

We thank the two referees very much for their constructive comments, and discuss them below. The referees' comments are shown in blue, and our responses in black. The main changes we performed are:

- We added an additional experiment in the manuscript to evaluate the ability of the Level-Set Method to conserve volume, and
- we significantly rewrote many sections of the manuscript, to improve clarity. Our conclusions remain unchanged.

1 Referee 1 (Guillaume Jovet):

1.1 General comments

1.1.1

RC: This study introduces a Level-Set Method (LSM) in order to follow dynamically the migration of the calving front in the Ice Sheet System Model (ISSM). Several experiments are led on the Jakobshavn Isbrae ice stream. In particular, the authors study its geometrical response after applying different perturbations to a given calving rate, which itself accounts for seasonality. The results show that the model is capable to reproduce the subsequent change (retreat, thinning, steepening, acceleration) of the ice stream under enhanced calving rates consistently to physical observations. Moreover, the model reproduces well how the whole system stabilizes after releasing the perturbation, showing a reversible calving front. This is an interesting paper which presents original results. LSM are well-designed methods to track complex geometries with possible changes in topologies. This makes it fully relevant for calving fronts. Overall, I found convincing that ice flow models should indeed incorporate such (or similar) ad-hoc tracking methods of the calving front for projections of future contributions to SLR. However, I think the paper can still be improved before to be published.

1.1.2

RC: I have two main concerns, plus lists of specific and technical comments, which I hope will help the authors to improve this article. My two main concerns are:

LSM can greatly deal with complex interfaces including changes of topology. However, LSM also have counterparts, namely, i) they are usually not mass conserving unless solved by finite volume (this is not the case here) ii) gradients of φ near the interface $\varphi = 0$ tend to flatten after few iterations (if nothing is done) so that the interface gets less and less accurate with time. (I did experiment both problems in a previous work). A classical trick to deal with the last issue is to regularly regularize φ by solving another Hamilton-Jacobi PDE, which admits the signed distance to the interface $\varphi = 0$ as a solution. Indeed, re-initializing with the signed distance function ensures to have safe gradients equal or close to 1. It would be worth to further discuss numerical issues when solving the KCFC, and in particular, to include in the

test-setup (Appendix) the two following checks: i) how the numerical volume of the moving disk change over time, and if it is controllable by the mesh size (as this is the case for the advection velocity) ii) how the gradient of the LS ϕ near the interface $\phi = 0$ behave after few iterations.

AC: i) The Level-Set Method (LSM) as implemented here is not mass conserving. We mention this in the introduction now, and have added a simple test in the appendix, which shows that the method causes between 0.2% and 1% volume loss over 100 years, depending on mesh element size.

ii) From our experience there is no need for reinitialization of the LSF with the kind of experiments presented here. Calving rates are scaled such that over the time span of one year the front velocity adds up to zero (see experiment description). Only strong calving rate perturbations are able to alter this, but even the strongest perturbations (experiment C8) leave the gradient of the LSF in the vicinity of the calving front in the order of one to ten, see plot below. We see, however, that reinitialization of the LSF will become necessary for longer-term runs or discontinuous calving front speeds.

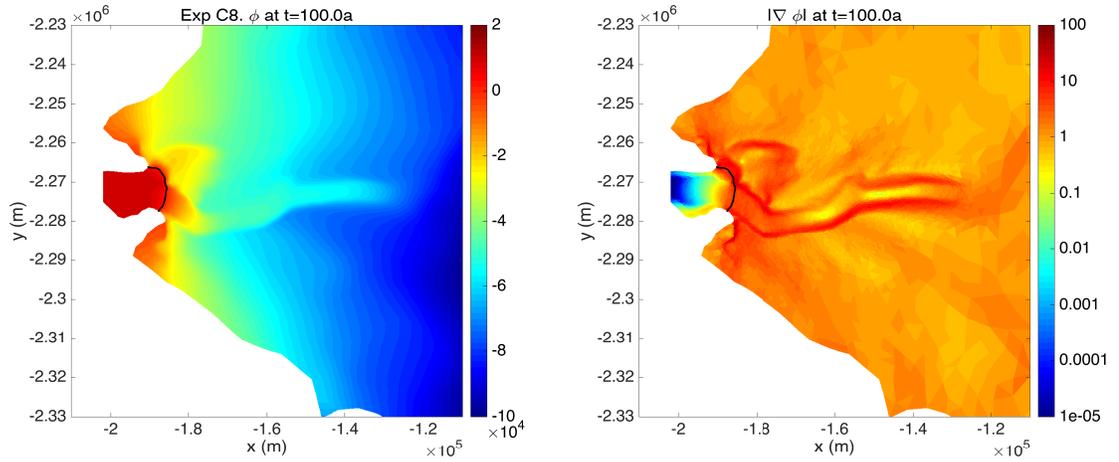


Figure 1: LSF (left) and a log-plot of the norm of its gradient (right) after 100 years of experiment C8. The black line denotes the calving front position.

iii) We listed the numerical issues solving the KCFC in section 2.3 of the submitted manuscript.

1.1.3

RC: The authors describe well the experiments setup, and also introduce a measure P for the time integration of the applied perturbation. Although the discussion part explains well all ice flow mechanisms and how they feedback each other, the direct outcomes of Experiments A, B and C in term of impact of the perturbation w.r.t. p_0 and Δt are little discussed. In addition, we expect that the measure P you introduced (arbitrarily) takes its sense after being corroborated with the results, but this come later on in a single sentence and without clear evidence. I therefore recommend the authors to better emphasize all the outcomes of Experiments A, B and C (in particular the consequences of changes in parameters p_0 and Δt) in the discussion (and in the conclusion as well).

AC: We see that the introduction of P is not justified, and replaced it by $\Delta t(1 - p_0)$. We

have put more emphasis on p_0 and Δt in the discussion and results presented in figures 7 and 8.

1.2 Specific comments

1.2.1

RC: abstract The abstract is not very efficient. In particular, the 5 first sentences should be made further concise so that the “Here, we present ...” comes earlier.

AC: Done.

1.2.2

RC: 1.3-4 p5488 “It is well suited ... partial differential equations”. I don’t see the point. PDEs are by nature challenging to implement in parallel since partial derivatives couple nodes by contrast to systems of ODEs (for instance).

AC: What we intended to say was that the solution of the KCFC does not require new numerical tools, but can be achieved using already present (parallel) solver routines. We corrected the sentence in the introduction.

1.2.3

RC: 1.5-10 p.5491 Quantities should be more rigorously introduced. E.g. the time interval $[0, \infty)$ comes before the time variable t is introduced, “then” in “if ... , then x belongs to ...” are in fact are all “ \Leftrightarrow ”. In addition, i is defined as an abstract domain, and one has to wait 3 more pages before it is said that i corresponds to the ice domain.

AC: Done.

1.2.4

RC: 1.10 p.5492 This is an interesting point to use the LSM horizontally while keeping vertically the traditional ice thickness. You should motivate your choice even if this is easy to guess for ice modellers. From a general LSM perspective, this is not obvious.

AC: We have included a motivation for this approach in section 2.2.

1.2.5

RC: eq. (9) should be motivated, or at least say that by requesting $n \cdot \nabla S = 0$, we want to be sure that S keeps constant at the interface (or the calving front) when following the normal unit, which points outside the ice domain. In what is it important that the thickness and the velocity keep constant in the neighbourhood of the calving front?

AC: Done, see also sections 2.2 and 2.3 of the manuscript, where we have included explanatory sentences.

1.2.6

RC: 1.10-11 p.5494 “The strongly . . . updates”, any evidence to support this statement? “High viscosity of ice makes that computing the BC exactly at the front or at one mesh size distance does makes a big difference”, is that what you mean? This doesn’t look obvious to me.

AC: We found the sentence to be not correct, and have therefore removed it.

1.2.7

RC: 1.28 p.5498 Last sentence of Section 3: This measure P should come later. It makes no sense to introduce it in Section 3.

AC: As mentioned in point 1.1.3 we exclude P from the manuscript.

1.2.8

RC: 18-14 p-5499 several times, “local high” or “topographic high” should be “local maximum”?

AC: We defined the term “local high“ in the results section, to address readers with either mathematical or geoscientific background.

1.2.9

RC: At that point, I would briefly recall for unaware readers that grounding line on retrograde slopes are usually unstable and briefly explain why (+ references).

AC: Done, see discussion.

1.2.10

RC: 113-19 p.5500 You mostly comment the time derivative of ΔVol , maybe drawing the derivative instead of the function ΔVol would make more sense for Fig. 8?

AC: With this figure we want to show the qualitative impact of calving on the total ice volume. Annual volume change rates can be estimated well from the annual steps in the graphs. On the other hand we don’t think that it would be intuitively clear to the reader to time-integrate the volume change rate in order to obtain the total volume loss. Alos, due to the seasonally changing calving rate the time derivative of the ice volume would oscillate even stronger than the ice volume itself. Figure 8 is of secondary importance to the manuscript, and we think that the space in the manuscript for a second figure is not justified. We therefore would like to maintain this presentation of the results.

1.2.11

RC: 1.17 p.5500 “Enhanced calving causes additional . . . measure P”, how did you corroborate “Enhanced calving causes additional . . . measure P" Vol to P? Is it a coarse/visual estimate from Fig 8? If P proves to be a good measure, you should show it more accurately.

AC: We included a subplot to Figure 8, that shows $\Delta Vol \simeq \Delta t(1 - p_0)$.

1.2.12

RC: The expression “ice modelling” comes often and stands for “ice flow modelling”. I find “ice modelling” too general. It would be better replaced by a more precise expression. Also, “calving front” sounds to me more common term than “ice front”, which is adopted in this paper.

AC: Done.

1.3 Technical comments

1.3.1

RC: 1.9 p.5490 “B the ice viscosity parameters”, this is confusing, one might think that B is the viscosity.

AC: The terminology question around this parameter has been brought to our attention by other reviewers several times. Terms like for example “ice stiffness parameter”, used by MacAyeal (1989), can be confused with similar terms originating in elasticity theory. We therefore chose this denotation.

1.3.2

RC: 1.17 p.5491 you should recall the meaning of the acronym “LSF” in Section 2.2.

AC: Done.

1.3.3

RC: 1.17 p.5491 “We can propagate the unit ...” → “We can define the unit ...”?

AC: We replaced “propagate“ by ”extend“.

1.3.4

RC: p.5492 You could maybe rename eq. (7) into (KCFC)?

AC: We agree with the referee that the KCFC is the most important equation of this manuscript. For reasons of consistency we would prefer to maintain the numbering order, however.

1.3.5

RC: 1.1 p.5493 “we need to propagate them ...”) “we need to prolong/extend them ...”?

AC: Done.

1.3.6

RC: 1.14 p.5493 “semi-implicit finite difference scheme”, I think “finite difference” comes implicitly with “semi-implicit” and doesn’t need to be said.

AC: We replace “semi-implicit finite difference scheme” by “semi-implicit time-stepping scheme”.

1.3.7

RC: 1.23 p. 5493 “using the Continuous Galerkin”, I would remove “Continuous Galerkin” since the statement also applies to “Discontinuous Galerkin”, the problem is the subgrid scale, not the type of approximation functions, isn’t it?

AC: Done.

1.3.8

RC: 1.6 p. 5494 “Then we consider ... of ice”, is this sentence correct?

AC: We rephrased the sentence.

1.3.9

RC: 1.16 p.5494 Sentences normally never start by a mathematical symbol.

AC: Done.

1.3.10

RC: 1.28 p.5494 How “correct” must be understood? Be more precise.

AC: We included a clarifying sentence in the manuscript at that position, and the Appendix.

1.3.11

RC: 1.29 p.5494 “cancels out over time”? you mean over mesh refinements?

AC: We refer to the point above.

1.3.12

RC: 1.15 p.5495 Some readers might be more familiar with the acronym CFL, so I would employ both.

AC: Done.

1.3.13

RC: eq. (10) You should replace the oblique symbol by a true letter to denote the kind of mask function which applies the calving only near the front. Also, better not use a dot \cdot for a simple multiplication by contrast to scalar products.

AC: Done.

1.3.14

RC: 1.16 p.5497 I don't see why π multiplies the sin.

AC: We include the factor π to attain a one year time integral of the seasonal calving rate scaling $s(t)$ of 1: $\int_0^1 s(t) dt = \int_0^1 \max(0, \pi \sin(2\pi t)) dt = 1$.

1.3.15

RC: 1.8 p.5498 "Increased ... retreats", you say twice the same thing in the same sentence.

AC: Done.

1.3.16

RC: 1.11-12 p.5498 "Resulting ... (Fig.2)", consider rephrasing.

AC: Done.

1.3.17

RC: 1.15 p.5498 say that this refers to Exp. C4.

AC: Done.

1.3.18

RC: 1.26 p.5498 "peaking at the point of further retreat" sounds redundant.

AC: We rephrased the sentence.

1.3.19

RC: 1.12 p.5499 "discrete location", could you clarify me what you mean by "discrete"?

AC: We see that the adjective is not appropriate, so it is dropped.

1.3.20

RC: 1.8 p.5502 “The non-linear rheology softens” could be further accurate like “The ice rheology softens” or “The Glen’s flow rheology softens” since “non-linear” could also mean the other sense (higher strain implies more viscous).

AC: Done.

1.3.21

RC: 1.20 p.5504 the velocity would better read $v = (\cos(\pi/4), \sin(\pi/4))\text{km a}^{-1}$ (without dot · and without mixing unit in the definition).

AC: Done.

1.3.22

RC: 1.25 p.5504 What means the “standard deviation of the numerical error”, standard deviation means you have a large number of point? I would have expect to simply consider one norm of the error with respect to mesh size.

AC: We have rephrased the sentence.

1.3.23

RC: Figs 5,7 It would be simpler to print “exp. A, B1, B2 and B3” on each figure instead of using intermediary letters a), b), c) and d).

AC: Done.

1.3.24

RC: Fig. 7 Even if this is for improving the readability, I’m not sure I like the shift by 0.5 factor because the curve gets wrong. What about simply splitting the y axis into several ones (shifted each other)?

AC: We added a second y-axis in the plot in the colour of the velocity value to the right hand plot column.

1.3.25

RC: Suggestion: “along-trough” \Rightarrow “longitudinal”, “across-trough” \Rightarrow lateral/transversal.

AC: We would like to highlight that the profiles are oriented along and across the deep trough, which happens to be roughly parallel and transverse to the lat/lon degree grid. However, the description “longitudinal”/“lateral/transversal” does not capture the characteristics of the tracks accurately. We therefore prefer to maintain this description.

2 Referee 2 (Jeremy Bassis):

2.1 General Appreciation

2.1.1

RC: This manuscript describes a numerical algorithm based on the Level Set Method to track the calving front position of glaciers and ice sheets and applies the model to study the response Jakobshavn Isbrae to calving rate perturbations. The study addresses two long standing problems in glaciology: The first problem involves finding a numerical algorithm that allows the calving front to consistently evolve in 2D. The second involves specifying how the glacier responds to an evolving calving front position and geometry. (There is a third issue, specifying the ‘rule’ that determine how the calving front evolves, but that is a different problem from the one considered in this manuscript.) Most studies that have examined glacier retreat have either used flowline models that can easily track the calving front position using semi-Lagrangian approaches. An exception to this is the PISM-Potsdam model which uses a sub-grid tracking algorithm to allow the calving front to advance and retreat. Hence, at least in my opinion devising an accurate and efficient method that allows the calving front to evolve remains an important challenge. Overall, portions of this manuscript provide valuable and innovative contributions to the field of ice sheet modeling and this manuscript should be published. I do, however, have some suggestions to improve the manuscript.

Foremost amongst these is that I would like to see the manuscript focus more tightly focused on introducing and testing the level set method and appropriate re-titled (e. g. , A level set method to track terminus position in 2D ice sheet models or something like this). I realize that many readers may be turned off by a primarily technical manuscript. However, I think there is a community of ice sheet models that will be highly interested in these results.

AC: Done.

2.1.2

RC: Furthermore, it is my opinion that the authors need to provide a little bit more demonstration of the method to highlight its limitations and show that it is not plagued by numerical issues before I have confidence in the model predictions. Moreover, as I will explain in more detail in later comments, the experiments have some problems that lead me to question the insight that they provide about Jakobshavn. The experiments are, however, well designed to demonstrate the level set method as part of a more general proof-of-concept so I think relatively minor changes would allow the manuscript to be acceptable for publication provided the connection to Jakobshavn is de-emphasized.

2.2 More detailed comments on the Level Set Method

2.2.1

RC: The mathematical description of the level set method is in general adequate, but it would helpful to provide readers with a better conceptual basis. For example, my understanding of the level set method is that the function φ , defined in Equation 4 is a hypersurface. In 2D, the intersection of the surface φ with the plane defined by $z = 0$ corresponds to the boundary of the domain. I would encourage the authors to provide a simple diagram illustrating this.

AC: We intended figure 3 to clarify this, but realize that contour lines of the LSF are missing in order to do this. We therefore added contour lines of the LSF to this figure.

2.2.2

RC: Equation 4, at first glance, also seems to be ill-defined. The text states that w is the velocity of the boundary, defined in this case as $\varphi = 0$. If this is the case then w would appear to only be defined on the boundary (calving front) and not in the interior of the domain, as implied by Equation 4. A more intuitive approach for me begins with the more general equation:

$$\varphi(t) = c \tag{1}$$

where c is an arbitrary contour level of φ . Taking the material derivative yields,

$$\frac{d\varphi(t)}{dt} + w \cdot \nabla\varphi = 0, \tag{2}$$

where w is the vector velocity at point (x,y) . From this it is apparent that the level set equations just correspond to the material derivatives of any fixed contour c of the level set function and the above equation tells us how *all* contours evolve.

AC: We understand that the derivation and motivation of the Level-Set Equation was not clear enough in the manuscript. We now include the proposed derivation for the Level-Set Equation in the manuscript.

2.2.3

RC: Irrespective to this discrepancy, the next question is how do you specify w in Equation 4? As the authors define it, w corresponds to the difference between the ice flow velocity u and a calving rate c , where $w = u - c$ (Forgive me for mixing notation here as I adopt the authors notation for calving rate in contradiction to the notation I used earlier where c denoted a fixed contour of φ). Specifying the velocity u is easy as velocity is defined everywhere. Specifying a calving rate globally (and in particular, far from the terminus) where calving does not occur is less intuitive. In general we can write c is a function of local properties of the calving front, global properties of the calving front (like, say, an integral over some portion of the calving front) and variables that are independent of the calving front. As I understand it, the method the authors adopt cannot accommodate global dependencies and calving rate must be parameterized in terms of local properties of the front and independent variables. Furthermore, the ‘calving rate’ is defined everywhere, even in the interior of the glacier/ice sheet where calving is prohibited? I would like to see some more clarification on this to make sure I understand what the authors are saying.

AC: We’d like to point out that we are not using a calving rate parameterization in this manuscript, but need to provide both a 2D calving rate field and ice velocity. Ice velocities therefore need to be extrapolated onto the ice-free domain, as noted in section 2.2 of the manuscript. However, we see that we have not been clear enough here, and rephrased the explanation in section 2.2 and the motivation for the choice of calving rate in section 3.2. for more clarity.

2.2.4

RC: The advantage that Equation (4) provides is that it allows the calving front to be defined and evolved by advecting a continuous variable φ . The trade-off is that now one must solve

an additional hyperbolic equation, which is non-trivial. This presents additional technical problems.

AC: We refer to point 1.2.2, and the last paragraph of section 2.3 in the manuscript.

2.2.5

RC: Even if φ is initially defined in a way that it is smooth and differentiable, there is no guarantee that it will maintain differentiability as it evolves. A trivial example that illustrates this is advecting a sinusoidal curve with fixed velocity v . Even this simple example develops kinks in finite time. As I understand it, the authors deal with this problem by smoothing out the contours by adding diffusion. But this creates its own problems: too much diffusion will act to smooth out spatial structure in calving fronts. Moreover, it is necessary to ensure that the scheme used to solve the equations conserves mass.

AC: We discuss the reinitialization of the LSF and the loss of mass-conservation in point 1.1.2. Spatial resolution of calving front features which we can resolve is on the scale of the front element diameter (500 m). The artificial diffusion we use is small, especially when compared to current uncertainties in observed calving rates.

2.2.6

RC: The above observations motivates my most significant suggestion: I would like the authors to include additional test cases to assess performance of the method. I provide a couple suggestions below. A simple test to perform is to see how well the model is able to reproduce a sinusoidally shaped calving front advection with constant velocity v as more and more cycles in the sinusoid (are added e.g, set the calving front to $x = \sin(Nmx/L$, and vary N assuming periodic boundary conditions in x with velocity v in the y -direction). A related issue is proving that the model conserves mass (or at least reasonably approximates conservation of mass). There are a variety of tests that that authors could apply to this. One test that could potentially be very revealing is showing that the model is able to reproduce the advance of confined ice tongue with zero lateral shear along the margins. This case is essentially one-dimensional and there is a well known analytic solution for ice thickness, strain rate and ice velocity for constant accumulation rate. In the absence of calving, it would be possible to test to see if the numerical model is able to reproduce the change in terminus position of the predicted analytic solution, along with appropriate mass conservation diagnostics.

AC: The proposed setup, found as “Ice Shelf Ramp” in for example Greve and Blatter (2009), is a 1D setup, for which an analytical solution exists. ISSM is able to numerically reproduce the analytical solution (see Fig. 2). However, since the thickness of the ice tongue is non-linear once it is allowed to expand freely, numerical and analytical solutions are then hard to compare. We have plotted the evolution of the ice volume in the manuscript, Fig. 11, and refer to point 1.1.2.

2.2.7

RC: Similarly, given the extrapolation and lack of a sub-grid parameterization of calving front position, I would like to see that the model can produce advance and retreat symmetrically for

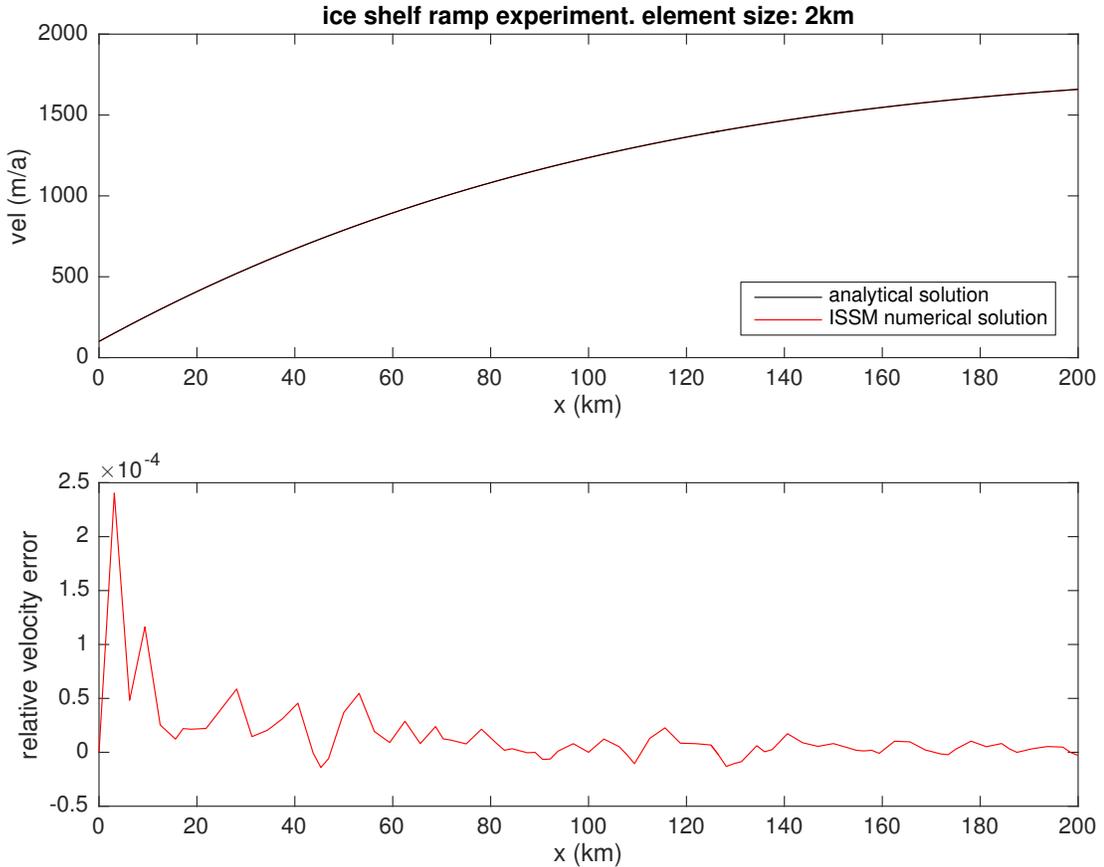


Figure 2: Top plot: Analytical and numerical solution to the Ice Shelf Ramp setup. Bottom plot: Relative error of the numerical solution.

an imposed calving rate. My guess is that the model will do very well in all this diagnostics, but it would be helpful to readers to see this more explicitly.

AC: This is a good point and we think the circle experiment in Appendix 1 addresses this question.

2.3 Experiments

2.3.1

RC: My understanding here is that the authors use an inverse method to obtain basal traction coefficients in the sliding law based on observed ice thickness and velocity at some time where they have observations. The authors then spin up the model to steady-state prior to the onset of their experiments. However, because the input data corresponded to a time when the actual glacier was far from state-state, their modeled glacier evolves into a new state that is very different from the state they started with and very different from any observed Jakobshavn state. I'm skeptical that we can learn much about the dynamics of Jakobshavn if all simulations start with an ice sheet configuration very different from that observed. The choices that the authors made are defensible, but the glacier modeled is really Jakobshavn-like

and at best a crude representation of the actual glacier as it has been observed.

AC: We refer to our addition to section 3.1 for arguments why the qualitative results of this section however do remain valid for Jakobshavn Isbræ.

2.3.2

RC: Moreover, the primarily results found (acceleration in deep water, stabilization on high points on the bed) are already well known as a generic consequence of form of the calving law used here. This is a numerical feat worth celebrating, as previous results were obtained for flow line models, but is not surprising by itself.

AC: We de-emphasized those results in the manuscript, and added references to the respective literature, see results and discussion part.

2.3.3

RC: Finally, as I understand it the authors specify a calving law that depends exclusively on water depth. This is fine for a proof-of-concept, but for a study that attempts to provide insight into glacier behavior I would like to see some justification for this. Is this calving law tuned to observations? Does it fit the original Brown et al. (1982) empirical fit? If the calving rate function is based on its agreement with observations of Jakobshavn, then the fact that the terminus position mimics observations is merely a consequence of tuning an empirical formula and cannot be used as evidence the model is performing appropriately. Similarly, given the fact that the geometry of the glacier is very different from Jakobshavn based on the spin up, what does it mean for the model to be able to reproduce the observed terminus behavior?

AC: We refer to point 2.2.2 and section 3.2 in the manuscript for motivation of the chosen calving rate field. For the fact that both observed and modelled calving front positions show qualitative agreement, see also an added comment at the end of the discussion.

2.3.4

RC: These points become less relevant if the authors decide to focus on more generic results, but I would be much more comfortable if the authors started from a state more closely resembling Jakobshavn prior to retreat and provided some physical justification for their calving law if they want convince readers that their results provide intuition about the dynamics of Jakobshavn.

AC: We refer to the answer in point 2.3.1 above.

2.4 Minor comments

2.4.1

RC: It would helpful to see a table with experiments described so that readers can quickly see the differences.

AC: Done.

2.4.2

RC: Line 20, page 5498, why is ice front configuration in quotes?

AC: Done.

2.4.3

RC: page 2518: What is a floor line??

AC: We meant flow line, of course. Done.

2.4.4

RC: The assumption that sea water pressure defines the effective pressure at the bed seems an especially dire approximation far from the calving front. I understand that this is done because you need something and the model, presumably, doesn't include subglacial hydrology. However, I wonder if it wouldn't be best to ignore the dependence of sliding on subglacial water pressure or at least show that the model predictions are not sensitive to this assumption.

AC: Using basal effective pressure as a factor in the basal stress boundary condition is commonly used in ice sheet modelling. We agree that this is a crude approximation far from the grounding line, and have added a note in section 2.1.

2.4.5

RC: page 5492 and Figure 4: I'm not sure that I fully understand the calving rate terminology used here. What the authors call "calving rate" is defined as a two dimensional field. However, my interpretation of Equation 6 and conventional glaciological terminology would suggest that the calving rate (or calving velocity) is only defined at the calving front. See my earlier comments on this.

AC: We refer to the earlier point 2.2.2 and section 3.2 in the manuscript for motivation of the chosen calving rate field.

2.4.6

RC: Page 5494: The authors activate or deactivate an entire element as being "ice filled" if one of the vertices intersects with the hyperplane. However, ice thickness and velocity appear to be extrapolated assuming continuous values outside of the ice sheet domain. Does this conserve mass?

AC:

We discuss the question of volume conservation in point 1.1.2 and the Appendix, and motivate the choice of extrapolation in sections 2.2 and 2.3 of the manuscript.

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Modelling ~~the dynamic response of~~ calving front dynamics using a Level-Set Method: Application to Jakobshavn Isbræ, West Greenland, ~~to calving rate perturbations~~

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Abstract

Calving is a major ~~means~~ mechanism of ice discharge of the Antarctic and Greenland Ice Sheets. ~~The breaking off of icebergs changes the ice front configuration of marine terminating glaciers, which affects the~~, and a change in calving front position affects the ~~entire~~ stress regime of ~~their upstream areas. Recent observations show the close correlation between the ice front position and the behaviour of many outlet glaciers. However, modelling of a glacier subject to calving poses various challenges. No universal calving rate parametrisation is known, and tracking of a moving ice front and the related boundary conditions in~~ marine terminating glaciers. The representation of calving front dynamics in a two or three spatial dimensions is ~~dimensional ice sheet model remains~~ non-trivial. Here, we present the theoretical and technical framework for a Level-Set Method, an implicit boundary tracking scheme, which we ~~implemented~~ implement into the Ice Sheet System Model (ISSM). ~~The~~ This scheme allows us to study the dynamic response of a drainage basin to user-defined ~~front ablation~~ calving rates. We apply the method ~~in a suite of experiments~~ to Jakobshavn Isbræ, a major marine terminating outlet glacier of the western Greenland Ice Sheet. The model robustly reproduces the high sensitivity of the glacier to ~~frontal ablation in form of calving. We~~ calving, and we find that enhanced calving ~~is able to trigger~~ triggers significant acceleration of the ice stream. Upstream acceleration is sustained through a combination of ~~various feedback~~ mechanisms. However, both lateral stress and ice influx ~~into the trough~~ are able to stabilise the ice stream. This study ~~contributes to the present discussion on causes and effects of the continued~~ provides new insights into the ongoing changes occurring at Jakobshavn Isbræ, and emphasises that the incorporation of ~~seasonal calving~~ moving boundaries and dynamic lateral effects, not captured in flowline models, is key for realistic model projections of ~~future global~~ sea level rise on centennial time scales.

1 Introduction

Calving of icebergs is a major mean of ice discharge for marine terminating glaciers around the world. It accounts for about ~~50%~~ half of the ice discharge of the Greenland and Antarctic Ice Sheets (Cuffey and Paterson, 2010; Rignot et al., 2013). ~~Calving causes ice~~ This process causes calving front retreat, which leads to reduced basal and lateral stress and results in upstream flow acceleration.

[Figure 1 about here.]

~~Understanding calving dynamics remains challenging because of the diversity of factors that can trigger calving events. Bathymetry, tides and storm swell, as well as sea ice cover and temperatures of both sea water and air are possible factors influencing calving rates. However, their effect, their respective share and their interplay seem to vary from glacier to glacier, and is not well understood. Therefore, no universal calving rate parametrisation, which can be used in macroscopic ice sheet models, exists to date (Benn et al., 2007; Cuffey and~~

In order to assess the impact of calving on the dynamics of ~~an outlet glacier~~ outlet glaciers using an ice sheet model, we need to ~~implement boundary conditions at the dynamically evolving ice~~ include a dynamically evolving calving front. This ~~poses a second technical challenge as the ice front position needs to be tracked, and the related boundary conditions need to be updated accordingly.~~ requires tracking the calving front position and adjusting the boundary conditions accordingly. Addressing these issues is rather straightforward for 1D-flowline or 2D-flowband models (Nick et al., 2009; Vieli and Nick, 2011), where the ~~ice front can be~~ calving front is tracked along the ~~floor line~~ flowline. However, this type of ~~models~~ model lacks the consistent representation of ~~mechanisms like lateral momentum transport and ice influx of converging ice streams~~ lateral momentum transfer and lateral ice influx from tributaries for example, which have to be parametrised instead. This parametrisation may neglect feedback effects important for simulations on decadal to centennial time scales, e.g. catchment area entrainment (Larour et al., 2012a).

~~Therefore, it is desirable to implement~~ It is therefore critical to include a front tracking scheme in 2D-horizontal and 3D models, which has been addressed by only a few ice sheet models ~~only, e.g. Winkelmann et al. (2011).~~ (e.g. Jouvet et al., 2008; Winkelmann et al., 2011). Various approaches to model the evolution of the shape of ice exist. Explicit methods track the position of a set of points, which represent the ice calving front. They require a complex technical framework to allow for geometric operations like folding and intersection of the continuum boundary, tracking singularities in curvature, and determining the position of a point in space relative to the modelled continuum. ~~On the other hand, a suite of implicit boundary tracking methods exists, e.g.~~ Alternatively, the Level-Set Method (LSM) by Osher and Sethian (1988). ~~The LSM (LSM Osher and Sethian, 1988)~~ represents the continuum boundary implicitly by a contour, or “level-set”, of a scalar-valued so-called “Level-Set Function” (LSF). It easily handles topological changes of the modelled continuum, like splitting and merging ~~of its parts. It is well suited for the use in a parallel architecture, since it is.~~ The LSM is based on a partial differential equation ~~.~~ This allows for application to large similar to the mass transport equation solved by ice sheet models. This makes the method straightforward to implement, and allows for the application to continental scale ice sheet simulations. ~~The method~~ Although the method does not necessarily conserve volume accurately, it is well established in Continuum Fluid Mechanics (Chang et al., 1996; Groß et al., 2006), ~~and can be used with various numerical schemes, like the Finite Differences Method or the Finite Element Method (FEM).~~ A LSM has been applied to ice flow modelling in test cases (Pralong and Funk, 2004), but not to real ice sheets yet.

Understanding calving dynamics remains challenging because of the diversity of factors involved in calving events. Bathymetry, tides and storm swell, as well as sea ice cover and temperatures of both sea water and air are possible factors influencing calving rates. However, their effect, their respective share and their interplay seem to vary from glacier to glacier, and are not well understood (Cuffey and Paterson, 2010). Therefore, no universal calving rate parametrisation exists to date (Benn et al., 2007), and we rely here on user-defined calving rates. However, incorporating calving rate parametrisations in the LSM is straightforward.

Jakobshavn Isbræ is a major marine terminating glacier in West Greenland, which drains about 6.5% of the Greenland Ice Sheet (Zwally et al., 2011). It is characterised by two ~~tributaries~~branches, which today terminate into ~~an ice-choked fjord of about a~~ 30 km ~~length~~long ice-choked fjord (Figs. 1 and 2). The southern ~~tributary~~branch exhibits high flow velocities, which are confined to a narrow, deep trough of about 5 km width. The trough retrogradely slopes inland to a maximum depth of about 1700 m below sea level (Gogineni et al., 2014). ~~It, and~~ discharges most of the ice of the drainage basin (Rignot and Mouginot, 2012). Observations have shown that the fast flowing areas of Jakobshavn Isbræ exhibit a weak bed with a ~~considerable~~basal layer of temperate, soft ice (Lüthi et al., 2002). ~~Most of these areas' horizontal motion is due to basal~~Basal sliding and shear in this layer ~~cause most of these areas' horizontal motion~~. A large fraction of the ice stream's momentum ~~has to be~~is transferred to the adjacent ice sheet by lateral stress. It is thus well justified to use the two-dimensional shelfy-stream approximation (SSA, MacAyeal, 1989) ~~for model simulations of to simulate~~ this glacier.

~~The basal topography of Jakobshavn Isbræ and its velocity field permit to roughly~~We partition the catchment area into the confined, deep and fast-flowing ~~troughs~~trough (“ice stream”) and the surrounding ~~parts of low velocity flow~~slow-moving ice (“ice sheet”). Those areas are separated by pronounced shear margins ~~to on~~on either side of the ice stream.

[Figure 2 about here.]

Until the late 1990s, ~~the glacier~~Jakobshavn Isbræ had a substantial floating ice tongue, which extended well into the ~~ice~~fjord, and was fed by both ~~tributaries~~. ~~The position of the ice front~~branches. ~~The calving front position~~ remained fairly constant ~~over time since the 1960s~~from 1962 to the 1990s (Sohn et al., 1998), and the glacier exhibited negligible seasonal variations in flow speed (Echelmeyer and Harrison, 1990). ~~The glacier started an ongoing~~In the 1990s, the glacier started a phase of acceleration, thinning and retreat, that followed the breakup of its ice tongue. Seasonal variations in ~~ice~~calving front position and flow velocity increased sharply (Joughin et al., 2004, 2008). ~~The glacier is currently far from equilibrium, making it~~Today, the glacier is one of the fastest ice streams in the world~~and~~.

It is still far from equilibrium and is a major contributor to global sea level rise (Howat et al., 2011; Joughin et al., 2014). Observations ~~confirm that the current ice stream dynamics are mainly controlled by the position of the ice front~~ suggest that the calving front position is a major control on the ice stream dynamics (Podrasky et al., 2012; Rosenau et al., 2013; Moon et al., 2014).

Various hypotheses have been proposed to explain the mechanisms behind ~~the~~ this change. All identify the breakup of the floating ice tongue as the initial ~~triggering mechanism~~ trigger of this dramatic chain of events, but different ~~possible~~ mechanisms have been proposed to explain the sustained acceleration, thinning and retreat of the glacier. Studies by Joughin et al. (2012) and Habermann et al. (2013) propose loss of buttressing and changes in basal conditions as the main cause behind the ongoing acceleration. On the other hand, van der Veen et al. (2011) argue that ~~the acceleration has to be accompanied by significant~~ weakening of the lateral shear margins has significantly amplified the upstream acceleration. ~~Various~~ Several modelling studies of the glacier, which use 1D-flowline and 2D-flowband models, project unstable retreat of the glacier along its southern trough for up to 60 km inland within the next century (Vieli and Nick, 2011; Joughin et al., 2012; Nick et al., 2013). Other modelling studies argue that this type of ice stream ~~displays stable behaviour is stable~~ as long as it is ~~being~~ fed by the surrounding ice sheet (Truffer and Echelmeyer, 2003). However, numerical 2D planview modelling efforts of Jakobshavn Isbræ ~~in the horizontal plane~~ so far lacked the representation of a dynamically evolving ice calving front. Hence, the hypotheses could not be tested in a satisfactory manner.

We present here a ~~theoretical and technical framework for a LSM used~~ LSM-based framework to model the dynamic evolution of ~~an ice~~ a calving front. This method ~~tracks the ice front related boundary conditions and~~ is a step towards ~~a more~~ better physical representation of ice calving front dynamics in 2D and 3D ice sheet models. We ~~implemented~~ the describe the implementation of the method into the Ice Sheet System Model (ISSM, Larour et al., 2012b), a parallel, ~~state-of-the-art FEM~~ state-of-the-art ice sheet model, and apply it here to Jakobshavn Isbræ in order to model ~~the~~ its dynamic response to pertur-

bations in calving rate ~~and conclude on its sensitivity to ice front retreat over the next 120 years.~~

2 Theory

2.1 Ice Flow Model

5 We employ the SSA on both floating and grounded ice. It neglects all vertical shearing but includes membrane stresses. The ice viscosity, μ , follows Glen's flow law (Glen, 1958):

$$2\mu = B\dot{\epsilon}_e^{\frac{1-n}{n}} \quad (1)$$

Here, $n = 3$ is Glen's flow law coefficient, B the ice viscosity parameter, and $\dot{\epsilon}_e$ the effective strain rate. We apply a Neumann stress boundary condition at the ice-air and ice-water
10 interface, corresponding to zero air pressure and hydrostatic water pressure, respectively. A linear friction law links basal shear stress, σ_b , to basal sliding velocity, v_b , on grounded ice:

$$\sigma_b = -\alpha^2 N v_b, \quad (2)$$

where α denotes the basal friction parameter. We calculate the effective basal pressure, N ,
15 assuming that sea water pressure applies everywhere at the glacier base, which is a crude approximation far from the grounding line. The ice thickness, H ~~evolves~~, evolves over time according to the mass transport equation:

$$\frac{\partial H}{\partial t} = -\nabla \cdot (H\mathbf{v}) + a_s + a_b. \quad (3)$$

Here, \mathbf{v} is the depth-averaged horizontal ice velocity, and a_s and a_b are the surface and
20 basal mass balance, respectively. We determine the grounding line position using hydrostatic equilibrium, and treat it with a sub-element parametrisation (Seroussi et al., 2014). We

refer the reader to ~~Greve and Blatter (2009), Cuffey and Paterson (2010) and Larour et al. (2012b), respectively, for further details on ice flow modelling and its implementation~~ Larour et al. (2012b) for details on the solution of these equations in ISSM.

[Table 1 about here.]

5 2.2 Level-Set Method

Let Ω be a computational domain in ~~two- or three-dimensional~~ 2D or 3D space, and φ a real, differentiable function on ~~$\Omega \times [0, \infty)$~~ $\Omega \times \mathbb{R}_+$, called “Level-Set Function” (LSF). For any $c \in \mathbb{R}$, we define the contour, or “ c -level-set”, of φ by $\varphi(x, t) = c$. Taking its material derivative yields the “Level-Set Equation” (LSE):

$$10 \quad \frac{\partial \varphi}{\partial t} + \mathbf{w} \cdot \nabla \varphi = 0. \quad (4)$$

This Hamilton-Jacobi type partial differential equation describes how level-sets move with the local value of the velocity \mathbf{w} , which is called level-set velocity. We need to provide an initial condition $\varphi_0(\mathbf{x}) = \varphi(\mathbf{x}, t = 0)$ to solve eq. (4).

We use φ to partition Ω ~~at time t~~ into three disjoint subdomains: the ice domain, $\Omega_i(t)$, its complement, $\Omega_c(t)$, and their common boundary, $\Gamma(t)$.

$$\begin{cases} \varphi(\mathbf{x}, t) < 0 \Leftrightarrow \mathbf{x} \in \Omega_i(t) \\ \varphi(\mathbf{x}, t) = 0 \Leftrightarrow \mathbf{x} \in \Gamma(t) \\ \varphi(\mathbf{x}, t) > 0 \Leftrightarrow \mathbf{x} \in \Omega_c(t) \end{cases}$$

We omit the time dependence of these sets in the ~~following, unless stated otherwise. Let \mathbf{x} be a point in Ω . If $\varphi(\mathbf{x}, t) < 0$, then \mathbf{x} belongs to Ω_i . If $\varphi(\mathbf{x}, t) = 0$, then \mathbf{x} lies on Γ . If $\varphi(\mathbf{x}, t) > 0$, then \mathbf{x} belongs to Ω_c .~~ remainder of this article. By construction, Γ , the ~~0-contour~~ or “0-level-set” of φ , separates Ω_i and Ω_c .

A particle at the boundary Γ moves with the boundary speed w . This motivates the “Level-Set Equation” (LSE):

$$\frac{\partial \varphi}{\partial t} + \mathbf{w} \cdot \nabla \varphi = 0.$$

This Hamilton-Jacobi type partial differential equation describes the transport of the ice boundary across Ω . We need to provide an initial field $\varphi_0(\mathbf{x}) = \varphi(\mathbf{x}, t=0)$ on Ω to solve equation (4). We can propagate the unit surface normal \mathbf{n} on Γ , the outward-pointing unit normal of Γ , onto Ω using the LSF by:

$$\mathbf{n} = \frac{\nabla \varphi}{|\nabla \varphi|}. \quad (5)$$

By definition, \mathbf{n} always points into Ω_c . For more details on the Level-Set Method and its applications, we refer to Osher and Sethian (1988) and Sethian (2001).

[Figure 3 about here.]

The boundary position of an ice sheet evolves with a velocity, which is the sum of the ice velocity \mathbf{v} and an ablation velocity $\mathbf{a} = -a^\perp \mathbf{n}$. The ablation rate $a^\perp = -\mathbf{a} \cdot \mathbf{n}$, a^\perp , is non-negative in compliance with current glaciology terminology, and is the difference between ice boundary velocity \mathbf{w} the level-set velocity and ice velocity \mathbf{v} projected along \mathbf{n} :

$$a^\perp = (\mathbf{v} - \mathbf{w}) \cdot \mathbf{n}. \quad (6)$$

It follows that the ice boundary is stationary ($\mathbf{w} \cdot \mathbf{n} = 0$) if and only if $a^\perp = \mathbf{v} \cdot \mathbf{n}$, i.e. the ablation rate matches the ice velocity perpendicular to the ice boundary. The ablation rate a^\perp can be an input taken from observations or a suitable needs to be prescribed, either based on observations or through a parametrisation.

Note that no limitations have been made so far with respect to the dimension of the problem in this section. Accordingly, the method could be applied to model the evolution of the glacier thickness and lateral extent simultaneously (Pralong and Funk, 2004). However, here we use the LSM to model only the horizontal extent of the ice sheet. Its vertical extent is described by the mass transport equation (3). ~~We~~ ISSM relies on vertically extruded meshes and the vertical motion of the ice boundary is accurately captured by redistributing the horizontal layers of the mesh. Using the LSM in the vertical dimension would not only significantly complicate its implementation, but it would also reduce the accuracy and precision of the ice boundary tracking (see also section 2.3).

For simplicity, we assume in the remainder of the article that lateral ablation occurs in the form of calving, with a calving velocity $c = -c^\perp \mathbf{n}$, ~~and that calving itself is~~. Calving itself is assumed to be a quasi-continuous process, consisting of frequent, but small calving events. With (5) and (6), equation (4) becomes:

$$\frac{\partial \varphi}{\partial t} + \mathbf{v} \cdot \nabla \varphi = c^\perp |\nabla \varphi|, \quad (7)$$

which is also known as “Kinematic Calving Front Condition” (KCFC, Greve and Blatter, 2009). ~~The calving rate needs~~ Both the calving rate and ice velocity need to be provided ~~as~~ a ~~on the entire 2D field on computational domain Ω , that can also vary with time in order to solve the KCFC.~~ An example of a calving rate field ~~can be seen in~~ will be given in section 3.2, and is shown in Fig. 4. The ~~ice~~ KCFC implies that all level-sets of φ , including the calving front Γ is advected at any, ~~move at a given time with the local~~ move at a given time with the local sum of the horizontal ice velocity and ~~local calving rate values at the current ice front position~~ calving rate along the normal \mathbf{n} (Fig. 3). We define the ~~“calving flux” to be the ice volume calving at the ice front per unit time~~ calving flux Q_{cf} as the ice flux crossing the calving front:

$$Q_{cf} = \int_{\Gamma} c^\perp(r) H(r) dr. \quad (8)$$

Calving rate and velocity need to be given on the entire computational domain Ω in order to solve the KCFC. Since field variables like velocity and thickness are only available on Ω_i , we need to propagate them continuously and need to be extended onto Ω_c . We extrapolate such a field variable. Any scalar field, S , is extrapolated onto Ω_c by solving:

$$\mathbf{n} \cdot \nabla S = 0, \quad (9)$$

while keeping S fixed on Ω_i . This type of extrapolation has the tendency to preserve $|\nabla \varphi| = \mathcal{O}(1)$, when we use the extrapolated ice velocity field to solve the KCFC (Zhao et al., 1996).

2.3 Implementation

ISSM relies on the FEM Finite Element Method (FEM) to solve partial differential equations. It applies a Continuous Galerkin FEM using triangular (2D) and prismatic (3D) Lagrange finite elements, and uses anisotropic mesh refinement to limit the number of degrees of freedom while maximizing spatial resolution in regions of interest.

We discretise the KCFC (7) and extrapolation equation (9) using linear finite elements on the same mesh as the one used to model the ice dynamics. We stabilise both equations with artificial diffusion (Donea and Huerta, 2003), which after thorough testing proved to be the most robust stabilisation scheme. We integrate over time using a semi-implicit finite difference time-stepping scheme. We solve the KCFC (7), and the field equations for ice flow modelling in a decoupled fashion. The KCFC is solved first with input data from the last previous time step. We then update the numerical ice domain Ω_i^h using the new LSF as described below, and update boundary conditions accordingly. We finally Finally, we solve the momentum balance and the mass transport equation on Ω_i the updated ice domain.

The 0-level-set of φ , Γ , does in general not coincide with the finite element mesh edges due to its implicit representation. The ice front Γ intersects a number of elements (“front elements”) with a hyperplane, which divides them into an ice-filled and an ice-free domain part (Fig. 3). Ice velocity and ice thickness exhibit a discontinuity at subgrid scale here;

~~which we cannot resolve using the Continuous Galerkin FEM.~~ This has various implications on the numerical level. When assembling the system stiffness matrices for ice flow modelling, exclusive integration over the ice-filled part of the element would be required. The stress boundary condition at the ice calving front would have to be applied at the intersecting hyperplane. Currently, ISSM is not capable of resolving those submesh scale processes.

Therefore, we ~~opt to~~ either fully activate or deactivate an a mesh element at every time step. Only active elements are considered for the numerical discretisation of the respective field equations. We activate a mesh an element if at least one of its vertices is in Ω_i or Γ . ~~Then we consider the entire element to consist of,~~ and the element is then considered to be entirely filled with ice. We flag the element as ice free if it lies entirely inside Ω_c , and it is deactivated ~~at this time step~~. As a consequence, the numerical ice front calving front, Γ^h , runs along mesh edges, and ~~updates of Γ^h occur~~ is updated in a discontinuous manner ~~-(Fig. 3)~~. We apply the stress boundary condition along Γ^h for numerical consistency. ~~The strongly viscous ice rheology efficiently dampens effects of these discrete boundary condition updates.~~ Ice Calving front normals on Γ and Γ^h may differ significantly in direction. However, stress components tangential to n cancel out along Γ^h , so that the integrated stress exerted at the ice calving front is close to the one applied along Γ . For all further calculations where a normal is involved, like extrapolation, the normal to the LSF (5) is used.

The ~~ice domain Ω_i~~ numerical calving front is by definition ~~a subset of the numerical ice domain Ω_i^h~~ . ~~Γ^h is thus displaced~~ further downstream than Γ . This may lead to slightly higher resistive lateral stress at the ice calving front, whose magnitude depends on the excess ice area of the intersected front element and the front geometry. We ~~choose~~ use a fine mesh resolution in the vicinity of the ice calving front to limit this effect.

We extrapolate the ice calving front thickness onto the ice-free ~~vertices of the front elements domain~~ using equation (9). This yields realistic ice thickness and ice thickness gradients across the front elements, that would otherwise lead to overestimated driving stress and underestimated water pressure at the ice-ocean interface. If not corrected, those two

effects unrealistically increase ice velocities at the ice calving front, which then feed back into the mass transport and LSM schemes.

We present ~~an idealized test setup~~ two experiments for validation of the LSM module in Appendix A. ~~It LSM implemented here in the Appendix.~~ The first experiment shows that the ice margin is advected with ~~correct speed. A small error is due to the underlying unstructured mesh used, but~~ the prescribed level-set velocity w . The linear representation of the LSF on an unstructured mesh causes a small error in the exact level-set position, which depends on element size and cancels out over time. The ~~module second~~ test shows that errors in volume conservation introduced by the LSM decrease with finer mesh resolution, and are below 0.2 % after 100 years for a mesh resolution of 1 km. In the application to Jakobshavn Isbræ, we use a front element size of 0.5 km. The potential volume loss inherent to this implementation of the LSM is thus far below current uncertainties of model input data.

Inclusion of the LSM requires additional computational effort for the extrapolation of field variables, to solve the KCFC, and for extra iterations of the momentum balance solver, since the stress boundary ~~condition at the ice front does often change~~ conditions at the calving front change frequently. Its amount depends on the flow approximation ~~used~~ and especially on whether the model setup is close to a stable configuration or not. Using ~~the SSA approximation~~ SSA, the additional computational cost ~~amounts to~~ reaches up to 25%, of which 11% is caused by the solution of the ~~LSM module for this experiment~~ KCFC.

3 Data and model setup

3.1 Jakobshavn Isbræ Model Setup

We use Jakobshavn Isbræ's drainage basin from Zwally et al. (2011) to generate a 2D-horizontal finite element mesh with element size varying from 500 m in the fjord and areas of fast flow to 10 km inland (Fig. 4). We choose this high mesh resolution to minimise ice calving front discretisation errors, and to resolve the fjord and the deep trough accurately in

the model. The resulting mesh has about 10,000 vertices and 19,000 elements. Due to high flow velocities, the ~~numerical~~ Courant-Friedrich-Lewy condition (~~Courant et al., 1928~~) (CFL, Coura) a time step ~~length~~ on the order of days.

We use ~~ice bedrock bed~~ topography from Morlighem et al. (2014), derived using a mass conservation approach (Morlighem et al., 2011). The ice surface elevation is taken from GIMP (Howat et al., 2014). ~~Ice~~, and ~~ice~~ thickness is the difference between ice surface and ice base elevation. Bathymetry of the ice-choked fjord of Jakobshavn Isbræ is difficult to measure and currently poorly known. As a first order estimate, we apply a parabolic profile of 800 m depth along the ice fjord, fitted via spline interpolation to known topography data. We rely on Ettema et al. (2009) ~~to force for~~ the surface mass balance. Their surface temperatures are used to calculate the ice viscosity parameter, B_i , following an Arrhenius relationship (Cuffey and Paterson, 2010) ~~once~~ at the beginning of the model simulations. Basal mass balance is set to zero and no thermal model is run. All these forcings are kept constant over time.

We infer a basal friction coefficient, α_b , in (2) using an adjoint-based inversion (MacAyeal, 1993; Morlighem et al., 2010) of ~~existing InSAR-derived~~ surface velocities from 2009 (Rignot and Mouginot, 2012). In regions like the ~~ice~~ fjord, where there is no ice today, we apply an ~~averaged area-averaged~~ value of $\alpha = 30 \text{ a}^{1/2} \text{m}^{-1/2}$. At the margins of the computational domain we prescribe zero horizontal ice velocities in order to prevent mass flux across this boundary. ~~α~~ ~~The friction parameter~~ is kept fixed over time for all model simulations.

Inconsistencies in model input data cause sharp readjustments of the glacier state at the beginning of each simulation, which would make it difficult to distinguish between such effects and those of the applied forcing (Seroussi et al., 2011). Therefore, we relax the ~~ice surface model~~ prior to the experiments using a fixed ~~ice front, whose position and orientation is set~~, ~~piecewise linear LSF~~ φ_0 , ~~whose 0-level-set corresponds~~ to the mean annual ~~ice calving~~ front position of ~~2009-2009~~ (Fig. 4). Since the glacier in this configuration is far from steady state, model relaxation causes considerable thinning across the glacier's catchment area. In order not to deviate too much from present day's geometric setting we choose a 100 year relaxation time ~~interval~~ ~~period~~. Note that the grounding line retreats during the

relaxation due to dynamic thinning, so that the glacier forms a new floating ice tongue. This ice tongue extends about 15 km to a ~~topographic high~~ local topographic maximum in the southern trough and 3 km into the northern one (Fig. 4). The relaxed geometry constitutes the initial state for our experiments.

Due to this deviation in geometry, providing quantitative insights into Jakobshavn Isbræ is beyond the scope of this study. However, the main characteristics of the ice stream (e.g. its large drainage basin and the narrow outlet channel) are preserved, so that the results presented in this paper qualitatively represent the behaviour of Jakobshavn Isbræ.

[Figure 4 about here.]

3.2 Description of experiments

For simplicity, frontal ablation occurs ~~in the experiments~~ exclusively in the form of calving ~~in the experiments~~. We set

$$c_0^\perp = \underline{l \cdot q} |v_0| \quad (10)$$

as a basic calving rate estimate, motivated by the small observed angle between v and n at the ~~ice front~~ calving front ($v \approx |v|n$). Then $w \cdot n = v \cdot n - c \cdot n \approx 0$, so that we can expect this calving rate estimate to yield a stationary calving front, if applied to a geometry that is in steady state. Here, v_0 denotes the velocity field at the end of the geometry relaxation run, ~~continued extended~~ onto Ω_c (Fig. 4). ~~l is a continuous function, which is~~ The continuous function q is equal to 1 in areas where the ~~bedrock bed~~ lies below -300 m, and linearly drops to 0 in areas of positive ~~bedrock bed~~ elevation. It prevents calving ~~onto to occur in~~ areas with a glacier bed above sea level, ~~motivated by observations from as suggested by observations of~~ tidewater glaciers (Brown et al., 1982). We ~~expect this calving rate to yield a stationary ice front, if applied to a geometry in steady state.~~ We scale c_0^\perp over time with a scaling function ~~$s = s(t)$~~ s , which allows for the representation of seasonal ~~calving~~ cycles, and a perturbation function ~~$p = p(t)$~~ p to modify the calving rate for some period of

time, respectively. The applied calving rate ~~then is $c^\perp = c^\perp(\mathbf{x}, t) = c_0^\perp(\mathbf{x}) \cdot s(t) \cdot p(t)$ is then:~~
 ~~$c^\perp(\mathbf{x}, t) = c_0^\perp(\mathbf{x}) s(t) p(t)$.~~

We perform three suites of experiments in order to analyse the impact of the calving rate on the glacier's dynamics. The ~~ice calving~~ front is now allowed to freely evolve in response to c^\perp . All experiments run for ~~a total time of~~ 120 years.

In experiment A, we keep ~~calving the calving rate~~ constant over time, i.e. we ~~let set~~ both $s(t) = p(t) = 1$. Hence, $c^\perp(\mathbf{x}, t) = c_0^\perp(\mathbf{x})$. This experiment, although not physically motivated, is used to evaluate whether a stable ~~ice calving~~ front position can be reached using the LSM, ~~and for comparison to the experiments described below.~~

In experiment suites B and C, we ~~mimic seasonal calving represent the seasonal cycle~~ by scaling c_0^\perp ~~over time by a factor by~~ $s(t) = \max(0, \pi \sin(2\pi(t/L - \phi_0)))$, with ~~a~~ phase shift $\phi_0 = 4/12$ and ~~period $L = 1$ a~~ ~~period $L = 1$ a~~. We perturb the calving ~~cycle rate~~ during a limited ~~perturbation duration~~ ~~duration~~, Δt , with a perturbation strength $p_0 \geq 0$: $p(t) =$

$$\begin{cases} p_0, & \text{if } t_0 < t < t_0 + \Delta t, \text{ and} \\ 1, & \text{else.} \end{cases}$$

We start the perturbation at $t_0 = 20$ a for all experiments. In experiment suite B ~~we choose~~ ~~we perform 5 experiments with~~ $\Delta t = 1$ a ~~and vary~~, ~~while varying~~ p_0 ~~in steps of 1~~ from 0 to 4. ~~On the other hand, in C by increments of 1.~~ In experiment suite C, we keep $p_0 = 2$ fixed, and ~~choose set~~ Δt as 2, 4 and 8 years. We use the notation $B_{\langle p_0 \rangle}$ and $C_{\langle \Delta t \rangle}$ to identify ~~the~~ single experiments, e.g. ~~B2-B1~~ for experiment B with perturbation strength $p_0 = 2$. ~~Experiment B1 represents in this experiment suite $p_0 = 1$, which represents~~ the case of unperturbed periodic calving. ~~It B1~~ is used as a control run the other experiments can be compared to. ~~We introduce $P = (1 - p_0) \cdot \Delta t$, which is a measure of the time-integrated deviation in applied calving rate relative to B1.~~ ~~Table 2 lists all the experiments performed here.~~

[Table 2 about here.]

4 Results

[Figure 5 about here.]

~~We present ice~~ Fig. 5 shows calving front positions for ~~several experiments in Fig. 5.~~ experiments A, B1, B2 and C4. Under constant calving rate forcing, the ~~ice front essentially~~ calving front remains at a stable position after minor readjustments in the first decade of the simulation. In experiment A, the ~~ice calving~~ front undergoes gradual retreat over time. When we perturb the calving rate, the ~~ice front migrates. Increased calving rates cause ice front~~ retreat: ~~calving front~~ migrates, and higher calving rates lead to larger retreats. The retreat is highest in areas of fast ~~ice flow, but~~ flow, and strongly decreases towards the ice stream margins. This yields the characteristic concave shape of a retreating ~~ice front. Resulting ice~~ fronts ~~calving front. The modelled calving front~~ positions and their ~~characteristics do agree~~ well with ice fronts obtained from ~~shape are in good agreement with~~ observations (Fig. 2). ~~Ice front positions of all~~ The retreat rate during continued phases of calving decreases to zero, so that the calving front reaches a new stable position 9 km upstream of its initial position (Fig. 5d). In experiments B and C ~~readvance after the perturbation interval to a~~ position which is indistinguishable from the one of the calving front returns to a similar position as in the unperturbed experiment B1 ~~in one to two decades~~ within ten to twenty years after the perturbation stops.

[Figure 6 about here.]

~~Figure~~ Fig. 6 shows ice velocity, geometry and strain rates ~~along two tracks~~ for experiment C4 ~~along two lines~~, which go along and across the southern trough, respectively (Fig. 4). ~~We observe that the ice front position, its ice velocity and thickness as well as the position of the grounding line form an ensemble, which quickly adjust to each other under constant mean annual calving rates to reach a characteristic “ice front configuration”. The variables of the ice front configuration mirror the minor adjustment of the ice front position~~ During the first 20 years prior to the perturbation. ~~Only~~, the ice thickness in the floating part decreases ~~up to by about~~ 100 metres ~~once the ice front is allowed to freely adjust.~~

(Fig. 6a). Enhanced calving causes the modelled ice stream to rapidly thin, accelerate and retreat. However, the ice front can be very thick at the end of the calving season, as the glacier has retreated back into areas of high ice thickness. Ice velocities respond to this change by increasing over time as the ice front retreats and peaking at the point of furthest retreat. The response is stronger with larger and longer perturbations, as expected. As the calving front retreats during the perturbation, the ice velocity increases (Fig. 6b), and the ice thickness adjusts accordingly (Fig. 6a). The ice thinning leads to a fast retreat of the grounding line in the regions of locally retrograde bed, and temporarily stabilises over local along-trough topographic maxima, referred to as “local highs”. The southern trough has many local highs, which act as pinning points and are critical for flow dynamics, in agreement with earlier results from Vieli and Nick (2011). The acceleration of the ice stream extends well upstream tens of kilometres upstream, to areas of grounded ice. Both thinning and acceleration mainly occur in (Fig. 6b). Thinning and acceleration are strongest over the ice stream itself, and diffuse, and spread out to the surrounding ice sheet in a strongly dampened fashion. These thinning and acceleration patterns increase surface and velocity gradients along and across the ice stream, especially in the shear margins. Here, the ice stream acceleration gradually increases (Fig. 6c), where the effective strain rates gradually increase up to a factor of 4 in experiment C4 (Fig. 6d), which corresponds to a drop in viscosity of about 60% (equation 1). This substantially weakens the mechanical coupling between the ice stream and the surrounding ice sheet substantially. The grounding line position lies downstream of a local high of the basal topography at any given time of the experiment, even if the general slope of the basal topography is retrograde. It is hardly affected by small calving rate perturbations. However, large perturbations cause discontinuous migration of its position to the next topographic high upstream. Their discrete locations form the set of stable grounding line positions along the deep part of the tributaries. Discontinuous retreat of the grounding line causes drastic, but short-lived flow acceleration, which indicates that the basal topographic highs act as pinning points, even if basal friction coefficient α and effective basal pressure N in the troughs are low.

[Figure 7 about here.]

Timelines of ice front position, ice velocity and thickness at the ice front Fig. 7 shows the intra-annual variability of ice properties at the calving front and grounding line for experiments A, B1, as well as effective strain rate and position of the first grounded point along trough are shown in Fig. 7. They allow us to study the interannual behaviour of the ice front configuration in more detail. B2 and C4. All shown variables reflect the characteristics of the applied calving rate forcing. They stay constant. The constant calving rate applied in experiment A, and oscillate periodically with a constant amplitude around an annual mean value in experiment suites B and C.

Prior to the perturbation, the ice front position varies leads to a steady configuration (Fig. 7a and 7b). For an unperturbed periodic calving rate forcing (Fig. 7c and 7d), the calving front position oscillates around a constant annual mean value by ± 3 km along-stream, while the grounding line position remains unchanged at kilometre 29. Ice velocities and front thickness thickness at the calving front act in phase to the ice with the calving front position, while the response of strain rates at the grounding line is slightly delayed. Ice velocities vary delayed by about a month. The ice velocity varies by $\pm 20\%$, which corresponds to about ± 2 km a^{-1} , the ice thickness by $\pm 13\%$, or ± 100 m, and effective grounding line strain rates strain rates at the grounding line by $\pm 7\%$, or ± 0.1 a^{-1} . The grounding line position is stable at kilometre 29 of the along-trough profile. In

The response to a two-fold increase in calving rate, the ice calving rate perturbation scales with p_0 and Δt . When the calving rate doubles (B2, C4), the calving front retreats initially at an average rate of 4.5 km a^{-1} . The calving front stabilizes 9 km upstream for longer perturbations (Fig. 7d). This retreat rate decreases to zero for longer perturbations, so that the ice front approaches a new stable position upstream. Variation of the ice front position during the retreat doubles to up to g). The intra-annual variability of the calving front position doubles to ± 6.5 km. Average. The grounding line position is hardly affected by small calving rate perturbations, but large perturbations trigger fast retreats of several kilometres, which in turn cause drastic, but short-lived flow accelerations (Figs. 7g and 7h). The annual average ice velocity increases by about 10% , but its variation also almost and its intra-annual variability doubles to $\pm 38\%$. Occasional velocity spikes occur related to

~~ungrounding from pinning points. The ice front thins on average by a third, with oscillations (Fig. 7h). The mean calving front thickness decreases by 30% towards the end of the perturbation of experiment C4, and experiences large variations up to $\pm 75\%$. This high thickness variation variability is due to the calving back front retreating into areas of thick ice in summer followed by stretching and thinning during ice calving front advance in the late winter season. Variation of the grounding line position remains small during the perturbation, disrupted only by its discontinuous retreat for large perturbations. winter. For small perturbations (Fig. 7c), variation, variations of effective strain rates here quadruples at the grounding line quadruple to $\pm 25\%$ (Fig. 7f). Once the calving rate perturbation ceases, the ice front configuration displays a striking reversibility in all its variables. stops, all variables display remarkable reversibility.~~

When calving is temporarily turned off (experiment B0, not shown here), the response of the glacier is reversed: the calving front advances, creating a convex ice tongue. Meanwhile, the ice stream decelerates, thickens, and the grounding line advances. After the perturbation, the glacier retreats into a state slightly thicker and faster than the one of experiment B1.

[Figure 8 about here.]

~~Finally, Fig. 8a shows the evolution of the total ice volume as modelled against control run B1. The mean annual volume change due to ice volume with respect to experiment B1, the control run. The glacier in experiment B1 continues to lose volume at an average rate of $-22.8 \text{ km}^3 \text{ a}^{-1}$ due to the ongoing geometry relaxation of experiment B1 is $-22.8 \text{ km}^3 \text{ a}^{-1}$, varying by 10.5 km^3 per year. Experiment A undercuts this value by $-0.4 \text{ km}^3 \text{ a}^{-1}$ on average related. In experiment A, Jakobshavn Isbræ loses an additional $0.4 \text{ km}^3 \text{ a}^{-1}$, which corresponds to the gradual retreat of its ice the calving front. Enhanced calving causes additional volume loss proportional to perturbation measure P . It amounts to $-35.7 \text{ km}^3 \text{ a}^{-1} \Delta t (1 - p_0)$, the measure of the time-integrated calving rate perturbation (Fig. 8b). If the calving rate is doubled, the additional volume loss reaches $35.7 \text{ km}^3 \text{ a}^{-1}$ in the first year of perturbation of experiment C8, but decreases with time, as the ice calving front thins and retreats into areas of lower calving rates. Those numbers agree well with recent ice discharge obser-~~

vations (Howat et al., 2011). ~~After the~~ Over the first decade after the perturbation, all modelled glaciers recover ~~40-60~~ 40 to 60% of the volume ~~difference in the first decade, because during the readvance the thin ice front strongly reduces the calving flux. The calving flux converges to the value of experiment B1 once the ice front configuration approaches the state of the~~ deviation to the control run. ~~The ice volume now has to be replenished by excess surface accumulation, which takes longer than the 120 years of simulation time presented here. Calving rate perturbations therefore leave a lasting imprint on the ice sheet.~~

~~Results of experiments of decreased calving are not shown here. They exhibit the exact inverse pattern to the one described above for enhanced calving : the ice front advances during the perturbation, creating a convex ice tongue. Meanwhile, the ice stream decelerates and thickens, causing grounding line advance and volume increase. After the perturbation period, the glacier calves back into a state slightly thicker and faster than the one of B1.~~

5 Discussion

~~The characteristics of the applied calving rate determine the behaviour of~~ The applied calving rate determines the behaviour of the ice front. ~~The ice front will reach a stable configuration if calving front and the ice stream. In our simulations, larger perturbation strengths p_0 lead to faster calving front retreats. In the case of long perturbations (experiments C4 and C8), the calving front reaches a new stable position. A stable calving front position requires the calving rate~~ exceeds to be larger than the ice velocity ~~from some point along stream~~ onward. It will either retreat back or advance to this point. If no such point exists, if the calving front advances, and similarly, if the calving front retreats, the ice front will undergo either unconfined advance or retreat, depending on the magnitude of the calving rate. calving rate needs to be lower than the ice velocity.

~~Adjustments of the ice front configuration to the current calving rate change the local driving stress, which lead to corresponding changes in thickness and surface gradient, and vice versa. The mass transport mechanism allows the thickness change to diffuse~~

upstream. In case of ice front thinning and surface steepening, the increased driving stress leads to enhanced mass flux, causing further upstream thinning, steepening and acceleration.

We observe multiple secondary feedback mechanisms in addition to the mass transport mechanism, that determine the way the ice sheet adjusts to calving rate forcings. Several mechanisms determine how the model responds to the calving rate forcing. First, a calving rate increase leads to a retreat of the calving front position, ice stream acceleration and dynamic thinning in the vicinity of the terminus. Second, this dynamic thinning increases surface slopes and therefore the driving stress. As the glacier locally speeds up, the ice thinning propagates upstream. The ice stream thins much faster than the surrounding ice sheet, which steepens the surface across the shear margins. Lateral inflow of ice into the ice stream hence increases until it balances the calving flux. This limits the thinning of the ice stream. Thinning of the ice stream leads to reduced basal effective pressure and in turn leads to grounding line retreat and reduction in basal effective pressure, which both reduce basal drag significantly. Detachment of the base from pinning points in the vicinity of the grounding line. We showed that grounding line retreat leads to short-lived, but drastic increases in ice flux. These mechanisms are. This mechanism is qualitatively the same as the ones one described in Vieli and Nick (2011) and Joughin et al. (2012).

The ice front lengthens during its retreat, which increases the total driving stress exerted by the ice front. Side arms of the main tributaries are now calving directly into the fjord. This increases calving flux Q_{cf} , which additionally thins and steepens areas in the immediate ice front vicinity. Several pinning points along the retrograde trough of the southern branch, as well as the lateral stress transfer and mass influx prevent the modelled ice stream from being prone to the Marine Ice Sheet Instability (Weertman, 1974; Schoof, 2007), a hypothesis which states that grounding line positions are unstable on retrograde slopes. This corroborates earlier results by Gudmundsson et al. (2012), who presented examples of stable grounding line positions on retrograde beds. However, due to large uncertainties in the input data, and since some physical processes are not represented in our experiments, evaluation of this question for Jakobshavn Isbræ is beyond the scope of this study.

~~The steepening of the surface slopes across the shear margins caused by ice stream thinning results in increased driving stress, which increases lateral ice inflow into the ice stream. This in turn limits surface steepening and grounding line retreat along the trough. The feedbacks described above cause a net rise in volume flux towards the ice front and enable quick adjustment of the glacier in response to changes of the ice front configuration.~~ A third mechanism is related to the calving front lengthening during its retreat (e.g. figure 5). The lengthening causes tributaries of the main ice stream to calve directly into the fjord, thereby increasing the calving flux Q_{cf} (equation 8) and thinning of the terminus vicinity.

~~Acceleration of~~ Finally, the ice stream ~~and surface steepening in its vicinity strongly increase~~ accelerates faster than the surrounding ice sheet, which increases strain rates at the shear margins ~~and the grounding line. The non-linear rheology of ice softens~~. This reduces the ice viscosity in these areas, which mechanically decouples the ice stream from the ice sheet, allowing the ice stream to accelerate further, ~~and to soften its shear margins more and more~~. This positive feedback ~~is only~~ confines the initial thinning to the ice stream, and is controlled by the rate at which ice ~~is able to enter~~ enters the ice stream ~~to sustain the driving stress. We consider this mechanism to be substantial for sustaining the longer term~~. This mechanism is essential for enabling ice stream acceleration tens of kilometres upstream of the grounding line, since large fractions of the ice stream's driving stress ~~is~~ are balanced by lateral stress. This corroborates force balance arguments produced earlier by van der Veen et al. (2011).

~~Reduced lateral momentum transfer decreases lateral mass influx. This makes the surrounding ice sheet less sensitive to short periods of enhanced calving. For this reason, calving induced thinning mainly occurs in the ice stream, which limits the rate at which additional ice can be discharged.~~

~~Conversely, in case of no seasonal calving cycle of experiment A,~~ In experiments A and B1, we apply the same annual mean calving rate. However, due to the lack of seasonal cycle in calving rate the mechanical coupling between the ice stream and ice sheet is higher than compared to experiment B1. ~~Ice in experiment A. The ice~~ stream velocity is therefore lower, which causes net ice ~~causing gradual calving~~ front retreat and ~~corresponding additional ice~~

5 volume loss. This illustrates that ~~ice sheet models which do not include a seasonal calving cycle may overestimate mass loss of glaciers. Moreover, we do advise against the use of ice models, which do not incorporate~~ volume change estimates from models with and without seasonal cycles of calving may differ. Our results suggest that including both a dynamically evolving ~~ice front nor the related lateral effects, for~~ calving front as well as seasonal cycles are critical for accurate projections of future contributions of ice sheets to global sea level rise on decadal to centennial time scales.

10 Response mechanisms not covered here will likely include feedbacks ~~in~~ with damage mechanics and thermodynamics due to the increased strain rates. During longer perturbations, ice surface lowering will probably affect the surface mass balance and the drainage basin outline.

15 The reversibility of the ~~ice calving~~ front configuration after the calving rate perturbation is a robust feature across all experiments. ~~We see the~~ The short duration of the perturbation, the prescribed calving rates ~~and the geometric setting of the ice stream to be,~~ and the geometry of the glacier are responsible for this behaviour. ~~Volume change relative to experiment B1 is mainly due to the migration of the ice front and the according change in lateral ice sheet extent. However, the additional ice discharge~~ The volume change in all experiments never exceeds 0.1% of the ~~total~~ initial glacier volume in the experiments shown here. ~~This volume loss can be easily balanced by the vast~~ Once the perturbation stops, the surrounding ice sheet ~~. Since calving rates are not coupled to ice dynamics, the ice front configuration is able to quickly recover once this forcing is set back to its initial value~~ continues to replenish the ice stream, which allows for its quick recovery.

25 The modelled glacier response ~~due~~ to enhanced calving is in good qualitative agreement with observations ~~-,~~ which corroborates that calving is a major control on this glacier. The similar shape of the modelled and observed calving front suggests that calving rates are indeed proportional to its flow speed during the glacier's current retreat. However, the reversibility of the ~~ice~~ modelled calving front position is in ~~stark~~ contrast to Jakobshavn Isbræ's ~~ongoing observed retreat. Therefore, the actual calving rates must have stayed increased~~ actual behaviour. Sustained high calving rates are therefore necessary to explain

the continued retreat of the glacier, as our results suggest that the glacier would have readvanced otherwise. Accurate model input data, representation of all relevant physical processes and incorporation of a suitable calving rate parametrisation will be necessary for quantitative analysis of this dynamic ice stream.

6 Conclusions

~~We presented~~ In this study, we present the theoretical framework for coupling a LSM Level-Set Method (LSM) to ice dynamics and ~~implemented it into the~~ implement it into ISSM. The LSM ~~proved~~ proves to be a robust method for modelling the dynamic evolution of ~~an ice front~~ subject to ablation. We applied the a calving front. We apply this technique to Jakobshavn Isbræ using prescribed calving rates. ~~We find its dynamics to be~~, and we find that the glacier is highly sensitive to ~~the applied calving rate~~ this forcing, which agrees well with observations.

~~The calving rate perturbation strongly affects the ice flow inland through various mechanisms. Changes~~ Calving rate perturbations strongly affect the ice stream through several linked mechanisms. First, changes in calving rate cause ice calving front migration and alter ~~its~~ the ice discharge. ~~The~~ Second, the resulting thickness change at the ~~ice front~~ diffuses upstream through a coupling between the stress balance and mass transport mechanism. Ice stream thinning reduces basal drag by means of calving front spreads out to the surrounding ice sheet. Third, thinning-induced grounding line retreat ~~and reduction of effective basal pressure. Rheological~~ causes further ice stream acceleration and creates a positive feedback. Finally, shear margin weakening caused by the ice stream acceleration decreases lateral drag ~~resistive to flow. These two positive feedback mechanisms are able to sustain prolonged~~ resisting ice flow. This positive feedback mechanism sustains significant acceleration of the ice stream ~~, and are only controlled by the rate at which ice can enter the ice stream. However, the vast~~ tens of kilometres upstream of the grounding line.

The surrounding ice sheet is barely affected by short periods of enhanced calving. It stabilises the ice stream and allows for quick reversibility of the ~~ice front configuration through~~

~~lateral mass calving front position through lateral ice~~ influx and stress transfer once ~~we set~~ the calving rates ~~are set~~ back to their initial ~~value~~. ~~The importance of the ice values.~~

Since the calving front position and ~~lateral effects for dynamic lateral effects~~ are critical to simulate and understand the behaviour of ~~this type of glacier lets us advise against~~ the use of models, which do not represent these mechanisms, for projections of future ice discharge.

The method presented here is able to close a major gap present in ISSM and various other marine terminating glaciers, the inclusion of moving boundaries in planview and 3D models is key for realistic sea level rise projections on centennial time scales. This method is a step towards better physical representation of calving front dynamics in ice sheet models. It enables a multitude of future studies in combination with e.g. thermodynamics and damage mechanics, and will improve our understanding of outlet glaciers.

Appendix A: ~~Test-Setup~~

Appendix: Validation of the Level-Set Method

We present ~~a simple test setup~~ two simple test setups to validate the LSM. The first is designed to show the accurate advection and shape preservation properties of the method. The second setup aims to give an estimate for the volume change introduced by the LSM for different mesh resolutions.

1 Advection

Let Ω be a 50 km square with ~~an~~ the initial LSF as:

$$\varphi_0(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_0\|_2 - R,$$

where $\mathbf{x}_0 = (25, 25)$ km and $R = 12.5$ km, so that ~~our~~ the initial 0-level-set describes a circle in the middle of the domain. We prescribe a constant velocity of ~~$\mathbf{v} = 1 \text{ km a}^{-1} \cdot (\cos(\pi/4), \sin(\pi/4))$~~

$\mathbf{v} = (\cos(\pi/4), \sin(\pi/4)) \text{ km a}^{-1}$ everywhere. We advect φ_0 over 10 years with time steps of 0.1 a, and keep track of ~~its~~ the 0-level-set.

[Figure 9 about here.]

~~We plot Fig. 9 shows~~ the 0-level-set position at the beginning ~~every year in Fig. 9. The~~ of every year. The LSM preserves the initial circular shape ~~is preserved and advected with~~ constant speed, and can be used to model both advance and retreat of a calving front.

[Figure 10 about here.]

We measure the advection speed of the 0-level-set along the diagonal marked in white in Fig. 9. Fig. 10 shows the standard deviation of ~~its~~ the numerical error relative to the prescribed advection speed taken over time for different element sizes. The numerical error is ~~mainly related to the interpolated representation due to the linear interpolation~~ of the curved shape on an irregular mesh. This causes, which causes variations of the level-set velocity around the front velocity to vary around the mean value, which indeed equals the prescribed velocity prescribed value. The standard deviation of the error linearly decreases with mesh ~~width~~ resolution, and drops below 1% for elements sizes below 0.5 km. We therefore recommend using ~~an element size of a mesh resolution~~ below 1 km in the ice calving front vicinity for ice sheet simulations.

2 Volume conservation

Let Ω be a $200 \times 20 \text{ km}^2$ rectangle with an initial LSF given by:

$$\varphi_0(\mathbf{x}) = (1, 0) \cdot \mathbf{x} - 100 \text{ km}.$$

The initial lateral extent is thus a $100 \times 20 \text{ km}^2$ rectangle. The geometry corresponds to the Ice Shelf Ramp presented in Greve and Blatter (2009). The ice thickness linearly decreases from 400 m at the grounding line ($x = 0 \text{ km}$) to 200 m at the calving front ($x = 100 \text{ km}$). We

5 apply zero surface accumulation and basal melt, as well as zero grounding line velocity and free slip boundary conditions at $y = 0$ km and $y = 20$ km. The ice sheet spreads under its own weight for 100 years. Fig. 11 shows the evolution of the ice volume for different element sizes. All simulations show volume loss due to the free flux boundary condition at the numerical ice front, which is not entirely balanced by the volume added through the ice thickness extrapolation. The volume loss decreases with element size, and is below 0.2% of the initial ice volume after 100 years for an element size of 1 km. This volume loss is far below current uncertainties of other model input data.

[Figure 11 about here.]

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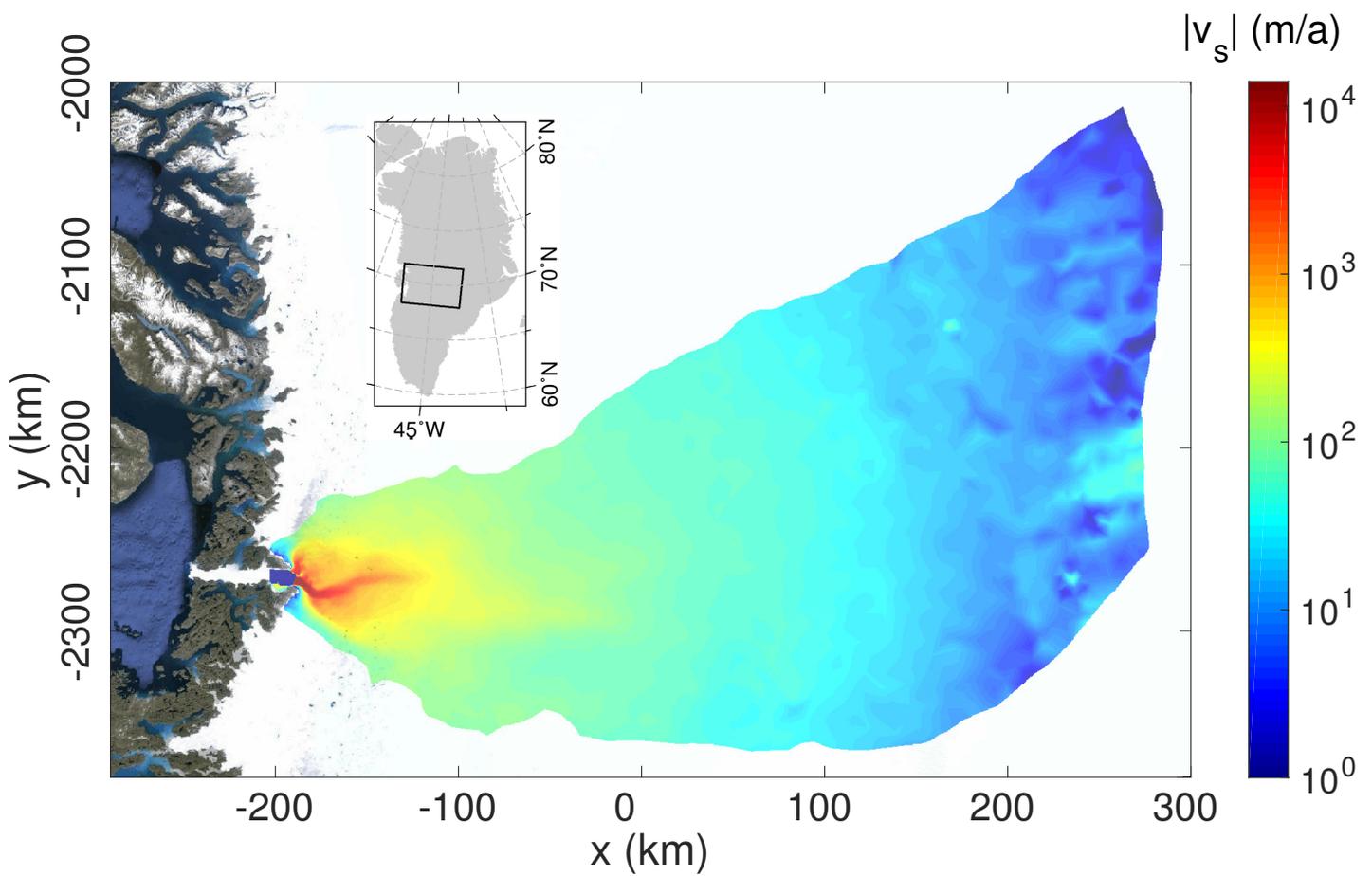


Figure 1. Observed ice surface velocities 2008/2009 (Rignot and Mouginot, 2012) of the Jakobshavn Isbræ drainage basin **used for modelling** (logarithmic scale). Background image from Google Earth ©.

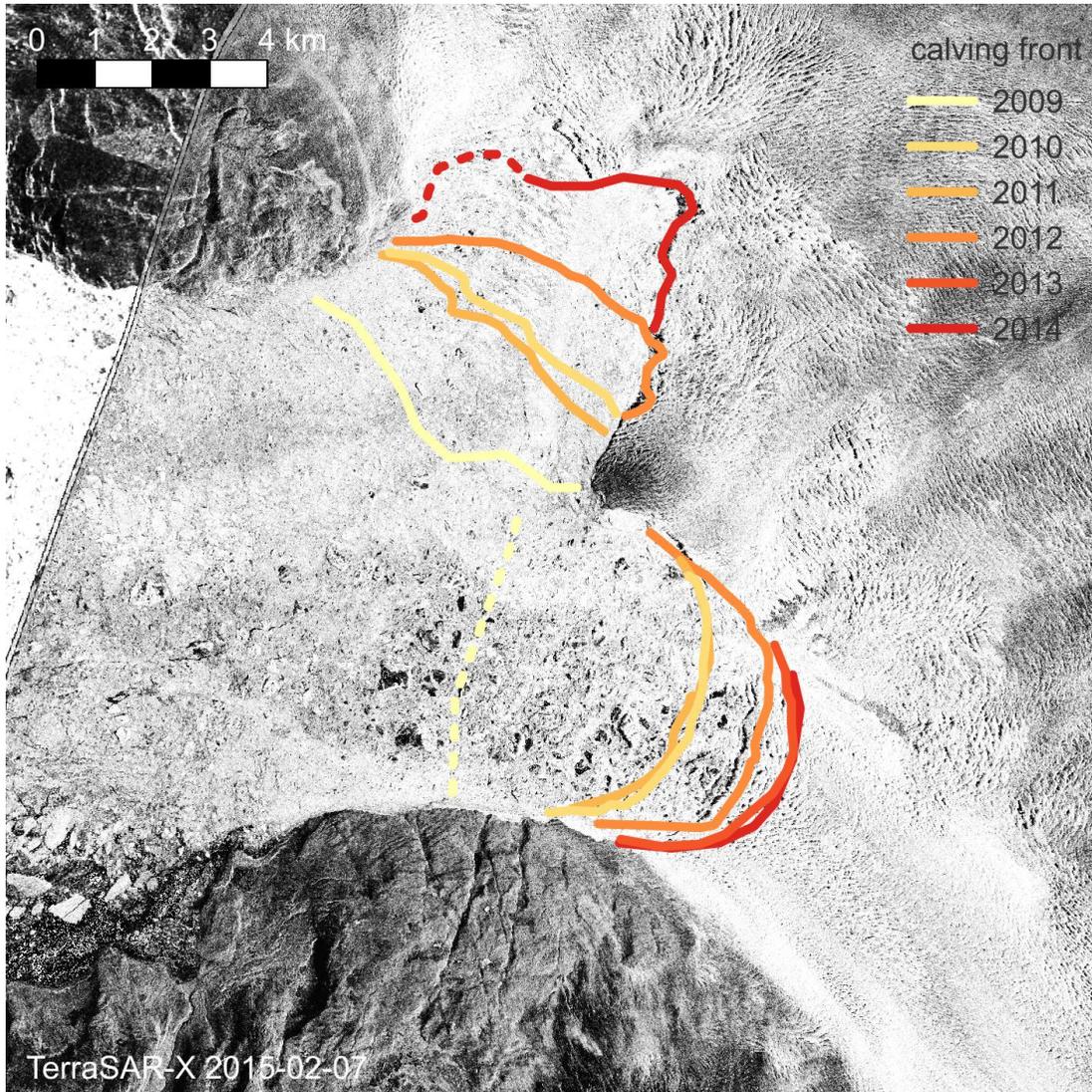


Figure 2. Winter (Feb-Mar) ice calving front positions from 2009 to 2014 superimposed overlaid on a TerraSAR-X scene from 2015-02-07 (© DLR). Striped-Dashed lines are used in case of ambiguous ice calving front positions.

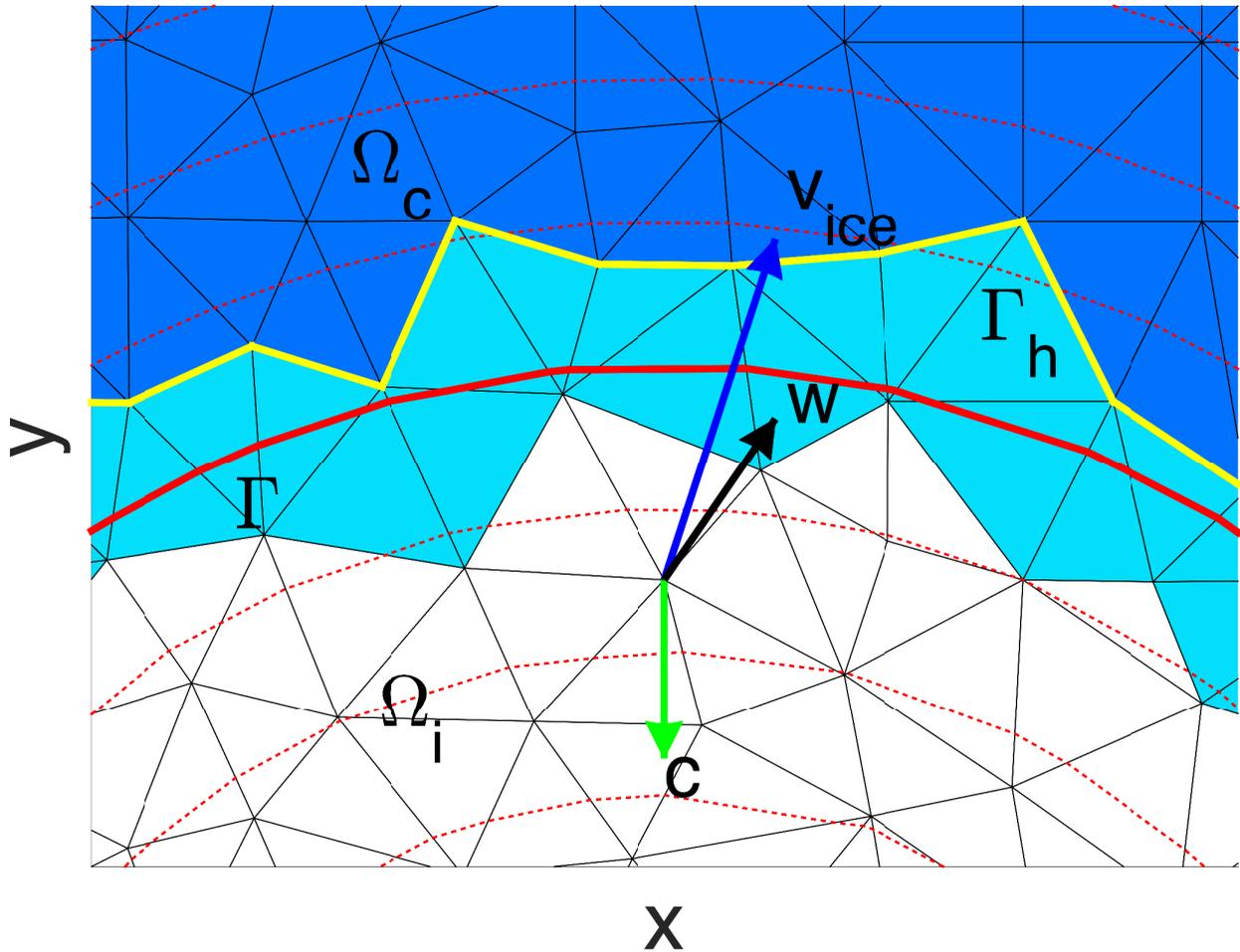


Figure 3. Schematic of the numerical ice margin. The red dashed lines denote different contour lines (level-sets) of the LSF φ . The thick red line marks the zero-level-set, Γ , the yellow line the numerical ice-calving front Γ^h . Dark blue triangles are ice-free elements, white ones are ice-filled ones and the light blue ones are the front elements. The three vectors show an example of the level-set velocity $w = v + c$ at a finite element node.

The three vectors show an example of the evaluation of the boundary velocity w at a finite element node.

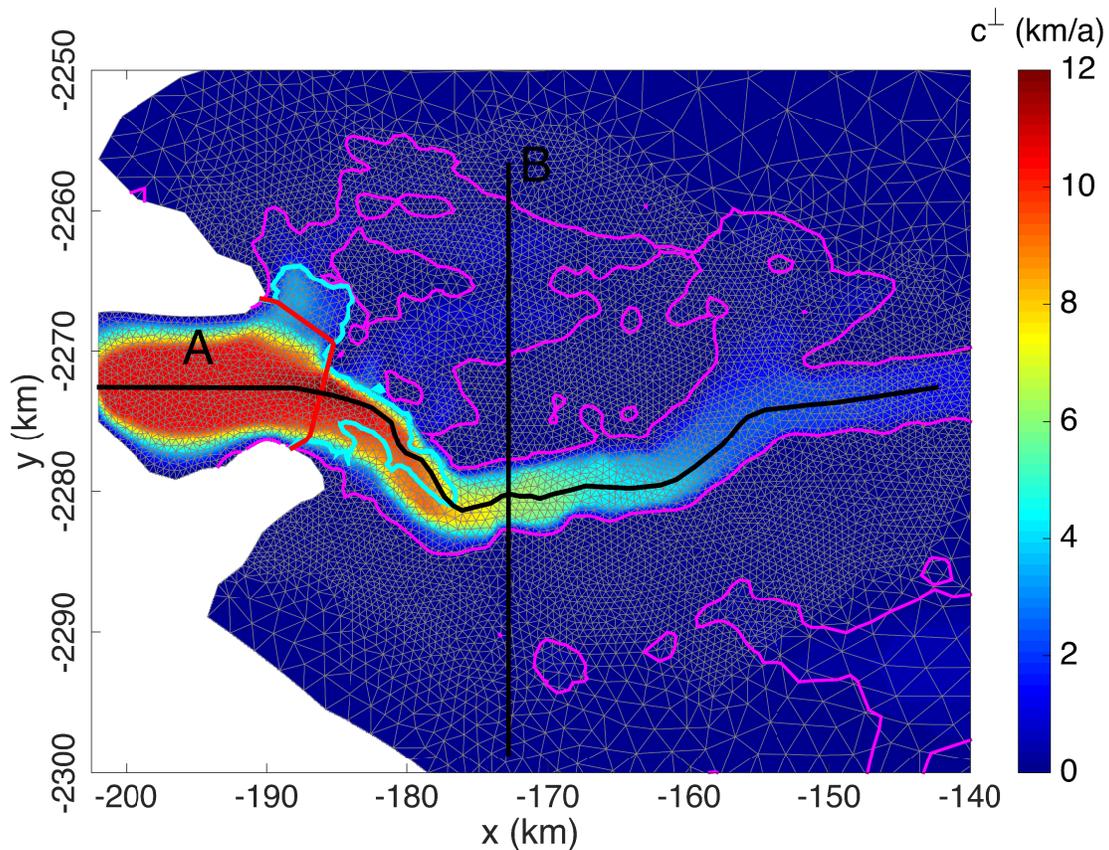


Figure 4. The applied calving Calving rate field c_0^\perp in the region of fast flow, which has been derived from modelled ice velocities at the end of the geometry-relaxation run. The red line indicates the zero level-set of the initial LSF used for geometry relaxation and as start position for the ice calving front during the experiments. The turquoise line marks the grounding line. Purple contours indicate zero bedrock elevation. Black lines are the tracks “along-trough” (A) and “across-trough” (B) profiles used in Fig. 6. The start points of tracks A and B are the western and northern end, respectively. The finite element mesh is displayed in grey.

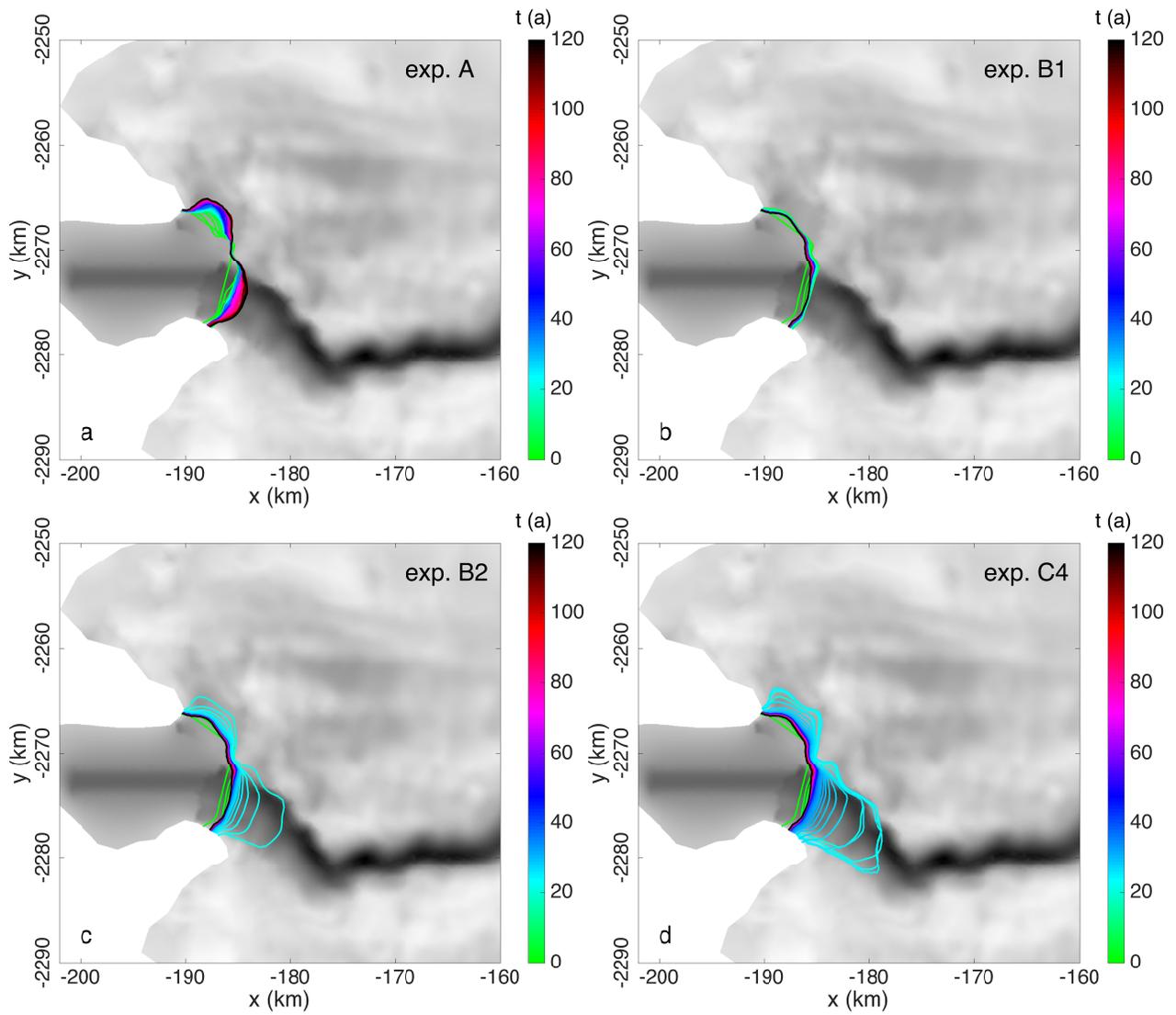


Figure 5. Ice Calving front position-positions for experiments A, B1, B2 and C4 at the start of each year, plotted over basal topography (grey).

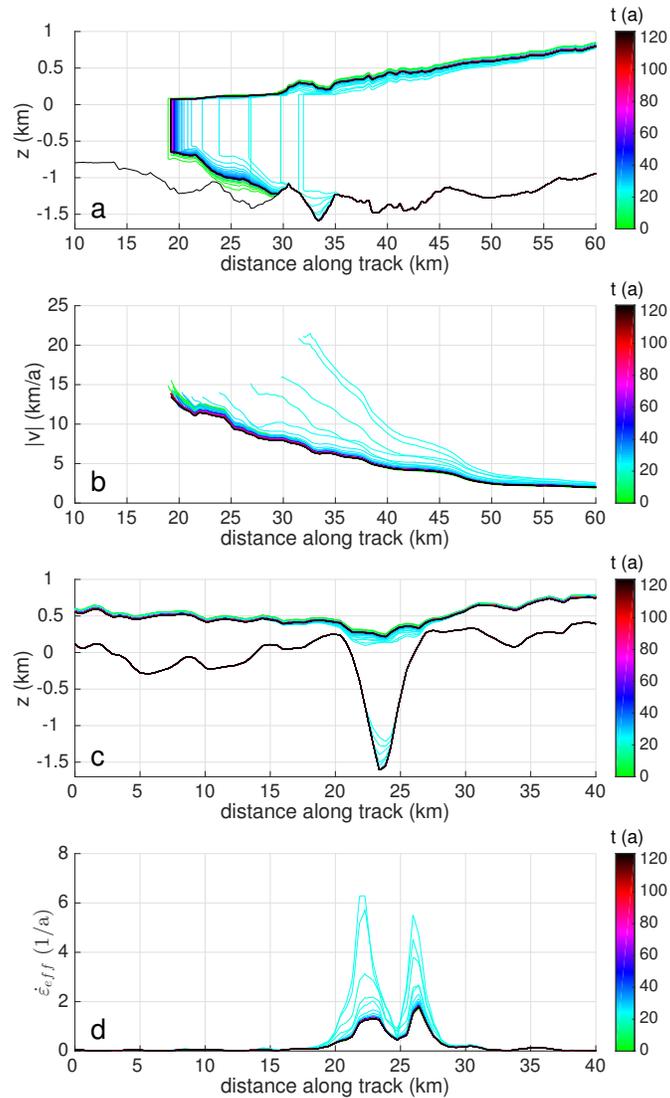


Figure 6. Section values of profiles for experiment C4 at the end of the calving season in October each year. a) ice geometry and b) ice velocity along-trough. c) ice geometry and d) effective strain rates across-trough. Positions of the tracks-lines are given in Fig. 4.

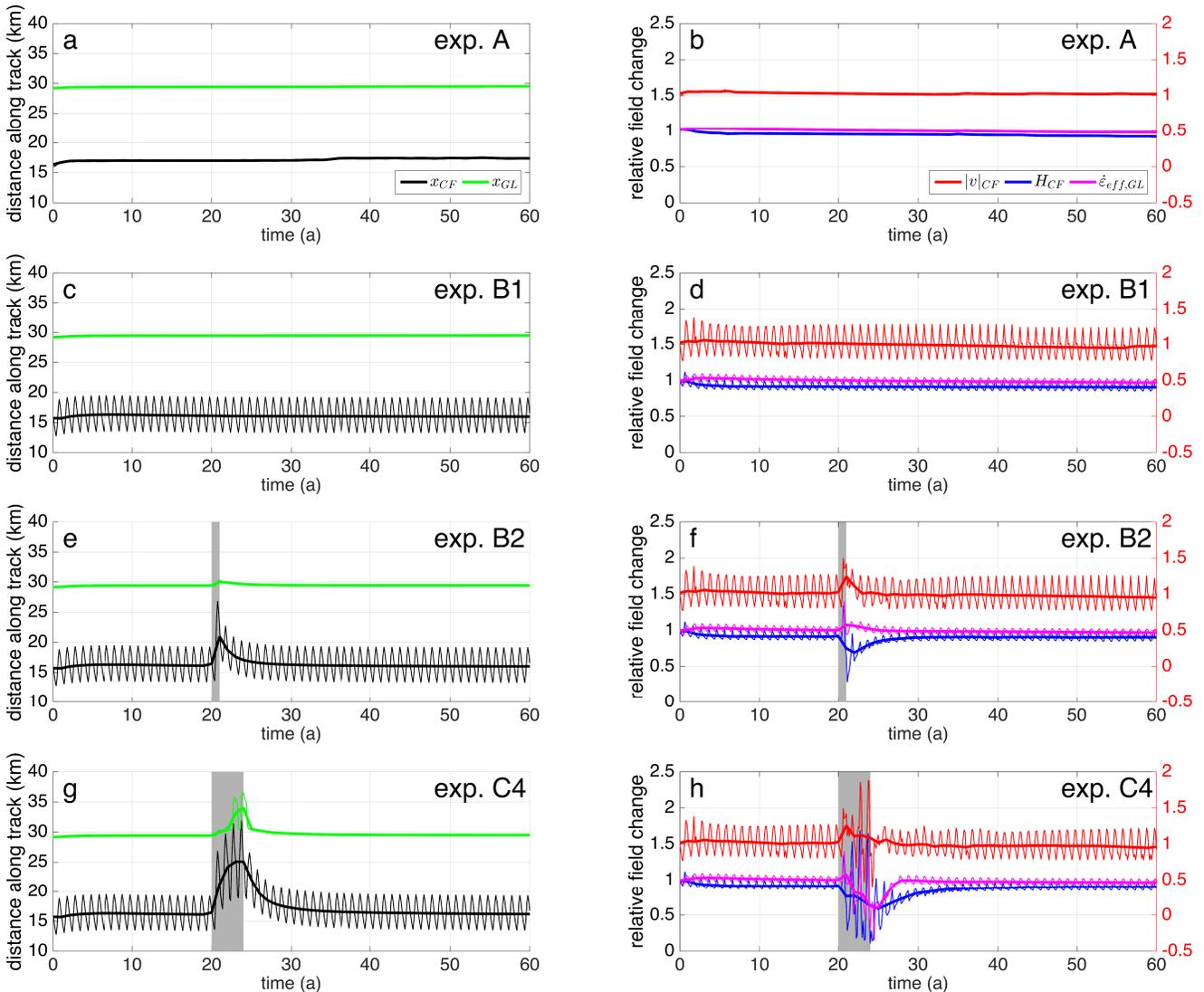


Figure 7. Ice Calving front and grounding line position-positions along-trough (left column), as well as values of ice calving front thickness, ice velocity and effective strain rate relative to their initial value along-trough (right column) over time for experiments a) A, b) B1, c) B2 and d) C4. Perturbation intervals are marked in grey. Relative values for ice velocity has have been shifted up by 0.5 up for better visibility (red y-axis).

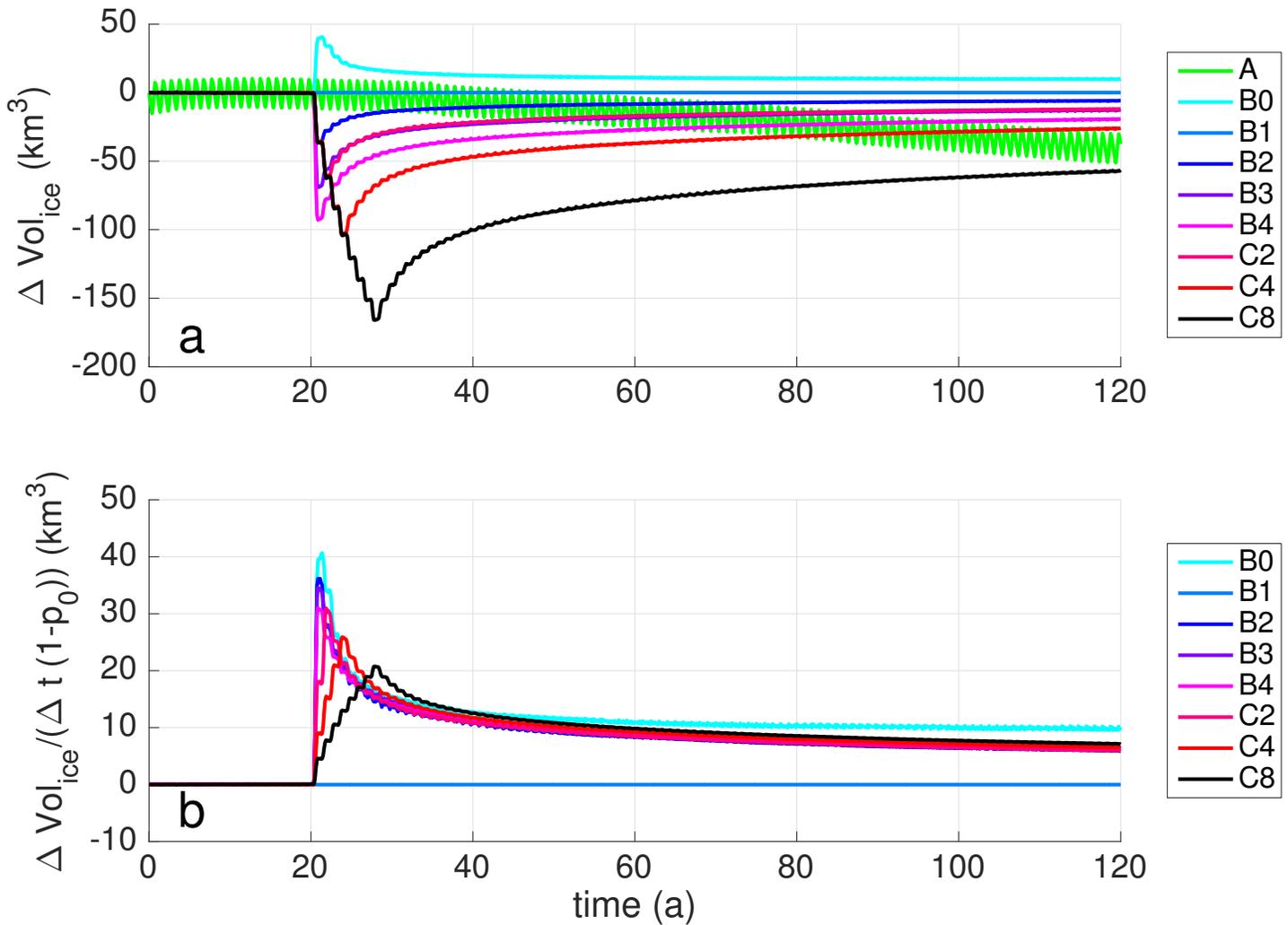


Figure 8. Difference a: Absolute difference in total ice volume of for the different simulations with respect to experiment B1 over time. The smooth non-oscillating ice volume profile of experiment A causes its difference to experiment B1 to oscillate. b: The volume differences from experiments B and C divided by $\Delta t(1 - p_0)$, the measure of the time-integrated calving rate perturbation.

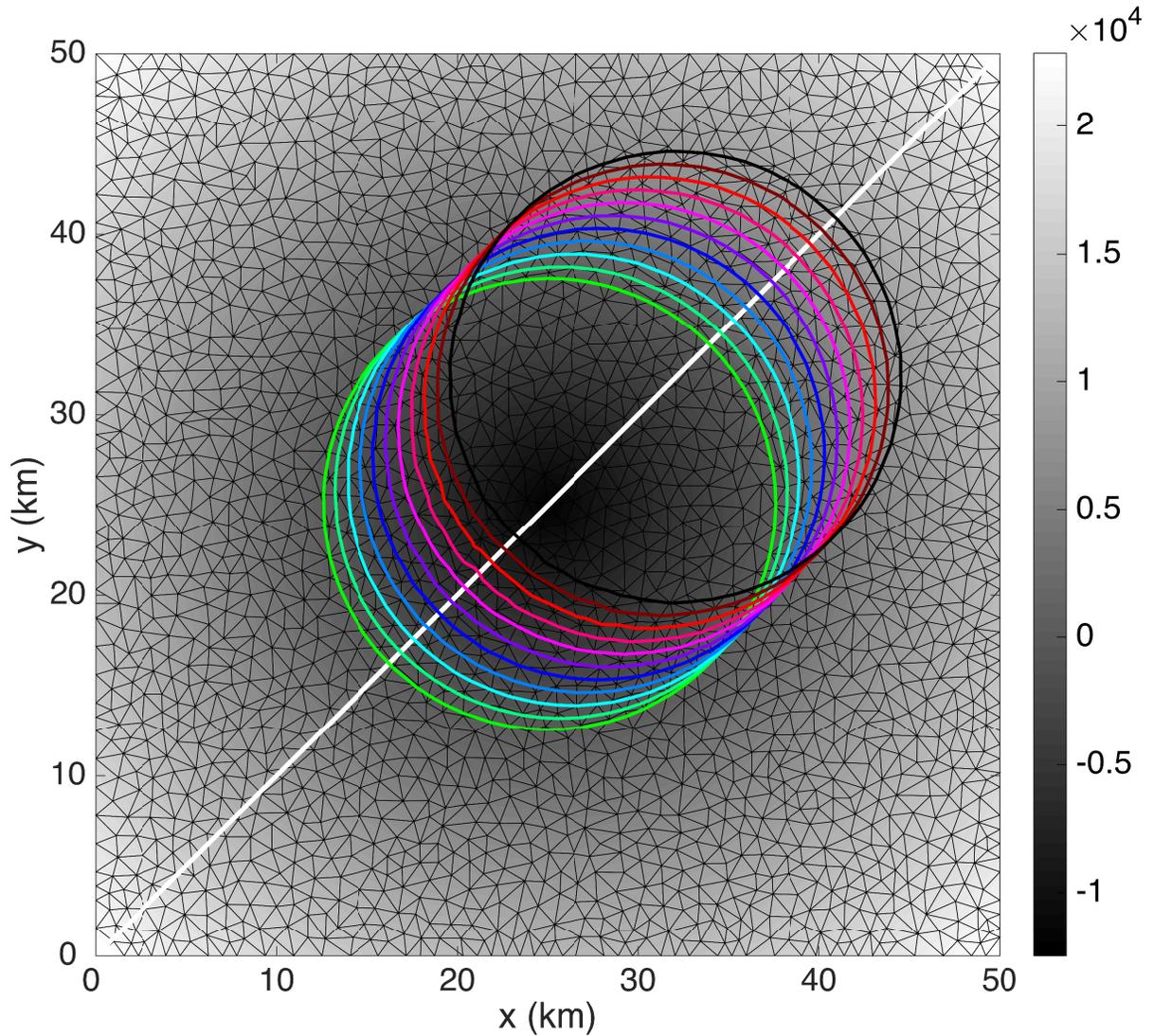


Figure 9. 0-level-set position Zero-level-set positions at the start of every year, plotted over φ_0 , which is shaded in grey scale. An example of the mesh with element size 1 km is marked in black. The white diagonal marks the straight line along which the velocity of the 0-level-set is tracked.

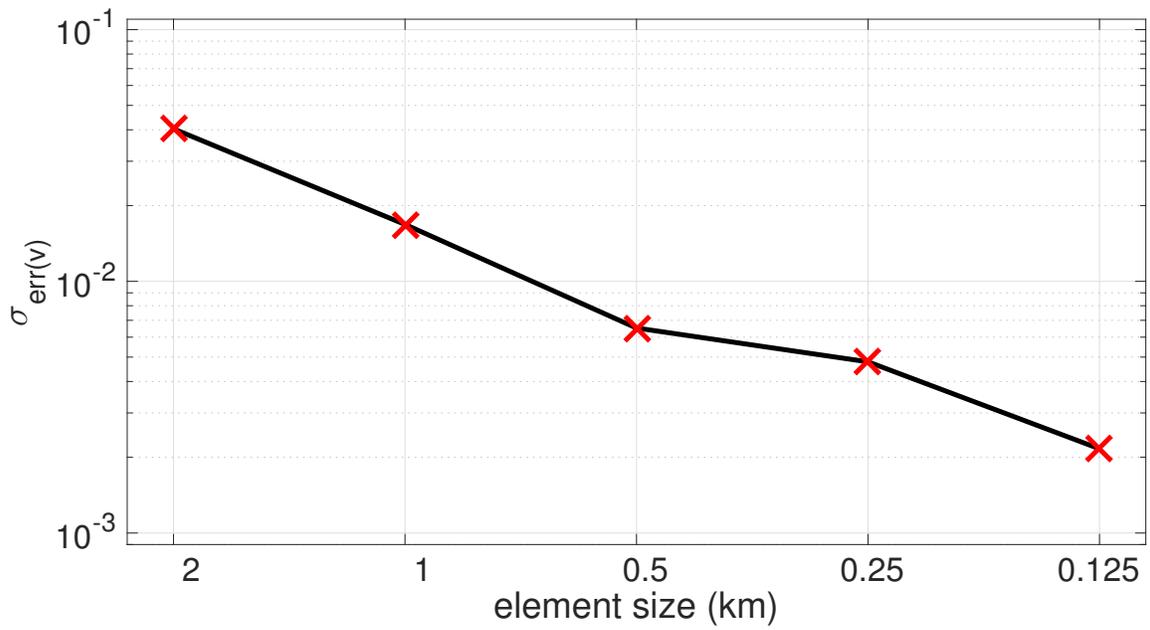


Figure 10. Standard deviation of the relative numerical error in advection velocity of the 0-level-set depending on mesh element size.

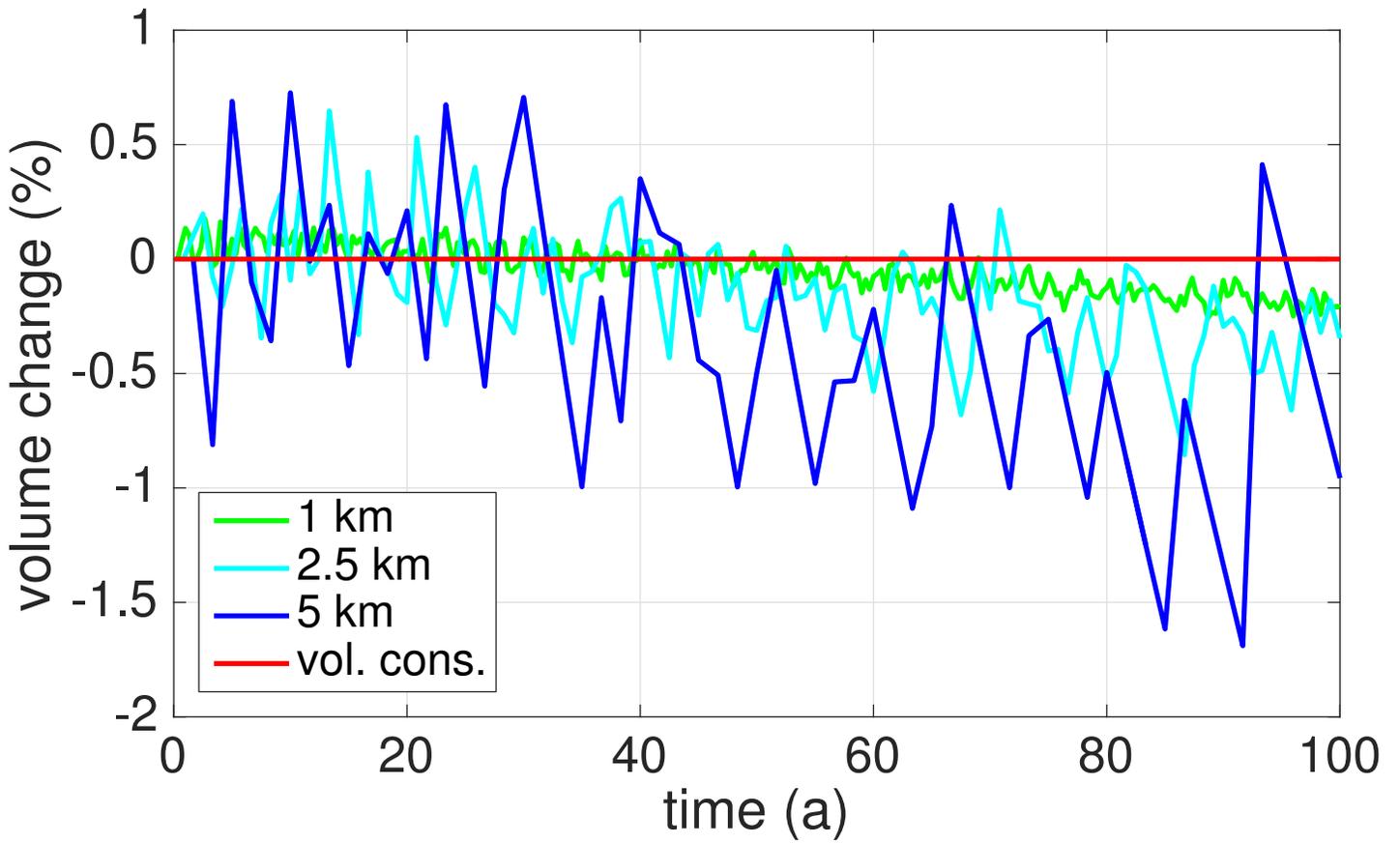


Figure 11. Evolution of the relative ice volume change for different element sizes. The red line shows volume conservation.

Table 1. Symbols and model parameters

Symbol	Quantity
μ	Ice viscosity
B	Ice viscosity parameter
$\dot{\epsilon}_e$	Effective strain rate
n	Glen's flow law parameter
α	Basal friction parameter
N	Effective basal pressure
\mathbf{v}	Depth-averaged <u>horizontal</u> ice velocity
H	Ice thickness
a_s	Surface mass balance
a_b	Basal mass balance
Ω	Computational domain
Ω_i	Ice domain
Ω_i^h Numerical ice domain Ω_c	Ice free domain
Γ	Ice boundary
Γ^h	Numerical ice boundary
φ	Level-Set Function
\mathbf{n}	Unit surface normal on Γ
\mathbf{w}	Ice front <u>Level-set</u> velocity
\mathbf{a}	Ablation velocity
a^\perp	Ablation rate
\mathbf{c}	Calving velocity
c^\perp	Calving rate
s	Scaling function
p	Perturbation function
Δt	Perturbation duration
p_0	Perturbation strength
L	Seasonal calving period length
ϕ_0	Phase shift
P Perturbation measure Q_{cf}	Calving flux

Table 2. Table of experiments

<u>Name</u>	<u>p_0</u>	<u>Δt</u>	<u>Name</u>	<u>p_0</u>	<u>Δt</u>	<u>Name</u>	<u>p_0</u>	<u>Δt</u>
<u>A</u>	<u>1</u>	<u>0</u>	<u>B2</u>	<u>2</u>	<u>1</u>	<u>C2</u>	<u>2</u>	<u>2</u>
<u>B0</u>	<u>0</u>	<u>1</u>	<u>B3</u>	<u>3</u>	<u>1</u>	<u>C4</u>	<u>2</u>	<u>4</u>
<u>B1</u>	<u>1</u>	<u>1</u>	<u>B4</u>	<u>4</u>	<u>1</u>	<u>C8</u>	<u>2</u>	<u>8</u>