Estimates of past climate change will therefore be exaggerated if typical AARs are assumed when reconstructing past climate from former moraines deposited by debris-covered glacial moraines glaciers.

Debris-covered glaciers glacier termini exhibit a wide range of responses to climate change (Scherler et al., 2011a). While almost all Himalayan debris-free glaciers are almost coherently retreating, Himalayan debris-covered glaciers glacier termini are not responding coherently to climate change despite a strong trend toward negative mass balance (e.g., Bolch et al., 2011; Benn et al., 2012). Some Himalayan debris-covered glaciers glacier termini are advancing, others are stationary, and yet others are retreating (e.g., Raper and Braithwaite, 2006; Scherler et al., 2011a; Benn et al., 2012; Banerjee and Shankar, 2013). However, there is a strong trend toward negative mass balance for most of these This discrepancy between debris-covered glaciers (Bolch et al., 2011; Benn et al., 2012)glacier mass balance and terminal response highlights the pressing need to understand the sometimes counterintuitive effects of debris on glacier response.

The direct effect of debris on glaciers is difficult to isolate on modern glaciers. In situ documentation of debris-covered glacier mass loss is made difficult by non-uniform debris thicknesses and the presence of scattered ice cliffs and surface ponds. As a result , in situ debris thickness, sub-debris melt rates, sub-debris ice thickness measurements and complete summer balances from debris-covered glaciers are sparse (WGMS, 2008). Measurements of englacial debris concentrations and distribution are yet more difficult to obtain , but vital for predicting debris-covered glacier response to climate change (e.g., Kirkbride and Deline, 2013). In addition, exploration of century-scale response of debris-covered glaciers to climate is limited by short satellite and observational periods (Bolch et al., 2011). Logistical realities therefore limit our ability to constrain feedbacks between debris deposition rates, the englacial environment, the supraglacial environment, ice dynamics, and climate changeand ice dynamics.

While logistics limit our ability to directly observe some feedbacks, many of the most provocative conclusions relating debris and glacier response are based on remotely-sensed data. Scherler et al., (2011b) provided an extensive inventory of remotely-sensed velocity and debris coverage data from 287 glaciers in High Asia. They inferred that several general patterns from these debris-covered glaciers: (1) hillslope debris flux onto glaciers correlates with the percentage of debris cover on glaciers; (2) debris-covered glacier AARs tend to be smaller than debris-free glaciers; and (3) surface debris perturbs velocity distributions on valley glaciers by shifting maximum glacier velocities up glacier, away from the terminus. These inferences highlight the effect of thick debris cover on valley glaciers and also act serve as targets for models of debris-covered glacier models glaciers.

Numerical models can help quantify feedbacks within the climate-debris-glacier debris-glacier-climate system (e.g., Konrad and Humphrey, 2000). Debris-covered glacier models have been used to explore the response of valley glaciers to (1) the constant steady input of debris (Konrad and Humphrey, 2000); (2) one-time landslide deposition of debris on glaciers (Vacco et al., 2010; Menounos et al., 2013); and (3) climate change (Naito et al., 2000; Banerjee and Shankar, 2013; Rowan et al., 2015). Konrad and Humphrey (2000) used a two-dimensional (2-D; long-valley-vertical) model with a constant surface slope to explore debris-covered glacier dynamics. In their model, debris was deposited on the glacier surface below the equilibrium line equilibrium-line altitude (ELA) and was then advected along the glacier surface. With high debris fluxes, simulated glaciers formed several-meter thick debris covers, which reduced sub-debris melt toward zero, and resulted in glaciers that never reached steady state. Numerical models have also shown that large landslides onto glaciers can lead to multiple-kilometer advances of the terminus (Vacco et al., 2010; Menuounos et al., 2013). Debris-covered glacier retreat response timescales have also been explored with a simplified debris-covered glacier model (Banerjee and Shankar, 2013). However, because of the Rowan et al. 2015 used a numerical model to forecast the response of the debris-covered Khumbu glacier, Nepal to climate change. But owing to the complexity of the debris-glacier-climate system, many feedbacks remain unexplored. it can be difficult to diagnose the effects of different processes on observable glacier responses. For example, both increased debris delivery to a glacier and a cooling climate could lead to glacier advances (e.g., Vacco et al., 2010; Menuounos et al., 2013). What approaches could we use to address these sorts of conundrums within the debris-glacier-climate system?

In this study, we isolate the effect of debris on valley glaciers independent of climate change.Debris fluxes, deposition rates, deposition zone widths, and deposition locations vary from glacier to glacier (Fig. 1), yet we know little about how changes in these debris related variables effect glaciers. So we ask: What about debris delivery controls glacier response ? Here we attempt to improve our understanding of the debris-glacier-climate system (and subsequently better project future glacier change) by isolating how debris effects glacier response, while holding climate steady. While significant effort has focused on glacier-climate interaction, less research has focused on isolating the effect of debris on glacier length (e.g., Konrad and Humphrey, 2000), and other basic measures of glacier response (e.g., change in glacier surface velocity due to debris deposition on the glacier). We explore debris-glacier interactions by isolating the role of debris in governing basic glacier dynamics and glacier length.

We use a simple glacier model to simulate hypothetical debris-covered glaciers. This new framework allows us to isolate the effects of debris on glacier response by controlling the potentially conflating effects of a variable bed, variable glacier width, or a temporally variable climate. To isolate the effect of debris, we start each simulation with a steady state debris-free (ssdf) glacier and impose a step change increase in debris deposition rate while holding climate steady. In many debris-covered glacier systems, debris is deposited in the accumulation zone, advected through the glacier~~following englacial flowpaths~~, and emerges in the ablation zone (e.g., Boulton and Eyles, 1979; Owen and Derbyshire, 1989; Benn and Owen, 2002; Benn et al., 2012). ~~In order to explore the response of glaciers to surface debris cover, we formulated a~~ Our new transient 2-D numerical model (x, z) ~~that couples~~ links debris deposition, englacial debris advection, debris emergence, surface debris advection, debris-melt coupling, debris removal from the glacier terminus, and shallow-ice-approximation dynamics (Figs. 1 and 2). ~~By coupling these components, we are able to~~ We provide a new terminus parameterization which allows for the use of steady state glacier length as a metric for comparison between simulated debris-covered glaciers. While real debris-covered glaciers may not reach steady state the concept is necessary for determining the sensitivity of debris-covered glaciers to changes in debris related parameters. Our intent is to determine which parameters and parameterizations are most important for capturing the response of glaciers to debris input. Here, we explore the sensitivity of hypothetical debris-covered glaciers to changes in ~~debris input~~ debris-input related variables (~~across the entire glacier) and~~ e.g., debris flux, debris deposition location, and debris deposition zone width). We also explore the sensitivity of debris-covered glaciers to different debris thickness-melt formulations. We compare our theory-based results ~~with~~ to the 'general trends' documented by Scherler et al. (2011b)~~'s dataset. To isolate~~ . By isolating the effect of debris ~~, we start each simulation with a steady state debris-free (ssdf) glacier and impose a step change increase in debris deposition rate. This~~ on glaciers, this study lays the ~~foundation for future modeling~~ theoretical foundation for efforts exploring the more complex response of debris-covered glaciers to climate change.

2 Theory and numerical methods

We employ a fully-transient 2-D finite difference numerical model (in downvalley and vertical, x and z) that can simulate the evolution of temperate valley glacier response to climate and debris. Forced by a time series of equilibrium-line altitudes (ELAs) and a prescribed mass balance gradient, the model calculates ice surface elevations above a longitudinal profile by solving equations for ice flux and mass conservation. The modeled longitudinal path represents the glacier centerline. A number of authors have used the shallow-ice-approximation (SIA) and basal sliding parameterizations in numerical glacier models (e.g., Nye 1965; Budd and Jensen, 1975; Oerlemans, 1986; MacGregor et al., 2000; Leysinger and Gudmundsson, 2004; Kessler et al., 2006). We employ a similar approach, but add a longitudinal stress coupling parameterization (Marshall et al., 2005). The model is efficient, allowing wide exploration of parameter space in simulations over thousands of years.

2.1 Conservation of ice mass

Mass conservation is at the core of the ice physics model. Assuming uniform ice density, and ignoring variations in the width of the glacier, ~~conservation of ice~~ ice conservation requires that

$$\frac{\partial H}{\partial t} = \dot{b} - \frac{\partial Q}{\partial x}, \tag{1}$$

where x is the distance along the glacier flowline, H is the local ice thickness, $\dot{b}$ is the local specific balance, and Q is the specific volume discharge of ice $[=]$ m^3 m^{-1} yr^{-1}. This requires a prescribed mass balance field, and a prescription of the ice physics governing ice discharge.

Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper |

2.2 Annual surface mass balance of ice in the absence of debris

We use a simple mass balance scheme that limits the number of parameters while honoring the essence of glacier surface mass balance. We combine surface accumulation and ablation into a single thresholded net mass balance profile as a function of elevation, z:

$$\dot{b}_z = \min\left(\frac{\mathrm{d}\dot{b}_z}{\mathrm{d}z}(Z_{\mathrm{ice}} - \mathrm{ELA}), \dot{b}_z^{\max}\right), \tag{2}$$

where $\frac{\mathrm{d}\dot{b}_z}{\mathrm{d}z}$ is the mass balance gradient with elevation, Z_{ice} is the ice surface elevation and $\dot{b}_z^{\max}$ is a maximum mass balance that accounts for the depletion of moisture available for precipitation at higher elevations. The annual surface mass balance of ice in the absence of debris is held steady for all simulations to isolate the effects of debris from those of climate change on glacier response.

2.3 Annual surface mass balance: effect of supraglacial debris

Sub-debris melt rate decreases ~~in an exponential or hyperbolic fashion~~ rapidly with increasing debris thickness (e.g., Østrem, 1959; Nicholson and Benn, 2006). For debris layers thinner than a critical thickness (~ 2 cm), surface debris can increase melt rates relative to bare ice. For debris thicknesses greater than ~ 2 cm, debris suppresses sub-debris melt rates relative to bare ice (e.g., Nicholson and Benn, 2006; Fig. 3). We assume that heat is transferred through the debris layer by conduction. Sub-debris melt should therefore vary inversely with debris thickness (i.e., be hyperbolic) ~~and change based on~~ as conduction is governed by the temperature gradient $\sim (T_{\mathrm{s}} - T_{\mathrm{ice}})/h_{\mathrm{debris}}$ (e.g., Nicholson and Benn, 2006). Here, $T_{\mathrm{ice}} = 0$. We neglect the melt-amplifying effects of very thin debris for simplicity and represent the damping of sub-debris melt rates with

$$b' = \dot{b}_z \left(\frac{h_*}{h_* + h_{\text{debris}}} \right), \tag{3}$$

where h_* is a characteristic length scale

$$h_* = \frac{k\overline{T}_{\text{s}}}{(1-\phi)\rho_{\text{i}} L f_{\text{pdd}} \overline{T}_{\text{a}}} \tag{4}$$

and k and ϕ are thermal conductivity and porosity of debris cover, ρ_{i} and L the density and latent heat of fusion of ice, $\overline{T}_{\text{s}}$ the average debris surface temperature, $\overline{T}_{\text{a}}$ the average screen-level air temperature, and f_{pdd} is a positive degree day factor relating air temperature and the bare ice melt rate (e.g. Mihalcea et al., 2006). In this formulation, sub-debris melt rates approach bare-ice melt rates as debris thins ($h_{\text{debris}} \ll h_*$), and asymptote towards zero melt asymptotes toward a hyperbolic dependence on debris thickness as debris thickens ($h_{\text{debris}} \gg h_*$). We use h_* values based on data from 15 studies (Fig. 3; $h_* = 0.066 \pm 0.029$ m (1σ), and ranges from 0.03 to 0.13 m). We For comparison, we also show the most likely exponential fit to the data for comparison to the most likely hyperbolic fit (Fig. 3). The exponential curve fit declines toward zero melt more rapidly than the hyperbolic fit (e.g., Konrad and Humphrey, 2000; Hagg et al., 2008). We neglect the effects of surface streams, thermokarst, and ice cliffs that can lead to complex local topography and melt rates within debris covers (e.g., Reid and Brock, 2014; Anderson, 2014).

2.4 Ice dynamics

Ice is transferred down valley by internal ice deformation and by basal motion. The ice discharge down glacier is:

$$Q = H\overline{u} \tag{5}$$

in which H is the local ice thickness and $\overline{u}$ is the depth-averaged bed parallel velocity that results from the sum of the ice deformation velocity and basal motion. The SIA reduces the momentum balance equations to expressions for vertical shear stress as a function of the local ice surface slope and ice thickness. The depth-averaged horizontal velocity due to internal deformation is

$$\overline{u}_{\text{def}} = \frac{2A}{n+2}(\rho_{\text{i}} g \alpha)^{n-1} H^n \tau_{\text{b}x}, \quad (6)$$

where ρ_{i} the density of ice, *g* the acceleration due to gravity, α the local ice surface slope, H the local ice thickness, $\tau_{\text{b}x}$ is the local basal shear stress, *A* is the creep parameter, and *n* is the flow law exponent (assumed to be 3). We assume that all ice is temperate, and *A* is therefore taken to be 24×10^{-25} [$\text{Pa}^{-3}\,\text{s}^{-1}$] (Cuffey and Paterson, 2010). In addition to internal deformation, temperate glaciers transfer mass via basal slip due to ice sliding over the bed and deformation of the bed itself. We assume that all basal slip is accomplished by sliding over bedrock, and follow the formulation of Kessler et al. (2006):

$$u_{\text{sliding}} = u_{\text{c}} e^{1 - \frac{\tau_{\text{c}}}{\tau_{\text{b}x}}} \quad (7)$$

~~where~~ in which u_{c} is a typical sliding velocity, and τ_{c} is the gravitational driving stress that gives rise to the typical sliding velocity. This sliding parameterization is not as sensitive to high τ_b values as many other sliding laws (e.g. Cuffed and Paterson, 2010), and provides a more conservative estimate of sliding velocities when $\tau_b > \tau_{\text{c}}$ (Kessler et al., 2006). We have ~~modified the SIA~~ also modified the SIA equations by including a parameterization of longitudinal stress coupling (after Marshall et al., 2005) and a shapefactor, f, that represents the effect of valley wall drag. The longitudinal coupling scheme modifies $\tau_{\text{b}x}$ to

$$\tau_{\text{b}x} = f\left(\rho_{\text{i}} g H \alpha + 4\overline{\eta} H \frac{\partial^2 u}{\partial x^2} \frac{\partial^2 \overline{u}}{\partial x^2} + 4\frac{\partial \overline{\eta} H}{\partial x} \frac{\partial u}{\partial x} \frac{\partial \overline{u}}{\partial x}\right), \quad (8)$$

where the effective viscosity, $\overline{\eta} = \frac{1}{2}[A\tau_{\text{E}}^{n-1}]^{-1}$ and $\overline{u}$ is the vertically averaged ice velocity. In the shallow ice approximation, τ_{E}, the effective stress, is approximated by the local $\tau_{\text{b}x}$

(after Cuffey and Paterson, 2010). We take $f = 0.75$ to approximate the effects of sidewall drag from a parabolic valley cross-section with a half-width 3 times the ice thickness (Cuffey and Paterson, 2010).

2.5 Ice velocity structure within the glacier

Horizontal and vertical velocity fields must be resolved within the glacier in order to advect englacial debris. We start by defining the horizontal velocity field within the glacier, and then employ continuity in an incompressible medium to calculate the associated vertical velocities. The $u(z)$ profile shape may be obtained from the analytic solution to flow of ice in a uniform channel with Glen's flow law rheology:

$$F = 5\left(\left(\zeta - 1.5\zeta^2\right) + \zeta^3 - \frac{1}{4}\zeta^4\right), \quad (9)$$

where ζ is the non-dimensional height z/H above the bed, and $F = \frac{u(z)}{\overline{u}_{\text{def}}}$ is the ratio of horizontal speed to mean deformation speed. The full horizontal velocity field is then characterized by

$$U_\zeta(x,\zeta) = \overline{u}_{\text{def}}(x)F + u_{\text{sliding}}(x) + u_{\text{coupling}}(x), \quad (10)$$

where u_{coupling} is the vertically-integrated velocity effect due to longitudinal stress coupling . and is determined by subtracting the original eqn. 6 from eqn. 6 modified by eqn. 8.

Vertical and horizontal velocity fields ($w(x,z)$ and $u(x,z)$) are related through the continuity equation for an incompressible fluid, which in two dimensions (x,z) is:

$$\frac{\partial w}{\partial z} = -\frac{\partial u}{\partial x}. \tag{11}$$

We then solve for the vertical velocity in each cell within each column by integrating vertically:

$$w = -\int_0^z \left(\frac{\partial u}{\partial x}\right) \mathrm{d}z, \tag{12}$$

employing the boundary condition that $w = 0$ at $z = 0$ (i.e., we assume no basal melt). ~~Vertical~~ In steady state, vertical velocities, w, at the glacier surface must be equal in magnitude and opposite in sign to the surface mass balance field, and are therefore directed downward at the ice surface in the accumulation zone, and upward in the ablation zone.

2.6 Debris deposition

Debris can be entrained in the glacier at either the upper glacier surface or at the glacier bed. Supraglacial debris deposition largely occurs by mass wasting from hillslopes above glaciers, while sub-glacial debris entrainment occurs through regelation and net freeze-on. Basal debris emergence at the glacier surface is typically limited to the glacier toe and likely plays a minor role in the formation of extensive debris covers (Benn and Evans, 2010). We focus on debris sourced from valley head and side walls. Headwall erosion rates are better constrained than subglacial entrainment rates and mass wasting from head and sidewalls is the primary process of debris delivery onto many valley glaciers (Messerli and Zurbuchen, 1968; Humlum, 2000 (European Alps); Owens and Derbyshire, 1989 (Karakoram); Ballantyne and Harris, 1994; Humlum, 2000 (West Greenland); Benn and Owen, 2002 (Himalaya); Humlum, 2005 (Svalbard); Arsenault and Meigs, 2005 (Southern Alaska); O'Farrell et al., 2009 (Southern Alaska); Benn and Evans, 2010;

Scherler et al., 2011b (High Asia)). The model replicates the deposition of debris onto the glacier surface leading to the formation of Ablation-dominantablation-dominant and Avalanche-typeavalanche-type medial moraines on the glacier surface (Benn and Evans, 2010). For simplicity, we neglect englacial thrusting and ice-stream interaction moraines (medial moraines associated with tributary junctions; see Eyles and Rogerson, 1978; Anderson, 2000; Benn and Evans, 2010). These cases can be treated in subsequent modeling that incorporates the 2-D planform complexities of valley glaciers.

Debris delivery to glacier surfaces can vary considerably from glacier to glacier, depending on glacier topology and above-glacier topography (e.g., Deline, 2009). We capture this complexity using four variables: the total debris flux to the glacier surface (d_{flux} [=] m^3 m^{-1} yr^{-1} $\dot{d}_{\text{flux}}$ [=] m^3 m^{-1} yr^{-1}), the debris deposition rate ($\dot{d}\dot{d}$[=]mm yr^{-1}), the debris deposition zone width ($\dot{d}_{\text{width}} d_{\text{width}}$[=]m), and the debris deposition location ($\dot{d}_{\text{loc}} d_{\text{loc}}$). In the model, d_{flux} $\dot{d}_{\text{flux}}$ is representative of the integrated effects of $\dot{d}$ and $\dot{d}_{\text{width}} d_{\text{width}}$.

Rock type, slope, and fracture density are significant factors determining hillslope erosion rates and therefore also control the debris deposition rate, $\dot{d}$ (e.g., Stock and Montgomery, 1999; Molnar et al., 2007). In the model, $\dot{d}$, is allowed to vary from 1 to 8,mmyr^{-1} and is steady within each simulation (Fig. 1b). Debris deposition rate depends on a number of site-specific variables:

$$\dot{d} = f_{\text{funneling}} f_{\text{hillslope}} \dot{\epsilon} \frac{H_{\text{wall}}}{\tan(\theta)\mathrm{d}x},$$

$$tan(\theta)dx,$$

((13)

where $f_{\text{funneling}}$ is a dimensionless factor capturing the effect of topographic funneling on debris deposition, $f_{\text{hillslope}}$ is the percentage of the headwall that is exposed bedrock, $\dot{\epsilon}$ e is the hillslope backwearing rate in $\mathrm{m\,yr^{-1}}$, H_{wall} is the height of the headwall, and θ is the headwall slope. The deposition rates explored in this study are viable deposition/hillslope erosion rates appropriate for typical headwall erosion rates (typically ranging between 0.5 and 2 $\mathrm{mm\,yr^{-1}}$), headwall heights, and headwall slopes for high-relief mountain environments (e.g., Heimsath and McGlynn, 2008; Ouimet et al., 2009; Scherler et al., 2011; Ward and Anderson, 2011). $\dot{d}_{\text{width}}$ defines the d_{width} defines the downvalley width of the deposition zone, the zone over which the debris is spread on the glacier surface (we employ a base width of 400 m; Table 1; Fig. 1b).

Debris is deposited on glaciers at locations where hillslope erosion processes are connected to the glacier surface. This requires high-relief topography above the glacier to provide the energy necessary to move the debris onto the glacier. In the model, we control the downvalley debris deposition location with the variable $\dot{d}_{\text{loc}} d_{\text{loc}}$, which we allow to vary from near the headwall to near the glacier terminus. $\dot{d}_{\text{loc}}$ d_{loc} defines the up-glacier end of the debris deposition zone.

2.7 Incorporation and advection of englacial debris

Debris deposited in the ablation zone is advected along the glacier surface, whereas debris deposited in the accumulation zone moves downward with the ice and is therefore incorporated into the glacier. Near-surface The near-surface debris concentration in the accumulation zone is defined as $C_0 = \frac{\dot{d}\rho_{\text{rock}} m_z \mathrm{d}t}{H}$, where m_z is the number of vertical slices the englacial advection scheme is divided into (H/m_z being the thickness of the slices) and $\mathrm{d}t$ is the model time interval. C_0 is therefore the mass concentration of debris in the surface-bounding cell.

Once embedded in the glacier, C, the concentration of englacial debris [=], will change only by straining of the icedebris is advected through the glacier following englacial flowpaths ($\frac{\partial C}{\partial t} = -\frac{\partial (uC)}{\partial x} - \frac{\partial (wC)}{\partial z}$). Taking an Eulerian point of view, the time rate of change of concentration of debris within ~~a parcel of ice is:~~ an ice cell (in our model) is:

$$\frac{\partial C}{\partial t} = -\frac{\partial (uC)}{\partial x} - \frac{\partial (wC)}{\partial z} - \frac{C}{h_\zeta}\frac{\partial h_\zeta}{\partial t} - \frac{uC}{h_\zeta}\frac{\partial h_\zeta}{\partial x} - \frac{\partial (wC)}{\partial z} - \frac{\partial (uC)}{\partial x}, \tag{14}$$

where h_ζ is the cell height in a given ice column ($h_\zeta = \frac{H}{m_z}$). The first and second terms represent changes in C due to advection in the vertical and the horizontal directions, respectively. The third term on the right hand side represents the rate of change of C due to vertical ~~strain of ice~~ice strain from the thinning or thickening of the glacier through time. Note that if the strain rate is negative, signifying vertical thinning of an ice column, debris concentration in ~~the ice~~ a cell will increase. The ~~second~~ fourth term represents the rate of change of C due to the longitudinal changes in glacier thickness. ~~The third and fourth terms represent changes in C due to advection in the vertical and the horizontal directions, respectively~~This term accounts for the fact that cells from one column to the next are not the same volume.

2.8 Advection of debris on the glacier surface and steady states

We track both the melt-out of englacial debris and the advection of supraglacial debris on the glacier surface. The rate of change of debris thickness on the glacier surface is captured by

$$\frac{\mathrm{d}h_{\text{debris}}}{\mathrm{d}t} = -\frac{Cb'}{(1-\phi)\rho_{\text{rock}}} - \frac{\partial u_{\text{surf}} h_{\text{debris}}}{\partial x}, \tag{15}$$

where h_{debris} is the debris thickness, ρ_{rock} is the density of the rock, ϕ is the porosity of supraglacial debris, and u_{surf} is the surface velocity of the glacier (after Konrad and Humphrey, 2000; Naito et al., 2000; Vacco et al., 2010). The first term on the right represents the addition of debris to the surface from melt of debris-laden ice. The second term represents the advection of debris down glacier.

Debris is transported off glacier by the wasting of debris down the terminal slope or by the backwasting of terminal ice cliffs (Konrad and Humphrey, 2000; Appendix A and B).

In the model we implement a triangular terminus wedge parameterization (after Budd and Jenssen, 1975; see Appendix A). The change of surface debris thickness with time ~~at the glacier toe is:~~ on the terminal wedge is:

$$\frac{\mathrm{d}h_{\text{debris}}^{\text{snout}}}{\mathrm{d}t}\ \frac{\mathrm{d}h_{\text{debris}}^{\text{term}}}{\mathrm{d}t} = -\frac{\mathrm{d}_{\text{flux}}^{\text{snout}}}{\mathrm{d}x}\ \frac{\dot{\mathrm{d}}_{\text{flux}}^{\text{term}}}{\mathrm{d}x_{\text{term}}} - \frac{C b'}{(1-\phi)\rho_{\text{rock}}} - \frac{\partial u_{\text{surf}} h_{\text{debris}}}{\partial x}, \tag{16}$$

where ~~$d_{\text{flux}}^{\text{snout}}$~~ $\dot{d}_{\text{flux}}^{\text{term}}$ is the debris flux into the foreland from the ~~toe~~terminus wedge [=] $\text{m}^3\,\text{m}^{-1}\,\text{yr}^{-1}$ and $\mathrm{d}x_{\text{term}}$ is the surface length of the terminal wedge. We use ~~$d_{\text{flux}}^{\text{snout}} = \dot{b}_z^{\text{snout}} h_{\text{debris}}^{\text{snout}}$~~ $\dot{d}_{\text{flux}}^{\text{term}} = \dot{b}_z^{\text{term}} h_{\text{debris}}^{\text{term}}$. Varying this parameterization has a minor effect on glacier length, but can have a considerable effect on the temporal evolution of the glacier as ~~d_{flux} must equal $d_{\text{flux}}^{\text{snout}}$~~ $\dot{d}_{\text{flux}}$ must equal $\dot{d}_{\text{flux}}^{\text{term}}$ for a simulated glacier to reach steady state (Appendix A). We explore the choice and effect of this parameterization in Appendix B.

3 Implementation and numerics

We now outline the order of calculations in the model. First, $\dot{b}_z$ and b' are calculated based upon elevation and debris thickness. Next, we use a second-order Runge–Kutta centered difference scheme to evolve $H(x,t)$, followed by the implementation of ~~the debris advection schemes. We also impose~~ an iterative "upstream" debris advection scheme following Smolarkiewicz, 1983. This iterative scheme imposes a two-step anti-diffusion correction algorithm to the advection scheme which greatly reduces numerical diffusion (Smolarkiewicz, 1983). We test advection scheme stability using the Courant–Friedrichs–Lewy (CFL) condition, which ensures that mass is not advected beyond adjacent cells in a single timestep. We implement a terminus wedge parameterization that allows simulated glaciers to advance to steady state (Appendix A). The time step, $\mathrm{d}t$, for ice-physics and debris advection is 0.01 years. All ice columns are segmented into m_z heights (i.e., $\zeta = 0 : (1/m_z) : 1$); in all results below we use $m_z = 20$ (Fig. 1b).

~~We~~ We impose a no flux boundary at the upper end of the glacier.

While our simulations are hypothetical we select the base model parameters to loosely represent the ablation zones of debris-covered glaciers in the Khumbu region of Nepal(. There is a wealth of debris-covered glacier research from this region, which assures that our parameter choices in the range of observed values (e.g., Kayastha et al., 2000; Bolch et al., 2011; Benn et al., 2012; Shea et al., 2015). Base simulations are run on a linear glacier bed with a basal slope of 8 % and a maximum bed elevation of 5200 m (Scherler, 2014). This simple bed geometry is used to insure that our results to do not conflate the effects of bed topography with the effects of debris. We use a $\frac{\mathrm{d}\dot{b}}{\mathrm{d}z} = 0.0075\,\mathrm{yr}^{-1}$, which is capped at $2\,\mathrm{m\,yr}^{-1}$ based on data from debris-free glaciers in the Khumbu region (Mera and Pokalde glaciers: after Wagnon et al., 2013). Our parameter exploration below shows that our conclusions are not influenced by our choice of base parameters from the ablation zones of debris-covered glaciers in the Khumbu region. All simulations start with an 8.7 km long steady state debris-free (ssdf) glacier with a steady ELA at 5000 m ($L_{\mathrm{ssdf}} =$ 8.7 km). In each simulation a step change increase in debris deposition rate is imposed at $t = 100$ years. The base parameter set uses $d_{\mathrm{flux}} = 3.2\,\mathrm{m}^3\,\mathrm{m}^{-1}\,\mathrm{yr}^{-1}$ $\dot{d}_{\mathrm{flux}} = 3.2\,\mathrm{m}^3\,\mathrm{m}^{-1}\,\mathrm{yr}^{-1}$, $\dot{d} = 8\,\mathrm{mm\,yr}^{-1}$, $\dot{d}_{\mathrm{width}}$ of 400 m, $\dot{d}_{\mathrm{loc}}$ and the location of debris input, d_{loc}, is 42 % from the headwall to the steady state of the distance between the headwall and the length of the debris-free glacier, L_{ssdf}.

4 Numerical experiments and results

We first demonstrate the transfer of debris between model components and demonstrate debris-covered steady state. We then explore the differences between the ssdf steady state debris-free (ssdf) glacier and debris-covered glaciers and explore relative importance of $\dot{d}$, $\dot{d}_{\mathrm{width}}$, $\dot{d}_{\mathrm{loc}}$, and d_{flux} d_{width}, d_{loc}, $\dot{d}_{\mathrm{flux}}$, and $\dot{d}^{\mathrm{term}}_{\mathrm{flux}}$ on glacier length. The effect of $\dot{d}^{\mathrm{term}}_{\mathrm{flux}}$ on the length and time evolution of the model is explored in Appendix B (see Fig. B1). We then test the sensitivity of the model to changes in h_* and ϕ. Last, we compare our results to data hypothetical simulations to 'general trends' observed from real debris-covered glaciersin High Asia.

4.1 Demonstration of debris-covered glacier steady state and conservation of debris

In order to compare steady state glacier lengths between simulations with different ~~d_{flux}~~ $\dot{d}_{\text{flux}}$ we track debris through the model. At any time in the simulation, the total debris mass that has been deposited on the simulated glacier must equal the total debris mass in the model:

$$M_{\text{input}} = M_{\text{englacial}} + M_{\text{surface}} + M_{\text{foreland}}, \tag{17}$$

where M_{input} is the total rock mass deposited on the glacier and accumulated over time, $M_{\text{englacial}}$ is the total englacial debris mass, M_{surface} is the total debris mass on the glacier surface, and M_{foreland} is the total mass deposited in the proglacial environment.

We use the base parameter set simulation to highlight the transfer of debris mass through the system (Fig. 4). Because debris is deposited in the accumulation zone near the ELA, in the base simulation, $M_{\text{englacial}}$ rapidly reaches steady state (Fig. 4). As the glacier extends, M_{surface} continues to increase at a declining rate as more surface debris is transferred into the foreland. The glacier reaches steady state when the glacier length, M_{surface}, and $M_{\text{englacial}}$ are steady and the rate of change of M_{foreland} is equal the rate of debris input to the glacier. Each model simulation presented conserves greater than 99 % of debris mass.

4.2 Comparison of modeled debris-free and debris-covered glaciers~~with a steady climate~~

We first highlight differences in length, and the patterns of ice discharge, Q, ice thickness, H, and surface speed, u_{surf}, between the ssdf glacier and ~~single simulated~~ its steady state debris-covered ~~glacier (~~counterpart, using the base parameter set ~~;~~ (Fig. 4). In this baseline case the steady state debris-perturbed glacier length is 175 % of L_{ssdf} (Fig. 5).

The debris thickness, h_{debris}, increases down glacier from the ~~point of initial~~ site of debris emergence, ~~$\dot{\epsilon}_{x_{\text{int}}}$~~ $x_{\epsilon_{\text{int}}}$, except near the ~~glacier toe where the $d^{\text{snout}}_{\text{flux}}$~~ terminal wedge where the $\dot{d}^{\text{term}}_{\text{flux}}$ parameterization reduces h_{debris} (Fig. 5–6). Down glacier from ~~$\dot{\epsilon}_{x_{\text{int}}}$~~ the site of debris emergence, $x_{\epsilon_{\text{int}}}$, gradients of Q, H, and u_{surf} are reduced relative to the debris-free glacier

(Fig. 6b and d). Debris-free patterns of Q and u_{surf} are convex up near the glacier terminus, while Q and u_{surf} from debris-covered termini are concave upward. The lowest gradients in Q, H, and u_{surf} occur near the glacier terminus where h_{debris} is thickest (excluding the terminal slope; Fig. 6).

4.2.1 ~~Comparison~~ Effect of ~~debris-covered glaciers with different~~ debris input ~~locations~~location

Debris input location (~~$\dot{d}_{loc}$~~d_{loc}) controls the englacial debris path. Debris deposited near the headwall is advected more deeply into the glacier than debris deposited near the ELA. Debris deposited near the ELA follows a shallow, short englacial path (Fig. 5). The original width of the debris band deposited in the accumulation zone, is reduced down glacier and then widens again near the surface in the ablation zone (Fig. 5). The debris band initially narrows due to the longitudinal straining of ice (Hooke and Hudleston, 1978; Cuffey and Paterson, 2010; Fig. 5a) and then widens due to feedbacks between the surface debris and ice dynamics.

In order to ~~highlight~~ show the effects of ~~$\dot{d}_{loc}$ on~~ d_{loc} on basic glacier properties (glacier length, Q, H, and u_{surf}), we highlight three simulations where ~~$d_{flux} = 3.2$~~we vary d_{loc} and hold all other debris-related parameters constant ($\dot{d}_{flux} = 3.2$ m^3 m^{-1} yr^{-1}, $\dot{d} = 8$ mm yr^{-1}, and $\dot{d}_{width} = 400$ m~~are held constant between runs and $\dot{d}_{loc}$ is varied. $\dot{d}_{loc}$~~). d_{loc} is varied from near the top of the glacier ~~(7from the headwall to L_{ssdf}; Figs. 5a and 6a and c), to near the ELA (42from the headwall to L_{ssdf}; Figs. 4, 5b, 6b and d), and~~ to near the debris-free glacier toe (~~98from the headwall to L_{ssdf}; Figs. 5c and 6c and e~~Fig. 5 and 6).

When debris is deposited or emerges where Q is large (near the ELA), glacier extension is greater than when debris is deposited/emerges where Q is small (near the headwall or the debris-free glacier terminus). Another way of stating this: If debris is deposited or emerges where Q_{free}/Q_{max} nears 1 glacier extension will be largest for a given glacier (Q_{free} refers to ice discharge from the ssdf glacier and Q_{max} is the maximum Q_{free} before debris is added to the glacier). Where Q_{free}/Q_{max} nears 0 glacier extension will be small.

We ran an additional 33 simulations (36 total) in which we vary d_{flux} and $\dot{d}_{\text{loc}}$ $\dot{d}_{\text{flux}}$ and d_{loc} (Fig. 7). Changes in d_{flux} are accomplished by changing $\dot{d}$ with $\dot{d}_{\text{width}}$ held constant Varying the debris deposition location while holding the debris flux constant results in a maximum of an 40% difference (for these 36 simulations; Table 2) in the resulting steady-state debris-covered glacier length. The importance of $\dot{d}_{\text{loc}}$ d_{loc} on glacier length increases with larger d_{flux} $\dot{d}_{\text{flux}}$ (Fig. 7). The general pattern seen in Fig. 7 is insensitive to changes in the linear bed slope. Debris emergence/deposition at smaller Q leads to larger max(h_{debris}). Increasing d_{flux} other parameters. Increasing $\dot{d}_{\text{flux}}$ leads to increases in max(h_{debris}) and the percentage of the glacier covered with debris (Fig. 8).

4.2.2 Sensitivity Effect of steady state glacier length to changes in debris deposition rateand , debris deposition zone deposit width, and debris flux

Increasing either the debris deposition rate ($\dot{d}$or $\dot{d}_{\text{width}}$) or the debris deposit width (d_{width}) leads to increases in d_{flux}, but their relative importance $\dot{d}_{\text{flux}}$, but the relative importance of $\dot{d}$ or d_{width} in governing glacier response is unclear. Does debris delivered to a small portion of a glacier at a high rate lead to a different length response than debris delivered to a glacier in a wide section but at a low rate? In order to parse the effects of $\dot{d}$ and $\dot{d}_{\text{width}}$ d_{width} on glacier length, we ran 36 simulations in which we vary $\dot{d}$, $\dot{d}_{\text{width}}$, and $\dot{d}_{\text{loc}}$. Steady state glacier length increases with $\dot{d}_{\text{width}}$ when $\dot{d}_{\text{loc}}$ and $\dot{d}$ are held constant (Fig. 9c). Steady state glacier length also increases with $\dot{d}$ when $\dot{d}_{\text{loc}}$ and $\dot{d}_{\text{width}}$ are held constant (Fig. 9d). Increasing either $\dot{d}$ or $\dot{d}_{\text{width}}$ effects the system similarly d_{width} (Fig. 9c and d9b for $d_{\text{loc}} = 42\%$). The dependence of glacier length on $\dot{d}$ and $\dot{d}_{\text{width}}$ is not linear effect is small, varying the contribution results in a maximum of a 4% difference in steady-state debris-covered glacier length for any given debris flux (Fig. 9). If we combine the effects of $\dot{d}$ and $\dot{d}_{\text{width}}$ by comparing the $d_{\text{flux}}$with steady state glacier length we see that steady state glacier length is primarily dependent on d_{flux} 9b; Table 2). In contrast, varying the debris flux, $\dot{d}_{\text{flux}}$, results in a maximum of 80% change in glacier length (Fig. 9e). Length enhancement by a factor of 9c; Table 2or more is viable for the range of d_{flux} explored).

4.3 Parameter sensitivityEffect of characteristic debris thickness and surface debris porosity

We explore the sensitivity of the model to changes in the characteristic debris thickness ($h_*$and) and surface debris porosity (ϕusing the base parameter set for other parameters and inputs). We vary h_* and ϕ, impose a step change increase in debris input to the ssdf glacier and compare the resulting steady state glacier lengths (Fig. 10). Simulated glacier length is highly sensitive to h_* (Fig. 10). For the same debris delivery variables, the more rapidly the melt rate is damped by debris (lower h_*), the longer the steady state glacier. Steady state debris-covered glacier length varies from 140 to 250by 110 % of relative to L_{ssdf} when h_* is varied from the extremes of 0.0035 to 0.165 m (160–215Table 2; 55 % for the 1σ range (0.037–0.095 m)). Glacier length is not as sensitive to the choice of debris porosity, ϕ (Fig. 10). Variation of Varying ϕ between the extreme range of 0 and 0.45 extremes of 0.18 and 0.43 (e.g., Bozhinskiy et al. 1986; Conway and Rasmussen, 2000) leads to lengths that range from 160 to 195vary 25 % extension from relative to L_{ssdf} (Table 2).

4.4 Comparison of model results with remote sensing derived datatrends observed from debris-covered glaciers

Our model results show that steady, high debris fluxes onto glaciers lead to glacier lengthening increased glacier lengths and high percentages of debris cover (Figs. 8 and 9). Remote-sensing derived measurements of u_{surf} and AAR provide provide general insight into valley glacier response to debris. We compare our model results to hypothetical results to the broad trends Scherler et al. , (2011b) 's inferred from their inventory of 287 debris-covered glacier surface velocitiesvelocity patterns, AARs, and debris cover percentagesfrom High Asia. While the Scherler dataset was collected from glaciers responding to persistent negative mass balance, the authors note that their inferences stand even 'when excluding stagnating glaciers.' This suggests that their observations represent 'general trends' relating debris to glacier response (e.g., increasing debris flux leads to reduced AARs and an up glacier shift of maximum glacier surface velocities).

Scherler et al. (2011b) ~~noted that~~ documented that higher debris cover percentage on glaciers correlates with steep above-glacier hillslopes. Because hillslope erosion rates and the percentage of exposed bedrock in the headwall increase with steeper slopes, it follows that increased debris input onto ~~the~~ a glacier should also increase both the glacier length and the percentage of the glacier covered with debris. Our ~~steady state~~ hypothetical model results confirm this inference and show ~~how changes in debris input variables can capture first-order trends from real debris-covered~~ that– independent of parameter selection (e.g., d_{loc}, h_*, bed slope)– higher debris flux leads to higher debris cover percentages on glaciers (Fig. 8 and 11).

Scherler et al. (2011b) showed that large debris cover percentages correspond with small AARs outside the typical range of 0.5–0.7 ~~seen on~~ from debris-free glaciers (e.g., Meier and Post, 1979). Our modeled steady state debris free glacier has an AAR of 0.5 (due to the piecewise-linear mass balance profile and constant width valley). In our model simulations, increases in ~~d_{flux}~~ debris flux lead to increases in both steady state glacier length, and ~~fractional debris cover~~ debris cover percentage independent of parameter selection (Fig. 11a). With a fixed ELA, the AAR must therefore decrease with an increased ~~d_{flux}~~ debris flux (Fig. 11a). Varying h_* (using the base parameter set~~with no changes in d_{flux} or $\dot{d}_{\text{loc}}$~~; Fig. 10) has a similar effect to varying ~~d_{flux}~~ debris flux (Fig. 11c and d). Changes in ~~$\dot{d}_{\text{loc}}$~~ the location of debris input lead to small changes in AAR but considerable changes in ~~fractional debris cover~~ debris cover percentage (Fig. 11a).

Scherler et al. (2011b) also showed that larger debris cover percentage correlated with ~~the ratio of average~~ lower ratios of average surface speed(u_{surf}) from the lower half of glaciers to the average u_{surf} from the upper half of glaciers. Increasing ~~d_{flux}~~ the debris flux in our model leads to lower u_{surf} in the lower half of glaciers relative to u_{surf} in the upper half of glaciers independent of parameter selection (Fig. 11b). Changing the location of debris input, ~~$\dot{d}_{\text{loc}}$~~d_{loc}, leads to small changes in the ratio of average u_{surf} but leads to large changes in the percentage of the glacier covered with debris. This highlights that debris flux and debris deposition location are important parameters for the specific response of a glacier to debris input.

~~While the simulations plot within the data from Scherler et al.~~ (In order to show the generality of inferences made by Scherler et al., 2011b~~), our steady state model results do not account for the full data spread (Fig. 11a). Our ssdf glacier has an AAR of 0.5. Adding debris to~~ , we also change the ~~model only reduces AARs. Simulations with initial ssdf glaciers with higher AARs could reproduce more of the data. The Scherler dataset was collected from glaciers responding to periods of negative mass balance.Reduced surface velocities under debris cover (not necessarily stagnant) – resulting from debris-covered glacier response to climate change – could account for the data with low debris cover percentages and low ratios of half length mean ice surface velocities (Fig. 11b).~~

bed slope in our hypothetical model. Changing the linear bed slope leads to similar relationships between debris cover %, AAR, and surface velocity~~to the simulations using the base bed slope~~. Notable differences occur primarily when the bed slope is reduced (Fig. 11c and d). With a reduced bed slope the initial debris-free steady state glacier is 3 times longer than the ~~ssdf~~ steady state debris free glacier. Even with the same hillslope debris fluxes as the simulations in Fig. 11a and b, the reduced bed slope leads to reduced asymmetry in the steady state debris-covered glacier surface velocities (Fig. 11d). With a linear mass balance profile and linear bed slope, changing the bed slope will have a similar effect to changing the mass balance gradient. The specific relationship of glacier response to debris is therefore also dependent on glacier size, bed slope, and the environmental mass balance gradient.

~~This model-data comparison shows thatviable changes in debris flux, debris deposition location, and h_* can cause changes in debris cover percentage, AAR, and glacier surface velocities that correspond with patterns observed~~ Ultimately, our exploration shows that, independent of parameter selection (e.g., not dependent on bed slope or mass balance profile selection), our model reproduces basic patterns inferred from real debris-covered glaciers~~. This~~ , which lends support to ~~the viability to~~ our model framework, while also providing quantitative, theoretical support to previous data-based ~~inferences~~observations.

5 Discussion

We explored the sensitivity of a new debris-covered glacier model to changes in various parameters and debris input related variables. We used a rigorous steady state glacier length definition to allow for the intercomparison of each simulation. Simulated glacier lengths are most sensitive to hillslope debris flux and the selection of the ~~characteristic debris thickness~~ debris thickness that characterizes the decline in melt rate beneath debris (Table 2). The location of debris deposition is important but plays a secondary role in setting glacier length. The time evolution of debris-covered glacier length is highly dependent on ~~$d^{\mathrm{snout}}_{\mathrm{flux}}$~~ $\dot{d}^{\mathrm{term}}_{\mathrm{flux}}$, although steady state glacier length is not (Appendix B; Fig. B1). Thick debris cover on glaciers from consistent debris input, independent of climate change, tends to (1) reverse and reduce mass balance gradients; (2) extend glaciers; (3) reduce AARs; and (4) reduce gradients of ice discharge, ice thickness, and surface velocity under debris cover. ~~Our model reproduces first-order~~ Independent of parameter selection, our simulations reproduce general relationships between debris cover percentages, AAR, and debris-perturbed surface velocity patterns from ~~High Asian~~ debris-covered glaciers.

5.1 The importance of debris flux and ~~h_*~~ characteristic debris thickness on steady state glacier length

Increases in hillslope debris flux (~~d_{flux}~~$\dot{d}_{\mathrm{flux}}$) lead to glacier extension (Figs. 8 and 9; Scherler et al., 2011b). But the rate and location of debris delivery to the surface ~~ought to~~ will vary widely due to local geologic and climatic settings. Our simulations show that ~~the flux of debris to the glacier surface, d_{flux},~~ debris flux is more important in determining the steady state debris-covered glacier length than $\dot{d}$, ~~$\dot{d}_{\mathrm{loc}}$, or $\dot{d}_{\mathrm{width}}$~~ d_{loc}, or d_{width} (Fig. 9~~). Debris delivery processes~~ ; Table 2). Processes of debris delivery to the glacier surface (e.g., deposition by avalanches, rockfall, the melt out of debris septa forming ice-stream interaction medial moraines, etc.) are first-order controls on the geometry of debris deposits on glaciers. Because ~~d_{flux}~~ debris flux trumps the importance of $\dot{d}$, ~~$\dot{d}_{\mathrm{loc}}$, and $\dot{d}_{\mathrm{width}}$~~ d_{loc}, and d_{width}, the specific debris delivery pathway ~~may be~~ is secondary to the debris flux in determining glacier length at least for this 2D case.

The effects of changing h_* are similar to the effects of varying the hillslope debris flux (Figs. 10 and 11). Establishing the importance of ~~d_{flux}~~ debris flux for individual glaciers requires that we constrain the variability of h_* from glacier to glacier: small changes in h_* can lead to large changes in steady state glacier length (Fig. 10). Simulations using an exponential debris thickness-melt curve (e.g., Konrad and Humphrey, 2000; Hagg et al., 2008) resulted in unrealistically long glaciers due to the rapid asymptote of melt towards zero ~~. The hyperbolic parameterization~~ (see Fig. 3). We argue that the hyperbolic parameterization (eqn. 3) is more physically defensible than the exponential~~parameterization if~~, as we assume that heat ~~is transferred through debris~~ transfer through debris is dominated by conduction.

Many paleoclimate estimates derived from glacial moraines neglect the potential effects of surface debris. Because debris ~~can have a strong effect on~~ strongly influences glacier length, independent of climate change, debris should be considered amongst temperature and precipitation as primary controls of paleoglacier lengths (e.g., Clark et al., 1994; Scherler, et al., 2011b). The effect of debris on paleoclimate estimates can be ~~mitigated~~ minimized by avoiding de-glaciated catchments with high-relief headwalls~~and~~, supraglacially sourced moraine sediments, or by using a debris-glacier-climate model to estimate the effect of debris on glacier extent.

5.2 The effect of steady debris input on patterns of Q, H and u_{surf}

In all debris-perturbed simulations, the mass balance gradient down-glacier from the location of initial debris emergence, ~~$\dot{\epsilon}_{x_{\text{int}}}$~~ $x_{\epsilon_{\text{int}}}$, reverses relative to the debris-free profile, decreases toward zero, and becomes more uniform (excluding the ~~toe cell~~terminal wedge; Fig. 5). This reversal results in a reduction of the surface mass balance b' relative to the ~~ssdf~~ steady-state debris free glacier (Fig. 6). Reducing b' toward zero reduces ice discharge gradients ~~. The glacier must extend in order to reach a steady state~~leading to glacier extension.

Thick debris reduces b' toward 0 and also makes b' more uniform (Fig. 5). This leads to ice discharge gradients that are reduced toward zero and become more uniform near the terminus (Fig. 5). Because $Q = H\overline{u}$, the surface velocity pattern follows a similar concave up pattern near the terminus where ice thicknesses are small and b' is close to zero (Fig. 6). Low ice thicknesses and thick debris near the terminus leads to low, nearly uniform surface velocities, independent of climate change (Fig. 6). While it is possible that debris cover can produce low velocity portions of glaciers independent of climate change, periods of negative mass balance can also lead to extensive portions of debris-covered glaciers with low surface velocities due to the largest increases in melt rates occurring near $\dot{\epsilon}_{x_{\text{int}}}$ $x_{\epsilon_{\text{int}}}$ (e.g., Kirkbride et al., 1993).

The ice discharge at $\dot{\epsilon}_{x_{\text{int}}}$ the point of debris emergence, $x_{\epsilon_{\text{int}}}$, controls the steady state glacier length and the down glacier patterns of ice discharge, ice thickness and u_{surf} surface velocity. In steady state, ice discharge at $\dot{\epsilon}_{x_{\text{int}}}$ $x_{\epsilon_{\text{int}}}$ represents the volume of ice per unit time that must be ablated between $\dot{\epsilon}_{x_{\text{int}}}$ $x_{\epsilon_{\text{int}}}$ and the terminus. Holding other debris related variables constant, if debris emerges where ice discharge is large high, the glacier will extend further because more glacier surface under thick debris (where melt rates are low and more uniform) is needed to ablate for ablation and match the large ice discharge at $\dot{\epsilon}_{x_{\text{int}}}$ $x_{\epsilon_{\text{int}}}$. If debris emerges where ice discharge is small the glacier does not extend as far because less area is needed under debris to match ice discharge at $\dot{\epsilon}_{x_{\text{int}}}$ $x_{\epsilon_{\text{int}}}$ (Fig. 6). The location of debris deposition/emergence relative to the ELA is therefore an important variable in the debris-glacier system, as it controls the relationship between debris cover percentage, AAR, and the pattern of surface velocities (Fig. 11).

The specific terminal pattern of ice discharge and thickness is controlled by the rate of debris removal from the toe. If $d^{\text{snout}}_{\text{flux}}$ terminal wedge (Appendix A and B; Fig. A1 and B1). If $\dot{d}^{\text{term}}_{\text{flux}}$ is high an ice cliff may persist at the toe leading to high melt rates and the pre-mature termination of a glacier when compared to a glacier with a low $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$. If the magnitude of $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ is low then the toe maybe may be drowned in debris, and the glacier may never reach steady state even with a steady climate. The glacier would continue to accumulate debris and slowly advance down valley with a slightly positive net mass balance (e.g., Konrad and Humphrey, 2000). It may be useful to consider if individual debris-covered glaciers are accumulating debris mass through time, losing debris mass through time, or potentially in steady quasi-steady state with regard to debris (Fig. 4).

The response time of the modeled glaciers is therefore dependent on the parameterization of $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ (Appendix B). A glacier with rapid debris removal at the toe margin will tend to reach a steady state much faster than a glacier with slow debris removal from the toe margin (Appendix B). Documenting the rates of debris removal at the toe is margin is therefore vital for modeling and understanding individual debris-covered glacier response.

In our steady state simulations, the ice thickness is increased up-glacier from the point of debris emergence , $\dot{\epsilon}_{x_{\text{int}}}$ (Fig. 6). The thickness perturbations caused by emerging debris are diffused up glacier, leading to lower ice surface slopes and greater ice thicknesses than on debris-free glaciers of comparable sizesforced by the same climate. The emergence of debris on a glacier can therefore perturb ice thickness both up and down glacier from $\dot{\epsilon}_{x_{\text{int}}}$the point of debris emergence. Debris cover decreases the surface mass balance and therefore also reduces the vertical component of englacial velocity; this leads to flow paths that are increasingly parallel to the surface (Konrad and Humphrey, 2000). Reducing ice melt ablation rates results in lower debris emergence rates, leading to the further advection of debris down-glacier and expansion of the zone of debris emergence (Fig. 5a). Debris emergence zones on real glaciers will therefore tend to be wider than debris deposition zones.

6 Future workPotential model improvements and future research

While we have explored first-order connections between glacier dynamics and debris deposition, additional components require investigation. Modeling the response of debris-covered glaciers to climate is the most pressing (e.g., Naito et al., 2000; Banerjee and Shankar, 2013; Rowan et al., 2015). The steady state results presented here can serve as initial conditions for future simulations exploring the response of debris-covered glaciers to climate change. Future efforts should ~~also~~ further explore the importance of glacier size, environmental mass balance gradient, and ~~bed slope~~ valley bedrock profile as they modulate the effect of debris on glacier response.

We assumed a steady debris input for simplicity. In reality, hillslope erosion in high-relief settings occurs through thresholded, mass wasting processes. The effect of temporal and spatial changes in debris deposition must be addressed through both empirical and theoretical approaches. Isolated, large landslides have been shown to suppress melt rates, change glacier surface slopes and perturb glacier surface velocity fields (Gardner and Hewitt, 1990; Reznichenko et al., 2011; Shugar et al., 2012). If debris inputs are allowed to vary in space and time, a complex glacier length history will likely result even with a steady climate. The specifics of that history will depend strongly on the frequency and magnitude of mass wasting events and to a lesser degree the ice discharge at the point of debris emergence.

Our modeling did not account for the planview dimension of glaciers. Debris advected into the glacier between tributaries emerges to form ice-stream interaction medial moraines. While the spatial widening of such moraines has been addressed (Anderson, 2000), the merging of these medial moraines results in debris thickening that we do not account for. Our present work lays the framework for such a 2-D planview model.

Ice cliffs and surface ponds are neglected in this study for simplicity but should be included in numerical models of glacier response to debris and climate change (e.g., Benn et al., 2012). Planview modeling of debris-covered glacier response is also needed (e.g., Menounos, et al., 2013; Rowan et al., 2015). The melt-enhancing effects of thin debris covers should be included in future modeling efforts. Environmental mass balance profiles and snow lines are not steady from year-to-year. The response of debris-covered glaciers to interannual climate variability must also be explored (Roe and O'Neal, 2009; Anderson et al., 2014). Debris covers and glacier lengths will fluctuate in response to this variability ~~because of~~ due to the feedbacks between the debris emergence, ice dynamics, and climate.

Debris advection through and on a glacier can take hundreds of years, leading to memory in the system (i.e., the glacier responds to debris input from hundreds of years ago). The response of individual debris-covered glaciers to climate change is therefore dependent on the distribution of debris on and in the glacier when the climate change occurs. Further constraint of englacial and surface debris is needed to understand predict the decadal to centennial response of present debris-covered glaciers to climate change.

7 Conclusions

Before modeling the response of debris-covered glaciers to a warming climate, it It is necessary to constrain how debris effects glaciers – independent of the effect of debris on glaciers so we can better predict the response of debris-covered glaciers to climate change. We provide a new framework to explore debris-covered glacier evolution and explore valley glacier sensitivity to debris input. Our simulations show that:

– For reasonable debris deposition fluxes, debris input can lead to glaciers that are many tens of percent longer than debris-free glaciers forced by the same climate but unperturbed by debris.

– Thick debris cover tends to reduce gradients of ice discharge, ice thickness, and surfaces velocities, independent of climate change.

– Debris-covered glacier length is highly sensitive to debris flux to the glacier surface. High surface debris fluxes can greatly increase glacier lengths relative to glaciers responding to the same climate without debris. Increases in debris flux lead to smaller AARs and larger debris covered fractions. Changes in the debris deposition zone width or the debris deposition rate are secondary to the total surface debris flux in governing the glacier geometry. This model provides a framework to quantify the effect of debris input on glacier length, and can therefore be used to estimate the effect of debris input on paleoclimate estimates derived from glacier models.

– The site of supraglacial debris deposition relative to the ELA modulates glacier response to debris. Steady debris input where ice discharge is high (near the ELA) leads to longer glaciers with greater fractional debris cover, whereas the same steady debris input where ice discharge is low (near the headwall or terminus) leads to shorter glaciers with smaller fractional debris cover.

- The importance of the mechanism of debris deposition onto glaciers (e.g., delivery by avalanching or by melt out of debris septa) is likely secondary to the importance of the total surface debris flux.

- Debris-covered glacier length is highly sensitive to the relationship between surface debris thickness and sub-debris melt. Our simulations support the use of capped hyperbolic debris thickness-melt curve fits(Eq. 3) instead of exponential fits.

- The rate and process of debris removal from the terminus exerts strong control on the time evolution of debris-covered glaciers, but only weakly controls influences the eventual steady-state length.

- Debris cover can perturb ice thicknesses and glacier surface slopes up-glacier from the debris-covered portion of the glacier. Thick debris cover can expand the zone of debris emergence. Debris deposition emergence zones will therefore be more narrow longer than zones of debris emergencedeposition.

Glacier response to debris cover is most sensitive to surface debris flux and the debris thickness-melt relationship. Our ability to predict the response of debris-covered glaciers to climate change, and to extract paleoclimate estimates from moraines in high-relief settings, is therefore highly dependent on our constraint of surface debris fluxes and debris thickness-melt relationship in the future and the past.

Appendix A

After the step change increase in debris deposition occurs, the steady-state debris free glacier evolves towards a debris-covered steady state. During this transition debris on the glacier surface is advected from cells with debris cover into debris-free cells. In our model, the debris thickness $h_{\text{debris}}(x,t)$ represents a layer of equal thickness on any cell. Debris thickens slower with a larger dx because the debris volume advected into a cell is spread over a larger area (due to the larger dx; dy=1; dy [=] m). There is therefore a timescale built into the thickening of debris in a cell that is dependent on dx. Increasing Because ablation rates are sensitive to debris-cover thickness, changing dx has an effect on glacier evolution. In order to test the effect of changing dx on the steady state debris-covered glacier length we increased dx from 100 (used in all simulations outside of this test) to 200 mleads . This test led to differences in steady state debris-covered glacier length that are which were less than 200 m even when debris flux is varied=. Because melt (Fig. 3) is highly sensitive to debris thickness, a newly formed glacier cell at the toe can be $\dot{d}_{\text{flux}}$ was varied. The dx dependence does not effect the conclusions we draw from this study.

Without a terminus wedge parameterization, simulated glaciers advancing toward steady state become trapped in false steady states. Without a terminus wedge parameterization a new glacier cell is exposed to melt rates un-perturbed by debris. As a result, the simulated glacier can be result, simulated glaciers become trapped in a steady length, although large amounts even though large volumes of ice are melted without the protection of debris. To correct this, we implement a triangular terminus parameterization (after terminal wedge parameterization for the last two grid points (the last ice-covered and the first ice-free grid point; Fig. A1; see Budd and Jenssen, 1975; Waddington, 1981) of the glacier which allows debris to cover the glacier terminus even when advancing or retreating. The volume and length of the terminal triangle wedge is based on ice mass conservation. The volume of the terminal wedge at time $t+\mathrm{d}t$ is the sum of the old snout terminus volume, the ablated volume under debris, and the volumetric flow past the last grid point. Equation (16) and dx_{term}, the surface length of the wedge, define the debris thickness on the terminal wedge. $\dot{d}^{\text{term}}_{\text{flux}}$ removes debris from the total volume of debris on the terminal debris wedge. A single environmental melt rate is calculated based on the mean elevation of the toe, and terminal wedge, and sub-debris ablation is calculated perpendicular to the surface of the triangle. Equation (16) and the surface length of the wedgedefine the debris thickness on the snoutwedge. When the snout terminal wedge length is greater than 2dx, the glacier advances wedge parameterization moves to the next cell down valley. If the snout terminal wedge is shorter than dx the glacier terminal wedge parameterization retreats one cell. Because the terminus parameterization allows the glacier to change length at the sub-dx scale, simulated glaciers avoid numerical traps and advance to true steady states. In this model, steady state occurs when $d_{\text{flux}} = d^{\text{snout}}_{\text{flux}}$ $\dot{d}_{\text{flux}} = \dot{d}^{\text{term}}_{\text{flux}}$ and the glacier length is steady.

Appendix B

Debris deposited on the glacier surface is removed from the glacier by ice cliff retreat or wasting down the terminal glacier slope. Unfortunately, the rates and processes of debris removal from glacier toes are poorly documented. We therefore explore parameterizations for the debris removal flux from the glacier ($d^{\text{snout}}_{\text{flux}}$$\dot{d}^{\text{term}}_{\text{flux}}$) and their effect on glacier length (using the base parameter set where d_{flux} $\dot{d}_{\text{flux}}$ = 3.2 m^3 m^{-1} yr^{-1}). Each simulation starts with the ssdf glacier followed by a step change increase in $d_{\text{flux}}$$\dot{d}_{\text{flux}}$. We consider $d^{\text{snout}}_{\text{flux}} = c$, $d^{\text{snout}}_{\text{flux}} = ch_{\text{debris}}$, and $d^{\text{snout}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ $\dot{d}^{\text{term}}_{\text{flux}} = c$, $\dot{d}^{\text{term}}_{\text{flux}} = ch_{\text{debris}}$, and $\dot{d}^{\text{term}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ where c is a constant that ranges between 0.1 and 10 and with variable units such that $d^{\text{snout}}_{\text{flux}}$ [=] m^3 m^{-1} yr^{-1} $\dot{d}^{\text{term}}_{\text{flux}}$ [=] m^3 m^{-1} yr^{-1}. Independent of the parameterization, $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ controls both the time needed to reach steady state as well as whether a simulated glacier can reach steady state (Fig. B1).

Large changes in $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ lead to minor changes in glacier length even after 5000 years, implying that the choice of the $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ parameterization would have a minor effect on the length results presented (Fig. B1). All three parameterizations lead to the same steady state length for low c values (190 % of L_{ssdf}).

If $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ cannot evolve to a state where $d^{\text{snout}}_{\text{flux}} = d_{\text{flux}}$$\dot{d}^{\text{term}}_{\text{flux}} = \dot{d}_{\text{flux}}$, surface debris thickens unrealistically and the glacier never reaches steady state. For $d^{\text{snout}}_{\text{flux}} = c$ $\dot{d}^{\text{term}}_{\text{flux}} = c$ the glacier will never reach steady state if c is less than 3.2 m^3 m^{-1} yr^{-1}. For $d^{\text{snout}}_{\text{flux}} = ch_{\text{debris}}$, and $d^{\text{snout}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ $\dot{d}^{\text{term}}_{\text{flux}} = ch_{\text{debris}}$, and $\dot{d}^{\text{term}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ the value of

$d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ changes through each simulation based on the debris thickness on the toe and the local debris-free melt rate. The $d^{\text{snout}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ $\dot{d}^{\text{term}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ parameter shows a wider length variation than the $d^{\text{snout}}_{\text{flux}} = ch_{\text{debris}}$ parameterization because $d^{\text{snout}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ $\dot{d}^{\text{term}}_{\text{flux}} = ch_{\text{debris}}$ parameterization because $\dot{d}^{\text{term}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ results in a wider range of $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ values due to the $\dot{b}_z$ term. To insure that steady state can be achieved in each simulation, we include the melt rate term in the $d^{\text{snout}}_{\text{flux}}$ $\dot{d}^{\text{term}}_{\text{flux}}$ parameterization (Fig. B1) that codifies an assumption that debris removal processes at the toe are in some fashion dependent on local air temperature and hence melt rates. We use $d^{\text{snout}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ $\dot{d}^{\text{term}}_{\text{flux}} = c\dot{b}_z h_{\text{debris}}$ for all simulations outside of this Appendix (with $c = 1$).

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Table 1. ~~Parameters~~ Parameter definitions and values.

Parameter	Name	Min	Base	Max	Units
ELA	Equilibrium-line altitude		5000		m
$\frac{\mathrm{d}\dot{b}_z}{\mathrm{d}z}$	Surface mass balance gradient		0.0075		yr^{-1}
b_{cap}	Maximum accumulation		2		$\mathrm{m\,yr}^{-1}$
Z_{max}	Maximum bed elevation		5200		m
α	Bed slope	4 %	8 %	20 %	
$\mathrm{d}t$	Time step		0.01		yr
$\mathrm{d}x$	Downvalley spatial discretization		100	200	m
$\mathrm{d}y$	Valley perpendicular spatial discretization		1		m
g	Gravity		9.81		ms^{-2}
n	Glen's constant		3		
A	Flow law parameter		2.4×10^{-24}		$\mathrm{Pa}^{-3}\,\mathrm{yr}^{-1}$
f	Shapefactor		0.75		
U_{c}	Critical sliding speed		5		$\mathrm{m\,yr}^{-1}$
τ_{c}	Reference basal shear stress		10^5		Pa
ρ_{ice}	Ice density		917		$\mathrm{kg\,m}^{-3}$
m_z	# of cells per ice column		20		
ρ_{rock}	Debris density		2650		$\mathrm{kg\,m}^{-3}$
h_*	Characteristic debris thickness	0.025	0.065	0.165	m
ϕ	Surface debris porosity	0.18	0.3	0.43	
$\dot{d}$	Debris deposition rate	1	8	8	$\mathrm{mm\,yr}^{-1}$
d_{loc}	Debris deposition location	7 %	42 %	98 %	
d_{width}	Debris deposit width	100	400	1600	m
$\dot{d}_{\mathrm{flux}}$	Debris flux onto the glacier	0.1	3.2	6.4	$\mathrm{m}^3\,\mathrm{m}^{-1}\,\mathrm{yr}^{-1}$
$\dot{d}_{\mathrm{flux}}^{\mathrm{term}}$	Debris flux off the glacier				$\mathrm{m}^3\,\mathrm{m}^{-1}\,\mathrm{yr}^{-1}$
L_{ssdf}	Steady state debris-free glacier length		8700		m

Table 2. Sensitivity of steady state glacier length to changes in debris-related parameters.

Parameter	Name	Max. % length change relative to L_{ssdf}
h_*	Characteristic debris thickness	110%
$\dot{d}_{\text{flux}}$	Debris flux onto the glacier	80%
d_{loc}	Debris deposition location	40%
ϕ	Surface debris porosity	25%
$\dot{d}_{\text{flux}}^{\text{term}}$	Debris flux off the glacier	25%*
$\dot{d}$ vs. d_{width}	Debris deposit location vs. width	4%

*results from the $\dot{d}_{\text{flux}}^{\text{term}} = c\dot{b}h_{\text{debris}}$ parameterization.

Figure_1_revisions-01.png

Figure 1. (a) Schematic of the debris-glacier system. Debris deposited on or emerging in the ablation zone reduces ~~melt~~ ablation rates (above the critical debris thickness) leading to the reduction in gradients of ice discharge and the lengthening of glaciers. **(b)** Schematic of the coupled debris-glacier model. Debris deposited on the glacier is either advected through the glacier and/or advected down the glacier surface. Englacial debris is advected using 2-D rectangular grid and coordinate transform. Ice physics and supraglacial debris advection is treated on a 1-D grid.

Figure_2_revisions-01.png

Figure 2. Flow chart of the elements connected in this debris-glacier model. Solid arrows represent the feedbacks we explore. Dashed arrows are neglected.

Figure_3_revisions-01.png

Figure 3. Compilation of curve fits to data from 15 melt rate vs. debris thickness studies (Østrem, 1959; Loomis, 1970; Khan, 1989; Mattson, et al., 1993; Lundstrom, 1993; Kayastha, et al., 2000; Lukas et al., 2005; Mihalcea, et al., 2006; Nicolson and Benn, 2006; Hagg, et al., 2008; Reid and Brock, 2010; Wang, 2011; Fyffe, 2012; Brook, et al., 2013; Anderson, 2014) (mean h_* is 0.066 ± 0.029 m (1σ), and ranges from 0.03 to 0.13 m). These curve fits are used to determine the parameter ranges in Table 1 for h_*. The best exponential fit is the mean of all the exponential curve fits; using sub-debris melt $= ae^{\frac{-h_{\text{debris}}}{b}}$ $a = 5.89$ cm day^{-1}, $b = 12.27$ cm.

Figure_4-01.png

Figure 4. Debris mass vs. time. The englacial debris mass reaches steady state rapidly because debris is deposited near the ELA and englacial advection paths are short. As debris emerges in the ablation zone M_{surface} increases nearly at the rate of debris input to the glacier. As the glacier nears a steady length the debris mass transferred to the glacier foreland increases. The glacier reaches steady state when ~~$d_{\text{flux}} = d^{\text{snout}}_{\text{flux}}$~~ $\dot{d}_{\text{flux}} = \dot{d}^{\text{term}}_{\text{flux}}$ and the glacier length is steady (see Appendix A).

Figure_5_revisions-01.png

Figure 5. Modeled glacier changes due to changes in $\dot{d}_{\mathrm{loc}}$ debris deposition location with $\dot{d}_{*}$ debris flux held constant. Englacial debris concentrations **(a–c)** and mass balance profiles **(d–f)** for three steady state debris-covered glacier simulations. $\dot{d}_{\mathrm{flux}} = 3.2\,\mathrm{m^3\,m^{-1}\,yr^{-1}}$ $\dot{d}_{\mathrm{flux}} = 3.2\,\mathrm{m^3\,m^{-1}\,yr^{-1}}$ for each panel. **(a)** $\dot{d}_{\mathrm{loc}}$ d_{loc} is 7 % of the steady state debris free glacier length (L_{ssdf}) from the head of the glacier. **(b)** $\dot{d}_{\mathrm{loc}}$ d_{loc} is 42 % to L_{ssdf}. **(c)** $\dot{d}_{\mathrm{loc}} = 98$ $d_{\mathrm{loc}} = 98$ % to L_{ssdf}. The increase in melt rate near the toe is related to the thinning of debris due to the $d^{\mathrm{snout}}_{\mathrm{flux}}$ $d^{\mathrm{term}}_{\mathrm{flux}}$ parameterization. $\dot{\epsilon}_{x_{\mathrm{int}}}$ $\mathrm{x}_{\epsilon_{\mathrm{int}}}$ is the point of initial debris emergence and $\dot{\epsilon}_{\mathrm{zone}}$ ϵ_{zone} is the length of the glacier over which englacial debris emerges.

Figure_6_revisions-01.png

Figure 6. Modeled ~~glacier~~ changes in ice fluxes, thicknesses and velocities due to ~~changes in~~ ~~$\dot{d}_{\mathrm{loc}}$~~debris deposition location. ~~$d_{\mathrm{flux}} = 3.2$~~$\dot{d}_{\mathrm{flux}} = 3.2$ m^3 m^{-1} yr^{-1} for each panel and other parameters excluding ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} are from the base set. **(a–c)** Comparison of h_{debris} and Q for the debris covered and debris free cases shown in Fig. 6. **(d–f)** Comparison of surface velocities and ice thicknesses for the debris covered and debris-free cases. **(a)** ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} is 7 % from the headwall to the steady state debris free glacier length (L_{ssdf}). **(b)** ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} is 42 % from the headwall to L_{ssdf}. **(c)** ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} is 98 % from the headwall to L_{ssdf}. **(d)** ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} is 7 % from the headwall to L_{ssdf}. **(e)** ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} is 42 % from the headwall to L_{ssdf}. **(f)** ~~$\dot{d}_{\mathrm{loc}}$~~ d_{loc} is 98 % from the headwall to L_{ssdf}.

Figure_7_revisions-01.png

Figure 7. Glacier length variations with changes in ~~d_{flux}~~ debris flux($\dot{d}_{\text{flux}}$) and ~~$\dot{d}_{\text{loc}}$~~debris deposition location(d_{loc}). Modeled glacier length is normalized by the steady state debris free glacier length (L_{ssdf}). Each string of connected markers represents simulations with the same debris flux ($\dot{d}_{\text{flux}}$). Changes in $\dot{d}_{\text{flux}}$ are accomplished by changing $\dot{d}$ with d_{width} held constant. The red markers indicate the ssdf glacier length. **(a)** Normalized glacier length relative to ~~$\dot{d}_{\text{loc}}$~~d_{loc}. **(b)** Normalized glacier length relative to $Q_{\text{free}}/Q_{\text{max}}$ at the point of debris emergence/deposition.

Fig_8_revised_again-01.png

Figure 8. Debris related results from 36 simulations varying ~~$\dot{d}_{\text{loc}}$~~ d_{loc} and ~~d_{flux}~~ $\dot{d}_{\text{flux}}$. All black circles are derived from steady state debris-covered glaciers. Red ~~circles show~~ circle shows results from the debris-free glacier. ~~**(a)** Dependence of debris cover percentage on d_{flux} and $\dot{d}_{\text{loc}}$.~~ Dashed lines connect simulations with the same ~~$\dot{d}_{\text{loc}}$. **(b)** Dependence of max(h_{debris}) on d_{flux}~~ d_{loc}.

Figure_9_new_rsa-01.png

Figure 9. Steady Dependence of steady state glacier length changes due to variations in on debris delivery to the glacier. Glacier lengths are normalized by the steady state debris free glacier length, L_{ssdf}. The bold Bold lines **(a, c–e)** connect results with the same parameters: $\dot{d}_{\mathrm{loc}}$ and $\dot{d}_{\mathrm{width}}$ are location of debris input, with d_{loc} fixed at 42 %and 400. **(a)** Steady state glacier length lengths from 36 simulations in which $\dot{d}$ debris flux, $\dot{d}_{\mathrm{flux}}$ and $\dot{d}_{\mathrm{loc}}$ d_{loc} are variedwith $\dot{d}_{\mathrm{width}}$, and d_{width} is fixed at 400 m. The multiple-dashed lines show the effect Vertical columns of changing $\dot{d}_{\mathrm{loc}}$points represent simulations in which debris location is varied and debris flux is held constant. The same results are presented in Fig. 7. **(b)** Steady state glacier length lengths from simulations where in which $\dot{d}_{\mathrm{width}}$ and $\dot{d}_{\mathrm{loc}}$ $\dot{d}$ are varied with $\dot{d}$ fixed at 8. **(c)** Length changes with $\dot{d}_{\mathrm{loc}}$ while d_{loc} remains fixed at 42 %while $\dot{d}_{\mathrm{width}}$ is varied. **(d)** Length changes The diameter of circle represents width of the debris deposition zone, its center representing the steady state glacier length. Clusters of circles are simulations with $\dot{d}_{\mathrm{loc}}$ fixed at 42while $\dot{d}$ is varied and $\dot{d}_{\mathrm{width}}$ is constantthe same debris flux. **(e)** **(c)** Steady state glacier length from 72 all simulations in **(a and b)** in which, $\dot{d}_{\mathrm{width}}$$d_{\mathrm{width}}$, $\dot{d}$, and $\dot{d}_{\mathrm{loc}}$ d_{loc} are varied. The maximum effect of varying $\dot{d}_{\mathrm{flux}}$ on steady state glacier length is 80 %.

Figure_10_revised-01.png

Figure 10. Sensitivity of steady state debris-covered glacier length to choices of characteristic debris thickness (h_*) and surface debris porosity (ϕ). The lines intersect at the base parameter set. Parameter ranges are extreme to highlight the possible range of effects of each parameter.

Figure_11_revisions-01.png

Figure 11. Comparison of our hypothetical steady state ~~debris-covered~~ debris-cover model output with data from 287 glaciers ~~in High Asia~~ showing broad patterns between debris and basic glacier properties (Scherler et al., 2011b). **(a)** The AAR compared to debris cover percentage, ~~d_{flux}~~ debris flux ($\dot{d}_{\text{flux}}$), and ~~$\dot{d}_{\text{loc}}$~~ debris deposition location (d_{loc}). **(b)** The ratio of the average surface speed of the lower 50 % of the glacier and the average surface speed of the upper 50 % of the glacier vs. debris cover percentage, ~~d_{flux}~~ $\dot{d}_{\text{flux}}$, and ~~$\dot{d}_{\text{loc}}$~~ d_{loc}. **(c, d)** Same data as **(a, b)**, but exploring the effect of changing the bed slope and h_*. The quadrangles show the area occupied by simulation results using the same ~~variables and~~ parameters from **(a, b)** but with lower and higher bed slopes. h_* results are from the parameter test where h_* is varied ~~, $\dot{d}_{\text{loc}}$ is 42and d_{flux} is 3.2~~(Fig. 10).

Fig_A1-01.png

Figure 12. Exploring various choices for the $d_{\text{flux}}^{\text{snout}}$ The terminal wedge parameterization . Glacier lengths are normalized by L_{ssdf}. Irrespective of and debris removal from the choice of model. Q_{in} is the $d_{\text{flux}}^{\text{snout}}$ parameterization ice discharge into the steady glacier length is nearly doubledterminal wedge. Circles represent simulations in which M_{surface} ($\dot{d}_{\text{flux}}^{\text{term}}$ is removed from the total volume of surface debris mass on the glacier) and glacier length did not reach steady state after 5000. For all simulations d_{flux} 3.2. All simulations presented outside of this plot use the $d_{\text{flux}}^{\text{snout}} = c\dot{b}h_{\text{debris}}$ parameterization with $c = 1$ (* in the figure)terminal wedge.

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Figure_B1_revisions-01.png

Figure 13. Exploring various choices for the $\dot{d}_{\text{flux}}^{\text{term}}$ parameterization. Glacier lengths are normalized by the steady state debris free glacier length (L_{ssdf}). Irrespective of the choice of the $\dot{d}_{\text{flux}}^{\text{term}}$ parameterization the steady glacier length is nearly doubled. For all simulations $\dot{d}_{\text{flux}}$ 3.2 $m^3\, m^{-1}\, yr^{-1}$. The glacier will never reach steady state for choices where $\dot{d}_{\text{flux}}^{\text{term}}$ cannot evolve to equal $\dot{d}_{\text{flux}}$. This occurs when $\dot{d}_{\text{flux}}^{\text{term}} = c$ and c is less than 3.2 $m^3\, m^{-1}\, yr^{-1}$. Circles represent simulations in which M_{surface} (the total debris mass on the glacier) and glacier length did not reach steady state after 5000 years. The time labels show how long it took for the glacier to reach steady state for the cases when $\dot{d}_{\text{flux}}^{\text{term}} = c\dot{b}h_{\text{debris}}$. All simulations presented outside of this plot use the $\dot{d}_{\text{flux}}^{\text{term}} = c\dot{b}h_{\text{debris}}$ parameterization with $c = 1$ (* in this figure).