Report 1:

Over the four main major points I raised in general comment, the authors only really gave answers to one (overall recommendations for the prediction method) in this revised version of the manuscript. The discussion about the value of lambda and its impact on the prediction time was not improved. The authors have not made any effort to adopt a probabilistic approach arguing that uncertainty on the data are insignificant in comparison to uncertainty on the fitting method. But why uncertainty on the fitting method could not be included in the probability calculation? Authors also argue that a probabilistic approach in too complex for real time estimation. I think it is not true, the problem is quite simple (fitting data with an analytical model...).

This is exactly what we have done...

We have fitted the data with analytical models (Eq. 2 and 3), and compared the respective uncertainties. In this new version, we have now improved the statistical analysis by including both uncertainty resulting from the data accuracy and the fitting method (see new Table 2). We have to admit that we did not fully understand the probabilistic method proposed by the reviewer: He proposed to define a probability density function for the threshold velocity and evaluate the density function of the rupture time for each threshold to finally obtain a final probability density function for the rupture time.

1. We have no way to know at which surface velocity the real break-off will occur.

2. The velocity threshold proposed in this paper to better evaluate the real break-off is just the time at which the modeled/fitted (with eq. 2 or 3) surface velocity exceed an arbitrary threshold. The critical time evaluated with additional surface threshold is also only depending on the initial fit; Once the parameters are evaluated to fit the data at best, they do not change when evaluating v50 or v100. In other words, $T_R(M)$ is INDEPENDANT of V_T .

The transferability of the method to another glacier is still not really discussed. The authors argue that all hanging glacier have similar geometry and the method already works for three of them. I think it is not enough when we know that the dependence of λ to geometry, grain size and loading are still unclear.

The reviewer is right, the dependence on λ to geometry, grain size and loading was still unclear. We reanalyzed the data according to the reviewer 2 proposition. The new results enabled to propose a better interpretation (see lines 245-274 and new figures 7a and 7b), that does not account for change in geometry, grain size or loading rate.

I still think the manuscript deserve publication in The Cryosphere but I am disappointed by the effort made by the author in this revised version. I would not recommend to reject the manuscript but I think this manuscript could be improved. It could be more that only a validation of a method already developed in Failletaz et al. [2008] if the author would make some effort on the treatment of the rupture time using the fitting method and better analyse the role of λ in the prediction (see also specific comment n°2).

OK, see changes

Specific comments

• Table 2 : why uncertainty on tc decrease when adding nose on the data ??

We only reported the uncertainties due to the data inaccuracy. Now we have also added the error produced by the fitting procedure. (See lines 224-230)

• Section 5.2 and figure 7 : Is the spectral peak at $\lambda=2d$ still exist after the first break off? It seems that the lomb periodogram for stakes 13 and 2 have been calculated on a time windows that include the period before the first break off. It means that the peak at $\lambda=2d$ arise from this pre-break off period and is maybe totally absent after the break off. It would mean that λ depend totally of the geometry and loading rate. The author have to clarify this.

As explained above, we have now investigated separately the period before and after the first break-off. The peak at $\lambda=2d$ is still present after the first break-off combined with other harmonics, indicating thus geometry and loading have probably no real influence on this behavior. See our new interpretation lines 245-274.

• Line 96-98: Same comment. I would say something here about "the correction technique".

The correction technique is further described in the results section lines 268-276

• Line 165: Say why and how you are accounting for flow direction?

See lines 268-276

• Line 216-218: The sentence is still unclear. "Scale" is repeated five times in one sentence ...

See lines 230-238

• Line 234: Fig 7 (not Fig 7b)

OK

Report 2:

The organization and readability of the manuscript have been improved. The authors have addressed some of the concerns and suggestions posed by each of the three reviewers. Many of the suggestions provided in the initial stage of review would have represented major changes to the manuscript. However, I would characterize the changes made by the authors as relatively minor. I view this as a missed opportunity for making a more substantial contribution to the literature on geohazard prediction, however I believe that with some additional minor changes the manuscript might be acceptable for publication in the Cryosphere.

1. The physical insight regarding the source of the log-periodic oscillations, specifically the initiation, interaction, and evolution of microcracks near the base of the glacier, is based partly on model results and partly (perhaps) on observations (e.g. Pralong and Funk, 2006). I think the damage accumulation model is quite plausible physically. However, the description in the manuscript is somewhat misleading in several places, as it sounds as if the location of failure ("typically a few meters above the bedrock", line 43; point 5 on lines 121-122) is observed. I made a similar point in my initial comments, and I'm not sure it was really addressed: did you gather any actual observations or have any other proof that this scale is "several meters" or otherwise not on the bedrock? What observations, and from where, indicate this length scale (if any)? Is this based entirely on the damage model of Pralong and Funk (2006)? (the authors of this reference clearly state that observations of the fracture process at the base of the glacier are not possible).

It is difficult to observe the rupture pattern, except on the presented picture (Fig. 2) and Fig 12d in Pralong and Funk (2006). Based on visual inspection of theses pictures (and also the pictures from the Weisshorn 2005 break-off event), it is clear that the rupture occurred within the ice immediately above the ice/bed interface. The most likely length scale is several meters above the bedrock. As this fact is problematic for the reviewer, we changed. See now line 122-126.

The extra description of the physical failure mechanism that the authors added (lines 114-125) is welcome, although this again seems to mix observations of the Whymper and Weisshorn events with model insights. I do not think that the authors are being intentionally misleading, but it should be made more clear which processes are based on observed (or observable) phenomena and which are based on insights gleaned from physical models.

This section is only based on our observations and does not rely on modeling results.

2. The 0.4 m/day velocity threshold suggested as a conservative estimate still seems arbitrary, since 0.5 m/day and 1.0 m/day thresholds were used in the analysis (and 0.5 m/day seemed to be quite good for predicting the break off). Why suggest a threshold for future forecasts if there is no insight in the manuscript that can be gleaned from the predictive skill of this threshold?

OK, following reviewers advice, we removed this 0.4m/day threshold.

3. The term "significantly" still needs to be removed in several places (line 193, line 198, line 320). Using this term to describe differences or improvements implies a statistical analysis that quantitatively demonstrated a statistically significant change. Since you did not perform such analyses, the use of this terminology is misleading.

OK, done.

4. The difference in fits between the power-law and log-periodic relations is quite negligible (Figs 5 and 6), in terms of the residuals. The differences are hardly greater than the measurement accuracy, and it is doubtful if a statistical analysis would show a statistically significant difference. I think you can make a plausible argument, on the basis of the damage accumulation modeling, that the log-periodic relation is more appropriate physically. However, I don't think that plotting the residuals makes much of a case here.

No, this is a key finding. The residuals are much lower for logperidic fit (on the order of data accuracy!), as shown in the new figures Fig. 5c and Fig 6c. Damage accumulation modeling just confirmed that the log-periodic relation is physically justified (Pralong, 2006).

5. Why did all of the results in Table 1 change? What have you done differently here? It's not clear from reading your responses to the reviews. Most of the changes are relatively small, but some of the parameter values changed quite a lot.

We just removed few outliers in the initial data.

6. It's not clear from Figure 7 that the large low-frequency peaks in Lomb power for Stakes 13 and 2 (f Lomb = 0.5) are after the first break off. These are clearly the strongest peaks, and which draw the reader's attention. In the text (lines 231-233) it reads as if you have focused on the smaller peaks at f Lomb = 2 d according to what you already expected (since these are the peaks you describe first, which seems odd if the reader looks at the figure and first sees the obvious and much bigger peaks for f Lomb = 0.5). If you can indicate dates somehow on the figure, or describe the bigger peaks first, then it might seem less like you are searching for an answer that you already know.

See the new section line 245-274.

7. Figure 8 (former Figure 7) is still quite confusing. There must be a better way of presenting these results, as I think this is a very important part of describing the real-time descriptive skill of the different methods and choices for the velocity threshold. There's a lot of good information in the figure, but I'm still struggling to understand it. The casual reader will invest much less time in figuring it out than I have.

We changed the figure and hope that it is clearer now (see new figure 8).

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Time forecast of a break-off event from a hanging glacier

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Abstract.

A cold hanging glacier located on the south face of the Grandes Jorasses (Mont Blanc, Italy) broke off on the 23rd and 29th September 2014 with a total estimated ice volume of 105.000 m³. Thanks to accurate surface displacement measurements taken up to the final break-off, this event was successfully predicted 10 days in advance, enabling local authorities to take the necessary safety

5 successfully predicted 10 days in advance, enabling local authorities to take the necessary safety measures. The break-off event also confirmed that surface displacements experience a power law acceleration along with superimposed log-periodic oscillations prior to the final rupture. This paper describes the methods used to achieve a satisfactory time forecast in real time and demonstrates, using a retrospective analysis, their potential for the development of early-warning systems in real time.

1 Introduction

Rockfalls, rock instabilities due to permafrost degradation, landslides, snow avalanches or avalanching glacier instabilities are gravity-driven rupture phenomena occurring in natural heterogeneous media. Such events have a potential to cause major disasters, especially when they are at the ori-

- 15 gin of a chain of processes involving other materials such as snow (snow avalanche), water (flood) and/or debris (mudflow). The reliable forecasting of such catastrophic phenomena combined with a timely evacuation of the endangered areas is often the most effective way to cope with such natural hazards. Unfortunately, accurate time prediction of such events remains a somewhat daunting task as (i) natural materials are heterogeneous, (ii) the heterogeneity is difficult to quantify and measure,
- 20 and (iii) the rupture is a non-linear process involving such heterogeneities. Although often located in a remote high-mountain environment, avalanching glacier instabilities offer an interesting starting point for investigating early-warning perspectives of break-off events, as a glacier consists of a single material (ice) lying on well-defined bedrock. This relative simplicity of the system allows the focus to be placed on the rupture processes leading to the initiation of the instability. Recently,
- 25 considerable efforts in monitoring, analyzing and modeling such phenomena have led to significant

advances in understanding the destabilization process and in improving early-warning perspectives (Faillettaz et al., 2015).

In general, it is possible to distinguish three types of avalanching glacier instabilities according to the thermal properties of their ice/bedrock interface (Faillettaz et al., 2011b, 2012, 2015). If temper-

- 30 ate, the presence of liquid water in the glacier plays a key role in the initiation and the development of the instability as its presence influences the basal properties of the ice/bedrock interface (diminution of friction, lubrication or loss of support). In such cases, several preliminary conditions to be fulfilled can be identified, but an accurate time forecast of an impending break-off event is still far from being possible. If the ice/bed interface experiences a transition from cold to temperate, the presence
- 35 of melt water may reduce the basal resistance, which promotes the instability. No clear and easily detectable precursory signs are known in this case, and the only way to infer any potential instability is to monitor the temporal evolution of the thermal regime. If the ice/bedrock is cold, glaciers are entirely frozen to their bedrock. This situation appears in the case of high altitude hanging glaciers located entirely in accumulation zone. The snow accumulation is mostly compensated by periodic
- 40 break-off of ice chunks (Pralong and Funk, 2006), occurring once a critical point in glacier geometry is reached. The instability results from the progressive accumulation of internal damage due to an increasing stress regime caused by glacier thickening. In this case, the rupture occurs within the ice, typically a few meters immediately above the bedrock (see Fig. 12 d in Pralong and Funk (2006)). The maturation of the rupture was shown to be associated with a typical time evolution of both sur-
- 45 face velocities (Faillettaz et al., 2008) and passive seismic activity (Faillettaz et al., 2011a). This characteristic time evolution can theoretically be used to predict the occurrence of a catastrophic event. This was done a posteriori with data obtained prior to the 2005 break-off of the Weisshorn glacier.

In this context, the Whymper glacier, a cold hanging glacier located at the Grandes Jorasses (Mont

- 50 Blanc, Alps, Italy), already broke off several times in the past, leading to major ice avalanches that occasionally reached the bottom of the valley. In autumn 2008, the glacier recovered its previous critical geometry from the year 1998 and a crevasse opened in the lower part of the tongueagain a critical crevasse appeared approximatively 100 meters upstream the frontal cliff, prompting the local authorities to initiate a monitoring program to enable a time forecast of the expected break-off
- 55 event. The glacier finally broke off causing no damage in autumn 2014, after more than 5 years of monitoring. The break-off event was successfully predicted 2-two weeks in advance.

The aim of this paper is to confirm the validity of the time forecast procedure first developed in 2005 on the Weisshorn glacier and to apply it here in real timebased on a unique data set of surface displacements up to the final break-off event. In all previous studies these records stopped several

60 days prior to the failure.

After describing the glacier and the monitoring system installed on the glacier, we analyze the time evolution of the surface displacement measurements in the context of a time forecast proce-

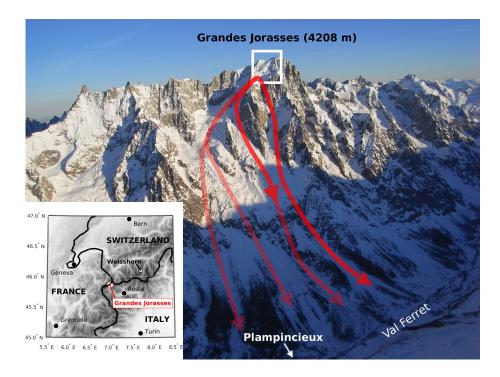


Figure 1. Global view of the Val Ferret with Grandes Jorasses. White rectangle highlights Whymper glacier. Light red lines indicate possible avalanche flow path (for more details see Margreth et al. (2011)), red lines indicate avalanche path from the 1998 break-off event. Inset shows geographical situation of the studied glacier.

dure. While comparing this break-off event with the Weisshorn event of 2005 we discuss the results obtained, with the goal of improving the understanding of this phenomenon.

65 2 Grandes Jorasses glacier

2.1 Study site

The Whymper glacier is located on the south face of the Grandes Jorasses (Mont Blanc massif, Italy) between 3900 and 4200 m asl (Fig. 1). The front of the glacier is about 90 m wide and its surface area amounts $25,000 \text{ m}^2$. This very steep cold hanging glacier (about 40 °) lies above the village

- of Planpincieux and the Italian Val Ferret, a famous and highly frequented tourist destination both in winter and summer. In 1997, six boreholes were drilled down to the bed and temperature profiles were measured, indicating basal temperatures below the freezing point (below $-1.6 \pm 0.4^{\circ}C$) at all locations (Pralong and Funk, 2006). Historical data and morphological evidence indicate that the glacier experienced recurrent break-off events that can be dangerous, particularly in winter, when
- 75 the initial ice avalanche can drag snow in its path. This hanging glacier periodically broke off in the past leading to large avalanches that flowed down into reached the bottom of the valley.

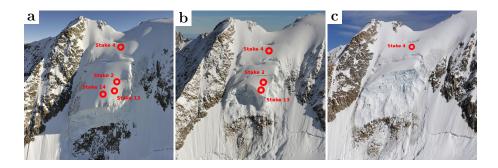


Figure 2. (a) Grandes Jorasses (Whymper) glacier before (23rd August 2014), (b) after the first break-off (23rd September 2014) and (c) after the second break-off (30th September 2014)

2.2 Break-off event history

The glacier broke off several times during last 100 years. Some of these events have been observed and reported:

- On 21 December 1952, after an intensive snowfall period, a huge avalanche was released below the Grandes Jorasses which destroyed a 200-year old forest and blocked the bottom of the Val Ferret over a distance of more than 1 km. The avalanche volume was estimated at more than 1,000,000 m³. It is not clear whether the snow avalanche was triggered by an ice avalanche from the Whymper glacier.
- In August 1993 and July 1996, the glacier released ice avalanches of 80,000 and 24,000 m³, respectively. These ice avalanches did not reach the bottom of the valley.
 - The last major break-off event occurred in the night of 31th May to 1st June 1998. Almost the entire Whymper glacier (around 150,000 m³) broke off at one time and the triggered ice avalanche reached the bottom of the valley, fortunately without causing damage (Fig. 1). According to Pralong and Funk (2006) the formation of the upper crevasse was observed 2.5 years before failure .

2.3 Present monitoring: 2009-2014

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and GPS as well as close-range photogrammetry (Margreth et al., 2011). Two reflectors set on the 95 rock on both sides of the glacier were used as reference, and several reflectors mounted on stakes were directly drilled into the ice, so that their exact positions could be monitored (Fig. 2). Because of instrument problems, the seismic activity unfortunately could not be monitored as initially planned.

Starting in 2010, surface displacements were continuously recorded at several stakes at 2-hour intervals (when the prisms were visible, i.e., good weather conditions) with the aim to timely detect

The survey primarily consisted of surface displacement measurements with an automatic total station

100 an impending ice avalanche (Margreth et al., 2011). Using the same correction technique as described by Faillettaz et al. (2008) (section 4.1), the surface displacements could be determined with an accuracy better than 1 cm, allowing surface velocities to be inferred.

In parallel to the monitoring program, a safety concept for the valley floor was developed considering several scenarios of falling ice volumes. The different ice avalanche scenarios were simulated

105 using the 2-dimensional calculation model RAMMS (Christen et al., 2010). The necessary safety measures were defined according to the local avalanche danger level and the potential volume of a

130

break-off event (Margreth et al., 2011).

2.4 The 2014 break-off event

From 2010 on, surface displacements were surveyed without interruption. The Whymper glacier
finally broke off with an estimated ice volume similar to the 1998 event (about 105,000 m³). Contrary to the 1998 event, the glacier broke off in two events on September 23rd and on September 29th 2014, without reaching the valley (Fig. 2). At the final break-off, 4 reflectors were still active, 2 of them in place for more than 2 years. Despite poor weather conditions between the 16th and 21st of September, the monitoring was operational up to the final break-off. By chance, there was one

115 reflector on each of the two unstable parts and one on the stable part, (Fig. 2).

Striking qualitative analogies with those of the 2005 Weisshorn event (Faillettaz et al., 2008) can be highlighted.

- 1. This steep cold hanging glacier experiences periodic break-off events.
- The geometrical configuration of the glacier is similar before each break-off, with an upper crevasse spanning the whole glacier width and a clear thickening of the glacier towards its tongue.
 - The upper crevasse marks a clear distinction between a stable upper part (where Stake 4 is located) and a downstream unstable part (where the other reflectors were located, Fig. 2, section 4.1). A crude estimation of the volume of the unstable part is thus possible.
- Downstream of this crevasse, surface displacements experience a typical acceleration prior break-off, whereas upstream this crevasse constant velocities are recorded (Stake 4 in Fig. 3).
 - 5. The rupture did not occur at took place immediately above the ice/bedrock contact but a few meters above it, interface, probably within the ice (Fig. 2), possibly because of . However, this observation remains imprecise since no length scale is available. Therefore no definitive conclusions on the fracture location can be drawn from this observation. Note that a similar observation on fracture location was mentioned for the Weisshorn break-off event, probably due to bedrock irregularities (Pralong and Funk, 2006).

5

6. The whole break-off occurred in two steps; a minor section at the left side of the glacier was released first.

135 **3** Previous findings on cold glacier break-off

Based on a retrospective analysis, the main conclusion drawn by Flotron (1977) and Röthlisberger (1981) was that the forecast of a break-off event from a hanging glacier was possible using surface displacements alone. The principle is to fit the characteristic acceleration of the surface motion with a power law behavior of the form:

140
$$s(t) = s_0 + u_s t - a(t_c - t)^{\theta},$$
 (1)

where s(t) is the displacement (in meters) at time t (in days), s_0 a constant in meters, u_s the constant velocity of the upstream stable part (in md⁻¹), t_c the critical time (in days), $\theta < 0$ (without units) and a (in md^{- θ}) the parameters characterizing the acceleration. In this way, the critical time t_c , i.e., time at which the theoretical displacement becomes infinite, could be evaluated using such

- 145 empirical law. Although the break-off event would necessarily occur earlier, this critical time represents the upper limit of the break-off timing. Moreover, an oscillating pattern superimposed on the power law acceleration of the surface displacements was evidenced prior to the 2005 Weisshorn event (Pralong et al., 2005; Faillettaz et al., 2008). This peculiar glacier dynamics was shown to be a log-periodic oscillating process superimposed on this acceleration (for appearance and interpretation).
- 150 see <u>Pralong (2006) and Faillettaz et al. (2015)</u>). The time evolution of the surface displacement measurements can be described with the following equation (after Sornette and Sammis, 1995; Pralong et al., 2005):

$$s(t) = s_0 + u_s t - a(t_c - t)^{\theta} \left[1 + C \sin\left(2\pi \frac{\ln(t_c - t)}{\ln(\lambda)} + D\right) \right],$$
(2)

where C is the relative amplitude (without units), λ the logarithmic frequency (in days) and D the phase shift of the log-periodic oscillation (without units).

Thanks to a combined analysis of surface displacement and seismic measurement, Faillettaz et al. (2011a) were able to obtain a coherent quantitative picture of the damage evolution process developing before the 2005 Weisshorn break-off. They have suggested three regimes in the evolution of the failure process leading to the break-off event:

- 160 (i) A first stable phase related to a self-organizing regime, where diffuse damage accumulates within the glacier, with a proliferation of dislocation-like defects.
 - (ii) A transitional phase where the damage process goes on, micro-cracks grow and start merging in a homogeneous way. Log-periodic oscillations appear and reveal the hierarchical structure of the fracture process under development.

165 (iii) A catastrophic regime where damage clusters are randomly activated. Damage clusters interact and merge with a preferential direction (i.e. preparing the final rupture pattern), in contrast to the previous regime.

4 Results

4.1 Surface displacements and associated velocities

- 170 Fig. 3 shows the corrected surface displacements and Fig. 4 the associated derived surface velocities of the 4 stakes (Fig 2) prior to the break-off. The associated derived surface velocities are computed taking the surface displacements (smoothed over 5 points) interpolated on a regular time step of one day. Note that Stakes 13 and 14 have more than 2 years of nearly continuous measurements and the position of Stake 13 was surveyed up to the final break-off event on September 29th.
- 175 After processing Because of the long distance between the theodolite and the reflectors, a small error in angle measurement has considerable impact on the accuracy of the calculated position. Following Faillettaz et al. (2008), two corrections were applied to the raw data with to improve the accuracy. First, the distance was corrected using two reference points located near the glacier and additionally accounting for flow direction, the accuracy on the rock next to the glacier to compensate
- 180 the meteorological disturbances of the air temperature, humidity and pressure. Second, assuming that a material point moves along a straight stream line, the reflector position can be determined by using only the measured distance, as each measurement is associated with a unique position on the line. Finally the error of the results was estimated to be less than a centimeter (Faillettaz et al., 2008). Note that this constitutes a unique dataset not only because of the great accuracy and long mea-
- 185 surement period but also due to available surface displacement data up to a few hours prior to the break-off event. Whereas surface velocities at Stake 4 are approximately constant (Fig. 4), the three other stakes show a clear acceleration which is typical for an unstable situation. According to this observation we can expect that the glacier section around Stake 2, 13 and 14 will break-off, while the section around Stake 2 will remain stable (section 2.4).

190 4.2 Application to forecasting

Previous findings (section 3) were applied in order to forecast the breaking-off event in real time. As soon as a significant increase in velocity was detected, the same procedure was followed as in Faillettaz et al. (2008). We periodically fitted surface displacements of all stakes to a power law (Eq. 1) and a log periodic oscillating behavior (Eq. 2). The nonlinear least-squares curve-fitting

195 was performed using the Levenberg-Marquardt algorithm. Because the results depend on the initial parameter estimates, especially t_c and θ , we have systematically used different initial values with a prescribed bound and selected the results corresponding to the best root mean squared error and the degree-of-freedom adjusted coefficient of determination.

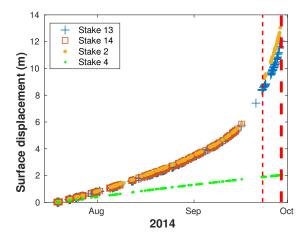


Figure 3. Surface displacements of the 4 stakes before the break-off using 19 July 2014 as reference (when Stake 2 and Stake 4 were installed). Vertical red dashed lines indicate the occurrence of the two break-offs, on 23rd and 29th September 2014. Interrupted lines indicate a period of bad weather conditions without measurements. Note that Stake 14 was not surveyed after 16th of September 2014, i.e., one week before the first break-off.

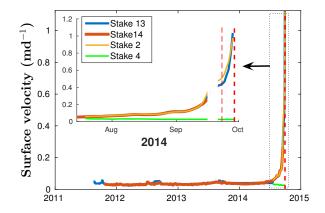


Figure 4. Surface velocities for 4 stakes since 2012. Inset shows a closer view during the same period as in Fig 3.

Fig. 5a and 6a show both power law (Eq. 1) and log-periodic (Eq. 2) fits using the last month

of available data, i.e up to 16th September for Stake 14 and 29th September for Stake 13. As both fits are barely distinguishable, we have also plotted on Fig. 5b and 6b the residuals to the power law fit and show the associated log-periodic fit (minus the power law fit) as a dashed gray line ; Table 1 contains the values of the parameters in Eq. 2, taking λ = 2 d. Note that measurements are available up to the final break-off for 3 prisms (i.e., Stake 13, Stake 2 and Stake 4) and stopped on 16 September for Stake 14, i.e., 8 days before the first break-off.

It appears that the power law behavior describes well the surface displacements with a maximum discrepancy of about 5 cm for Stake 14 (8 days before break-off), about the same order of magnitude as the one observed during the 2005 Weisshorn event (Fig. 5). However, residuals indicate an oscillating pattern. When using the log-periodic function (Eq. 2), the agreement between measured and

- 210 fitted values (dashed gray line) becomes significantly better, with an accuracy of the order of magnitude of the measurement accuracy (less than a centimeter). Results show that the critical time can be expected around the 3rd October for both stakes, which is fairly close to the observed break-off. Note that such an approach can be used to investigate how far in advance a reliable time forecast is possible (see section 5.4).
- 215 However, even if Stake 14 is located on a section that broke off earlier, no significant differences could be detected. Our approach is not able to detect whether the break-off will occur all at once or as successive small events.

Now when considering the entire dataset for Stake 13 (where measurements could be recorded up to the break-off) using the same method, it appears that the amplitudes of the oscillations superim-

220 posed on the power law acceleration become even larger close to the break-off - they reach values up to 30-40 cm (Fig. 6). Such a broad oscillating pattern has never been observed before, confirming that the jerky motion of the glacier (with oscillating nature) might have a physical origin (see Section 5.2).

5 Discussion

225 5.1 Influence of data accuracy on the final result

To assess how the data accuracy influences the time forecast of the break-off, we artificially added two uniformly distributed random noise (between -1 and 1 cm and between -5 and 5 cm) to our dataset and to analyze how the obtained critical time depends on our fitting method. To ensure good statistical representation, this procedure was performed 100 times on Stake 14 up to 16.09.2014 and

230 on Stake 13 up to the final break-off (29.09.2014). Results (see table 2) show that the forecasting errors are mainly due are shown in table 2 where errors (i.e., 95% confidence interval) associated to the additional noise (in bracket) and to the fitting procedure . Errors due to the data accuracy are about one order of magnitude lower than those generated by the fitting procedureare also reported.

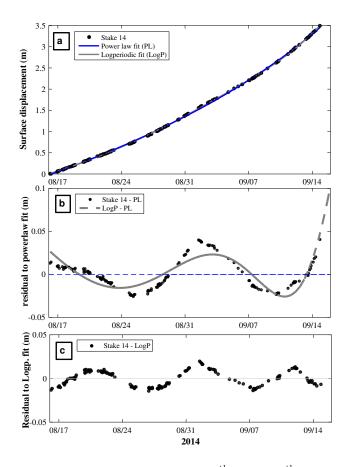


Figure 5. a. Surface displacements of Stake 14 for the period 16th August- 16th September (last measurement of Stake 14) and the associated power law (in blue) and log-periodic fit (in gray). b. Residuals (in meters) to the power law fit (in blue) for the same period. Log-periodic fit is also shown with dashed gray line. Values for the parameters are shown in Table 1. c. Residuals (in meters) to the logperiodic fit for the same period.

These results show that (i) data accuracy does not influence the value of the forecast (less than 0.5
day), (ii) data accuracy directly influences the confidence of the fit, (decreasing data accuracy affects drastically the confidence interval of the fit) (iii) The confidence interval of the fit is reduced if data (even with low accuracy) can be collected up to the break-off. Therefore, the data accuracy directly determines how far in advance a satisfying forecast can be achieved (see section 5.4).

5.2 Appearance of log-periodic behavior

240 The origin of the log-periodic oscillating behavior is likely due to a Discrete Scale Invariance (DSI), .DSI is a weaker kind of scale invariance according to which the system obeys scale invariance only at a specific scaling factor scale (Sornette and Sammis, 1995; Sornette, 1998; Zhou and Sornette, 2002a; Sornette, 2006). Whereas the hallmark of Continuous Scale Invariance is the existence of power law, the signature of DSI is the presence of power laws with complex exponents which mani-

Table 1. Values of the estimated coefficients of Equation 2 with $\lambda = 2$ d and the root-mean-square error (RMSE) of the fit, first two columns corresponding to the parameters of the fit used in Fig. 5 for the period 16.08-16.09, the last two columns corresponding to the parameters of the fit used in Fig. 6 for the period 30.08- 30.09. t_c is given in days after the first day of the investigated period.

Parameter	Units	Stake 13 (16.08 - 16.09)	Stake 14 (16.08 - 16.09)	Stake 13 (29.08 - 29.09)	Stake 2 (29.08 - 29.09)
t_c	d	48.02 ± 5.13	48.11 ± 5.6	41.93 ± 0.91	41.80 ± 2.18
	date	03-Oct-2014	03-Oct-2014	10-Oct-2014	10 -Oct-2014
θ	-	-0.24	-0.25	-0.99	-0.98
s_0	m	$-1.47 \cdot 10^4$	-1.47×10^4	-2.03×10^4	-1.48×10^{4}
u_s	md^{-1}	2.00×10^{-2}	2.00×10^{-2}	2.99×10^{-2}	2.27×10^{-2}
a	$\mathrm{md}^{-\theta}$	27.88	27.72	141.73	164.42
C	-	2.9×10^{-3}	2.3×10^{-3}	3.0×10^{-2}	2.0×10^{-2}
D	-	2.25	1.97	6.13	0.06
RMSE	$\mid md^{-1}/m$	8.7×10^{-3}	7.9×10^{-3}	3.05×10^{-2}	2.51×10^{-2}

Table 2. Critical time t_c evaluated using our initial dataset and with additional uniformly distributed random noise of ± 1 cm and ± 5 cm (over 100 realizations). The errors (i.e., 95% confidence interval) resulting for the additional noise (in brackets) and from the fit are also reported.

Stake	Initial data	\pm 1cm noise	±5cm noise				
13	$t_c = 41.93 \pm 0.91 \; \mathrm{d}$	$t_c = 41.90 \pm 0.038 \ t_c = 41.90 [\pm 0.038] \pm 0.9 \ d$	$t_c = 41.86 \pm 0.041 \ t_c = 41.86 [\pm 0.041] \pm 1.2 \ d$				
14	$t_c = 48.11 \pm 5.6 \; \mathrm{d}$	$\frac{t_c = 48.55 \pm 0.56}{t_c} \underbrace{t_c = 48.55 [\pm 0.56] \pm 8.9}_{t_c} d$	$\underline{t_c = 49.33 \pm 0.55} \underbrace{t_c = 49.33 [\pm 0.55] \pm 34.2}_{c} d$				
Critical	Critical time t_c evaluated with our dataset and with a uniformly distributed random noise of ± 1 cm and ± 5						

cm.

- 245 fests itself in data by log-periodic corrections to scaling. Several mechanisms may lead to this partial breaking of the continuous symmetry. Thanks to a combined analysis of surface displacements and seismic measurements, Faillettaz et al. (2011a) suggest that it results from the dynamic interactions between newly developed micro-cracks, as shown by Huang et al. (1997) and Sahimi and Arbabi (1996).
- To identify the log-frequency, we analyzed the data in the same way as Faillettaz et al. (2008) with a Lomb periodogram analysis (Press, 1996; Zhou and Sornette, 2002b), which is designed to analyze non-uniformly sampled time series. This method enables us to determine f_{Lomb} as a function of $cos(2\pi f_{Lomb}t)$. The parameter λ in Equation 2 can then be evaluated easily as $\lambda = e^{1/f_{Lomb}t}$. Unfortunately, the critical time t_c has to be known to perform this analysis, i.e., this analysis can only
- 255 be performed a posteriori . It clearly shows (Fig. 7a) peaks in Lomb power (power spectral density) at $\lambda \sim 2$ d shows a Lomb periodogram analysis for the three analyzed points, stakes accounting for their displacements before the first break-off using the final break-off as the critical time. A common

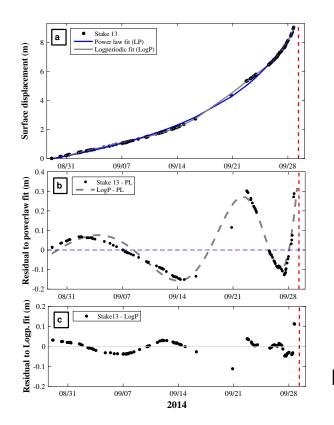


Figure 6. a Surface displacement of Stake 13 for the period 29th August 2014 - 29th September 2014 and associated power law (in blue) and logperiodic (in gray) fits. b. Residuals (in meters) to the power law fit (black point) for the same period. Log-periodic fit is shown with gray dashed line. Values of the parameters are shown in Table 1. c. Residuals (in meters) to the logperiodic fit for the same period.

peak is clearly visible at $f_{Lomb} = 1.45 \pm 0.1 \leftrightarrow \lambda \sim 2 \pm 0.1$, thus confirming that the oscillating behavior is not a measurement artefact but has physical origins, such as the merge of newly developed

260 micro-cracks. Note that this λ value is compatible with previous findings on such types of break-off (Faillettaz et al., 2008) but also with other phenomena such growth processes (Sornette et al., 1996), earthquakes (Sornette and Sammis, 1995) or financial crashes (Sornette and Johansen, 2001).

Interestingly, when analyzing only the data collected after the first break-off, i.e., for Stakes 2 and 13, another strong log frequency appears at $\lambda \sim 7.4$ d for Stakes 2 and 13 $\lambda_2 \sim 4.45 \pm 1$ d in

- 265 addition to the previous peak at $\lambda_1 \sim 2$ (Fig. 7b), after the first. A similar peak was also observed when analyzing the Weisshorn 2005 break-off. The reason for the appearance of such peak both the appearance and the value of such subharmonic peak at λ_2 after the first break-off is not clear, but is probably induced by the occurrence of the first break-off thatchanges the geometry of the glacier: Using experimental data, Moura et al. (2005, 2006) suggested that grain size and loading
- 270 rate directly influence log-periodic oscillations. A possible explanation would thus be that the first release led to a sudden change in the global loading.

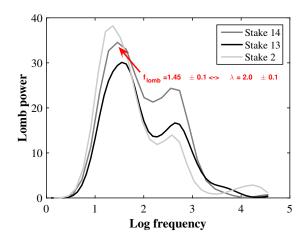


Figure 7a. Lomb periodogram for Stakes 13, 14 and 2 before the first break-off event as well as the corresponding log frequencies (λ) of the peaks.

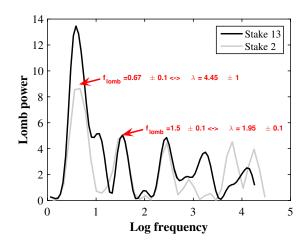


Figure 7b. Lomb periodogram for Stakes 13 and 2 after the first break-off event as well as the corresponding log frequencies (λ) of the peaks.

However, Sornette et al. (1996) and Huang et al. (1997) suggested that, for growth of population of cracks oriented in one direction, such subharmonic frequencies appear naturally and are arbitrary powers λ^n of the preferred scaling ratio λ . Such a mechanism could thus explain the peak at

- 275 $\lambda_2 \sim \times \lambda_1^2$. This peak clearly occurred after the first break-off, suggesting that this event is at the origin of its appearance. The vibrations generated by this ice mass release and its consecutive avalanche might generate a sudden additional increase in the internal damage of the remaining section of the glacier, i.e., loading rate change, introducing another subharmonic log frequency and perturbing the overall behavior of the remaining section of the glacier where Stakes 2 and (where
- 280 Stake 13 stand. Lomb periodogram for Stakes 13 and 2 (in inset) as well as the corresponding log frequencies (λ) of the peaks. Lomb periodogram for Stake 14 as well as the corresponding log frequencies (λ) of the peaks. and 4 are located). This supplementary external load perturbs the hierarchical self-organization of the micro-cracks, presumably promoting other subharmonic peaks.
- 285 Another possible explanation for the appearance of log-periodic behavior with different harmonic peaks after the first break-off could result from a perturbation in the hierarchy of cracking: The pure log-periodicity assumes a single discrete hierarchy. It seems that the first large rupture may lead to a nonlinear distortion of the subsequent development of the hierarchy of cracking, with a drift in the log-frequency. This kind of phenomenon was modeled using a nonlinear second-order (or third order) London expression of the Log Deviation for explication to the subsequent to the provide the second-order (or the second order).
- 290 third-order) Landau expansion of the Log Periodic Power Law (LPPL) formalism for application to financial markets (Johansen and Sornette, 1999; Zhou and Sornette, 2004).

5.3 Accurate determination of break-off occurrence

As critical time t_c given by power law or log-periodic fit indicates when surface displacements become theoretically infinite. However, the break-off event is expected before t_c . When fitting in real time the surface displacements with both power law and log-periodic behavior, it is not only possible to assess the critical time but also the time at which the derived velocities are expected to reach a given threshold (for example 50 cm d⁻¹ or 1 m d⁻¹). Fitting and estimating the time at which the velocity reaches a given threshold provides a more accurate way to predict the break-off event. We developed a software based on this idea by fitting in real time the measurements with both power law and log-periodic behavior and thus provide an estimate of the break-off time.

According to our knowledge, it is not possible to know in advance the velocity at which breakoff will occur. However, from previous events (Weisshorn 1973 and 2005 event, Flotron (1977); Röthlisberger (1981); Faillettaz et al. (2008)), it seems that break-off occurs between 50 cm d⁻¹ to 1.2 m d^{-1} , but this is based on a restricted number of events.

Taking threshold surface velocities of 50 cm d⁻¹ and 1 m d⁻¹, our analysis (using Eq. 2) performed every days from the 12 September to 16 September suggested that break-off could occur between the 23 September ($v_{th} = 50 \text{ cm d}^{-1}$) and the 29 September ($v_{th} = 1 \text{ m d}^{-1}$). Note that the two breaking-off events occurred exactly at these two days, which were forecasted 10 days in advance. Following this analysis, alert was immediately sent to the authorities leading them to close

the endangered area one week before the event. Note that the definition of the velocity threshold has 310 an influence on the prediction itself, as we saw nearly one week is needed for the glacier to accelerate from 50 cm d^{-1} to 1 m d^{-1} . The precise prediction would also not only be based on a correct fit of the surface displacement data but also on a guess of this parameter. We suggest to choose $40 \text{ cm } \text{d}^{-1}$ as a conservative threshold to define a safe break-off danger time interval. It is not yet clear which value has to be considered according to the results from the events analyzed so far. 315

5.4 How far in advance are time forecasts possible?

Surface displacements were analyzed retrospectively based on the last month of data for each stake, and the critical time as well as the time at which the fitted velocity reached 50 ${
m cm~d^{-1}}$ (v_{50}) and 1 m d⁻¹ (v_{100}) were plotted as a function of the time (Fig. 8). Associated errors (Fig. 8 bottomright panels) account for the fitting procedure.

First, the prediction is better when using log-periodic fit than power law fit. This retrospective analysis shows that the prediction is correct after 12 September, i.e., 11 and 17 days before the break-off with a confidence interval becoming less than than 10 days with a log-periodic fit.

- This analysis points out the great prediction potential and early warning perspective of this 325 method, as the exact time of the break-off could be forecasted almost 2 weeks in advance. Note that both the power law and the log-periodic fits become less accurate after the first break-off for Stake 13.13 (Fig. 8b). This might be related to the sudden change in glacier geometry that may influence the surface displacements at Stake 13. occurrence of the first break-off that had possibly changed the hierarchical organization of the internal damage (see section 5.2). However, note that the time at
- which a velocity of $1 \text{ m d}^{-1}(v_{100})$ is expected remains unaffected, still pointing at September 29th. 330

5.5 Overall recommendations

320

According to the knowledge gained from the different studies on Weisshorn, Mönch and Grandes Jorasses glaciers, accurate data are required to forecast an impeding break-off event. As the am-

- 335 plitudes of the log-periodic oscillations are increasing towards the break-off (from 5 cm one week before the break-off to 40 cm at the break-off), the confidence of the time forecast strongly depends on the precision of the surveying data. To ensure a satisfactory forecast about one week in advance, a surveying accuracy better than half of the expected log-periodic amplitudes, i.e., 2.5 cm, is required. In this study an accuracy of 1 cm was achieved with an automatic total station (Leica theodolite
- TM1800 combined with the DI3000S Distometer). The sampling rate needs to be adapted to the 340 oscillating pattern in order to enable its detection. Moreover, in such rapid changing meteorology where clouds can momentarily hindered measurements, several measurements need to be performed

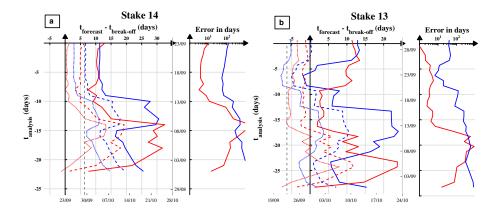


Figure 8. TopLeft: Thick lines: evaluated critical time t_c for power law (blue) and log-periodic (red) fit for Stakes 13-14 (lefta) and 14 Stakes 13 (rightb) as a function of the time of analysisprior to the break-off event ($t_{analysis}$). Interrupted lines indicate time at which estimated derived velocity from power law and log-periodic fit reaches 50 cm d⁻¹ (dashed lines, v_{50}) and 1 m d⁻¹ (dot-dash line, v_{100}). Horizontal grey lines represent the observed The break-off (corresponding to 0 on x-axis) occurred on the 23rd September (a) and 29th September (b). Bottom Vertical grey line represents the observed break-off. Right: Error in days on critical time fitted with power law (blue) and log-periodic (red) estimated from the 95% confidence interval as a function of the time prior to the break-off event ($t_{analysis}$). Errors on v_{50} and v_{100} are similar to the errors on critical time, as they are directly derived from these fits.

each days. A sampling rate of 2 hours was chosen in this study, ensuring thus several opportunities to obtain data every day. This technique can be performed in near real time and several measure-ments can be performed every day with a sufficient accuracy. Note that GPS measurements would

- be a valuable alternative but this technique requires a long acquisition time and additional processing to achieve to required accuracy. Although independent of weather conditions, the power supply and data transmission are problems to be solved. This procedure based on power law/log-periodic oscillations regression requires at least two measurements points on the potentially unstable part of
 the glacier, so that the time evolution of surface motion at different points could be compared. It also ensures that the results are not affected by stake/prism stability issues.
 - An alternative surveying technique is terrestrial Insar. The advantage of this technique is that no installation on the glacier (potentially dangerous) is required. However the data accuracy which can be expected with this monitoring system is not completely clear yet (Preiswerk et al., 2016)

355 6 Conclusions

Grandes Jorasses glacier broke off twice, on 23rd and 29th September 2014. In 2008, as it was suspected that a large part of this glacier is becoming unstable, a monitoring program was initiated. At the time of the break-off, 4 stakes covering a large part of the glacier enabled surface displacement measurements up to the time of the break-off. By regularly analyzing the dataset, it was possible to

360 forecast the event ten days in advance. In the following the local authorities closes the endangered area up to the final rupture.

It was possible to confirm for an impeding ice fall that a time series of surface displacements exhibits strong log-periodic oscillations superimposed on a global power law acceleration, as first discovered for the Weisshorn event (Faillettaz et al., 2015). In the immediate vicinity of the break-

365 off, such oscillations reached an amplitude of more than 40 cm, almost one order of magnitude larger than revealed in previous findings. By fitting our recorded surface displacements, the critical time, i.e. time at which surface displacement become infinite, can be determined. Using this critical time value as an upper bound, a good time forecast could be achieved.

The inferred surface velocities immediately prior the two events were 0.5 m d⁻¹ and 1.2 m d⁻¹, 370 in the same range as for the Weisshorn event, suggesting that break-off of a cold hanging glacier could occur as soon as surface velocities reached 0.5 m d⁻¹. We showed that evaluating the time at which extrapolated velocities (based on the log-periodic fit) reach a prescribed threshold (0.5 m d⁻¹ and 1 m d⁻¹) provides a significantly better forecast. However, in the present case, surface velocity increased from 50 to 100 cm/d in the order of one week. In practice, we suggest to use a

- 375 critical velocity of v=0.4.0.5 m d⁻¹ to determine the period of highly likely break-off occurrence. A retrospective analysis based on this method showed that an accurate prediction of the phenomenon can be achieved two weeks before its occurrence using the last month of surface displacement data and 0.5 m d⁻¹ and 1 m d⁻¹ as velocity thresholds. Although enabling a crude estimation of the total unstable ice volume, this point based surveying procedure is not appropriate to determine whether the
- 380 unstable ice mass will fall down in one event or disaggregate and give rise to several smaller events, as no differences in the evolution of surface displacements were detected. This has consequences for the risk evaluation, as the resulting ice avalanche (and also the chain of processes resulting from its release) depends on the falling ice volume. To conclude, our results suggest that the presented monitoring and data processing techniques exploiting the log-periodic oscillating behavior can be
- applied in real time to forecast a break-off event on any cold unstable hanging glacier.

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